Soft gluon resummation in $t\bar{t}$ production at hadron colliders

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Outline

- basics of soft gluon resummation
- applications at NNLL

Based on series of papers on resummation at NNLL in SCET with
V. Ahrens, A. Ferroglia, M. Neubert, and L. L. Yang

arXiv:1106.6051 [hep-ph]
arXiv:1105. 5824 [hep-ph] +computer program for total cross section
arXiv:1103.0550 [hep-ph]
JHEP 1009 (2010) 097 (arXiv:1003.5827 [hep-ph])
Phys.Lett. B687 (2010) 331-337 (arXiv:0912.3375 [hep-ph])

Top-quark pair production beyond NLO

Two routes:

1) NNLO in fixed order

- goal: calculate all $\mathcal{O}(\alpha_s^2)$ corrections to Born processes

2) soft gluon resummation

Berger, Contopanagos; Bonciani, Catani, Mangano, Nason; Kidonakis, Laenen, Sterman; Kidonakis, Laenen, Moch, Vogt; Langenfeld, Moch, Uwer '08, '09; Czakon, Mitov, Sterman '09; Kidonakis '10; Beneke et. al. '09; Ahrens, Ferroglia, Neubert, BP, Yang ('10, '11)

- applies to total cross section and differential distributions
- goal: calculate (presumably dominant) logarithmic corrections to all orders
- extended from NLL to NNLL in last few years
- often implemented as "approximate NNLO" formulas

Idea of resummation

1) a given $d\hat{\sigma}$ has double logs from real emission in soft limit $\lambda \to 0$

2) using counting $L \equiv \ln \lambda \sim 1/\alpha_s$, can re-organize the perturbative series as

$$\begin{aligned} d\hat{\sigma} \quad &\sim \quad \overbrace{\overbrace{1 + \alpha_s(L^2 + L + 1 + \mathcal{O}(\lambda))}^{NLO} + \alpha_s^2(L^4 + L^3 + L^2 + L}^{approx.NNLO} + 1 + \mathcal{O}(\lambda)) + \mathcal{O}(\alpha_s^3) \\ &\sim \quad \exp\Big(\underbrace{\underbrace{Lg_1(\alpha_s L) + g_2(\alpha_s L)}_{NLL} + \alpha_s g_3(\alpha_s L)}_{NNLL} + \cdots\Big) \underbrace{C(\alpha_s)}_{\text{constants}} + \mathcal{O}(\lambda) \end{aligned}$$

- resummed formulas (NLL, NNLL, etc.) exponentiate large logs (and can be derived using RG equations in effective theory)
- approximate NNLO formulas keep logarithmic parts of full correction

Resummation in three soft limits

$$p_i(p_1) + p_j(p_2) \to t(p_3) + \bar{t}(p_4) + X(k) \qquad (p_i, p_j \in \{q, g\})$$

Name	Observable	Soft limit
production threshold	σ	$\beta = \sqrt{1 - 4m_t^2/\hat{s}} \to 0$
single-particle-inclusive (1PI)	$d\sigma/dp_T dy$	$s_4 = (p_4 + k)^2 - m_t^2 \to 0$
pair-invariant-mass (PIM)	$d\sigma/dM_{t\bar{t}}dy$	$(1 - z) = 1 - M_{t\bar{t}}^2/\hat{s} \to 0$

$$d\sigma = PDFs \otimes \left[d\hat{\sigma}_{\text{NNLL}}^{\text{leading, soft}}(\ln^n \lambda) + \mathcal{O}(\lambda) \right]; \quad \lambda \in \{\beta,\, s_4,\, 1-z\}$$

- resummation for partonic cross sections is useful if singular terms in soft limit are enhanced compared to $\mathcal{O}(\lambda)$ terms after convolution with PDFs ("dynamical threshold enhancement")

NNLL CALCULATIONS

1) production threshold ($\beta \to 0$, total cross section only)

- Langenfeld, Moch, Uwer '08, '09; Czakon, Mitov, Sterman '09; Beneke, Czakon, Falgari, Mitov, Schwinn '09
- method for joint resummation of soft gluon and Coulomb terms ($\sim 1/\beta$) developed in Beneke, Falgari, Schwinn '09

2) PIM kinematics ($M_{t\bar{t}}$ and y distributions in $t\bar{t}$ frame)

- in SCET Ahrens, Ferroglia, Neubert, BP, Yang '10

3) 1PI kinematics (p_T and y distributions in lab frame)

- Kidonakis '10
- in SCET Ahrens, Ferroglia, Neubert, BP, Yang '11

Applications

- total cross section
  - comparison of results in production threshold, PIM, and 1PI limits using "approximate NNLO formulas"
- forward-backward asymmetry
- differential cross sections

Results from approximate NNLO formulas

$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008 90% CL

1) production threshold (result from HATHOR Aliev et. al. '10)

	Tevatron	LHC (7 TeV)
$\sigma_{\rm NLO}$ (pb)	$6.72^{+0.36\,+0.37}_{-0.76\,-0.24}$	$159^{+20\,+8}_{-21\,-9}$
$\sigma_{\rm NNLO\,approx}$	$7.11^{+0.30\,+0.4}_{-0.40\,-0.3}$	$164^{+3\,+9}_{-9\,-9}$

2) 1PI threshold (m_t=173 GeV) Kidonakis '10

$\sigma_{\rm NNLO\,approx,\,1PI}$	$7.08^{+0.00\,+0.36}_{-0.24\,-0.24}$	$163^{+7\,+9}_{-5\,-9}$

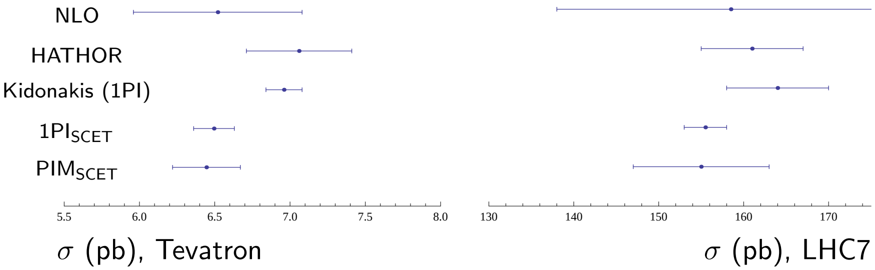
3) PIM and 1PI threshold in SCET Ahrens et.al. '10, '11

$\sigma_{\rm NNLO\ approx,\ 1PI_{SCET}}$	$6.63^{+0.00\,+0.33}_{-0.27\,-0.24}$	$155^{+3\,+8}_{-2\,-9}$
$\sigma_{\rm NNLO\ approx,\ PIM_{SCET}}$	$6.62^{+0.05\,+0.33}_{-0.40\,-0.24}$	$155^{+8\,+8}_{-8\,-9}$

Note: large discrepancy between PIM and 1PI kinematics observed in Kidonakis et. al. '01 not present in SCET calculation.

NNLO approximations at Tevatron and LHC7

$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008



- scale variation not necessarily good indication of uncertainties from subleading terms in soft limits
- 1PI$_{\text{SCET}}$ and PIM$_{\text{SCET}}$ uses insights from SCET to reorganize expansion in soft limit compared to 1PI and PIM used in Kidonakis et '01
- study of NLO cross sections hints that PIM$_{\text{SCET}}$ and 1PI$_{\text{SCET}}$ receive smaller power corrections away from soft limit (backup slides)

Forward-backward asymmetry

$$A_{\mathrm{FB}}^{i} = \frac{N_t(y^i > 0) - N_t(y^i < 0)}{N_t(y^i > 0) + N_t(y^i < 0)} = \frac{\Delta\sigma_{\mathrm{FB}}}{\sigma}$$

Tevatron:

- total asymmetry measured in $i = p\bar{p}$ or $t\bar{t}$ rest frame
- also with cuts on $M_{t\bar{t}}$ and $\Delta y = y_t - y_{\bar{t}}$ in $t\bar{t}$ frame

LHC:

- $A_{\mathrm{FB}} = 0$, because initial state is symmetric
- can define non-vanishing (differential) charge asymmetries

THE FORWARD-BACKWARD ASYMMETRY IN THE STANDARD MODEL

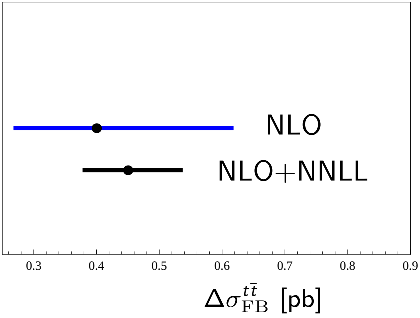
$$A_{\mathrm{FB}} = \frac{\Delta\sigma_{\mathrm{FB}}}{\sigma} = \frac{\alpha_s^3 \Delta\sigma^{(0)}_{\mathrm{FB},q\bar{q}} + \alpha_s^4 \Delta\sigma^{(1)}_{\mathrm{FB},q\bar{q}} + \dots}{\alpha_s^2\sigma^{(0)} + \alpha_s^3\sigma^{(1)} + \dots} = \alpha_s A^{(0)}_{\mathrm{FB}} + \dots$$

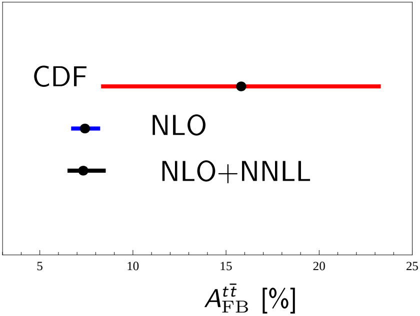
Status of Standard Model calculations for A_{FB}

- $A^{(0)}_{\mathrm{FB}}$ in [Kuhn,Rodrigo 1998] (will call "NLO" since uses $d\sigma$ at NLO)
- (mixed QCD)-electroweak corrections
  [Kuhn, Rodrigo 1998],[Bernreuther, Si 2010], [Hollik, Pagani 2011]
- A_{FB} to NLO + NLL in $t\bar{t}$ frame with cuts on $M_{t\bar{t}}$
  [Almeida, Sterman, Vogelsang 2008]
- A_{FB} to NLO + NNLL with cuts on Δy, $M_{t\bar{t}}$ in $t\bar{t}$ frame, cuts on y in lab frame, also differential charge asymmetry at LHC
  [Ahrens, Ferroglia, Neubert, BP, Yang 2011]

Total asymmetry in $t\bar{t}$ frame

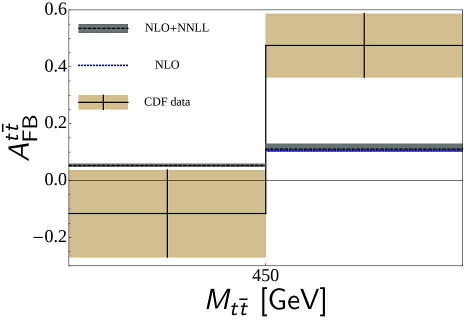
$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008 90% CL





- resummation roughly halves scale dependence in $\Delta\sigma_{\rm FB}$ and σ compared to NLO, but scale dependence of $A_{\rm FB}$ somewhat larger
- theory and experiment agree at about 1σ

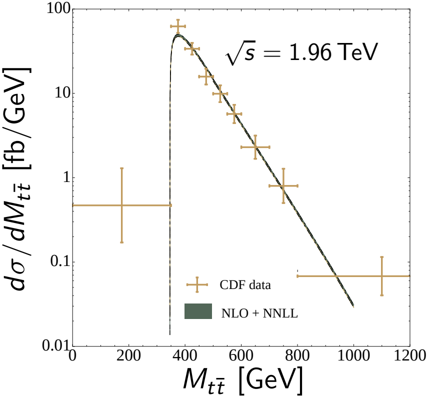
A_{FB} WITH CUTS ON $M_{t\bar{t}}$

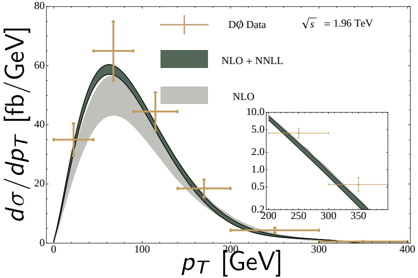


	$M_{t\bar{t}} \leq 450$ GeV		$M_{t\bar{t}} > 450$ GeV	
	$\Delta\sigma_{\mathrm{FB}}^{tt}$ [pb]	A_{FB}^{tt}[%]	$\Delta\sigma_{\mathrm{FB}}^{tt}$ [pb]	A_{FB}^{tt}[%]
CDF		$-11.6^{+15.3}_{-15.3}$		$47.5^{+11.2}_{-11.2}$
NLO	$0.17^{+0.08+0.02}_{-0.05-0.00}$	$5.3^{+0.3+0.1}_{-0.4-0.1}$	$0.21^{+0.12+0.02}_{-0.07-0.00}$	$10.6^{+1.1+0.3}_{-0.8-0.1}$
NLO+NNLL	$0.21^{+0.04+0.02}_{-0.03-0.00}$	$5.2^{+0.7+0.1}_{-0.5-0.0}$	$0.24^{+0.05+0.02}_{-0.04-0.00}$	$11.1^{+1.9+0.3}_{-1.0-0.0}$

Resummation at NNLL is mild effect [Ahrens,Ferroglia,Neubert,BP,Yang]

Invariant mass and p_T distributions at NLO+NNLL vs. Tevatron data





- $M_{t\bar{t}}$ and p_T distributions at NLO+NNLL from [Ahrens et. al.]
- normalization and shape of distributions consistent with data
- can also study rapidity distributions (but no measurements so far)

Summary

- Total and differential cross sections in top-pair production at hadron colliders have been known at NLO for 20 years.

- Two routes beyond NLO
  - NNLO in fixed order (in progress)
  - soft gluon resummation to NLO+NNLL $\leftrightarrow$ approximate NNLO (known in three different soft limits: production threshold, 1PI, PIM)

- NLO+NNLL calculations have reduced scale uncertainties compared to NLO, but can question whether these reliably estimate uncalculated contributions in different limits

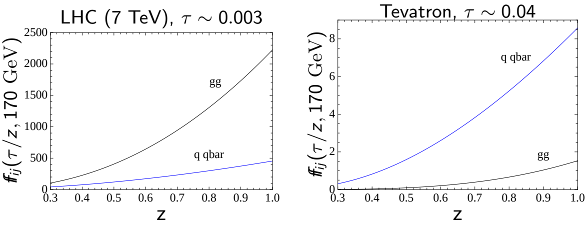
- Higher-order QCD corrections from soft gluon resummation do not explain current discrepancies in A_{FB} at the Tevatron backup slides

Dynamical threshold enhancement

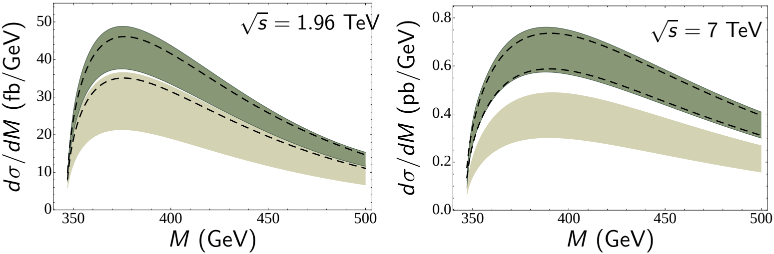
$$\frac{d\sigma}{dM_{t\bar{t}}} \sim \sum_{i,j} \int_\tau^1 \frac{dz}{z}\, ff_{ij}(\tau/z, \mu_f) \left[\delta(1-z) C_0^{ij} + \sum_m \sum_{n \le 2m-1} \alpha_s^m d_{mn}^{ij} \left[\frac{\ln^n(1-z)}{1-z} \right]_+ + \dots \right]$$

Leading terms in $z \longrightarrow 1$ limit dominant if:

- $\tau = M_{t\bar{t}}^2/s \longrightarrow 1$ (high invariant mass)
- $ff_{ij}(\tau/z, \mu)$ largest as $z \longrightarrow 1$, even if τ not close to 1 ("dynamical threshold enhancement")



Dominance of soft gluon corrections at NLO



- green band = exact fixed order at NLO ($\mu_f = 200$, 800 GeV)
- dashed lines = leading terms for $z \to 1$ at NLO ($\mu_f = 200$, 800 GeV)

Soft gluon corrections dominate cross section even at low $M_{t\bar{t}}$

Approximate NNLO formulas

$$\hat{\sigma}_{ij}^{\mathrm{NNLO\,approx.}}(\beta,\mu_f) = \alpha_s^2\hat{\sigma}_{ij}^{(0)} + \alpha_s^3\hat{\sigma}_{ij}^{(1)} + \alpha_s^4\hat{\sigma}_{ij}^{(2),\,\mathrm{approx}}$$

$$\hat{\sigma}_{ij=q\bar{q},\,gg}^{(2)\,\mathrm{approx\,p.t.}} = \sum_{m=1}^{4} d_{2m}^{\mathrm{p.t.}} \ln^m\beta + \frac{1}{\beta}\left(c_{22}^{\mathrm{p.t.}}\ln^2\beta + c_{11}^{\mathrm{p.t.}}\ln\beta + c_{10}^{\mathrm{p.t.}}\right) + \frac{c_{20}^{\mathrm{p.t.}}}{\beta^2} + \hat{R}'^{\,\mathrm{p.t.}}(\beta)$$

$$\hat{\sigma}_{ij=q\bar{q},gg}^{(2)\,\mathrm{approx.\,1PI}} = \int dp_T dy \left\{ \sum_{m=0}^{3} d_m^{\mathrm{1PI}} \left[\frac{\ln^m(2E_s(s_4)/\mu_f)}{s_4}\right]_+ + c^{\mathrm{1PI}}\delta(s_4) + \hat{R}'^{\,\mathrm{1PI}}(s_4)\right\}$$

$$\hat{\sigma}_{ij=q\bar{q},gg}^{(2)\,\mathrm{approx.\,PIM}} = \int dM_{t\bar{t}} dy \left\{ \sum_{m=0}^{3} d_m^{\mathrm{PIM}} \left[\frac{\ln^m(2E_s(z)/\mu_f)}{1-z}\right]_+ + c^{\mathrm{PIM}}\delta(1-z) + \hat{R}'^{\,\mathrm{PIM}}(z)\right\}$$

- pieces in blue determined exactly from NNLL (or Coulomb res. for p.t.)
- in 1PI and PIM, the logs depend on soft energy E_s. Two schemes

Ahrens et al '10,'11	$\mathrm{PIM_{SCET}}$: $2E_s(z) = \frac{M_{t\bar{t}}(1-z)}{\sqrt{z}}$	$\mathrm{1PI_{SCET}}$: $2E_s(s_4) = \frac{s_4}{\sqrt{m_t^2+s_4}}$
Kidonakis et al '01	PIM: $2E_s(z) \approx M_{t\bar{t}}(1-z)$	1PI: $2E_s(s_4) \approx s_4/m_t$

Comparing soft limits

If subleading terms in soft limit are known, the production, 1PI, and PIM formulas agree. For instance, at NLO

$$
\begin{aligned}
\hat{\sigma}^{(1)}_{ij=q\bar{q},\,gg} &= \sum_{m=1}^{2} d_m^{\mathrm{p.t.}} \ln^m \beta + \frac{C^{\mathrm{p.t.}}}{\beta} + c^{\mathrm{p.t.}} + \hat{R}^{\,\mathrm{p.t.}}(\beta) \\
&= \int dp_T\, dy \Big\{ \sum_{m=0}^{1} d_m^{\mathrm{1PI}} \left[\frac{\ln^m(2E_s(s_4)/\mu_f)}{s_4} \right]_+ + c^{\mathrm{1PI}} \delta(s_4) + \hat{R}^{\mathrm{1PI}}(s_4) \Big\} \\
&= \int dM_{t\bar{t}}\, d\theta \Big\{ \sum_{m=0}^{1} d_m^{\mathrm{PIM}} \left[\frac{\ln^m(2E_s(z)/\mu_f)}{1-z} \right]_+ + c^{\mathrm{PIM}} \delta(1-z) + \hat{R}^{\mathrm{PIM}}(z) \Big\}
\end{aligned}
$$

By evaluating NLO corrections using only leading pieces in soft limits and comparing with exact answer, know size of $\int_{PS}[\hat{R}]$. Will compare using

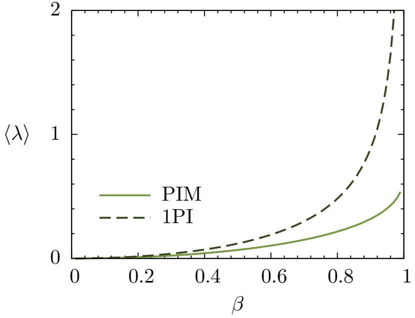
$$
\frac{d\sigma}{d\beta} = \frac{1}{s} \frac{8\beta}{(1-\beta^2)^2} \sum_{ij} ff_{ij}\left(\frac{\hat{s}}{s}, \mu_f\right) m_t^2\, \hat{\sigma}_{ij}\left(\frac{4m_t^2}{\hat{s}}, \mu_f\right); \quad \hat{s} = 4m_t^2/(1-\beta^2)
$$

- if $\hat{R}$ is not small at NLO, weaker case for it to be at NNLO

The β-distribution and total cross section

$$\frac{d\sigma}{d\beta} = \frac{1}{s}\frac{8\beta}{(1-\beta^2)^2}\sum_{ij} f\!f_{ij}\left(\frac{\hat{s}}{s},\mu_f\right) m_t^2\,\hat{\sigma}_{ij}\left(\frac{4m_t^2}{\hat{s}},\mu_f\right); \quad \hat{s} = 4m_t^2/(1-\beta^2)$$

- $f\!f_{ij}(y,\mu_f) = \int_y^1 \frac{dx}{x}\, f_{i/h_1}(x,\mu_f)\, f_{j/h_2}(y/x,\mu_f)$ are parton luminosities
- $\hat{\sigma}_{ij} \sim \int_{\mathrm{PS}}[\mathrm{Tr}[\mathbf{H}\,\mathbf{S}]_{\mathrm{ij}} + \mathcal{O}(\lambda)], \qquad \lambda = \{E_s(z)/M,\ E_s(s_4)/m_t\}$



- for $\beta \to 0$, gluon emission is soft, 1PI, PIM, and exact QCD should agree
- for larger β, expect power corrections to be more important in 1PI kinematics than in PIM kinematics

Approximate vs. exact NLO (red = $\beta \to 0$)