SOFT GLUON RESUMMATION IN $t\bar{t}$ PRODUCTION AT HADRON COLLIDERS

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basics of soft gluon resummation
applications at NNLL

Based on series of papers on resummation at NNLL in SCET with
V. Ahrens, A. Ferroglia, M. Neubert, and L. L. Yang
Top-quark pair production beyond NLO

Two routes:

1) NNLO in fixed order
   - goal: calculate all $\mathcal{O}(\alpha_s^2)$ corrections to Born processes

2) soft gluon resummation
   Berger, Contopanagos; Bonciani, Catani, Mangano, Nason; Kidonakis, Laenen, Sterman; Kidonakis, Laenen, Moch, Vogt; Langenfeld, Moch, Uwer ’08, ’09; Czakon, Mitov, Sterman ’09; Kidonakis ’10; Beneke et. al. ’09; Ahrens, Ferroglia, Neubert, BP, Yang (’10, ’11)
   - applies to total cross section and differential distributions
   - goal: calculate (presumably dominant) logarithmic corrections to all orders
   - extended from NLL to NNLL in last few years
   - often implemented as “approximate NNLO” formulas
Idea of resummation

1) a given \( d\hat{\sigma} \) has double logs from real emission in soft limit \( \lambda \to 0 \)

2) using counting \( L \equiv \ln \lambda \sim 1/\alpha_s \), can re-organize the perturbative series as

\[
\begin{aligned}
d\hat{\sigma} &\sim 1 + \alpha_s (L^2 + L + 1 + \mathcal{O}(\lambda)) + \alpha_s^2 (L^4 + L^3 + L^2 + L + 1 + \mathcal{O}(\lambda)) + \mathcal{O}(\alpha_s^3) \\
&\sim \exp \left( Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots \right) C(\alpha_s) + \mathcal{O}(\lambda)
\end{aligned}
\]

- resummed formulas (NLL, NNLL, etc.) exponentiate large logs (and can be derived using RG equations in effective theory)
- approximate NNLO formulas keep logarithmic parts of full correction
## Resummation in Three Soft Limits

\[ p_i(p_1) + p_j(p_2) \rightarrow t(p_3) + \bar{t}(p_4) + X(k) \quad (p_i, p_j \in \{q, g\}) \]

<table>
<thead>
<tr>
<th>Name</th>
<th>Observable</th>
<th>Soft limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>production threshold</td>
<td>(\sigma)</td>
<td>(\beta = \sqrt{1 - 4m_t^2/\hat{s}} \rightarrow 0)</td>
</tr>
<tr>
<td>single-particle-inclusive (1PI)</td>
<td>(d\sigma/dp_T , dy)</td>
<td>(s_4 = (p_4 + k)^2 - m_t^2 \rightarrow 0)</td>
</tr>
</tbody>
</table>
| pair-invariant-mass (PIM)         | \(d\sigma/dM_{t\bar{t}} \, dy\) | \((1 - z) = 1 - M_{t\bar{t}}^2/\hat{s} \rightarrow 0\)

\[ d\sigma = PDFs \otimes \left[ d\hat{\sigma}_{\text{NNLL}}^{\text{leading, soft}} \left( \ln^n \lambda + \mathcal{O}(\lambda) \right) ; \quad \lambda \in \{\beta, s_4, 1 - z\} \right] \]

- resummation for partonic cross sections is useful if singular terms in soft limit are enhanced compared to \(\mathcal{O}(\lambda)\) terms after convolution with PDFs ("dynamical threshold enhancement")
NNLL calculations

1) production threshold ($\beta \rightarrow 0$, total cross section only)
   - Langenfeld, Moch, Uwer '08, '09; Czakon, Mitov, Sterman '09;
     Beneke, Czakon, Falgari, Mitov, Schwinn '09
   - method for joint resummation of soft gluon and Coulomb terms ($\sim 1/\beta$)
     developed in Beneke, Falgari, Schwinn '09

2) PIM kinematics ($M_{t\bar{t}}$ and $y$ distributions in $t\bar{t}$ frame)
   - in SCET Ahrens, Ferroglia, Neubert, BP, Yang '10

3) 1PI kinematics ($p_T$ and $y$ distributions in lab frame)
   - Kidonakis '10
   - in SCET Ahrens, Ferroglia, Neubert, BP, Yang '11
Applications

- total cross section
  - comparison of results in production threshold, PIM, and 1PI limits using “approximate NNLO formulas”
- forward-backward asymmetry
- differential cross sections
**Results from approximate NNLO formulas**

\[ m_t = 173.1 \text{ GeV}, \quad m_t/2 < \mu_f = \mu_r < 2m_t, \quad \text{MSTW2008 90\% CL} \]

1) production threshold (result from HATHOR Aliev et. al. '10)

<table>
<thead>
<tr>
<th>( \sigma_{\text{NLO}} ) (pb)</th>
<th>Tevatron</th>
<th>LHC (7 TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{\text{NNLO approx}} )</td>
<td>6.72( ^{+0.36+0.37}_{-0.76-0.24} )</td>
<td>159( ^{+20+8}_{-21-9} )</td>
</tr>
</tbody>
</table>

2) 1PI threshold (\( m_t=173 \text{ GeV} \)) Kidonakis '10

\[ \sigma_{\text{NNLO approx, 1PI}} \]

\[ 7.08^{+0.00+0.36}_{-0.24-0.24} \quad 163^{+7+9}_{-5-9} \]

3) PIM and 1PI threshold in SCET Ahrens et.al. '10, '11

| \( \sigma_{\text{NNLO approx, 1PI}_{\text{SCET}}} \) | 6.63\( ^{+0.00+0.33}_{-0.27-0.24} \) | 155\( ^{+3+8}_{-2-9} \) |
| \( \sigma_{\text{NNLO approx, PIM}_{\text{SCET}}} \) | 6.62\( ^{+0.05+0.33}_{-0.40-0.24} \) | 155\( ^{+8+8}_{-8-9} \) |

Note: large discrepancy between PIM and 1PI kinematics observed in Kidonakis et. al. '01 not present in SCET calculation.
NNLO approximations at Tevatron and LHC7

$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008

- NLO
- HATHOR
- Kidonakis (1PI)
- $1\text{PI}_{\text{SCET}}$
- $\text{PIM}_{\text{SCET}}$

$\sigma \text{ (pb), Tevatron}$

$\sigma \text{ (pb), LHC7}$

- scale variation not necessarily good indication of uncertainties from subleading terms in soft limits
- $1\text{PI}_{\text{SCET}}$ and $\text{PIM}_{\text{SCET}}$ uses insights from SCET to reorganize expansion in soft limit compared to 1PI and PIM used in Kidonakis et '01
- study of NLO cross sections hints that $\text{PIM}_{\text{SCET}}$ and $1\text{PI}_{\text{SCET}}$ receive smaller power corrections away from soft limit (backup slides)
**Forward-backward asymmetry**

\[
A^i_{\text{FB}} = \frac{N_t(y^i > 0) - N_t(y^i < 0)}{N_t(y^i > 0) + N_t(y^i < 0)} = \frac{\Delta \sigma_{\text{FB}}}{\sigma}
\]

Tevatron:
- total asymmetry measured in \( i = p\bar{p} \) or \( t\bar{t} \) rest frame
- also with cuts on \( M_{t\bar{t}} \) and \( \Delta y = y_t - y_{\bar{t}} \) in \( t\bar{t} \) frame

LHC:
- \( A_{\text{FB}} = 0 \), because initial state is symmetric
- can define non-vanishing (differential) charge asymmetries
The forward-backward asymmetry in the Standard Model

\[ A_{FB} = \frac{\Delta \sigma_{FB}}{\sigma} = \frac{\alpha_s^3 \Delta \sigma_{FB,q\bar{q}}^{(0)} + \alpha_s^4 \Delta \sigma_{FB,q\bar{q}}^{(1)} + \ldots}{\alpha_s^2 \sigma^{(0)} + \alpha_s^3 \sigma^{(1)} + \ldots} = \alpha_s A_{FB}^{(0)} + \ldots \]

Status of Standard Model calculations for \( A_{FB} \)

- \( A_{FB}^{(0)} \) in [Kuhn, Rodrigo 1998] (will call “NLO” since uses \( d\sigma \) at NLO)
- (mixed QCD)-electroweak corrections
  [Kuhn, Rodrigo 1998], [Bernreuther, Si 2010], [Hollik, Pagani 2011]
- \( A_{FB} \) to NLO + NLL in \( t\bar{t} \) frame with cuts on \( M_{t\bar{t}} \)
  [Almeida, Sterman, Vogelsang 2008]
- \( A_{FB} \) to NLO + NNLL with cuts on \( \Delta y, M_{t\bar{t}} \) in \( t\bar{t} \) frame, cuts on \( y \) in lab frame, also differential charge asymmetry at LHC
  [Ahrens, Ferroglia, Neubert, BP, Yang 2011]
Total asymmetry in $t\bar{t}$ frame

$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008 90\% CL

- Resummation roughly halves scale dependence in $\Delta\sigma_{FB}$ and $\sigma$ compared to NLO, but scale dependence of $A_{FB}$ somewhat larger
- Theory and experiment agree at about 1\%
$A_{FB}$ WITH CUTS ON $M_{t\bar{t}}$

<table>
<thead>
<tr>
<th></th>
<th>$M_{t\bar{t}} \leq 450$ GeV</th>
<th></th>
<th>$M_{t\bar{t}} &gt; 450$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta \sigma_{FB}^{tt}$ [pb]</td>
<td>$A_{FB}^{tt}$ [%]</td>
<td>$\Delta \sigma_{FB}^{tt}$ [pb]</td>
</tr>
<tr>
<td>CDF</td>
<td>-11.6$^{+15.3}_{-15.3}$</td>
<td>-11.6$^{+15.3}_{-15.3}$</td>
<td>47.5$^{+11.2}_{-11.2}$</td>
</tr>
<tr>
<td>NLO</td>
<td>0.17$^{+0.08+0.02}_{-0.05-0.00}$</td>
<td>5.3$^{+0.3+0.1}_{-0.4-0.1}$</td>
<td>0.21$^{+0.12+0.02}_{-0.07-0.00}$</td>
</tr>
<tr>
<td>NLO+NNLL</td>
<td>0.21$^{+0.04+0.02}_{-0.03-0.00}$</td>
<td>5.2$^{+0.7+0.1}_{-0.5-0.0}$</td>
<td>0.24$^{+0.05+0.02}_{-0.04-0.00}$</td>
</tr>
</tbody>
</table>

Resummation at NNLL is mild effect [Ahrens,Ferroglia,Neubert,BP,Yang]
**Invariant mass and $p_T$ distributions at NLO+NNLL vs. Tevatron data**

- $M_{t\bar{t}}$ and $p_T$ distributions at NLO+NNLL from [Ahrens et. al.]
- normalization and shape of distributions consistent with data
- can also study rapidity distributions (but no measurements so far)
Total and differential cross sections in top-pair production at hadron colliders have been known at NLO for 20 years.

Two routes beyond NLO
- NNLO in fixed order (in progress)
- Soft gluon resummation to NLO+NNLL ↔ approximate NNLO
  (known in three different soft limits: production threshold, 1PI, PIM)

NLO+NNLL calculations have reduced scale uncertainties compared to NLO, but can question whether these reliably estimate uncalculated contributions in different limits.

Higher-order QCD corrections from soft gluon resummation do not explain current discrepancies in $A_{FB}$ at the Tevatron.
backup slides
Dynamical threshold enhancement

$$\frac{d\sigma}{dM_{t\bar{t}}} \sim \sum_{i,j} \int_{\tau}^{1} \frac{dz}{z} f_{ij}(\tau/z, \mu_f) \left[ \delta(1-z)C_{0}^{ij} + \sum_{m} \sum_{n \leq 2m-1} \alpha_{s}^{m} d_{mn}^{ij} \left[ \frac{\ln^{n}(1-z)}{1-z} \right] + \ldots \right]$$

Leading terms in $z \to 1$ limit dominant if:

- $\tau = M_{t\bar{t}}^2/s \to 1$ (high invariant mass)
- $f_{ij}(\tau/z, \mu)$ largest as $z \to 1$, even if $\tau$ not close to 1 ("dynamical threshold enhancement")

![Graphs showing $f_{ij}(\tau/z, 170 \text{ GeV})$ for LHC (7 TeV) and Tevatron (7 TeV), with $\tau \sim 0.003$ for LHC and $\tau \sim 0.04$ for Tevatron.](image)
Dominance of soft gluon corrections at NLO

- green band = exact fixed order at NLO ($\mu_f = 200, 800$ GeV)
- dashed lines = leading terms for $z \to 1$ at NLO ($\mu_f = 200, 800$ GeV)

Soft gluon corrections dominate cross section even at low $M_{t\bar{t}}$
Approximate NNLO Formulas

\[
\hat{\sigma}_{ij}^{\text{NNLO approx.}}(\beta, \mu_f) = \alpha_s^2 \hat{\sigma}_{ij}^{(0)} + \alpha_s^3 \hat{\sigma}_{ij}^{(1)} + \alpha_s^4 \hat{\sigma}_{ij}^{(2), \text{approx}}.
\]

\[
\hat{\sigma}_{ij}^{(2), \text{approx p.t.}} = q\bar{q}, gg = \sum_{m=1}^4 d_{2m}^{\text{p.t.}} \ln^m \beta + \frac{1}{\beta} \left( c_{22}^{\text{p.t.}} \ln^2 \beta + c_{11}^{\text{p.t.}} \ln \beta + c_{10}^{\text{p.t.}} \right) + \frac{c_{20}^{\text{p.t.}}}{\beta^2} + \hat{R}'(\beta).
\]

\[
\hat{\sigma}_{ij}^{(2), \text{approx. 1PI}} = q\bar{q}, gg = \frac{\int d\rho_T dy \left\{ \sum_{m=0}^3 d_m^{\text{1PI}} \left[ \ln^m \left( \frac{2E_s(s_4)/\mu_f}{s_4} \right) \right] + c^{\text{1PI}} \delta(s_4) + \hat{R}'^{\text{1PI}}(s_4) \right\}}{1 - z} + c^{\text{PIM}} \delta(1 - z) + \hat{R}'^{\text{PIM}}(z).
\]

\[
\hat{\sigma}_{ij}^{(2), \text{approx. PIM}} = q\bar{q}, gg = \frac{\int dM_{\tilde{t}\tilde{t}} dy \left\{ \sum_{m=0}^3 d_m^{\text{PIM}} \left[ \ln^m \left( \frac{2E_s(z)/\mu_f}{1 - z} \right) \right] + c^{\text{PIM}} \delta(1 - z) + \hat{R}'^{\text{PIM}}(z) \right\}}{1 - z} + c^{\text{PIM}} \delta(1 - z) + \hat{R}'^{\text{PIM}}(z).
\]

- Pieces in blue determined exactly from NNLL (or Coulomb res. for p.t.)
- In 1PI and PIM, the logs depend on soft energy \( E_s \). Two schemes

| Ahrens et al '10,'11 | PIM_{SCET}: \( 2E_s(z) = \frac{M_{\tilde{t}\tilde{t}}(1-z)}{\sqrt{z}} \) | 1PI_{SCET}: \( 2E_s(s_4) = \frac{s_4}{\sqrt{m_t^2 + s_4}} \) |
| Kidonakis et al '01 | PIM: \( 2E_s(z) \approx M_{\tilde{t}\tilde{t}}(1 - z) \) | 1PI: \( 2E_s(s_4) \approx s_4 / m_t \) |
Comparing soft limits

If subleading terms in soft limit are known, the production, 1PI, and PIM formulas agree. For instance, at NLO

\[
\hat{\sigma}_{ij=q\bar{q}, gg}^{(1)} = \sum_{m=1}^{2} d_m^{p.t.} \ln^m \beta + \frac{C^{p.t.}}{\beta} + c^{p.t.} + \hat{R}^{p.t.}(\beta)
\]

\[
= \int dp_T dy \left\{ \sum_{m=0}^{1} d_m^{1PI} \left[ \ln^m \left( \frac{2E_s(s_4)/\mu_f}{s_4} \right) \right] + c^{1PI} \delta(s_4) + \hat{R}^{1PI}(s_4) \right\}
\]

\[
= \int dM_{t\bar{t}} d\theta \left\{ \sum_{m=0}^{1} d_m^{PIM} \left[ \ln^m \left( \frac{2E_s(z)/\mu_f}{1 - z} \right) \right] + c^{PIM} \delta(1 - z) + \hat{R}^{PIM}(z) \right\}
\]

By evaluating NLO corrections using only leading pieces in soft limits and comparing with exact answer, know size of \( \int_{PS}[\hat{R}] \). Will compare using

\[
\frac{d\sigma}{d\beta} = \frac{1}{s} \frac{8\beta}{(1 - \beta^2)^2} \sum_{ij} f_{ij} \left( \frac{\hat{s}}{s}, \mu_f \right) m_t^2 \hat{\sigma}_{ij} \left( \frac{4m_t^2}{\hat{s}}, \mu_f \right); \quad \hat{s} = 4m_t^2/(1 - \beta^2)
\]

- if \( \hat{R} \) is not small at NLO, weaker case for it to be at NNLO
The $\beta$-distribution and total cross section

$$\frac{d\sigma}{d\beta} = \frac{1}{s} \frac{8\beta}{(1 - \beta^2)^2} \sum_{ij} f_{ij}(\frac{\hat{s}}{s}, \mu_f) m_t^2 \hat{\sigma}_{ij}(\frac{4m_t^2}{\hat{s}}, \mu_f); \quad \hat{s} = 4m_t^2/(1 - \beta^2)$$

- $f_{ij}(y, \mu_f) = \int_y^1 \frac{dx}{x} f_{i/h_1}(x, \mu_f) f_{j/h_2}(y/x, \mu_f)$ are parton luminosities
- $\hat{\sigma}_{ij} \sim \int_{PS} [\text{Tr}[H S]_{ij} + \mathcal{O}(\lambda)], \quad \lambda = \{E_s(z)/M, E_s(s_4)/m_t\}$

- for $\beta \rightarrow 0$, gluon emission is soft, 1PI, PIM, and exact QCD should agree
- for larger $\beta$, expect power corrections to be more important in 1PI kinematics than in PIM kinematics
Approximate vs. exact NLO (red = $\beta \to 0$)

\[ \sqrt{s} = 1.96 \text{ TeV} \]
\[ q\bar{q}\text{-channel} \]
\[ \mu_f = m_t \]

\[ \sqrt{s} = 7 \text{ TeV} \]
\[ q\bar{q}\text{-channel} \]
\[ \mu_f = m_t \]

\[ \sqrt{s} = 1.96 \text{ TeV} \]
\[ gg\text{-channel} \]
\[ \mu_f = m_t \]

\[ \sqrt{s} = 7 \text{ TeV} \]
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