

New Aspects of Non Commutative Space-time Cosmology

By N.Mebarki

Laboratoire de Physique Mathématique et Subatomique
Mentouri University, Constantine, Algeria

NCG Cosmology ?...

Motivations

Space-time non commutativity could be significant at Planck scale (same scale where quantum gravity effects become important)

and

Very sensitive to search for signatures in cosmological observations

Maybe:

Observed anisotropies of cosmic microwave background (CMB) and accelerated expansion of universe may be caused by non commutativity of space-time

Matter-antimatter asymmetry etc....

General Formalism

Assume Non commutative structure of space-time determined by condition:

$$[X^\mu, X^\nu] = i\Theta^{\mu\nu}$$

introduce star product “*” between functions

$$(f * g)(x) = f(x) \text{Exp}(\frac{i}{2}\Theta^{\mu\nu}\partial_\mu\partial_\nu)g(x)$$

Gauge fields (Spin Connection) for non commutative space - time

$$\hat{\omega}_\mu^{AB} = \omega_\mu^{AB} - i\Theta^{\nu\rho}\omega_{\nu\rho}^{AB} + \Theta^{\nu\rho}\Theta^{\lambda\tau}\omega_{\nu\rho\lambda\tau}^{AB} + \dots$$

$$\omega_{\mu\nu\rho}^{AB} = \frac{1}{4}\{\omega_\nu, \partial_\rho\omega_\mu + R_{\rho\mu}\}^{AB}$$

$$\omega_{\mu\nu\rho\lambda}^{AB} = \frac{1}{32}\left\{\begin{array}{l} \{\omega_\lambda, \partial_\nu\{\omega_\lambda, \partial_\rho\omega_\mu + R_{\rho\mu}\}\} + 2\{\omega_\lambda, [R_{\nu\rho}, R_{\mu\lambda}]\} \\ - \{\omega_\lambda, [\omega_\nu, D_\rho R_{\nu\mu} + \partial_\rho R_{\nu\mu}]\} - \{\{\omega_\nu, \partial_\rho\omega_\lambda + R_{\rho\lambda}\}\partial_\nu\omega_\mu + R_{\nu\mu}\} \end{array}\right\}^{AB}$$

$$+ 2[\partial_\nu\omega_\lambda, \partial_\rho(\partial_\nu\omega_\mu + R_{\nu\mu})]$$

$$\text{Where } [\alpha, \beta]^{AB} = \alpha^{AC}\beta_C^{B\mu} - \beta^{AC}\alpha_C^{B\mu}$$

$$D_\mu R_{\rho\sigma}^{AB} = \partial_\mu R_{\rho\sigma}^{AB} + (\omega_\mu^{AC}R_{\rho\sigma}^{DB} + \omega_\mu^{BC}R_{\rho\sigma}^{DA})\eta_{CD}$$

Non-zero components of tetrad fields:

$$\hat{e}_\mu^a = e_\mu^a - i\Theta^{\nu\rho}e_{\nu\rho}^a + \Theta^{\nu\rho}\Theta^{\lambda\tau}e_{\nu\rho\lambda\tau}^a + \dots$$

$$\text{Where } e_{\mu\nu\rho}^a = \frac{1}{4}\{\omega_\nu^a\partial_\rho e_\mu^d + (\partial_\rho\omega_\mu^a + R_{\mu\rho}^{ad})e_\nu^d\}\eta_{cd}$$

$$e_{\mu\nu\rho\lambda}^a = \frac{1}{32}\left\{\begin{array}{l} 2\{R_{\nu\rho}, R_{\mu\lambda}\}^{ab}e_\lambda^c - \omega_\lambda^{ab}(D_\rho R_{\mu\lambda}^{cd} + \partial_\rho R_{\mu\lambda}^{cd})e_v^m\eta_{dm} \\ - \{\omega_\nu, D_\rho R_{\mu\lambda} + \partial_\rho R_{\mu\lambda}\}^{ab}e_\lambda^c - \partial_\tau\{\omega_\nu, \partial_\rho\omega_\mu + R_{\mu\rho}\}^{ab}e_\lambda^c \\ - \omega_\lambda^{ab}\partial_\tau\{\omega_\nu^a\partial_\rho e_\mu^m + (\partial_\rho\omega_\mu^{ad} + R_{\mu\rho}^{ad})e_\nu^m\}\eta_{dm} \\ + 2\partial_\nu\omega_\lambda^{ab}\partial_\rho e_\mu^c - 2\partial_\rho(\partial_\nu\omega_\mu^{ab} + R_{\nu\mu}^{ab})\partial_\nu e_\lambda^c \\ - \{\omega_\nu, \partial_\rho\omega_\lambda + R_{\rho\lambda}\}^{ab}\partial_\nu e_\mu^c \\ + (\partial_\nu\omega_\mu^{ab} + R_{\nu\mu}^{ab})(\omega_\nu^{cd}\partial_\rho e_\mu^m + (\partial_\rho\omega_\lambda^{cd} + R_{\mu\lambda}^{cd})e_\nu^m)\eta_{dm} \end{array}\right\}_{abc}$$

Notice Complex Vierbein and Spin Connection

NCG Curvature tensor of the Riemann-Cartan space-time:

$$\hat{F}_{\mu\nu}^{ab} = F_{\mu\nu}^{ab} + i\Theta^{\rho\sigma}F_{\mu\nu\rho\sigma}^{ab} + \Theta^{\rho\sigma}\Theta^{\kappa\tau}F_{\mu\nu\rho\tau\kappa\sigma}^{ab} + O(\Theta^3),$$

Where

$$F_{\mu\nu}^{ab} \equiv R_{\mu\nu}^{ab} - \partial_\mu\omega_\nu^{ab} + (\omega_\mu^{ac}\omega_\nu^{db} - \omega_\mu^{dc}\omega_\nu^{ab})\eta_{cd} + 4\lambda^2(\delta_\mu^{ab}\delta_\nu^{cd} - \delta_\mu^{cd}\delta_\nu^{ab})e_\mu^a e_\nu^b,$$

$$F_{\mu\nu\rho\tau}^{ab} = \partial_\mu\omega_{\nu\rho}^{ab} + (\omega_\mu^{ac}\omega_{\nu\rho}^{db} + \omega_\mu^{dc}\omega_{\nu\rho}^{ab} + \omega_\nu^{ab}\omega_{\mu\rho}^{cd} - \frac{1}{2}\partial_\rho\omega_\mu^{ac}\partial_\tau\omega_\nu^{db})\eta_{cd} - (\mu \leftrightarrow \nu)$$

$$F_{\mu\nu\rho\sigma}^{ab} = \partial_\mu\omega_{\nu\rho}^{ab} + (\omega_\mu^{ac}\omega_{\nu\rho}^{db} + \omega_{\mu\rho}^{ac}\omega_{\nu\sigma}^{db} + \omega_\nu^{ab}\omega_{\mu\sigma}^{cd} - \frac{1}{2}\partial_\rho\omega_\mu^{ac}\partial_\sigma\omega_\nu^{db})\eta_{cd} - (\mu \leftrightarrow \nu),$$

NCG Scalar curvature

$$\hat{F} = \hat{e}_a^\mu * \hat{F}_{\mu\nu}^{ab} * \hat{e}_b^\nu$$

Where

$$\hat{F} = F + \Theta^{\rho\sigma}\Theta^{\kappa\tau}(e_a^\mu F_{\mu\nu\rho\kappa}^{ab}e_b^\nu + e_{\mu\rho}^\kappa F_{\mu\nu\kappa\sigma}^{ab}e_b^\nu + e_a^\mu F_{\mu\nu\sigma\kappa}^{ab}e_\nu^\kappa - e_{\mu\rho}^\kappa F_{\mu\nu\kappa\sigma}^{ab}e_{\sigma\rho}^\nu) + O(\Theta^4).$$

$$\text{Introduce a real metric } \hat{g}_{\mu\nu} = \frac{1}{2}\eta_{ab}(\hat{e}_\mu^a * \hat{e}_\nu^b + \hat{e}_\nu^a * \hat{e}_\mu^b)$$

Start from a commutative spherically symmetric, isotropic and homogeneous FRW metric

consider a flat space case K=0 Use Dimensionless variables

$$\hat{r} = r/r_0 + \hat{t}/t_0$$

$$R(t) = \hat{t}^\beta$$

Some of Non-zero components of deformed metric:

$$\hat{g}_{11} = \hat{t}^{2\beta} + \frac{\Theta^2}{64}\beta^2(-18\hat{t}^{2\beta-2} + 21\hat{t}^2\beta^2\hat{t}^{2(3\beta-2)})$$

$$\hat{g}_{14} = \hat{g}_{41} = \frac{\Theta^2}{128}\beta^2\hat{r}^{2\beta-1}(28 - 25\beta)$$

$$\hat{g}_{22} = \hat{r}^2\hat{t}^{2\beta} + \frac{\Theta^2}{64}\hat{t}^{2\beta}(-45\hat{r}^2\beta^2\hat{t}^{2(\beta-1)} + 21\hat{r}^4\beta^4\hat{t}^{4(\beta-1)} + 4)$$

$$\hat{g}_{44} = -1 - \frac{\Theta^2}{32}\beta(\beta-1)(\hat{t}^{2(\beta+1)} + 18\hat{r}^2\beta^3\hat{t}^{4(\beta-1)})$$

Some of NCG components of spin connection

$$\hat{\omega}_{\mu}^{12} = \frac{-i}{4}\Theta(\beta\hat{t}^{\beta-1})^2, -1 + \frac{\Theta^2}{128}(\beta\hat{t}^{\beta-1})^2[33(\hat{r}\beta\hat{t}^{\beta-1})^2 + 4], 0, \frac{-i}{4}\Theta\beta^2(\beta-1)\hat{t}^{2\beta-3}$$

$$\hat{\omega}_{\mu}^{13} = \left(0, 0, -\sin\theta + \frac{i}{2}\Theta\hat{r}\cos\theta(\beta\hat{t}^{\beta-1})^2 + \frac{\Theta^2}{64}\beta^2\hat{t}^{2\beta-1}\sin\theta(\hat{r}\beta\hat{t}^{\beta-1})^2 + 6, 0\right)$$

$$\hat{\omega}_{\mu}^{14} = \left(-\beta\hat{t}^{\beta-1} + \frac{1}{64}\Theta^2(\beta\hat{t}^{\beta-1})^3[8 + 7(\hat{r}\beta\hat{t}^{\beta-1})^2], \frac{i}{2}\hat{r}\hat{t}^{\beta-1}(\beta\hat{t}^{\beta-1})^3, 0, -\frac{3\Theta^2}{32}\hat{r}\beta^3(\beta-1)\hat{t}^{3\beta-4}\right)$$

$$\hat{\omega}_{\mu}^{21} = \left(-\frac{i}{4}\Theta(\beta\hat{t}^{\beta-1})^2, -1 - \frac{29\Theta^2}{128}\hat{r}^2(\beta\hat{t}^{\beta-1})^4, 0, \frac{i}{2}\Theta\beta^2(\beta-1)\hat{t}^{2\beta-3}\right)$$

$$\hat{\omega}_{\mu}^{22} = \left(\frac{15}{128}\Theta^2\hat{r}(\beta\hat{t}^{\beta-1})^4, -i\Theta(\beta\hat{t}^{\beta-1})^2, 0, \frac{\Theta^2}{64}\beta^2(\beta-1)[8\hat{r}^{2\beta-3} + 23(\hat{r}\beta)^2\hat{t}^{4\beta-5}]\right)$$

One can show that: Dynamical evolving Black Holes

Location Trapping dynamical horizon

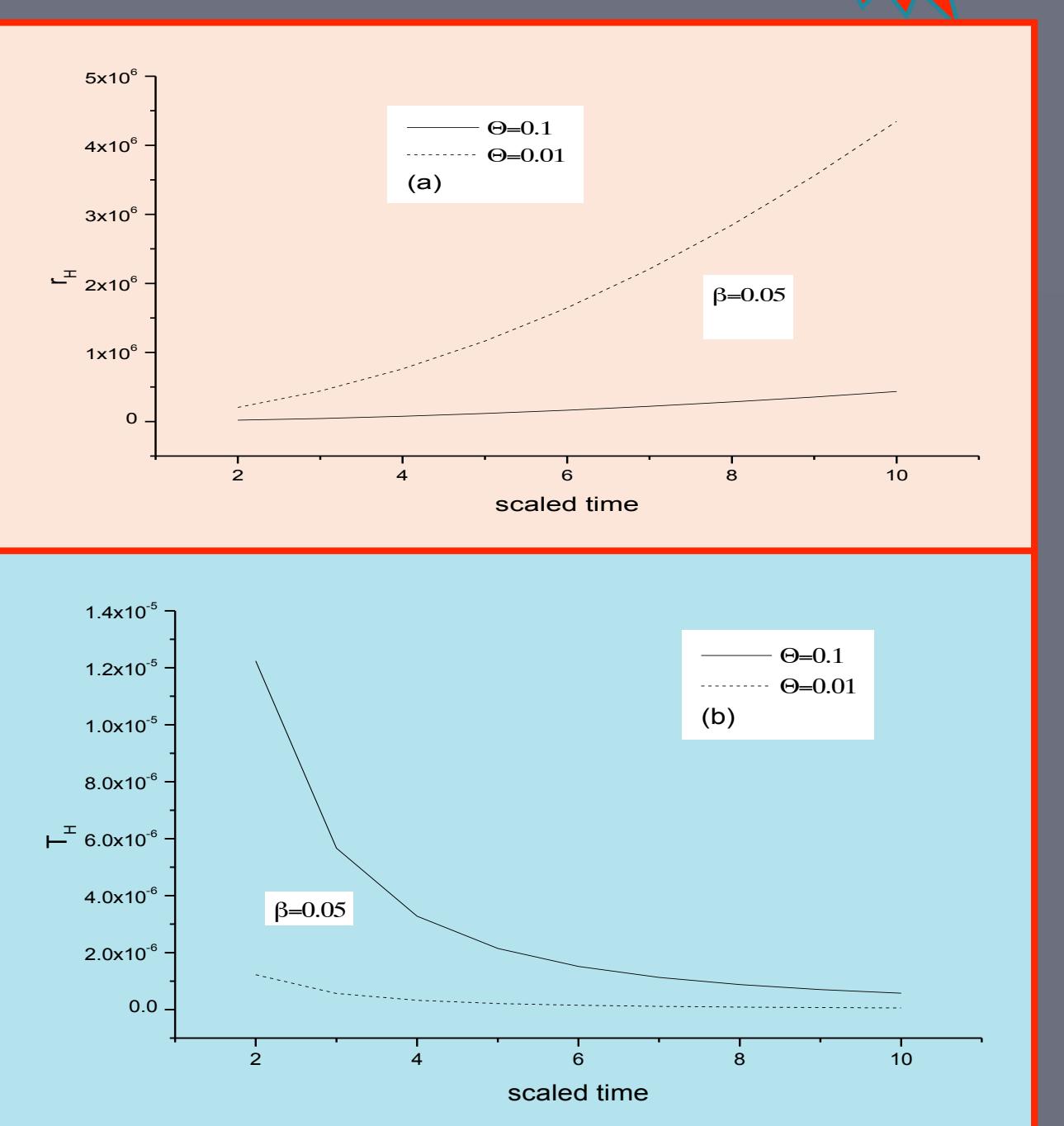
Hawking Temperature

$$r_H \approx \sqrt{\frac{32 - \Theta^2\beta(1-\beta)\hat{t}^{2(\beta+1)}}{18\Theta^2\beta^4(1-\beta)\hat{t}^{4(\beta-1)}}}$$

$$T_{\text{Hawking}} \approx \frac{1}{64}\sqrt{\frac{9}{2}\Theta}\sqrt{[32 - \Theta^2\beta(1-\beta)\hat{t}^{2(\beta+1)}]\beta^4(1-\beta)\hat{t}^{4(\beta-1)}}$$

Notice This is a pure NCG Effect

New!



First Unexpected Result

NCG predicts new kind of dynamical black holes

Due space-time deformation

Second New Interesting Result

Accelerated Expansion of Universe without Dark Energy Scenario

Due to inhomogeneity and anisotropy generated by NCG

In fact

$$q_T = \frac{-\partial^2 \hat{R}}{\hat{R}^2 t^2} = -\frac{1}{H_T^2} \left[(\Sigma_1 + \Sigma_2 + \Sigma_3) a^2 + (\Sigma_4 + \Sigma_5) a\dot{a} \right]$$

Deceleration parameter/time evolution

$$q_L = -\frac{\partial^2 \hat{R}}{\hat{R}^2 t^2} = -\frac{1}{H_L^2} [\Sigma_1 - 2\Omega\Sigma_2 - \Omega\Sigma_7]$$

Deceleration parameter/space evolution

$$H_T = H_T(r', t') = \frac{1}{\hat{R}} \frac{\partial \hat{R}}{\partial t'} = \left(\dot{\Lambda} - \frac{\Lambda'}{\Omega} \right) \frac{a}{2\Lambda}$$

Hubble parameter/time evolution

$$H_L = H_L(r', t') = \frac{1}{\hat{R}} \frac{\partial \hat{R}}{\partial r'} = \frac{\dot{\Lambda}}{2\Lambda}$$

Where $\Omega = \frac{2\hat{g}_{11} + \hat{g}_{14} + \hat{g}_{41}}{2\hat{g}_{44} + \hat{g}_{14} + \hat{g}_{41}}$

$$\Sigma_3 = \frac{\dot{\Omega}\Lambda'}{2\Omega^2\Lambda}$$

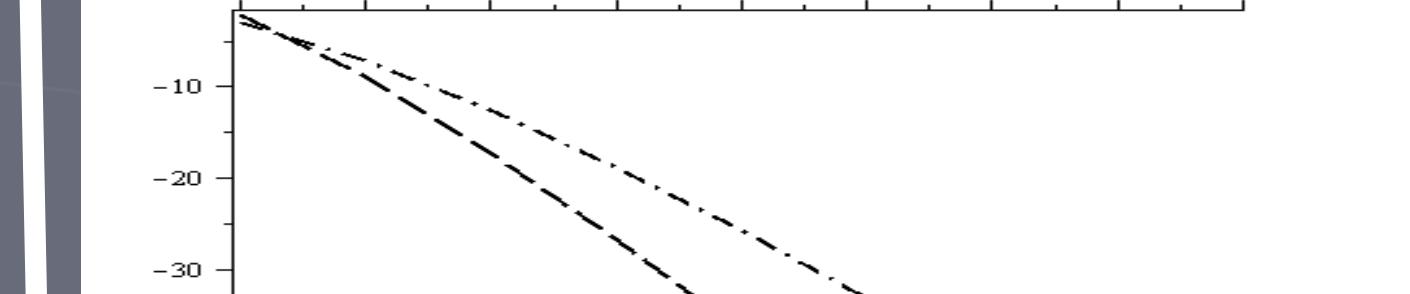
$\pm \Omega$

$$a = \left[-\Omega^2 \hat{g}_{44} - \hat{g}_{11} + \Omega(\hat{g}_{14} + \hat{g}_{41}) \right]$$

$$\Sigma_1 = \frac{\ddot{\Lambda}}{2\Lambda} - \frac{\dot{\Lambda}^2}{\Lambda^2}, \quad \Sigma_2 = -\frac{1}{4\Omega} \left(2\frac{\dot{\Lambda}}{\Lambda} - \frac{\dot{\Lambda}\Lambda'}{\Lambda^2} \right)$$

$$\Sigma_4 = \frac{\dot{\Lambda}}{2\Lambda}, \quad \Sigma_5 = -\frac{\Lambda'}{2\Omega\Lambda}$$

$$\Sigma_6 = -\Omega\Sigma_2, \quad \Sigma_8 = \frac{\Omega'}{\Omega} \Sigma_5, \quad \Lambda = \hat{g}_{44} + \hat{g}_{11} + \hat{g}_{14} + \hat{g}_{41}$$



More Useful New NCG Results

1) If one can write NCG FRW like equation as:

$$\frac{2\ddot{R}}{R} = -\left(\tilde{P} + \frac{1}{3}\tilde{\rho}\right), \quad \tilde{P} = P_{\text{ord}} + \Theta^2 P_1, \quad \tilde{\rho} = \rho_{\text{ord}} + \Theta^2 \rho_1$$

Get time and space dependent variable State parameter

$$\tilde{P} = w(r, t)\tilde{\rho}, \quad \tilde{\rho} = \rho_{\text{ord}} + \Theta^2 \rho_1$$

2) If one can write NCG FRW like equation as:

$$\frac{2\ddot{R}}{R} = \frac{2}{3}\tilde{\Lambda} + (P_{\text{ord}} + \frac{1}{3}\rho_{\text{ord}}), \quad \tilde{\Lambda} \propto \Theta^2$$

NCG Induced Running cosmological constant

Leptogenesis from a Non Commutative FRW Like Model

To explain

Origin lepton number asymmetry

Many mechanisms were proposed

Affleck-Dine

Fermions propagation in a curved space-time or around a black hole

Lorentz and CPT violating scenarios in context of Riemann-Cartan space-time

Starting from a commutative isotropic, homogenous and spherical symmetric FRW universe</p