Towards exact field theory results for the Standard Model on fractional D6-branes

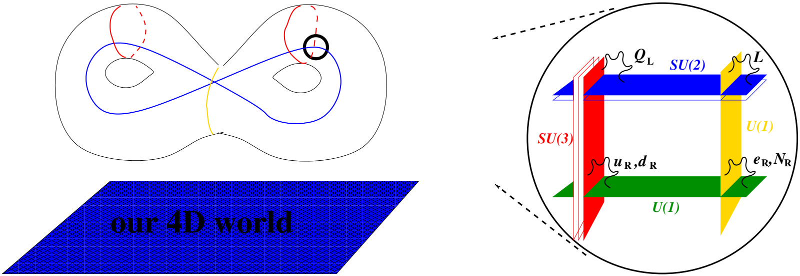
Gabriele Honecker (JG|U Mainz)

based on work in progress (to appear soon)
and **NPB829** (2010) 225 (`arXiv:0910.0843[hep-th]`) with Florian Gmeiner

EPS-HEP 2011, Grenoble, 23 July 2011

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Motivation



- Explicit *globally consistent* models on fractional D6-branes on toroidal orbifolds $T^6/\mathbb{Z}_{2N}$ G.H., Ott '04; Gmeiner, G.H. '07-09
  rigid D6-branes expected on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ Förste, G.H. *et al.* in progress; ...
- Gauge & Yukawa couplings, Kähler metrics ... known for **untilted six-torus** only (& partial results for **untilted** $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ without/with torsion) Cremades, Ibáñez, Marchesano '03; Cvetic, Papadimitriou '03; Abel, Schofield/Goodsell '04/05; Lüst, (Mayr, Richter), Stieberger '03('04); Akerblom, Blumenhagen, Lüst, Schmidt-Sommerfeld '07; ...

Motivation

- need exact results for some **vanishing angle** in $\boxed{\mathbb{Z}_2}$ presence & **tilted tori** starting point: gauge thresholds in Gmeiner, G.H. '09
  - **(anti)symmetric matter** contributes differently to gauge kinetic function on tilted tori
  - **one vanishing angle**: only $\mathcal{N}=1$ SUSY if $\mathbb{Z}_2$ acts
    - $\boxed{T^6/\mathbb{Z}_{2N}}$: the SM on $T^6/\mathbb{Z}_6'$ and $T^6/\mathbb{Z}_6$
    - $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}}$ with discrete torsion: expected models on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6$ and $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$
  - different number of **complex structure moduli** (e.g. less *bulk* from orbifolded six-torus, several *exceptional* ones)
  - **compact expressions**, which are valid for any known torus orbifold, admit direct comparisons
- need gauge kinetic function for **physical U(1)s** like $U(1)_Y = \frac{1}{2}\left(\frac{1}{3}U(1)_a + U(1)_c + U(1)_d\right)$

Gauge couplings in string & field theory

- $\mathcal{N}=1$ SUSY result of **1-loop string** calculation: with gauge threshold Δ_a

$$\frac{8\pi^2}{g_a^2(\mu)} = \frac{8\pi^2}{g^2_{a,\mathrm{string}}} + \frac{b_a}{2}\ln\left(\frac{M^2_{\mathrm{string}}}{\mu^2}\right) + \frac{\Delta_a}{2}$$

via magnetic background field method Untilted tori for T^6: Lüst, Stieberger '03; Akerblom *et al.* '07; Untilted tori for $T^6/\mathbb{Z}_2\times\mathbb{Z}_2$ with torsion: partial results by Blumenhagen, Schmidt-Sommerfeld '07; $T^6/\mathbb{Z}_{2N}$ and **tilted tori**: Gmeiner, G.H. '09

- $\mathcal{N}=1$ SUSY **field theory**:

$$\frac{8\pi^2}{g_a^2(\mu)} = 8\pi^2\ \Re(\mathrm{f}_a) + \frac{b_a}{2}\ln\left(\frac{M^2_{\mathrm{Planck}}}{\mu^2}\right) + \frac{b_a + 2\,C_2(G_a)}{2}\ \mathcal{K} + C_2(G_a)\ \ln[g_a^{-2}(\mu^2)] - \sum_a C_2(R_a)\ \ln\det K_{R_a}(\mu^2)$$

- derive **gauge kinetic function** f_a, **Kähler metrics** K_{R_a}, Kähler potential $\mathcal{K}$ by matching string & field theory

Tree-Level Gauge Couplings & 1-Loop Redefinitions

- **tree-level**:

$$\frac{1}{g_a^2} = \frac{e^{-\phi_4}}{2\pi}\sqrt{\prod_{i=1}^3 V_{aa}^{(i)}} \stackrel{\text{SUSY}}{=} \begin{cases} S\,X_a^0 - \sum_{i=1}^3 U_i X_a^i & T^6 \\ S\,X_a^0 - U\,X_a^1 & T^6/\mathbb{Z}_6' \\ S\,X_a^0 & T^6/\mathbb{Z}_6 \end{cases} = \Re(f_a^{\text{tree}})$$

  - with definition of **dilaton & complex structure moduli**

$$(S, U_i)_{\text{tree}} \equiv \frac{e^{-\phi_4}}{2\pi} \times \begin{cases} \left(1/\sqrt{\prod_{i=1}^3 r_i}\,,\ \sqrt{r_j r_k / r_i}\right) & T^6 \\ \left(\frac{c_{\text{lattice}}}{\sqrt{r}}\,,\ \frac{4}{3^{3/2} c_{\text{lattice}}}\sqrt{r}\right) & T^6/\mathbb{Z}_6' \\ (c_{\text{lattice}}\,,\ \emptyset) & T^6/\mathbb{Z}_6 \end{cases}$$

- **1-loop massless strings**:

$\frac{b_a}{2}\ln\left(\frac{M_{\text{string}}}{\mu}\right)^2 \leftrightarrow \frac{b_a}{2}\left[\ln\left(\frac{M_{\text{Planck}}}{\mu}\right)^2 + \mathcal{K}\right]$ provided by

$\left(\frac{M_{\text{string}}}{M_{\text{Planck}}}\right)^2 = e^{2\phi_4} = \frac{e^{2\phi_{10}}}{\sqrt{\text{Vol}_{CY_3}}}$ and

$$\mathcal{K} = -\ln(S)^\alpha - \sum_k \ln(U_k)^\alpha - \sum_{i=1}^3 \ln T_i + \ldots \quad \text{with } \alpha = \begin{cases} 1 & T^6 \\ 2 & T^6/\mathbb{Z}_6' \\ 4 & T^6/\mathbb{Z}_6 \end{cases}$$

- **1-loop massive strings** provide
  - Kähler metrics & correction to gauge kinetic functions
  - field redefinitions of moduli

1-Loop Results from Massive Strings on e.g. $T^6/\mathbb{Z}_6'$

$\mathbb{Z}_2$ subgroup: $\boxed{N\vec{v} = (\frac{1}{2}, 0, -\frac{1}{2})}$, distinguish *relative* angles $(\vec{\phi}_{ab})$:

- $\boxed{(\vec{0}), \mathcal{N} = 2 \text{ SUSY}}$: parallel D6-branes contribute lattice sum: holomorphic part $\rightsquigarrow \delta_b \mathrm{f}_a^{1-\mathrm{loop}}$; non-hol. $\rightsquigarrow$ Kähler metrics K_{R_a}
- $\boxed{(\phi, 0, -\phi), \mathcal{N} = 2}$: hol. $\rightsquigarrow \delta_b \mathrm{f}_a^{1-\mathrm{loop}}$; non-hol. $\rightsquigarrow K_{R_a}$
- $\boxed{(0, \phi, -\phi), \mathcal{N} = 1}$: lattice sum on $T^2_{(1)}$ ($\rightsquigarrow \delta_b \mathrm{f}_a^{1-\mathrm{loop}}, K_{R_a}$)
  & field redefinitions
- $\boxed{(\vec{\phi}_{ab}), \mathcal{N} = 1}$: D6-branes at angles $\rightsquigarrow$ non-hol. $\ln(\Gamma(\phi_{ab}))$ terms & field redefinitions
- **crucial** for interpretation: **global prefactor** b_a in $\Delta_{ab} \rightsquigarrow$ identical K_{R_a} for each multiplet, in particular $R_a = \mathbf{Anti}_a, \mathbf{Sym}_a$

Results for gauge thresholds I: Bifundamentals

- lattice sums:
  - $\Lambda_{0,0}(v;V) \equiv \ln(2\pi v V) + [2\ln\eta(iv) + c.c.]$

    with two-torus volume v and (length)^2 of 1-cycle V
  - $\Lambda(\sigma,\tau,v) \equiv -\frac{\pi}{2} v\sigma^2 + \left[2\ln\frac{\vartheta_1(\frac{\tau - iv\sigma}{2}, iv)}{\eta(iv)} + c.c.\right]$

    with *relative* displacements & Wilson lines (σ,τ)
  - define $b_a \equiv \tilde{b}_a\, \delta_{\sigma^i,0}\delta_{\tau^i,0}$ for D6-branes parallel on two-torus $T^2_{(i)}$

$b_{SU(N_a)}$ and gauge thresholds for bifundamentals and adjoints on $T^6/\mathbb{Z}_{2N}$ with $\mathbb{Z}_2 \equiv \mathbb{Z}_2^{(2)}$

SUSY	$(\phi^{(1)}_{ab},\phi^{(2)}_{ab},\phi^{(3)}_{ab})$	$b^{\mathbb{Z}_2}_{SU(N_a)} = \sum_b b^{\mathcal{A}}_{ab} + \ldots = \sum_b \frac{N_b}{2}\varphi^{ab} + \ldots$	$\Delta^{\mathbb{Z}_{2N}}_{SU(N_a)} = N_b\,\tilde{\Delta}_{ab} + \ldots$
2	$(0,0,0)$	$-\frac{I^{\mathbb{Z}_2,(1\cdot 3)}_{ab} N_b}{2}\,\delta_{\sigma^2_{ab},0}\delta_{\tau^2_{ab},0}$	$-b^{\mathcal{A}}_{ab}\,\Lambda_{0,0}(v_2;V^{(2)}_{ab})$ $-\tilde{b}^{\mathcal{A}}_{ab}\left(1-\delta_{\sigma^2_{ab},0}\delta_{\tau^2_{ab},0}\right)\Lambda(\sigma^2_{ab},\tau^2_{ab},v_2)$
2	$(\phi,0,-\phi)$	$\frac{N_b}{2}\,\delta_{\sigma^2_{ab},0}\,\delta_{\tau^2_{ab},0}\left(\lvert I^{(1\cdot 3)}_{ab}\rvert - I^{\mathbb{Z}_2,(1\cdot 3)}_{ab}\right)$	$-b^{\mathcal{A}}_{ab}\,\Lambda_{0,0}(v_2;V^{(2)}_{ab})$ $-\tilde{b}^{\mathcal{A}}_{ab}\left(1-\delta_{\sigma^2_{ab},0}\delta_{\tau^2_{ab},0}\right)\Lambda(\sigma^2_{ab},\tau^2_{ab},v_2)$
1	$(0,\phi,-\phi)$	$\frac{N_b}{2}\,\delta_{\sigma^1_{ab},0}\,\delta_{\tau^1_{ab},0}\,\lvert I^{(2\cdot 3)}_{ab}\rvert$	$-b^{\mathcal{A}}_{ab}\,\Lambda_{0,0}(v_1;V^{(1)}_{ab})$ $-\tilde{b}^{\mathcal{A}}_{ab}\left(1-\delta_{\sigma^1_{ab},0}\delta_{\tau^1_{ab},0}\right)\Lambda(\sigma^1_{ab},\tau^1_{ab},v_1)$ $+\frac{N_b I^{\mathbb{Z}_2}_{ab}}{2}\left(\mathrm{sgn}(\phi) - 2\phi\right)\ln(2)$
1	$(\phi^{(1)},\phi^{(2)},\phi^{(3)})$ with $\sum_{k=1}^3 \phi^{(k)} = 0$	$\frac{N_b}{4}\left(\lvert I_{ab}\rvert + \mathrm{sgn}(I_{ab})\, I^{\mathbb{Z}_2}_{ab}\right)$	$b^{\mathcal{A}}_{ab}\,\mathrm{sgn}(I_{ab})\sum_{i=1}^3 \ln\left(\frac{\Gamma(\lvert\phi^{(i)}_{ab}\rvert)}{\Gamma(1-\lvert\phi^{(i)}_{ab}\rvert)}\right)^{\mathrm{sgn}(\phi^{(i)}_{ab})}$ $+\frac{N_b}{2} I^{\mathbb{Z}_2}_{ab}\left(\mathrm{sgn}(I_{ab}) + \mathrm{sgn}(\phi^{(2)}_{ab}) - 2\phi^{(2)}_{ab}\right)\ln(2)$

From gauge thresholds to Kähler metrics I: Bifundamentals

$\boxed{\vec{\phi}_{ab} = (\vec{0}), (0_i, (\phi)_j, (-\phi)_k)}$: has $\mathcal{N} = 2$ or 1 SUSY on $T^6/\mathbb{Z}_{2N}$

- 1-loop correction to **gauge kinetic function** for arbitrary (σ, τ)

$$\delta_b \mathrm{f}_a^{1-\mathrm{loop}} = -\frac{b_{ab}^{\mathcal{A}}}{4\pi^2} \times \begin{cases} \ln \eta(iv) & (\sigma,\tau) = (0,0) \\ \ln \frac{\vartheta_1(\frac{\tau - iv\sigma}{2}, iv)}{\eta(iv)} & \neq (0,0) \end{cases}$$

- **Kähler metrics:**

$$K_{\mathbf{Adj}_a}^{(2)} = \frac{2\pi\, e^{\phi_4}}{v_2} \sqrt{\frac{V_{aa}^{(1)} V_{aa}^{(3)}}{V_{aa}^{(2)}}}$$

and

$$K_{(\mathbf{N}_a, \overline{\mathbf{N}}_b)} = 2\pi e^{\phi_4} \sqrt{\frac{V_{aa}^{(i)}}{v_j v_k}}$$

- Comparison with other orbifolds:
  - T^6 and $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ without torsion: $\mathcal{N} = 4$ for $\vec{\phi}_{ab} = (\vec{0})$: no $\delta_b \mathrm{f}_a^{1-\mathrm{loop}}$, $K_{\mathbf{Adj}_a}^{(i)}$ derived from scattering amplitudes $\surd$
  - $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ with discrete torsion: $\mathcal{N} = 1$ for $\vec{\phi}_{ab} = (\vec{0})$: present result differs from Blumenhagen, Schmidt-Sommerfeld '07

Bifundamentals cont'd

other $(\phi^{(1)}_{ab}, \phi^{(2)}_{ab}, \phi^{(3)}_{ab})$ is always $\mathcal{N}=1$ SUSY

- **Kähler metrics:**

$$K_{(\mathbf{N}_a,\overline{\mathbf{N}}_b)} = \frac{2\pi\, e^{\phi_4}}{\sqrt{\prod_{i=1}^3 v_i^{1+\mathrm{sgn}(I_{ab})\phi^{(i)}_{ab}}}} \sqrt{\prod_{i=1}^3 \left(\frac{\Gamma(|\phi^{(i)}_{ab}|)}{\Gamma(1-|\phi^{(i)}_{ab}|)}\right)^{\frac{\mathrm{sgn}(\phi^{(i)}_{ab})}{\mathrm{sgn}(I_{ab})}}}$$

- **1-loop field redefinitions** (here: Volume$(T^2_{(i)}) \equiv v_i \equiv T_i$)
  - dilaton and *bulk* compl. structures ($i = 3, 1, \emptyset$ for T^6, $T^6/\mathbb{Z}_6'$, $T^6/\mathbb{Z}_6$)

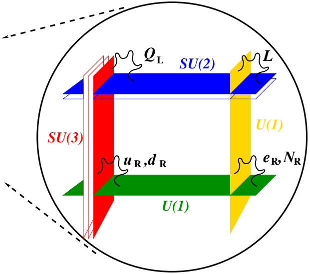
$$S^L = S - \frac{1}{16\pi^2}\sum_b N_b Y^0_b \sum_{j=1}^3 \phi^{(j)}_b \ln T_j \qquad U^L_i = U_i + \frac{1}{16\pi^2}\sum_b N_b Y^i_b \sum_{j=1}^3 \phi^{(j)}_b \ln T_j$$

with $\Pi_b = \sum_{i=0}^{h_{21}} \left(X^i_b\, \Pi^{\rm even}_i + Y^i_b \Pi^{\rm odd}_i\right)$

  - *exceptional* complex structures cf. Angelantonj, Condeescu, Dudas, Lennek '09

$$W^L_\alpha = W_\alpha + \frac{1}{16\pi^2}\sum_b N_b Y^\alpha_b \sum_{j=1}^3 \phi^{(j)}_b \ln T_j$$

Examples for Bifundamentals & Adjoints



On the **ABa** lattice on $T^6/\mathbb{Z}_6'$ Gmeiner, G.H. '07-09

- aa sector $\boxed{\vec{\phi} = (\vec{0})}$: one adjoint $\mathbf{8}+\mathbf{1}$ of $U(3)$
- ac sector $\boxed{\pi(0, \frac{1}{2}, -\frac{1}{2})}$: $2\,\mathbf{u}_R$
- $a(\theta^2 c)$ sector $\boxed{\pi(-\frac{1}{3}, -\frac{1}{6}, \frac{1}{2})}$: $1\,\mathbf{u}_R$

- ab' sector $\boxed{\pi(\frac{1}{6}, -\frac{1}{2}, \frac{1}{3})}$: $2\,\mathbf{Q}_L$
- $a(\theta b')$ sector $\boxed{\pi(-\frac{1}{2}, \frac{1}{6}, \frac{1}{3})}$: $2\,\mathbf{Q}_L$
- $a(\theta^2 b')$ sector $\boxed{\pi(-\frac{1}{6}, -\frac{1}{6}, \frac{1}{3})}$: $1\,\overline{Q}_L$
- ...all possible configurations of angles appear!

Results for gauge thresholds II: (Anti)Symmetrics

$b_{SU(N_a)}$ **and gauge thresholds for symmetrics and antisymmetrics:** $T^6/\mathbb{Z}_{2N}$ **with** $\mathbb{Z}_2 \equiv \mathbb{Z}_2^{(2)}$

SUSY	$(\phi^{(1)}_{aa'}, \phi^{(2)}_{aa'}, \phi^{(3)}_{aa'})$	$b^{\mathbb{Z}_{2N}}_{SU(N_a)} = b^{\mathcal{A}}_{aa'} + b^{\mathcal{M}}_{aa'} + \ldots = \frac{N_a}{2}\left(\varphi^{\mathbf{Sym}_a} + \varphi^{\mathbf{Anti}_a}\right) + \left(\varphi^{\mathbf{Sym}_a} - \varphi^{\mathbf{Anti}_a}\right) + \ldots$	$\Delta^{\mathbb{Z}_{2N}}_{SU(N_a)} = N_a\,\tilde{\Delta}_{aa'} + \Delta_{a,O6} + \ldots$
2	$(0,0,0)$ $\uparrow\uparrow \Omega\mathcal{R}$	$-\delta_{\sigma^2_{aa'},0}\delta_{\tau^2_{aa'},0} \times \left(\frac{N_a I^{\mathbb{Z}_2,(1\cdot3)}_{aa'}}{2} + \lvert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2,(1\cdot3)}_a\rvert\right)$	$-\left(b^{\mathcal{A}}_{aa'} + b^{\mathcal{M}}_{aa'}\right)\ \Lambda_{0,0}(v_2; V^{(2)}_{aa'}) - (4\,b_2)\ b^{\mathcal{M}}_{aa'}\ \ln\left(\frac{\eta(2iv_2)}{\vartheta_4(0,2iv_2)}\right) - 2\,b^{\mathcal{M}}_{aa'}\ \ln(2)$ $-\left(1-\delta_{\sigma^2_{aa'},0}\delta_{\tau^2_{aa'},0}\right)\left[\tilde{b}^{\mathcal{A}}_{aa'}\ \Lambda(\sigma^2_{aa'},\tau^2_{aa'},v_2) + \tilde{b}^{\mathcal{M}}_{aa'}\ \Lambda(\sigma^2_{aa'},\tau^2_{aa'},\tilde{v}_2)\right]$
2	$(0,0,0)$ $\uparrow\uparrow \Omega\mathcal{R}\mathbb{Z}_2^{(2)}$	$-\delta_{\sigma^2_{aa'},0}\delta_{\tau^2_{aa'},0} \times \left(\frac{N_a I^{\mathbb{Z}_2,(1\cdot3)}_{aa'}}{2} + \lvert\tilde{I}^{\Omega\mathcal{R},(1\cdot3)}_a\rvert\right)$	$-\left(b^{\mathcal{A}}_{aa'} + b^{\mathcal{M}}_{aa'}\right)\ \Lambda_{0,0}(v_2; V^{(2)}_{aa'}) - (4\,b_2)\ b^{\mathcal{M}}_{aa'}\ \ln\left(\frac{\eta(2iv_2)}{\vartheta_4(0,2iv_2)}\right) - 2\,b^{\mathcal{M}}_{aa'}\ \ln(2)$ $-\left(1-\delta_{\sigma^2_{aa'},0}\delta_{\tau^2_{aa'},0}\right)\left[\tilde{b}^{\mathcal{A}}_{aa'}\ \Lambda(\sigma^2_{aa'},\tau^2_{aa'},v_2) + \tilde{b}^{\mathcal{M}}_{aa'}\ \Lambda(\sigma^2_{aa'},\tau^2_{aa'},\tilde{v}_2)\right]$
1	$(0,0,0)$ $\perp \Omega\mathcal{R}$ on $T_1 \times T_2$	$-\left(\frac{N_a I^{\mathbb{Z}_2,(1\cdot3)}_{aa'}\ \delta_{\sigma^2_{aa'},0}\delta_{\tau^2_{aa'},0}}{2} + \lvert\tilde{I}^{\Omega\mathcal{R},(1\cdot2)}_a\rvert + \lvert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2,(2\cdot3)}_a\rvert\right)$	$-b^{\mathcal{A}}_{aa'}\ \Lambda_{0,0}(v_2; V^{(2)}_{aa'}) + \lvert\tilde{I}^{\Omega\mathcal{R},(1\cdot2)}_a\rvert\ \Lambda_{0,0}(\tilde{v}_3; 2\tilde{V}^{(3)}_{aa'}) + \lvert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2,(2\cdot3)}_a\rvert\ \Lambda_{0,0}(\tilde{v}_1; 2\tilde{V}^{(1)}_{aa'})$ $-\tilde{b}^{\mathcal{A}}_{aa'}\ \left(1-\delta_{\sigma^2_{aa'},0}\delta_{\tau^2_{aa'},0}\right)\ \Lambda(\sigma^2_{aa'},\tau^2_{aa'},v_2)$
1	$(0,0,0)$ $\perp \Omega\mathcal{R}$ on $T_2 \times T_3$	$-\frac{N_a I^{\mathbb{Z}_2,(1\cdot3)}_{aa'}\ \delta_{\sigma^2_{aa'},0}\delta_{\tau^2_{aa'},0}}{2} + \tilde{I}^{\Omega\mathcal{R},(2\cdot3)}_a + \tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2,(1\cdot2)}_a$	$-b^{\mathcal{A}}_{aa'}\ \Lambda_{0,0}(v_2; V^{(2)}_{aa'}) + \lvert\tilde{I}^{\Omega\mathcal{R},(2\cdot3)}_a\rvert\ \Lambda_{0,0}(\tilde{v}_1; 2\,\tilde{V}^{(1)}_{aa'}) + \lvert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2,(1\cdot2)}_a\rvert\ \Lambda_{0,0}(\tilde{v}_3; 2\,\tilde{V}^{(3)}_{aa'})$ $-\tilde{b}^{\mathcal{A}}_{aa'}\ \left(1-\delta_{\sigma^2_{aa'},0}\delta_{\tau^2_{aa'},0}\right)\ \Lambda(\sigma^2_{aa'},\tau^2_{aa'},v_2)$
2	$(\phi,0,-\phi)_{\phi\neq\pm\frac{1}{2}}$ $\uparrow\uparrow\left(\Omega\mathcal{R} + \Omega\mathcal{R}\mathbb{Z}_2^{(2)}\right)$ on T_2	$\delta_{\sigma^2_{aa'},0}\ \delta_{\tau^2_{aa'},0}\left[\frac{N_a\left(\lvert I^{(1\cdot3)}_{aa'}\rvert - I^{\mathbb{Z}_2,(1\cdot3)}_{aa'}\right)}{2} - \lvert\tilde{I}^{\Omega\mathcal{R},(1\cdot3)}_a\rvert - \lvert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2,(1\cdot3)}_a\rvert\right]$	$-\left(b^{\mathcal{A}}_{aa'} + b^{\mathcal{M}}_{aa'}\right)\ \Lambda_{0,0}(v_2; V^{(2)}_{aa'}) - (4\,b_2)\ b^{\mathcal{M}}_{aa'}\ \ln\left(\frac{\eta(2iv_2)}{\vartheta_4(0,2iv_2)}\right) - 2\,b^{\mathcal{M}}_{aa'}\ \ln(2)$ $-\left(1-\delta_{\sigma^2_{aa'},0}\delta_{\tau^2_{aa'},0}\right) \times \left[\tilde{b}^{\mathcal{A}}_{aa'}\ \Lambda(\sigma^2_{aa'},\tau^2_{aa'},v_2) + \tilde{b}^{\mathcal{M}}_{aa'}\ \Lambda(\sigma^2_{aa'},\tau^2_{aa'},\tilde{v}_2)\right]$
1	$(\phi,0,-\phi)_{\phi\neq\pm\frac{1}{2}}$ $\perp\left(\Omega\mathcal{R} + \Omega\mathcal{R}\mathbb{Z}_2^{(2)}\right)$ on T_2	$\frac{N_a\left(\lvert I^{(1\cdot3)}_{aa'}\rvert - I^{\mathbb{Z}_2,(1\cdot3)}_{aa'}\right)}{2}\delta_{\sigma^2_{aa'},0}\ \delta_{\tau^2_{aa'},0}$	$-b^{\mathcal{A}}_{aa'}\ \Lambda_{0,0}(v_2; V^{(2)}_{aa'}) - \tilde{b}^{\mathcal{A}}_{aa'}\ \left(1-\delta_{\sigma^2_{aa'},0}\delta_{\tau^2_{aa'},0}\right)\ \Lambda(\sigma^2_{aa'},\tau^2_{aa'},v_2)$ $+\left(\lvert\tilde{I}^{\Omega\mathcal{R}}_a\rvert + \lvert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2}_a\rvert\right)\ \ln(2)$
1	$(0,\phi,-\phi)$ $\uparrow\uparrow \Omega\mathcal{R}$ on T_1	$\frac{N_a}{2}\lvert I^{(2\cdot3)}_{aa'}\rvert - \lvert\tilde{I}^{\Omega\mathcal{R},(2\cdot3)}_a\rvert$	$-\left(b^{\mathcal{A}}_{aa'} + b^{\mathcal{M}}_{aa'}\right)\ \Lambda_{0,0}(v_1; V^{(1)}_{aa'}) - (4\,b_1)\ b^{\mathcal{M}}_{aa'}\ \ln\left(\frac{\eta(2iv_1)}{\vartheta_4(0,2iv_1)}\right)$ $+\left(\frac{N_a I^{\mathbb{Z}_2}_{aa'}}{2}\ (\mathrm{sgn}(\phi) - 2\,\phi) + \lvert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2}_a\rvert + 2\,\lvert\tilde{I}^{\Omega\mathcal{R},(2\cdot3)}_a\rvert\right)\ \ln(2)$
1	$(0,\phi,-\phi)$ $\perp \Omega\mathcal{R}$ on T_1	$\frac{N_a}{2}\lvert I^{(2\cdot3)}_{aa'}\rvert - \lvert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2,(2\cdot3)}_a\rvert$	$-\left(b^{\mathcal{A}}_{aa'} + b^{\mathcal{M}}_{aa'}\right)\ \Lambda_{0,0}(v_1; V^{(1)}_{aa'}) - (4\,b_1)\ b^{\mathcal{M}}_{aa'}\ \ln\left(\frac{\eta(2iv_1)}{\vartheta_4(0,2iv_1)}\right)$ $+\left(\frac{N_a I^{\mathbb{Z}_2}_{aa'}}{2}\ (\mathrm{sgn}(\phi) - 2\,\phi) + \lvert\tilde{I}^{\Omega\mathcal{R}}_a\rvert + 2\,\lvert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2,(2\cdot3)}_a\rvert\right)\ \ln(2)$
1	$(\phi^{(1)},\phi^{(2)},\phi^{(3)})$ $0 < \lvert\phi^{(i,j)}\rvert \leq \lvert\phi^{(k)}\rvert < 1$ $\mathrm{sgn}(\phi^{(i)}) = \mathrm{sgn}(\phi^{(j)})$ $\neq \mathrm{sgn}(\phi^{(k)})$	$\frac{N_a\left(\lvert I_{aa'}\rvert + \mathrm{sgn}(I_{aa'})\,I^{\mathbb{Z}_2}_{aa'}\right)}{4} + \frac{c^{\Omega\mathcal{R}}_a\,\lvert\tilde{I}^{\Omega\mathcal{R}}_a\rvert + c^{\Omega\mathcal{R}\mathbb{Z}_2}_a\,\lvert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2}_a\rvert}{2}$	$\left(b^{\mathcal{A}}_{aa'} + b^{\mathcal{M}}_{aa'}\right)\ \mathrm{sgn}(I_{aa'})\ \sum_{i=1}^{3}\ln\left(\frac{\Gamma(\lvert\phi^{(i)}_{aa'}\rvert)}{\Gamma(1-\lvert\phi^{(i)}_{aa'}\rvert)}\right)^{\mathrm{sgn}(\phi^{(i)}_{aa'})}$ $+\left(\frac{N_a I^{\mathbb{Z}_2}_{aa'}}{2}\ \left(\mathrm{sgn}(I_{aa'}) + \mathrm{sgn}(\phi^{(2)}_{aa'}) - 2\phi^{(2)}_{aa'}\right) + \lvert\tilde{I}^{\Omega\mathcal{R}}_a\rvert + \lvert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2}_a\rvert\right)\ \ln(2)$

From gauge thresholds to Kähler metrics II: Anti/Sym

$$\boxed{\vec{\phi}_{aa'} = (\vec{0}), (0_i, (\phi)_j, (-\phi)_k)}$$: has $\mathcal{N} = 2$ or 1 SUSY on $T^6/\mathbb{Z}_{2N}$

- **gauge kinetic function** on **tilted tori**: $\tilde{v} \equiv \frac{v}{1-b}$ $\quad (b \in \{0, \frac{1}{2}\})$

$$\delta_{a'} \mathrm{f}_a^{1-\mathrm{loop}} = \frac{-1}{4\pi^2} \begin{cases} b^{\mathcal{A}}_{aa'} \ln \eta(iv) + b^{\mathcal{M}}_{aa'} \ln \eta(i\tilde{v}) & (\sigma, \tau) = (0,0) \\ b^{\mathcal{A}}_{aa'} \ln \frac{\vartheta_1(\frac{\tau - iv\sigma}{2}, iv)}{\eta(iv)} + b^{\mathcal{M}}_{aa'} \ln \frac{\vartheta_1(\frac{\tau - i\tilde{v}\sigma}{2}, i\tilde{v})}{\eta(i\tilde{v})} & \neq (0,0) \end{cases}$$

with e.g. $\ln \eta(i\tilde{v}) = \ln \eta(iv) + 4b \ln \frac{\eta(2iv)}{\vartheta_4(0,2iv)}$

$\rightsquigarrow$ extra contribution from Möbius strip besides $(b^{\mathcal{A}}_{aa'} + b^{\mathcal{M}}_{aa'}) \ln \eta(iv)$

- **Kähler metrics**: $\tilde{V} \equiv 2(1-b)V \rightsquigarrow \ln(2\tilde{v}\tilde{V}) = \ln(vV) + 2\ln 2$ gives same shape as for bifundamentals:

$$\boxed{K^{(i)}_{\mathbf{Anti}_a/\mathbf{Sym}_a} = 2\pi e^{\phi_4} \sqrt{\frac{V^{(i)}_{aa}}{v_j v_k}}}$$

Anti/Sym cont'd

other $\vec{\phi}_{aa'}$ is always $\mathcal{N} = 1$

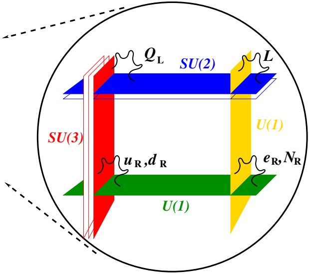
- $\ln(\Gamma(|\phi|)$ terms identical to bifundamental case $\rightsquigarrow$ same **Kähler metrics** for **(anti)symmetrics**:

$$K_{\textbf{Anti/Sym}_a} = \frac{2\pi\, e^{\phi_4}}{\sqrt{\prod_{i=1}^3 v_i^{1+\mathrm{sgn}(I_{aa'})\phi^{(i)}_{aa'}}}} \sqrt{\prod_{i=1}^3 \left(\frac{\Gamma(|\phi^{(i)}_{aa'}|)}{\Gamma(1-|\phi^{(i)}_{aa'}|)} \right)^{\frac{\mathrm{sgn}(\phi^{(i)}_{aa'})}{\mathrm{sgn}(I_{aa'})}}}$$

- same shape for field redefinitions
- different **constant terms** from intersections with O6-planes $\rightsquigarrow$ interpretation?

- Interpretation of $\frac{N_a I^{\mathbb{Z}_2}_{aa'}}{2} \left(\mathrm{sgn}(I_{aa'}) + \mathrm{sgn}(\phi^{(2)}_{aa'}) - 2\phi^{(2)}_{aa'} \right) \ln(2)$
  - additional term in field redefinition of *exceptional* moduli???
  - corrections to Kähler metrics *only at* $\mathbb{Z}_2$ *fixed points*?
  - extra contributions to gauge kinetic functions *from* $\mathbb{Z}_2$ *fixed points*?

Examples for (anti)symmetrics



The $T^6/\mathbb{Z}_6'$ example Gmeiner, G.H. '07-09

- $\boxed{\vec{\phi}_{cc'} = (\vec{0}),\text{ i.e. } \uparrow\uparrow \Omega\mathcal{R}}$ for brane (θc)
- $\boxed{\pi(-\frac{1}{6}, 0, \frac{1}{6})}$ for brane $(\theta^2 b)$
- $\boxed{\pi(0, \frac{1}{2}, -\frac{1}{2})}$ for brane (θa)
- $\boxed{\pi(\frac{1}{2}, 0, -\frac{1}{2}),\text{ i.e. } \uparrow\uparrow \Omega\mathcal{R}\mathbb{Z}_2^{(2)}}$ for brane (θd)
- ... all possible angles occur

Special cases of $\Omega\mathcal{R}$ invariant D6-branes: $Sp(2N)$ groups
[$\Omega\mathcal{R}$-invariance: consider exceptional 3-cycles]

- hidden sector $Sp(6) \uparrow\uparrow a$ or $Sp(2) \uparrow\uparrow d$
- $Sp(2)_c \rightarrow U(1)_c$ broken by displacement on $T^2_{(2)}$: need to consider continuous (σ_c^2, τ_c^2)!

Conclusions & Outlook

- completed list of **gauge kinetic functions & Kähler metrics @ 1-string loop** G.H. to appear soon '11
  - for arbitrary (untilted & **tilted**) background lattices
  - with arbitrary (discrete & **continuous**) Wilson lines & displacement moduli
  - for all possible (non)-**vanishing** SUSY angles
  - for physical U(1)s *($\rightsquigarrow$ see article)*
  - including all constants properly

Outlook

- Selection rules & $Y_{ijk} \sim e^{-\text{Area}_{ijk}}$ dependence of Yukawa couplings for $T^6/\mathbb{Z}_6'$ model G.H., Vanhoof **to appear soon** '11
- complete worldsheet instanton sum for *holomorphic* part of Yukawas with $\mathbb{Z}_2$ symmetry still missing
- study (numerical) parameter dependence of couplings in view of phenomenology at M_{weak}
- new models on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6^{(\prime)}$ Förste, G.H. *et al.* in progress