b-quark mass and B decay constant from $\mathrm{N_f} = 2$ lattice QCD simulations

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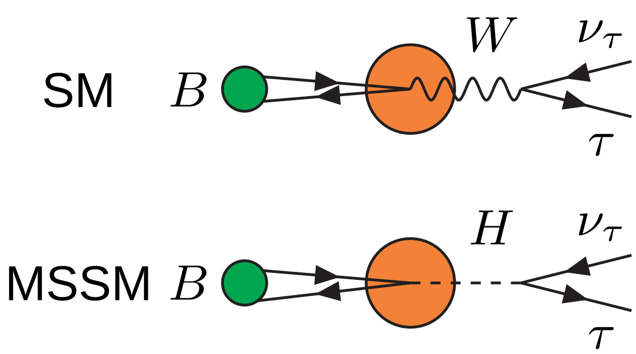
EPS-HEP '11, Grenoble, 21 - 27 July 2011

- Hints of New Physics in the b sector?
- Heavy flavour physics on the lattice
- B spectrum and b quark mass
- f_B decay constant

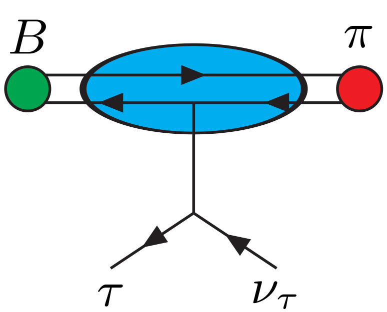
Hints of New Physics in the b sector?

V_{ub} puzzle and ($Br(B \to \tau\nu)$, $\sin 2\beta$) discrepancy



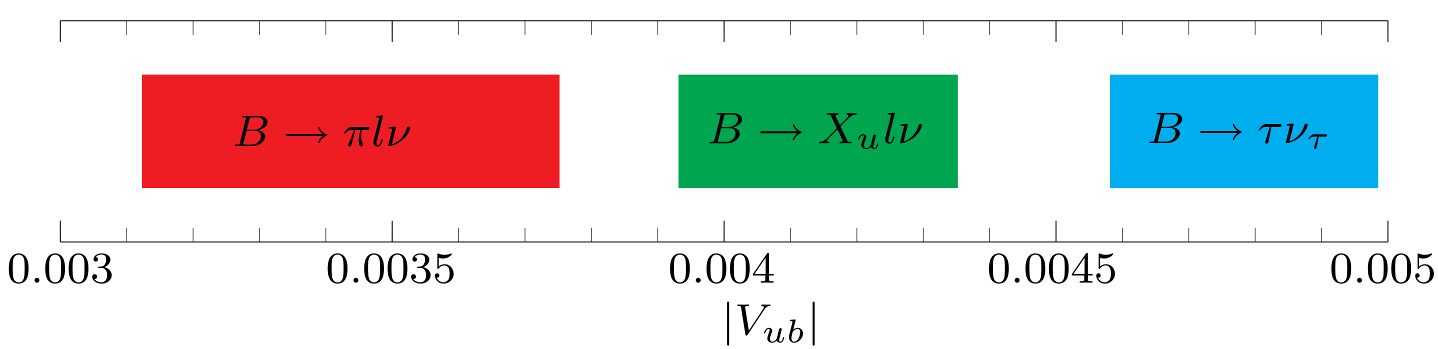
$$\Gamma(B^- \to \tau\nu_\tau) = \frac{G_F^2 |V_{ub}|^2}{8\pi} f_B^2 m_\tau^2 m_B \times \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \left|1 + \frac{m_B^2}{m_b m_\tau} C_{\rm NP}^\tau\right|^2$$

$\delta(f_B^2) \sim 30$ %



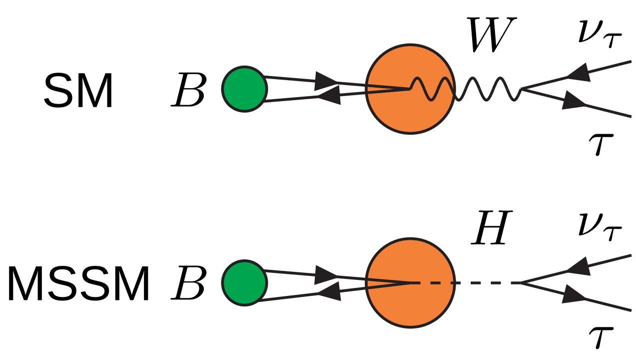
$$\langle \pi(p')|\bar{u}\gamma_\mu b|B(p)\rangle = \left(p_\mu + p'_\mu - q_\mu \frac{m_B^2 - m_\pi^2}{q^2}\right) f_+(q^2) + q_\mu \frac{m_B^2 - m_\pi^2}{q^2} f_0(q^2)$$

$$\Gamma(B \to \pi l\nu) \propto |V_{ub}|^2 \int_0^{q^2_{\rm max}} f_+^2(q^2) dq^2$$



Hints of New Physics in the b sector?

V_{ub} puzzle and $(Br(B \to \tau\nu), \sin 2\beta)$ discrepancy



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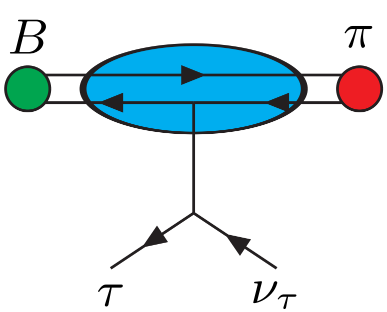

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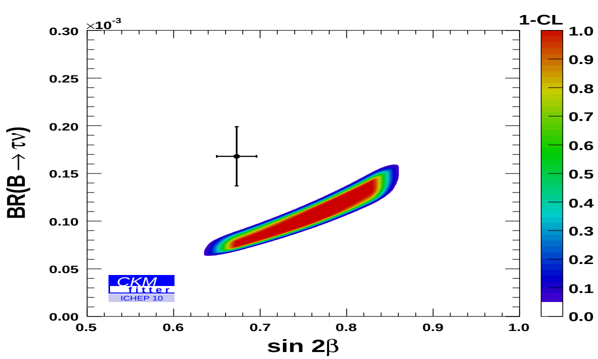
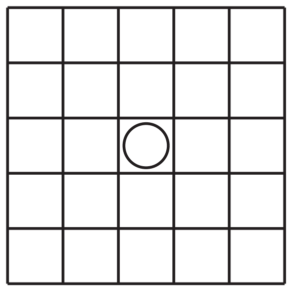


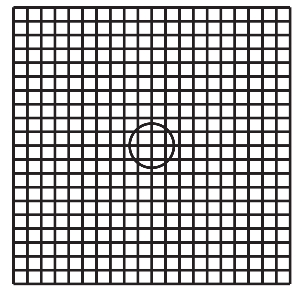
From global fits $\sin 2\beta$ and $Br(B \to \tau\nu_\tau)$ are areas of discrepancy with the SM ($\sim 3\ \sigma$ and $\sim 2.5\ \sigma$)

Heavy flavour physics on the lattice

Systematics coming from large discretisation effects ($\Lambda_{\mathrm{Compt}} \sim 1/m_Q$).



Cut-off Effects



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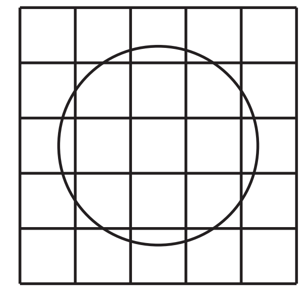
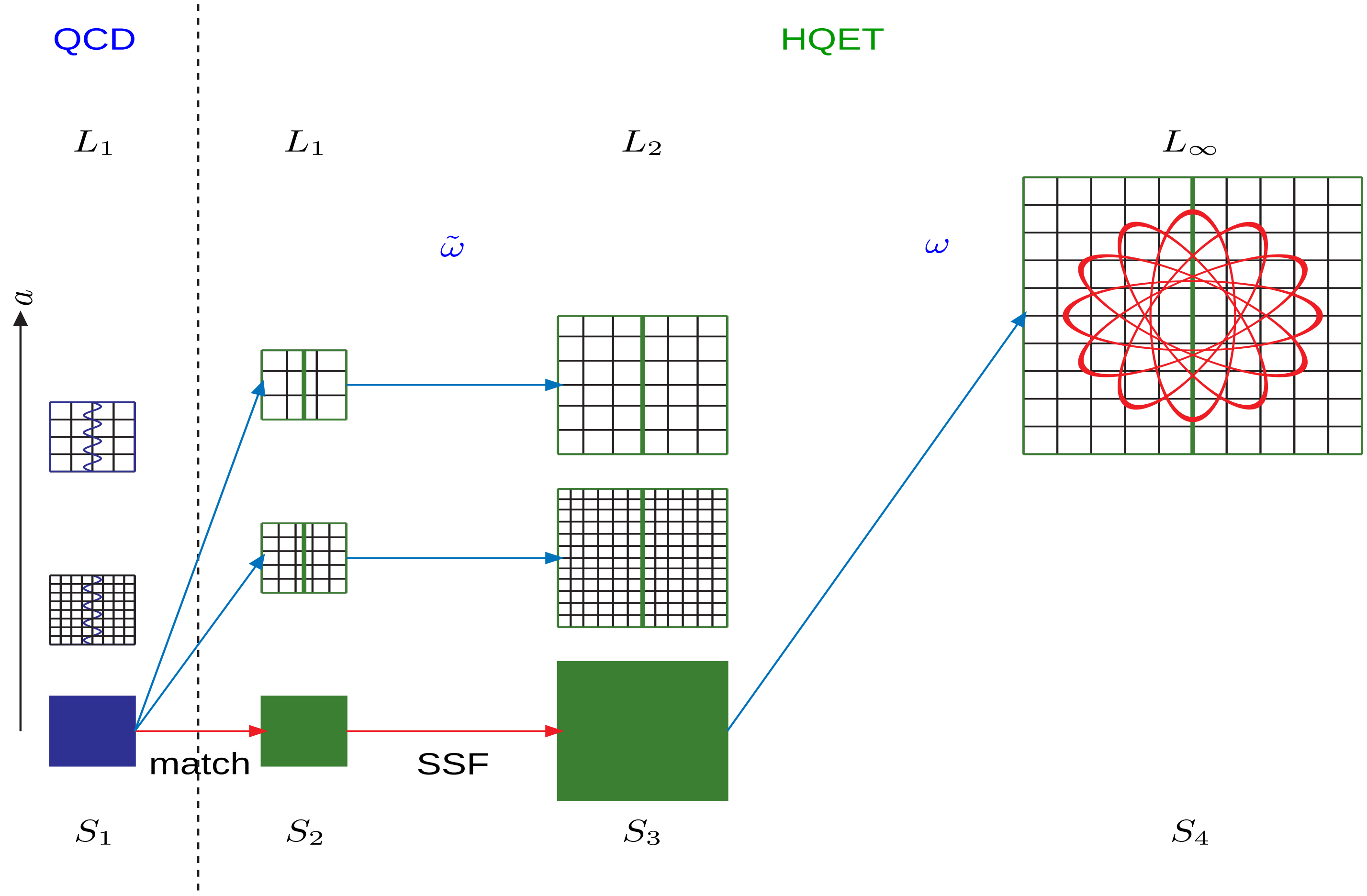


cut-off effects

Several strategies are proposed in the literature to deal with those cut-off effects:

- Use NRQCD to describe the heavy quark [P. Lepage and B. Thacker, '91]; though, no continuum limit when the theory is regularised on the lattice
- Define an action with counterterms that are tuned to get $\mathcal{O}(a)$, $\mathcal{O}(am_Q)$ and $\mathcal{O}(\alpha_s a m_Q)$ improvements [A El Khadra *et al*, '96]
- Computation within Heavy Quark Effective Theory, the effective couplings are determined non perturbatively by imposing matching conditions between QCD and HQET [J. Heitger and R. Sommer, '03]

Sketch of the strategy



Ultraviolet divergences of HQET are absorbed in the ω coefficients, determined from a Schrödinger Functional set up (Dirichlet boundary conditions in time).

Hadronic matrix elements are extracted with a particular care to excited states (solution of the Generalised Eigenvalue problem on a matrix of correlators).

Small volume part of the strategy

$$\mathcal{L}^{\mathrm{HQET},1/\mathrm{m}} = \mathcal{L}^{\mathrm{stat}} + m_{\mathrm{bare}}\mathcal{O}^{\mathrm{c.t.}} - \omega_{\mathrm{kin}}\mathcal{O}^{\mathrm{kin}} - \omega_{\mathrm{mag}}\mathcal{O}^{\mathrm{mag}}$$
$$A_0^{\mathrm{HQET},1/\mathrm{m}} = Z_A^{\mathrm{HQET}}[A_0^{\mathrm{stat}} + c_A^{(1)}A_0^{(1)} + c_A^{(2)}A_0^{(2)}]$$

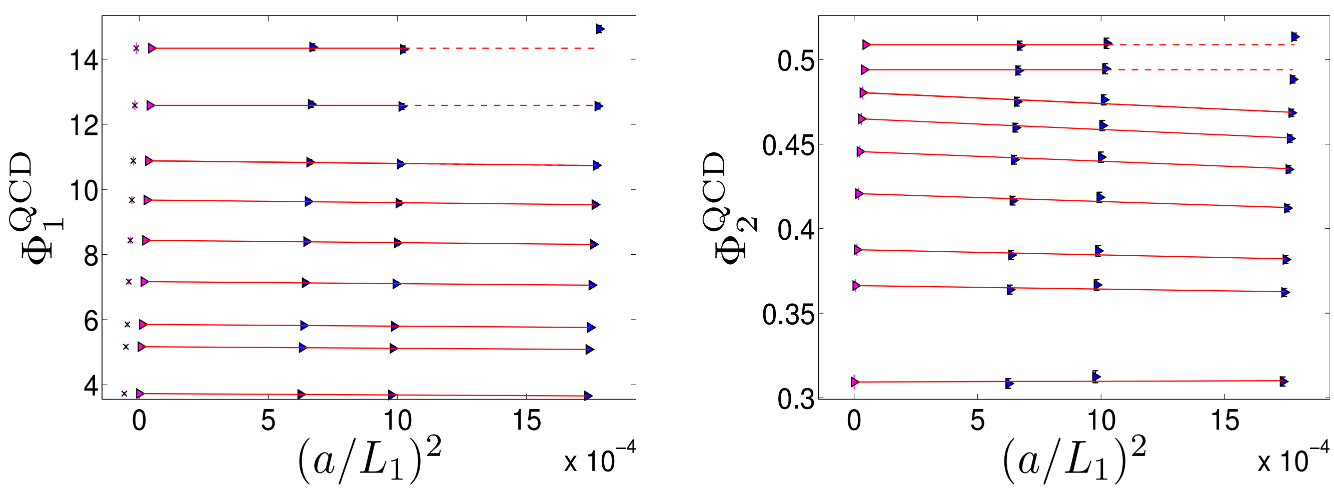
$$\mathcal{O}^{\mathrm{kin}} = \bar{\psi}_h \mathbf{D}^2\psi_h \qquad \mathcal{O}^{\mathrm{mag}} = \bar{\psi}_h \sigma\cdot\mathbf{B}\psi_h$$
$$A_0^{(0)} = \bar{\psi}_l\gamma_0\gamma^5\psi_h \qquad A_0^{(1)} = \bar{\psi}_l\tfrac{1}{2}\gamma^5\gamma_i(\nabla_i - \overleftarrow{\nabla}_i)\psi_h \qquad A_0^{(2)} = -\tfrac{1}{2}(\partial_i + \partial_i^*)A_i^{\mathrm{stat}}$$

$$\underbrace{\Phi_i^{\mathrm{QCD}}}_{\text{cont lim}}(L_1) = \underbrace{\varphi_{ij}^{\mathrm{HQET}}(L_1)\tilde{\omega}_j}_{\text{finite } a}$$

$$\Phi_{AA}(t) \equiv Z_A^2 \sum_{\vec{x}} \langle (\bar{\psi}_b\gamma_0\gamma^5\psi_l)(\vec{x},t)(\bar{\psi}_l\gamma_0\gamma^5\psi_b)(0)\rangle$$

$$\Phi_{AA}(t) = e^{-m_{\mathrm{bare}}t}(Z_A^{\mathrm{HQET}})^2\left[\varphi_{AA}^{\mathrm{stat}}(t) + \omega_{\mathrm{kin}}\varphi_{AA}^{\mathrm{kin}}(t) + \omega_{\mathrm{mag}}\varphi_{AA}^{\mathrm{mag}}(t) + C_A^{(1)}[\varphi_{A\delta A}(t) + \varphi_{\delta AA}(t)]\right]$$

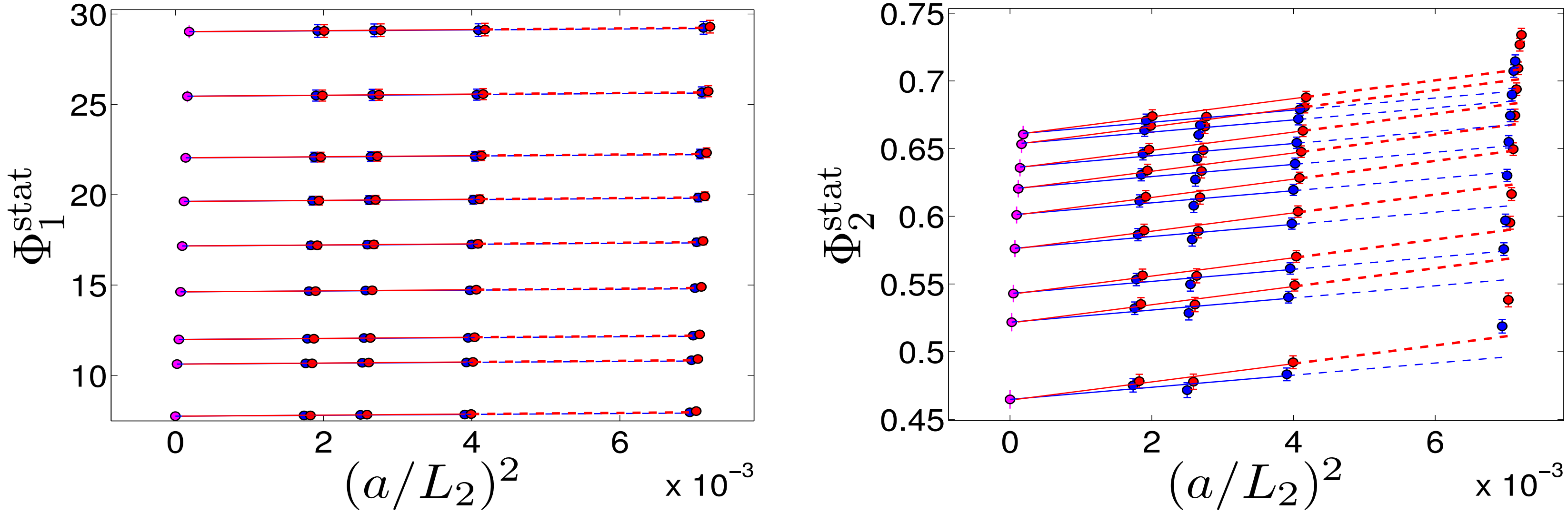
Extrapolation to the continuum limit of $\Phi_1^{\mathrm{QCD}} \equiv$ "m_B" and $\Phi_2^{\mathrm{QCD}} \equiv \ln($"$f_B\sqrt{m_B}$"$)$



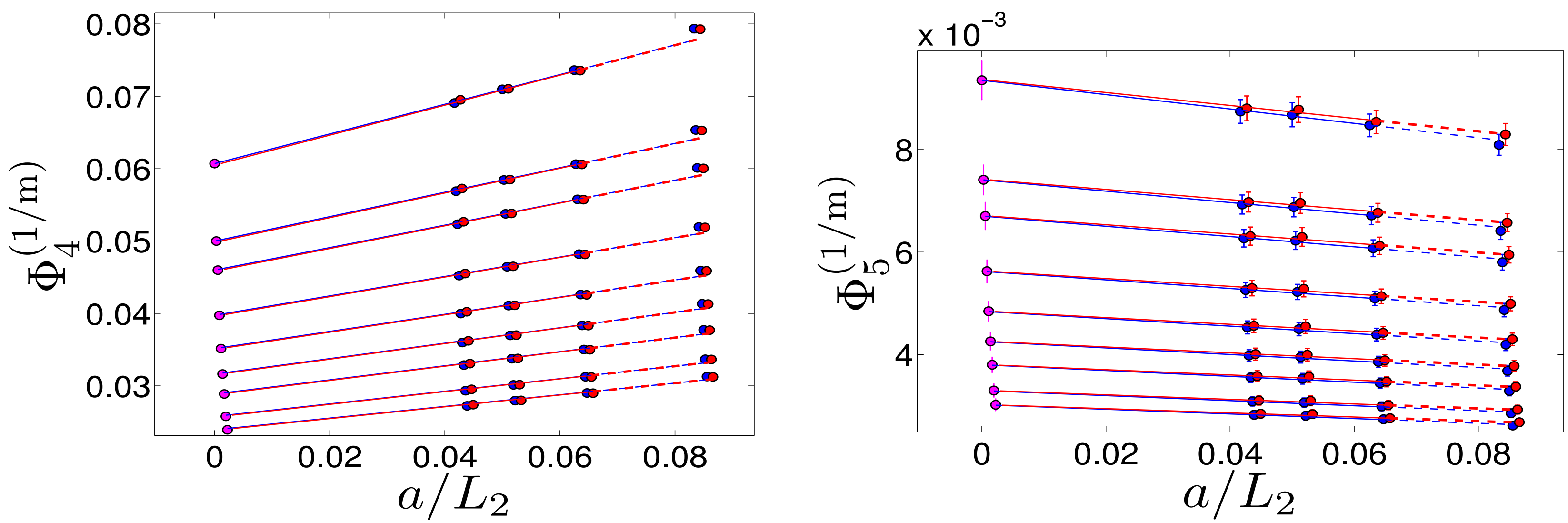
Evolution of the observables through Step Scaling functions from L_1 to $L_{\inf} = s^k L_1$ where long-distance physics dominates; in practice $s = 2$ and $k = 1$

$$\Phi_i(L_2) = \lim_{a \to 0} \Sigma_{ij}(a) \Phi_j(L_1)$$

Extrapolation to the continuum limit of $\Phi_1^{\rm stat}(L_2)$ and $\Phi_2^{\rm stat}(L_2)$



Extrapolation to the continuum limit of observables used to determine $\omega_{\rm kin}$ and $\omega_{\rm mag}$



Suppression of excited states contribution to hadronic quantities

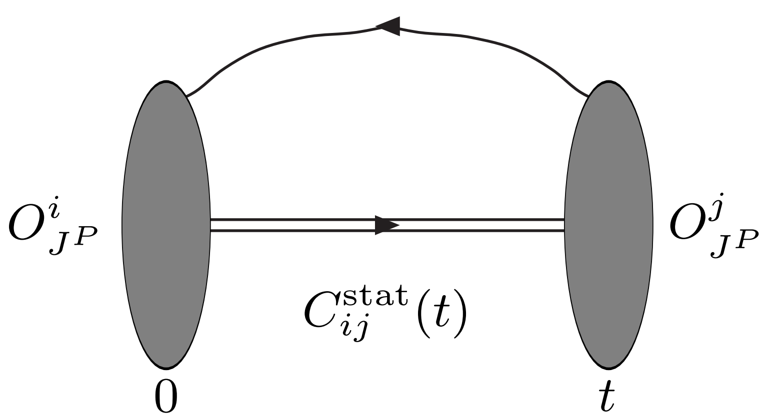
Contribution of excited states to correlators efficiently suppressed by solving a Generalised Eigenvalue Problem [C. Michael, '85; M. Lüscher and U. Wolff, '90] [ALPHA, B. B. *et al*, '09]

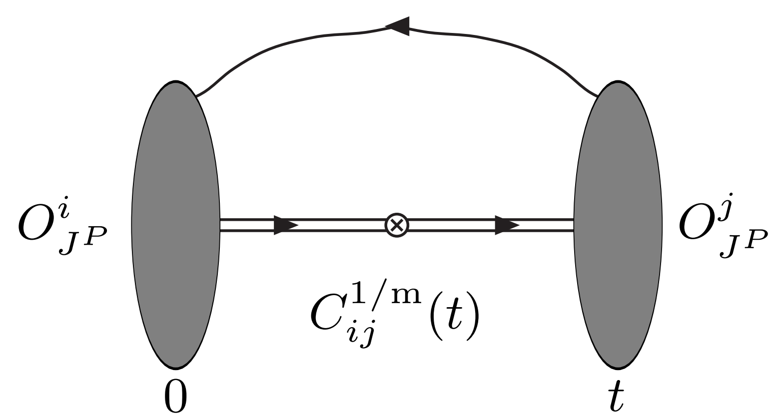
– Compute an $N \times N$ matrix of correlators $C^{ij}_{PP}(t) = \sum_{\vec{x},\vec{y}} \langle \Omega | \mathcal{T}[O^i_{J^P}(\vec{x},t) O^j_{J^P}(\vec{y},0)] | \Omega \rangle$ with $O^i_{J^P}(\vec{x},t) = \sum_{\vec{z}} \bar{q}(\vec{x},t) [\Gamma \times \Phi(|\vec{x}-\vec{z}|)]^i_{J^P} q(\vec{z},t)$

– Solve the generalised eigenvalue problem $C^{ij}(t)\, v^j_n(t,t_0) = \lambda_n(t,t_0)\, C^{ij}(t_0)\, v^j_n(t,t_0)$

$$aE^{\rm eff}_n(t,t_0) = -\ln\left(\frac{\lambda_n(t+a,t_0)}{\lambda_n(t,t_0)}\right)$$

$$Q^{\rm eff}_n(t,t_0) = \frac{O^i(t) v^i_n(t,t_0)}{\sqrt{v^i_n(t,t_0) C^{ij}(t) v^j_n(t,t_0)}} \left(\frac{\lambda_n(t_0+a,t_0)}{\lambda_n(t_0+2a,t_0)}\right)^{t/2a}$$

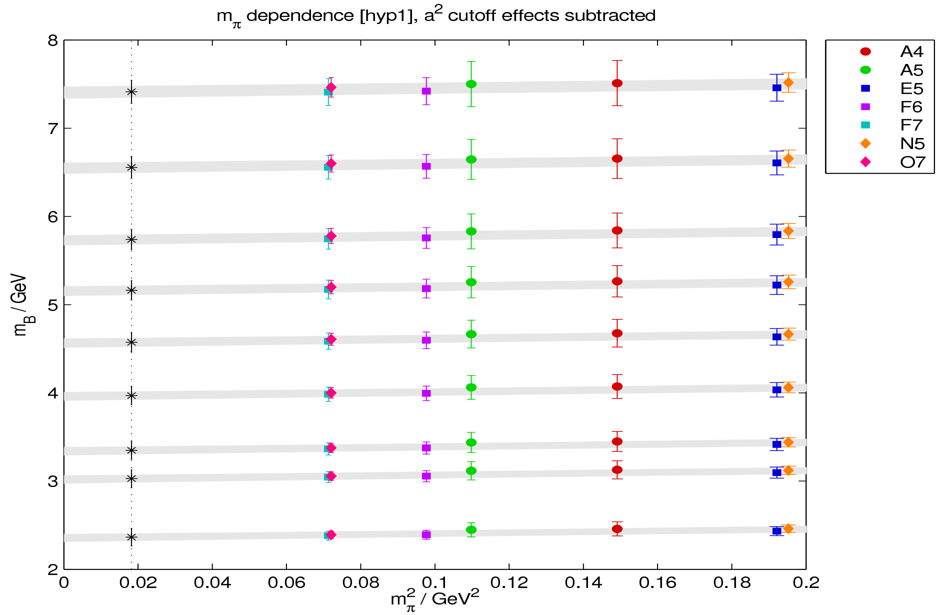


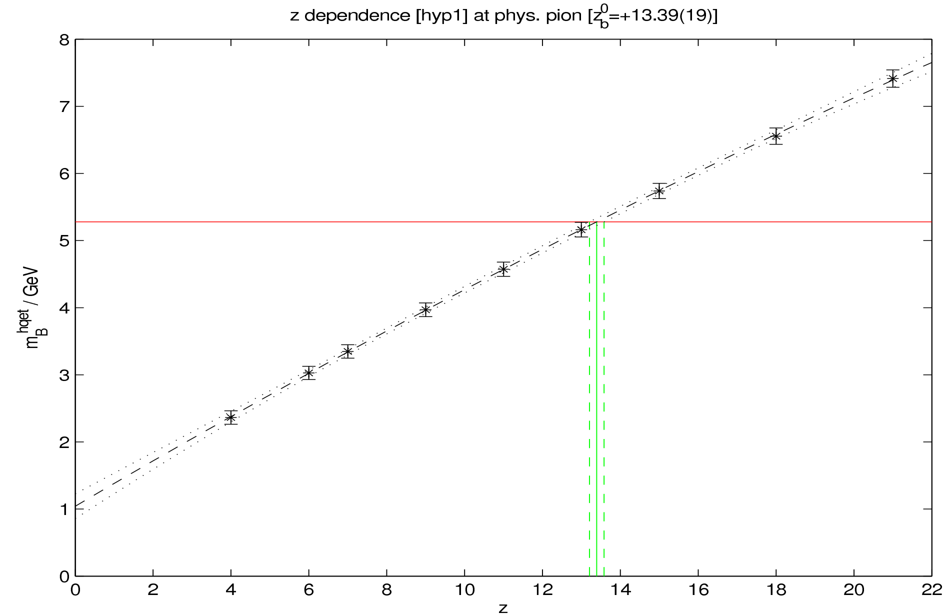


Estimate the $1/m$ corrections in HQET to static energies and matrix elements using GEVP is not an issue; it is enough to determine $\lambda^{\rm stat}_n$ and $v^{\rm stat}_n$

B spectrum and b quark mass

Hadronic matrix elements extracted at 3 lattice spacings (0.05 fm, 0.065 fm, 075 fm)
Pion mass in the range [250 - 400] MeV; $Lm_\pi > 4$





m_B data well described by a linear fit in m_π^2; the NLO term in m_π^3 of HMχPT cannot be observed; quadratic fit in $z \equiv L_1 M_b$ fully satisfactory.

$$\bar{m}_b(m_b) = \underbrace{4.234(76)_{\text{stat}}(56)_{\text{renorm}}(14)_{\text{scale}}\ \text{GeV}}_{\text{Preliminary}}$$

$\bar{m}_b(m_b)^{\mathrm{N_f=2\ \ TM}} = 4.29(14)$ GeV [P Dimopoulos *et al*, '11]

$\bar{m}_b(m_b)^{\text{sum rules}} = 4.163(16)$ GeV [K. Chetyrkin *et al*, '09]

Interesting to compare $\bar{m}_b(m_b)$ measured with the B spectrum with its counterpart estimated from the B_s spectrum in a partially quenched set up (need to know κ_s).

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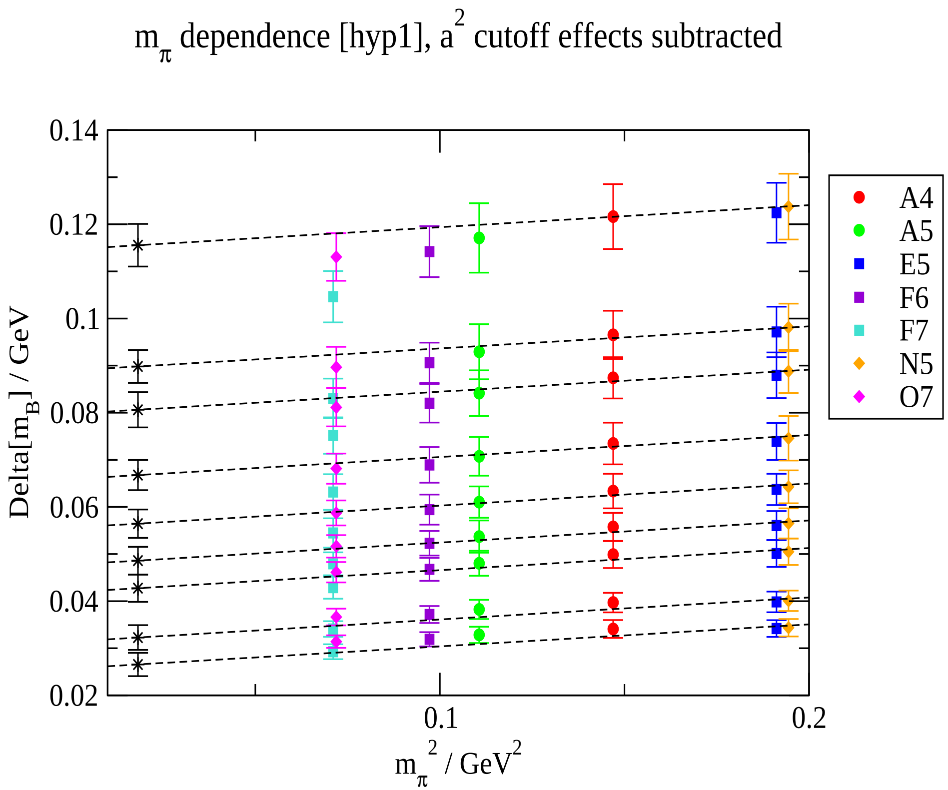

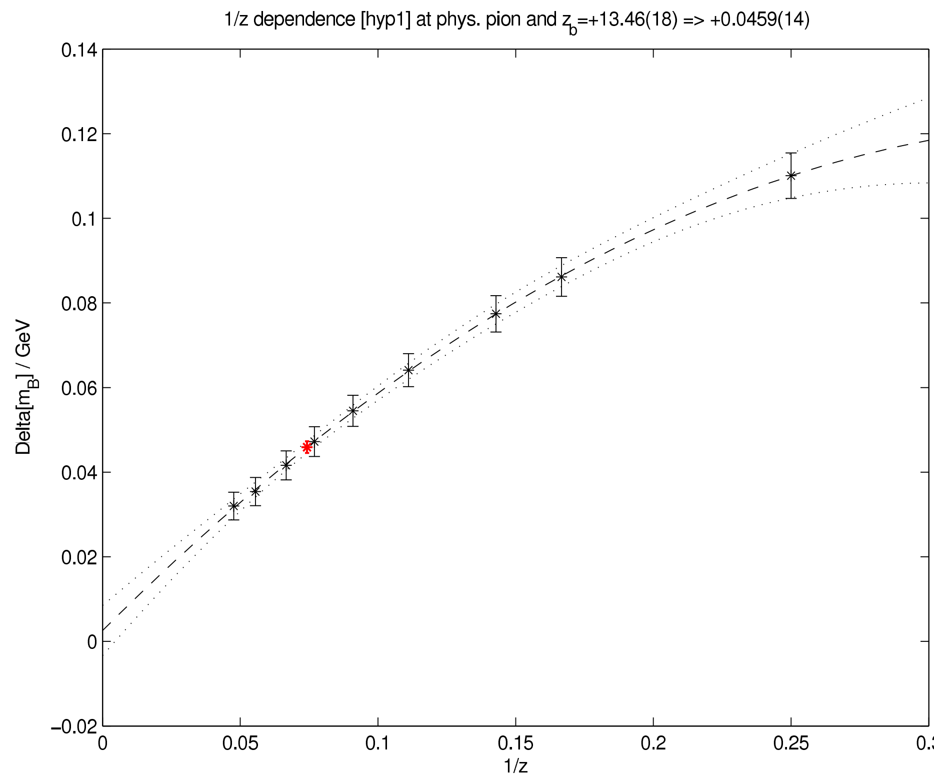

Hyperfine mass splitting data well described by a linear fit in m_π^2

Quadratic fit in $1/z$ excellent

$m_{B^*} - m_B = \underbrace{46.4(1.3)_{\text{stat}}(1.0)_{\text{scale}}\ \text{MeV}}_{\text{Preliminary}}$ in full agreement with experiment

B decay constant

Hadronic matrix elements extracted at 3 lattice spacings (0.05 fm, 0.065 fm, 075 fm)
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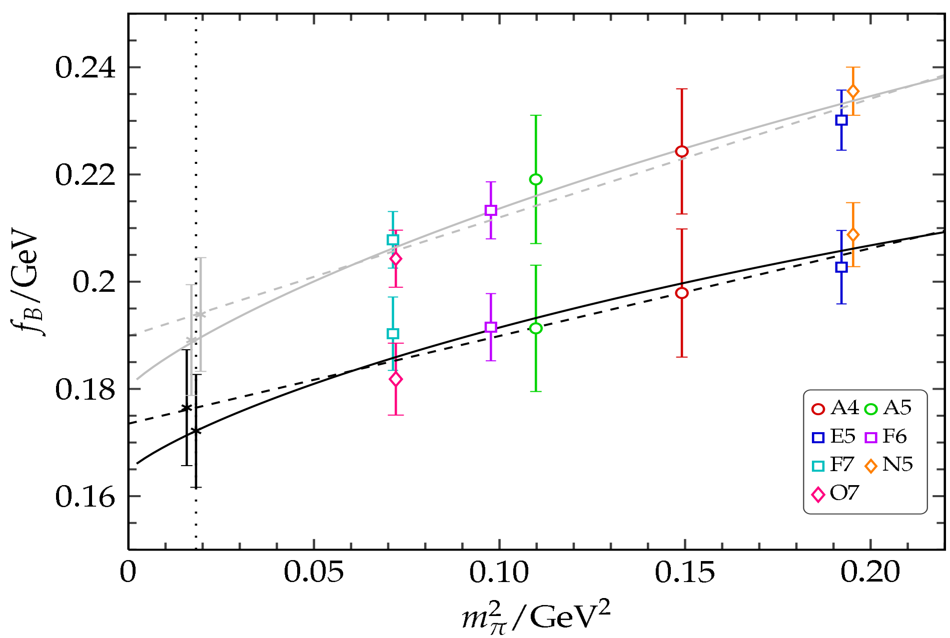
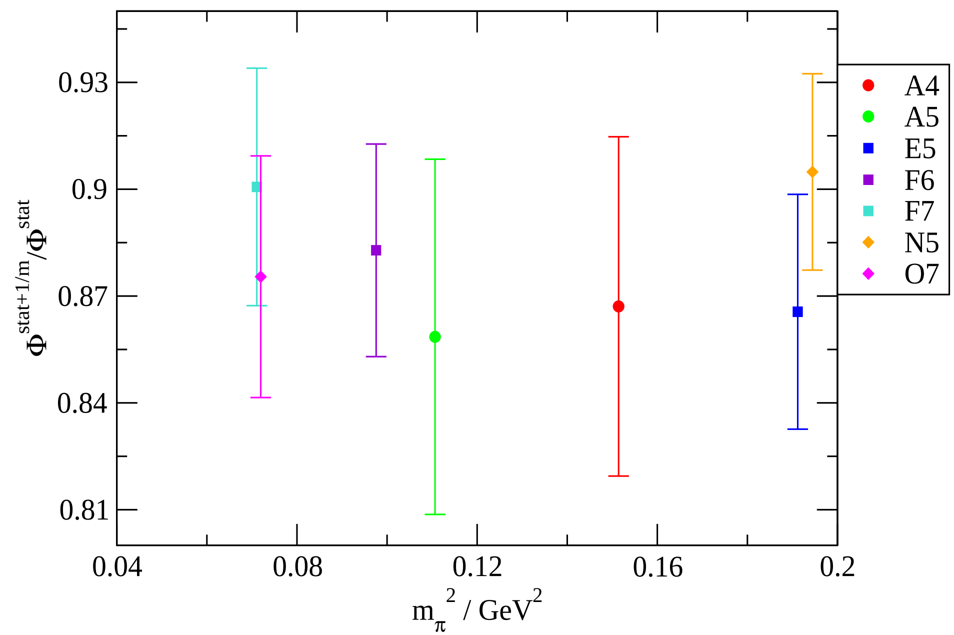
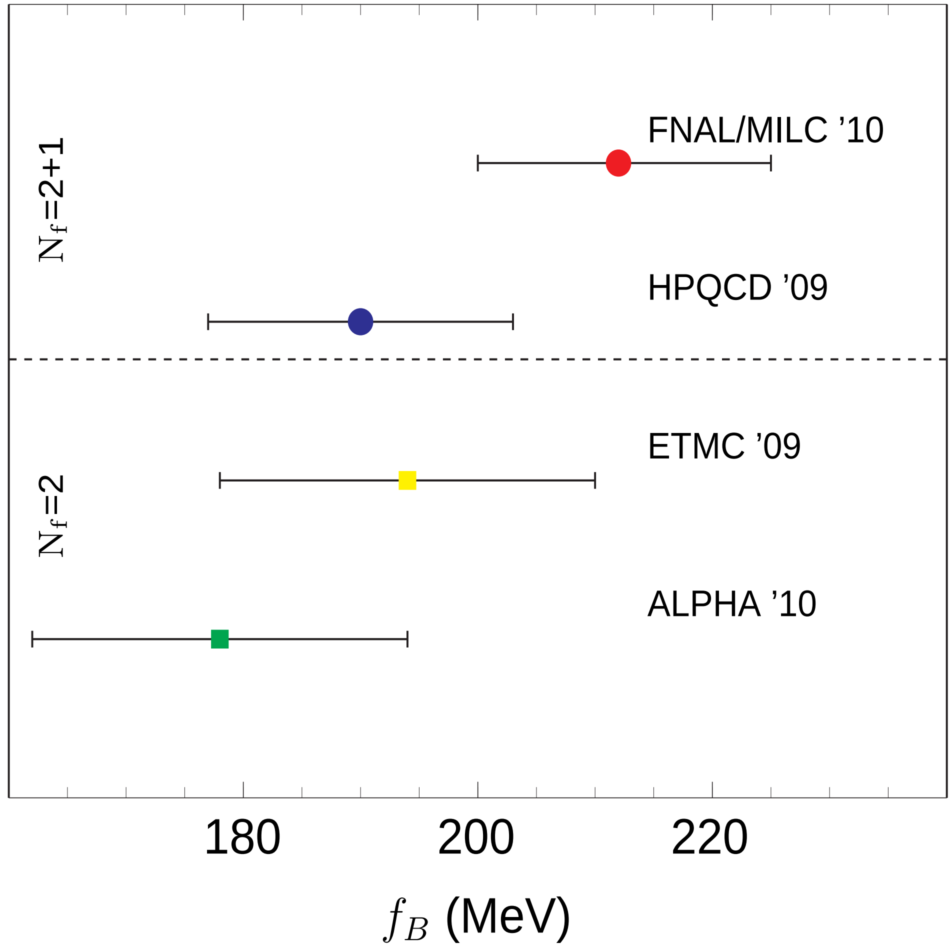




B decay constant data well described by a linear fit in m_π^2; however adding the NLO in $m_\pi^2 \ln m_\pi^2$ does not hurt

$$|f_B^{1/m}/f_B^{\mathrm{stat}}| \sim 10\% \qquad f_B = \underbrace{175(10)_{\mathrm{stat}}(5)_{\mathrm{HM}\chi\mathrm{PT}}(6)_{\mathrm{scale}}\ \mathrm{MeV}}_{\text{Preliminary}}$$



Pretty low values of $f_B \Longrightarrow$ hint of NP in $B \to \tau\nu$ not in contradiction with lattice data...

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