Hints of New Physics in the $b$ sector?
Heavy flavour physics on the lattice
$B$ spectrum and $b$ quark mass
$f_B$ decay constant
Hints of New Physics in the $b$ sector?

$V_{ub}$ puzzle and $(Br(B \to \tau \nu), \sin 2\beta)$ discrepancy

\[
\Gamma(B^- \to \tau \nu) = \frac{G_F^2 |V_{ub}|^2}{8\pi} f_B^2 m^2_{\tau} m_B \\
\times \left(1 - \frac{m^2_{\tau}}{m^2_B}\right)^2 \left|1 + \frac{m^2_B}{m_b m_{\tau}} C_{NP}\right|^2
\]

$\delta(f^2_B) \sim 30 \%$

\[
\langle \pi(p') |\bar{u}\gamma_{\mu}b |B(p)\rangle = \left(p_{\mu} + p'_{\mu} - q_{\mu} \frac{m^2_B - m^2_{\pi}}{q^2}\right) f_+(q^2) \\
+ q_{\mu} \frac{m^2_B - m^2_{\pi}}{q^2} f_0(q^2)
\]

\[
\Gamma(B \to \pi \nu) \propto |V_{ub}|^2 \int_{0}^{q^2_{\text{max}}} f^2_+(q^2) dq^2
\]
Hints of New Physics in the $b$ sector?

$V_{ub}$ puzzle and $(Br(B \to \tau \nu)), \sin 2\beta$ discrepancy

$$\Gamma(B^- \to \tau \nu) = \frac{G_F^2 |V_{ub}|^2}{8\pi} \frac{f_B^2 m_B^2 m_B}{B_m^2 m_B} \left(1 - \frac{m_B^2}{m_B^2}\right)^2 \left|1 + \frac{m_B^2}{m_B m_\tau} C_{NP}\right|^2$$

$$\delta(f_B^2) \sim 30\%$$

$$\langle \pi(p') | \bar{u} \gamma_{\mu} b | B(p) \rangle = \left(p_{\mu} + p'_{\mu} - q_{\mu} \frac{m_B^2 - m_\pi^2}{q^2}\right) f_+(q^2) + q_{\mu} \frac{m_B^2 - m_\pi^2}{q^2} f_0(q^2)$$

$$\Gamma(B \to \pi l \nu) \propto |V_{ub}|^2 \int_0^{q_{\text{max}}^2} f_+(q^2) dq^2$$

From global fits $\sin 2\beta$ and $Br(B \to \tau \nu)$ are areas of discrepancy with the SM ($\sim 3\sigma$ and $\sim 2.5\sigma$)
Heavy flavour physics on the lattice

Systematics coming from large discretisation effects ($\Lambda_{\text{Compt}} \sim 1/m_Q$).

Several strategies are proposed in the literature to deal with those cut-off effects:

- Use NRQCD to describe the heavy quark [P. Lepage and B. Thacker, '91]; though, no continuum limit when the theory is regularised on the lattice

- Define an action with counterterms that are tuned to get $O(a)$, $O(am_Q)$ and $O(\alpha_s am_Q)$ improvements [A El Khadra et al, '96]

- Computation within Heavy Quark Effective Theory, the effective couplings are determined nonperturbatively by imposing matching conditions between QCD and HQET [J. Heitger and R. Sommer, '03]
Ultraviolet divergences of HQET are absorbed in the $\omega$ coefficients, determined from a Schrödinger Functional set up (Dirichlet boundary conditions in time).

Hadronic matrix elements are extracted with a particular care to excited states (solution of the Generalised Eigenvalue problem on a matrix of correlators).
Small volume part of the strategy

\[ \mathcal{L}^{\text{HQET, } 1/m} = \mathcal{L}^{\text{stat}} + \frac{m_{\text{bare}}}{m} \mathcal{O}^{c.t.} - \omega_{\text{kin}} \mathcal{O}^{\text{kin}} - \omega_{\text{mag}} \mathcal{O}^{\text{mag}} \]

\[ A^{\text{HQET, } 1/m}_0 = Z_A^{\text{HQET}} [A^{\text{stat}}_0 + c_A^{(1)} A^{(1)}_0 + c_A^{(2)} A^{(2)}_0] \]

\[ \mathcal{O}^{\text{kin}} = \bar{\psi}_h D^2 \psi_h \quad \mathcal{O}^{\text{mag}} = \bar{\psi}_h \sigma \cdot B \psi_h \]

\[ A^{(0)}_0 = \bar{\psi}_l \gamma_0 \gamma^5 \psi_h \quad A^{(1)}_0 = \bar{\psi}_l \frac{1}{2} \gamma^5 \gamma_i (\nabla_i - \bar{\nabla}_i) \psi_h \quad A^{(2)}_0 = -\frac{1}{2} (\partial_i + \partial^*_i) A^{\text{stat}}_i \]

\[ \Phi_i^{\text{QCD}}(L_1) = \varphi_{ij}^{\text{HQET}}(L_1) \tilde{\omega}_j \]

\[ \Phi_{AA}(t) \equiv Z_A^2 \sum_{\bar{x}} \langle (\bar{\psi}_b \gamma_0 \gamma^5 \psi_l)(\bar{x}, t)(\bar{\psi}_l \gamma_0 \gamma^5 \psi_b)(0) \rangle \]

\[ \Phi_{AA}(t) = e^{-m_{\text{bare}} t} (Z_A^{\text{HQET}})^2 \left[ \varphi_{AA}^{\text{stat}}(t) + \omega_{\text{kin}} \varphi_{AA}^{\text{kin}}(t) + \omega_{\text{mag}} \varphi_{AA}^{\text{mag}}(t) + C_A^{(1)} [\varphi_{\delta A}(t) + \varphi_{\delta AA}(t)] \right] \]

Extrapolation to the continuum limit of \( \Phi_1^{\text{QCD}} \equiv "m_B" \) and \( \Phi_2^{\text{QCD}} \equiv \ln("f_B \sqrt{m_B}"") \)
Evolution of the observables through Step Scaling functions from $L_1$ to $L_{\text{inf}} = s^k L_1$ where long-distance physics dominates; in practice $s = 2$ and $k = 1$

\[ \Phi_i(L_2) = \lim_{a \to 0} \sum_{ij}(a)\Phi_j(L_1) \]

Extrapolation to the continuum limit of $\Phi_{\text{stat}}^1(L_2)$ and $\Phi_{\text{stat}}^2(L_2)$

Extrapolation to the continuum limit of observables used to determine $\omega_{\text{kin}}$ and $\omega_{\text{mag}}$
Suppression of excited states contribution to hadronic quantities

Contribution of excited states to correlators efficiently suppressed by solving a Generalised Eigenvalue Problem [C. Michael, '85; M. Lüscher and U. Wolff, '90] [ALPHA, B. B. et al, '09]

Compute an $N \times N$ matrix of correlators $C_{ij}^{PP}(t) = \sum_{\vec{x}, \vec{y}} \langle \Omega | T [O_{JP}^i(\vec{x}, t) O_{JP}^j(\vec{y}, 0)] | \Omega \rangle$ with $O_{JP}^i(\vec{x}, t) = \sum_{\vec{z}} \bar{q}(\vec{x}, t)[\Gamma \times \Phi(|\vec{x} - \vec{z}|)]^i_{JP} q(\vec{z}, t)$

Solve the generalised eigenvalue problem $C_{ij}^{ij}(t) v_n^j(t, t_0) = \lambda_n(t, t_0) C_{ij}^{ij}(t_0) v_n^j(t, t_0)$

$$aE_n^{\text{eff}}(t, t_0) = - \ln \left( \frac{\lambda_n(t + a, t_0)}{\lambda_n(t, t_0)} \right)$$

$$Q_n^{\text{eff}}(t, t_0) = \frac{C_{ij}^{ij}(t) v_n^j(t, t_0)}{\sqrt{v_n^i(t, t_0) C_{ij}^{ij}(t) v_n^j(t, t_0)}} \left( \frac{\lambda_n(t_0 + a, t_0)}{\lambda_n(t_0 + 2a, t_0)} \right)^{t/2a}$$

Estimate the $1/m$ corrections in HQET to static energies and matrix elements using GEVP is not an issue; it is enough to determine $\lambda_n^{\text{stat}}$ and $v_n^{\text{stat}}$
**B spectrum and b quark mass**

Hadronically matrix elements extracted at 3 lattice spacings (0.05 fm, 0.065 fm, 075 fm)

Pion mass in the range [250 - 400] MeV; $Lm_\pi > 4$

$m_B$ data well described by a linear fit in $m_\pi^2$; the NLO term in $m_\pi^3$ of HM$\chi$PT cannot be observed; quadratic fit in $z \equiv L_1 M_B$ fully satisfactory.

$$\tilde{m}_b(m_b) = 4.234(76)_{\text{stat}}(56)_{\text{renorm}}(14)_{\text{scale}} \text{ GeV}$$

Preliminary

$$\tilde{m}_b(m_b)^{N_f=2 \ TM} = 4.29(14) \text{ GeV} \ [P \ Dimopoulos \ et \ al, \ '11]$$

$$\tilde{m}_b(m_b)^{\text{sum rules}} = 4.163(16) \text{ GeV} \ [K. \ Chetyrkin \ et \ al, \ '09]$$

Interesting to compare $\tilde{m}_b(m_b)$ measured with the $B$ spectrum with its counterpart estimated from the $B_s$ spectrum in a partially quenched set up (need to know $\kappa_s$).
**B spectrum and b quark mass**

Hadron mass extracted at 3 lattice spacings (0.05 fm, 0.065 fm, 0.075 fm)

Pion mass in the range [250 - 400] MeV; \( L m_\pi > 4 \)

\[ m_\pi \text{ dependence [hyp1], } a^2 \text{ cutoff effects subtracted} \]

Hyperfine mass splitting data well described by a linear fit in \( m_\pi^2 \)

Quadratic fit in \( 1/z \) excellent

\[
m_{B^*} - m_B = 46.4^{(1.3)}_{(1.0)} \text{MeV in full agreement with experiment}
\]

Preliminary
**B decay constant**

Hadronic matrix elements extracted at 3 lattice spacings (0.05 fm, 0.065 fm, 0.075 fm)
Pion mass in the range [250 - 400] MeV; $Lm_\pi > 4$

\[ f_B = 175(10)_{\text{stat}}^{(5)}_{\text{HM}}_{\chi_{\text{PT}}}(6)_{\text{scale}} \text{ MeV} \]

B decay constant data well described by a linear fit in $m_\pi^2$; however adding the NLO in $m_\pi^2 \ln m_\pi^2$ does not hurt

\[ |f_B^{1/m}/f_B^{\text{stat}}| \sim 10\% \]

Preliminary
Pretty low values of $f_B \Rightarrow$ hint of NP in $B \rightarrow \tau\nu$ not in contradiction with lattice data...

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