CP violation and cold electroweak baryogenesis in the Standard Model

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CP violation in the Standard Model

- Resides in Yukawa couplings; requires at least three fermion families.
- For three families exactly one complex phase in the CKM matrix V.
- All CP-violating effects proportional to the *Jarlskog invariant* [1],

$$J = \left|\mathrm{Im}(V_{ij}V_{k\ell}V^*_{i\ell}V^*_{kj})\right| \approx 3\times 10^{-5}.$$

- No CP violation in case of horizontal degeneracy of quark masses.

(Cold) electroweak baryogenesis

- Current baryon asymmetry normalized to the CMB photon density:

$$n_B/n_\gamma \approx 6\times 10^{-10}.$$

- *Sakharov conditions* for baryon asymmetry generation in early Universe:
  - Baryon number violation.
  - C and CP violation.
  - Departure from thermal equilibrium.
- Problems with the "standard" electroweak baryogenesis scenario:
  - Particle physics lower bound on the Higgs mass implies a crossover electroweak phase transition ⇒ not far enough off equilibrium.
  - Perturbatively, CP-violating effects suppressed by the *Jarlskog determinant* $J\Delta/v^{12} \approx 10^{-24}$, where $v \approx 246$ GeV is the Higgs expectation value and [2]

$$\Delta = (m_u^2-m_c^2)(m_c^2-m_t^2)(m_t^2-m_u^2)(m_d^2-m_s^2)(m_s^2-m_b^2)(m_b^2-m_d^2).$$

- Cold electroweak baryogenesis scenario:
  - Satisfies the off-equilibrium condition by means of low-scale inflation [3].
  - Electroweak transition triggered by the Higgs coupling to the inflaton at the end of the inflation period, well below the electroweak scale.
  - Thanks to low temperature, infrared enhancement invalidates the naive perturbative estimate and allows for sizable CP violation effects [4, 5].

Results available in literature

- General strategy: integrate out quarks and simulate the resulting effective theory for Standard Model bosons numerically on the lattice.
- Use *derivative expansion* to identify the leading CP-violating operators.
- Smit [4] showed that there is no CP violation up to the *fourth order*; no CP violation is thus induced by the P-odd anomalous term in the action.
- Two independent calculations of CP-violating operators at *sixth order*:
  - [6] use *worldline formalism* and find CP-odd, P-odd (C-even) contributions.
  - [7] use *method of symbols* and find only CP-odd, P-even contributions; first CP-odd, P-odd contribution only appears at the next, eighth order [8].
- The two available calculations give qualitatively different results.
- Moreover, all previous calculations were restricted to zero temperature.
- Goal of our project: resolve the discrepancy and extend the results to nonzero temperature.

Some technical details

- Calculate Tr log of the Dirac operator in background gauge and Higgs fields.
- Equivalently sum one-quark-loop graphs with external gauge or Higgs legs.
- Perform an expansion in number of external gauge legs and derivatives.
- *Method of symbols*: convenient way to calculate traces of differential operators. For a (matrix) function $M(x)$ and a (covariant) derivative D_x,

$$\mathrm{Tr}\, f(M(x),D_x) = \int \frac{\mathrm{d}^d x\, \mathrm{d}^d p}{(2\pi)^d}\mathrm{tr}\left[f(M(x),D_x+\mathrm{i}p)\mathbb{1}\right].$$

- Loses manifest covariance due to appearance of "free" covariant derivatives.
- *Method of covariant symbols* [7] makes the expansion manifestly covariant already on the level of the integrand,

$$\mathrm{Tr} f(M(x),D_x) = \int \frac{\mathrm{d}^d x\, \mathrm{d}^d p}{(2\pi)^d}\mathrm{tr}\left[f(\overline{M}(x),\overline{D}_x)\mathbb{1}\right],$$

$$\overline{M} = M + \mathrm{i}[D_\alpha, M]\frac{\partial}{\partial p_\alpha} - \frac{1}{2}[D_\alpha,[D_\beta,M]]\frac{\partial^2}{\partial p_\alpha \partial p_\beta} + \cdots,$$

$$\overline{D}_\mu = \mathrm{i}p_\mu + \frac{\mathrm{i}}{2}[D_\alpha, D_\mu]\frac{\partial}{\partial p_\alpha} - \frac{1}{3}[D_\alpha,[D_\beta,D_\mu]]\frac{\partial^2}{\partial p_\alpha \partial p_\beta} + \cdots.$$

- Apart from a rescaling factor, Higgs field appears in the result as $\varphi_\mu = \frac{1}{\phi}\partial_\mu\phi$.
- Charged weak boson fields appear in the result in covariant derivatives,

$$W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu \pm g' B_\mu W^\pm_\nu.$$

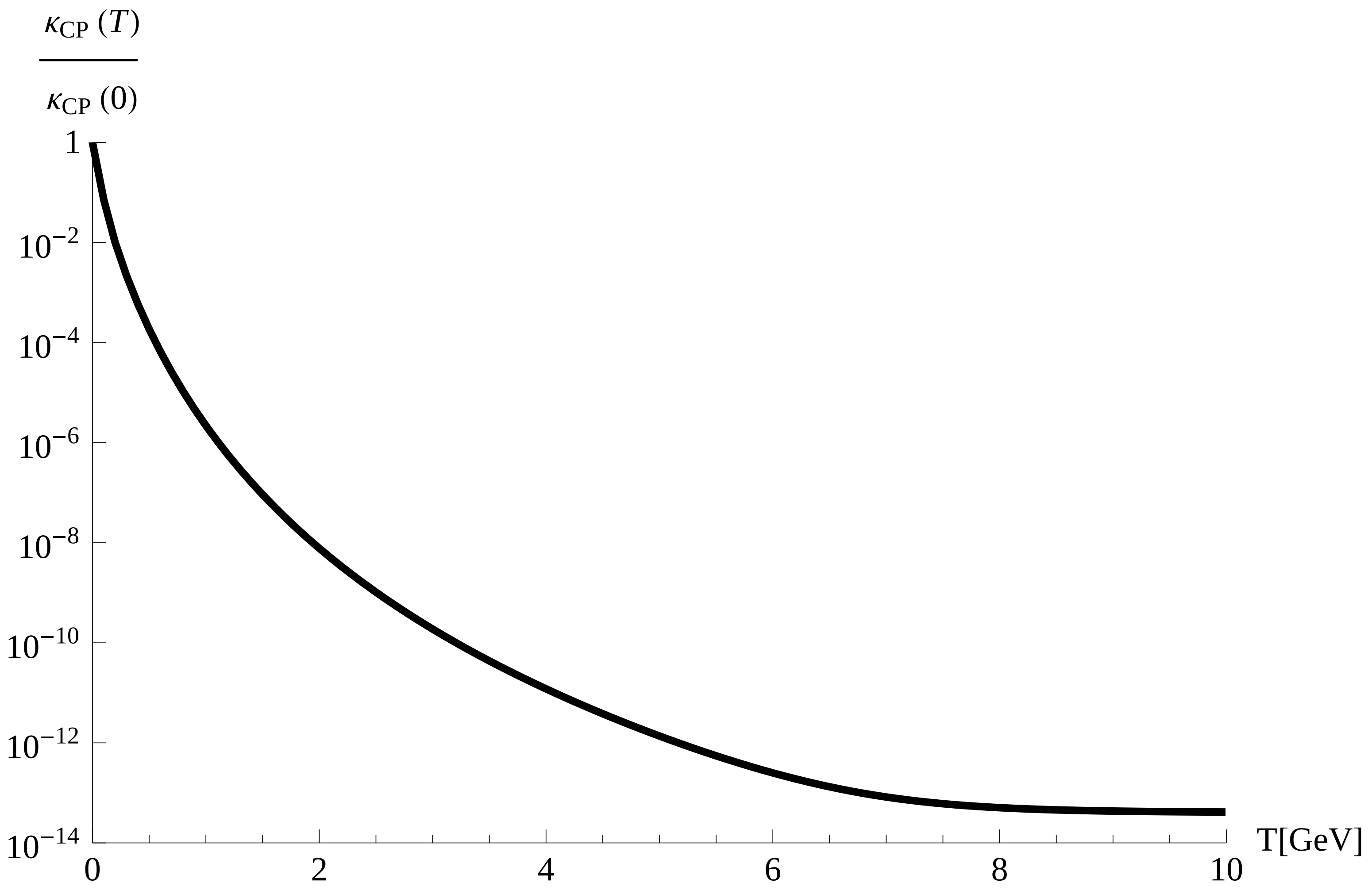
Our result at zero temperature

- Our calculation fully confirms the result of [7].
- The Euclidean effective action for the Standard Model bosons acquires first CP-violating contributions at the sixth order of the derivative expansion:

$$\begin{aligned}
\Gamma_{\mathrm{eff}} &= -\frac{\mathrm{i}}{2}N_c J G_{\mathrm{F}}\kappa_{\mathrm{CP}}\int \mathrm{d}^4 x \left(\frac{v}{\phi}\right)^2 (\mathcal{O}_0+\mathcal{O}_1+\mathcal{O}_2),\\
\mathcal{O}_0 &= -\frac{1}{3}(W^+)^2 W^-_{\mu\mu}W^-_{\nu\nu} + \frac{5}{3}(W^+)^2 W^-_{\mu\nu}W^-_{\mu\nu} - \frac{1}{3}(W^+)^2 W^-_{\mu\nu}W^-_{\nu\mu} +\\
&\quad + \frac{4}{3}W^+_\mu W^+_\nu W^-_{\mu\alpha}W^-_{\alpha\nu} - \frac{2}{3}W^+_\mu W^+_\nu W^-_{\mu\alpha}W^-_{\nu\alpha} - 2W^+_\mu W^+_\nu W^-_{\alpha\mu}W^-_{\alpha\nu} +\\
&\quad + \frac{4}{3}W^+_\mu W^+_\nu W^-_{\mu\nu}W^-_{\alpha\alpha} - \mathrm{c.c.},\\
\mathcal{O}_1 &= \frac{8}{3}(Z_\mu+\varphi_\mu)\left[(W^+)^2 W^-_\mu W^-_{\nu\nu} - (W^+)^2 W^-_\nu W^-_{\mu\nu} - (W^+)^2 W^-_\nu W^-_{\nu\mu} -\right.\\
&\quad \left. - (W^+\cdot W^-)W^+_\mu W^-_{\nu\nu} + (W^+\cdot W^-)W^+_\nu W^-_{\mu\nu} + W^+_\mu W^+_\nu W^-_\alpha W^-_{\alpha\nu}\right] - \mathrm{c.c.},\\
\mathcal{O}_2 &= 4(Z_\mu Z_\nu + \varphi_\mu\varphi_\nu)\left[(W^+)^2 W^-_\mu W^-_\nu - (W^-)^2 W^+_\mu W^+_\nu\right] -\\
&\quad - \frac{16}{3}(Z\cdot\varphi)\left[(W^+\cdot W^-)^2 - 2(W^+)^2(W^-)^2\right] + \frac{4}{3}(Z_\mu\varphi_\nu + Z_\nu\varphi_\mu)\times\\
&\quad \times\left[(W^+)^2 W^-_\mu W^-_\nu + (W^-)^2 W^+_\mu W^+_\nu - 2(W^+\cdot W^-)(W^+_\mu W^-_\nu + W^+_\nu W^-_\mu)\right],\\
\kappa_{\mathrm{CP}} &= \frac{\Delta}{G_{\mathrm{F}}}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}(p^2)^3 \prod_{f=1}^{6}\frac{1}{(p^2+m_f^2)^2} \approx 3\times 10^2.
\end{aligned}$$

Generalization to nonzero temperature

- First preliminary result: naive generalization of κ_{CP} to a sum-integral.



- $\kappa_{\mathrm{CP}}(T)$ drops rapidly as temperature crosses quark mass thresholds.
- A more detailed analysis of finite temperature effects is under way:
  - Deal with nontrivial topology of time dimension versus gauge invariance.
  - Generate the list of rotationally-invariant gauge-invariant CP-odd operators.
  - Evaluate the corresponding thermal sum-integrals.
  - Is there any temperature-induced CP-odd, P-odd term at fourth order?

Conclusions and outlook

- Within the cold electroweak baryogenesis scenario, Standard Model seems capable to generate sufficient baryon asymmetry in the early Universe.
- Using the zero-temperature value of κ_{CP} leads to baryon asymmetry *four orders of magnitude* larger than the observed value [5].
- The steep decrease of $\kappa_{\mathrm{CP}}(T)$ constrains the model; more work needed here.
- Still have a chance to explain the observed asymmetry with known physics!

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