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The spectrum of quarks in QCD2

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Gauge invariant Green's functions

Gauge invariant Green's functions are the natural ingredients to be used to investigate the physical properties of observables in gauge theories. Need appropriate treatment with Wilson loops.
For quarks, in the fundamental representation of $SU(N_c)$, the gauge invariant two-point Green's function is defined as

$$S_{\alpha\beta}(x,x';C_{x'x}) = -\frac{1}{N_c}\langle\overline{\psi}_\beta(x')\,U(C_{x'x};x',x)\,\psi_\alpha(x)\rangle,$$

where U is a path-ordered gluon field phase factor along a line $C_{x'x}$ joining a point x to a point x':

$$U(C_{x'x};x',x) = Pe^{-ig\int_x^{x'} dz^\mu A_\mu(z)}.$$

Green's functions with polygonal lines

Green's functions with paths along polygonal lines are of particular interest. They can be decomposed into the succession of straight line segments. Straight line segments have Lorentz invariant forms. Easy classification of polygonal lines according to the number of segments. For polygonal lines with n sides and $n-1$ junction points $y_1, y_2, \ldots, y_{n-1}$ between the segments, we define:

$$S_{(n)}(x,x';y_{n-1},\ldots,y_1) = -\frac{1}{N_c}\,\langle\overline{\psi}(x')U(x',y_{n-1})\ldots U(y_1,x)\psi(x)\rangle,$$

where each U is along a straight line segment.

For one straight line, one has:

$$S_{(1)}(x,x') \equiv S(x,x') = -\frac{1}{N_c}\,\langle\overline{\psi}(x')\,U(x',x)\,\psi(x)\rangle.$$

Quark propagator in the external gluon field

A two-step quantization. One first integrates with respect to the quark fields. This produces in various terms the quark propagator in the presence of the gluon field. Then one integrates with respect to the gluon field through Wilson loops.

We use for the quark propagator in the extenal gluon field, $S(A)$, a representation which involves phase factors along straight lines together with the full quark Green's function. Generalization of a representation introduced by Eichten and Feinberg, 1981, for heavy quarks. $S(A)$ is expanded around the following gauge covariant quantity:

$$S(x,x')\Big[U(x,x')\Big]^{a}_{\ b}.$$

[$S(x,x')$ is the gauge invariant Green's function along one straight line segment.]

Integrodifferential equation

Using then the quark equations of motion and the functional relations between Green's functions, one establishes the following integrodifferential equation for the Green's function $S(x,x')$:

$$(i\gamma.\partial_{(x)} - m)S(x,x') = i\delta^4(x-x') + i\gamma^\mu \Big\{ K_{2\mu}(x',x,y_1)\, S_{(2)}(y_1,x';x)$$

$$+ \sum_{n=3}^{\infty} K_{n\mu}(x',x,y_1,\ldots,y_{n-1})\, S_{(n)}(y_{n-1},x';x,y_1,\ldots,y_{n-2}) \Big\},$$

where the kernel K_n contains globally n derivatives of Wilson loops with a $(n+1)$-sided polygonal contour and also the Green's function S and its derivative.

The Green's functions $S_{(n)}$ themselves are related to the simplest Green's function S with functional relations.

Two-dimensional QCD

Many simplifications in two-dimensional QCD at large N_c. In two dimensions, Wilson loop averages are exponential functionals of the areas of the surfaces enclosed by the contours. At large N_c, crossed diagrams and quark loop contributions disappear. ('t Hooft, 1974.)

Equation of S with the lowest-order kernel becomes an exact equation. In two dimensions, the second-order derivative of the logarithm of the Wilson loop average is a delta-function.

$$(i\gamma.\partial - m)S(x) = i\delta^2(x) - \sigma\gamma^\mu(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})x^\nu x^\beta$$

$$\times\Big[\int_0^1 d\lambda\lambda^2 S((1-\lambda)x)\gamma^\alpha S(\lambda x) + \int_1^\infty d\xi S((1-\xi)x)\gamma^\alpha S(\xi x)\Big].$$

$$S(p) = \gamma.pF_1(p^2) + F_0(p^2).$$

$$S(x) = \frac{1}{2\pi}\Big(\frac{i\gamma.x}{r}\widetilde{F}_1(r) + \widetilde{F}_0(r)\Big), \quad r = \sqrt{-x^2}.$$

One obtains two coupled equations. Their resolution proceeds through several steps, based mainly on the spectral representation and the related analyticity properties.

The equations can be solved explicitly.

The covariant functions $F_1(p^2)$ and $F_0(p^2)$ are:

$$F_1(p^2) = -i\frac{\pi}{2\sigma}\sum_{n=1}^{\infty} b_n \frac{1}{(M_n^2 - p^2)^{3/2}},$$

$$F_0(p^2) = i\frac{\pi}{2\sigma}\sum_{n=1}^{\infty}(-1)^n b_n \frac{M_n}{(M_n^2 - p^2)^{3/2}}.$$

The threshold singularities or branch points $M_1^2,\ M_2^2, \ldots,\ M_n^2, \ldots$ are labelled with increasing values with respect to the index n; in particular $M_1 > m$.

For large n:

$$M_n^2 \simeq \sigma\pi n, \quad b_n \simeq \frac{\sigma^2}{M_n}, \quad \text{for } \sigma\pi n \gg m^2.$$

In $x-$space ($r = \sqrt{-x^2}$):

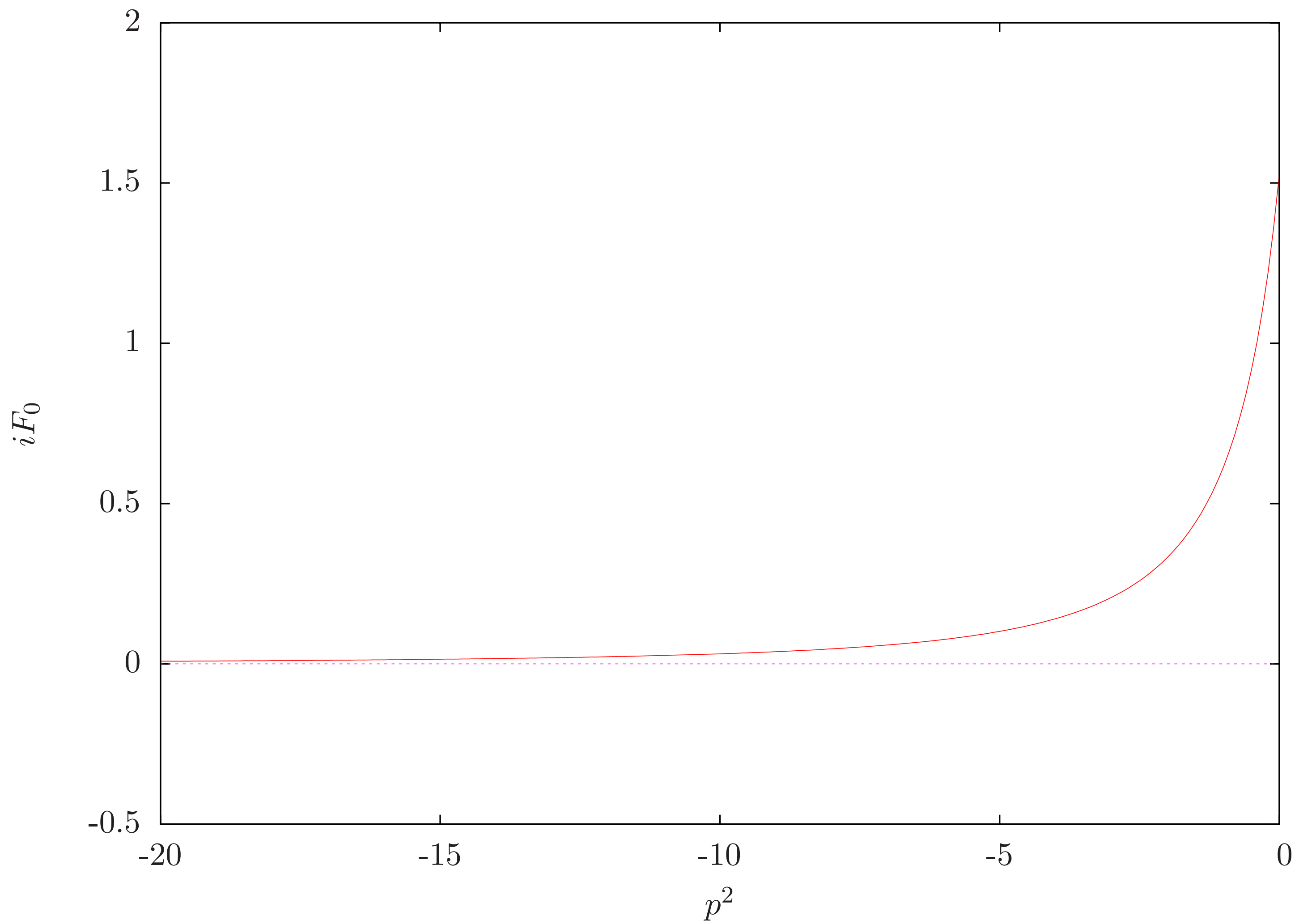
$$\widetilde{F}_1(r) = \frac{\pi}{2\sigma}\sum_{n=1}^{\infty} b_n e^{-M_n r}, \quad \widetilde{F}_0(r) = \frac{\pi}{2\sigma}\sum_{n=1}^{\infty}(-1)^{n+1} b_n e^{-M_n r}.$$

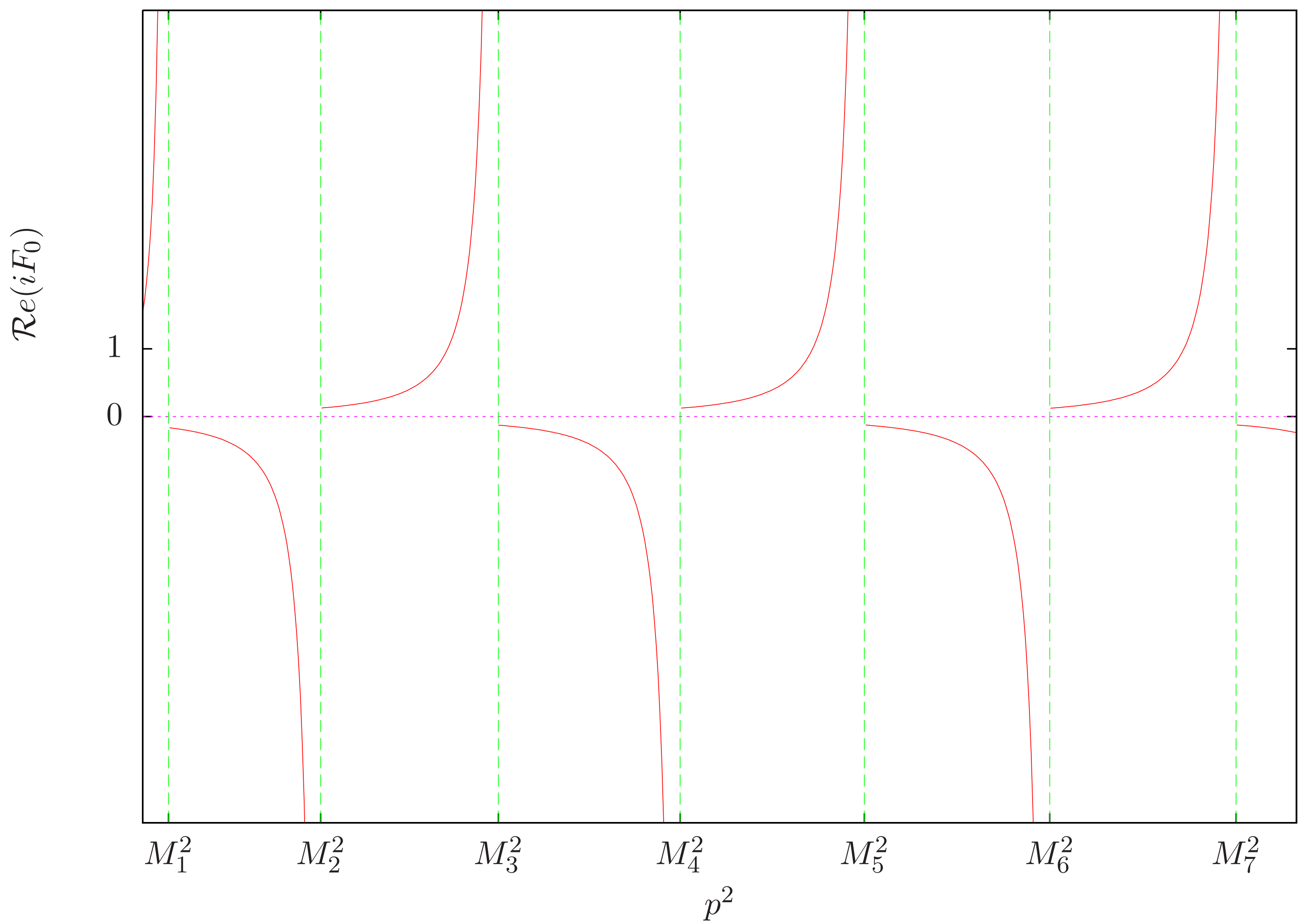
Asymptotic behaviors:

$$F_1(p^2) \underset{|p^2|\to\infty}{=} \frac{i}{p^2},$$

$$F_0(p^2) \underset{|p^2|\to\infty}{=} \frac{im}{p^2}, \qquad m \neq 0,$$

$$F_0(p^2) \underset{|p^2|\to\infty}{=} \frac{2i\sigma}{N_c} \frac{\langle \overline{\psi}\psi \rangle}{(p^2)^2}, \qquad m = 0.$$


2
1.5
1
0.5
0
-0.5
iF_0
-20
-15
-10
-5
0
p^2


$\mathcal{R}e(iF_0)$
1
0
M_1^2
M_2^2
M_3^2
M_4^2
M_5^2
M_6^2
M_7^2
p^2

Conclusion

1) The spectral functions are infrared finite and lie on the positive real axis of p^2. No singularities in the complex plane or on the negative real axis have been found. $\Longrightarrow$ Quarks contribute with positive energies.

2) The singularities are represented by an infinite number of threshold type singularities, characterized by positive masses M_n ($n = 1, 2, \ldots$). The corresponding singularities are stronger than simple poles and this feature might prevent observability of quarks as free particles.

3) The threshold masses M_n represent dynamically generated masses and maintain the scalar part of the Green's function at a nonzero value.