The Asymptotic Safety Program for Quantum Gravity

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Introduction

standard model of particle physics:

- describes: electromagnetic/strong/weak force + interactions with matter
- extremely well tested
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theoretical basis: quantum field theory
- includes only relevant and marginal couplings
  \[\rightarrow\text{renormalizable quantum field theory}\]
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General Relativity:

- describes: gravity + interactions with matter
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theoretical basis: classical theory

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu} + 8\pi G_N T_{\mu\nu}
\]

- Newton constant $G_N$ has negative mass-dimension
  \[\implies\] perturbatively non-renormalizable quantum field theory
General Relativity: perturbatively non-renormalizable

perturbative quantization of General Relativity:

- $G_N$ has negative mass-dimension:
  - infinite number of counterterms
  - General Relativity is perturbatively non-renormalizable

Possible conclusions:

a) Treat General Relativity as effective field theory:

- compute corrections in $E^2/M_{Pl}^2 \ll 1$ (independent of UV-completion)
- breaks down at $E^2 \approx M_{Pl}^2$
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   - supersymmetry, extra dimensions, ... 
   - possibly: extension of QFT-framework
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Wilsonian renormalization and asymptotic safety

basic concepts
Wilson’s modern picture of renormalization

central idea: integrate out quantum fluctuations shell-by-shell in momentum-space
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implementation:

- action with scale-dependent couplings \((G_N, \Lambda, \ldots)\):
  \[ g_i(k) \]

- scale-dependence governed by \(\beta\)-functions:
  \[ k \partial_k g_i = \beta_{g_i}(\{g_i\}) \]
Ensuring good UV-behavior: fixed points of the RG-flow

amplitudes depend on dimensionless couplings only

- RG-flow for dimensionless running couplings:
  \[ g_i(k) \]

Fixed points \( g_i^* \):

- \( \beta \)-functions vanish:
  \[ \beta g_i(\{g_i^*\}) = 0 \]
  \( g_i^* \) remain finite

- RG-trajectory captured by fixed point in UV:
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**Concepts associated with UV-fixed points:**
- trajectories emanating from fixed point in UV
  \( \equiv \text{span UV critical surface} \)
- predictivity:
  \( \equiv \text{UV critical surface has finite dimension} \)
Renormalization: asymptotic freedom and asymptotic safety

Wilsonian formulation:

- UV fixed points allow two classes of renormalizable Quantum Field Theories

- **Gaussian Fixed Point (GFP):**
  - perturbatively renormalizable field theories
  - UV-limit: free theory
  - asymptotic freedom (example: QCD)
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  ◦ asymptotic safety

Wilsonian picture: generalization of perturbative renormalization

asymptotic safety as predictive as asymptotic freedom
**Examples: Asymptotically Safe Theories**

Theories with non-Gaussian UV fixed point

- $O(N)$-sigma model ($d = 2 + \epsilon$)  
  - critical exponents of Heisenberg ferromagnets  
  [Brézin, Zinn-Justin ’76]

- Gross-Neveu model ($d = 2 + \epsilon$)  
  [Gawedzki, Kupiainen ’85]

- Grosse-Wulkenhaar model (non-commutative $\phi^4$-theory)  
  [Grosse, Wulkenhaar ’05; Disertori, Gurau, Magnen, Rivasseau ’07]

- Gravity in $2 + \epsilon$ dimensions  
  [Christensen, Duff; Gastmans, Kalosh, Truffin ’78]
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Weinberg’s asymptotic safety conjecture (1979):  

gravity in $d = 4$ has non-Gaussian UV fixed point
Renormalizing gravity

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Testing asymptotic safety:

Functional Renormalization Group Equations (FRG)

Causal Dynamical Triangulations (CDT)
Functional Renormalization Group Equation for gravity


scale-dependence of $\Gamma_k$ governed by exact RG equation

$$k \partial_k \Gamma_k[\phi, \bar{\phi}] = \frac{1}{2} \text{Tr} \left[ \left( \frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi} + R_k \right)^{-1} k \partial_k R_k \right]$$

- $R_k(p^2) = \text{IR momentum-cutoff at scale } k$
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limits of the RG-flow:

- $k = \Lambda$: initial (boundary) condition
  $$\Gamma_{k=\Lambda} = \Gamma_\Lambda$$

- $k = 0$: all quantum fluctuations integrated out
  $$\Gamma_{k=0} = \Gamma$$

$$\Gamma = \Gamma_\Lambda + \lim_{k \to 0} \int_\Lambda^k d\hat{k} \partial_{\hat{k}} \Gamma_{\hat{k}} \left[ \Gamma_{\hat{k}}^{(2)}, R_{\hat{k}} \right]$$

in between:

- regulator ensures finiteness of flow
Functional Renormalization Group Equation for gravity


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$$k \partial_k \Gamma_k[\phi, \bar{\phi}] = \frac{1}{2} \text{Tr} \left[ \left( \frac{\delta^2 \Gamma_k}{\delta \phi \delta \bar{\phi}} + \mathcal{R}_k \right)^{-1} k \partial_k \mathcal{R}_k \right]$$

renormalizability:

- if $\Gamma_{\Lambda \to \infty} = \Gamma_*$ exists, $\Gamma_*$ qualifies as fundamental theory
- perturbatively renormalizable theory: $\Gamma_*$ is free theory (e.g. QCD)
- non-perturbatively renormalizable theory: $\Gamma_*$ is interacting
- non-renormalizable: $\Gamma_*$ does not exist

- predictivity: provided by fixed point
Theory space underlying the Functional Renormalization Group

\[ \Gamma_\infty = \Gamma_* \sim \text{bare action} \]
\[ \Gamma_0 = \Gamma \quad \text{effective action} \]
Non-perturbative approximation: derivative expansion of $\Gamma_k$

- caveat: FRGE cannot be solved exactly

$\iff$ gravity: need non-perturbative approximation scheme
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- expand $\Gamma_k$ in derivatives and truncate series:

$$\Gamma_k[\Phi] = \sum_{i=1}^{N} \bar{u}_i(k) O_i[\Phi]$$

$\implies$ substitute into FRGE

$\implies$ projection of flow gives $\beta$-functions for running couplings

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- testing the reliability:
  - within a given truncation:
    cutoff-scheme dependence of physical quantities ($= \text{vary } R_k$)
  - stability of results within extended truncations
Letting things flow

The Einstein-Hilbert truncation
The Einstein-Hilbert truncation: setup

Einstein-Hilbert truncation: two running couplings: $G(k), \Lambda(k)$

\[ \Gamma_k = \frac{1}{16\pi G(k)} \int d^4x \sqrt{g} \left[ -R + 2\Lambda(k) \right] + S^{gf} + S^{gh} \]

- project flow onto $G$-$\Lambda$-plane
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- project flow onto $G-\Lambda$-plane

explicit $\beta$-functions for dimensionless couplings $g_k := k^2 G(k), \lambda_k := \Lambda(k) k^{-2}$

- Particular choice of $R_k$ (optimized cutoff)

$$k \partial_k g_k = (\eta_N + 2) g_k,$$

$$k \partial_k \lambda_k = - (2 - \eta_N) \lambda_k - \frac{g_k}{2\pi} \left[ 5 \frac{1}{1-2\lambda_k} - 4 - \frac{5}{6} \frac{1}{1-2\lambda_k} \eta_N \right]$$

- anomalous dimension of Newton’s constant:

$$\eta_N = \frac{gB_1}{1-gB_2}$$

$$B_1 = \frac{1}{3\pi} \left[ 5 \frac{1}{1-2\lambda} - 9 \frac{1}{(1-2\lambda)^2} - 7 \right], \quad B_2 = -\frac{1}{12\pi} \left[ 5 \frac{1}{1-2\lambda} + 6 \frac{1}{(1-2\lambda)^2} \right]$$
Einstein-Hilbert truncation: Fixed Point structure

\( \beta \)-functions for \( g_k := k^2 G(k) \), \( \lambda_k := \Lambda(k) k^{-2} \)

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microscopic theory \( \iff \) fixed points of the \( \beta \)-functions

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\beta_g(g^*, \lambda^*) = 0 , \quad \beta_\lambda(g^*, \lambda^*) = 0
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- Gaussian Fixed Point:
  - at \( g^* = 0, \lambda^* = 0 \iff \text{free theory} \)
  - saddle point in the \( g\)-\( \lambda \)-plane
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- non-Gaussian Fixed Point (\( \eta_N^* = -2 \)):
  - at \( g^* > 0, \lambda^* > 0 \iff \text{“interacting” theory} \)
  - UV attractive in \( g_k, \lambda_k \)
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Asymptotic safety: non-Gaussian Fixed Point is UV completion for gravity
### Einstein-Hilbert truncation: Stability properties

<table>
<thead>
<tr>
<th>Ref.</th>
<th>$g^*$</th>
<th>$\lambda^*$</th>
<th>$g^<em>\lambda^</em>$</th>
<th>$\theta' \pm i\theta''$</th>
<th>gauge</th>
<th>$\mathcal{R}_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMS</td>
<td>0.902</td>
<td>0.109</td>
<td>0.099</td>
<td>2.52 $\pm$ 1.78i</td>
<td>geometric</td>
<td>II, opt</td>
</tr>
<tr>
<td>RS</td>
<td>0.403</td>
<td>0.330</td>
<td>0.133</td>
<td>1.94 $\pm$ 3.15i</td>
<td>harmonic</td>
<td>I, sharp</td>
</tr>
<tr>
<td>LR</td>
<td>0.272</td>
<td>0.348</td>
<td>0.095</td>
<td>1.55 $\pm$ 3.84i</td>
<td>harmonic</td>
<td>I, exp</td>
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<tr>
<td></td>
<td>0.344</td>
<td>0.339</td>
<td>0.117</td>
<td>1.86 $\pm$ 4.08i</td>
<td>Landau</td>
<td>I, exp</td>
</tr>
<tr>
<td>L</td>
<td>1.17</td>
<td>0.25</td>
<td>0.295</td>
<td>1.67 $\pm$ 4.31i</td>
<td>Landau</td>
<td>I, opt</td>
</tr>
<tr>
<td>CPR</td>
<td>0.707</td>
<td>0.193</td>
<td>0.137</td>
<td>1.48 $\pm$ 3.04i</td>
<td>harmonic</td>
<td>I, opt</td>
</tr>
<tr>
<td></td>
<td>0.556</td>
<td>0.092</td>
<td>0.051</td>
<td>2.43 $\pm$ 1.27i</td>
<td>harmonic</td>
<td>II, opt</td>
</tr>
<tr>
<td></td>
<td>0.332</td>
<td>0.274</td>
<td>0.091</td>
<td>1.75 $\pm$ 2.07i</td>
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Einstein-Hilbert truncation: NGFP in $d = 2 + \epsilon$

$\beta$-functions continuous in $d \iff$ reproduce perturbative fix point in $d = 2 + \epsilon$
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NGFP in $d = 4 \iff$ analytic continuation of NGFP in $d = 2 + \epsilon$
Einstein-Hilbert-truncation: the phase diagram
where FRG and CDT meet

spectral dimension of space-time
Spectral dimension for classical manifolds

- Heat-equation: diffusion of scalar test particle on manifold with metric $g$

$$\partial_T K_g(x, x'; T) = \Delta_g K_g(x, x'; T)$$

- define average return probability

$$P_g(T) \equiv \frac{1}{V} \int d^d x \sqrt{g(x)} K_g(x, x; T)$$

$$= \frac{1}{V} \text{Tr} \left[ \exp(T \Delta_g) \right]$$

$$= \left( \frac{1}{4\pi T} \right)^{d/2} \sum_{n=0}^{\infty} A_n T^n$$

- asymptotic expansion: space-time dimension seen by diffusion process

$$d = -2 \frac{d \ln P_g(T)}{d \ln T} \bigg|_{T=0}$$
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$P(T)$: accessible in CDT and FRG!
Spectral dimension of QEG space-times

• in QEG: metric of manifold is $k$-dependent
  \[ \Rightarrow \text{diffusion process “with momentum } k \text{” sees metric } \langle g_{\mu\nu} \rangle_k \]
  \[ \Rightarrow \text{diffusion equation and return probability will become } k\text{-dependent} \]

• Computation of the spectral dimension:
  1. determine $k$-dependence of $\Delta(k)$
  2. solve the $k$-dependent heat equation
  3. evaluate “quantum return probability” $P(T)$
  4. obtain spectral dimension

\[
D_s = -2 \left. \frac{d \ln P(T)}{d \ln T} \right|_{T=0}
\]
Spectral dimension $\mathcal{D}_s$ of QEG space-times

- spectral dimension

$$D_s = -2\left. \frac{d \ln P(T)}{d \ln T} \right|_{T=0}$$

- Quantum return probability:

$$P(T) = \int \frac{d^d p}{(2\pi)^d} \exp[-p^2 F(p^2) T], \quad F(p^2) = \frac{\Lambda(p)}{\Lambda(k_0)}$$

  - classical regime: no running, $F(p^2) = 1$:

    $$P(T)|_{T=0} \propto T^{-d/2} \implies D_s = d$$

  - fixed point regime: $\Lambda(p) \propto p^2 \rightarrow F(p^2) \propto p^2$:

    $$P(T)|_{T=0} \propto T^{-d/4} \implies D_s = d/2$$
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\[

\text{\emph{d = 4}: QEG predicts continuous change of fractal dimension} \\
D_s = 4 \text{ macroscopically} \implies D_s = 2 \text{ microscopically}
\]
Spectral dimension $D_s$ of QEG space-times

Flow of spectral dimension along a typical RG-trajectory
Matching FRG and 3d-lattice-simulations


Semiclassical regime: parameterize leading quantum corrections:

\[ \Lambda(p) = \Lambda_0 \left(1 + a p^\delta\right) \]

FRG-prediction: \( \delta = 3 \quad \approx \quad \) Lattice fit: \( \delta \approx 3.3 \)
Matching FRG and 3d-lattice-simulations


\[ \Lambda(p) = \Lambda_0 \left( 1 + a p^\delta \right) \]

FRG-prediction: \( \delta = 3 \quad \approx \quad \text{Lattice fit: } \delta \approx 3.3 \)

can compare and fit CDT Data and RG-results
Summary

gravitational asymptotic safety program

UV completion of gravity provided by non-trivial RG fixed point

Functional Renormalization Group equations:

- all computations support the existence of this fixed point
- UV-critical surface has finite dimension $\iff$ predictivity

Causal Dynamical Triangulations

- spectral dimension allows to compare lattice and continuum results!
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