The Asymptotic Safety Program for Quantum Gravity

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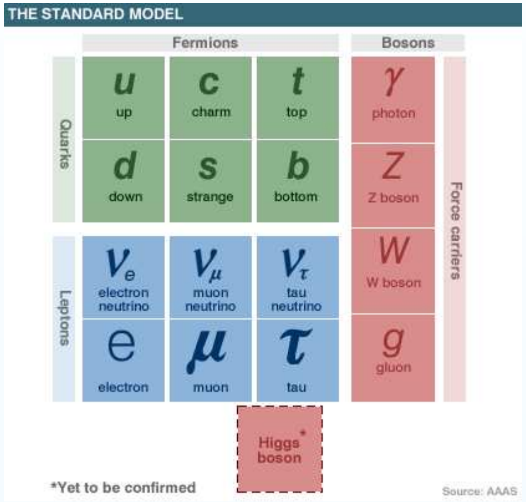
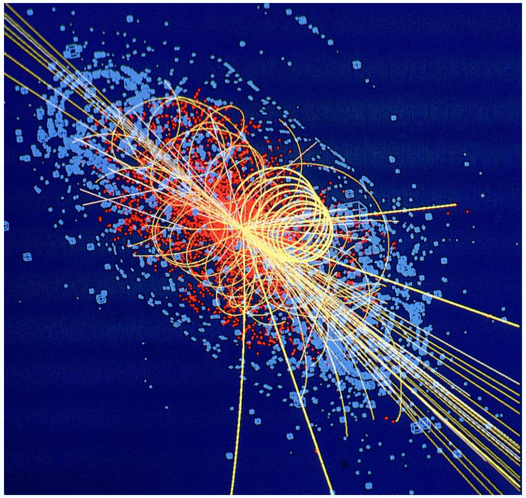
Introduction

standard model of particle physics:





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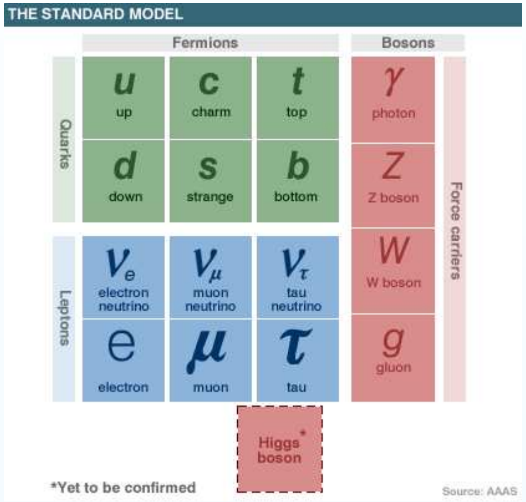

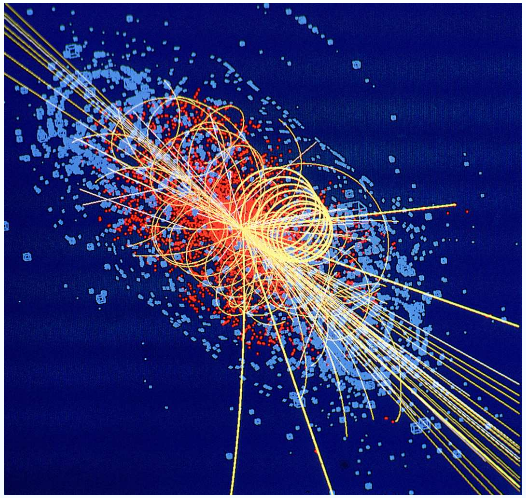

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theoretical basis: quantum field theory

- includes only relevant and marginal couplings

  $\Longrightarrow$ renormalizable quantum field theory

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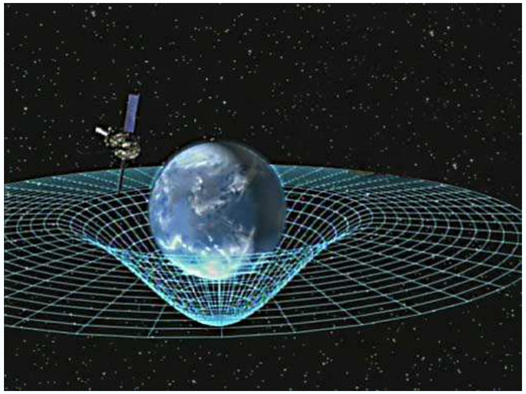
General Relativity:

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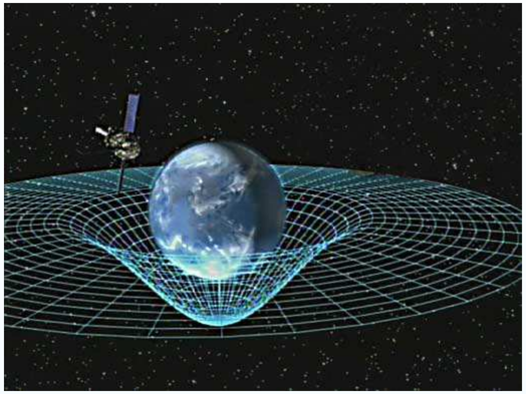

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theoretical basis: classical theory

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{space-time curvature}} = \underbrace{-\Lambda g_{\mu\nu} + 8\pi G_N T_{\mu\nu}}_{\text{matter content}}$$

- Newton constant G_N has negative mass-dimension

  $\Longrightarrow$ perturbatively non-renormalizable quantum field theory

General Relativity: perturbatively non-renormalizable

perturbative quantization of General Relativity:

- G_N has negative mass-dimension:
  - infinite number of counterterms
  - General Relativity is perturbatively non-renormalizable

Possible conclusions:

a) Treat General Relativity as effective field theory:

- compute corrections in $E^2/M_{\rm Pl}^2 \ll 1$ (independent of UV-completion)
- breaks down at $E^2 \approx M_{\rm Pl}^2$

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- supersymmetry, extra dimensions, ...
- possibly: extension of QFT-framework

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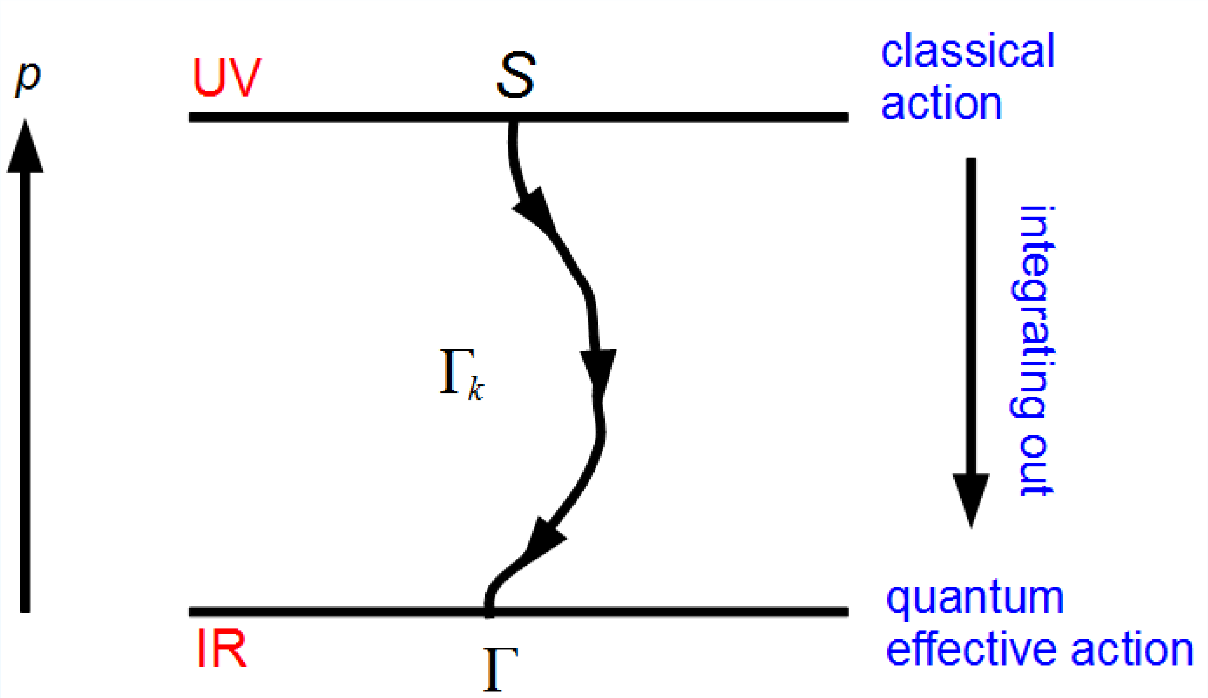
Wilsonian renormalization and asymptotic safety

basic concepts

Wilson's modern picture of renormalization

central idea: integrate out quantum fluctuations shell-by-shell in momentum-space



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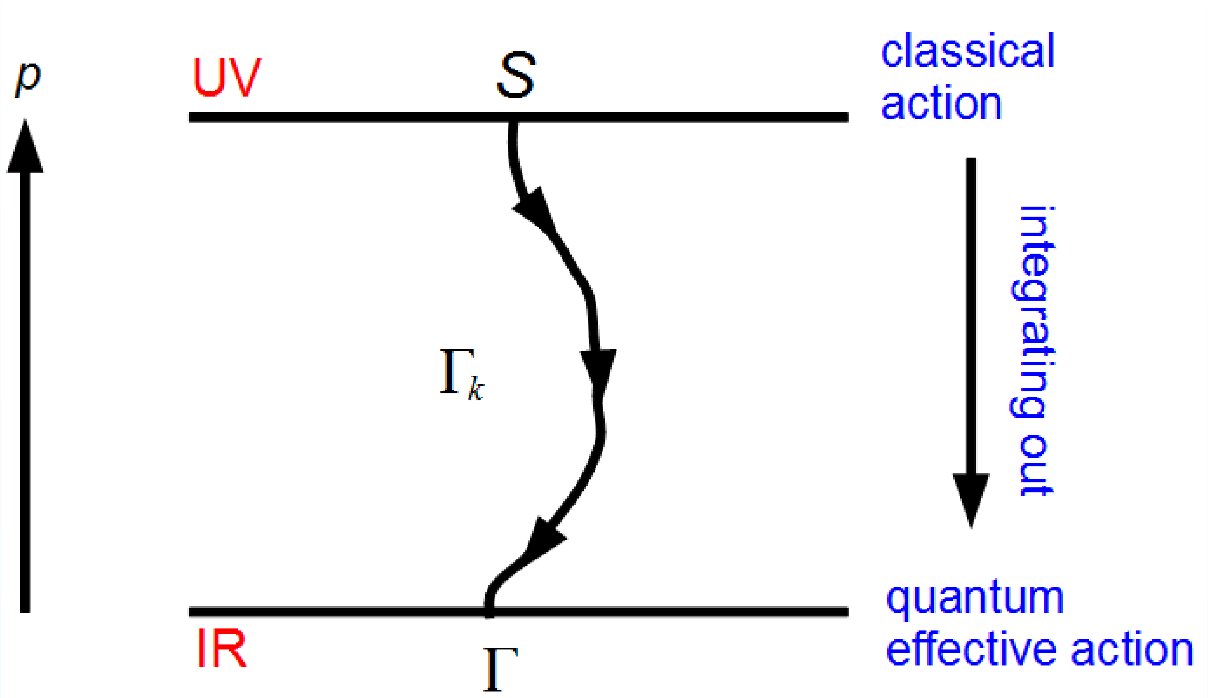

implementation:

- action with scale-dependent couplings $(G_N, \Lambda, \ldots)$: $g_i(k)$
- scale-dependence governed by β-functions: $k\partial_k g_i = \beta_{g_i}(\{g_i\})$

Ensuring good UV-behavior: fixed points of the RG-flow

amplitudes depend on dimensionless couplings only

- RG-flow for dimensionless running couplings: $g_i(k)$

Fixed points g_i^*:

- β-functions vanish: $\beta_{g_i}(\{g_i^*\}) \stackrel{!}{=} 0$

  g_i^* remain finite

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  $\Longrightarrow$ physical quantities remain free of unphysical divergences

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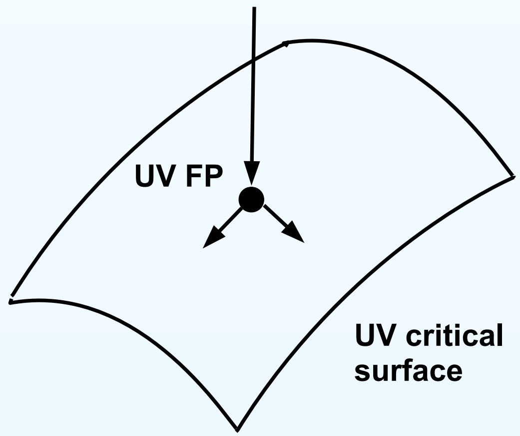
Concepts associated with UV-fixed points:

- trajectories emanating from fixed point in UV

  $\equiv$ span UV critical surface

- predictivity:

  $\equiv$ UV critical surface has finite dimension



Renormalization: asymptotic freedom and asymptotic safety

Wilsonian formulation:

- UV fixed points allow two classes of renormalizable Quantum Field Theories

- Gaussian Fixed Point (GFP):
  - perturbatively renormalizable field theories
  - UV-limit: free theory
  - asymptotic freedom (example: QCD)

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Wilsonian picture: generalization of perturbative renormalization

asymptotic safety as predictive as asymptotic freedom

Examples: Asymptotically Safe Theories

Theories with non-Gaussian UV fixed point

- $O(N)$-sigma model ($d = 2 + \epsilon$)

  [Brézin, Zinn-Justin ’76]

  - critical exponents of Heisenberg ferromagnets

- Gross-Neveu model ($d = 2 + \epsilon$)

  [Gawedzki, Kupiainen ’85]

- Grosse-Wulkenhaar model (non-commutative ϕ^4-theory)

  [Grosse, Wulkenhaar ’05; Disertori, Gurau, Magnen, Rivasseau ’07]

- Gravity in $2 + \epsilon$ dimensions

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Gravity

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Testing asymptotic safety:

Functional Renormalization Group Equations (FRG)

Causal Dynamical Triangulations (CDT)

Functional Renormalization Group Equation for gravity

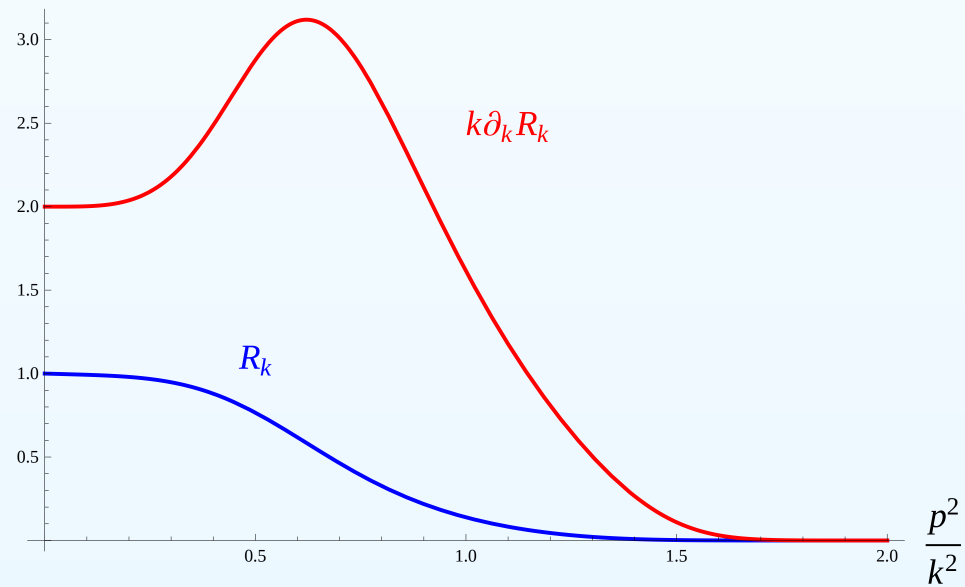
[C. Wetterich, Phys. Lett. **B301** (1993) 90]

[M. Reuter, Phys. Rev. D **57** (1998) 971, hep-th/9605030]

scale-dependence of Γ_k governed by exact RG equation

$$k\partial_k \Gamma_k[\phi,\bar{\phi}] = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

- $\mathcal{R}_k(p^2)$ = IR momentum-cutoff at scale k



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limits of the RG-flow:

- $k = \Lambda$: initial (boundary) condition $\qquad \Gamma_{k=\Lambda} = \Gamma_\Lambda$
- $k = 0$: all quantum fluctuations integrated out $\qquad \Gamma_{k=0} = \Gamma$

$$\Gamma = \Gamma_\Lambda + \lim_{k \to 0} \int_\Lambda^k d\hat{k}\, \partial_{\hat{k}} \Gamma_{\hat{k}} \left[\Gamma^{(2)}_{\hat{k}}, \mathcal{R}_{\hat{k}} \right]$$

in between:

- regulator ensures finiteness of flow

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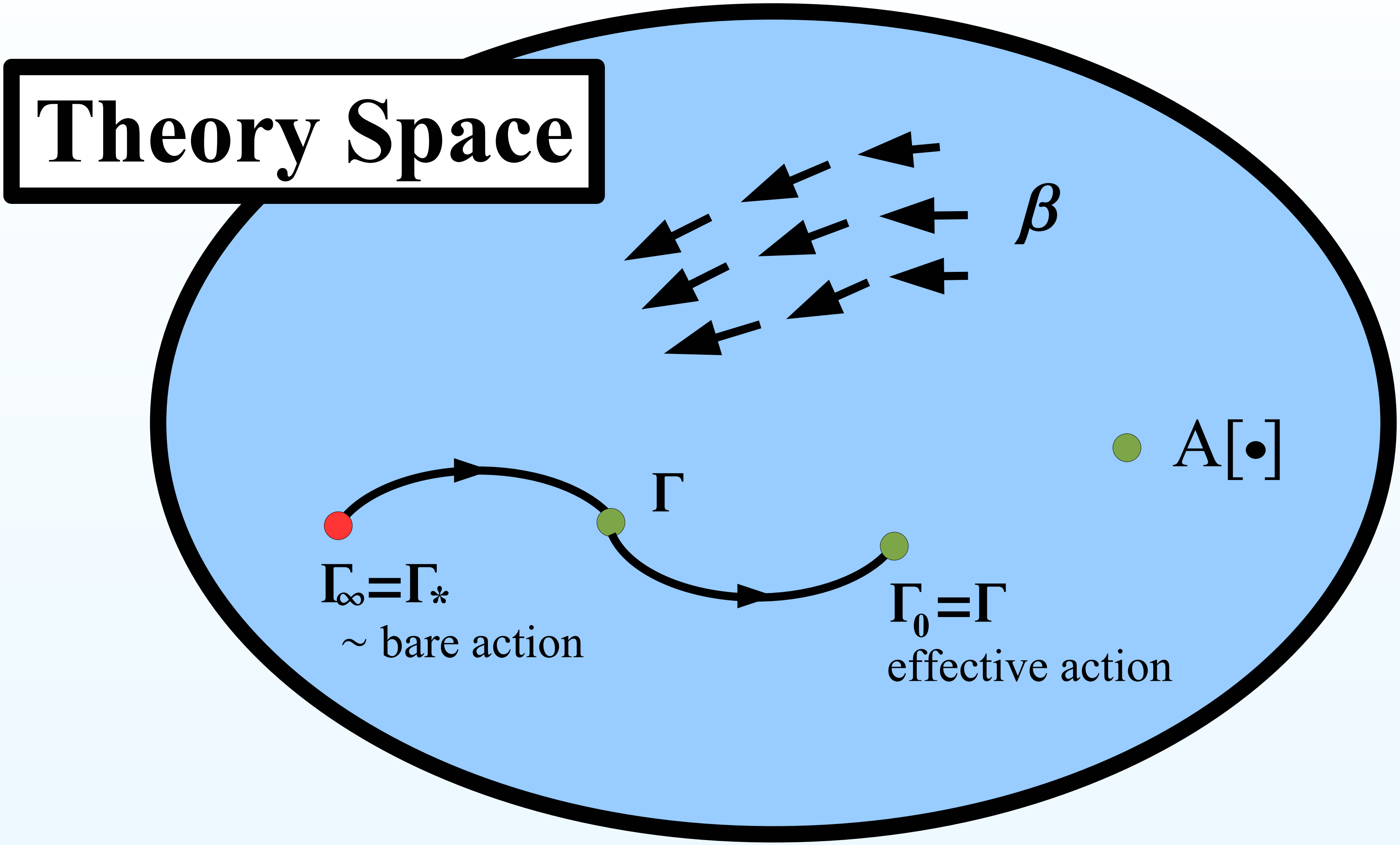
renormalizability:

- if $\Gamma_{\Lambda \to \infty} = \Gamma_*$ exists, Γ_* qualifies as fundamental theory
  - perturbatively renormalizable theory: Γ_* is free theory (e.g. QCD)
  - non-perturbatively renormalizable theory: Γ_* is interacting
  - non-renormalizable: Γ_* does not exist
- predictivity: provided by fixed point

Theory space underlying the Functional Renormalization Group



Non-perturbative approximation: derivative expansion of Γ_k

- caveat: FRGE cannot be solved exactly

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- expand Γ_k in derivatives and truncate series:

$$\Gamma_k[\Phi] = \sum_{i=1}^{N} \bar{u}_i(k)\, \mathcal{O}_i[\Phi]$$

$\Longrightarrow$ substitute into FRGE

$\Longrightarrow$ projection of flow gives β-functions for running couplings

$$k\partial_k \bar{u}_i(k) = \beta_i(\bar{u}_i; k)$$

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- testing the reliability:
  - within a given truncation:
    cutoff-scheme dependence of physical quantities (= vary $\mathcal{R}_k$)
  - stability of results within extended truncations

Letting things flow

The Einstein-Hilbert truncation

The Einstein-Hilbert truncation: setup

Einstein-Hilbert truncation: two running couplings: $G(k), \Lambda(k)$

$$\Gamma_k = \frac{1}{16\pi G(k)} \int d^4x \sqrt{g} \left[-R + 2\Lambda(k)\right] + S^{\rm gf} + S^{\rm gh}$$

- project flow onto G-$\Lambda-$plane

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explicit β-functions for dimensionless couplings $g_k := k^2 G(k)\,,\ \lambda_k := \Lambda(k) k^{-2}$

- Particular choice of $\mathcal{R}_k$ (optimized cutoff)

$$k\partial_k g_k = (\eta_N + 2) g_k\,,$$

$$k\partial_k \lambda_k = -(2-\eta_N)\,\lambda_k - \tfrac{g_k}{2\pi}\left[5\,\tfrac{1}{1-2\,\lambda_k} - 4 - \tfrac{5}{6}\,\tfrac{1}{1-2\lambda_k}\eta_N\right]$$

- anomalous dimension of Newton's constant:

$$\eta_N = \frac{g B_1}{1 - g B_2}$$

$$B_1 = \tfrac{1}{3\pi}\left[5\,\tfrac{1}{1-2\lambda} - 9\,\tfrac{1}{(1-2\lambda)^2} - 7\right]\,,\ B_2 = -\tfrac{1}{12\pi}\left[5\,\tfrac{1}{1-2\lambda} + 6\,\tfrac{1}{(1-2\lambda)^2}\right]$$

Einstein-Hilbert truncation: Fixed Point structure

β-functions for $g_k := k^2 G(k)\,,\ \lambda_k := \Lambda(k) k^{-2}$

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- non-Gaussian Fixed Point ($\eta_N^* = -2$):
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  - UV attractive in g_k, λ_k

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Asymptotic safety: non-Gaussian Fixed Point is UV completion for gravity

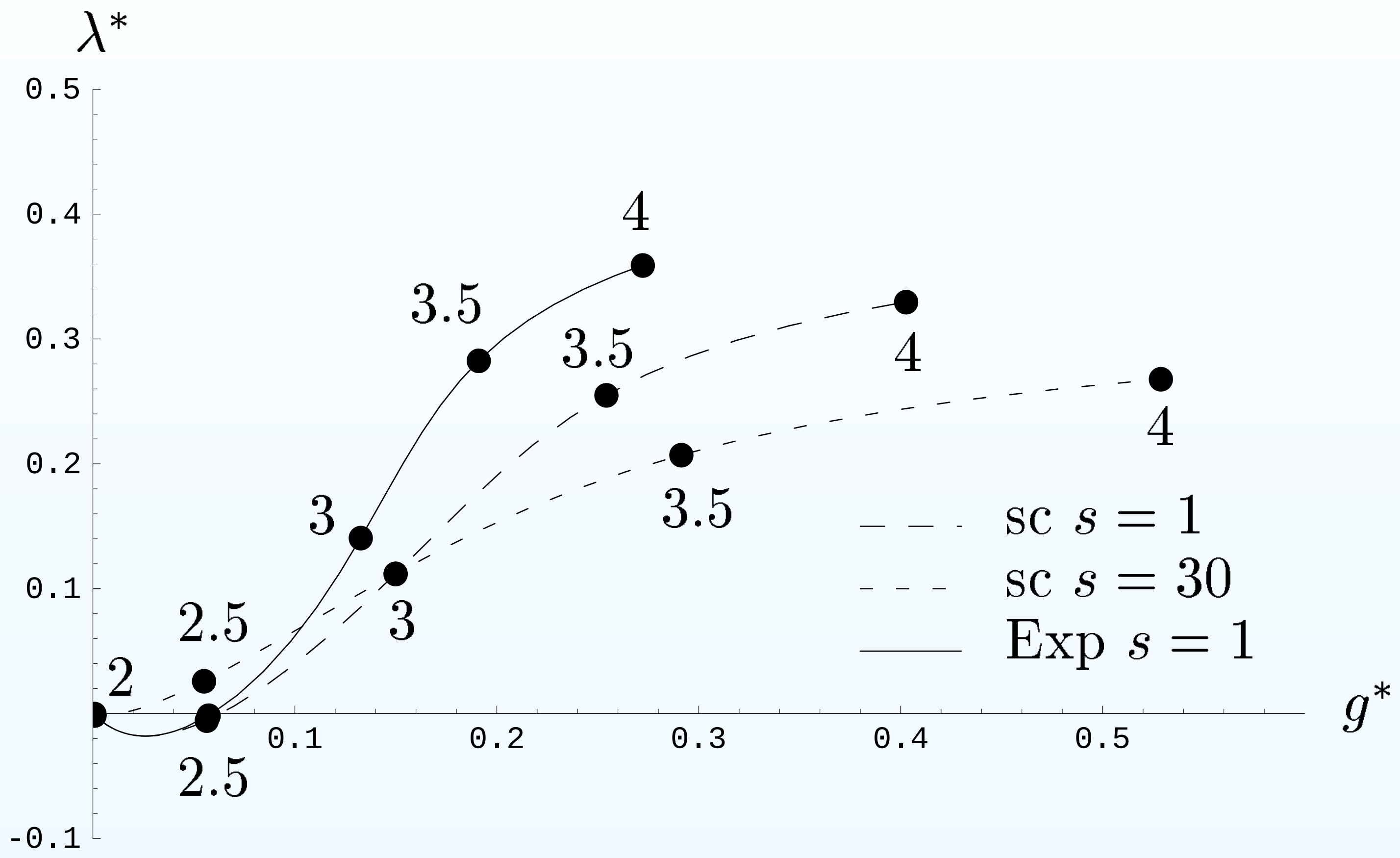
Einstein-Hilbert truncation: Stability properties

Ref.	g^*	λ^*	$g^*\lambda^*$	$\theta' \pm i\theta''$	gauge	$\mathcal{R}_k$
BMS	0.902	0.109	0.099	$2.52 \pm 1.78i$	geometric	II, opt
RS	0.403	0.330	0.133	$1.94 \pm 3.15i$	harmonic	I, sharp
LR	0.272	0.348	0.095	$1.55 \pm 3.84i$	harmonic	I, exp
	0.344	0.339	0.117	$1.86 \pm 4.08i$	Landau	I, exp
L	1.17	0.25	0.295	$1.67 \pm 4.31i$	Landau	I, opt
CPR	0.707	0.193	0.137	$1.48 \pm 3.04i$	harmonic	I, opt
	0.556	0.092	0.051	$2.43 \pm 1.27i$	harmonic	II, opt
	0.332	0.274	0.091	$1.75 \pm 2.07i$	harmonic	III, opt

BMS: Benedetti, Machado, Saueressig, 2009.
RS: Reuter, Saueressig, 2002.
LR: Lauscher, Reuter, 2002.
L: Litim, 2004.
CPR: Codello, Percacci, Rahmede, 2009.

Einstein-Hilbert truncation: NGFP in $d = 2 + \epsilon$

β-functions continuous in $d \Longleftrightarrow$ reproduce perturbative fix point in $d = 2 + \epsilon$



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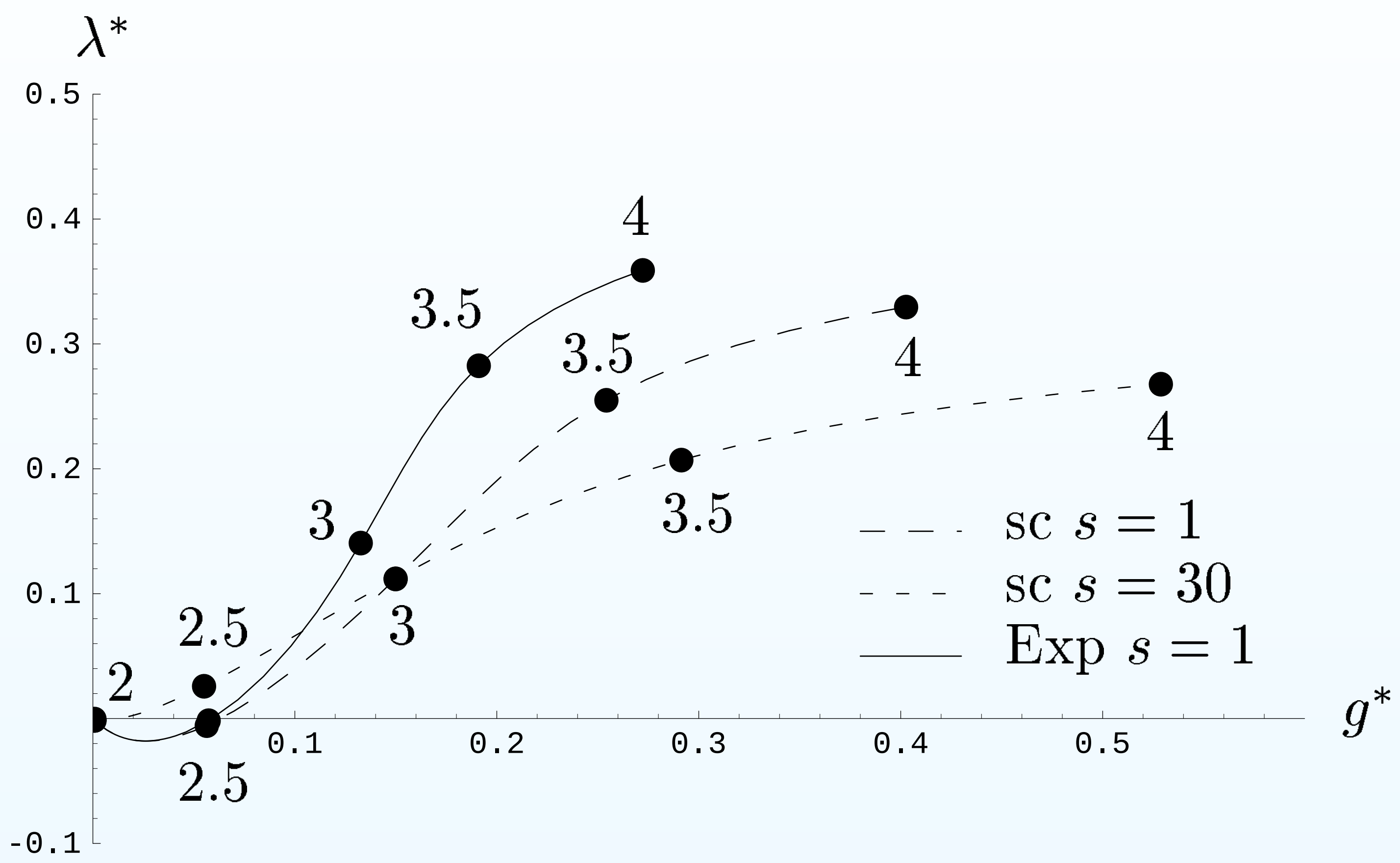
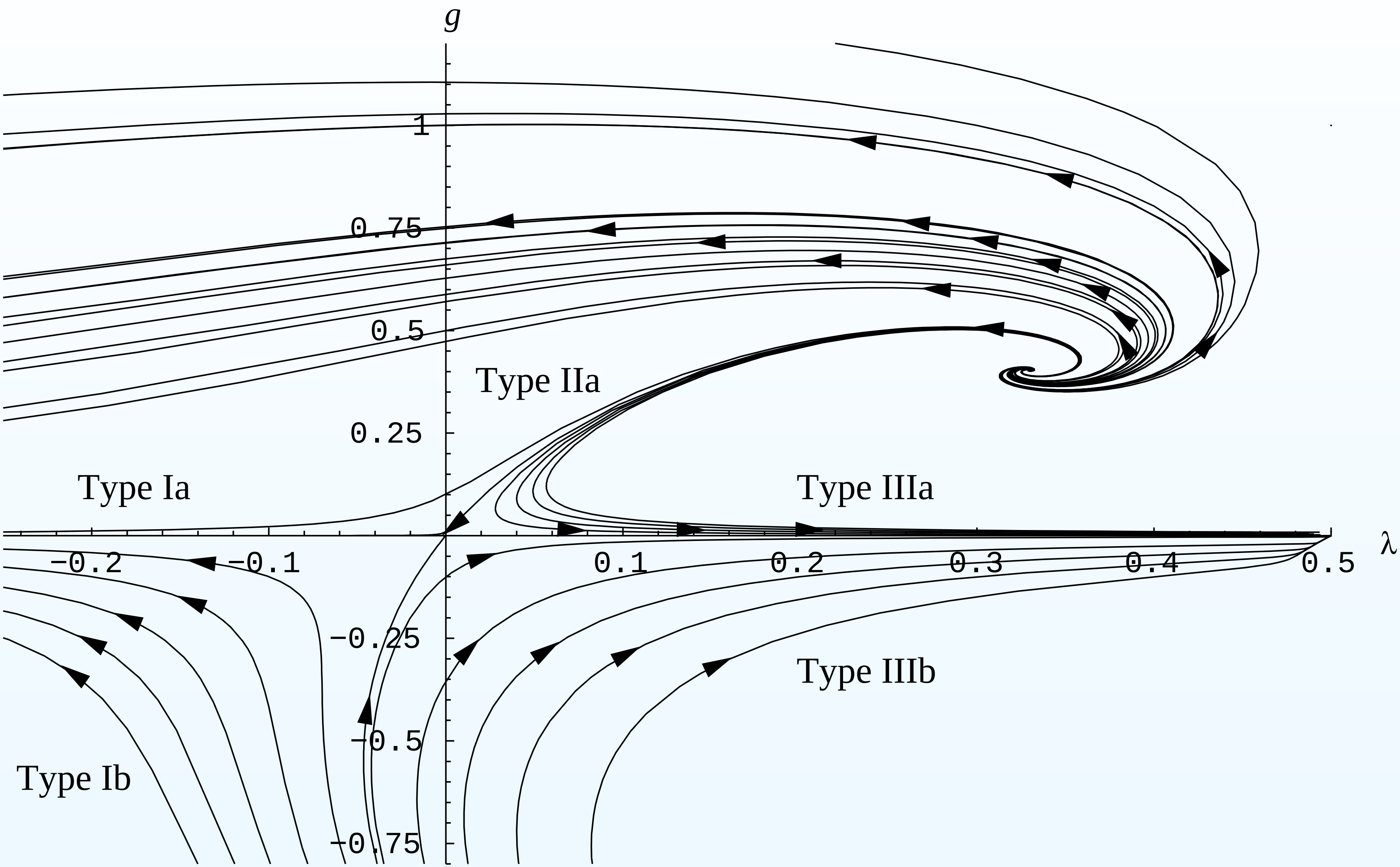


NGFP in $d = 4 \Longleftrightarrow$ analytic continuation of NGFP in $d = 2 + \epsilon$

Einstein-Hilbert-truncation: the phase diagram



where FRG and CDT meet

spectral dimension of space-time

Spectral dimension for classical manifolds

- Heat-equation: diffusion of scalar test particle on manifold with metric g

$$\partial_T K_g(x,x';T) = \Delta_g K_g(x,x';T)$$

- define average return probability

$$\begin{aligned} P_g(T) &\equiv \frac{1}{V} \int d^d x \sqrt{g(x)}\, K_g(x,x;T) \\ &= \frac{1}{V} \mathrm{Tr}\left[\exp(T\Delta_g)\right] \\ &= \left(\frac{1}{4\pi T}\right)^{d/2} \sum_{n=0}^{\infty} A_n T^n \end{aligned}$$

- asymptotic expansion: space-time dimension seen by diffusion process

$$d = -2 \frac{d \ln P_g(T)}{d \ln T}\bigg|_{T=0}$$

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$P(T)$: accessible in CDT and FRG!

Spectral dimension of QEG space-times

- in QEG: metric of manifold is k-dependent

  $\Longrightarrow$ diffusion process "with momentum k" sees metric $\langle g_{\mu\nu} \rangle_k$

  $\Longrightarrow$ diffusion equation and return probability will become k-dependent

- Computation of the spectral dimension:

  1. determine k-dependence of $\Delta(k)$
  2. solve the k-dependent heat equation
  3. evaluate "quantum return probability" $P(T)$
  4. obtain spectral dimension

$$\mathcal{D}_s = -2 \frac{d \ln P(T)}{d \ln T} \bigg|_{T=0}$$

Spectral dimension $\mathcal{D}_s$ of QEG space-times

- spectral dimension

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- Quantum return probability:

$$P(T) = \int \frac{d^d p}{(2\pi)^d} \exp[-p^2 F(p^2) T] , \quad F(p^2) = \Lambda(p)/\Lambda(k_0)$$

  - classical regime: no running, $F(p^2) = 1$:

$$P(T)|_{T=0} \propto T^{-d/2} \Longrightarrow \mathcal{D}_s = d$$

  - fixed point regime: $\Lambda(p) \propto p^2 \to F(p^2) \propto p^2$:

$$P(T)|_{T=0} \propto T^{-d/4} \Longrightarrow \mathcal{D}_s = d/2$$

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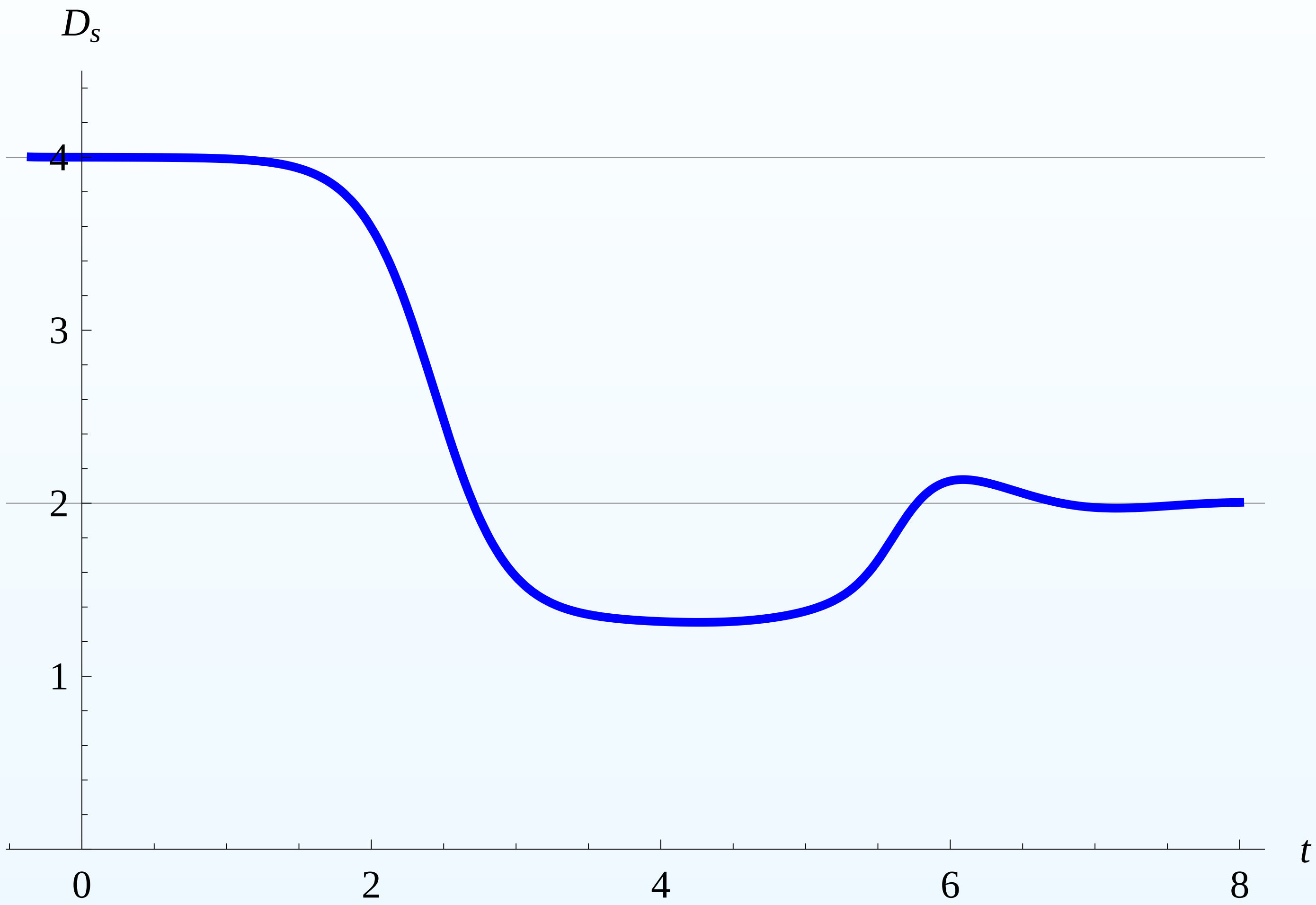
$d = 4$: QEG predicts continuous change of fractal dimension

$\mathcal{D}_s = 4$ macroscopically $\Longrightarrow$ $\mathcal{D}_s = 2$ microscopically

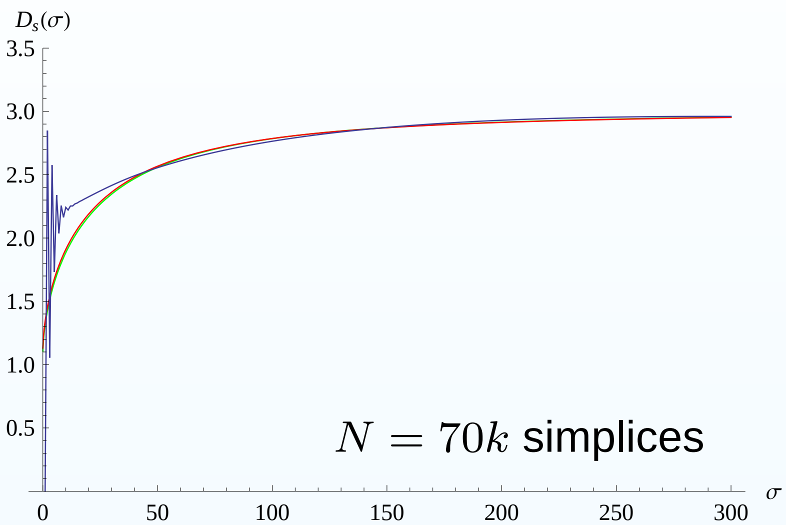
Spectral dimension $\mathcal{D}_s$ of QEG space-times

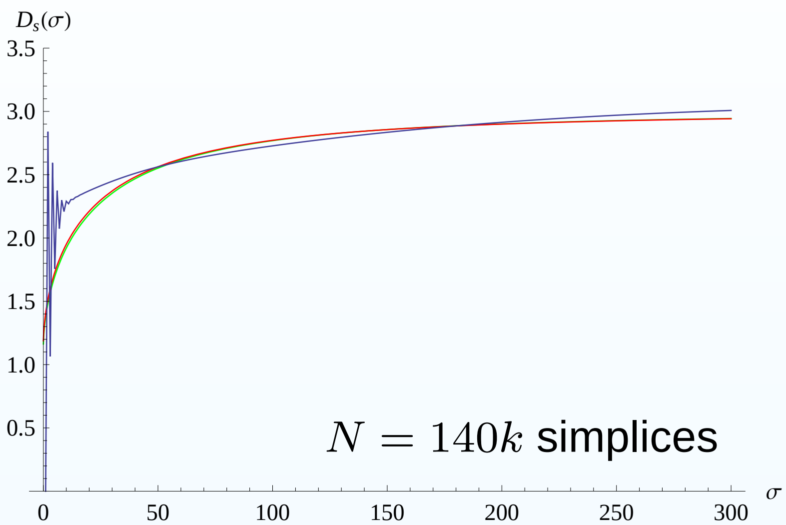
Flow of spectral dimension along a typical RG-trajectory



Matching FRG and 3d-lattice-simulations

[3d-CDT-Data: D. Benedetti and J. Henson, Phys. Rev. D80 (2009) 124036]





Semiclassical regime: parameterize leading quantum corrections:

$$\Lambda(p) = \Lambda_0 \left(1 + a\, p^\delta\right)$$

FRG-prediction: $\delta = 3$ $\approx$ Lattice fit: $\delta \approx 3.3$

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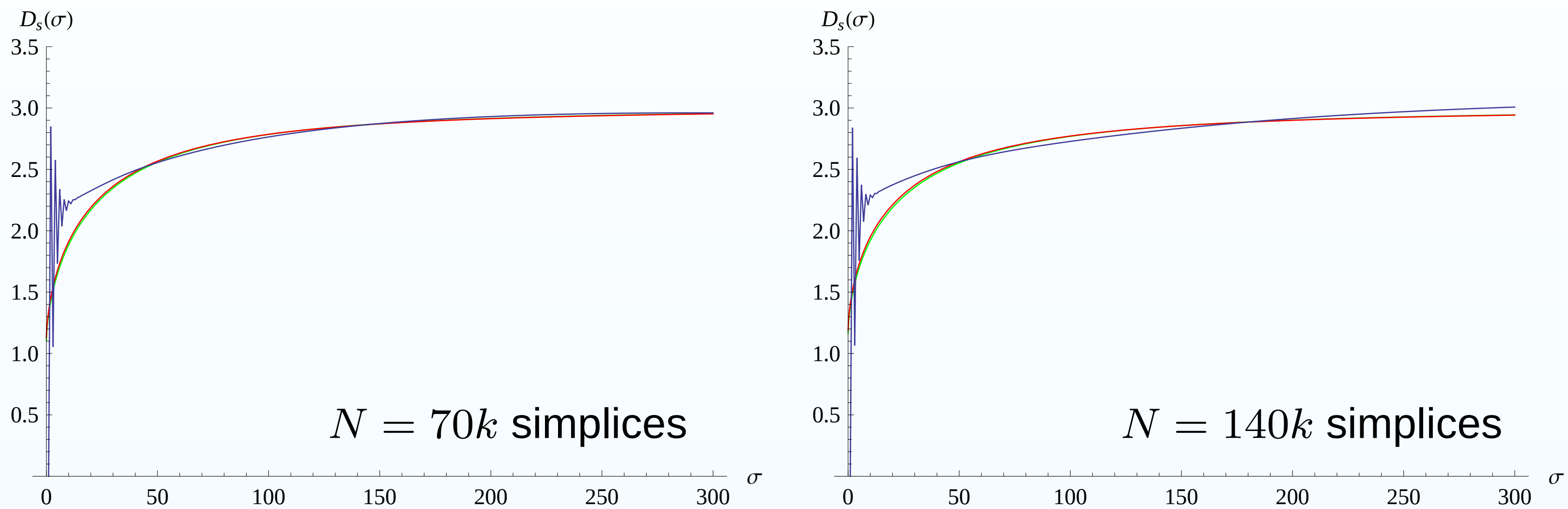

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can compare and fit CDT Data and RG-results

Summary

gravitational asymptotic safety program

UV completion of gravity provided by non-trivial RG fixed point

Functional Renormalization Group quations:

- all computations support the existence of this fixed point
- UV-critical surface has finite dimension $\Longleftrightarrow$ predictivity

Causal Dynamical Triangulations

- spectral dimension allows to compare lattice and continuum results!

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Thank you!