# Top-quark pair production at two loops

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# The top quark at hadron colliders

- **top quark**:
  - heaviest observed elementary particle: $m_t \approx 173$ GeV $\sim \Lambda_{\text{EW}}$

    $\Rightarrow$ Yukawa coupling $y_t \approx 1$
  - rapid electroweak decay: $\tau_t \approx 5 \cdot 10^{-25}$ s $\ll \tau_{\text{had}} \sim 10^{-24}$ s

    $\Rightarrow$ no bound states, $t$ polarisation imprint on decay particles
- **open questions** with special role of top quark:
  - electroweak symmetry breaking (coupl. to new resonances, $m_t$ crucial for EWPT)
  - flavour patterns, ...
- **Tevatron** and **LHC**: pair production total cross-section and distributions

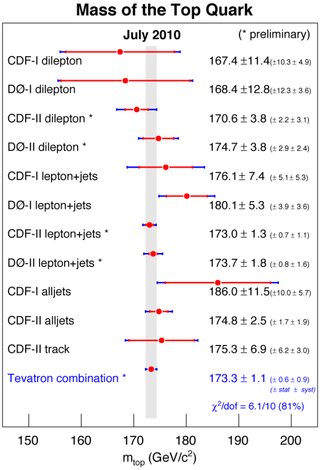
$$p\bar{p}, pp \to t\bar{t}X$$

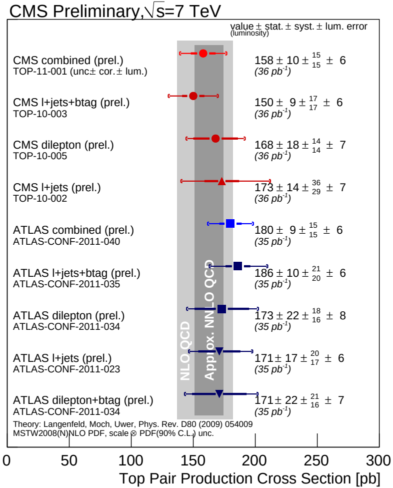
top decays $t \to bW^+$ ($\to bq\bar{q}'$ or $\to bl^+\nu_l$) lead to search channels:

$$\begin{aligned} p\bar{p}, pp \to t\bar{t}X &\quad\to\quad l_1^+ + l_2^- + j_b + j_{\bar{b}} + p_T^{\text{miss}} + (n \geq 0) \text{ jets} \\ p\bar{p}, pp \to t\bar{t}X &\quad\to\quad l_1^\pm + j_b + j_{\bar{b}} + p_T^{\text{miss}} + (n \geq 2) \text{ jets} \\ p\bar{p}, pp \to t\bar{t}X &\quad\to\quad j_b + j_{\bar{b}} + (n \geq 4) \text{ jets} \end{aligned}$$

- **precision** measurement and prediction important:
  - precise $m_t$ determination
  - background to missing $p_T$ signatures from new physics (MSSM, extra-dim., ...)

# Tevatron and LHC measurements



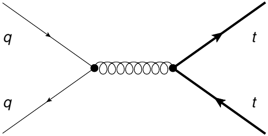




- $A_{FB}$: see talks by **S. Westhoff, B. Pecjak**
- experimental error at LHC already at NLO accuracy
- $\Rightarrow$ **need NNLO** (PDF errors below NLO accuracy)

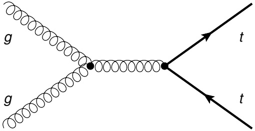
# Tree level processes for top pair production

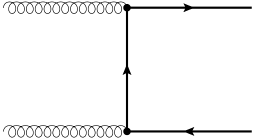
Top quark pair production can be calculated in **perturbative QCD**:
factorise PDFs and consider the 2 parton level processes:

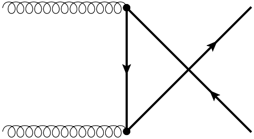
$q\bar{q} \to t\bar{t}$ channel (dominant at Tevatron $\sim 85\%$):




$gg \to t\bar{t}$ channel (dominant at LHC $\sim 90\%$):





# NLO and approximate higher orders

**NLO QCD** corrections to top pair production are known for a while:
Nason, Dawson, Ellis (1988-1990), Beenakker, Kuijf, van Neerven, Smith (1989), Beenakker, van Neerven, Meng, Schuler (1991), Mangano, Nason, Ridolfi (1992) Bernreuther, Brandenburg, Si, Uwer (2004)
for LHC $\sim$ 14% total uncertainty, not sufficient for expected experimental $\sim 5\,\%$

**Mixed QCD-EW** corrections in both channels (smaller than current QCD uncertainties):
Beenakker *et al.* (1994), Bernreuther, Fuecker, and Si (2005-2008), Kühn, Scharf, and Uwer (2005-2006), Moretti, Nolten, and Ross (2006)

**Threshold** and related resummations:
NLL: Berger, Contopanagos (1995,1996), Kidonakis and Sterman (1997), Bonciani *et al.* (1998), Kidonakis *et al.*(2001), Kidonakis, Vogt (2003), Banfi, Laenen (2005), Beneke, Falgari, Klein, Schwinn (2010),
NNLL from SCET: Ahrens, Ferroglia, Neubert, Pecjak, Yang (2011, 2011, 2011)
non-relativ. QCD at threshold: Kiyo, Kühn, Moch, Steinhauser, Uwer (2009)
"NNLO approx.": $\log\beta$, Coulomb, exact scale dep. Moch, Uwer (2008), Kidonakis, Vogt (2008), Langenfeld, Moch, Uwer (2009)

NLO $pp \to WWbb$ with **finite top width**:
Denner, Dittmaier, Kallweit, Pozzorini (2010), Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek (2010)

# Ingredients for full NNLO calculation

- **VV**: two-loop ME for $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$
- **RV**: one-loop ME for $t\bar{t}$ + 1 parton
  Dittmaier, Uwer, Weinzierl '07
- **RR**: tree level ME for $t\bar{t}$ + 2 partons
- **subtraction terms**: up to 2 unresolved partons needed
  Gehrmann-De Ridder, Ritzmann '09, Daleo et al. '09, Boughezal et al. '10, Glover, Pires '10; Czakon '10, '11, Anastasiou, Herzog, Lazopoulos '10

consider $2 \to 2$ ingredients:

$$\sum_{\text{spin,colour}} |\mathcal{M}|^2 = 16\pi^2\alpha_s^2 \left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{A}_2 + \mathcal{O}(\alpha_s^3)\right]$$

with

$$\mathcal{A}_0 = \left\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \right\rangle$$
$$\mathcal{A}_1 = 2\,Re \left\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \right\rangle$$
$$\mathcal{A}_2 = \left\langle \mathcal{M}^{(1)} | \mathcal{M}^{(1)} \right\rangle + 2\,Re \left\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \right\rangle$$

$\left\langle \mathcal{M}^{(1)} | \mathcal{M}^{(1)} \right\rangle$: Kniehl, Körner, Merebashvili, Rogal '05-'08, Anastasiou, Aybat '08

## Steps toward complete NNLO calculation

two-loop × tree interference: $2\,Re\left\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \right\rangle$

**small mass expansion**:

- Czakon, Mitov, Moch (2006) for $q\bar{q}$, $gg$

**IR poles**:

- Ferroglia, Neubert, Pecjak, Yang (2009) for $q\bar{q}$, $gg$

$q\bar{q}$ with **full dependence** on $s$, $t$, $m_t$, $\mu$:

- numerical result for all contributions:
  Czakon (2008)
- analytical result for fermionic:
  Bonciani, Ferroglia, Gehrmann, Maitre, Studerus (2008)
- analytical result for leading $N_c$:
  Bonciani, Ferroglia, Gehrmann, Studerus (2009)

$gg$ with **full dependence** on $s$, $t$, $m_t$, $\mu$:

- analytical result for leading $N_c$:
  Bonciani, Ferroglia, Gehrmann, A.v.M., Studerus (2010)
- analytical result for light fermionic:
  Bonciani, Ferroglia, Gehrmann, A.v.M., Studerus (in preparation)
- numerical result for all contributions:
  Czakon, Bärnreuther (in preparation)

# Outline

1 Introduction

2 Analytical two-loop calculation

3 Conclusions

# Gauge invariant subsets in two-loop contributions

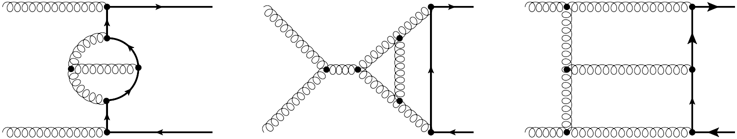
*gg* **channel**: 789 two-loop diagrams (+ ghost init.) contrib. to 16 coeff.:

$$2\,Re\left\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \right\rangle = 2C_F N_c \big( N_c^3 \mathbf{A} + N_c B + \frac{1}{N_c} C + \frac{1}{N_c^3} D$$
$$+ N_c^2 n_l E_l + n_l F_l + \frac{n_l}{N_c^2} G_l + N_c n_l^2 H_l + \frac{n_l^2}{N_c} I_l$$
$$+ N_c^2 n_h E_h + n_h F_h + \frac{n_h}{N_c^2} G_h + N_c n_h^2 H_h + \frac{n_h^2}{N_c} I_h$$
$$+ N_c n_l n_h H_{lh} + \frac{n_l n_h}{N_c} I_{lh} \big)$$

$q\bar{q}$ **channel**: 218 two-loop diagrams, 10 coefficients

example: for **leading $N_c$ coefficient $A$** we need:

- 300 two-loop diagrams (+ ghost initiated), e.g.:



- two independent ratios of scales
- up to: 4-point, 7 propagators, 4 loop momenta in numerator

# Organisation of the calculation

## Recipe

1. generate **Feynman diagrams** with `QGRAF`
2. build **interference** terms
3. **reduce** scalar integrals to masters with **parallel Laporta**
4. **solve masters** with differential equations
5. **renormalize**: $\overline{MS}$, pole mass

⇒ **analytical result** in terms of generalized polylogarithms, allows fast **numerical evaluation**, **expansions**, ...

various tasks automatized in computer program Reduze 2
A.v.M., Studerus (open source, in preparation)

# Reductions via IBP identities

## integration by part (IBP) identities

$$\int d^d k^{(1)} \dots d^d k^{(n)} \frac{\partial}{\partial k_\mu^{(i)}} \left( k^{(j)} \frac{1}{D_1^{n_1} \cdots D_N^{n_N}} \right) = 0,$$

$$\int d^d k^{(1)} \dots d^d k^{(n)} \frac{\partial}{\partial k_\mu^{(i)}} \left( p^{(j)} \frac{1}{D_1^{n_1} \cdots D_N^{n_N}} \right) = 0$$

where $p^{(i)}$ are external momenta and e.g. $D_1 = (k^{(1)} - k^{(2)} - p^{(1)})^2 - m^2, \dots$

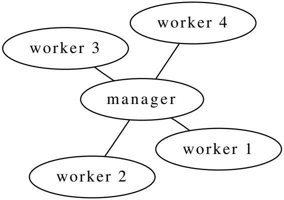
- leads to **linear relations** between different Feynman integrals
- exploit for **systematic reduction** to a few master integrals (MIs)
  Chetyrkin, Tkachov (1981)

reduction techniques:

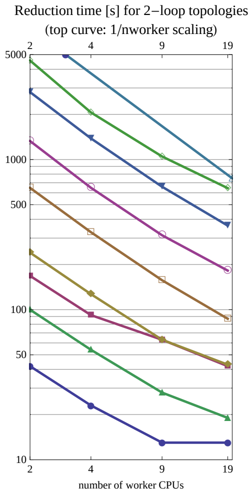
- Laporta: order integrals, reduce lin. system
  **public codes**: Anastasiou: `AIR`, Smirnov: `FIRE`, Studerus: `Reduze`
- Smirnov, Smirnov: S-basis for differential operators
  **public code**: `FIRE`
- Gluza, Kajda, Kosower: Gram-determinants, Gröbner basis for *n*-tuples

# Reduze 2: parallelisation of Laporta's algorithm

- **full parallelisation** (MPI):
  - multiple sectors
  - within one sector
- dynamical **load balancing**:




A.v.M., Studerus (in preparation)




(example: subtopologies of massive double box)

## Reduze 2

**reduction:**

- fully parallelised algebraic reductions
- generation of differential equations for masters

**topological analysis:**

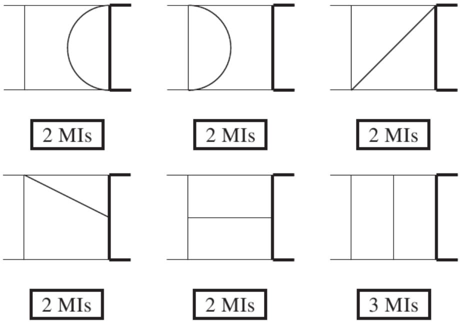
- finding shift of diagram loop momenta to sectors
- finding shift relations between sectors

**other features:**

- written in `C++`, OO design, `MPI` for parallel., `YAML` for config
- uses `GiNaC` by Bauer, Frink, Kreckel (open source, free)
- uses transactional database BerkleyDB (open source, free)
- optional: `Fermat CAS` by Lewis (closed source, non-free)
- `QGRAF` input
- `FORM`, `Mathematica` output
- resume of aborted reduction runs
- computation of QCD diagram interferences up to masters
- to be released as open source, free
  A.v.M., Studerus (in preparation)

# Some needed master integrals

for leading colour coefficients e.g. these four-point masters are needed:




Bonciani, Ferroglia, Gehrmann, Studerus (2009)

## Solving masters with differential equations

1. derive **differential equations** (derivatives taken at integrand level)

$$\frac{\partial}{\partial s} I(s,t) = \sum c_j I_j, \qquad \frac{\partial}{\partial t} I(s,t) = \sum d_j I_j$$

2. **solve** order by order in $\epsilon = (4-d)/2$ up to a constant
   - for more than 1 master integral: need decoupling of diff. eq.
3. **fix constant** with regularity cond., Mellin-Barnes for special kinematics
   - `MB.m` by Czakon, A. Smirnov (2005), for planar: `AMBRE.m` by Gluza, Kajda, Riemann (2007)

result in terms of GPLs (Remiddi, Gehrmann; Goncharov):

### Definition of generalised polylogarithms (GPLs)

$$G(\vec{0}_n; x) = \frac{1}{n!}\log^n(x),$$

$$G(a; x) = \int_0^x dt\, \frac{1}{t-a}, \qquad \text{for } a \in \mathbb{C}, \quad a \neq 0$$

$$G(a, \vec{b}; x) = \int_0^x dt\, \frac{1}{t-a} G(\vec{b}; t), \qquad \text{for } a \in \mathbb{C}, \quad a \neq 0$$

- $G(a;x) = \log\left(\frac{a-x}{a}\right)$, $\quad G(\vec{0}_{n-1}, 1; x) = Li_n(x)$, $\quad G(\vec{0}_n, \vec{1}_p; x) = S_{n,p}(x)$, HPLs
- shuffle algebra, symbols (see $N = 4$ remainder func), ...
- here: need 2-dim. GPLs up to weight 4, e.g.: $G\left(-1, \frac{1}{z}, 0, -1; x\right)$

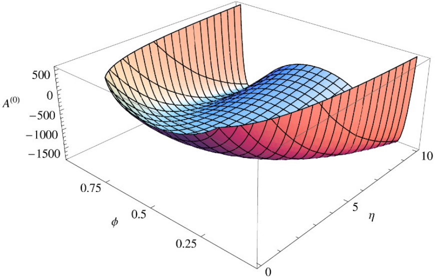
# Results for $gg$ leading $N_c$: finite part of $A$

- full analytical result
- fast **numerical program** with GiNaC's GPLs by Vollinga, Weinzierl (2005)

for $gg$ leading $N_c$: $A = \frac{A^{(-4)}}{\varepsilon^4} + \frac{A^{(-3)}}{\varepsilon^3} + \frac{A^{(-2)}}{\varepsilon^2} + \frac{A^{(-1)}}{\varepsilon} + A^{(0)}$

$$A^{(0)} = \left( \frac{4(7 - 26y - 9y^2)}{(1+y)(1+z)} + \ldots \right) G\left(-1, \frac{1}{z}, 0, -1; x\right) \qquad \text{with } y = -\frac{t}{m^2}, \quad z = -\frac{u}{m^2}, \quad x = \frac{\sqrt{s} - \sqrt{s - 4m^2}}{\sqrt{s} + \sqrt{s - 4m^2}}$$

+ some more GPLs, also of lower weights



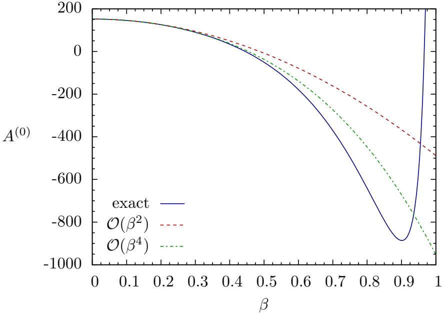

with $\eta = \frac{s}{4m^2} - 1,$

$$\phi = -\frac{(t - m^2)}{s}$$

Bonciani, Ferroglia, Gehrmann, A.v.M., Studerus (2010)

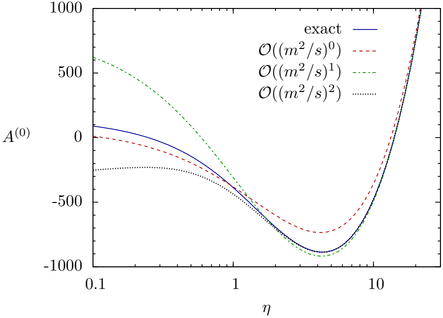
# Results for $gg$ leading $N_c$: expansions

threshold expansion:




$\beta = \sqrt{1 - 4m_t^2/s}$,
c.m. scatt. angle $= \pi/2$

small mass expansion:

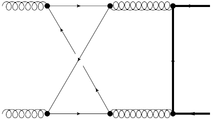



$\eta = \frac{s}{4m_t^2} - 1$,
c.m. scatt. angle $= \pi/2$

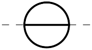
(careful with phase space for small $m$ expansion:
don't introduce forw.-backw. asymmetry, $\phi = -(t - m_t^2)/s = const$ )

Bonciani, Ferroglia, Gehrmann, A.v.M., Studerus (2010)

# Remaining corrections

- **massless fermion loops in $gg$:**
   etc.,
  non-planar diagrams, known techniques,
  Bonciani, Ferroglia, Gehrmann, A.v.M., Studerus (in preparation)

- **gluonic subleading $N_c$ in $qq$ and $gg$:**
  more non-planar diagrams

- **closed top loop in $gg$:**
  subtopologies involve elliptic functions:
  
  sunset for $p^2 \neq m^2 \Rightarrow K(z) = \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-zx^2)}}$ (Laporta, Remiddi '04)
  beyond GPLs, need new techniques

# Outline

# Conclusions

- top quark pair production: essential to test Standard Model and new physics
- matching expected experimental precision for $t\bar{t}$: complete NNLO QCD prediction
- virtual corrections:
  - $q\bar{q} \to t\bar{t}$: numerical result + leading-$N_c$/fermionic analytical ✓
  - $gg \to t\bar{t}$: leading-$N_c$ analytical ✓
- real radiation (substraction terms): progress by several groups
- complete (fast) NNLO QCD code: some work left to do ...