Pion elastic and $\pi \to \gamma\gamma^*$ form factors in a broad range of momentum transfers

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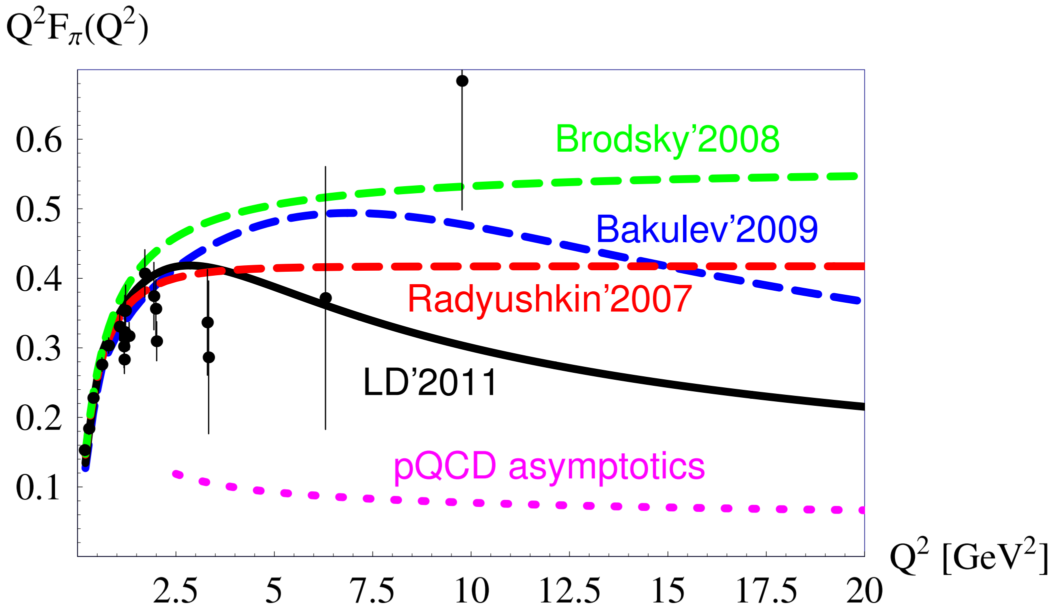
We study $F_\pi(Q^2)$ and $F_{\pi\gamma}(Q^2)$ making use of the local-duality (LD) version of QCD sum rules.

To probe the accuracy of the LD sum rule, we consider in parallel to QCD a potential model with an interaction consisting of Coulomb and confining parts: in this case, the exact form factor may be calculated from the solutions of the Schroedinger equation and confronted with the result from the quantum-mechanical LD sum rule, thus probing the accuracy of the method.

We argue that the LD sum rule is expected to give reliable predictions for $F_\pi(Q^2)$ and $F_{\pi\gamma}(Q^2)$ in the region $Q^2 \geq 6$ GeV^2.

Based on "Accuracy of local-duality sum rules for the pion elastic form factor" arXiv:1103.3781 and work on $F_{\pi\gamma}$ in progress.

Theoretical description of the pion form factor at $Q^2 \sim 5 - 50$ GeV^2 in QCD is a complicated problem:



No conclusive results have been obtained and we still have a strong discrepancy between the results from various versions of QCD sum rules.

Our goal is to study $F_\pi(Q^2)$ and $F_{\pi\gamma}(Q^2)$ making use of the so-called local-duality version of QCD sum rules.

The value of any hadron observable, e.g. the form factor, extracted from QCD sum rules depends on two ingredients:

(i) the field-theoretic calculation of the OPE series for the relevant correlator.

(ii) the details of the implementation of quark-hadron duality.

The error introduced by the implementation of quark-hadron duality is very hard to control.

Local-duality (LD) model - specific implementation of quark-hadron duality in QCD sum rules.

- **The pion elastic form factor: one starts with the Borel sum rules for three-point function**

$$\langle 0|T\, j^5_\alpha j_\mu j^5_\beta|0\rangle$$

Implementing quark-hadron duality in the standard way — as the low-energy cut on the perturbative contributions to the correlators — one obtains the following sum rule in the chiral limit:

$$f_\pi^2 F_\pi(Q^2) = \int_0^{s_{\rm eff}(\tau,Q^2)} ds_1 \int_0^{s_{\rm eff}(\tau,Q^2)} ds_2 e^{-\frac{(s_1+s_2)\tau}{2}} \Delta_{\rm pert}(s_1, s_2, Q^2) + \frac{\langle\alpha_s G^2\rangle}{24\pi}\tau + \frac{4\pi\alpha_s\langle\bar{q}q\rangle^2}{81}(13 + Q^2\tau)\tau^2 + \cdots$$

$\Delta_{\rm pert}$ **are double spectral densities of 3-point diagrams of the perturbation theory.**

<u>Problem:</u> **Power corrections in three-point function rise with Q. To apply sum-rule at large Q: one of the few possibilities is just set $\tau = 0$.**

The Local – duality (LD) limit is $\tau \to 0$. Then ALL power corrections vanish.

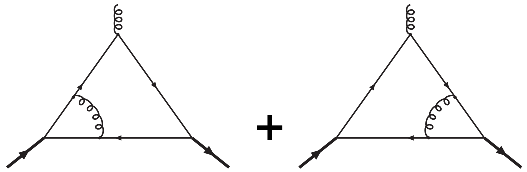
$$F_\pi(Q^2) = \frac{1}{f_\pi^2} \int\limits_0^{s_{\rm eff}(Q^2)} ds_1 \int\limits_0^{s_{\rm eff}(Q^2)} ds_2\, \Delta_{\rm pert}(s_1, s_2, Q^2).$$

The problem is now how to determine $s_{\rm eff}(Q^2)$.

Properties of the spectral functions

- **Vector Ward identity at $Q^2 = 0$ relates 3-point and 2-point functions.**

- **Factorization at $Q^2 \to \infty$: the leading $1/Q^2$ behavior of the spectral function is given by**



If we set

$$s_{\rm eff}(Q^2 = 0) = \frac{4\pi^2 f_\pi^2}{1 + \alpha_s/\pi} \qquad s_{\rm eff}(Q^2 \to \infty) = 4\pi^2 f_\pi^2,$$

then the form factor obtained from the LD sum rule satisfies the correct normalization at $Q^2 = 0$ and reproduces the asymptotic behavior according to the factorization theorem for the form factor at $Q^2 \to \infty$.

The two values are not far from each other, construct an interpolation function $s_{\rm eff}(Q^2)$ for all Q^2.

The local – duality model for hadron elastic form factors :

a. Based on a dispersive three-point sum rule at $\tau = 0$ (i.e. infinitely large Borel mass parameter). In this case all power corrections vanish and the details of the non-perturbative dynamics are hidden in one quantity — the effective threshold $s_{\rm eff}(Q^2)$.

b. Makes use of a model for $s_{\rm eff}(Q^2)$ based on a smooth interpolation between its values at $Q^2 = 0$ determined by the Ward identity and at $Q^2 \to \infty$ determined by factorization. Since these values are not far from each other, the details of the interpolation are not essential.

Obviously, the LD model for the effective continuum is a crude model which does not take into account the details of the confinement dynamics. The only property of theory relevant for this model is factorization of hard form factors.

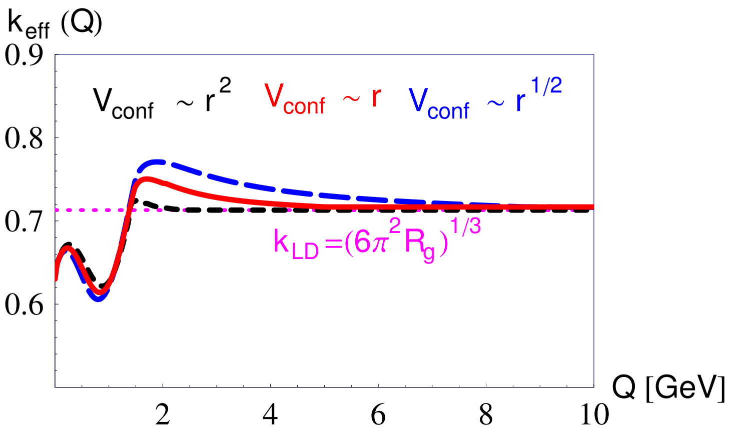
The model may be tested in quantum mechanics for the case of the potential containing the Coulomb and Confining interactions.

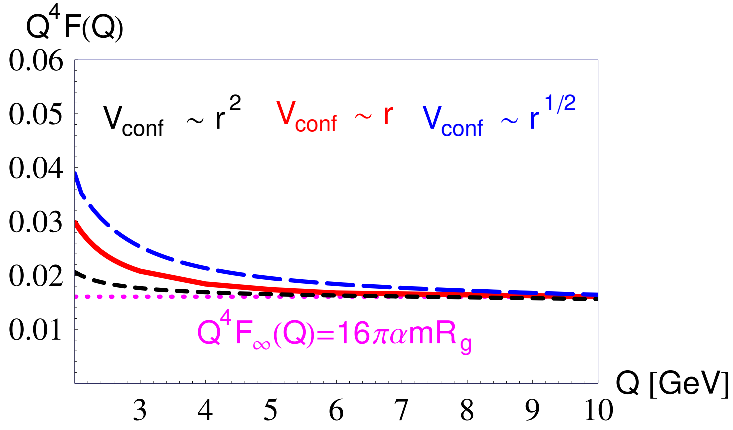
- **The form factor satisfies factorization theorem similar to QCD. LD sum rules is very similar to QCD; the spectral densities are calculated from diagrams of NR field theory.**

- **The exact form factor may be calculated and confronted with LD model, probing its accuracy.**

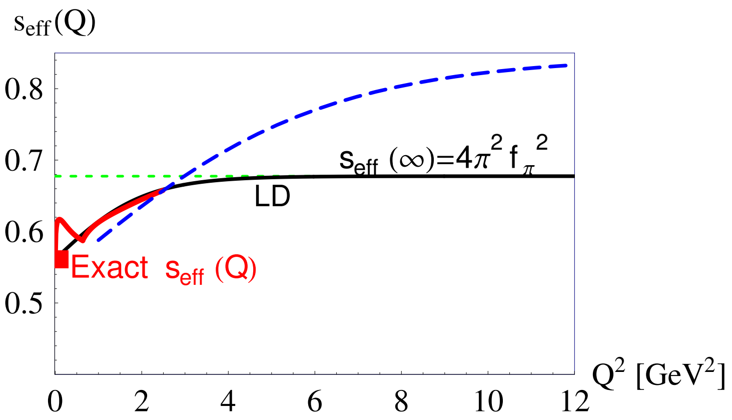
Elastic form factor

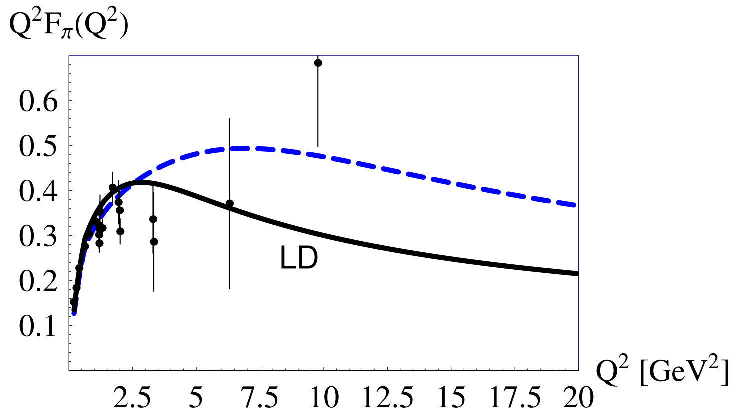
Results for elastic form factor in quantum-mechanical potential model





Results for elastic pion form factor in QCD:





$P \to \gamma\gamma^*$ transition form factor

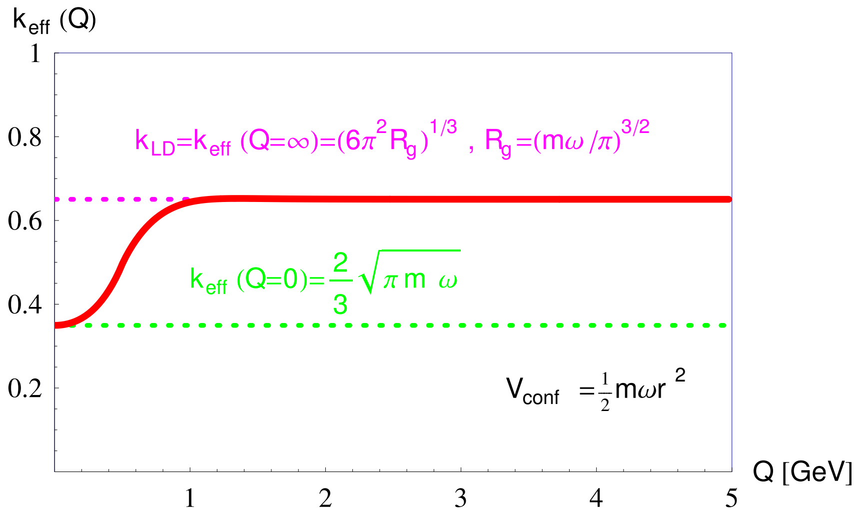
A Borel sum rule for

$$\langle 0|T j_\alpha j_\beta j^5_\mu|0\rangle$$

at $\tau = 0$:

$$F_{\pi\gamma}(Q^2) = \frac{1}{f_\pi} \int\limits_0^{s_{\rm eff}(Q^2)} \Delta_{\rm pert}(s, Q^2, Q'^2 = 0)\, ds, \qquad s_{\rm eff}(Q^2 \to \infty) \to 4\pi^2 f_\pi^2.$$

Quantum mechanics:



$\eta,\ \eta' \to \gamma\gamma^*$ transition form factor

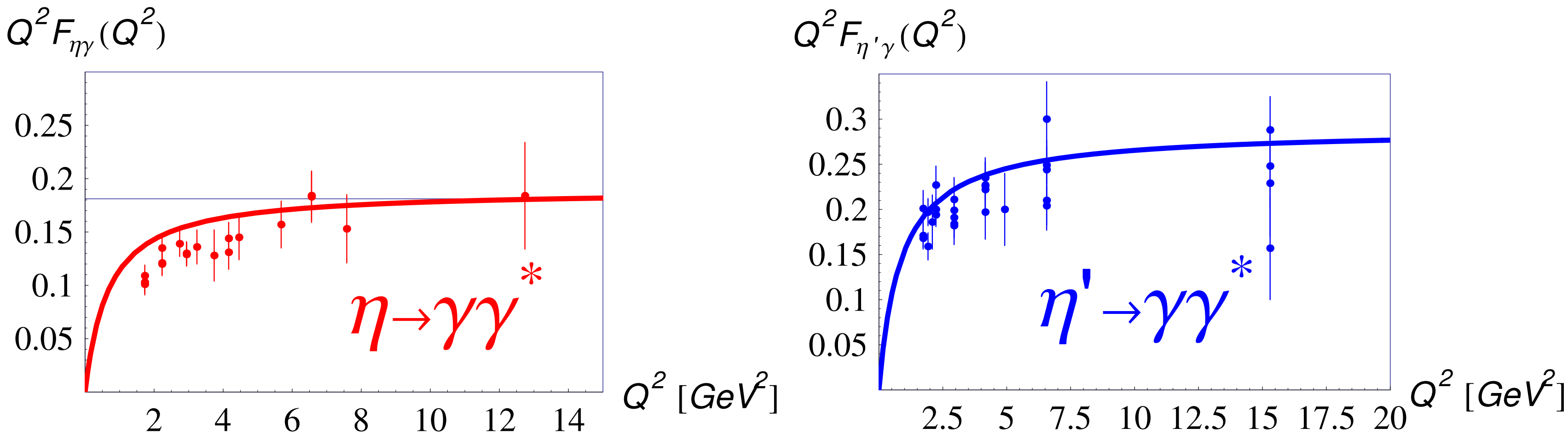
Mixing scheme by Feldmann et al:

$$F_{\eta\gamma} = \cos(\phi)F_{n\gamma} - \sin(\phi)F_{s\gamma}, \quad F_{\eta'\gamma} = \sin(\phi)F_{n\gamma} + \cos(\phi)F_{s\gamma}, \quad \phi \simeq 38^0$$

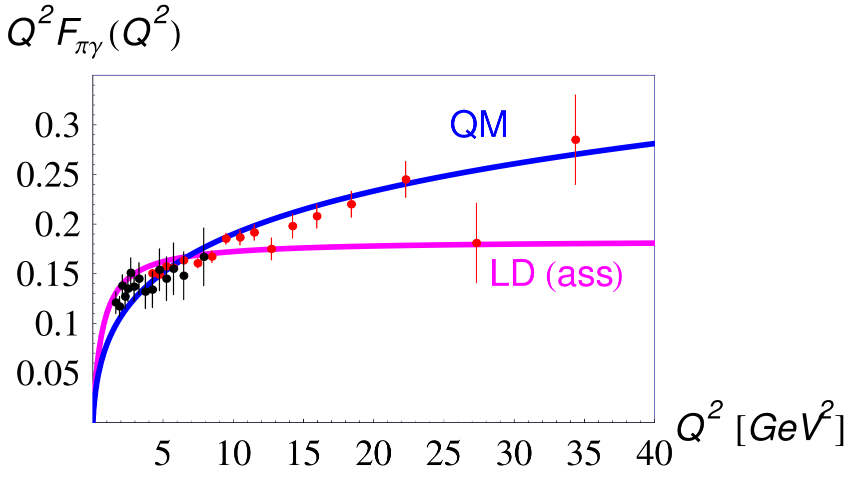
where

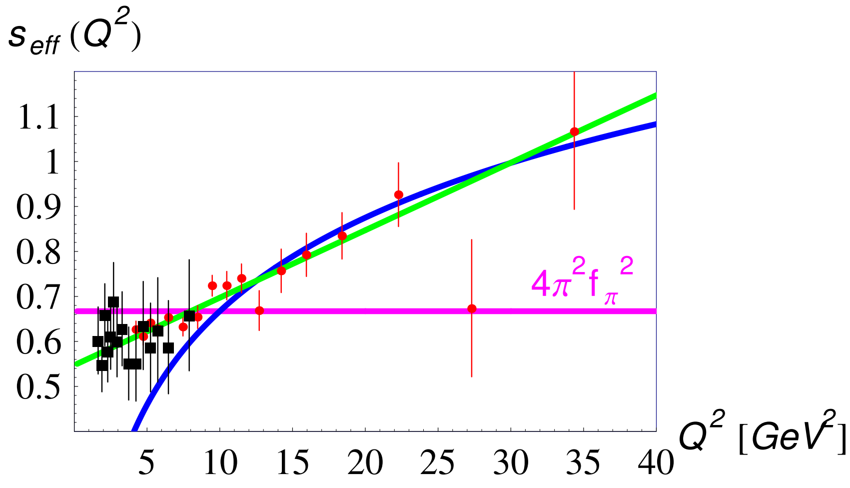
$$F_{n\gamma}(Q^2) = \frac{1}{f_n}\int_0^{s_{\rm eff}^{(n)}(Q^2)} \Delta_n(s, Q^2)\, ds, \qquad F_{s\gamma}(Q^2) = \frac{1}{f_s}\int_0^{s_{\rm eff}^{(s)}(Q^2)} \Delta_s(s, Q^2)\, ds,$$

Two separate effective thresholds: $s_{\rm eff}^{(n)} = 4\pi^2 f_n^2, \quad s_{\rm eff}^{(s)} = 4\pi^2 f_s^2, \quad f_n \simeq 1.07 f_\pi, \quad f_s \simeq 1.36 f_\pi$.



$\pi^0 \to \gamma\gamma^*$ transition form factor





Puzzle: why nonstrange components in η, η' and π^0 should behave so much differently?

Summary and conclusions

We investigated the pion form factors by means of a LD model which may be formulated in any theory where hard exclusive amplitudes satisfy the factorization theorem (in essence, any theory where the interaction behaves as a Coloumb-like interaction at small distances and as a confining interaction at large distances).

Our main conclusions are as follows:

• For the elastic form factor, independently of the details of the confining interaction, the predictions of the LD model exhibit maximal deviations from the exact form factor in the region $Q^2 \approx 4-8$ GeV^2. As Q increases further, the accuracy of the LD model increases rather fast. For arbitrary confining interaction, the LD model gives very accurate results for $Q^2 \geq 20-30$ GeV^2.

The accurate data on the pion form factor indicate that the LD limit for the effective threshold $s_{\rm eff}(\infty) = 4\pi^2 f_\pi^2$ is reached already at relatively low values $Q^2 = 5-6$ GeV^2; thus, large deviations from the LD limit at $Q^2 = 20-50$ GeV^2 appear to us unlikely.

• For the $P \to \gamma\gamma^*$ form factors, the LD model should work well in the region $Q^2 \geq$ a few GeV^2. At small Q^2 the deviation of the exact threshold from the LD threshold is large.

The LD predictions work well for $\eta \to \gamma\gamma^*$ and $\eta' \to \gamma\gamma^*$ form factors.

For $\pi \to \gamma\gamma^*$ form factor the present BaBar data indicate extreme violation of local duality prompting a linearly rising effective threshold.