Hadronic matrix elements for exclusive rare *B*-meson decays

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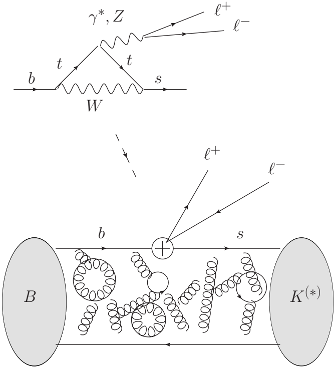
FCNC exclusive decays: $B \to K^{(*)}\ell^+\ell^-$, $B \to K^*\gamma$

- in Standard Model, $b \to s$ transitions , via t, W, Z loops



$$\boxed{H_{eff} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)\,,}$$

$$O_{9(10)} = \frac{\alpha_{em}}{2\pi}[\bar{s}\gamma_\mu(1-\gamma_5)b]\ell\gamma^\mu(\gamma_5)\ell,$$

$C_9(m_b) \simeq 4.4$, $C_{10}(m_b) \simeq = -4.7$,

$$O_7 = -\frac{m_b}{8\pi^2}[\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b]F^{\mu\nu},$$

$C_7(m_b) \simeq -0.3$

O_{1-6} - 4-quark , O_8- quark-gluon also contribute !

$B \to K^{(*)}\,\ell^+\ell^-$, $B \to K^*\gamma$, the observables

- decay amplitudes:
  $A(B \to K^{(*)}\ell^+\ell^-) = \langle K^{(*)}\ell^+\ell^- \mid H_{eff} \mid B\rangle$
- all decay distributions, BR's, asymmetries, etc.
  determined by $|A(B \to K^{(*)}\ell^+\ell^-)|^2 \times \{\textit{phase space}\}$
- $A(B \to K^*\gamma) = \langle K^{(*)}\gamma \mid H'_{eff} \mid B\rangle$ H'_{eff} without $O_{9,10}$
  important contributions are the same as in
  $A(B \to K^{(*)}\ell^+\ell^-)$ at $q^2 = 0$,
- $B \to K^{(*)}\ell^+\ell^-$ contributes to inclusive $B \to X_s\ell^+\ell^-$, but !
  in the theory, the inclusive width is defined and treated differently.

  For exclusive decays we need hadronic matrix elements

$B \to K, K^*$ form factors

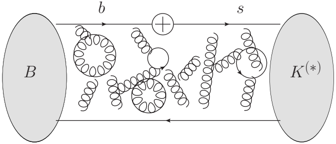
- contributions of $O_{9,10}$ and O_7 factorize, e.g.,

$$\langle K^{(*)}(p)|C_9 O_9|B(p+q)\rangle$$

$$= \frac{\alpha_{em}}{2\pi} C_9 \langle K^{(*)}(p)|\bar{s}\gamma_\mu(1-\gamma_5)b|B(p+q)\rangle(\bar{\ell}\gamma_\mu\ell)$$

$$\Downarrow$$

$B \to K$ and $B \to K^*$
form factors



- form factors depend on $q^2 = (p_{\ell^+} + p_{\ell^-})^2$,
  $0 < q^2 < (m_B - m_{K^{(*)}})^2$ in $B \to K^{(*)}\ell^+\ell^-$, $q^2 = 0$ in $B \to K^*\gamma$
- dominated by nonperturbative quark-gluon interactions,

Calculating the form factors in QCD

- Lattice QCD with growing accuracy:
  currently, $B \to \pi, K$ at large q^2 , $n_f = 3$,
  $B \to \rho, K^*$ only quenched

- non-lattice method: QCD Light-cone sum rules (LCSR)

  - outline:
  correlation function in QCD via light-cone OPE
  $\Rightarrow$ hadronic dispersion relation $\Rightarrow$ ground state contribution
  $\Rightarrow$ form factor

  - main input:
  quark masses, α_s,
  light-cone distribution amplitudes (DA's) of $\pi, K, B, ...$

Status and accuracy of LCSR calculations

- the region $q^2 \leq 12 - 15$ GeV^2 accessible,
  complementing the lattice FF's
- $B \to \pi, K$ form factors
  [G.Duplancic, AK, Th.Mannel, B.Melic, N.Offen (2008)] ,
  [A.K, Th.Mannel, N.Offen, Y-M. Wang (2011)]
  [G.Duplancic, B.Melic (2008)], *[A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]*
- estimated uncertainties for $B \to \pi, K$ $\pm(12 - 15)\%$
- $B_{(s)} \to \rho$, ω, K^*, ϕ form factors, DA's of K^* and ρ
  [P.Ball, R.Zwicky (2005)]
- $B \to \pi, K, \rho, K^*, ...$
  LCSR with B-meson distribution amplitudes
  [A.K., Th. Mannel, N.Offen (2005)]
  LCSR in SCET [F. De Fazio, Th. Feldmann and T.Hurth (2006)]

$B \to K, K^{(*)}$ form factors from LCSR

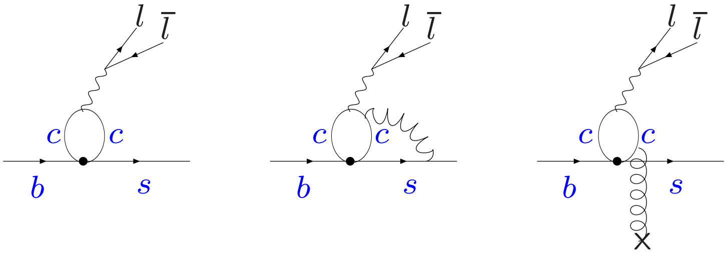
[A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]

form factor	$F^i_{BK^{(*)}}(0)$	b^i_1	$B_s(J^P)$	input at $q^2 < 12$ GeV^2
f^+_{BK}	$0.34^{+0.05}_{-0.02}$	$-2.1^{+0.9}_{-1.6}$	$B^*_s(1^-)$	
f^0_{BK}	$0.34^{+0.05}_{-0.02}$	$-4.3^{+0.8}_{-0.9}$	no pole	LCSR
f^T_{BK}	$0.39^{+0.05}_{-0.03}$	$-2.2^{+1.0}_{-2.00}$	$B^*_s(1^-)$	with K DA's
V^{BK^*}	$0.36^{+0.23}_{-0.12}$	$-4.8^{+0.8}_{-0.4}$	$B^*_s(1^-)$	
$A_1^{BK^*}$	$0.25^{+0.16}_{-0.10}$	$0.34^{+0.86}_{-0.80}$	$B_s(1^+)$	
$A_2^{BK^*}$	$0.23^{+0.19}_{-0.10}$	$-0.85^{+2.88}_{-1.35}$	$B_s(1^+)$	LCSR
$A_0^{BK^*}$	$0.29^{+0.10}_{-0.07}$	$-18.2^{+1.3}_{-3.0}$	$B_s(0^-)$	with B DA's
$T_1^{BK^*}$	$0.31^{+0.18}_{-0.10}$	$-4.6^{+0.81}_{-0.41}$	$B^*_s(1^-)$	
$T_2^{BK^*}$	$0.31^{+0.18}_{-0.10}$	$-3.2^{+2.1}_{-2.2}$	$B_s(1^+)$	
$T_3^{BK^*}$	$0.22^{+0.17}_{-0.10}$	$-10.3^{+2.5}_{-3.1}$	$B_s(1^+)$	

using BCL *[Bourrely,Caprini, Lellouch(2008)]* parameterization, b_1-slope parameter

Charm-loops and other complications

- a combination of the $(\bar{s}c)(\bar{c}b)$ weak interaction ($O_{1,2}$) and e.m.interaction $(\bar{c}c)(\bar{\ell}\ell)$ “mimicking FCNC”
- Charm-loop effect:

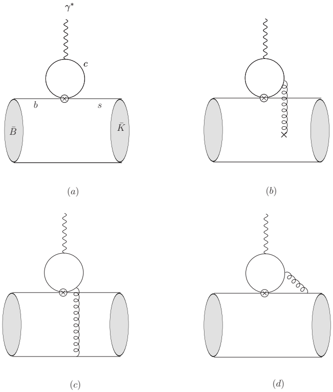


- similar u, d, s, c, b-quark loops (O_{3-6} operators), u-loops from $O^u_{1,2}$ (CKM suppressed in $b \to s$),
- "weak annihilation" $\oplus$ photon emission
- factorization is lost !
- $A(B \to K^{(*)}\ell^+\ell^-)$ include additional hadronic matrix elements, not simply form factors

Charm loop turns charmonium

- at $q^2 \to m_{J/\psi}, \dots$ an on-shell hadronic state:
  $B \to J/\psi K \otimes J/\psi \to \ell^+\ell^-$
- other ψ-levels (charmonia with $J^P = 1^-$),
  open-charm states $B \to \bar{D}DK \to K\ell^+\ell^-$,
  ($\bar{c}c$ states with the masses up to $m_B - m_K^{(*)}$)
- J/ψ and $\psi(2S)$ bins are subtracted from the q^2-distribuiton data in $B \to K^{(*)}\ell^+\ell^-$
- the effect of intermediate virtual $\bar{c}c$ states remains at
  $q^2 \ll m^2_{J/\psi}$ (nonperturbative at $q^2 \sim 4m_c^2$)

Charm-loop in $B \to K^{(*)}\ell^+\ell^-$



- factorizable c-quark loop
$C_9 \to C_9 + (C_1 + 3C_2)g(m_c^2, q^2)$

- perturbative gluons $\to$ (nonfactorizable) corrections being factorized in $O(\alpha_s)$ and added to C_9
[M. Beneke,T. Feldmann,D. Seidel (2001)]

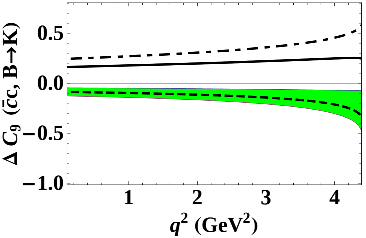
- taking into account the soft gluons (low-virtuality, nonvanishing momenta) emitted from the c-quark loop
[A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]
- light-cone expansion $\Rightarrow$ nonlocal effective $\bar{s}Gb$ operator
$\sim 1/(4m_c^2 - q^2)$-suppression
- the $B \to K^{(*)}$ hadronic matrix elements of this operator calculated using the same LCSR method as for $B \to K^{(*)}$ form factors

Charm-loop effect in $B \to K\ell^+\ell^-$ in terms of ΔC_9

- the effective coefficient $C_9(\mu = m_b) \simeq 4.4$
  a process-dependent correction to be added:

$$\Delta C_9^{(\bar{c}c, B\to K)}(q^2) = (C_1 + 3C_2)\, g(m_c^2, q^2) + 2C_1 \frac{32\pi^2}{3} \frac{\tilde{A}(q^2)}{f_{BK}^+(q^2)}$$

$\Delta C_9(0) = 0.17^{+0.09}_{-0.18}$,
$(\mu = m_b = 4.2\text{GeV})$



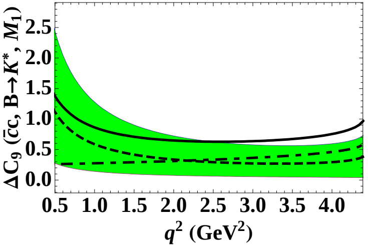
Charm-loop effect for $B \to K^* \ell^+ \ell^-$

- factorizable part determined by the three $B \to K^*$ form factors $V^{BK^*}(q^2)$, $A_1^{BK^*}(q^2)$, $A_2^{BK^*}(q^2)$,
- three kinematical structures for the nonfactorizable part:

$$\Delta C_9^{(\bar{c}c, B \to K^*, V)}(q^2) = (C_1 + 3C_2)\, g(m_c^2, q^2) \\ -2C_1 \frac{32\pi^2}{3} \frac{(m_B + m_{K^*})\widetilde{A}_V(q^2)}{q^2 V^{BK^*}(q^2)} \,,$$

- nonfactorizable part enhances the effect, $1/q^2$ factor

$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(1.0\text{GeV}^2) = 0.7^{+0.6}_{-0.4}$
$\Delta C_9^{(\bar{c}c, B\to K^*, A_1)}(1.0\text{GeV}^2) = 0.8^{+0.6}_{-0.4}$
$\Delta C_9^{(\bar{c}c, B\to K^*, A_2)}(1.0\text{GeV}^2) = 1.1^{+1.1}_{-0.7}$



Charm-loop effect in $B \to K^*\gamma$

- By-product of our calculation for $B \to K^*\ell^+\ell^-$ at $q^2 = 0$
- factorizable part vanishes,
  nonfactorizable part yields a correction to $C_7^{eff}(m_b) \simeq -0.3$
  in the two inv. amplitudes:

$$C_7^{eff} \to C_7^{eff} + [\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_{1,2}\,,$$

$$\left[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}\right]_1 \simeq \left[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}\right]_2 = (-1.2^{+0.9}_{-1.6}) \times 10^{-2}\,,$$

- the previous results in the local OPE limit , LCSR with K^* DA:

$$[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_1^{BZ} = (-0.39 \pm 0.3) \times 10^{-2}\,,$$
$$[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_2^{BZ} = (-0.65 \pm 0.57) \times 10^{-2}\,. \qquad (1)$$

[P.Ball, G. W. Jones and R. Zwicky (2007)]

- our result in the local limit is closer to 3-point sum rule estimate: [A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]

Accessing large q^2 with dispersion relation

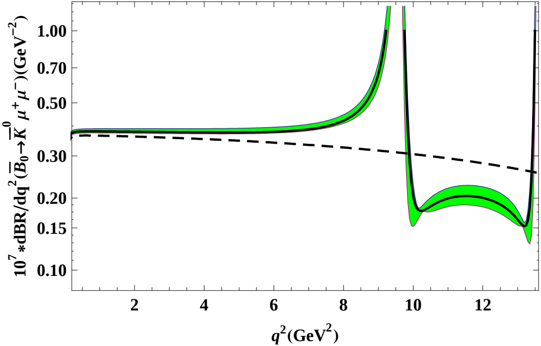
- analyticity of the hadronic matrix element in q^2, dispersion relation:

$$\mathcal{H}^{(B\to K)}(q^2) = \mathcal{H}^{(B\to K)}(0) + q^2\Big[\sum_{\psi=J/\psi,\psi(2S)} \frac{f_\psi A_{B\psi K}}{m_\psi^2(m_\psi^2 - q^2 - i m_\psi \Gamma_\psi^{tot})} + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s-q^2-i\epsilon)}\Big]$$

- absolute values of the residues $|f_\psi A_{B\psi K}|$ from exp. data
- the integral over $\rho(s)$ fitted as an effective pole
- using LCSR result at $q^2 \ll 4m_c^2$ as an input
- the whole $q^2 \leq m^2_{\psi(2S)}$ region accessed
- need more data on $B \to \psi K^{(*)}$, $B \to \bar{D}DK^{(*)}$

Influence on the observables for $B \to K\ell^+\ell^-$

- adding the calculated $\Delta C_9(q^2)$ to the $(C_9)_{FCNC}$ in the decay amplitude
- differential distribution in q^2 with (solid) and without (dashed) charm-loop effect

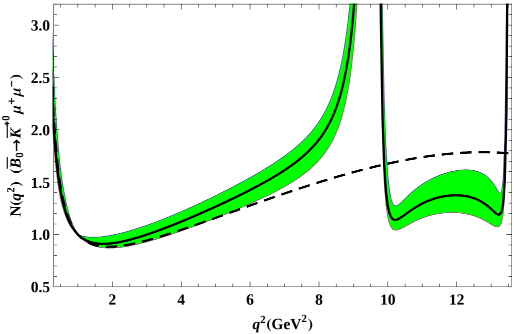


green shaded - estimated uncertainty

(of the LCSR result for soft-gluon effect)

Observables for $B \to K^*\ell^+\ell^-$

- predicted shape in q^2 with (solid) and without (dashed) charm-loop effect

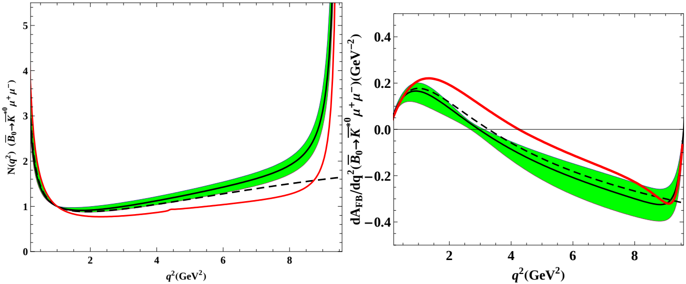


decay width q^2=distribution normalized at $q^2 = 1.0$ GeV^2

no uncertainties of form factors shown!

Outlook and preliminary results

- work in progress: all soft-gluon effects (penguin operators, WA), improving $B \to V$ form factors, adding the hard-gluon nonfactorizable contributions from QCDF
- (very preliminary !): adding the hard-gluon nonfactorizable effects calculated in *[M. Beneke,T. Feldmann,D. Seidel (2001)]* (red curve, central input)



- FB asymmetry mainly shifted by the hard-gluon effects
- towards SM prediction for the observables in $B_s \to P(V)\ell^+\ell^-$ at $q^2 \leq m_\psi^2$ including all hadronic effects