

# Hadronic matrix elements for exclusive rare $B$ -meson decays

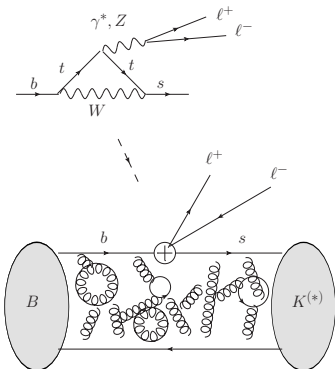
Alexander Khodjamirian



parallel session "Flavour Physics and Fundamental Symmetries"  
EPS HEP 2011, Grenoble

# FCNC exclusive decays: $B \rightarrow K^{(*)} \ell^+ \ell^-$ , $B \rightarrow K^* \gamma$

- in Standard Model,  $b \rightarrow s$  transitions, via  $t, W, Z$  loops



$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu),$$

$$O_{9(10)} = \frac{\alpha_{em}}{2\pi} [\bar{s} \gamma_\mu (1 - \gamma_5) b] \ell \gamma^\mu (\gamma_5) \ell,$$

$$C_9(m_b) \simeq 4.4, C_{10}(m_b) \simeq -4.7,$$

$$O_7 = -\frac{m_b}{8\pi^2} [\bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b] F^{\mu\nu},$$

$$C_7(m_b) \simeq -0.3$$

$O_{1-6}$  - 4-quark,  $O_8$ - quark-gluon also contribute !

$B \rightarrow K^{(*)} \ell^+ \ell^-$ ,  $B \rightarrow K^* \gamma$ , the observables

- decay amplitudes:

$$A(B \rightarrow K^{(*)} \ell^+ \ell^-) = \langle K^{(*)} \ell^+ \ell^- | H_{\text{eff}} | B \rangle$$

- all decay distributions, BR's, asymmetries, etc.

determined by  $|A(B \rightarrow K^{(*)} \ell^+ \ell^-)|^2 \times \{\text{phase space}\}$

- $A(B \rightarrow K^* \gamma) = \langle K^* \gamma | H'_{\text{eff}} | B \rangle$   $H'_{\text{eff}}$  without  $O_{9,10}$

important contributions are the same as in

$A(B \rightarrow K^{(*)} \ell^+ \ell^-)$  at  $q^2 = 0$ ,

- $B \rightarrow K^{(*)} \ell^+ \ell^-$  contributes to inclusive  $B \rightarrow X_s \ell^+ \ell^-$ , but !  
in the theory, the inclusive width is defined and treated differently.

For exclusive decays we need **hadronic matrix elements**

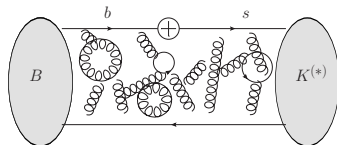
# $B \rightarrow K, K^*$ form factors

- contributions of  $O_{9,10}$  and  $O_7$  factorize, e.g.,

$$\begin{aligned} & \langle K^{(*)}(p) | C_9 O_9 | B(p+q) \rangle \\ &= \frac{\alpha_{em}}{2\pi} C_9 \langle K^{(*)}(p) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B(p+q) \rangle (\bar{\ell} \gamma_\mu \ell) \end{aligned}$$

↓

$B \rightarrow K$  and  $B \rightarrow K^*$   
form factors



- form factors depend on  $q^2 = (p_{\ell^+} + p_{\ell^-})^2$ ,  
 $0 < q^2 < (m_B - m_{K^{(*)}})^2$  in  $B \rightarrow K^{(*)} \ell^+ \ell^-$ ,  $q^2 = 0$  in  $B \rightarrow K^* \gamma$
- dominated by **nonperturbative** quark-gluon interactions,

# Calculating the form factors in QCD

- Lattice QCD with growing accuracy:  
currently,  $B \rightarrow \pi, K$  at large  $q^2$ ,  $n_f = 3$ ,  
 $B \rightarrow \rho, K^*$  only quenched
- non-lattice method: QCD Light-cone sum rules (LCSR)
  - outline:  
correlation function in QCD via light-cone OPE  
 $\Rightarrow$  hadronic dispersion relation  $\Rightarrow$  ground state contribution  
 $\Rightarrow$  form factor
  - main input:  
quark masses,  $\alpha_s$ ,  
light-cone distribution amplitudes (DA's) of  $\pi, K, B, \dots$

# Status and accuracy of LCSR calculations

- the region  $q^2 \leq 12 - 15 \text{ GeV}^2$  accessible, complementing the lattice FF's
- $B \rightarrow \pi, K$  form factors  
*[G.Duplancic, A.K, Th.Mannel, B.Melic, N.Offen (2008)] ,*  
*[A.K, Th.Mannel, N.Offen, Y-M. Wang (2011)]*  
*[G.Duplancic, B.Melic (2008)], [A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]*
- estimated uncertainties for  $B \rightarrow \pi, K \pm(12 - 15)\%$
- $B_{(s)} \rightarrow \rho, \omega, K^*, \phi$  form factors, DA's of  $K^*$  and  $\rho$   
*[P.Ball, R.Zwicky (2005)]*
- $B \rightarrow \pi, K, \rho, K^*, \dots$   
**LCSR with B-meson distribution amplitudes**  
*[ A.K., Th. Mannel, N.Offen (2005)]*  
LCSR in SCET *[ F. De Fazio, Th. Feldmann and T.Hurth (2006)]*

# $B \rightarrow K, K^{(*)}$ form factors from LCSR

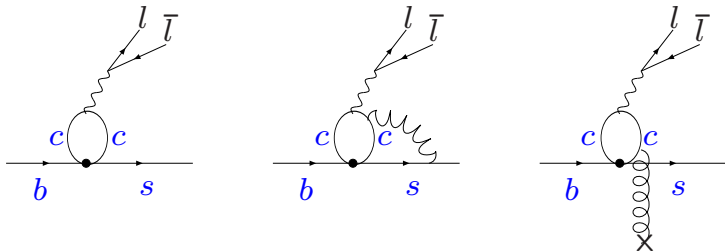
[A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]

form factor	$F_{BK^{(*)}}^i(0)$	$b_1^i$	$B_s(J^P)$	input at $q^2 < 12 \text{ GeV}^2$
$f_{BK}^+$	$0.34^{+0.05}_{-0.02}$	$-2.1^{+0.9}_{-1.6}$	$B_s^*(1^-)$	LCSR with $K$ DA's
$f_{BK}^0$	$0.34^{+0.05}_{-0.02}$	$-4.3^{+0.8}_{-0.9}$	no pole	
$f_{BK}^T$	$0.39^{+0.05}_{-0.03}$	$-2.2^{+1.0}_{-2.00}$	$B_s^*(1^-)$	
$V^{BK^*}$	$0.36^{+0.23}_{-0.12}$	$-4.8^{+0.8}_{-0.4}$	$B_s^*(1^-)$	LCSR with $B$ DA's
$A_1^{BK^*}$	$0.25^{+0.16}_{-0.10}$	$0.34^{+0.86}_{-0.80}$	$B_s(1^+)$	
$A_2^{BK^*}$	$0.23^{+0.19}_{-0.10}$	$-0.85^{+2.88}_{-1.35}$	$B_s(1^+)$	
$A_0^{BK^*}$	$0.29^{+0.10}_{-0.07}$	$-18.2^{+1.3}_{-3.0}$	$B_s(0^-)$	
$T_1^{BK^*}$	$0.31^{+0.18}_{-0.10}$	$-4.6^{+0.81}_{-0.41}$	$B_s^*(1^-)$	
$T_2^{BK^*}$	$0.31^{+0.18}_{-0.10}$	$-3.2^{+2.1}_{-2.2}$	$B_s(1^+)$	
$T_3^{BK^*}$	$0.22^{+0.17}_{-0.10}$	$-10.3^{+2.5}_{-3.1}$	$B_s(1^+)$	

using BCL [*Bourrely, Caprini, Lellouch(2008)*] parameterization,  $b_1$ -slope parameter

# Charm-loops and other complications

- a combination of the  $(\bar{s}c)(\bar{c}b)$  weak interaction ( $O_{1,2}$ ) and e.m.interaction  $(\bar{c}c)(\bar{\ell}\ell)$  "mimicking FCNC"
- Charm-loop effect:



- similar  $u, d, s, c, b$ -quark loops ( $O_{3-6}$  operators),  $u$ -loops from  $O_{1,2}^u$  (CKM suppressed in  $b \rightarrow s$ ),
- "weak annihilation"  $\oplus$  photon emission
- factorization is lost !
- $A(B \rightarrow K^{(*)}\ell^+\ell^-)$  include additional hadronic matrix elements, not simply form factors



# Charm loop turns charmonium

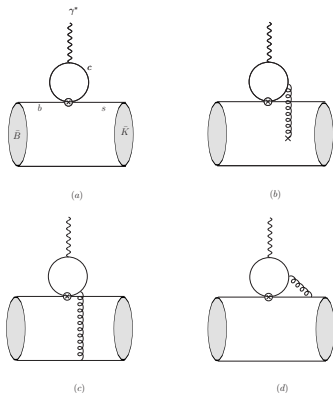
- at  $q^2 \rightarrow m_{J/\psi}, \dots$  an on-shell hadronic state:  
 $B \rightarrow J/\psi K \otimes J/\psi \rightarrow \ell^+ \ell^-$
- other  $\psi$ -levels (charmonia with  $J^P = 1^-$ ),  
open-charm states  $B \rightarrow \bar{D}DK \rightarrow K\ell^+\ell^-$ ,  
( $\bar{c}c$  states with the masses up to  $m_B - m_K^{(*)}$ )
- $J/\psi$  and  $\psi(2S)$  bins are subtracted from the  $q^2$ -distribution data in  $B \rightarrow K^{(*)}\ell^+\ell^-$
- the effect of intermediate virtual  $\bar{c}c$  states remains at  
 $q^2 \ll m_{J/\psi}^2$  (nonperturbative at  $q^2 \sim 4m_c^2$ )

# Charm-loop in $B \rightarrow K^{(*)} \ell^+ \ell^-$

- factorizable c-quark loop  
 $C_9 \rightarrow C_9 + (C_1 + 3C_2)g(m_c^2, q^2)$

- perturbative gluons  $\rightarrow$   
 (nonfactorizable) corrections  
 being factorized in  $O(\alpha_s)$   
 and added to  $C_9$

[M. Beneke, T. Feldmann, D. Seidel (2001)]



- taking into account the **soft gluons** (low-virtuality, nonvanishing momenta) emitted from the c-quark loop

[A.K. Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]

- light-cone expansion  $\Rightarrow$  nonlocal effective  $\bar{s}Gb$  operator  
 $\sim 1/(4m_c^2 - q^2)$ -suppression
- the  $B \rightarrow K^{(*)}$  hadronic matrix elements of this operator calculated using the same **LCSR method** as for  $B \rightarrow K^{(*)}$  form factors

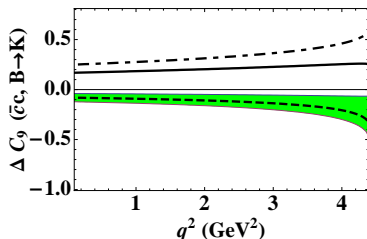
# Charm-loop effect in $B \rightarrow K\ell^+\ell^-$ in terms of $\Delta C_9$

- the effective coefficient  $C_9(\mu = m_b) \simeq 4.4$   
a process-dependent correction to be added:

$$\Delta C_9^{(\bar{c}c, B \rightarrow K)}(q^2) = (C_1 + 3C_2) g(m_c^2, q^2) + 2C_1 \frac{32\pi^2}{3} \frac{\tilde{A}(q^2)}{f_{BK}^+(q^2)}$$

$$\Delta C_9(0) = 0.17^{+0.09}_{-0.18},$$

( $\mu = m_b = 4.2\text{GeV}$ )



## Charm-loop effect for $B \rightarrow K^* \ell^+ \ell^-$

- factorizable part determined by the three  $B \rightarrow K^*$  form factors  $V^{BK^*}(q^2)$ ,  $A_1^{BK^*}(q^2)$ ,  $A_2^{BK^*}(q^2)$ ,
- three kinematical structures for the nonfactorizable part:

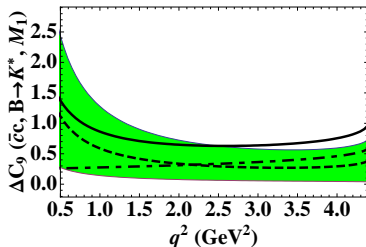
$$\Delta C_9^{(\bar{c}c, B \rightarrow K^*, V)}(q^2) = (C_1 + 3C_2) g(m_c^2, q^2) - 2C_1 \frac{32\pi^2}{3} \frac{(m_B + m_{K^*}) \tilde{A}_V(q^2)}{q^2 V^{BK^*}(q^2)},$$

- nonfactorizable part enhances the effect,  $1/q^2$  factor

$$\Delta C_9^{(\bar{c}c, B \rightarrow K^*, V)}(1.0 \text{ GeV}^2) = 0.7^{+0.6}_{-0.4}$$

$$\Delta C_9^{(\bar{c}c, B \rightarrow K^*, A_1)}(1.0 \text{ GeV}^2) = 0.8^{+0.6}_{-0.4}$$

$$\Delta C_9^{(\bar{c}c, B \rightarrow K^*, A_2)}(1.0 \text{ GeV}^2) = 1.1^{+1.1}_{-0.7}$$



## Charm-loop effect in $B \rightarrow K^* \gamma$

- By-product of our calculation for  $B \rightarrow K^* \ell^+ \ell^-$  at  $q^2 = 0$
- factorizable part vanishes, nonfactorizable part yields a correction to  $C_7^{\text{eff}}(m_b) \simeq -0.3$  in the two inv. amplitudes:

$$C_7^{\text{eff}} \rightarrow C_7^{\text{eff}} + [\Delta C_7^{(\bar{c}c, B \rightarrow K^* \gamma)}]_{1,2},$$

$$[\Delta C_7^{(\bar{c}c, B \rightarrow K^* \gamma)}]_1 \simeq [\Delta C_7^{(\bar{c}c, B \rightarrow K^* \gamma)}]_2 = (-1.2_{-1.6}^{+0.9}) \times 10^{-2},$$

- the previous results in the local OPE limit, LCSR with  $K^*$  DA:

$$\begin{aligned} [\Delta C_7^{(\bar{c}c, B \rightarrow K^* \gamma)}]_1^{\text{BZ}} &= (-0.39 \pm 0.3) \times 10^{-2}, \\ [\Delta C_7^{(\bar{c}c, B \rightarrow K^* \gamma)}]_2^{\text{BZ}} &= (-0.65 \pm 0.57) \times 10^{-2}. \end{aligned} \quad (1)$$

[P.Ball, G. W. Jones and R. Zwicky (2007)]

- our result in the local limit is closer to 3-point sum rule estimate: [A.K., G. Stoll, R. Rueckl, D. Wyler (1997)]

# Accessing large $q^2$ with dispersion relation

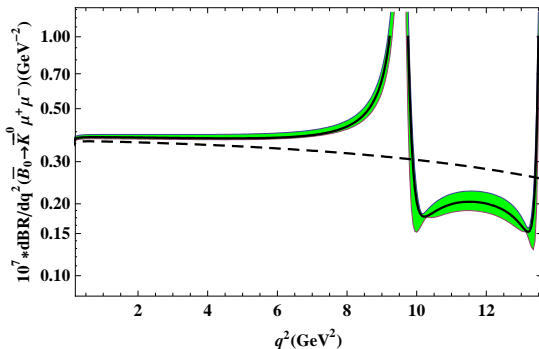
- analyticity of the hadronic matrix element in  $q^2$ ,  
dispersion relation:

$$\mathcal{H}^{(B \rightarrow K)}(q^2) = \mathcal{H}^{(B \rightarrow K)}(0) + q^2 \left[ \sum_{\psi=J/\psi, \psi(2S)} \frac{f_\psi A_{B\psi K}}{m_\psi^2 (m_\psi^2 - q^2 - im_\psi \Gamma_\psi^{tot})} + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s - q^2 - i\epsilon)} \right]$$

- absolute values of the residues  $|f_\psi A_{B\psi K}|$  from exp. data
- the integral over  $\rho(s)$  fitted as an effective pole
- using LCSR result at  $q^2 \ll 4m_c^2$  as an input
- the whole  $q^2 \leq m_{\psi(2S)}^2$  region accessed
- need more data on  $B \rightarrow \psi K^*$ ,  $B \rightarrow \bar{D} D K^*$

# Influence on the observables for $B \rightarrow K\ell^+\ell^-$

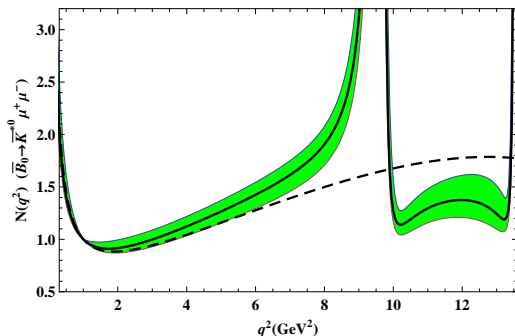
- adding the calculated  $\Delta C_9(q^2)$  to the  $(C_9)_{FCNC}$  in the decay amplitude
- differential distribution in  $q^2$  with (solid) and without (dashed) charm-loop effect



green shaded - estimated uncertainty  
(of the LCSR result for soft-gluon effect)

# Observables for $B \rightarrow K^* \ell^+ \ell^-$

- predicted shape in  $q^2$  with (solid) and without (dashed) charm-loop effect

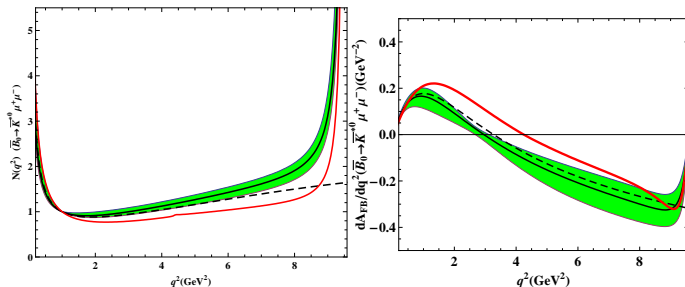


decay width  $q^2$ -distribution normalized at  $q^2 = 1.0 \text{ GeV}^2$   
no uncertainties of form factors shown!



# Outlook and preliminary results

- **work in progress:** all soft-gluon effects (penguin operators, WA), improving  $B \rightarrow V$  form factors, adding the **hard-gluon** nonfactorizable contributions from QCDF
- **(very preliminary !):** adding the hard-gluon nonfactorizable effects calculated in [M. Beneke, T. Feldmann, D. Seidel (2001)] (red curve, central input)



- FB asymmetry mainly shifted by the hard-gluon effects
- towards SM prediction for the observables in  $B_s \rightarrow P(V)\ell^+\ell^-$  at  $q^2 \leq m_{\psi}^2$  including all hadronic effects