Some phenomenological aspects of Loop Quantum Cosmology

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Why going beyond GR ?

Dark energy (and matter) / quantum gravity

• Observations : the acceleration of the Universe
• Theory : singularity theorems

Successful techniques of QED do not apply to gravity. Something new has to be invented.

Which gedankenexperiment ? (as is QM, SR and GR) Which paradoxes ?

Quantum black holes and the early universe are privileged places to investigate such effects !

* Entropy of black holes
* End of the evaporation process, IR/UV connection
* the Big-Bang

Many possible approaches : strings, covariant approaches (effective theories, the renormalization group, path integrals), canonical approaches (quantum geometrodynamics, loop quantum gravity), etc. See reviews par C. Kiefer

I will focus on LQG
The observed acceleration

SNLS, Astier et al.

WMAP, 5 ans

SDSS, Eisenstein et al. 2005

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\[ \Lambda / 8\pi G \sim 10^{-47} \text{GeV}^4 \]

\[
H^2 = \frac{8\pi G}{3} \left( \sum_a \rho_a + \rho_{DE} \right) - \frac{k}{a^2},
\]

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \sum_a (\rho_a + 3p_a) + \rho_{DE} + 3p_{DE} \right).
\]

\[
a(t) = a(t_0) + \dot{a}|_{t_0}(t - t_0) + \frac{\ddot{a}|_{t_0}}{2}(t - t_0)^2 + \frac{\dddot{a}|_{t_0}}{6}(t - t_0)^3 + \ldots.
\]

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<tr>
<th>Level</th>
<th>Geometrical Parameter</th>
<th>Physical Parameter</th>
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| 1     | \( H(z) \equiv \frac{4\pi G}{3} \) | \( \rho_m(z) = \rho_{tot}(1+z)^3 \),
|       |                      | \( \rho_{DE} = \frac{4\pi G}{3} - \rho_m \) |
| 2     | \( q(z) \equiv -\frac{\dot{a}}{a} \) | \( V(z), T(z) \equiv \frac{\dot{\phi}}{2} \), 
|       | \( \rho(z) \right|_{\Lambda C D M} \equiv -1 + \frac{3}{2}\Omega_m(z) \) | \( \Omega_V = \frac{8\pi G}{3H^2}, \quad \Omega_T = \frac{8\pi G}{3H^2} \) |
| 3     | \( \tau(z) \equiv \frac{s^2}{\dot{s}^2}, \quad s \equiv \frac{\tau}{3(\tau - 1/2)} \) | \( \Pi(z) \equiv \dot{\phi}V', \quad \Omega_n = \frac{8\pi G}{3H^2} \)
|       | \( \{r,s\} \right|_{\Lambda C D M} \equiv \{1,0\} |
« Can we construct a quantum theory of spacetime based only on the experimentally well confirmed principles of general relativity and quantum mechanics? » L. Smolin, hep-th/0408048

**DIFFEOMORPHISM INVARIANCE**

Loops (solutions to the WDW) = space
- Mathematically well defined
- Singularities
- Black holes

In QFT, one need $U(O)$ where $O$ is a region of $(M,g)$ to define (anti)commutation relations between spacelike separated regions!!!
How to build Loop Quantum Gravity?
1) If you are a relativist...

- Foliation $\rightarrow$ space metric and conjugate momentum
- Constraints (difféomorphism, hamiltonian + SO(3))
- Quantization « à la Dirac » $\rightarrow$ WDW $\rightarrow$ Ashtekar variables
- « smearing » $\rightarrow$ holonomies and fluxes

See e.g. the book « Quantum Gravity » by C. Rovelli
How to build Loop Quantum Gravity?

2) If you are a particle physicist...

- Think of lattice QCD
- Define a graph and the Hilbert space: $L^2(G^L/G^N)$. The Fock space is obtained by taking the appropriate limit.
- In gravity you do the same: $H \Gamma = L^2[SU(2)^L/SU(2)^N]$. Then $\tilde{H} \Gamma = H \Gamma / \sim$ (automorphism group)
- Define « natural » operators on $L^2[SU(2)]$
- Gauge invariance + Penrose theorem lead to a simple geometrical interpretation in the classical limit.
- Define the spin-network basis (diagonolizes the area and volume operators)

See e.g. the book « Quantum Gravity » by C. Rovelli

- Mathematically well defined
- Singularities
- Black holes
Experimental tests

- High energy gamma-ray (Amélio-Camelia et al.)

Not very conclusive however
Experimental tests

- Discrete values for areas and volumes (Rovelli et al.)

- Observationnal cosmology (..., et al.)

LQC:

- IR limit
- UV limit (bounce)
- inflation
WDW vs LQC

WDW: The IR test is passed with flying colors. But the singularity is not resolved.

In LQC, no Big Bang, no new principle, no new principle required, « other side » opened, huge « quantum geometrical » effects @ $10^{94}$ g/cm$^3$

Plots from Ashtekar
LQC: a few results

- The volume of the Universe takes its minimum value at the bounce and scales as $p(\Phi)$.
- The recollapse happens at $V_{\text{max}}$ which scales as $p(\Phi)^{(3/2)}$. GR is OK.
- The states remain sharply peaked for a very large number of cycles. Determinism is kept even for an infinite number of cycles.
- The dynamics can be derived from effective Friedmann equations

\[
\left( \frac{\dot{a}}{a} \right)^2 = (8\pi G \rho/3) \left( 1 - \frac{\rho}{\rho_{\text{crit}}} \right)
\]

- The LQC correction naturally comes with the correct sign. This is non-trivial.
- Furthermore, one can show that the upper bound of the spectrum of the density operator coincides with $\rho_{\text{crit}}$

$\rightarrow$ Role of the high symmetry assumed? (string entropy?)
LQC & inflation

- Inflation
  - success (paradoxes solved, perturbations, etc.)
  - difficulties (no fundamental theory, initial conditions, etc.)

- LQC
  - success (background-independant quantization of GR, BB Singularity resolution, good IR limit)
  - difficulties (very hard to test !)

Could it be that considering both LQC and inflation within the same framework allows to cure simultaneously all the problems ?

Bojowald, Hossain, Copeland, Mulryne, Numes, Shaeri, Tsujikawa, Singh, Maartens, Vandersloot, Lidsey, Tavakol, Mielczarek …….
First approach:
LQC corrections to the modes in a classical background

« standard » inflation
- decouples the effects
- happens after superinflation

Second approach: Taking into account the background modifications

H changes sign in the KG equation $\phi'' + 3H\phi' + m^2\phi = 0$

→ Inflation inevitably occurs!

Mielczarek, Cailleteau, Grain, A.B., Phys. Rev. D, 81, 104049, 2010
A tricky horizon history...

Physical modes may cross the horizon several times...

Computation of the primordial power spectrum:
- Bogolibov transformations
- Full numerical resolution

- The power is suppressed in the infra-red (IR) regime. This is a characteristic feature associated with the bounce.
- The UV behavior agrees with the standard general relativistic picture.
- Damped oscillations are superimposed with the spectrum around the "transition" momentum $k^*$ between the suppressed regime and the standard regime.
- The first oscillation behaves like a "bump" that can substantially exceed the UV asymptotic value.
Effective description with a Bogoliubov transformation:
- Frequency of the oscillations controlled by $\Delta(\eta)$, the width of the bounce
- Amplitude of the oscillations controlled by $k_0$, the effective mass at the bounce

Fundamental description:
- $R$ driven by the field mass
- $k^*$ driven by initial conditions

Initial conditions are critical
CMB consequences...

A.B., Grain et al.
CMB consequences

If the scalar spectrum is assumed not to be affected: one needs $x < 2 \times 10^{-6}$ to probe the model

Is a $N > 78$ inflation probable? What is the probability to be compatible with WMAP data?

YES. Good news from Ashtekar and Sloan!

Can B-mode be used to distinguished with string inflation?

I think yes.
Anomaly-free vector algebra for holonomy corrections

\[ \{ S_{\text{tot}}[N_1], S_{\text{tot}}[N_1] \} = 0, \]
\[ \{ D_{\text{tot}}[N_1^a], D_{\text{tot}}[N_2^a] \} = 0, \]
\[ \{ S_{\text{tot}}[N], D_{\text{tot}}[N^a] \} = \frac{\tilde{N}}{\sqrt{\tilde{P}}} B D^Q[N^a] \]
\[ + \frac{\tilde{N}}{\kappa \sqrt{\tilde{P}}} \int_{\Sigma} d^3 x \delta N^c \delta^k_c \left( \partial_d \delta E^d_k - \delta E^d_k \right) A \]
\[ + [\cos(\nu_2 \tilde{\mu} \gamma_k) - 1] \frac{\sqrt{\tilde{P}}}{2} \left( \frac{\tilde{\pi}^2}{2\tilde{p}^3} - V(\tilde{\varphi}) \right) \times \]
\[ \times \int_{\Sigma} d^3 x \tilde{N} \partial_c (\delta N^a) \delta^j_a \delta E^c_j \]
\[ + \frac{\tilde{\pi}}{\tilde{p}^{3/2}} \int_{\Sigma} d^3 x \tilde{N} (\partial_a \delta N^a) \delta \pi \]
\[ - \tilde{p}^{3/2} V_{\varphi}(\tilde{\varphi}) \int_{\Sigma} d^3 x \tilde{N} (\partial_a \delta N^a) \delta \varphi \]

Including matter

The counterterms can be computed together with the integers. As expected \(v_2=0\) (no diffeo correction).

→ The algebra is determined (on need \(B=0\) – and of course \(A=0\))
Perspective I: closing the algebra for scalar modes (with holonomy corrections)

\[
\{H_G^0[N], D_G^0[N^\alpha]\} = \frac{N}{\sqrt{p}} \left[ \frac{\sin v_2 \mu \gamma k}{v_2 \mu \gamma} + \frac{\sin v_1 \mu \gamma k}{v_1 \mu \gamma} - 2 \frac{\sin 2 \mu \gamma k}{2 \mu \gamma} \right] D_G^0[N^\alpha] \\
+ \frac{1}{\kappa} \int \Sigma d^3x \frac{N}{\sqrt{p}} A^{HD}(\partial_\alpha \delta N^\alpha) \delta E^k_\beta \delta^k_c
\]  

(12)

The solution seems to be uniquely determined.

Complementary to Bojowald, Hossain, Kagan and Shankaranarayanan

Cailleteau, Mielczarek, A.B., Grain, preliminary
Perpsectives II (tensor modes) :
IV + holonomy for background + modes

\[ \dot{\Phi} = \frac{d\Phi}{dt} = \{\Phi, p_\Phi\} \frac{\partial H_m}{\partial p_\Phi} = D \frac{p_\Phi}{\bar{p}^2} \]

\[ \dot{p}_\Phi = \frac{dp_\Phi}{dt} = -\{\Phi, p_\Phi\} \frac{\partial H_m}{\partial \Phi} = -\bar{p}^2 \frac{3}{2} V_\Phi. \]

\[ \ddot{\Phi} = \dot{D} \frac{p_\Phi}{\bar{p}^2} + D \frac{\dot{p}_\Phi}{\bar{p}^2} - 3 \hat{\bar{p}} D \frac{p_\Phi}{\bar{p}^2} \]

\[ = \frac{\dot{D}}{D} \frac{D p_\Phi}{\bar{p}^2} - D \partial_\Phi V(\Phi) - 3H D \frac{p_\Phi}{\bar{p}^2} \]

\[ = \frac{\dot{D}}{D} \dot{\Phi} - D \partial_\Phi V(\Phi) - 3H \dot{\Phi} \]

Simulation in progress

A.B., Cailleteau, Grain, in progress
Toward a loop – inflation paradigm?