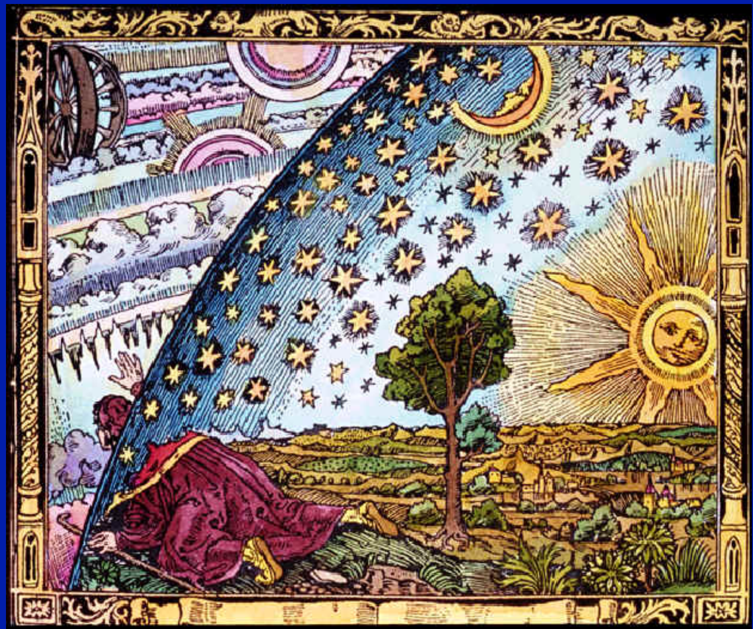
Some phenomenological aspects of Loop Quantum Cosmology



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Why going beyond GR ?

Dark energy (and matter) / quantum gravity

- Observations : the acceleration of the Universe
- Theory : singularity theorems

Successful techniques of QED do not apply to gravity. Something new has to be inveted.

Which gedenkenexperiment ? (as is QM, SR and GR) Which paradoxes ?

Quantum black holes and the early universe are privileged places to investigate such effects !

* Entropy of black holes

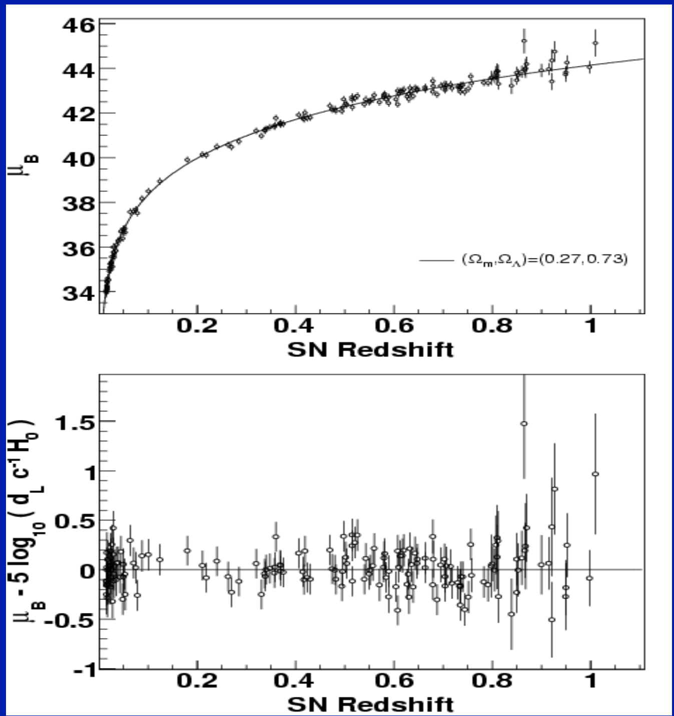
* End of the evaporation process, IR/UV connection

* the Big-Bang

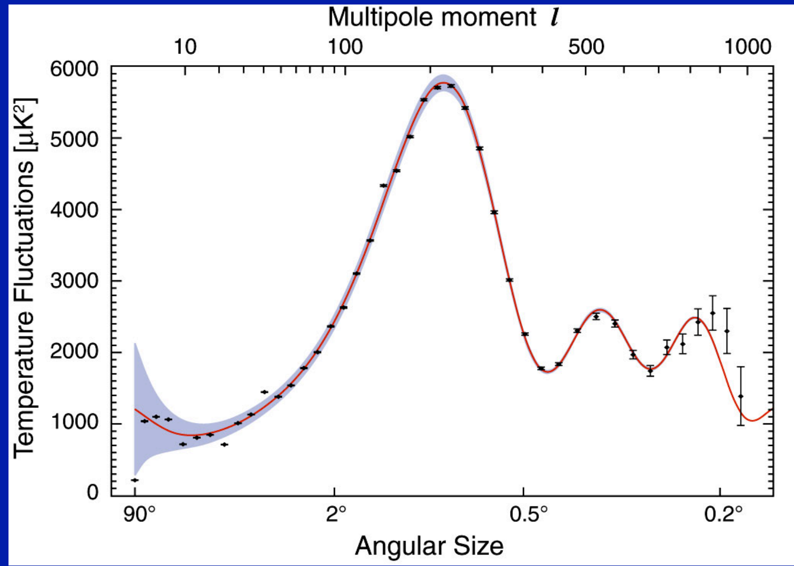
→ Many possible approaches : strings, covariant approaches (effective theories, the renormalization group, path integrals), canonical approaches (quantum geometrodynamics, loop quantum gravity), etc. See reviews par C. Kiefer

→ I will focus on LQG

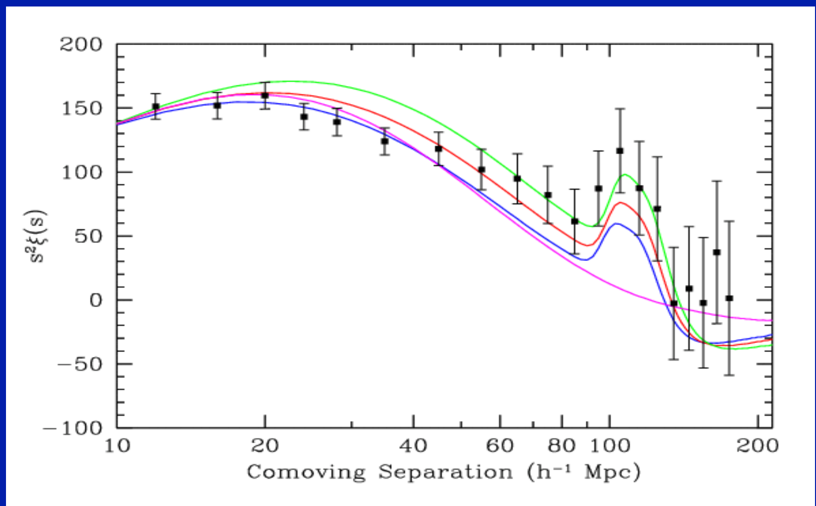
The observed acceleration



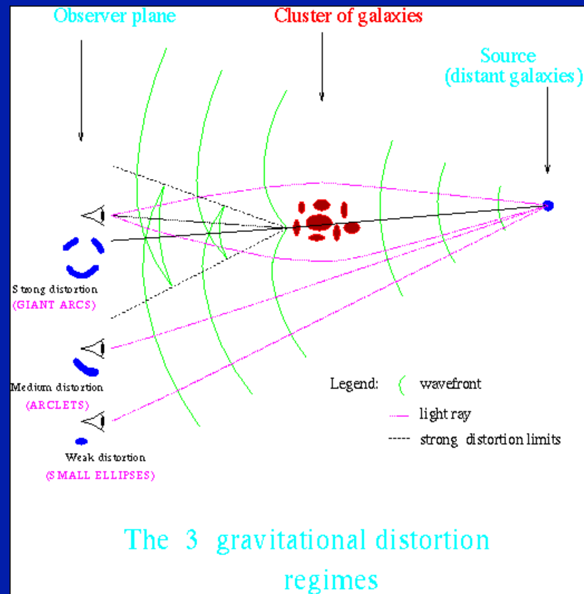
SNLS, Astier et al.



WMAP, 5 ans



SDSS, Eisenstein et al. 2005



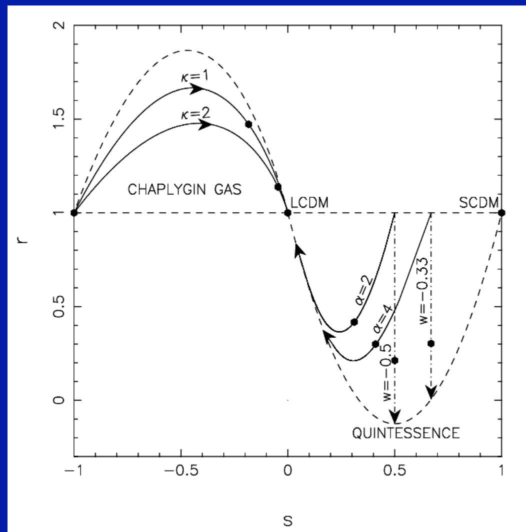
$$\Lambda / 8\pi G \sim 10^{-47}\, GeV^4$$

$$H^2 = \frac{8\pi G}{3}\left(\sum_a \rho_a + \rho_{DE}\right) - \frac{k}{a^2} \,,$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\sum_a (\rho_a + 3p_a) + \rho_{DE} + 3p_{DE}\right)$$

$$a(t) = a(t_0) + \dot{a}\big|_0 (t - t_0) + \frac{\ddot{a}\big|_0}{2}(t - t_0)^2 + \frac{\dddot{a}\big|_0}{6}(t - t_0)^3 + \dots \,.$$

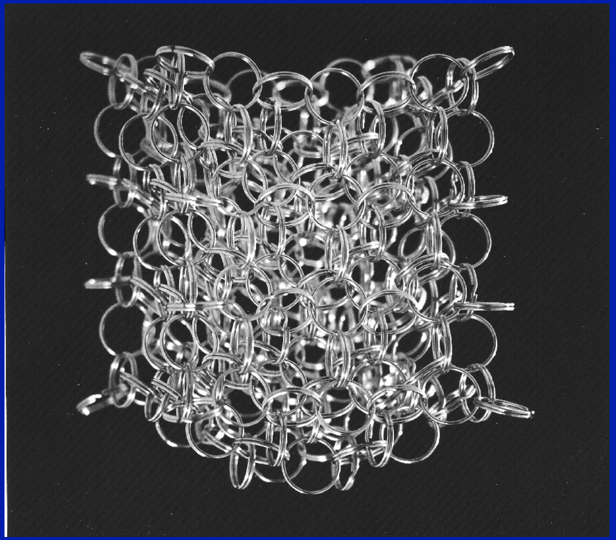
Level	Geometrical Parameter	Physical Parameter
1	$H(z) \equiv \frac{\dot{a}}{a}$	$\rho_m(z) = \rho_{0m}(1+z)^3$, $\rho_{\rm DE} = \frac{3H^2}{8\pi G} - \rho_m$
2	$q(z) \equiv -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{d\log H}{d\log(1+z)}$ $q(z)\big\vert_{\Lambda\rm CDM} = -1 + \frac{3}{2}\Omega_m(z)$	$V(z)$, $T(z) \equiv \frac{\dot{\phi}^2}{2}$, $w(z) = \frac{T-V}{T+V}$, $\Omega_V = \frac{8\pi GV}{3H^2}$, $\Omega_T = \frac{8\pi GT}{3H^2}$
3	$r(z) \equiv \frac{\dddot{a}a^2}{\dot{a}^3}$, $s \equiv \frac{r-1}{3(q-1/2)}$ $\{r,s\}\big\vert_{\Lambda\rm CDM} = \{1,0\}$	$\Pi(z) \equiv \dot{V} = \dot{\phi}V'$, $\Omega_\Pi = \frac{8\pi G\dot{V}}{3H^3}$



Alam et al., MNRAS 344 (2003) 1057

Not that easy in the UV limit…

« Can we construct a quantum theory of spacetime based only on the experimentally well confirmed principles of general relativity and quantum mechanics ? » L. Smolin, hep-th/0408048



DIFFEOMORPHISM INVARIANCE

Loops (solutions to the WDW) = space
-Mathematically well defined
-Singularities
-Black holes

In QFT, one need U(O) where O is a region of (M,g) to define (anti)commutation relations between spacelike separated regions !!!

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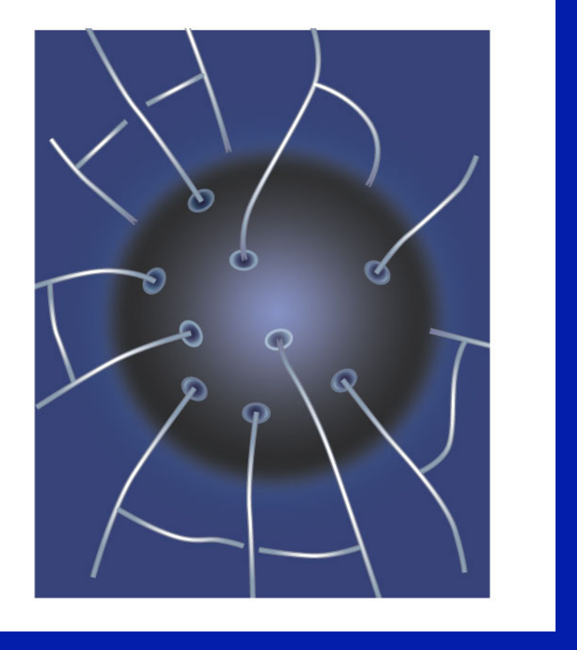
How to build Loop Quantum Gravity ?
1) If you are a relativist…

- **Foliation → space metric and conjugate momentum**
- **Constraints (difféomorphism, hamiltonian + SO(3))**
- **Quantization « à la Dirac » → WDW → Ashtekar variables**
- **« smearing » → holonomies and fluxes**



See *e.g.* the book « Quantum Gravity » by C. Rovelli

How to build Loop Quantum Gravity ?
2) If you are a particule physicist…

- Think of lattice QCD
- Define a graph and the Hilbert space : L2(G^L/G^N). The Fock space is obtain by taking the appropriate limit.
- In gravity you do the same : HΓ = L2[SU(2)^L/SU(2)^N]. Then H̃Γ = HΓ/ ~ (automorphism group)
- Define « natural » operators on L2[SU2]
- Gauge invariance + Penrose theorem lead to a simple geometrical interpretation in the classical limit.
- Define the spin-network basis (diagonolizes the area and volume operators)

See *e.g.* the book « Quantum Gravity » by C. Rovelli

-Mathematically well defined
-Singularities
-Black holes

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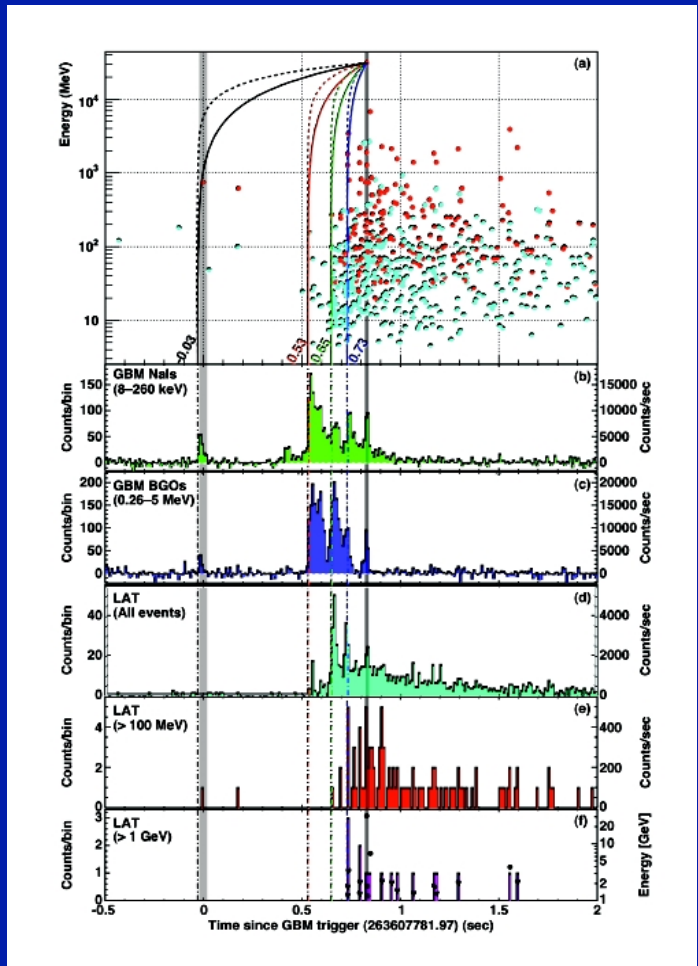
Experimental tests

- **High energy gamma-ray** **(Amélino-Camelia et al.)**



Not very conclusive however

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Experimental tests

- **Discrete values for areas and volumes** **(Rovelli et al.)**

- **Observationnal cosmology** **(…, et al.)**

LQC :

- IR limit
- UV limit (bounce)
-inflation

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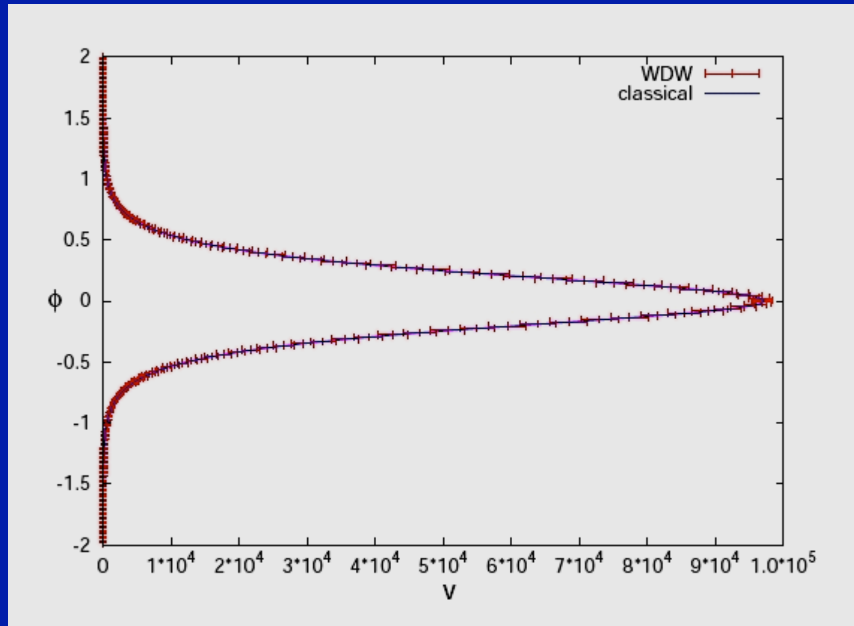
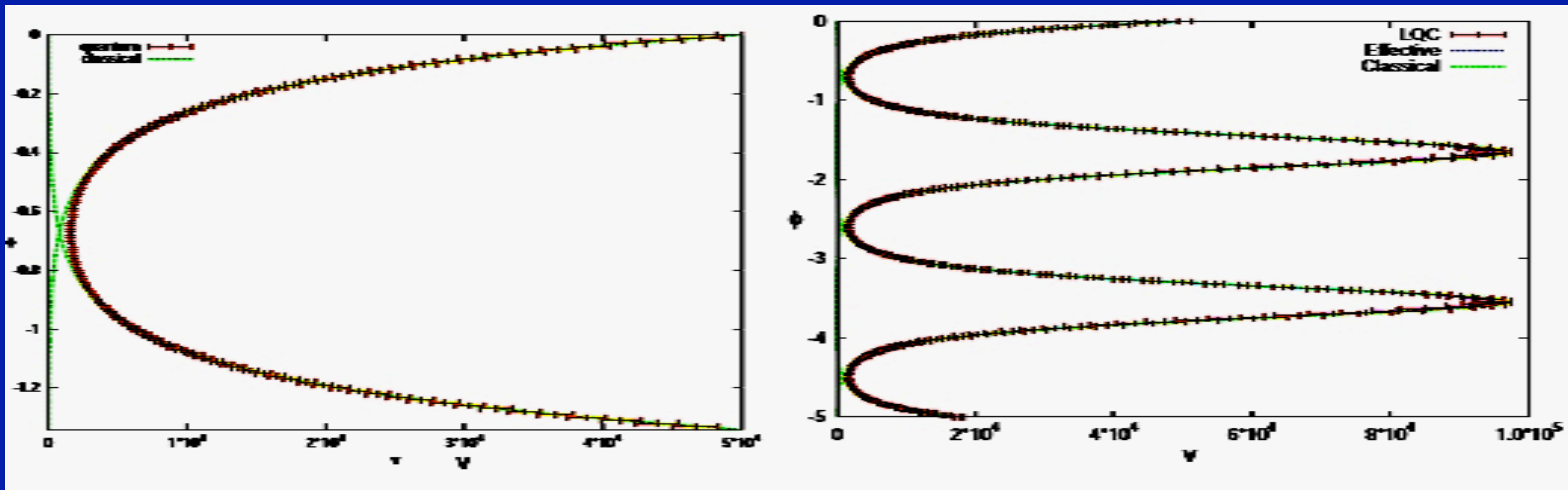
WDW vs LQC

WDW : The IR test is pased with flying colors. But the singularity is not resolved.→



In LQC, no Big Bang, no new principle, no new principle required, « other side » opened, huge « quantum geometrical » effects @ 10^94 g/cm^3



Plots from Ashtekar

LQC: a few results

- The volume of the Universe takes its minimum value at the bounce and scales as p(Φ)
- The recollapse happens at Vmax which scales as p(Φ)^(3/2). GR is OK.
- The states remain sharply peaked for a very large number of cycles. Determinism is kept even for an infinite number of cycles.
- The dynamics can be derived from effective Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = (8\pi G\,\rho/3)\left(1 - \frac{\rho}{\rho_{\rm crit}}\right)$$

- The LQC correction naturally comes with the correct sign. This is non-trivial.
- Furthermore, one can show that the upper bound of the spectrum of the density operator coincides with ρ_{crit}

→ Role of the high symmetry assumed ? (string entropy ?)

LQC & inflation

-Inflation

- success (paradoxes solved, perturbations, etc.)

- difficulties (no fundamental theory, initial conditions, etc.)

-LQC

- success (background-independant quantization of GR, BB Singularity resolution, good IR limit)

- difficulties (very hard to test !)

Could it be that considering both LQC and inflation within the same framework allows to cure simultaneously all the problems ?

Bojowald, Hossain, Copeland, Mulryne, Numes, Shaeri, Tsujikawa, Singh, Maartens, Vandersloot, Lidsey, Tavakol, Mielczarek …….

First approach:
LQC corrections to the modes in a classical background

« standard » inflation

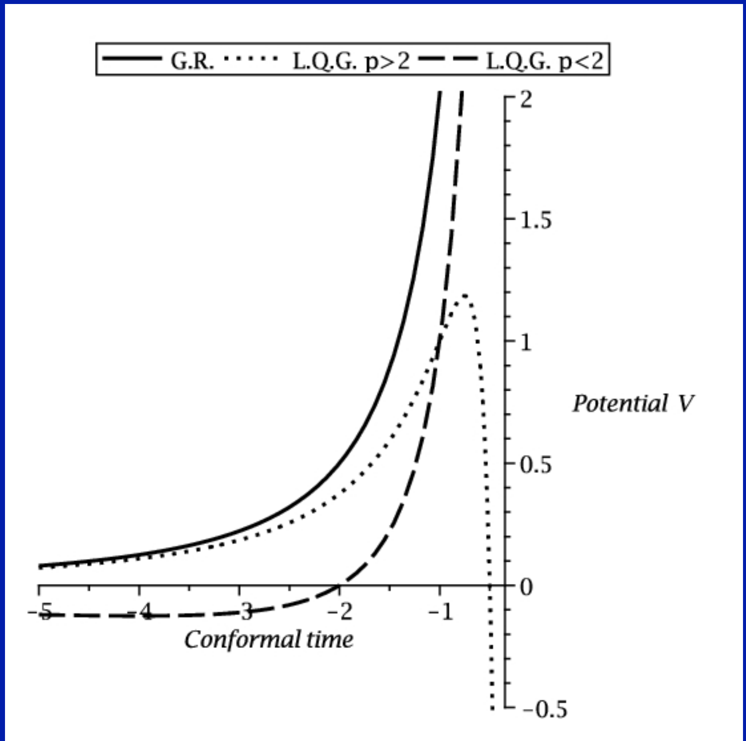
-decouples the effects
-happens after superinflation

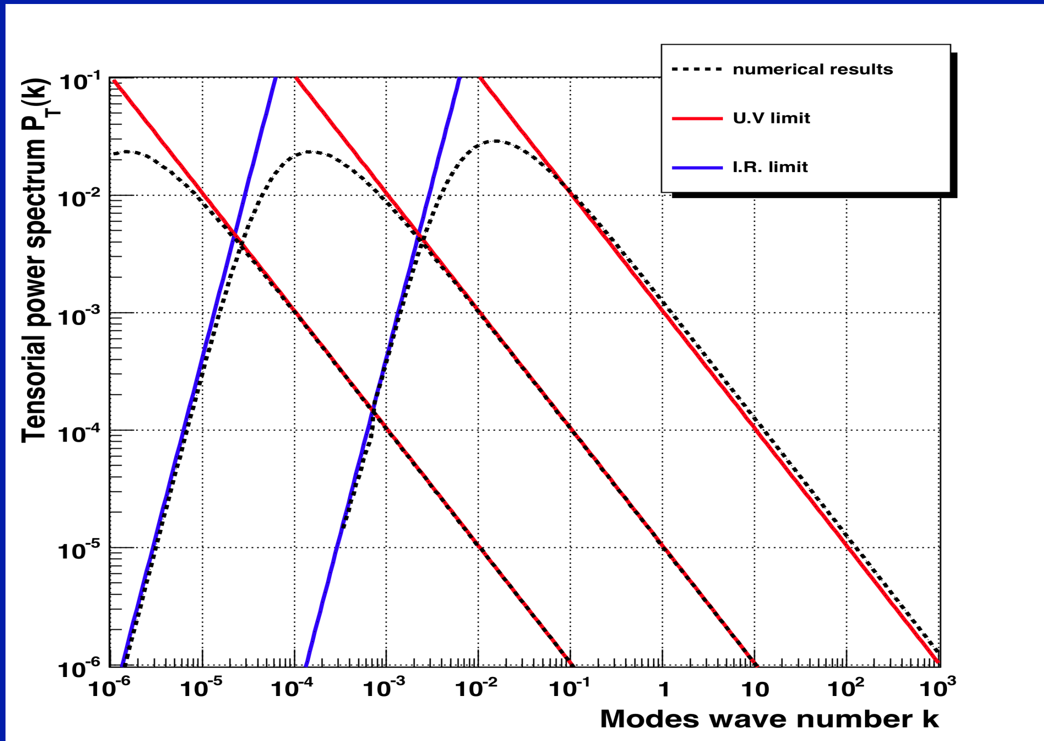
Bojowald & Hossain, Phys. Rev. D 77, 023508 (2008)

A.B. & Grain, Phys. Rev. Lett. , 102, 081321 (2009)

J. Grain, A.B., A. Gorecki, Phys. Rev. D , 79, 084015 (2009)

J. Grain, T. Cailleteau, A.B., Phys. Rev. D, 81, 024040 (2010)

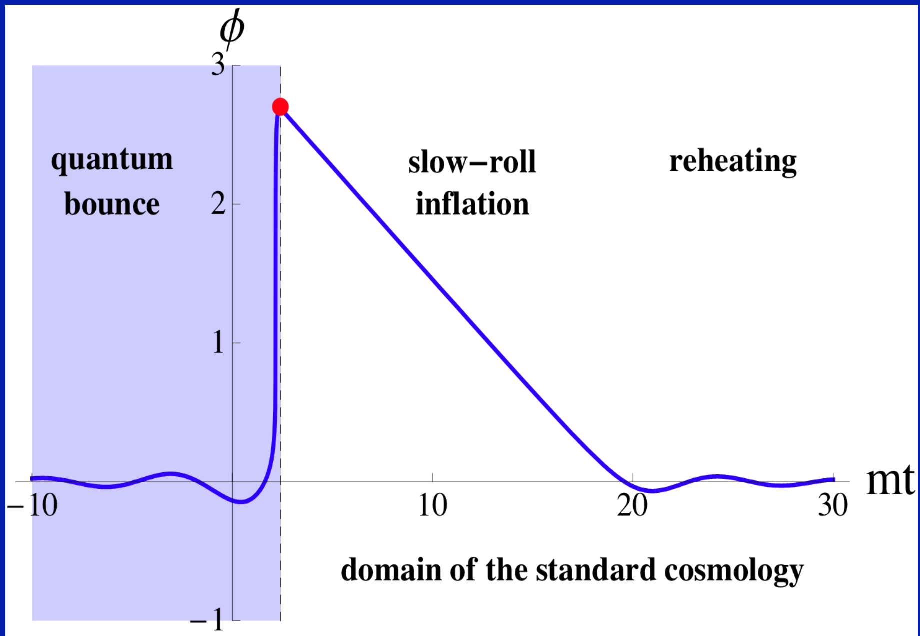


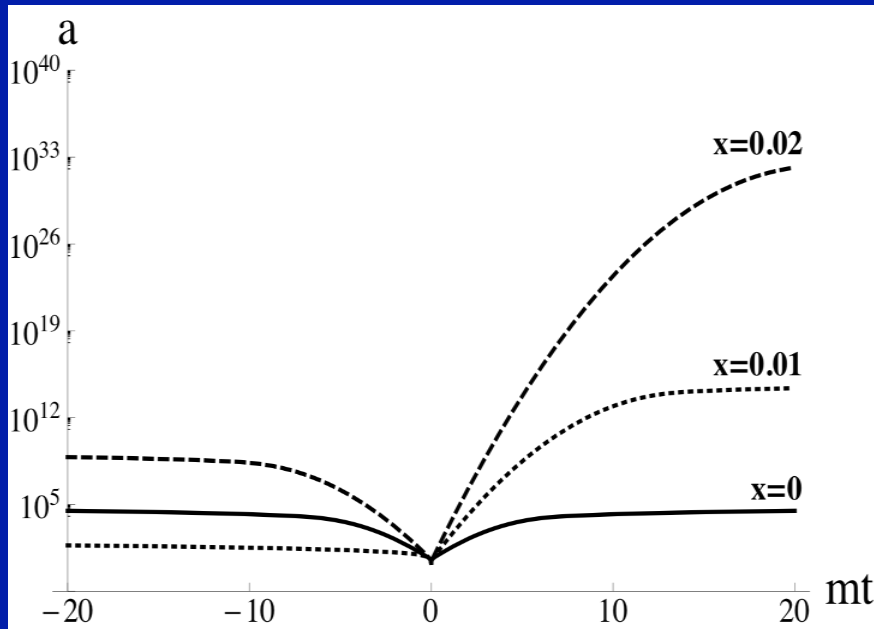


Second approach : Taking into account the background modifications

H changes sign in the KG equation $\phi'' + 3H\phi' + m2\phi = 0$

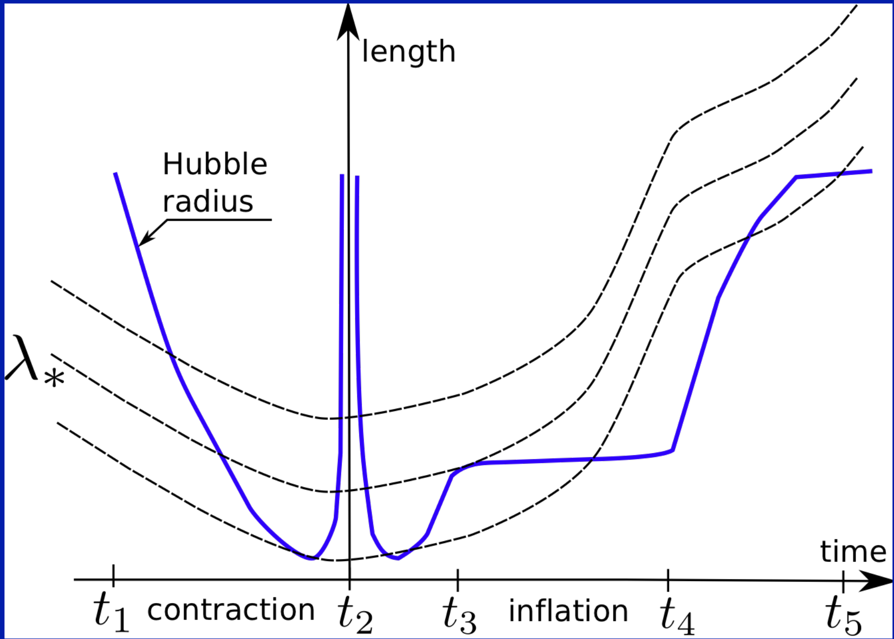
→ Inflation inevitably occurs !



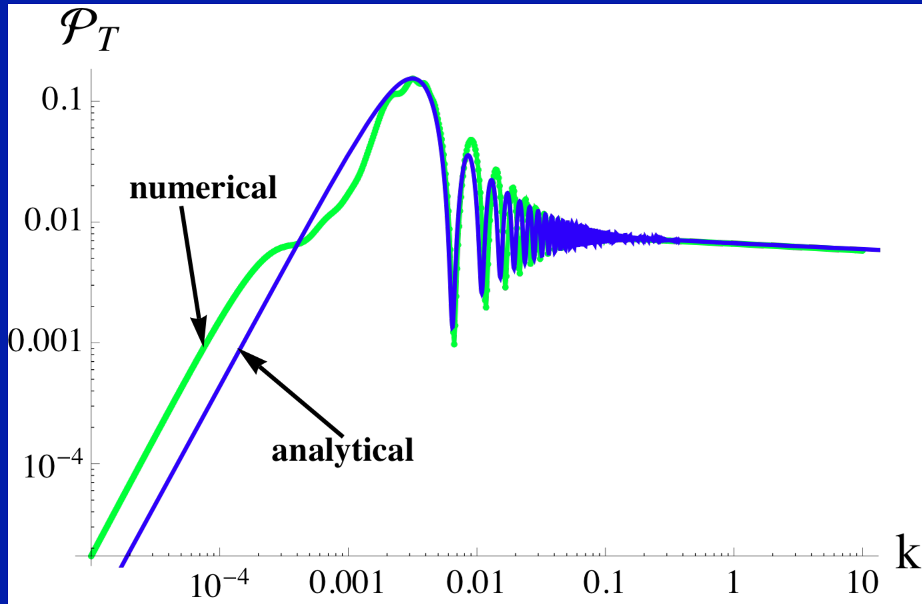


Mielczarek, Cailleteau, Grain, A.B., Phys. Rev. D, 81, 104049, 2010

A tricky horizon history…



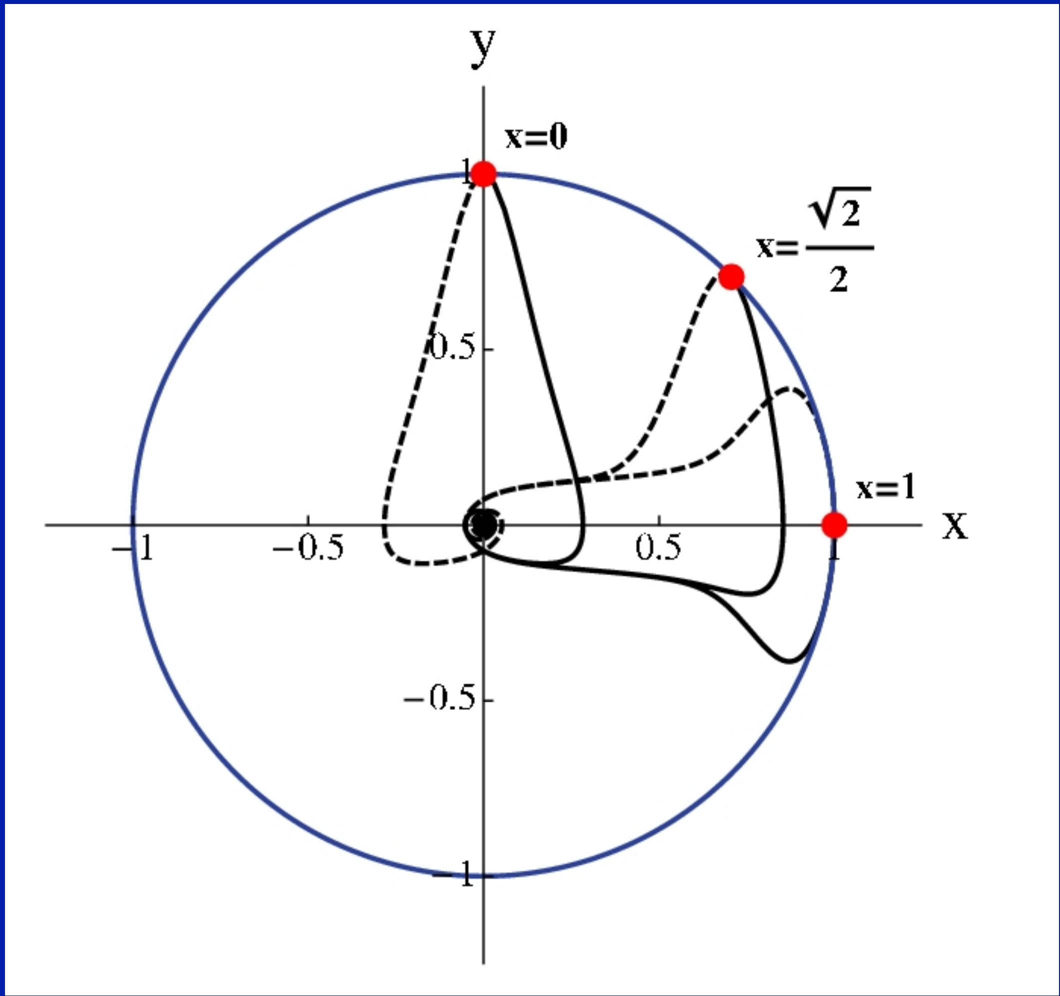
Physical modes may cross he horizon several times…



Computation of the primordial power spectrum:
-Bogolibov transformations
-Full numerical resolution

-The power is suppressed in the infra-red (IR) regime. This is a characteristic feature associated with the bounce
-The UV behavior agrees with the standard general relativistic picture.
-Damped oscillations are superimposed with the spectrum around the ”transition” momentum k* between the suppressed regime and the standard regime.
-The first oscillation behaves like a ”bump” that can substantially exceed the UV asymptotic value.

 Mielczarek, Cailleteau, Grain, A.B., Phys. Rev. D, 81, 104049, 2010



Effective description with a Bogoliubov transformation :

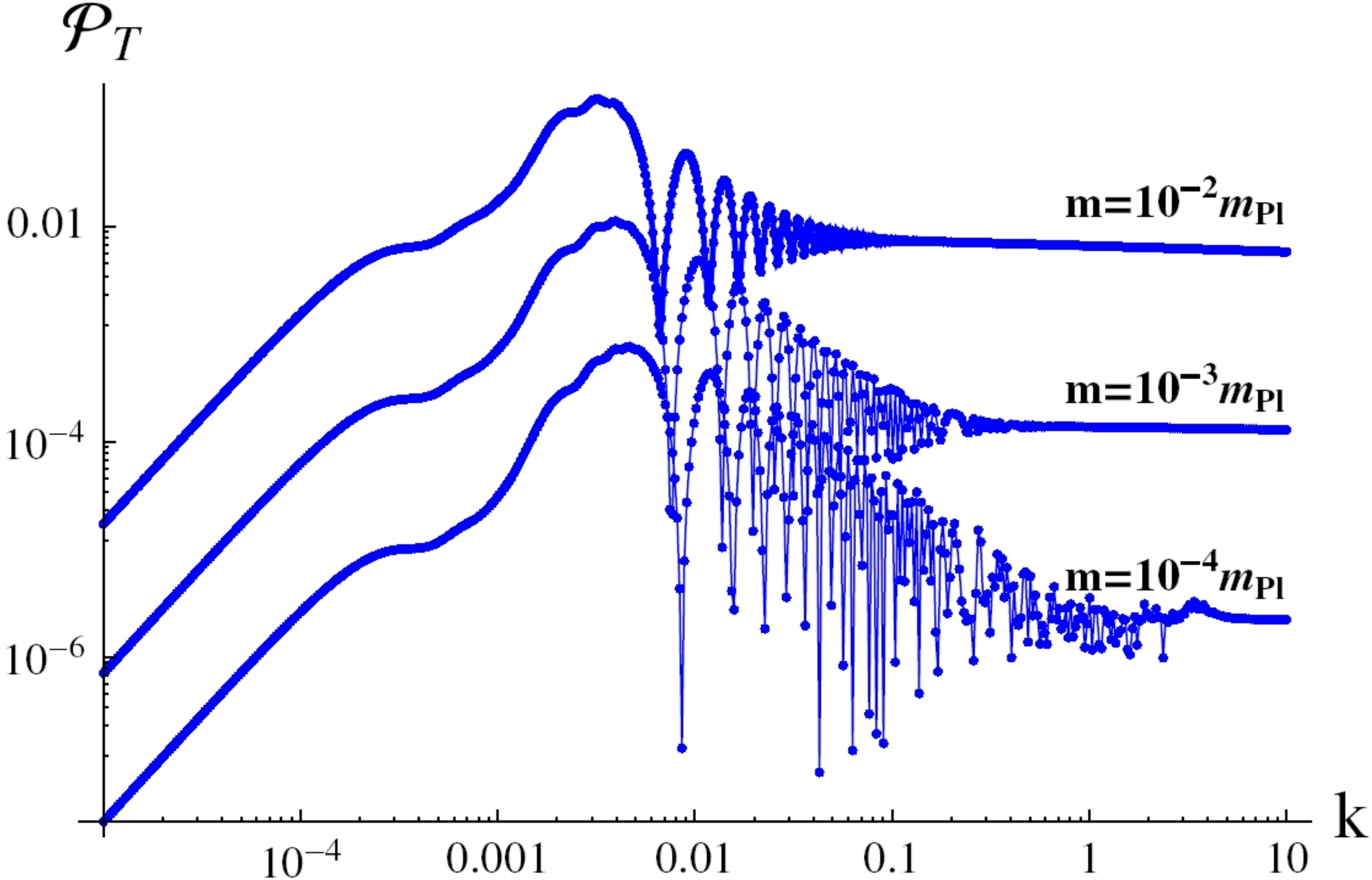
- Frequency of the oscillations controlled by Delta(eta), the width of the bounce
- Amplitude of the oscillations controlled by k0, the effective mass at the bounce

Fundamental description :

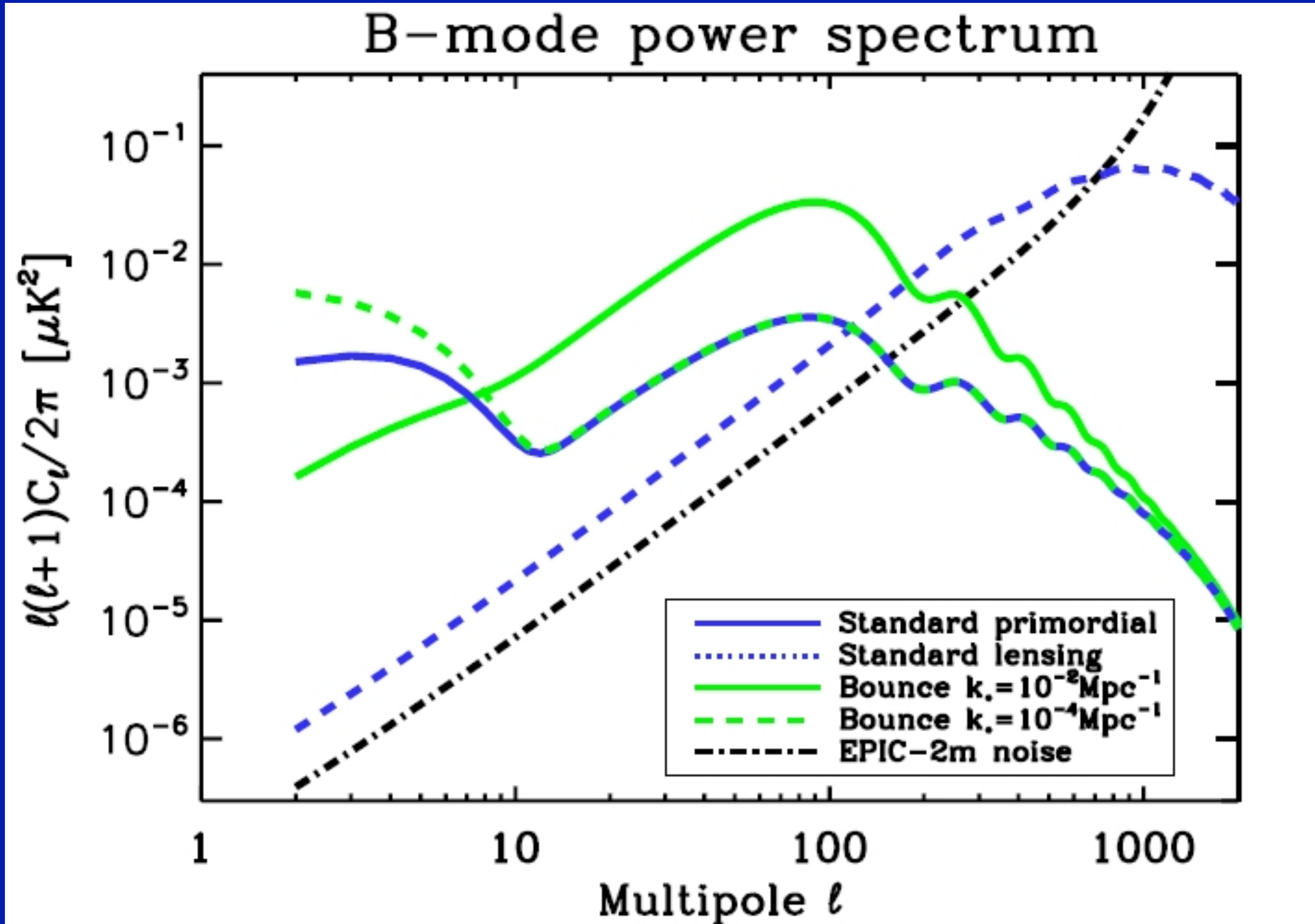
-R driven my the field mass

-k* driven by initial conditions

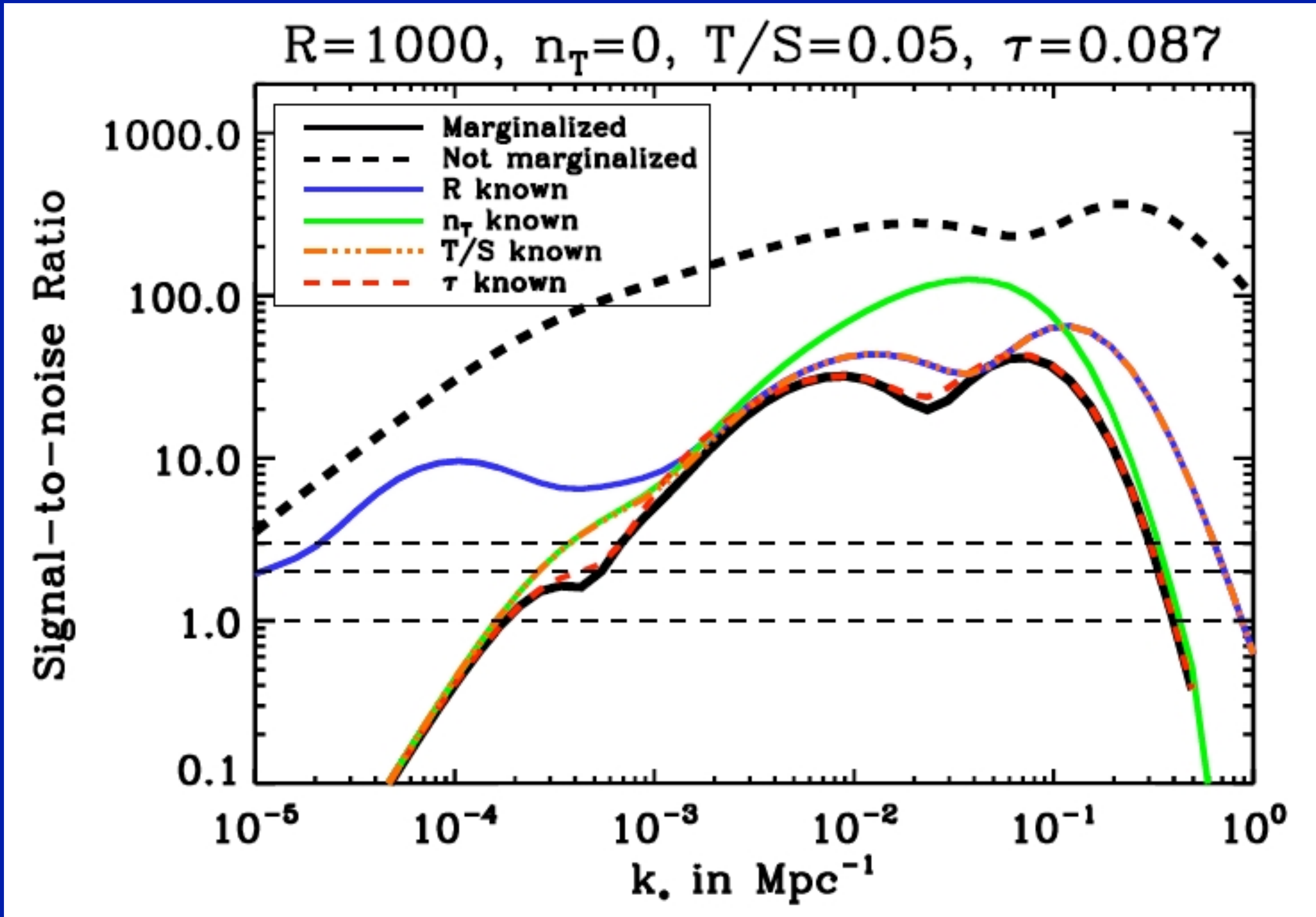
Initial conditions are critical


$\mathcal{P}_T$
0.01
10^{-4}
10^{-6}
$m=10^{-2}m_{Pl}$
$m=10^{-3}m_{Pl}$
$m=10^{-4}m_{Pl}$
k
10^{-4}
0.001
0.01
0.1
1
10

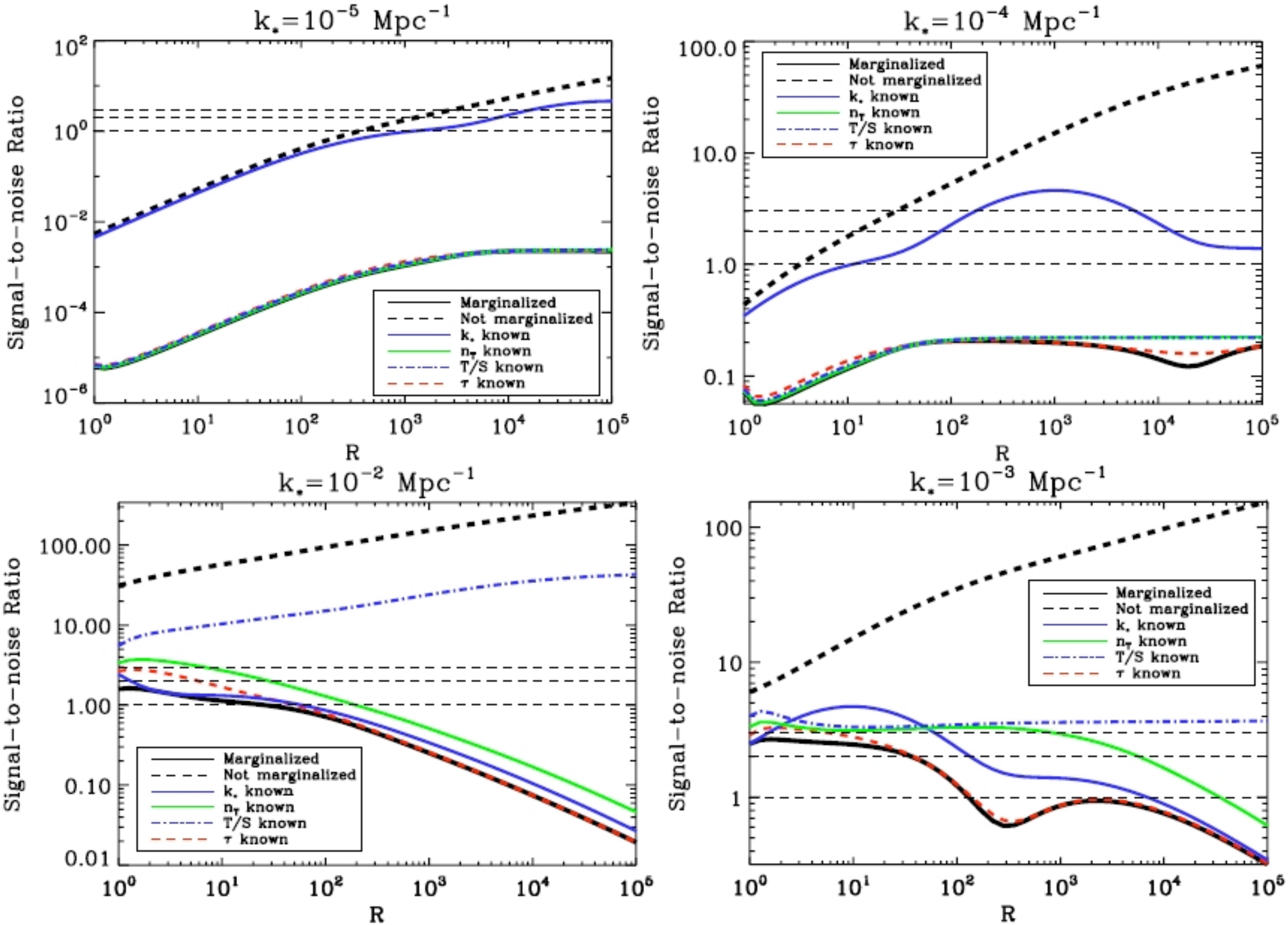
CMB consequences…



Grain, A.B., Cailleteau, Mielczarek, Phys. Rev. D, 82, 123520 (2010)

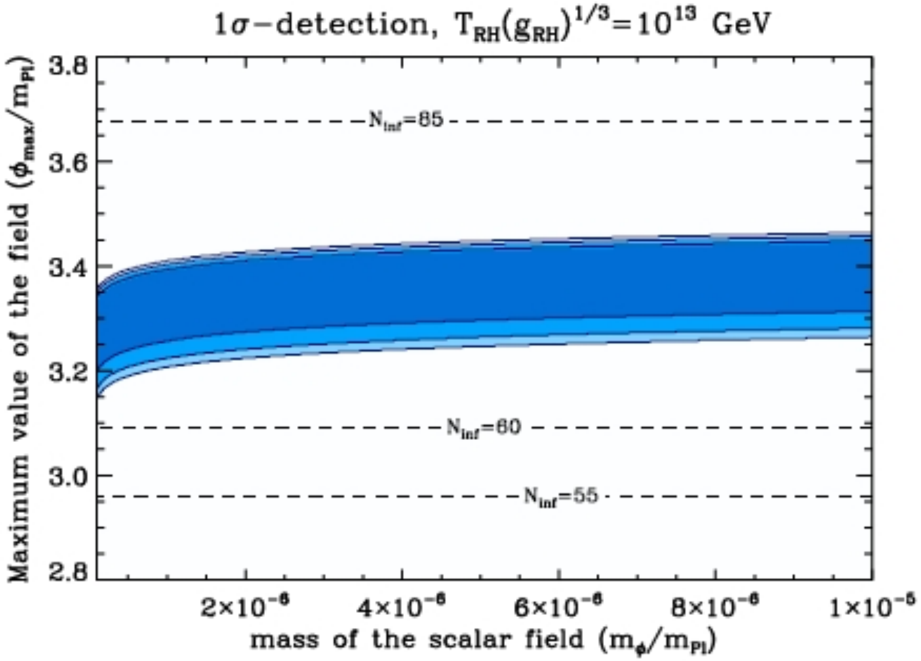


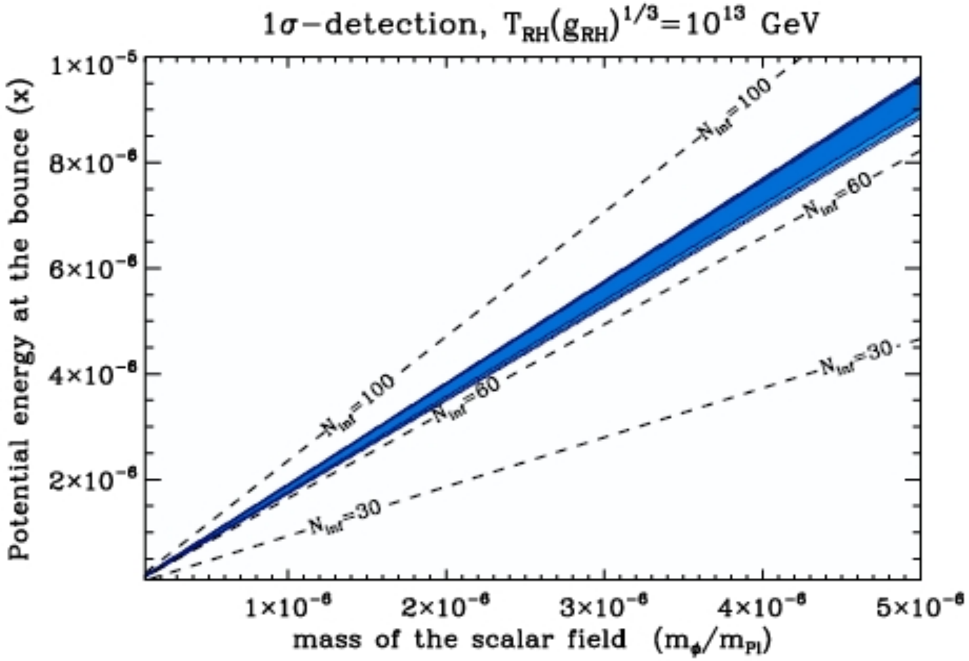
A.B., Grain et al.



$k_* = 10^{-5}$ Mpc^{-1}
$k_* = 10^{-4}$ Mpc^{-1}
$k_* = 10^{-2}$ Mpc^{-1}
$k_* = 10^{-3}$ Mpc^{-1}
Signal-to-noise Ratio
R
Marginalized
Not marginalized
k_* known
n_T known
T/S known
τ known

CMB consequences





If the scalar spectrum is assumed not to be affected : one needs x<2E-6 to probe the model

Grain, A.B., Cailleteau, Mielczarek, Phys. Rev. D, 82, 123520 (2010)

Is a N>78 inflation probable ? What is the probability to be compatible with WMAP data ?

YES. Good news from Ashtekar and Sloan !

Can B-mode be used to distinguised with string inflation ?

I think yes.

Anomaly-free vector algebra for holonomy corrections

$$\{S_{\rm tot}[N_1], S_{\rm tot}[N_1]\} = 0,$$
$$\{D_{\rm tot}[N_1^a], D_{\rm tot}[N_2^a]\} = 0,$$
$$\{S_{\rm tot}[N], D_{\rm tot}[N^a]\} = \frac{\bar{N}}{\sqrt{\bar{p}}}\mathcal{B}D^Q[N^a]$$
$$+\frac{\bar{N}}{\kappa\sqrt{\bar{p}}}\int_\Sigma d^3x\delta N^c\delta_c^k(\partial_d\delta E_k^d)\delta E_k^d\mathcal{A}$$
$$+[\cos(v_2\bar{\mu}\gamma\bar{k})-1]\frac{\sqrt{\bar{p}}}{2}\left(\frac{\bar{\pi}^2}{2\bar{p}^3}-V(\bar{\varphi})\right)\times$$
$$\times\int_\Sigma d^3x\bar{N}\partial_c(\delta N^a)\delta_a^j\delta E_j^c$$
$$+\frac{\bar{\pi}}{\bar{p}^{3/2}}\int_\Sigma d^3x\bar{N}(\partial_a\delta N^a)\delta\pi$$
$$-\bar{p}^{3/2}V_\varphi(\bar{\varphi})\int_\Sigma d^3x\bar{N}(\partial_a\delta N^a)\delta\varphi$$

Including matter

The counterterms can be computed together with the integers. As expected v2=0 (no diffo correction).

→ The algerbra is determined (on need B=0 – and of course A=0)

Perspective I : closing the algebra for scalar modes (with holonomy corrections)

$$\left\{H_G^Q[N], D_G^Q[N^a]\right\} = \frac{\bar{N}}{\sqrt{\bar{p}}}\left[\frac{\sin v_2\bar{\mu}\gamma\bar{k}}{v_2\bar{\mu}\gamma} + \frac{\sin v_1\bar{\mu}\gamma\bar{k}}{v_1\bar{\mu}\gamma} - 2\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}\right] D_G^Q[N^a] + \frac{1}{\kappa}\int_\Sigma d^3x \frac{\bar{N}}{\sqrt{\bar{p}}}\mathcal{A}^{HD}(\partial_d\delta N^c)\delta E_k^d\delta_c^k \tag{12}$$

where

$$\begin{aligned}\mathcal{A}^{HD} = \bar{k}^2\Bigg[&\frac{1}{2}\left(\frac{\sin x}{x}\right)^2(1+\cos v_2x) \\ &+ 2\left(\frac{\sin v_2x}{v_2x}\right)\left(\frac{\sin 2x}{2x} - \frac{\sin v_1x}{v_1x}\right) \\ &+ 2\left(\cos x - \frac{\sin x}{x}\right)\left(\frac{\sin x}{x}\right)\cos(v_2x)\left(\frac{\bar{p}}{\bar{\mu}}\frac{\partial\bar{\mu}}{\partial\bar{p}}\right) \\ &- \left(\frac{\sin v_2x}{v_2x}\right)^2 \\ &- 2\left(\cos v_2x - \frac{\sin v_2x}{v_2x}\right)\left(\frac{\sin 2x}{2x}\right)\left(\frac{\bar{p}}{\bar{\mu}}\frac{\partial\bar{\mu}}{\partial\bar{p}}\right)\Bigg].\end{aligned} \tag{13}$$

The solution seems to be uniquely determined.

Complementary to Bojowald, Hossain, Kagan and Shankaranarayanan

Cailleteau, Mielczarek, A.B., Grain, preliminary

Perpsectices II (tensor modes) :
IV + holonomy for background + modes

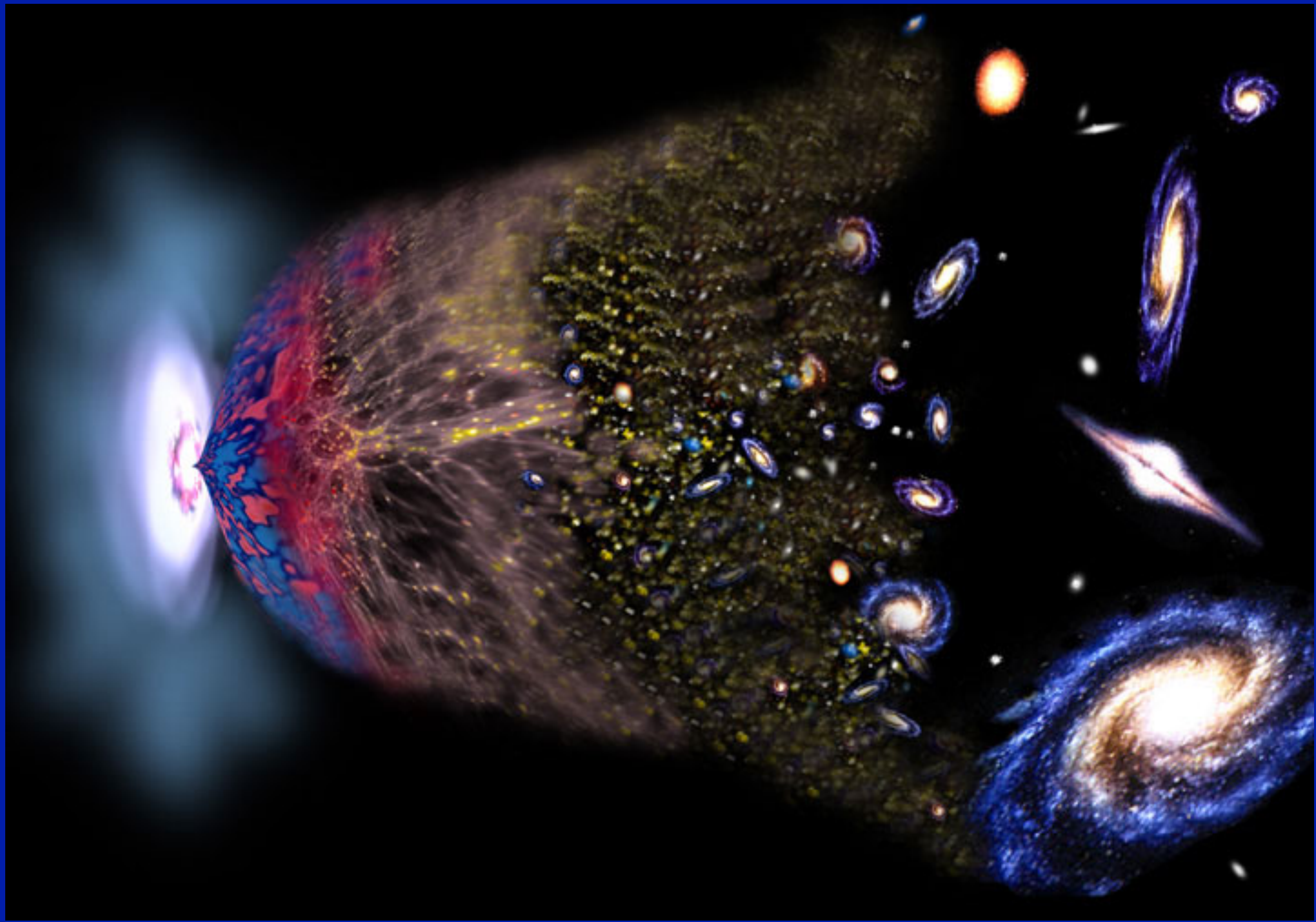
$$\dot{\Phi} = \frac{d\Phi}{dt} = \{\Phi, p_\Phi\}\frac{\partial H_m}{\partial p_\Phi} = D\frac{p_\Phi}{\bar{p}^{\frac{3}{2}}}$$

$$\dot{p_\Phi} = \frac{dp_\Phi}{dt} = -\{\Phi, p_\Phi\}\frac{\partial H_m}{\partial \Phi} = -\bar{p}^{\frac{3}{2}}V_{,\Phi}.$$

$$\begin{aligned}\ddot{\Phi} &= \dot{D}\frac{p_\Phi}{\bar{p}^{\frac{3}{2}}} + D\frac{\dot{p_\Phi}}{\bar{p}^{\frac{3}{2}}} - \frac{3}{2}\frac{\dot{\bar{p}}}{\bar{p}}D\frac{p_\Phi}{\bar{p}^{\frac{3}{2}}} \\ &= \frac{\dot{D}}{D}\frac{Dp_\Phi}{\bar{p}^{\frac{3}{2}}} - D\partial_\Phi V(\Phi) - 3HD\frac{p_\Phi}{\bar{p}^{\frac{3}{2}}} \\ &= \frac{\dot{D}}{D}\dot{\Phi} - D\partial_\Phi V(\Phi) - 3H\dot{\Phi}\end{aligned}$$

Simulation in progress

A.B., Cailleteau, Grain, in progress



Toward a loop – inflation paradigm ?