Lattice Flavour Physics

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lattice QCD errors

- in order to improve errors on hadronic matrix elements by using lattice techniques one has to pay (the currency is $TFlops \times year$)

L.Del Debbio, L.Giusti, M.Lüscher, R.Petronzio, N.T. JHEP 0702 (2007) 056

$$TFlops \times year = 0.03 \left(\frac{N_{conf}}{100}\right) \left(\frac{20\,MeV}{m_{ud}}\right) \left(\frac{L_t}{2L_s}\right) \left(\frac{L_s}{3\,fm}\right)^5 \left(\frac{0.1\,fm}{a}\right)^6$$

$$\sim 0.03 \left(\frac{N_{conf}}{100}\right) \left(\frac{20\,MeV}{m_{ud}}\right) \left(\frac{N_t \times N_s}{64 \times 32}\right)^{\sim 3}$$

- i.e., as a rule of thumb, we can say that fixed the pion mass and given a supercomputer we have a budget quantified in terms of number of points of our lattice. . .
- then we have to decide if to spend this budget for light quark physics (big volumes) or for heavy quark physics (small lattice spacings)
- important:
  - using this formula today is a conservative estimate: several other algorithmic improvements since 2007 (Lüscher deflation acceleration, etc.)
  - on the other hand sampling errors do enter our game and we are neglecting them to obtain our estimates
  - for a detailed discussion of these problems and for a proposal to solve them see (and references therein)

M. Lüscher, S. Schaefer arXiv:1105.4749

lattice QCD errors

- let's play the "lattice effective theory" game invented by:

  S.Sharpe @ Orsay 2004 "LQCD, present and future"
  V. Lubicz @ XI SuperB Workshop LNF 2009

- concerning continuum extrapolations, we imagine to simulate an $O(a)$ improved theory at a_{min} and $\sqrt{2}a_{min}$ and to extrapolate linearly in a^2

$$\mathcal{O}^{phys} = \mathcal{O}^{latt}\left\{1 + c_2(a\Lambda_{QCD})^2 + c_3(a\Lambda_{QCD})^3 + \ldots\right\} \quad \rightarrow \quad \frac{\Delta O}{O} = (2^{3/2} - 1)\, c_3\, (a_{light}\Lambda_{QCD})^3$$

$$\mathcal{O}^{phys} = \mathcal{O}^{latt}\left\{1 + c_2(am_h)^2 + c_3(am_h)^3 + \ldots\right\} \quad \rightarrow \quad \frac{\Delta O}{O} = (2^{3/2} - 1)\, c_3\, (a_{heavy}m_h)^3$$

- we assume $c_3 \sim 1$ (if $c_3 = 0$ usually c_4 large) and set the goal precision to 1%, getting

scale (GeV)	a (fm)	$N_t \times N_s$ @ 3fm	$Pflops \times y$	$N_t \times N_s$ @ 4fm	$Pflops \times y$
0.5	0.069	96×48	10^{-3}	128×64	2×10^{-3}
2.0	0.017	360×180	1	480×240	5
4.0	0.009	720×360	60	960×480	340

- today, large lattice collaborations have access to the computer power required to accommodate low energy scales, so. . .

(pseudoscalar) light meson's physics at 1% level today

BMW, arXiv:1011.2711

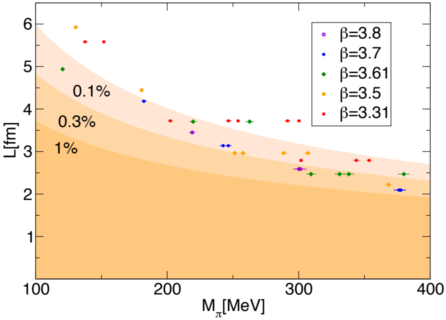
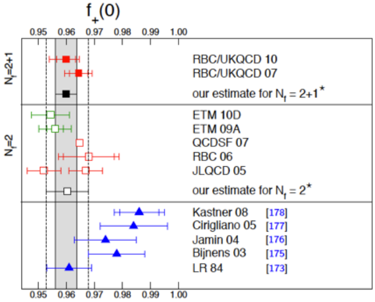


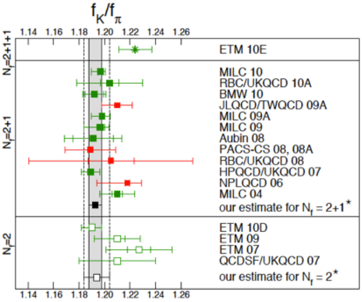
Figure 1: *Summary of our simulation points. The pion masses and the spatial sizes of the lattices are shown for our five lattice spacings. The percentage labels indicate regions, in which the expected finite volume effect [3] on M_π is larger than 1%, 0.3% and 0.1%, respectively. In our runs this effect is smaller than about 0.5%, but we still correct for this tiny effect.*

- from the previous slide we learn that (standard) light meson's observable should be under control now!
- chiral extrapolations are no more a source of concern in 2011 (not only BMW collaboration,. . .)
- . . . at least if one is spending his own budget for simulating big volumes

F_K/F_π & $F_+^{K\pi}(0)$ summary from FLAG

G.Colangelo et al. arXiv:1011.4408





$$F_+^{K\pi}(0) = 0.956(3)(4) \qquad \sim 0.5\%$$

$$\frac{F_K}{F_\pi} = 1.193(5) \qquad \sim 0.5\%$$

- are these error estimates reliable? i.e. can we trust our predictions?
- within the lattice community we could discuss all the life about that, but...

F_K/F_π & $F_+^{K\pi}(q^2)$ can be measured (within SM)

we do have a lot of precise experimental measurements in the quark flavour sector of the standard model that, combined with CKM unitarity (first row), allow us to measure hadronic matrix elements

a simple example from FLAVIAnet kaon working group M.Antonelli et al. Eur.Phys.J.C69

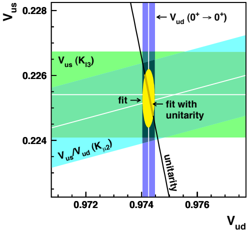
$$\begin{cases} \left|\frac{V_{us}F_K}{V_{ud}F_\pi}\right| = 0.27599(59) \\ \left|V_{us}F_+^{K\pi}(0)\right| = 0.21661(47) \end{cases} \qquad \begin{cases} |V_{ud}|^2 + |V_{us}|^2 = 1 \\ |V_{ud}| = 0.97425(22) \end{cases}$$

where $|V_{ud}|$ comes by combining 20 super-allowed nuclear β-decays and $|V_{ub}|$ has been neglected because smaller than the uncertainty on the other terms, combine to give

$$|V_{us}| = 0.22544(95)$$

$$F_+^{K\pi}(0) = 0.9608(46) \qquad F_+^{K\pi}(0)\Big|_{lattice} = 0.956(3)(4)$$

$$\frac{F_K}{F_\pi} = 1.1927(59) \qquad \frac{F_K}{F_\pi}\Big|_{lattice} = 1.193(5)$$



F_K/F_π & $F_+^{K\pi}(q^2)$ reducing the error

there are two sources of isospin breaking effects,

$$\underbrace{m_u \neq m_d}_{QCD} \qquad\qquad \underbrace{q_u \neq q_d}_{QED}$$

in the particular and (lucky) case of these observables, the correction to the isospin symmetric limit due to the difference of the up and down quark masses (QCD) can be estimated in *chiral perturbation theory*,

$$\begin{cases} F_+^{K\pi}(0) = 0.956(3)(4) \qquad \sim 0.5\% \\ \left(\frac{F_+^{K^+\pi^0}(q^2)}{F_+^{K^0\pi^-}(q^2)} - 1\right)_{QCD} = 0.029(4) \end{cases}$$

A. Kastner, H. Neufeld Eur.Phys.J.C57 (2008)

$$\begin{cases} \frac{F_K}{F_\pi} = 1.193(5) \qquad \sim 0.5\% \\ \left(\frac{F_{K^+}/F_{\pi^+}}{F_K/F_\pi} - 1\right)_{QCD} = -0.0022(6) \end{cases}$$

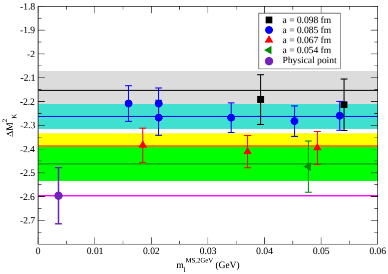
V. Cirigliano, H. Neufeld arXiv:1102.0563

reducing the error on these quantities without taking into account isospin breaking is useless. . .

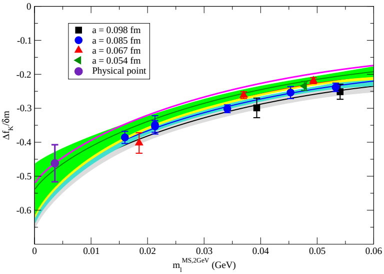
RM123 collaboration, PRELIMINARY!

$$\langle \mathcal{O} \rangle + \Delta \langle \mathcal{O} \rangle = \frac{\int DU\, e^{-S_g[U]-S_f[U]}\, \mathcal{O}}{\int DU\, e^{-S_g[U]-S_f[U]}} = \frac{\int DU\, e^{-S_g[U]-S_f^0[U]}\, (1+\Delta m S^3)\, \mathcal{O}}{\int DU\, e^{-S_g[U]-S_f^0[U]}\, (1+\Delta m S^3)} = \langle \mathcal{O} \rangle + \Delta m \langle S^3\, \mathcal{O} \rangle$$

Chiral extrapolation of ΔM^2_K



Chiral extrapolation of $\Delta f_K/\delta m$



taking as input

$$\Delta M_K = M_{K^0} - M_{K^+} - \Delta M_K^{QED} = -6.0(6)\ MeV$$

we get

$$(m_d - m_u)^{\bar{M}S,2GeV} = 2.28(6)(24)\ MeV$$

$$\left(\frac{F_{K^+}/F_{\pi^+}}{F_K/F_\pi} - 1 \right)_{QCD} = -0.0034(3)(3)$$

to be compared with the χ-pt estimate $-0.0022(6)$

B_K summary from FLAG

G.Colangelo et al. arXiv:1011.4408

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	running	B_K	$\hat{B}_K$
Kim 09	[252]	2+1	C	★	●	●	■	●	0.512(14)(34)	0.701(19)(47)
Aubin 09	[240]	2+1	A	●	★□	●	★	●	0.527(6)(21)	0.724(8)(29)
RBC/UKQCD 09	[253]	2+1	C	●	●	★	★	●	0.537(19)	0.737(26)
RBC/UKQCD 07A, 08	[84, 254]	2+1	A	■	●	★	★	●	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	[255]	2+1	A	■	●*	★	■	●	0.618(18)(135)	0.83(18)
ETM 09D	[256]	2	C	★	●	●	★	●	0.52(2)(2)	0.73(3)(3)
JLQCD 08	[250]	2	A	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
RBC 04	[257]	2	A	■	■	■†	★	●	0.495(18)	0.699(25)
UKQCD 04	[258]	2	A	■	■	■†	■	●	0.49(13)	0.69(18)

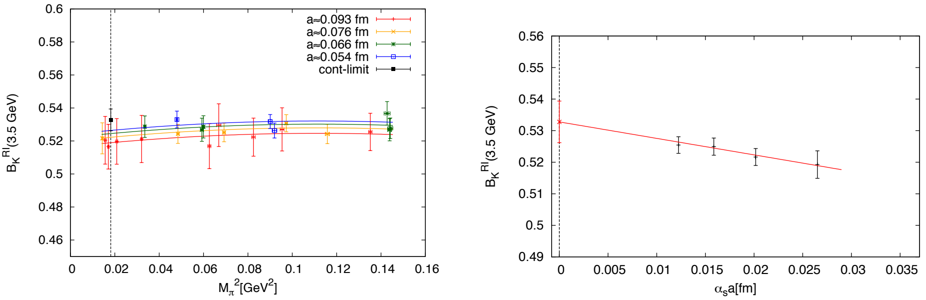
the average is obtained by considering $n_f = 2 + 1$ results only (no debate!) and is

$$B_K(2GeV) = 0.527(6)(21) \qquad \hat{B}_K = 0.724(8)(29) \ \sim 4\%$$

the error is bigger than 1% because the systematics due to the renormalization of the four fermion operator is $\sim 3\%$

latest B_K at 1%

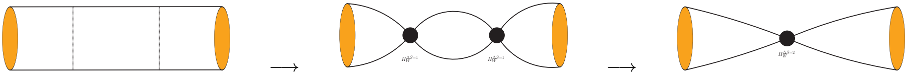
BMW collaboration arXiv:1106.3230



$$B_K(2GeV) = 0.569(6)(4)(6) \qquad \hat{B}_K = 0.779(8)(5)(8) \;\sim 1.6\%$$

- although Wilson-like fermions (wrong chirality mixings) small systematics from renormalization constants. . . (??)
- quite surprising!!. . . on the other hand, on large volumes ($\sim$ 6 fm), small lattice spacings ($\sim$ 0.05 fm) and physical pion masses one expects continuum-like behavior
- in better agreement with unitarity triangle analyses

can we do better?



B_K parametrizes the mixing of the neutral Kaons in the effective theory in which both the W bosons and the up-type quarks have been integrated out,

$$B_K(\mu) = \frac{\langle \bar{K}|\, H_W^{\Delta S=2}(\mu)\, |K\rangle}{\frac{8}{3} F_K^2 M_K^2}$$

in order to be used in ϵ_K formula, the figures in the previous slides have to be corrected for a factor parametrizing long distance contributions

A.Buras, D.Guadagnoli Phys.Rev. D78 (2008)
J.Laiho, E.Lunghi, R.S. Van de Water Phys.Rev. D81 (2010)

$$\hat{B}_K = \kappa_\epsilon\, \hat{B}_K^{lattice} \qquad\qquad \kappa_\epsilon \simeq 0.92$$

in order to do better on this process, we should be able to make a step backward and compute on the lattice the long distance contributions,

$$\langle \bar{K}|\, \mathrm{T} \left\{ \int d^4x\, H_W^{\Delta S=1}(x;\mu)\; H_W^{\Delta S=1}(0;\mu) \right\} |K\rangle$$

to this end, we should be able to make sense of the previous quantity in euclidean space

G.Isidori, G.Martinelli, P.Turchetti Phys.Lett. B633 (2006)

N. Crist arXiv:1012.6034

$\Delta I = 1/2\ K \to \pi\pi$ is coming...

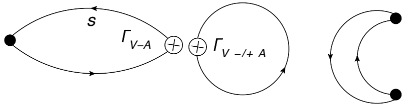
the RBC-UKQCD collaboration is putting a huge effort in the calculation of $K \to \pi\pi$ amplitudes

the key ingredients are the theoretical developments of the last few years

L.Lellouch, M.Lüscher Commun.Math.Phys.219 (2001)
D.Lin et al. Nucl.Phys.B619 (2001)
G.M.de Divitiis, N.T. hep-lat/0409154
C.h.Kim, C.T.Sachrajda, S.R.Sharpe Nucl.Phys.B727 (2005)
...

$$|A|^2 = 8\pi V^2 \frac{M_K^2}{q_\star^2} \left[\delta'(q_\star) + \phi'(q_\star)\right] |M|^2$$

among the remaining complications are disconnected diagrams



RBC+UKQCD collaborations PoS LATTICE2010, 313 (2010)

$$M_\pi = 145 MeV \qquad M_K = 519 MeV$$

$$\Re A_2 = 1.56(07)(25) \times 10^{-8} GeV$$
$$\Im A_2 = -9.6(04)(2.4) \times 10^{-13} GeV$$

RBC+UKQCD collaborations arXiv:1106.2714

$M_\pi = 420 MeV$ unphysical kinematics!

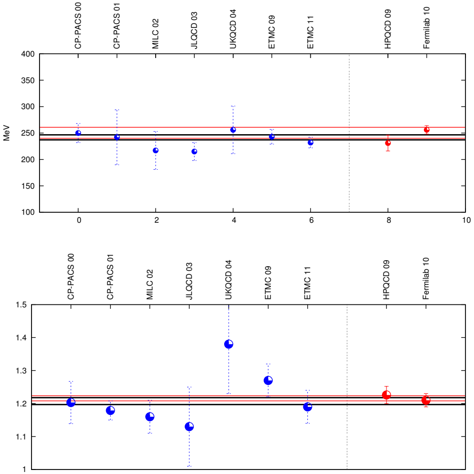
$$\Re A_0 = 3.0(9) \times 10^{-7} GeV$$
$$\Im A_0 = -2.9(2.2) \times 10^{-11} GeV$$

F_B & F_{B_s} averages

$$F_B^{N_f=2+1} = 205(12)\ MeV \qquad \sim 6\%$$

$$F_{B_s}^{N_f=2+1} = 250(12)\ MeV \qquad \sim 5\%$$

$$\frac{F_{B_s}}{F_B}^{N_f=2+1} = 1.215(19) \qquad \sim 1.5\%$$

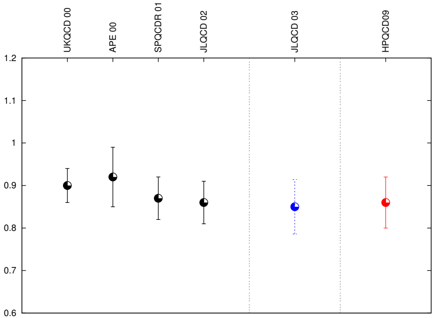


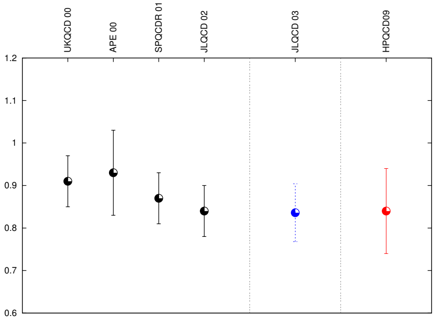
central values are consistent among $N_f = 2$ and $N_f = 2+1$ data sets

as a conservative estimate of the error, one can average $N_f = 2+1$ results

the true question is: are these reasonable estimates?

B_B & B_{B_s} averages





a single $N_f = 2 + 1$ calculation, that combines with F_{B_q} to give

$$F_{B_s}\sqrt{\hat{B}_{B_s}}^{N_f=2+1} = 233(14)\ MeV \quad \sim 6\% \qquad\qquad \xi_B^{N_f=2+1} = 1.237(32) \quad \sim 2.5\%$$

again, are these reasonable estimates?

we usually spend all our budget for big volumes

by simulating b-quarks on the same volumes that we use to extract light meson's physics we have to extrapolate in $1/m_h$, (linear extrapolation from m_h and $\sqrt{2}m_h$)

$$\mathcal{O}^{phys} \quad = \quad \mathcal{O}^{latt} \left\{ 1 + b_1 \frac{\Lambda_{QCD}}{m_h} + b_2 \left(\frac{\Lambda_{QCD}}{m_h} \right)^2 + \dots \right\} \to \frac{\Delta O}{O} = \frac{b_2}{2} \left(\frac{\Lambda_{QCD}}{m_h} \right)^2 \sim 2 \div 3\%$$

$$\to \quad \frac{\Delta O_B}{O_B} \propto \sqrt{a_n^2 \left(\frac{1}{\Lambda_{QCD} L} \right)^{2n} + b_2^2 \left(\frac{\Lambda_{QCD}}{m_h} \right)^4 + c_3^2 (am_h)^6} \quad \sim \quad 3 \div 4\%$$

this can be considered a rough estimate of the bigger errors on B mesons's observables

$N_t \times N_s$	$Pflops \times y$	scale (GeV)	a (fm)	L (fm)
96×48	10^{-3}	0.5	0.069	3 fm
96×48	10^{-3}	2.0	0.017	0.8 fm
96×48	10^{-3}	4.0	0.009	0.4 fm
360×180	1	0.5	0.069	12 fm
360×180	1	2.0	0.017	3 fm
360×180	1	4.0	0.009	1.5 fm

in case of b-physics it (may be) is convenient to change strategy and, given our budget and the scale we want to "accommodate" eventually to do finite volume calculations

step scaling method

(Guagnelli, Palombi, Petronzio, N.T. Phys.Lett.B546:237,2002)

$$\mathcal{O}(m_b, m_l) = \mathcal{O}(m_b, m_l; L_0) \underbrace{\frac{\mathcal{O}(m_b, m_l; 2L_0)}{\mathcal{O}(m_b, m_l; L_0)}}_{\sigma(m_b, m_l; L_0)} \frac{\mathcal{O}(m_b, m_l; 4L_0)}{\mathcal{O}(m_b, m_l; 2L_0)} \cdots$$

- step scaling functions, the σ's, have to be calculated at lower values of the high energy scale

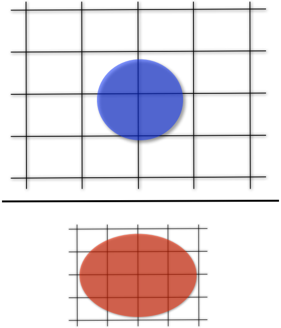
$$\mathcal{O}(m_b, m_l; L_0) \leftarrow m_b = m_b^{phys}$$

$$\sigma(m_b, m_l; nL_0) \leftarrow m_b \leq \frac{m_b^{phys}}{n}$$

- but extrapolating the step scaling functions is much easier than extrapolating the observable itself

$$\mathcal{O}(m_b, m_l; L) = \mathcal{O}^0(m_l; L)\left[1 + \frac{\mathcal{O}^1(m_l; L)}{m_b}\right]$$

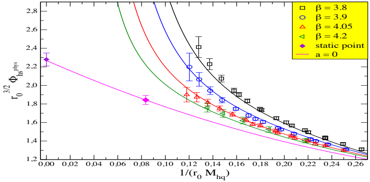
$$\sigma(m_b, m_l; L) = \frac{\mathcal{O}^0(m_l; 2L)}{\mathcal{O}^0(m_l; L)}\left[1 + \frac{\mathcal{O}^1(m_l; 2L) - \mathcal{O}^1(m_l; L)}{m_b}\right]$$



extrapolating $\mathcal{O}$ vs extrapolating finite volume effects

let's take the simplest example, $\Phi_{B_s} = f_{B_s}\sqrt{M_{B_q}}$

B.Blossier et al. PoS LAT2009 151



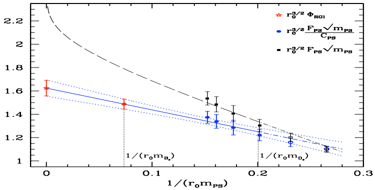
the standard approach to b-physics consists in:

- making simulations at "not so heavy" quark masses ($m_h \sim m_c$)
- extrapolating at the physical point ($m_h^{phys} = m_b$)
- constraining extrapolations with HQET (possibly non-perturbatively renormalized and matched)

$$\frac{\Phi_{B_q}}{C_{PS}} = f_q^0\left[1 + \frac{f_q^1}{m_b} + \ldots\right]$$

J. Heitger and R. Sommer JHEP 0402:022,2004

M. Della Morte et al. JHEP 0802:07,2008



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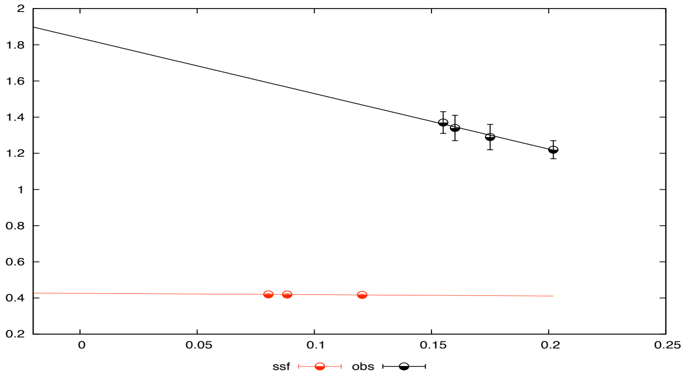

G.M.de Divitiis, M.Guagnelli, F.Palombi, R.Petronzio, N.T. Nucl.Phys.B672:372-386,2003

extrapolating $\mathcal{O}$ vs extrapolating finite volume effects

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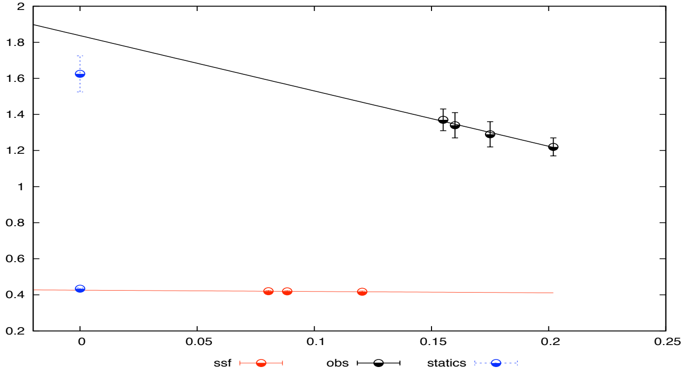
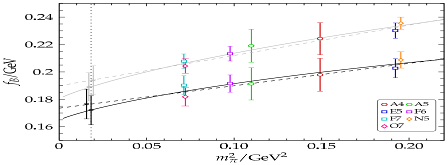


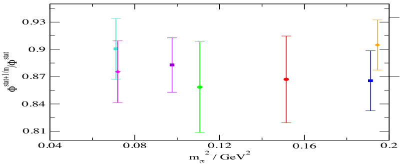
G.M.de Divitiis, M.Guagnelli, F.Palombi, R.Petronzio, N.T. Nucl.Phys.B672:372-386,2003
D.Guazzini, R.Sommer, N.T. JHEP 0801:076 (2008)

similar ideas have been developed in. . .

one does small volume simulations in order to non-perturbatively renormalize HQET and match it to QCD at $O(1/m)$:

see B.Blossier talk at this conference

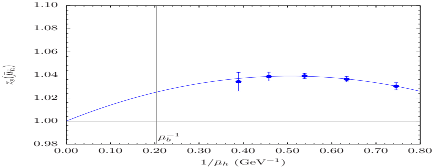


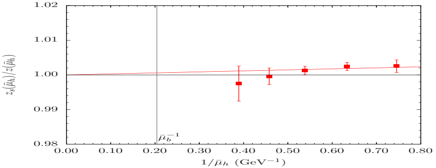


$$F_B^{ALPHA} = 175(10)(5)(6)\ MeV \quad \sim 7\%$$

one considers ratios of observables at fixed large volume but at different values of the heavy quark masses in such a way that the static limit is exactly known:

ETMC collaboration JHEP 1004:049 (2010),arXiv:1107.1441



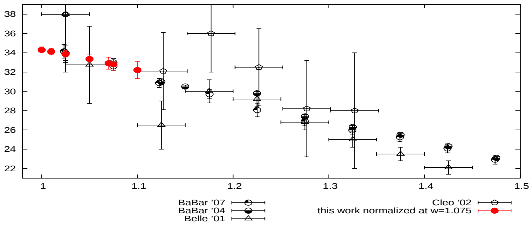


$$F_B^{ETMC} = 195(12)\ MeV \quad \sim 6\%$$

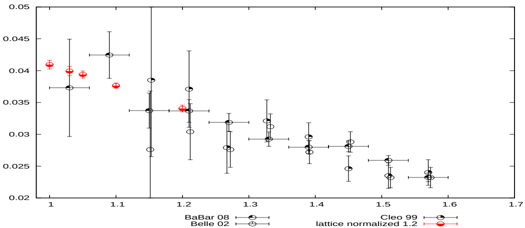
$$F_{Bs}^{ETMC} = 232(10)\ MeV \quad \sim 4\%$$

$B \to D^{(\star)} \ell \nu$ at $\omega > 1$

de Divitiis,Petronzio,N.T. Nucl.Phys.B807:373,2009
de Divitiis,Molinaro,Petronzio,N.T. Phys.Lett.B655:45,2007



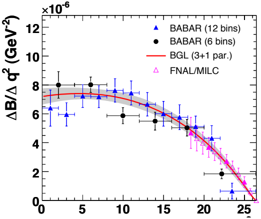
$$V_{cb}(@w = 1.075) \;=\; 37.4(8)(5) \times 10^{-3}$$



$$V_{cb}(@w = 1.2) \;=\; 38.4(9)(42) \times 10^{-3}$$

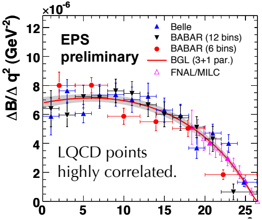
$B \to \pi \ell \nu$ & $B \to D^{(\star)} \ell \nu$ at $\omega = 1$

see M. Franco Sevilla talk at this conference

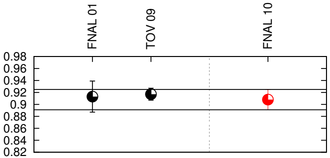


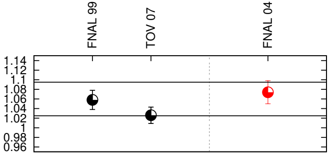
$$|V_{ub}| \times 10^{-3} = 3.13(14)(27) \quad \sim \quad 10\%$$

see P. Urquijo talk at this conference



$$|V_{ub}| \times 10^{-3} = 3.51(34) \quad \sim \quad 10\%$$





$$F(1) = 0.908(17) \quad \sim \quad 1.8\%$$
$$G(1) = 1.060(35) \quad \sim \quad 3\%$$

same analysis of Lubicz, Tarantino, arXiv:0807.4605 except for the updated value of $F(1)$ by Fermilab/MILC collaboration

outlooks

- concerning low energy quantities, such as pseudoscalar light meson's spectrum and matrix elements not requiring disconnected diagrams, lattice QCD entered the precision era (1% accuracy)
- in the low energy sector it's time to compute new quantities: isospin breaking, long distance contributions to weak matrix elements, rare decay rates...
- and to find new efficient *estimators* of in principle *simple* observables like vector meson's and barion's spectrum and matrix elements
- concerning heavy quark's observables, reducing current errors requires dedicated strategies, dedicated collaborations and dedicated computer resources
- attach the problem of non-leptonic decays of heavy ($M > M_K$) mesons