Heavy – quark masses and heavy – meson decay constants from QCD sum rules

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We present a sum-rule extraction of the decay constants of D, D_s, B, and B_s mesons from the two-point function of heavy-light pseudoscalar currents with the main emphasis on the uncertainties in these quantites, both related to the input QCD parameters and the intrinsic uncertainties of the method of sum rules.

[Based on LMS, Phys. Lett. B687, 48, 2010; Phys.Lett. B701, 82, 2011]

A QCD sum-rule calculation of hadron parameters involves two steps:

I. One calculates the operator product expansion (OPE) series for a relevant correlator; obtains the sum rule which relates this OPE to the sum over hadronic states.

We make use of the well-known three-loop OPE for the pseudoscalar correlator in $\overline{MS}$*-scheme*.

II. One extracts the parameters of the ground state by a numerical procedure.

NEW :

(a) Make use of a more accurate duality relation based on Borel-parameter-dependent threshold.

Allows a more accurate extraction of the decay constants and provides realistic estimates of the intrinsic (systematic) errors — those related to the sum-rule extraction procedure.

(b) Study the sensitivity of the extracted value of f_P *to the OPE parameters (quark masses, condensates,...). The corresponding error is referred to as OPE uncertainty, or statistical error.*

Basic object: OPE for $\Pi(p^2) = i\int dx e^{ipx}\langle 0|T\left(j_5(x)j_5^\dagger(0)\right)|0\rangle$, $\qquad j_5(x) = (m_Q + m)\bar{q}i\gamma_5 Q(x)$
and its Borel transform ($p^2 \to \tau$).

Quark – hadron duality implemented as a cut on perturbative correlator :

$$f_Q^2 M_Q^4 e^{-M_Q^2\tau} = \int\limits_{(m_Q+m_u)^2}^{s_{\rm eff}} e^{-s\tau}\rho_{\rm pert}(s,\alpha,\mu)\,ds + \Pi_{\rm power}(\tau,\mu) \equiv \Pi_{\rm dual}(\tau,\mu,s_{\rm eff})$$

The "dual" mass: $$M^2_{\rm dual}(\tau) = -\frac{d}{d\tau}\log\Pi_{\rm dual}(\tau, s_{\rm eff}(\tau)).$$

If quark-hadron duality is implemented properly, then $M_{\rm dual}$ **should be equal to** $M_Q \to$

$s_{\rm eff}$ is a function of τ (and μ) : $s_{\rm eff}\ (\tau,\ \mu)$

Taking into account the dependence of $s_{\rm eff}$ on τ
allows one to improve the accuracy of the duality approximation.

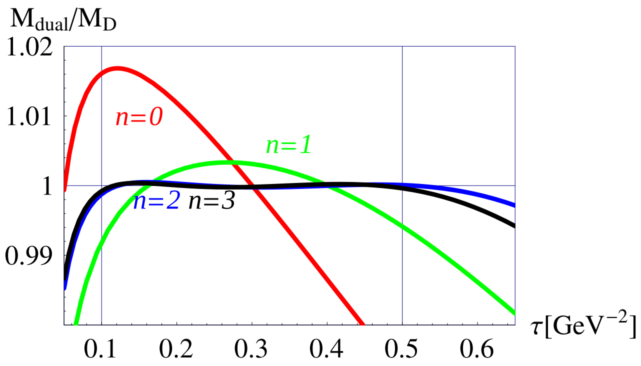
Obviously, in order to predict f_Q, **we need to fix** $s_{\rm eff}$.

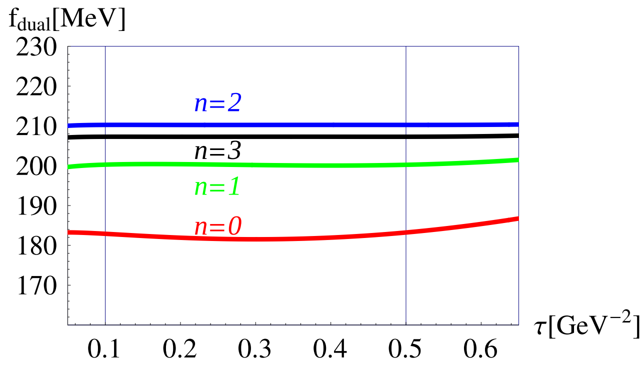
Our new algorithm for extracting ground – state parameters when M_Q is known

(i) Consider a set of Polynomial τ-dependent Ansaetze for $s_{\rm eff}$: $s_{\rm eff}^{(n)}(\tau) = \sum_{j=0}^{n} s_j^{(n)} \tau^j$ and calculate the dual mass for these τ-dependent thresholds. Since the dual mass is the τ-derivative of the dual correlator, the τ-dependence of $s_{\rm eff}$ is essential.

(ii) Minimize the squared difference between the "dual" mass $M_{\rm dual}^2(\tau)$ and the known value M_Q^2 in the working region of τ. This gives us the parameters of the effective continuum threshold.

(iii) Making use of the obtained thresholds, calculate the decay constants:



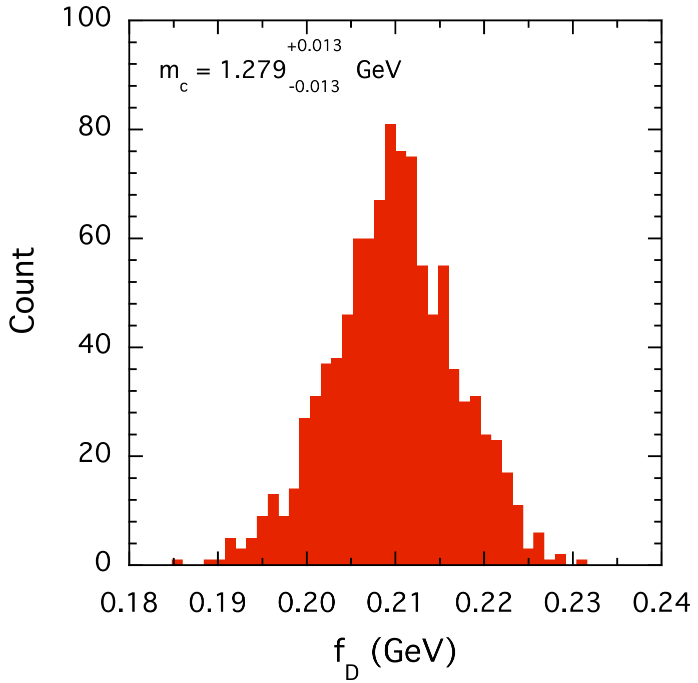


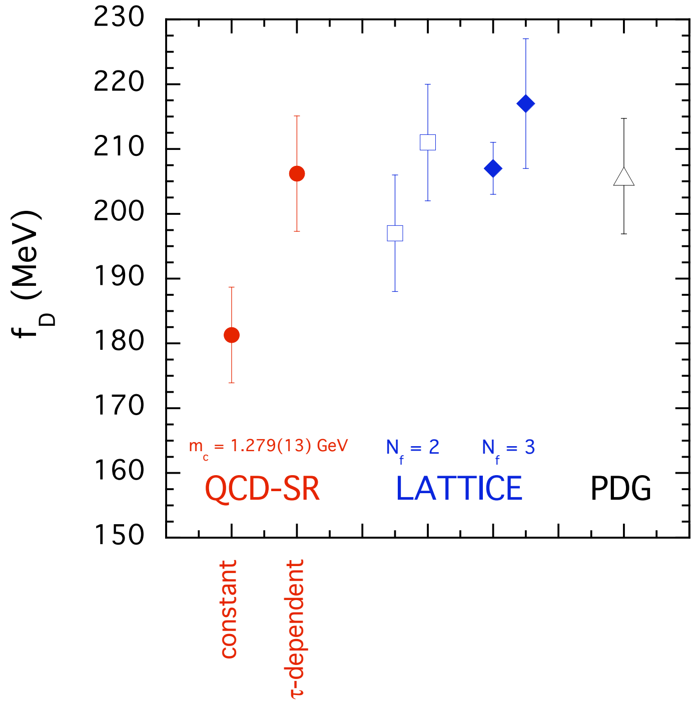
(iv) Take the band of values provided by the results corresponding to linear, quadratic, and cubic effective thresholds as the characteristics of the intrinsic uncertainty of the extration procedure.

[This prescription is based on the experience from quantum mechanics.]

Extraction of f_D

$m_c(m_c) = 1.279 \pm 0.013\,\mathrm{GeV}, \mu = 1 - 3\,\mathrm{GeV}.$





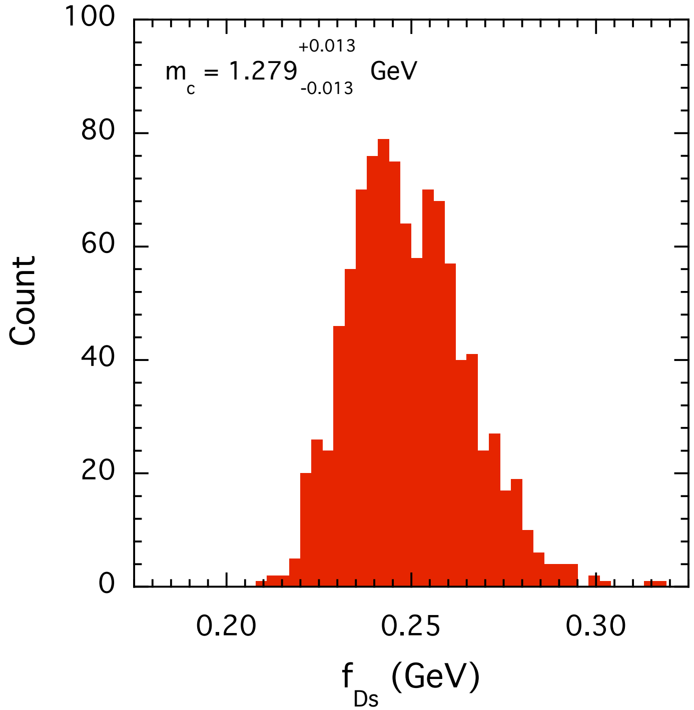
$f_D = 206.2 \pm 7.3_{OPE} \pm 5.1_{syst}$ MeV

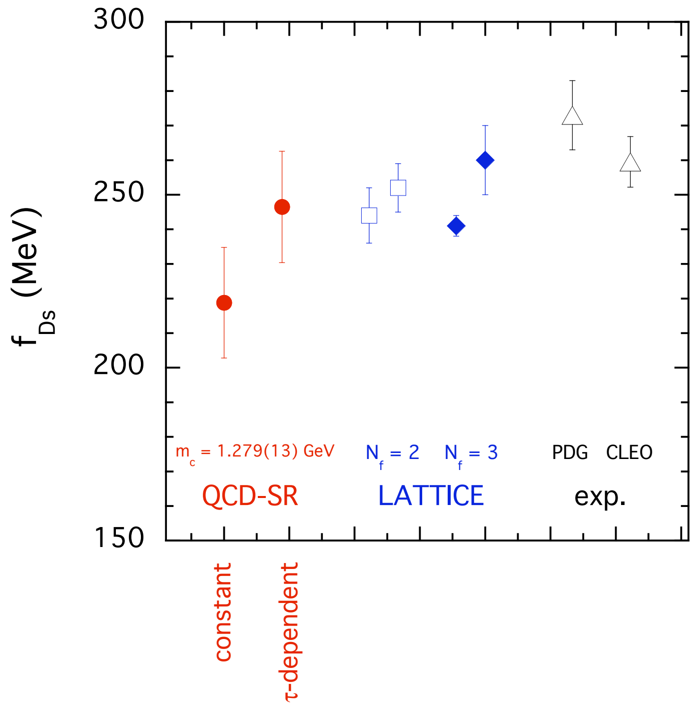
f_D (const) = $181.3 \pm 7.4_{OPE}$ MeV

The effect of τ-dependent threshold is visible!

Extraction of f_{Ds}

$m_c(m_c) = 1.279 \pm 0.013\,\mathrm{GeV}, \mu = 1 - 3\,\mathrm{GeV}.$

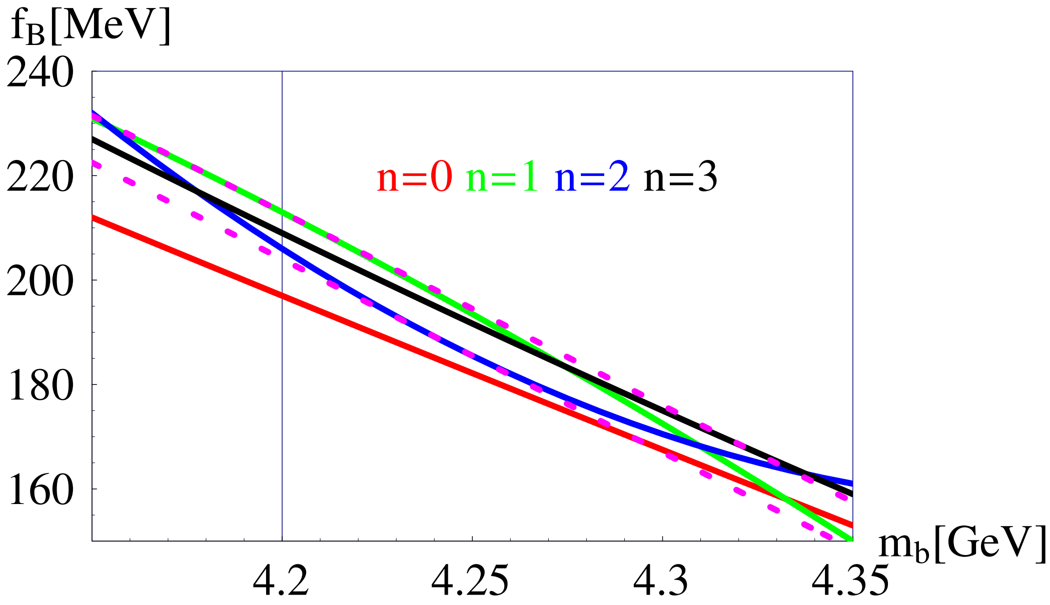




$f_{Ds}\,(\mathrm{const}) = 218.8 \pm 16.1_{OPE}$ MeV

$f_{D_s} = 246.5 \pm 15.7_{OPE} \pm 5_{syst}$ MeV

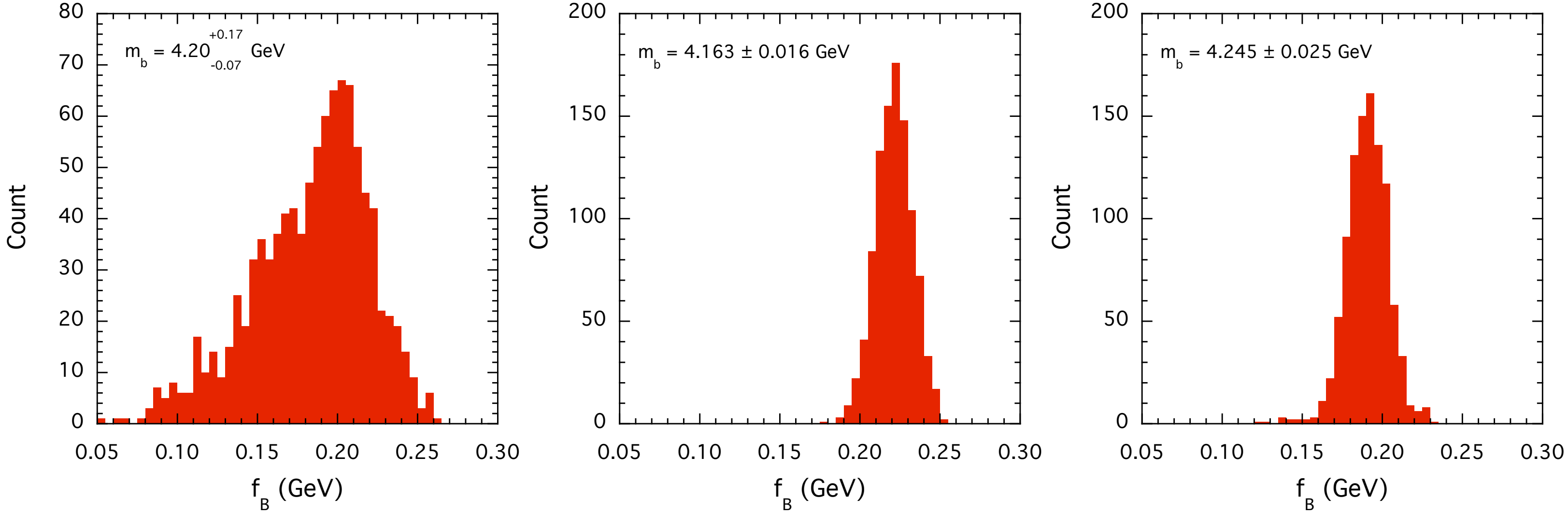
Extraction of f_B: a very strong sensitivity to $m_b(m_b)$



τ-dependent effective threshold:

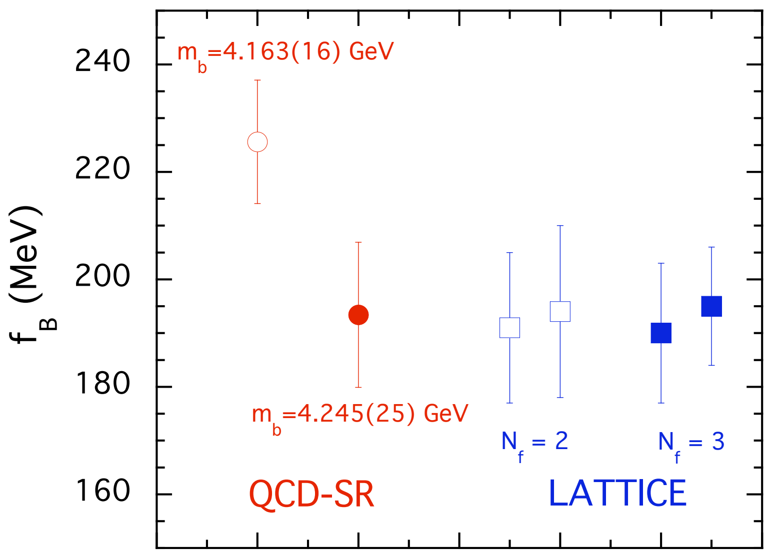
$$f_B^{\rm dual}(m_b, \langle\bar{q}q\rangle, \mu = m_b) = \left[208 \pm 4 - 37\left(\frac{m_b - \mathbf{4.2\ GeV}}{\mathbf{0.1\ GeV}}\right) + 4\left(\frac{\langle\bar{q}q\rangle^{1/3} - \mathbf{0.267\ GeV}}{\mathbf{0.01\ GeV}}\right)\right] MeV,$$

$\pm$ 10 MeV on $m_b \to \mp$ 37 MeV on f_B!



μ=2-8 GeV

The prediction of f_B seems not feasible without a very precise knowledge of m_b



$$m_b(m_b) = 4.163 \pm 0.016 \text{ GeV}:$$

$$f_B = 225 \pm 11.3_{OPE} \pm 4_{syst} \text{ MeV}$$

$$f_{Bs} = 265 \pm 17_{OPE} \pm 5_{syst} \text{ MeV}$$

$$m_b(m_b) = 4.245 \pm 0.025 \text{ GeV}:$$

$$f_B = 193 \pm 12.3_{OPE} \pm 5_{syst} \text{ MeV}$$

$$f_B = 232 \pm 18.36_{OPE} \pm 5_{syst} \text{ MeV}$$

Conclusions

• The effective continuum threshold $s_{\rm eff}$ is an essential ingredient of the method of sum rules which determines to a large extent the numerical values of the extracted hadron parameters. Finding a good criterion for fixing $s_{\rm eff}$ poses a serious problem in the method of sum rules.

• τ-dependence of $s_{\rm eff}$ emerges naturally when trying to make quark-hadron duality – implemented as a cut on the perturbative correlator – more accurate. We proposed a new algorithm for fixing τ-dependent $s_{\rm eff}(\tau)$. We have tested that our algorithm:
- leads to the extraction of more accurate values of bound-state parameters than the standard algorithms used in the context of sum rules before.
- allows one to probe realistic intrinsic uncertainties of the extracted parameters of the bound states.

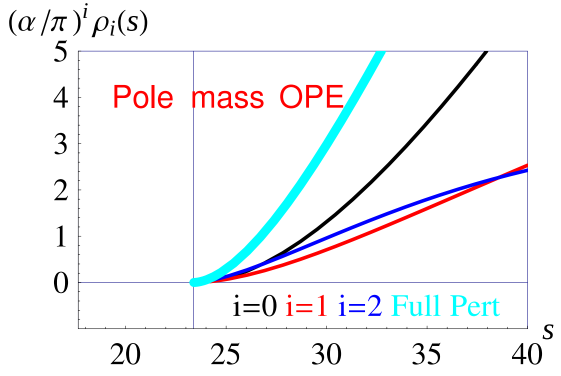
• We obtained predictions for the decay constants of heavy mesons f_Q which along with the "statistical" errors related to the uncertainties in the QCD parameters, <u>for the first time</u> include <u>realistic</u> "systematic" errors related to the uncertainty of the extraction procedure of the method of QCD sum rules.

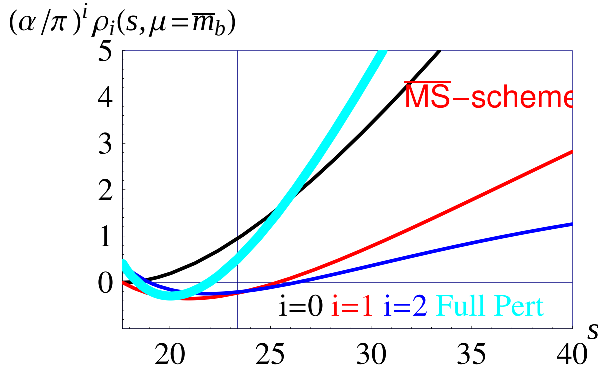
• For the B mesons, a strong correlation between $\bar{m}_b(\bar{m}_b)$ and f_B has been observed. Combining the lattice data for f_B and the results of our analysis leads to a relatively large value of b-quark mass $m_b(m_b) = 4.245 \pm 0.025$ GeV.

OPE : heavy – quark pole mass or running mass?

To α_s^2-accuracy, $m_{b,pole} = 4.83$ GeV $\leftrightarrow$ $m_b(m_b) = 4.20$ GeV:

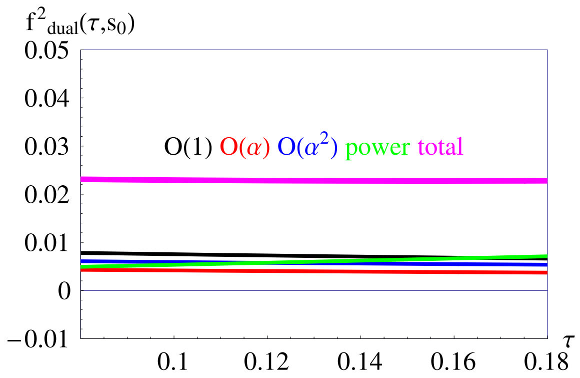
Spectral densities

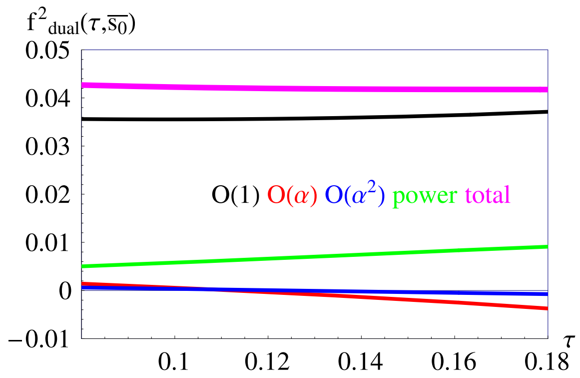




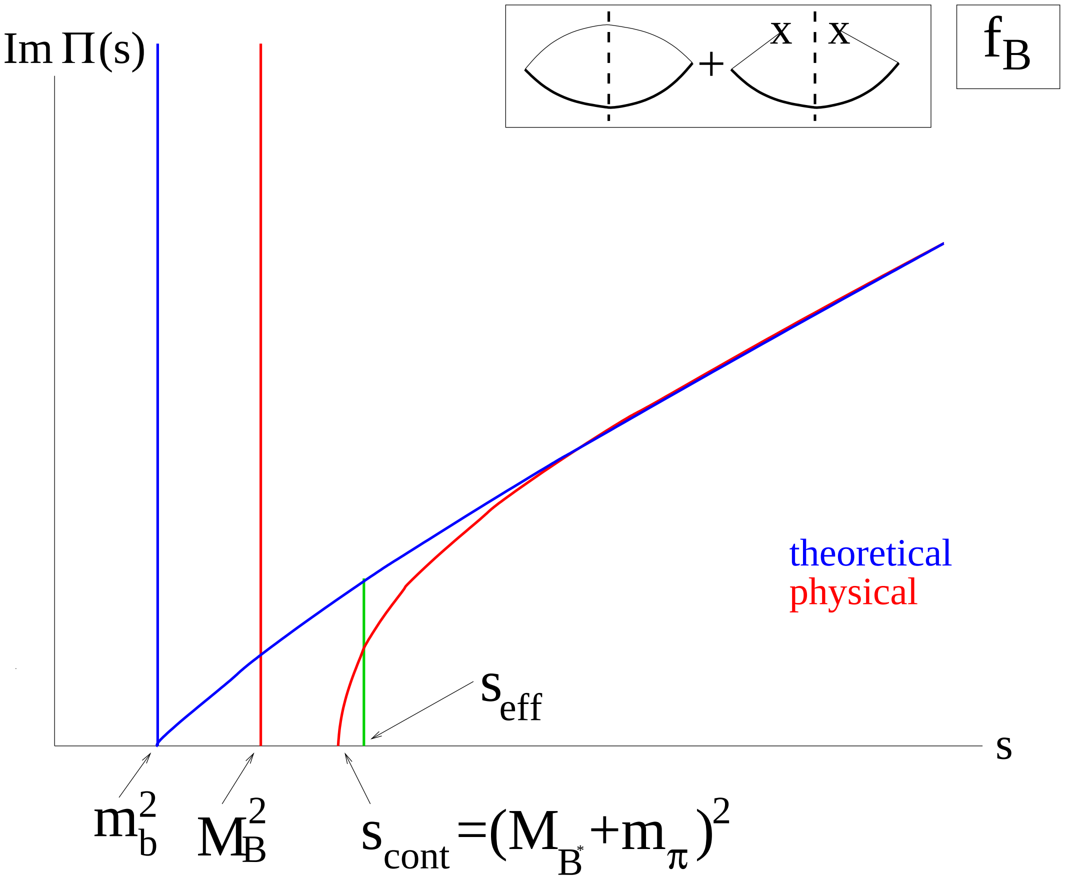
- **In pole mass scheme poor convergence of perturbative expansion**
- **In $\overline{MS}$ scheme the pert. spectral density has negative regions $\rightarrow$ higher orders NOT negligible**

Extracted decay constant





- **Decay constant in pole mass shows NO hierarchy of perturbative contributions**
- **Decay constant in $\overline{MS}$-scheme shows such hierarchy. Numerically, f_P using pole mass $\ll$ f_P using $\overline{MS}$ mass.**



Im Π(s)
x
x
+
f_B
theoretical
physical
s_{eff}
s
m_b^2
M_B^2
$s_{cont}=(M_{B^*}+m_\pi)^2$