Dark Energy and Modified Gravity

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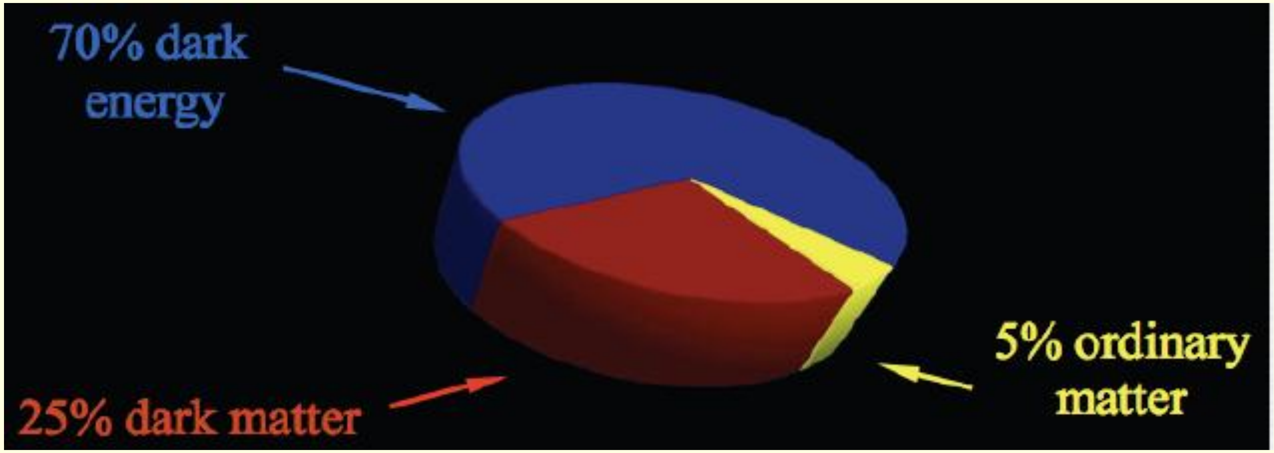
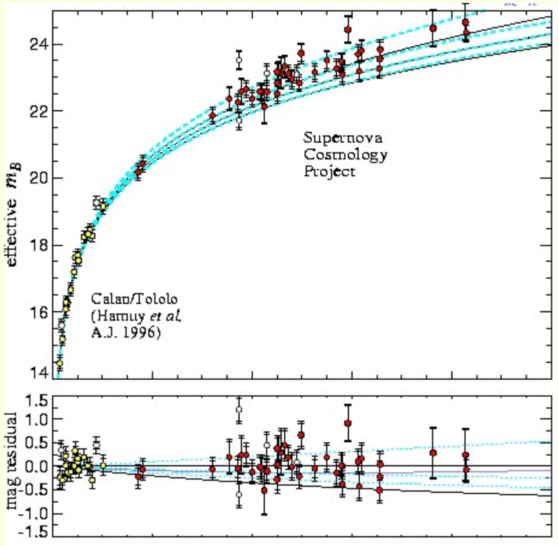
The Big Puzzle



Evidence: The Hubble Diagram

The explosion of high red-shift SN Ia (standard candles):

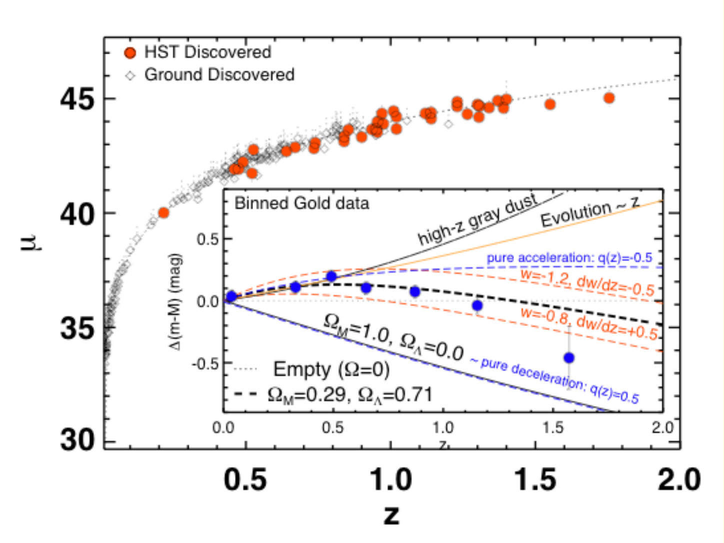
$$q_0 \equiv -\frac{a_0 \ddot{a}_0}{(\dot{a}_0)^2} \simeq -0.67 \pm 0.25$$



Within General Relativity, link to matter and dark energy

$$q_0 = -\Omega_\Lambda + \frac{1}{2}\Omega_m \sim -0.67$$

Dark Energy must exist!



The Cosmic Microwave Background

Fluctuations of the CMB temperature across the sky lead to acoustic peaks and troughs, snapshot of the plasma oscillations at the last scattering

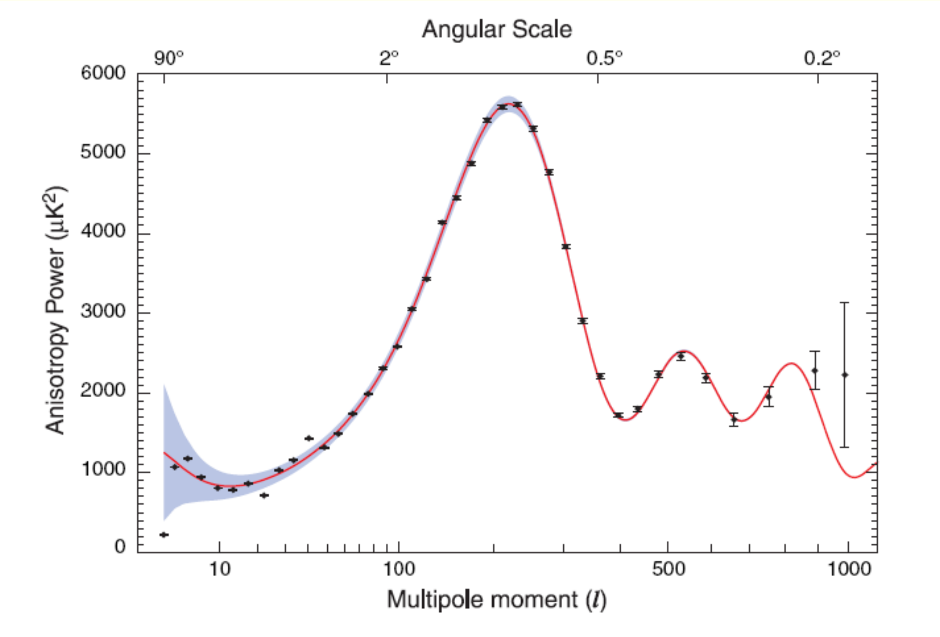
The position of the first peak:

$$l_1 \approx \frac{220}{\sqrt{\Omega_\Lambda + \Omega_m}}$$

The universe is spatially flat

$$\Omega_\Lambda + \Omega_m = 1$$

$$\Omega_\Lambda = \frac{2}{3}\left(\frac{1}{2} - q_0\right) \sim 0.78$$



WMAP data

Dark Energy Really?

In fact we are not absolutely certain that the acceleration of the universe is due to dark energy. On the contrary, the acceleration of the expansion of the universe may be interpreted in four different ways:

1) The acceleration is entirely due to the presence of a constant vacuum energy (cosmological constant).

$$\frac{1}{16\pi G_N}\int d^4x\sqrt{-g}(R-2\Lambda)$$

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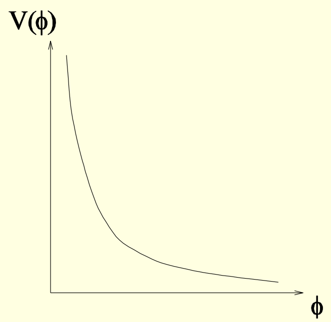
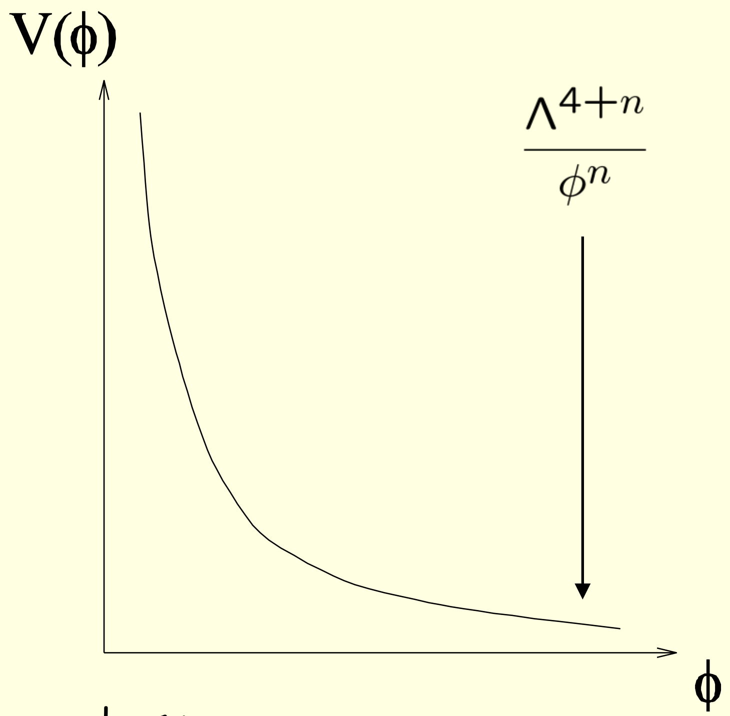
2) The acceleration results from the existence of a new type of matter: dark energy.

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}(R - \mathcal{L}_{DE})$$



Dark Energy

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)$$



Field rolling down a runaway potential, reaching large values now (typically Planck scale)

How Flat?

Energy density and pressure:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \;\; p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$w = \frac{p}{\rho}$$

$m \gg H_0$	very fast roll	$w \approx 1$	
$m \ll H_0$	slow roll	$w \approx -1$	cosmological constant
$m \approx H_0$	gentle roll	$w \neq -1$	dark energy

$$H_0 \approx 10^{-42}\mathrm{GeV}$$

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3) What is seen as acceleration is in fact a misinterpretation of data and really we must face a modification of gravity at large enough scales.

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}h(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$$

- An infinite class of modified gravity models can be considered:

$$S_{\mathsf{MG}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$$

- These Lagrangian field theories fall within the category of higher derivative theories.

- Ostrosgradski's theorem states that these theories are *generically* plagued with ghosts. Quantum mechanically, this implies an explosive behaviour with particles popping out of the vacuum continuously. In particular an excess in the gamma ray background.

- A large class is ghost-free though, the f(R) models:

$$S_{\mathrm{MG}} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} f(R)$$

f(R) totally equivalent to an effective field theory with gravity and scalars

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, e^{2\phi/\sqrt{6}m_{\rm Pl}}g_{\mu\nu})\right)$$

The potential V is directly related to f(R).

$$V(\phi) = m_{\rm Pl}^2\frac{Rf' - f}{2f'^2},\ \ f' = e^{-2\phi/\sqrt{6}m_{\rm Pl}}$$

Dark Energy Really?

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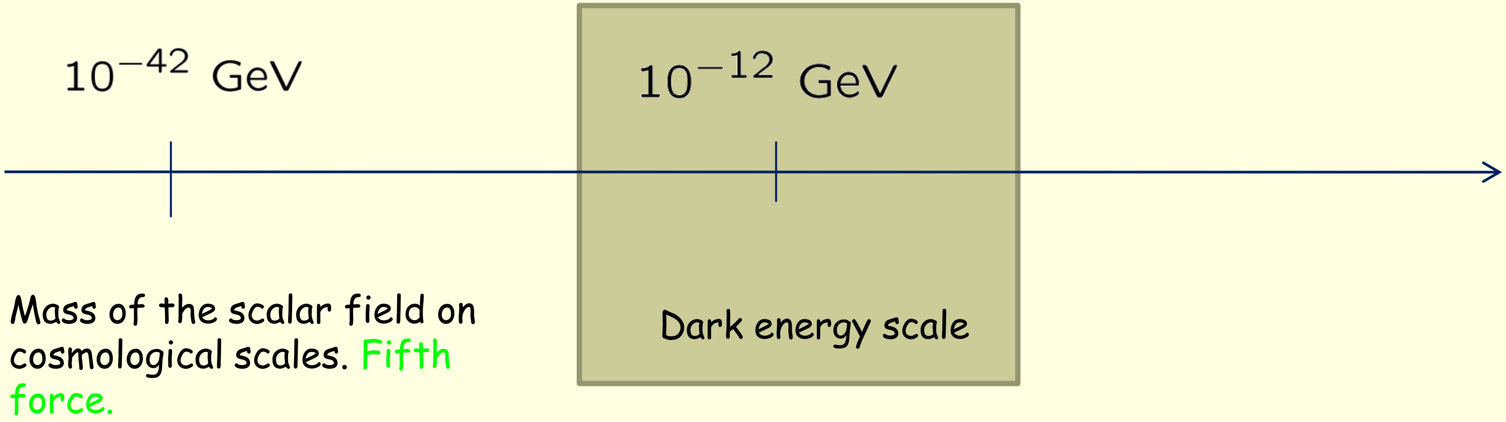
2) The acceleration results from the existence of a new type of matter: dark energy.

3) What is seen as acceleration is in fact a misinterpretation of data and really we must face a modification of gravity at large enough scales.

4) There is no real acceleration. We just live in a void surrounded by more matter. No copernican principle stands.

$$ds^2 = -dt^2 + \frac{R'^2(r,t)}{1-k(r)r^2}dr^2 + R^2(r,t)d\Omega^2$$

New Scales in Physics

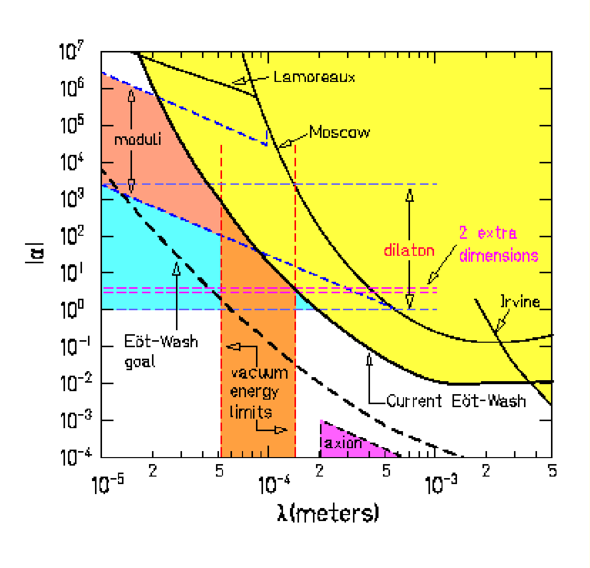


- Deviations from Newton's law are parametrised by:

$$\phi_N = -\frac{G_N}{r}(1 + 2\beta_\phi^2 e^{-r/\lambda})$$

The tightest constraint on β comes from the Cassini probe measuring the Shapiro effect (time delay):

$$\beta_\phi^2 \leq 1.210^{-5}$$



Three known mechanisms can accomodate usual gravity locally with deviations on larger scales:

i) The Vainshtein mechanism in the case of DGP gravity (and similar models like the Galileon). In this case, gravity is like GR locally, like a scalar-tensor theory at larger scales (and 5d at very large scales).

ii) The chameleon mechanism for scalar-tensor theories and f(R) models. Gravity is locally like GR, deviates from GR at intermediate distances and is like GR far away.

iii)The Damour-Polyakov mechanism (dilaton at strong coupling) or symmetron. Gravity is like GR locally and modified at all scales further away.

Chameleons

Chameleon field: field with a matter dependent mass

A way to reconcile gravity tests and cosmology

Nearly massless field on cosmological scales

Massive field in the laboratory

Chameleon Lagrangian

Effective field theories with gravity and scalars:

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, A^2(\phi)g_{\mu\nu})\right)$$

$$\beta_\phi = m_{\mathsf{Pl}}\frac{d\ln A}{d\phi}$$

Coupling to Photons

$$L_{\text{eff}} = \frac{1}{M_\gamma}\phi F_{ab}F^{ab}$$

$$\beta_\phi = \frac{m_{\text{Pl}}}{M_{\text{matter}}}$$

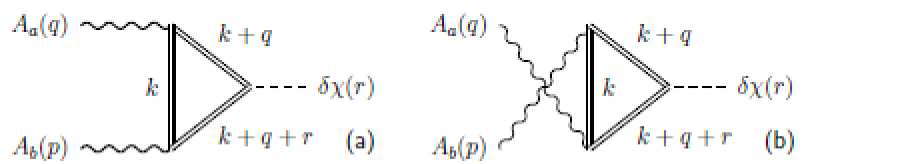


Figure 1. Diagrams contributing to the leading interaction between dark energy and the electroweak gauge bosons, which determine an effective operator acting on $A_a(q)A_b(p)\chi(r)$. Note that the momentum carried by χ is taken to flow into the diagram. Double lines represent a species of heavy fermion charged under $SU(2)\times U(1)$.

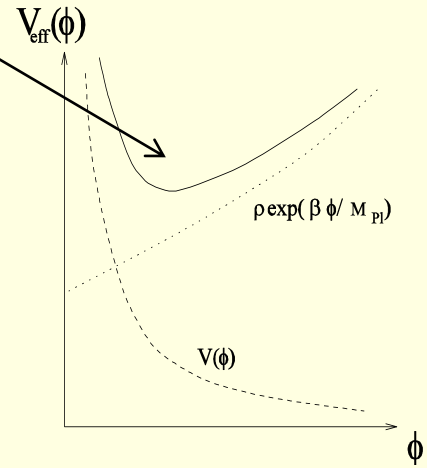
When the coupling to matter is universal, and heavy fermions are integrated out, a photon coupling is induced. Other contributions from conformal anomaly.

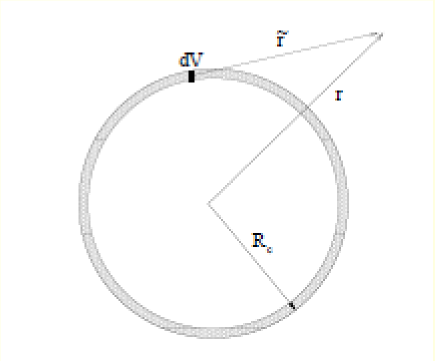
The effect of the environment

When coupled to matter, scalar fields have a matter dependent effective potential

$$V_{eff}(\phi) = V(\phi) + \rho_m A(\phi)$$

Environment dependent minimum





The field generated from deep inside is Yukawa suppressed. Only a thin shell radiates outside the body. Hence suppressed scalar contribution to the fifth force.

Large Scale Structure

At the background level, chameleon models and their siblings the f(R) models behave like a pure cosmological constant.

Fortunately, this is not the case at the perturbation level where the growth factor evolves like:

$$\delta'' + \mathcal{H}\delta' - \frac{3}{2}\mathcal{H}^2(1 + \frac{2\beta^2}{1 + \frac{m^2 a^2}{k^2}})\delta = 0$$

The new factor in the brackets is due to a modification of gravity depending on the comoving scale k.

This is equivalent to a scale dependent Newton constant.

Everything depends on the comoving Compton length:

$$\lambda_c = \frac{1}{ma}$$

Gravity acts in an usual way for scales larger than the Compton length

$$\delta \sim a$$

Gravity is modified inside the Compton length with a growth:

$$\delta \sim a^{\frac{\nu}{2}}, \quad \nu = \frac{-1 + \sqrt{1 + 24(1 + 2\beta^2)}}{2}$$

General Relativity ⇒ Modified gravity

$z = z^*$

Matter follows the geodesics of the Jordan frame metric:

$$ds^2 = e^{2\beta\kappa_4\phi}a^2(\eta)(-(1+2\phi_N)d\eta^2+(1-2\phi_N)dx^2)$$

There are two Newtonian potentials:

$$\psi = \phi_N + \kappa_4\beta\phi, \quad \Phi_N = \phi_N - \kappa_4\beta\phi$$

Photons are sensitive to the sum of the two potentials:

$$\eta_\theta \equiv \frac{\Phi_N}{\psi} = \frac{1-2\beta^2}{1+2\beta^2}$$

inside the Compton wave length.

Laboratory Experiments?

- Scalar fields could be experimentally detected. Different types of experiments have been proposed and carried out:
- Casimir force experiments could be sensitive to a new scalar field force.
- Helioscopes (CAST(CERN)……) could detect scalars emitted from the inner sun.
- Optical cavity experiments are looking for birefringence and afterglow effects (BMV , ALP (DESY), GammeV (Fermilab), ADMX (Seattle)……)
- Bouncing Neutrons here at the ILL in Grenoble

Atomic Precision Tests

In the electric field created by the nucleus, the scalar field satisfies:

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = -\frac{E^2}{M_\gamma} + \frac{m_N}{M}\delta$$

The scalar field is therefore:

$$\phi = -\frac{m_N}{4\pi M r} - \frac{Z^2\alpha}{8\pi M_\gamma r^2}$$

This gives a shift to the atomic energy levels

$$\delta H = \frac{m}{M}\phi \qquad \delta E_i = < i|\delta H|i >$$

This gives a shift to the 1s-2s difference:

$$\delta E_{1s-2s} = \frac{3m_N}{16\pi M^2 a_0} m_e + \frac{7\alpha}{32\pi a_o^2 M M_\gamma} m_e$$

The contribution to the Lamb shift:

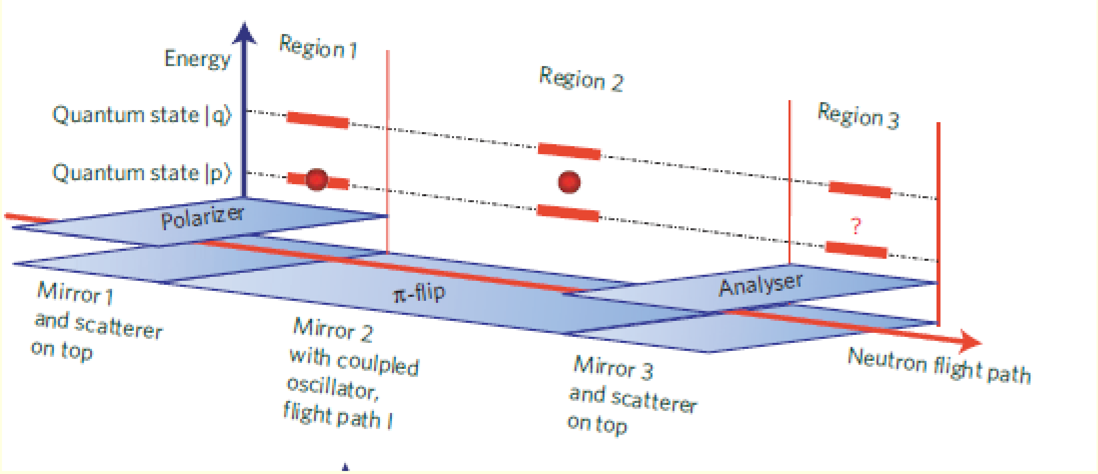
$$\delta E_{2s-2p} = \frac{Z^4 \alpha}{48\pi a_0^2 M M_\gamma} m_e$$

A stringent bound on the matter coupling can be deduced from the 1s-2s uncertainty:

$$M \geq 10\,\mathrm{TeV}$$

Atomic precision tests simply indicate that if scalars are around, they belong to beyond the standard model physics.

Bouncing Neutrons



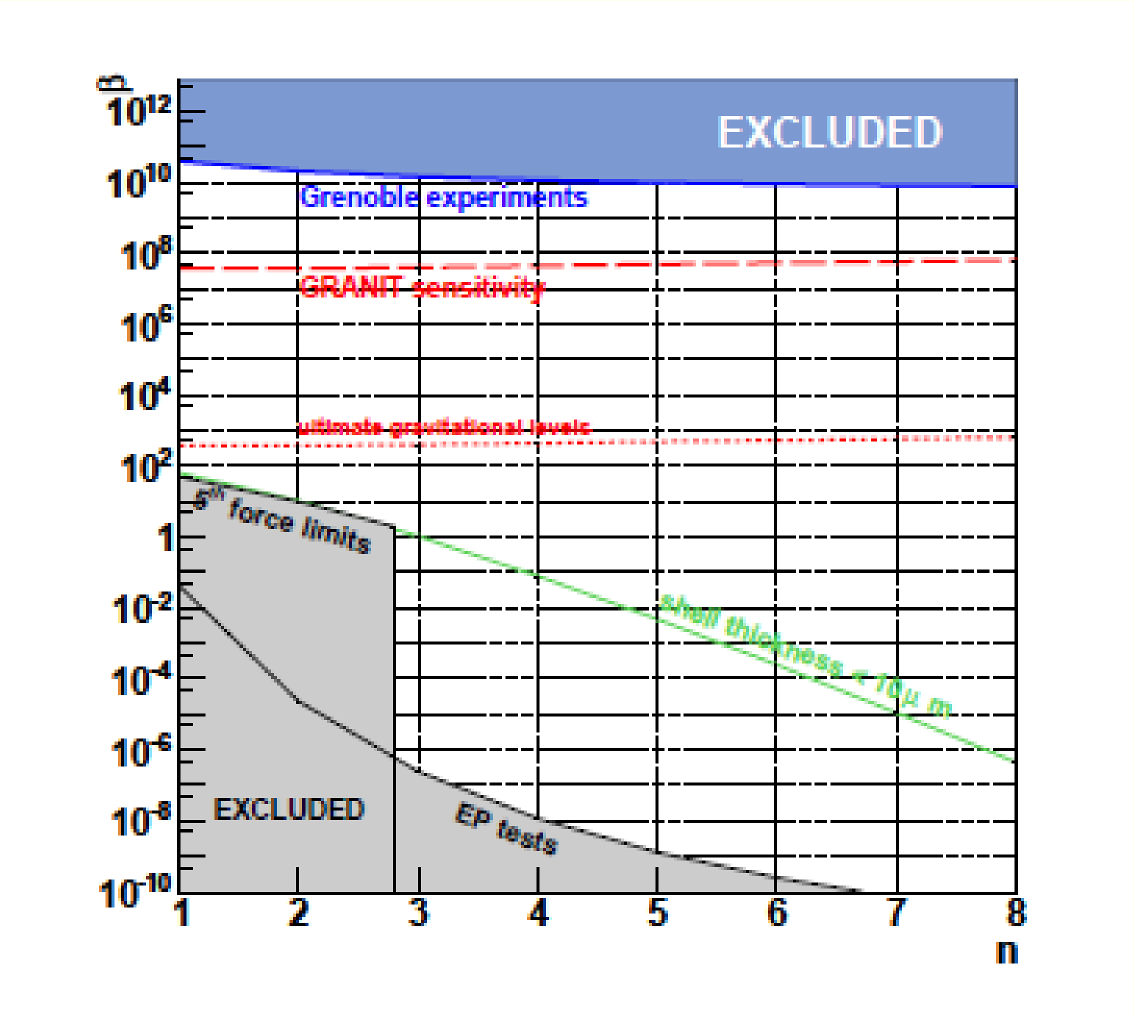
Ultra cold neutrons from a nuclear reactor fly over a mirror in the terrestrial gravitational fields. Their energy levels are quantised with Airy wave functions of extension a few microns (measured in ILL Grenoble (2002)). Perturbations by periodic magnetic fields induce a transition between two level states, hence a measurement of the energy level difference by observations of Rabi oscillations.

The chameleonic potential above the mirror perturbs the neutron energy levels:

$$\Phi = mgz + \beta \frac{m}{m_{\rm Pl}} \Lambda (\frac{2+n}{\sqrt{2}} \Lambda z)^{2/(n+2)}$$

The difference between the 1 and 3 energy levels will be measured at the 0.01 peV level in the next year. Ultimately a 1 per 10 million peV could be reached. As the extension of the unperturbed wave functions is not dissimilar from the dark energy scale, this sets limits on the coupling of chameleons to matter.

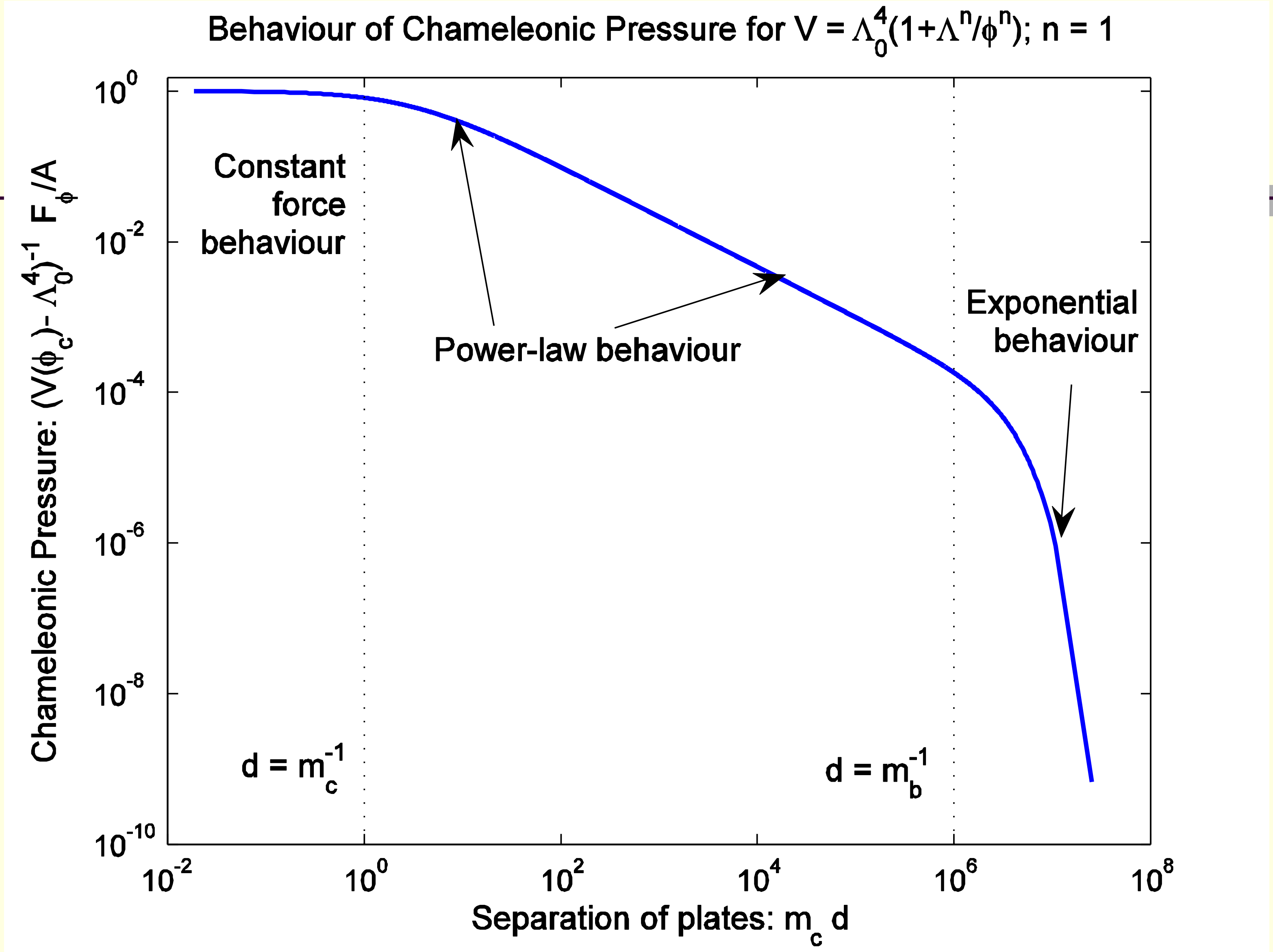
$$\Lambda^{-1} \sim 82 \mu m$$


β
10^12
10^10
10^8
10^6
10^4
10^2
1
10^-2
10^-4
10^-6
10^-8
10^-10
EXCLUDED
Grenoble experiments
GRANIT sensitivity
ultimate gravitational levels
5^th force limits
shell thickness < 10μ m
EXCLUDED
EP tests
1
2
3
4
5
6
7
8
n

Ph.B,G. Pignol

Casimir Force Experiments

- Measure force between
  - Two parallel plates



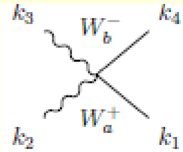
Potentially detectable in next generation of experiments d=10 microns

Coupling to the Standard Model

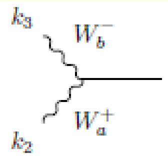
The coupling involves two unknown coupling functions (gauge invariance):

$$
\begin{aligned}
S &= -\frac{1}{4}\int d^4x\Big\{2B(\beta\chi)(\partial^a W^{+b} - \partial^b W^{+a})(\partial_a W_b^- - \partial_b W_a^-) + 4m_W^2 B_H(\beta_H\chi)W^{+a}W_a^- \\
&+ B(\beta\chi)(\partial^a Z^b - \partial^b Z^a)(\partial_a Z_b - \partial_b Z_a) + 2m_Z^2 B_H(\beta_H\chi)Z^a Z_a \\
&+ B(\beta\chi)(\partial^a A^b - \partial^b A^a)(\partial_a A_b - \partial_b A_a)\Big\},
\end{aligned}
$$

At one loop the relevant vertices are:



$$\mapsto \frac{\bar{B}''\beta^2}{2}\left[\eta^{ab}(k_2\cdot k_3 - \epsilon m_W^2) - k_2^b k_3^a\right],$$



$$k_1 \mapsto \bar{B}'\beta\left[\eta^{ab}(k_2\cdot k_3 - \gamma m_W^2) - k_2^b k_3^a\right],$$

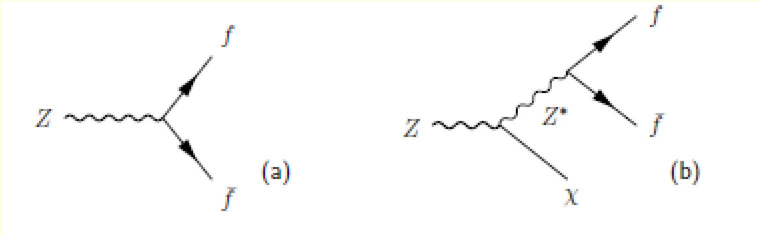
$$\epsilon = \frac{\bar{B}''_H}{\bar{B}''}\frac{\beta_H^2}{\beta^2}$$

$$\beta = \frac{1}{M}$$

$$\gamma = \frac{\bar{B}'_H}{\bar{B}'}\frac{\beta_H}{\beta}$$

Z-Width

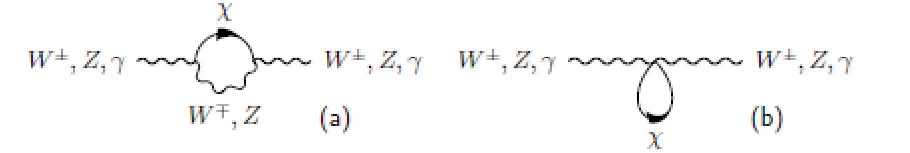
Dark energy scalars being very light and coupling to the Z boson may lead to an increase of the Z width (similar to neutrinos).



$$\frac{\Gamma(Z \to \chi f \bar{f})}{\Gamma(Z -> f\bar{f})} \approx \frac{1}{80\pi^3}\frac{M_Z^2}{M^2}$$

This leads to a weak bound on M greater than 60 GeV. Stronger bounds follow from precision tests.

Self-Energy Corrections I



The corrected propagator becomes:

$$\Delta(k^2) = \frac{1}{k^2 + M_A^2 - \Pi_{AA}^{(0)}(k^2)}$$

Measurements at low energy and the Z and W poles imply ten independent quantities. Three have to be fixed experimentally. One is not detectable hence six electroweak parameters: STUVWX

$$\Pi'_{\gamma\gamma}(q^2),\ \Pi'_{Z\gamma}(q^2), \Pi_{ZZ}(q^2),\ q^2 = 0, -M_Z^2$$

$$\Pi'_{ZZ}(-M_Z^2), \Pi'_{WW}(-M_W^2), \Pi_{WW}(0), \Pi_{WW}(-M_W^2)$$

Self-Energy Corrections II

The self energy can be easily calculated:

$$\Pi_{AA}(k^2) = \frac{\beta^2}{8\pi^2}\frac{\bar{B}'^2}{\bar{B}}\int_0^1 dx\,\Big\{\frac{2k^2+\gamma^2 M_A^2}{4}\left[\Lambda^2+\frac{\Lambda^2}{2}\frac{\Lambda^2}{\Lambda^2+\Sigma^2}-\Sigma^2\ln\left(1+\frac{\Lambda^2}{\Sigma^2}\right)\right]$$
$$+(xk^2+\gamma M_A^2)^2\left[-\frac{1}{2}\frac{\Lambda^2}{\Lambda^2+\Sigma^2}+\frac{1}{2}\ln\left(1+\frac{\Lambda^2}{\Sigma^2}\right)\right]$$
$$-\frac{\Omega}{2}(k^2+\epsilon M_A^2)\left[\frac{\Lambda^2}{2}-\frac{M_X^2}{2}\ln\left(1+\frac{\Lambda^2}{M_X^2}\right)\right]\Big\}$$

$$\Omega = \frac{B''B}{B'^2}$$

The self energy parameters all involve quadratic divergences:

$$\Pi_{AA}(k^2) = \frac{g^2}{M^2}[M_A^2\alpha_{0,A}+\alpha_{2,A}k^2+\alpha_{4,A}k^4+O(\frac{k^2}{M_W^2})]$$

For instance: $\alpha_0 = \frac{\Lambda^2}{4}(\frac{\gamma^2}{2}-\Omega\epsilon)+16(6\ln\frac{\Lambda^2}{M_A^2}-1)\gamma^2 M_A^2+\ldots$

The quadratic divergences cancel in all the precision tests

$$\alpha T = \frac{g^2}{M^2}(\alpha_{0,Z}-\alpha_{0,W})$$

Dark Energy Screening

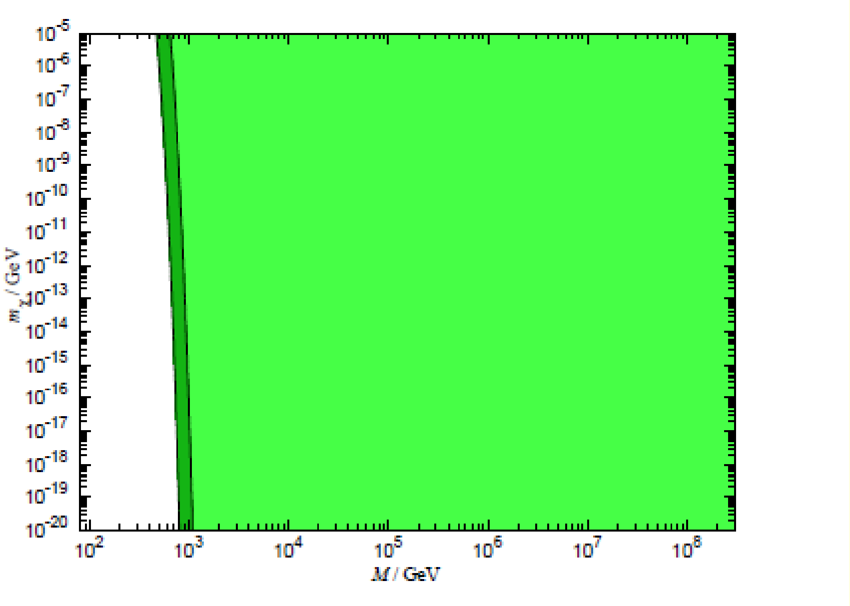
Vacuum Polarisation sensitive to the UV region of phase space where the difference between the W and Z masses is negligible

Gauge invariance implies the existence of only two coupling functions with no difference between the W and Z bosons in the massless limit (compared to the UV region)

No difference between the different particle vacuum polarisations, hence no effects on the precision tests.

Experimental Constraints

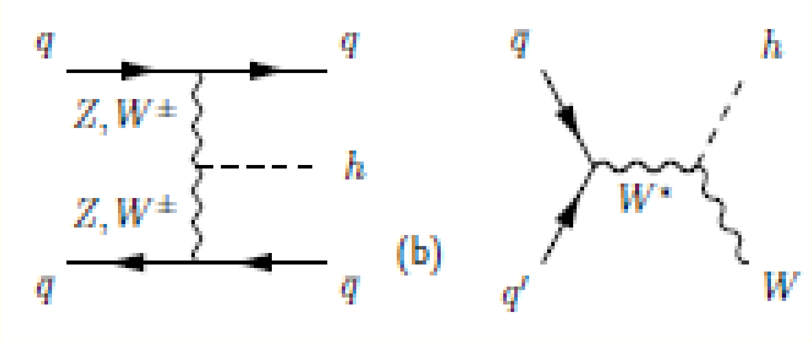
mass

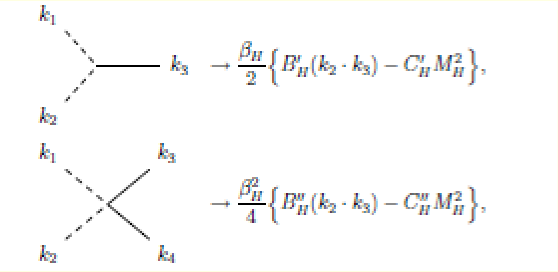


Inverse Coupling

Higgs Production

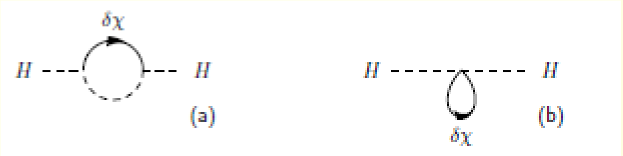
The dark energy scalar can also couple to the Higgs. This will influence the W fusion and W-Higgstrahlung production rates.





Higgs-scalar Feynman rules

$\Pi_{HH}(k^2)$



Higgs self-energy

↓

Wave function renormalisation

The new oblique parameter is:

$$\alpha R = \frac{d}{dk^2}\Pi_{HH}(k^2)|_{k^2=-M_H^2} + \frac{\Pi_{ZZ}(0)}{M_Z^2}$$

The Higgs production rate is then modified:

$$\frac{\Gamma(WW \to h)}{\tilde{\Gamma}(WW \to h)} = 1 + \alpha R$$

As expected, the dark energy correction is quadratically divergent depending on the cut-off of the theory. Hence, one might expect large deviations from the SM due to the presence of very light dark energy scalars.

$$\alpha R = \frac{\beta_H^2 \Lambda^2}{32\pi^2}\frac{B'^2_H}{B}[\frac{1}{2}(1+\frac{B}{B_H}) - 2\frac{B''_H B}{B'^2_H}] = \mathcal{O}(1)$$

Concluding?

Scalars ubiquitous in dark energy and modified gravity models.

Screening mechanisms in gravitational experiments

Scalars are actively sought for in laboratory experiments

Scalars would only show up in particle experiments when coupled to another scalar like the Higgs field.