Cosmological perturbations : going beyond the linear theory

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Introduction

What's the point ?

Precision Cosmology : two types of observations

- **C**osmic **M**icrowave **B**ackground experiments $\longrightarrow$ redshift $\geq$ 1000
- **L**arge **S**cale **S**tructure experiments $\longrightarrow$ redshift $\leq$ 10

Introduction

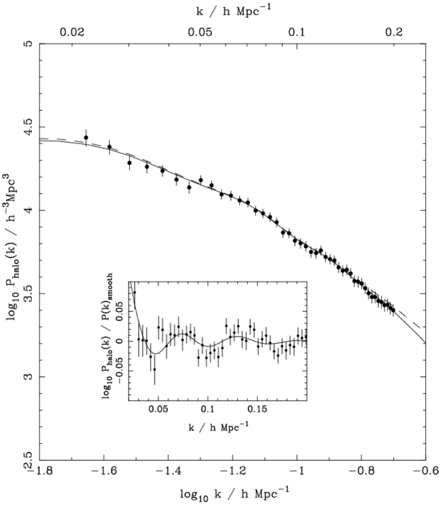
What's the point ?

Precision Cosmology : two types of observations

- **CMB** $\longrightarrow z \geq 1000 \longrightarrow$ Linear regime
- **LSS** $\longrightarrow z \leq 10 \longrightarrow$ Gravitational collapse : non linearities

LSS can put constraints on (among others)

- (total) neutrino mass,
- Nature of Dark Energy, through perturbations and Baryonic Acoustic Oscillation (BAO) scales.



The matter power spectrum $P(k)$ reconstructed from the data release 7 of the Sloan Digital Sky Survey by Reid et al. [0907.1659].

N-body, the ideal solution ?

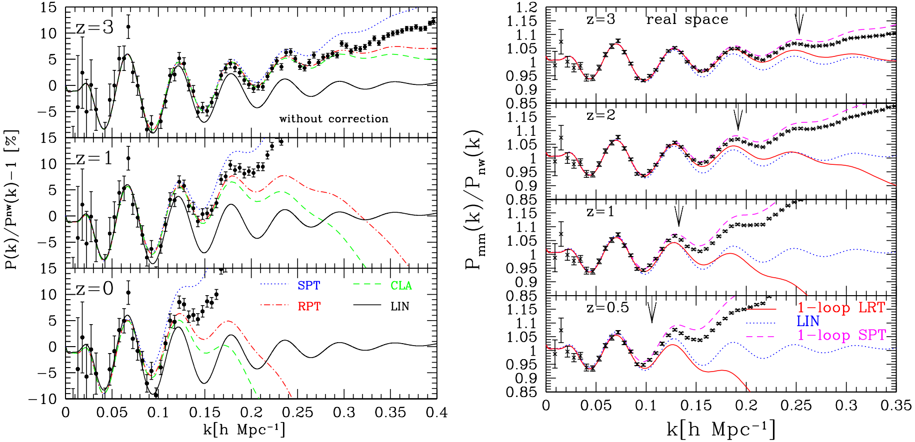


Figure: Predictions from Sato and Matsubara, at three years difference (0810.0813 and 1105.5007)

Notations

Perturbative decomposition

Perturbed quantities and equations of motion

$$\rho(\mathbf{x},\tau) \equiv \bar{\rho}(\tau)[1+\delta(\mathbf{x},\tau)]$$
$$\mathbf{v}(\mathbf{x},\tau) \equiv \mathcal{H}(\tau)\mathbf{x}+\mathbf{u}(\mathbf{x},\tau)$$
$$\phi(\mathbf{x},\tau) \equiv -\frac{1}{2}\frac{\partial \mathcal{H}}{\partial \tau}\mathbf{x}^2+\Phi(\mathbf{x},\tau)$$

$$\nabla^2\Phi(\mathbf{x},\tau) = \frac{3}{2}\Omega_m(\tau)\mathcal{H}^2(\tau)\delta(\mathbf{x},\tau)$$
$$\frac{d\mathbf{p}}{d\tau} = -am\nabla\Phi(\mathbf{x},\tau)$$

Collisionless Boltzmann equation

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{ma}\cdot\nabla f - ma\nabla\Phi\cdot\frac{\partial f}{\partial \mathbf{p}} = 0$$

Non-linear computation

Equations in Fourier space

Fully non-linear equations

$$\frac{\partial \delta(\mathbf{k},\tau)}{\partial \tau} + \theta(\mathbf{k},\tau) = -\iint d^3\mathbf{k_1} d^3\mathbf{k_2} \delta_D(\mathbf{k}-\mathbf{k_{12}}) \alpha(\mathbf{k_1},\mathbf{k_2}) \delta(\mathbf{k_2},\tau) \theta(\mathbf{k_1},\tau)$$

$$\frac{\partial \theta(\mathbf{k},\tau)}{\partial \tau} + \mathcal{H}(\tau)\theta(\mathbf{k},\tau) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta(\mathbf{k},\tau) = -\iint d^3\mathbf{k_1} d^3\mathbf{k_2} \delta_D(\mathbf{k}-\mathbf{k_{12}}) \beta(\mathbf{k_1},\mathbf{k_2}) \theta(\mathbf{k_1},\tau) \theta(\mathbf{k_2},\tau)$$

with the mode-coupling functions :

$$\alpha(\mathbf{p},\mathbf{q}) = \frac{(\mathbf{p}+\mathbf{q})\cdot\mathbf{p}}{p^2}$$

$$\beta(\mathbf{p},\mathbf{q}) = \frac{(\mathbf{p}+\mathbf{q})^2(\mathbf{p}\cdot\mathbf{q})}{2p^2q^2}$$

Non-linear computation

Standard Perturbation expansion (see Bernardeau et al. 0112551 for a complete review)

Decomposition, for an (unphysical) Einstein de Sitter universe

$$\delta(\mathbf{k},\tau) = \sum_{n=1}^{\infty} a^n(\tau)\delta_n(\mathbf{k}) \qquad ; \qquad \theta(\mathbf{k},\tau) = -\mathcal{H}(\tau)\sum_{n=1}^{\infty} a^n(\tau)\theta_n(\mathbf{k})$$

it gives rise to the following (and slightly cumbersome) expressions :

$$\delta_n(\mathbf{k}) = \int d^3\mathbf{q_1}\dots\int d^3q_n\delta_D(\mathbf{k}-\mathbf{q_{1\dots n}})F_n(\mathbf{q_1},\dots,\mathbf{q_n})\delta_1(\mathbf{q_1})\dots\delta_1(\mathbf{q_n})$$

$$\theta_n(\mathbf{k}) = \int d^3\mathbf{q_1}\dots\int d^3q_n\delta_D(\mathbf{k}-\mathbf{q_{1\dots n}})G_n(\mathbf{q_1},\dots,\mathbf{q_n})\delta_1(\mathbf{q_1})\dots\delta_1(\mathbf{q_n})$$

$\Longrightarrow$ hard to release the assumption on time-dependence of non-linear terms

Non-linear computation

Beyond the Standard approach

Renormalized Perturbation Theory (RPT by M. Crocce and R. Scoccimarro, 0509418)

Time Renormalization Group method (M. Pietroni, 0806.0971)

Non-linear computation

Beyond the Standard approach

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- quantities are evolved with time:
  no more assumptions on their behaviour!

Non-linear computation

TRG equations, in terms of spectra and bispectra

Variable change $\eta = log(a/a_{\rm ini})$, but no assumption on time dependence

$$\partial_\eta P_{ab}(\mathbf{k},\eta) = -\Omega_{ac}(\mathbf{k},\eta)P_{cb}(\mathbf{k},\eta) - \Omega_{bc}(\mathbf{k},\eta)P_{ac}(\mathbf{k},\eta)$$
$$+e^\eta \int d^3q\, [\gamma_{acd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k})B_{bcd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k},\eta)$$
$$+B_{acd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k},\eta)\gamma_{bcd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k},\eta)]$$

$$\partial_\eta B_{abc}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k},\eta) = -\Omega_{ad}(\mathbf{k},\eta)B_{dbc}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k},\eta)$$
$$-\Omega_{bd}(-\mathbf{q},\eta)B_{adc}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k},\eta)$$
$$-\Omega_{cd}(\mathbf{q}-\mathbf{k},\eta)B_{abd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k},\eta)$$
$$+2e^\eta\,[\gamma_{ade}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k})P_{db}(\mathbf{q},\eta)P_{ec}(\mathbf{k}-\mathbf{q},\eta)$$
$$\gamma_{bde}(-\mathbf{q},\mathbf{q}-\mathbf{k},\mathbf{k})P_{dc}(\mathbf{k}-\mathbf{q},\eta)P_{ea}(\mathbf{k},\eta)$$
$$\gamma_{cde}(\mathbf{q}-\mathbf{k},\mathbf{k},-\mathbf{q})P_{da}(\mathbf{k},\eta)P_{eb}(\mathbf{q},\eta)]$$

Small advertisement

Small advertisement

The new Boltzmann code is here...

- CLASS, for Cosmic Linear Anisotropy Solving System
- written in a clear, flexible and commented way in C
  by J. Lesgourgues (EPFL)
- New semi-analytical non-linear module released in June

Available at http://class-code.net

Results

Numerical predictions

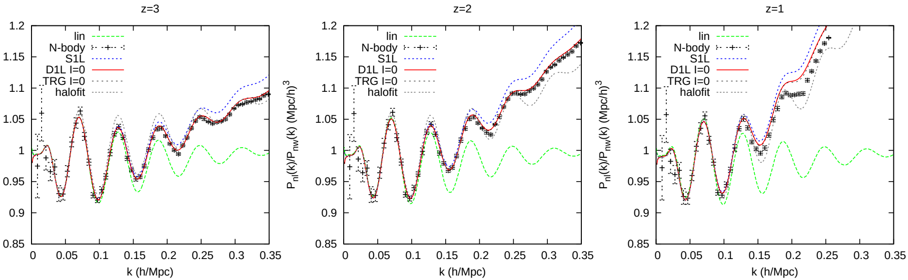

Figure: Comparison between the standard 1-loop (S1L), the dynamical 1-loop (D1L) and the renormalized computation (TRG) for different set of IC: here no initial bispectrum for D1L and TRG. Simulation results kindly provided by M. Sato and T. Matsubara.

Results

Numerical predictions

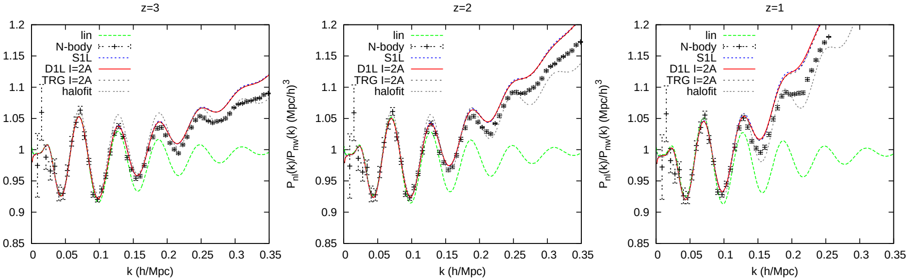

Figure: Here EdS initial bispectrum for D1L and TRG.

Results

Discussion

Main points

- Percent precision up to $k \simeq 0.35\ h/\text{Mpc}$ at $z = 2$, $k \simeq 0.18\ h/\text{Mpc}$ at z=1, with the Dynamical 1-loop.
- Extremely small computing time (some minutes),
- because renormalization seems not to matter too much !
- Can be extended to various cosmologies (WIP), however the Dynamical 1-loop would not work as fast: computing time up to some hours.

Conclusion

So far

- Crucial role of IC in both N-body simulations and perturbation theory,
- It seems that the importance of proper ICs has a major effect on mildly non-linear scale compared to higher loop corrections (for scale independent background cosmology, and percent precision),

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- Crucial role of IC in both N-body simulations and perturbation theory,
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To do

- ICs in N-body: not only internally but also cosmologically consistent, (computed within CLASS)
- Compare with LSS real data or weak lensing surveys,
- Include the code inside a MCMC code for parameter extraction (neutrino mass, warm dark matter contribution, more 'exotic' ideas...)