Intro to LQG

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Outline

Outline

Motivations

SU(2) singlets and polyhedra

Applications

Conclusions

Motivations

Einstein's equations:

$$R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g) = \frac{8\pi G}{c^4} T_{\mu\nu}$$

The source of spacetime curvature is the energy-momentum tensor of matter

What is the response of spacetime in situations where the quantum nature of matter is dominant?

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Structure of equations

$$A(g)\partial^2 g + B(g)(\partial g)^2 + C(g) = \frac{8\pi G}{c^4} T$$

		e-m analogy
• gauge part:	diffeos	$A_\mu \mapsto A_\mu + \partial_\mu \lambda$
• constrained part:	Newton's law	$\nabla \cdot E = \rho$
• degrees of freedom:	Two	Two spin-1 polarizations

Perturbative expansion

Perturbative approach:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$\Longrightarrow$ Two spin-2 polarizations, gravitational waves

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$\Longrightarrow$ tools of quantum field theory become available

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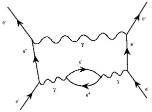
Case for background-independence

A paradigm shift

	kinematics	dynamics
QFT:	$\|p_i, h_i\rangle$	
	quanta: momenta, helicities, etc.	Feynman diagrams

A paradigm shift

kinematics | dynamics

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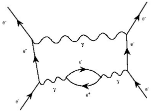

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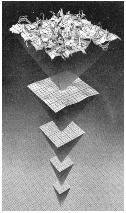
At the Planck scale:



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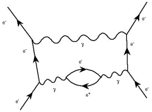

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everything takes place in spacetime ⇒ quanta make up space and evolve into spacetime

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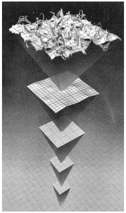

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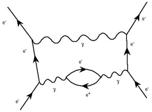
QFT:	$|p_i, h_i\rangle$	
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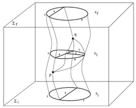
LQG:	$|\Gamma, j_e, i_v\rangle$	
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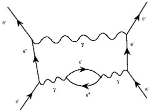
QFT:	$\vert p_i, h_i\rangle$	
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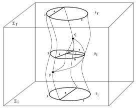
LQG:	$\vert\Gamma, j_e, i_v\rangle$	
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how do we recover semiclassical physics on a smooth spacetime?

Stating the problem

- LQG is a continuum theory with well-defined and interesting kinematics (spin networks, discrete spectra of geometric operators, etc.)
- Models for the dynamics exist
- Main open problem: how to test the theory and extract low-energy physics from it

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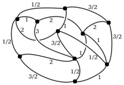

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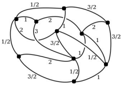

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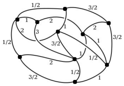

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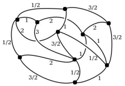

$\Longrightarrow$

Aim of the talk:

showing the link between LQG on a fixed graph and a notion of discrete geometry

Work in collaboration with L. Freidel, C. Rovelli, E. Bianchi and P. Doná

Fundamental coupling constants

Working hypothesis: the *connection* as a fundamental (and independent) variable[1]

$$g_{\mu\nu} \mapsto (g_{\mu\nu}, \Gamma^{\rho}_{\mu\nu})$$

Lowest dimension operators and their coupling constants:

$$\sqrt{-g}, \quad \sqrt{-g}g^{\mu\nu}R_{\mu\nu}(\Gamma),$$

$$\frac{\Lambda}{G} \qquad \frac{1}{G}$$

[1] more precisely: $(e^I_\mu, \omega^{IJ}_\mu)$.

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$$\frac{\Lambda}{G} \qquad \frac{1}{G} \qquad \frac{1}{\gamma G}$$

- Classically irrelevant in the absence of torsion:

$$\Gamma^\rho_{\mu\nu} = \left\{ {\rho \atop \mu\nu} \right\} \implies \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}(\Gamma(e)) = 0$$

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- Non-perturbative quantum role?

Area gap in LQG: $A_{min} = \frac{\sqrt{3}}{2} \gamma \ell_P^2$

[1] more precisely: $(e^I_\mu, \omega^{IJ}_\mu)$.

Canonical formulation: Ashtekar variables

Hamiltonian analysis very complicated (second class constraints)

Key simplification: Ashtekar-Barbero variables: $\Rightarrow$ First class constraints

- Densitised triad: E^a_i
- $SU(2)$ connection: A^i_a

($a = 1, 2, 3$ spatial indices, $i = 1, 2, 3$ $SU(2)$ indices)

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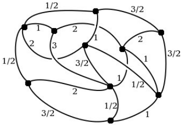
Related to ADM variables via a canonical transformation

$$(g_{ab}, K^{ab}) \Longrightarrow (A^i_a, E^a_i)$$

$$\{A^i_a(x), E^b_j(y)\} = \gamma\, G\, \delta^i_j \delta^b_a \delta^{(3)}(x, y)$$

Remarks:
- Same phase space of an SU(2) gauge theory
- the Immirzi parameter enters the Poisson structure

Spin networks and quantum geometry



QFT	LQG
$\mathcal{F} = \bigoplus_n \mathcal{H}_n$	$\mathcal{H} = \bigoplus_\Gamma \mathcal{H}_\Gamma$
$\vert n, p_i, h_i\rangle \rightarrow$ quanta of fields	$\vert\Gamma, j_e, i_v\rangle \rightarrow$ quanta of space

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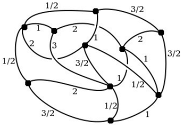

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Key result

geometric operators turn out to have discrete spectra
with minimal excitations proportional to the Planck length

- spins j_e on each edge:

  quanta of areas $\quad A(\Sigma) = \gamma \hbar G \sum_{e \in \Sigma} \sqrt{j_e(j_e+1)}$

- intertwiners i_v on each vertex:

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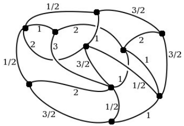

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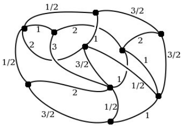
Spin networks and quantum geometry



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Three aspects of quantum geometry:

- discrete eigenvalues
- non-commutativity
- graph structure

A paradigm shift

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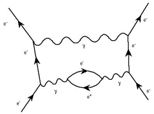
QFT:	$\vert n, p_i, h_i\rangle$	
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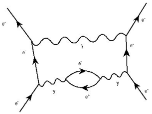
QFT:	$\|n, p_i, h_i\rangle$	
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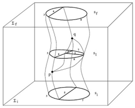
LQG:	$\|\Gamma, j_e, i_v\rangle$	
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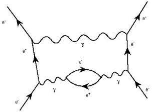
QFT:	$\|n, p_i, h_i\rangle$	
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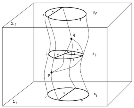
LQG:	$\|\Gamma, j_e, i_v\rangle$	
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	link to classical geometries? meaning of Γ?	what approximation?

Geometry on a single graph?

$$\{A_a^i(x), E_j^b(y)\} \qquad \longrightarrow \qquad \mathcal{H} = \bigoplus_{\Gamma} \mathcal{H}_\Gamma, \qquad |\Gamma, j_e, i_v\rangle$$

- Consider a single graph Γ, and the associated Hilbert space $\mathcal{H}_\Gamma$.
- This truncation captures only a finite number of degrees of freedom of the theory, thus states in $\mathcal{H}_\Gamma$ do not represent smooth geometries.
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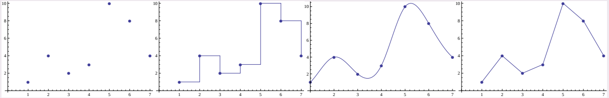

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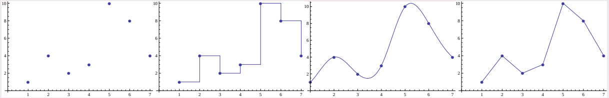

Can we interpret $\mathcal{H}_\Gamma$ as the quantization of a space of discrete geometries?

$$V^{(j_e)}, \qquad \mathcal{H}_v \equiv \mathrm{Inv}\left[\bigotimes_{e\in v} V^{(j_e)}\right], \qquad \mathcal{H}_\Gamma = \bigoplus_{j_e}\left[\bigotimes_v \mathcal{H}_v\right]$$

irrep of SU(2)

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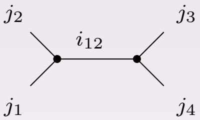
The building block of loop gravity: intertwiner space

$\mathcal{H}_v \equiv \mathrm{Inv}\left[\otimes_{e\in v} V^{(j_e)}\right]$

Operators: $\vec{J}_i,\ \vec{J}_i \cdot \vec{J}_j, \qquad i = 1 \dots F$

Only $F-3$ commuting operators: $\{J_1^2 \dots J_F^2, (J_1+J_2)^2 \dots\}$

Recoupling basis: $|j_1 \dots j_F, i_{12}, \dots\rangle$



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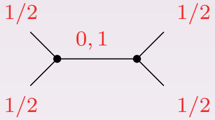
Only $F-3$ commuting operators: $\{J_1^2 \dots J_F^2, (J_1+J_2)^2 \dots\}$

Recoupling basis: $|j_1 \dots j_F, i_{12}, \dots\rangle$



The building block of loop gravity: intertwiner space

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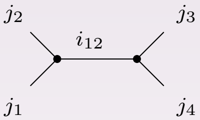

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Intertwiners and polyhedra 1

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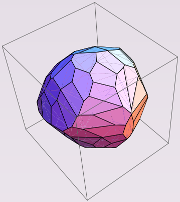
Polyhedra!



The connection is made in two steps:

polyhedra	$\mathcal{S}_F$	$\mathcal{H} = \mathrm{Inv}\left[\otimes_i V^{j_i}\right]$

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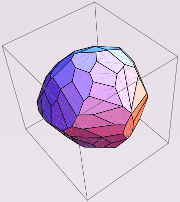

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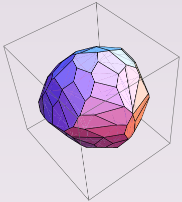

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[Kapovich and Millson '96, '01, Charles '08, Conrady and Freidel '08]
2. Points in this phase space represent bounded convex flat polyhedra in $\mathbb{R}^3$
[E.Bianchi,P.Doná,SS 1009.3402]

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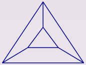
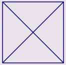
Dominant: 

Codimension 1: 

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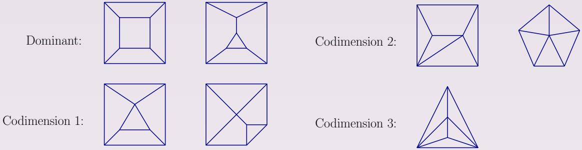

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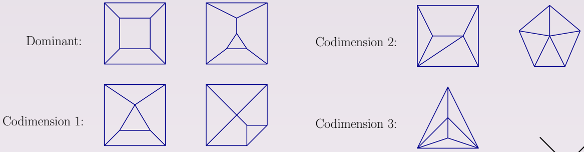
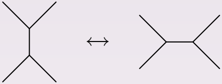


- The classes are all connected by 2-2 Pachner moves (they are all tessellations of the 2-sphere)



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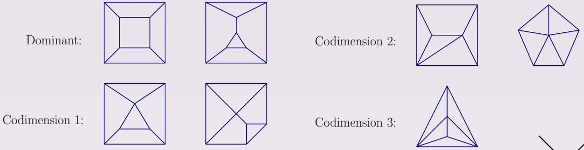

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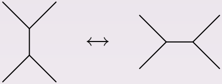
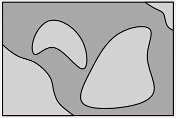


It is the configuration of normals to determine the class

- The phase space $\mathcal{S}_F$ can be mapped in regions corresponding to different classes.
  – *Dominant classes* have all 3-valent vertices.
  [maximal n. of vertices, $V = 3(F-2)$, and edges, $E = 2(F-2)$]
  – *Subdominant classes* are special configurations with lesser edges and vertices, and span measure zero subspaces.
  [lowest-dimensional class for maximal number of triangular faces]



Coherent intertwiners

polyhedra $\Longleftarrow$ $\mathcal{S}_F$ $\Longrightarrow$ $\mathcal{H} = \mathrm{Inv}\left[\otimes_i V^{j_i}\right]$

Geometric quantization to derive holomorphic coherent states for $\mathcal{H} = \mathrm{Inv}\left[\otimes_i V^{j_i}\right]$
[E. Livine and SS PRD ('07)]

Geometric operators $\hat{O}(\vec{J}_i)$ peaked on classical values $O(A_i n_i)$ with minimal uncertainties

$\Rightarrow$ states of semiclassical polyhedra

Polyhedra on the full graph

The Hilbert space:

$$\mathcal{H}_\Gamma = \bigoplus_{j_e} \left[\bigotimes_v \mathcal{H}_v \right]$$

is a quantization of the classical phase space

$$S_\Gamma = \times_e T^* S^1 \times_v \mathcal{S}_{F(v)}$$

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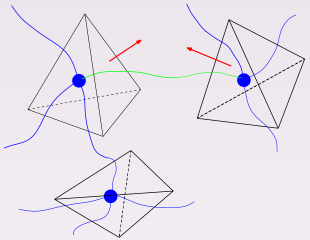
$$\mathcal{S}_F = \{ n_i \,|\, \sum_i j_i n_i = 0 \}$$



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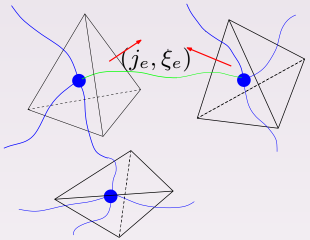

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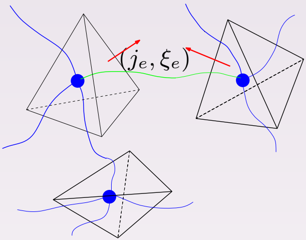

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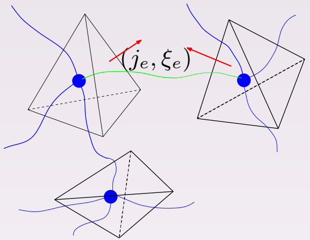

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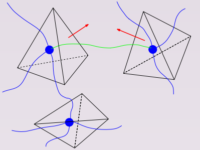
Twisted geometries: interpretation



For each point (A_e, E_e) on the phase space at fixed graph, there are infinite continuous metrics that can correspond to it

Twisted geometries are a particular choice of interpolating geometry associated with a cellular decomposition of the manifold dual to Γ:

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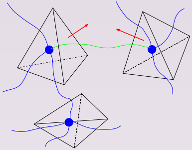

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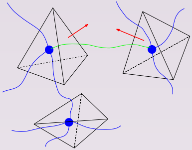

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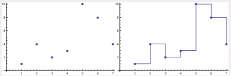
The geometries are twisted in the sense that they are well-defined locally (on each polyhedron), but are *discontinuous* at the intersections (the faces)



Overview

Twistor space

Twisted geometries $\Longleftrightarrow$ Loop gravity

$\downarrow$ *matching shapes reduction*

Regge calculus

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$\downarrow$ *matching area reduction*

Twisted geometries $\Longleftrightarrow$ Loop gravity

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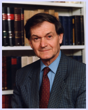
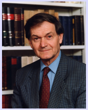


Spin networks ↙ ↘ Twistors

Overview



Spin networks → Twisted geometries

Twistors → Twisted geometries

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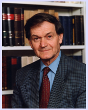


Spin networks   Twistors

Twisted geometries

Regge geometries

Outline

Motivations

SU(2) singlets and polyhedra

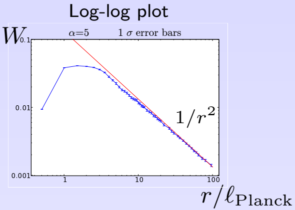
Applications

Conclusions

Some applications

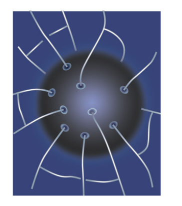
- Short scale dinamical regularization
  [SS, Livine, ...]

  2-point function in a toy model:
  expected large scale behaviour recovered,
  hints of new Planck scale physics



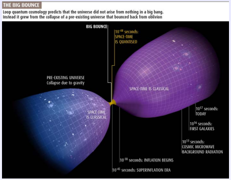
- Black holes
  [Ashtekar, Baez, Perez, Rovelli, ...]

  Interpretation of the BH entropy $S = \frac{A}{4G}$
  in terms of microstates corresponding to
  a unique macroscopic geometry
  but different quantum shapes



- Cosmology
  [Ashtekar, Bojowald, Rovelli, Barrau, ...]

  New repulsive force avoiding the singularity
  and creating a *quantum bounce*
  Modification of the Friedmann equations



Outline

Conclusions

- LQG is a continuum theory with well-defined and interesting kinematics (spin networks, discrete spectra of geometric operators, etc.)
- Models for the dynamics exist, defined graph by graph similar to scattering amplitudes in QFT
- Each graph represents quantum geometries, which we can visualize as a collection of fuzzy polyhedra
- The semiclassical limit should be recovered in the limit in which the polyhedra are much larger than the Planck scale (no fuzzy) and much smaller than the resolution scale (smooth geometry)
  - single graph level: connection with Regge calculus established
  - continuum limit: main open question!