Study of Frame Dependence of Response Functions

Giuseppina Orlandini

Department of Physics
University of Trento

Work done in collaboration with

Victor EFROS	**(Kurchatov Inst. Moscow)**
Winfried LEIDEMANN	**(Univ. of Trento)**
Ed TOMUSIAK	**(Univ. of Victoria)**

Outline:

- The response function **S(q,ω)**
- S(q,ω) in the non relativistic framework
- an ab initio method to calculate it (including the continuum exactly)
- Results on frame dependence
- How far in q is the n.r. calculation reliable?

A *familiar* object:

$$F(t) = \langle 0 | \Theta^{\dagger}(t)\, \Theta(t=0) | 0 \rangle$$

t = real time

$\Theta(t) =$ *field operators or creation/annihilation operators in Heisenberg representation*

In quantum field theory or many-body theory

it is called

Correlation Function or Two-Point Function

its Fourier Transform:

$$\chi(\omega) = \int e^{-i t \omega} F(t)\, dt =$$

$$= \langle 0 | \Theta^\dagger \frac{1}{[\omega - (H - E_0) + i\varepsilon]} \Theta | 0 \rangle$$

called

Linear response or Green Function

Spectral representation of $\chi(\omega)$:

$$\chi(\omega) = \sum_n \frac{|\langle n | \Theta | 0 \rangle|^2}{[\,\omega - (E_n - E_0) + i\,\varepsilon\,]}$$

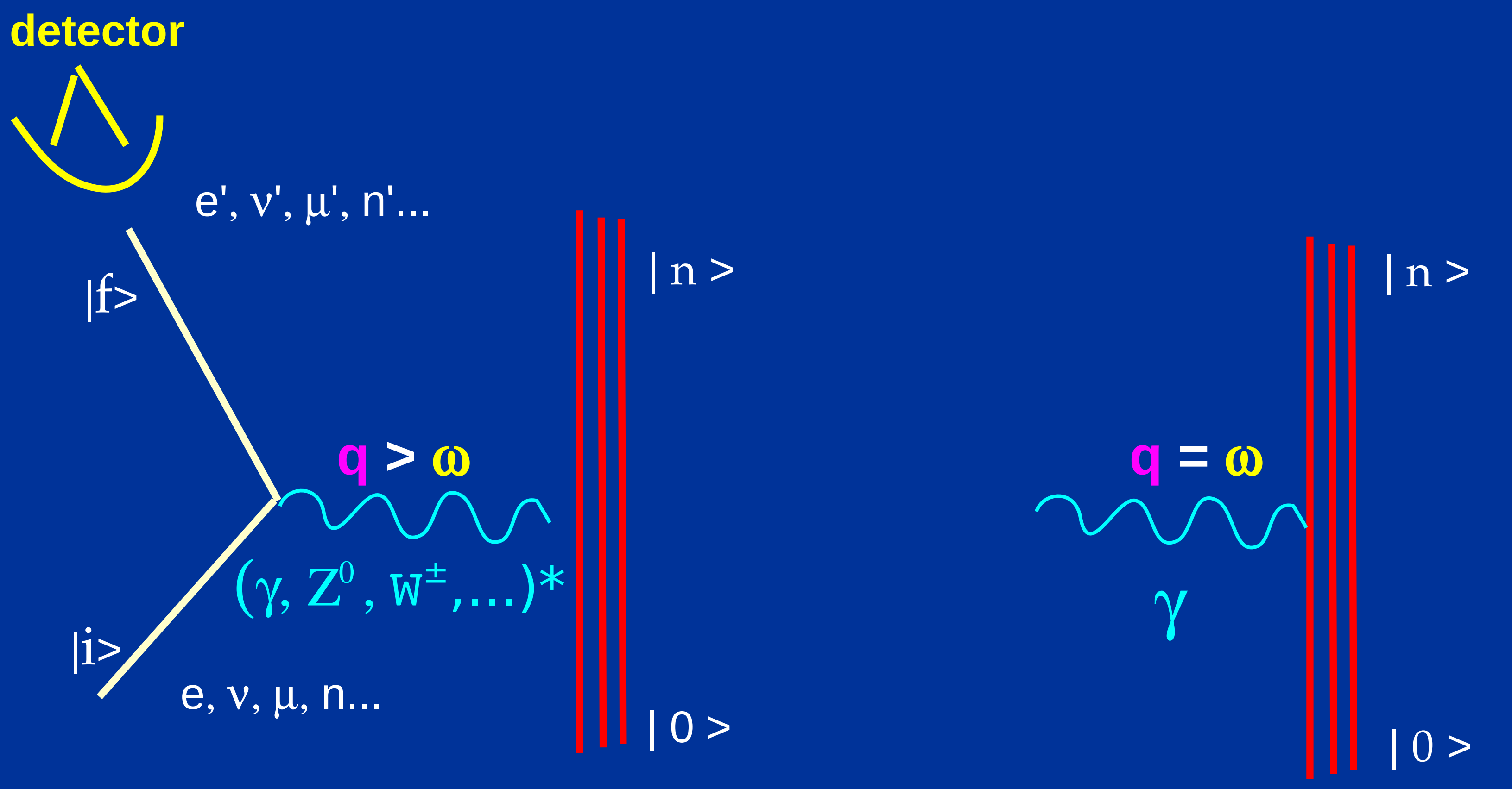
$(H | n \rangle = E_n | n \rangle)$

$$-\pi^{-1} Im\, \chi(\omega) = \sum\!\!\!\!\!\!\int_n |\langle n|\Theta|0\rangle|^2\, \delta(\omega - E_n + E_0)$$

A scattering observable:

$$-\pi^{-1} Im\, \chi(\omega) = \sum\!\!\!\!\!\!\int_n |\langle n|\Theta|0\rangle|^2\, \delta(\omega - E_n + E_0)$$

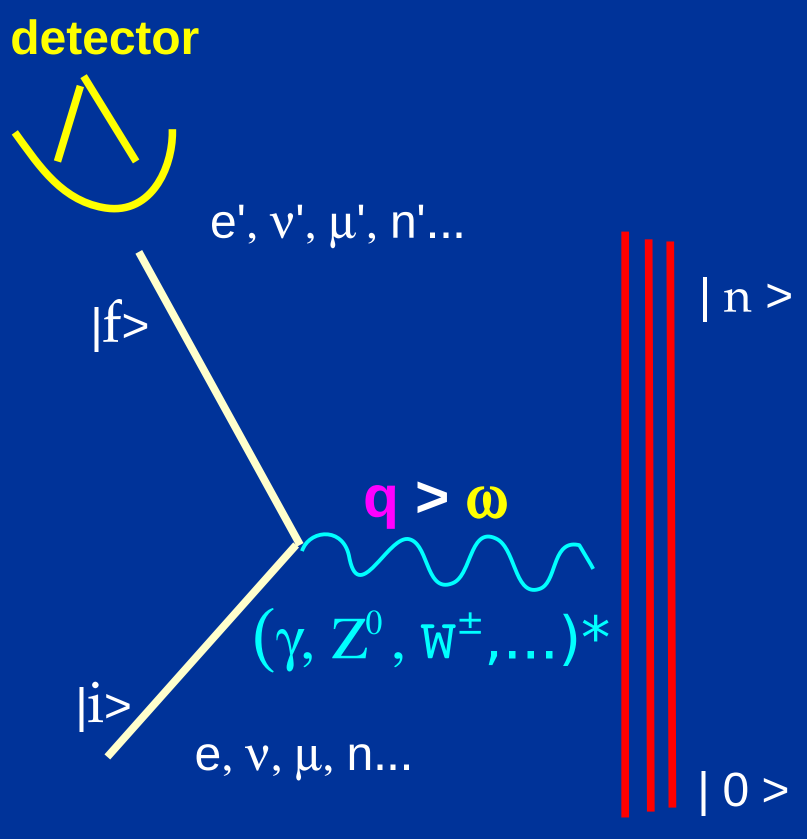
A very common example:
perturbation induced **inclusive** reactions

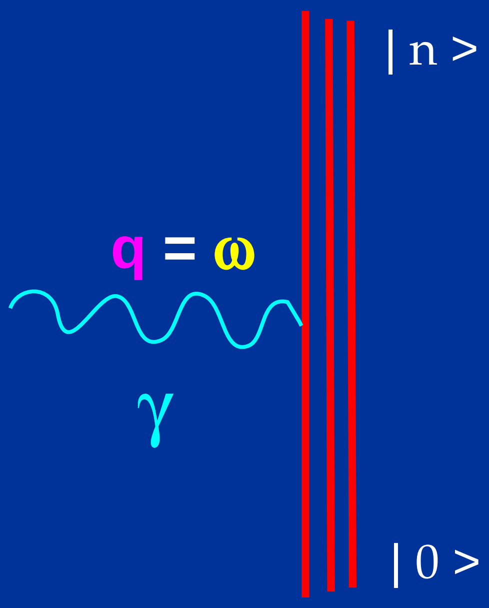


A very common example:

perturbation induced **inclusive** reactions

| n > can be in the continuum because the system can break into different channels !!!





The observable S(q,ω):

In perturbation induced inclusive reactions
cross sections are proportional to :

$$S(q,\omega) = \sum\!\!\!\!\!\!\int_n |\langle n|\Theta|0\rangle|^2\, \delta(\omega - E_n + E_0)$$

$$- \pi^{-1} Im\, \chi(\omega)$$

Θ depends on q

First remark:

$$F(t) = \langle 0 | \Theta^\dagger(t)\, \Theta(t=0) | 0 \rangle$$

t is the real time, however, for imaginary time **it = τ** one can proove that

1) $F^*(\tau) = F(\tau)$ i.e. **F(τ)** is real

2) $F(\tau) = \int e^{-\tau\omega}\, \mathbf{Im}\, \chi(\omega)\, d\omega$

i.e. **F(τ) is the Laplace transform of S(q, ω)**

Then in order to obtain **S(**q,**ω)** one could calculate

$F(\tau)$ *(Monte Carlo)*

and invert the **Laplace transform**

$$F(\tau) = \int e^{-\tau\omega} \, Im\,\chi(\omega) \, d\omega$$

second remark:

remember the Spectral representation

$$\chi(\omega) = \sum_n \frac{|< n | \Theta | 0 >|^2}{[\,\omega - (E_n - E_0) + i\,\varepsilon\,]}$$

$$(H\,|\,n> \; = E_n \,|\,n>)$$

$$Im\,\chi(\omega) = \lim_{\varepsilon \to 0} \sum_n \frac{\varepsilon\,|\langle n|\Theta|0\rangle|^2}{[(\omega - E_{n0})^2 + \varepsilon^2]}$$

$$S(q,\omega) = \sum_n \left|\langle n|\Theta|0\rangle\right|^2 \delta(\omega - E_n + E_0)$$

$$= \sum_n \frac{\varepsilon \; |< n \,|\, \Theta \,|\, 0 >|^2}{[(\omega - E_{n0})^2 + \varepsilon^2]}$$

equivalent to represent the delta-function

by a **Lorentzian** of width **ε**

$$\sum_n \frac{\varepsilon \, |\langle n | \Theta | 0 \rangle|^2}{[(\omega - E_{n0})^2 + \varepsilon^2]} = \Phi(\varepsilon, \omega)$$

Then in order to obtain **S(q,ω)** one could calculate

$$\Phi(\varepsilon,\omega)$$

as a function of finite $\varepsilon = \Gamma$ and extrapolate

for $\Gamma \longrightarrow 0$

or, alternatively...

Notice that:

$$\Phi(\Gamma,\omega) = \sum_n \frac{\Gamma\, |< n\,|\,\Theta\,|\,0 >|^2}{[(\omega - E_{n0})^2 + \Gamma^2]}$$

$$\Phi(\Gamma,\omega_0) = \int \frac{\Gamma\, S(q,\omega)}{[(\omega_0 - \omega)^2 + \Gamma^2]}\, d\omega$$

i.e. $\Phi(\Gamma,\omega_0)$ is the Lorentz transform of $S(q,\omega)$

Then in order to obtain **S(q,ω)** one could calculate

$$\Phi(\Gamma,\omega_0)$$

and invert the **Lorentz transform**

$$\Phi(\Gamma,\omega_0) = \int \frac{\Gamma\, S(q,\omega)}{[(\omega_0 - \omega)^2 + \Gamma^2]}\, d\omega$$

It is well known that the numerical inversion of the **Laplace** Transform is a (tremendous) **ill-posed** problem

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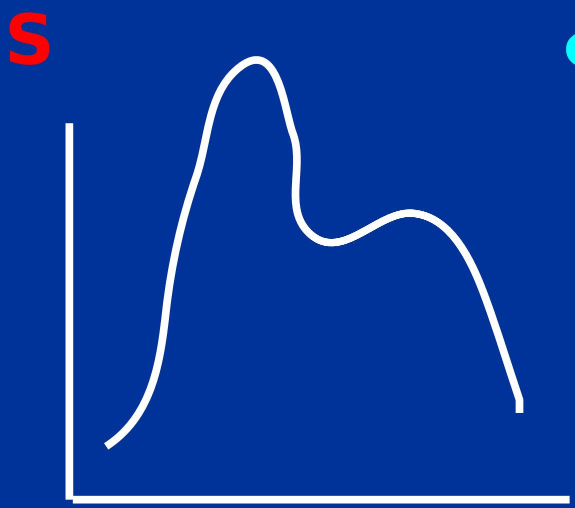

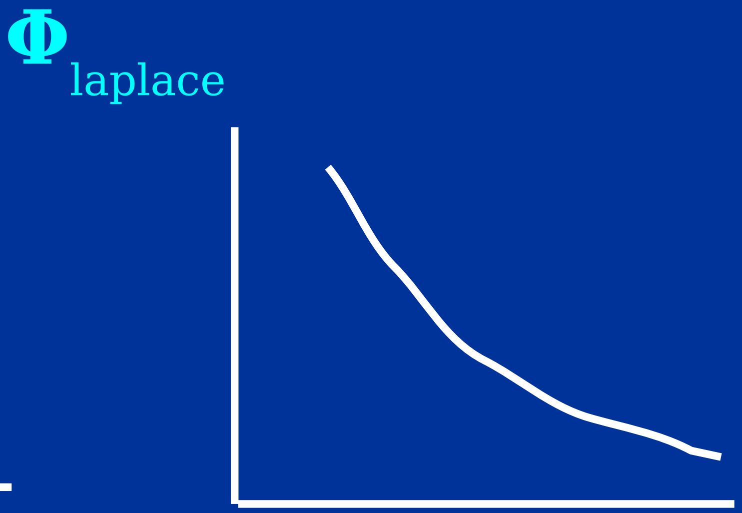

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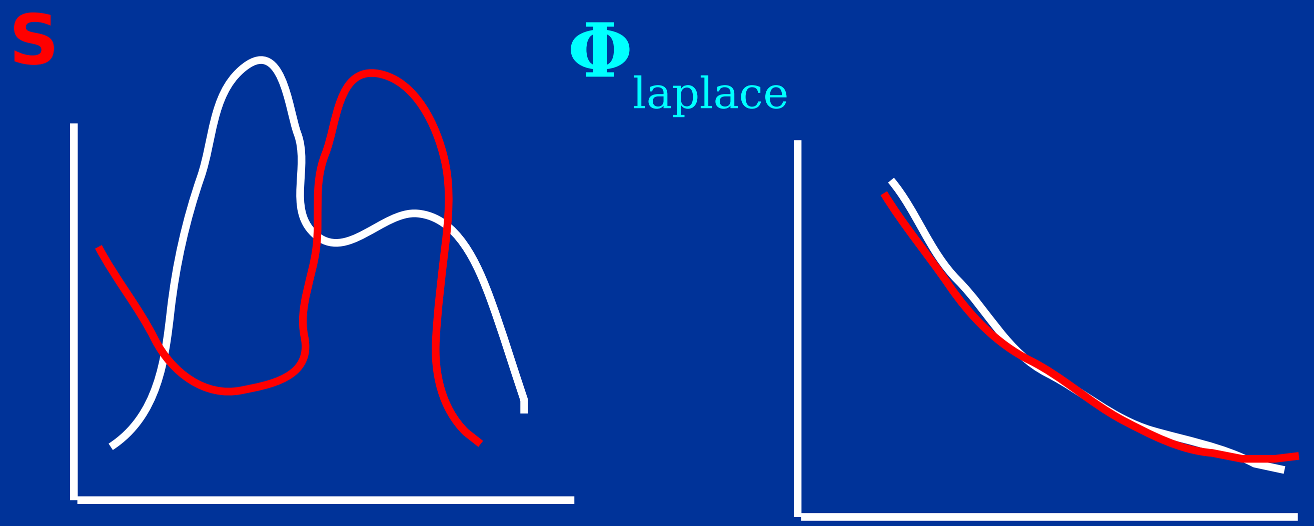

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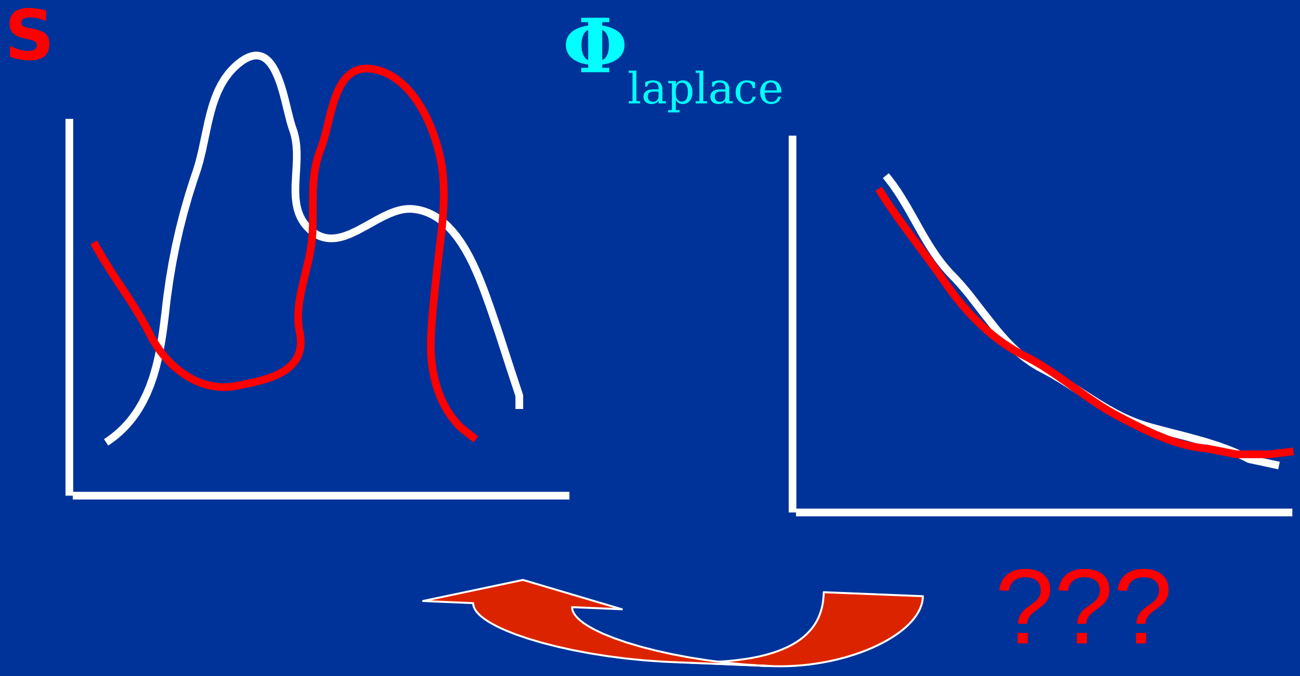
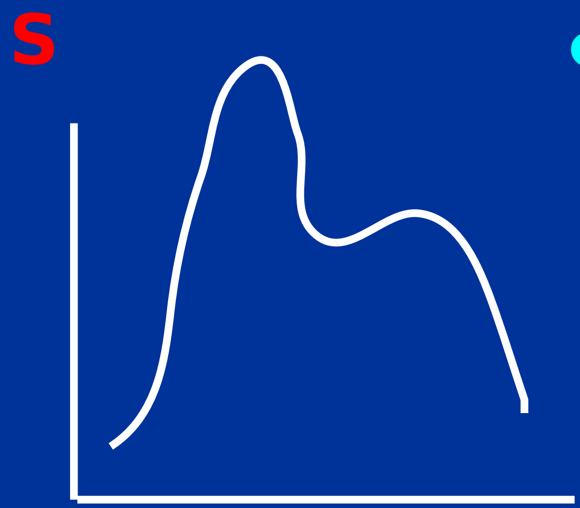
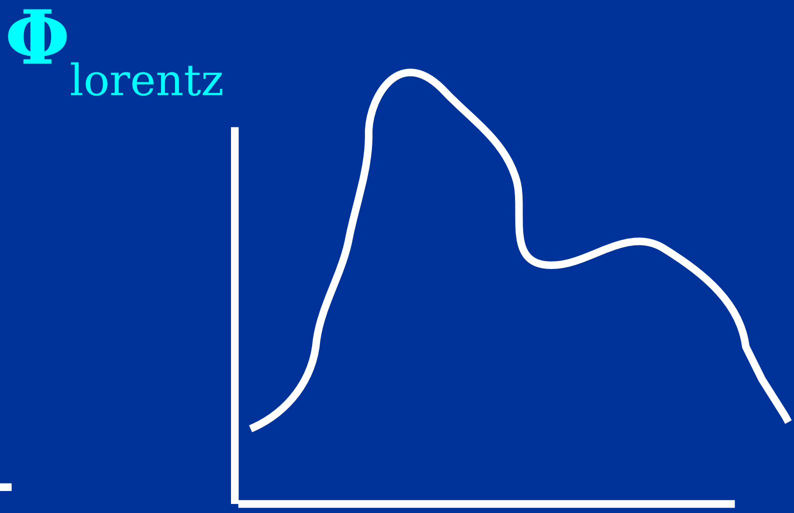


The numerical inversion of the **Lorentz** Transform is much more stable!





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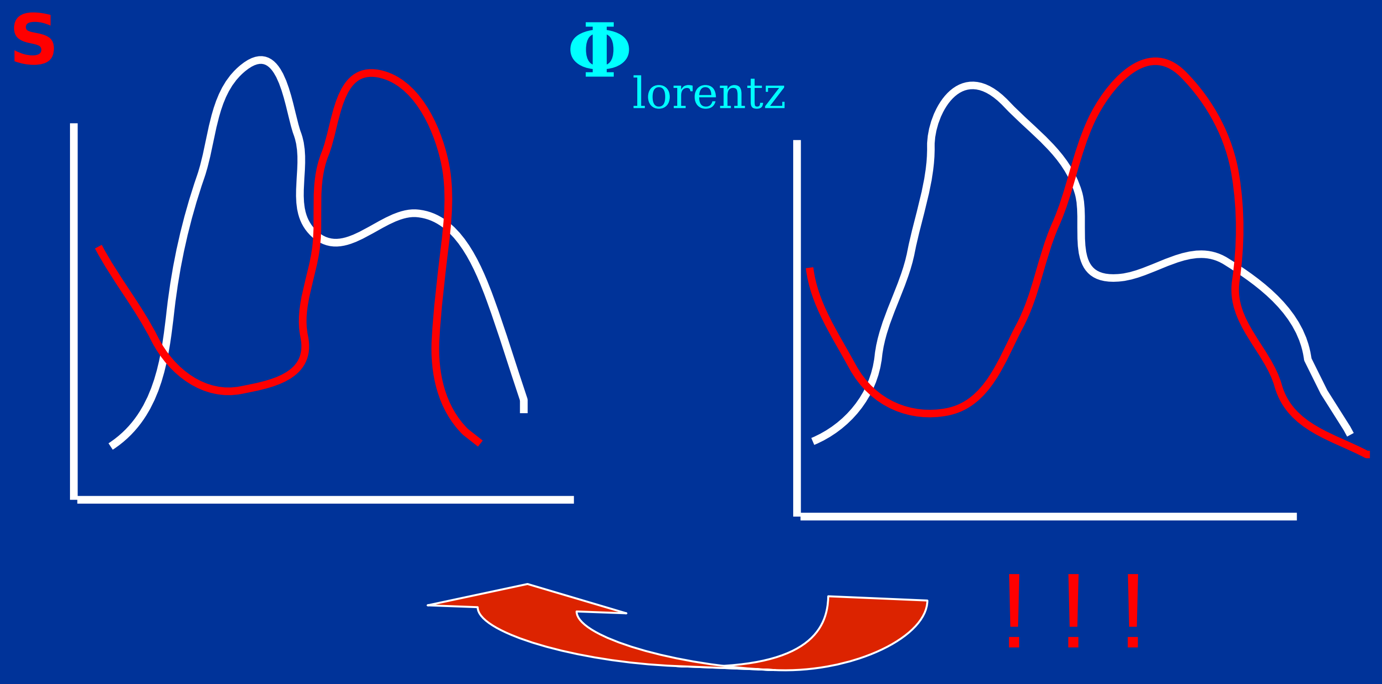


The Lorentz Integral Transform (LIT) method

First proposed in

V. D. Efros, W. Leidemann and G. Orlandini**,**

Phys. Lett. B338, 130 (1994)

Topical Review:

V. D. Efros, W. Leidemann, G. Orlandini and N. Barnea

"The Lorentz Integral Transform (LIT) method and its applications to perturbation induced reactions"

J. Phys. G: Nucl. Part. Phys. 34 (2007) R459-R528

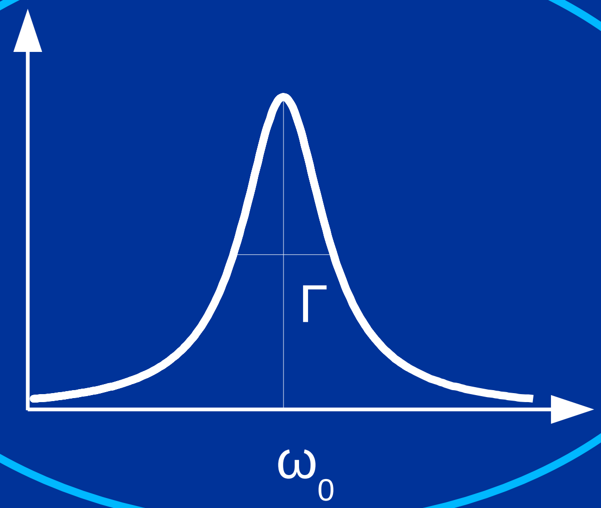
The Lorentz Kernel satisfies the two essential requirements :

N.1. one can calculate the integral transform

N.2 one is able to invert the transform, minimizing instabilities

Illustration of requirement N.1: one can calculate the integral transform

a **theorem** based on **closure** states that



$$\Phi(\omega_0,\Gamma) = \int S(q,\omega)\, L(\omega,\omega_0,\Gamma)\, d\omega = \left\langle \tilde{\Psi} | \tilde{\Psi} \right\rangle$$

$$|\tilde{\Psi}> = \frac{1}{(H - E_0 - \omega_0 + i\Gamma)}\, \Theta\, |0>$$

Proof of the theorem:

$$\Phi(\omega_0,\Gamma) = \int_{E_{th}^-}^{\infty} d\omega \frac{S(q,\omega)}{(\omega-\omega_0)^2+\Gamma^2}$$

$$= \int_{E_{th}^-}^{\infty} d\omega \frac{\sum_n |\langle n|\Theta|0\rangle|^2 \delta(\omega - E_n + E_0)}{(\omega-\omega_0-i\Gamma)(\omega-\omega_0+i\Gamma)}$$

$$= \sum_n <0|\Theta^\dagger \frac{1}{(E_n - E_0 - \omega_0 - i\Gamma)}|n> <n|\frac{1}{(E_n - E_0 - \omega_0 + i\Gamma)}\Theta|0>$$

Closure = 1

$$= \sum_n <0|\Theta^\dagger \frac{1}{(H - E_0 - \omega_0 - i\Gamma)}|n><n| \frac{1}{(H - E_0 - \omega_0 + i\Gamma)}\Theta|0>$$

$$= <0|\Theta^\dagger \frac{1}{(H - E_0 - \omega_0 - i\Gamma)} \frac{1}{(H - E_0 - \omega_0 + i\Gamma)}\Theta|0> .$$

$$= <\tilde{\Psi}|\tilde{\Psi}>$$

where $|\tilde{\Psi}> = \frac{1}{(H - E_0 - \omega_0 + i\Gamma)}\Theta|0>$

The LIT in practice:

1. $$|\tilde{\Psi}> = \frac{1}{(H - E_0 - \omega_0 + i\Gamma)} \Theta\,|0>$$

is found solving for fixed Γ and many ω_0

$$(H - E_0 - \omega_0 + i\Gamma)\,\tilde{\Psi} = \Theta\,|0\rangle$$

2. the overlap $\langle \tilde{\Psi} | \tilde{\Psi} \rangle$ is calculated

3. the transform is inverted

$$\langle \tilde{\Psi} | \tilde{\Psi} \rangle = \int S(\mathbf{q}, \omega) \mathbf{L}(\omega, \omega_0, \mathbf{\Gamma}) \, \mathrm{d}\omega$$

main point of the LIT :

Schrödinger-like equation with a source

$$\left(H - E_0 - \omega_0 + i\Gamma\right) \tilde{\Psi} = S$$

S = Θ |0>

main point of the LIT :

Schrödinger-like equation with a source

$$(H - E_0 - \omega_0 + i\Gamma)\,\tilde{\Psi} = S$$

Theorem:

The $\tilde{\Psi}$ solution is unique and has ***bound state*** asymptotic behavior

$$\langle \tilde{\Psi} | \tilde{\Psi} \rangle = \int [(\omega - \omega_0)^2 + \Gamma^2]^{-1}\, S(\mathbf{q},\omega)\, d\omega < \infty$$

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Schrödinger-like equation with a source

$$(H - E_0 - \omega_0 + i\Gamma)\,\tilde{\Psi} = S$$

Theorem:

The $\tilde{\Psi}$ solution is unique and has ***bound state*** asymptotic behavior

→ one can apply ***bound state methods***

The LIT method

- reduces the continuum problem to a bound state problem
- needs "only" a good method for bound state calculations (FY, HH, NCSM, ...???)
- has been benchmarked in systems (A=2,3) where one can solve the Schroedinger equation in the continuum
- has been successfully applied for A=4,6,7

Some interesting observables:

The **electron scattering cross section**, in particular the **L**ongitudinal **$R_L(q, \omega)$** and **T**ransverse **$R_T(q, \omega)$** response functions

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Charge density ρ

$$S(q,\omega) = \sum\!\!\!\!\!\!\int_n |\langle n|\Theta|0\rangle|^2 \,\delta(\omega - E_n + E_0)$$

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Current density J

$$S(q,\omega) = \sum\!\!\!\!\!\!\int_n |\langle n|\Theta|0\rangle|^2 \, \delta(\omega - E_n + E_0)$$

Dipole operator D

The **photoabsorption cross section**, $\sigma_\gamma(q = \omega)$

one example where we have a “good” theoretical situation and a “very bad” experimental one

Clear dependence of ab initio results on the potental

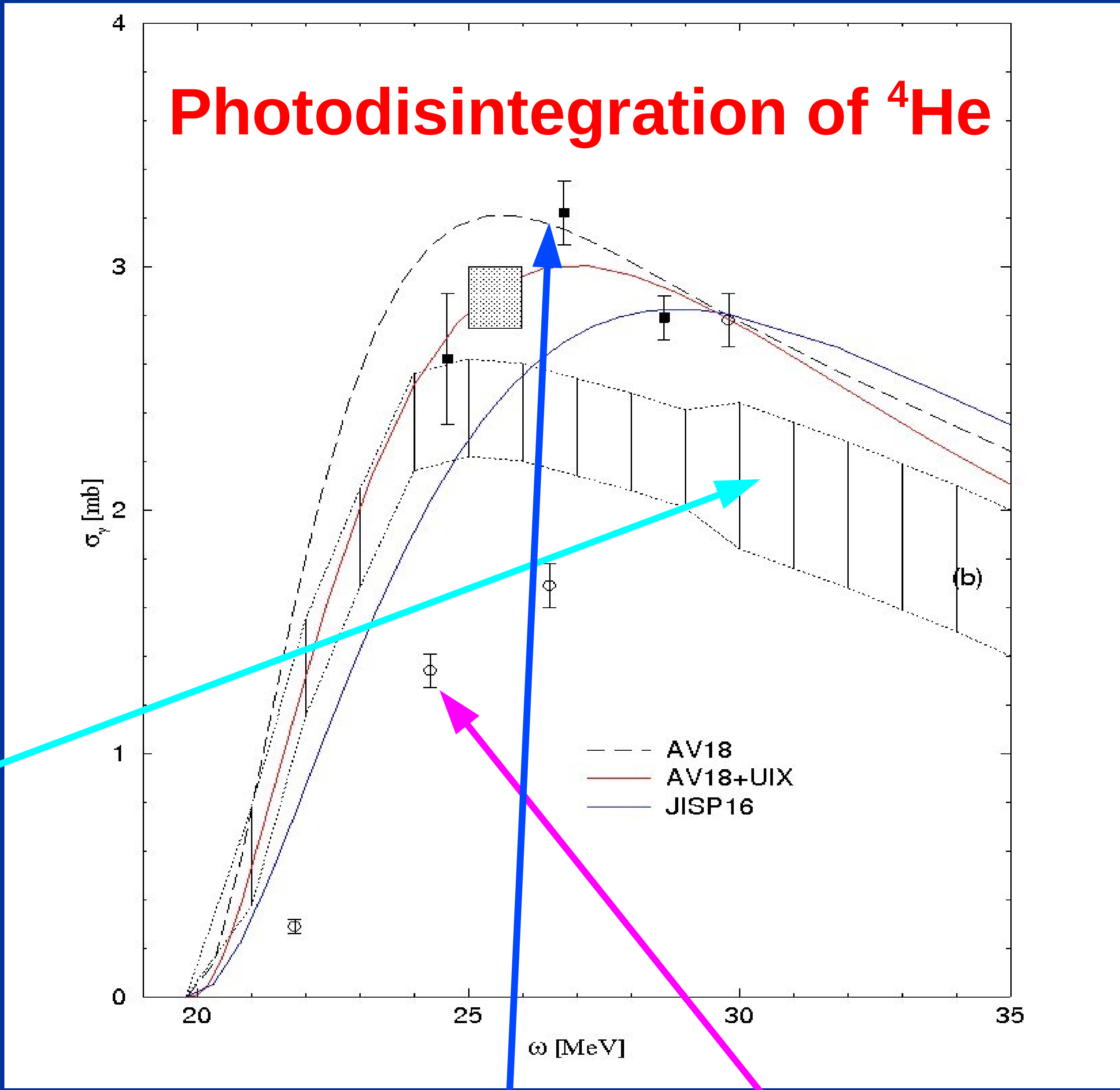
Very confused experimental situation

Data:

(γ , n) Berman et al. '80

+

(γ , p) Feldman et al. '90



additional exp data: Nilsson (2005), **Shima (2005)**

How important are relativistic effects as q increases?

One criteria to judge is the
frame dependence of the results

The electron scattering response functions in various frames

$$R_L^{fr}(q_{fr},\omega_{fr}) = \sum\!\!\!\!\!\!\int \left|\left\langle \Psi_f^{fr}|\rho(\mathbf{q}_{fr},\omega_{fr})|\Psi_i^{fr}\right\rangle\right|^2 \delta^4(P_f^{fr} - P_i^{fr} - Q^{fr}).$$

$$R_T^{fr}(q_{fr},\omega_{fr}) = \sum\!\!\!\!\!\!\int \left|\left\langle \Psi_f^{fr}|\mathbf{J}_T(\mathbf{q}_{fr},\omega_{fr})|\Psi_i^{fr}\right\rangle\right|^2 \delta^4(P_f^{fr} - P_i^{fr} - Q^{fr}).$$

VARIOUS FRAMES:

LAB: **initially nucleons have momenta** $\mathbf{p_i \cong 0}$

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ANTI-LAB: **initially nucleons have momenta** $\mathbf{p}_i \cong -\mathbf{q}/A$

(in q.e. the final momentum of the "active nucleon" $\mathbf{p}_f \cong \mathbf{q}\,(A-1)/A$ **)**

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BREIT: **initially nucleons have momenta** $p_i \cong - q/2A$

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ANB: initially nucleons have momenta $p_i \cong - q /2$

(in q.e. the final momentum of the "active nucleon" $p_f \cong q /2$)

They are connected to the response functions in the LAB frame (where they are measured !)

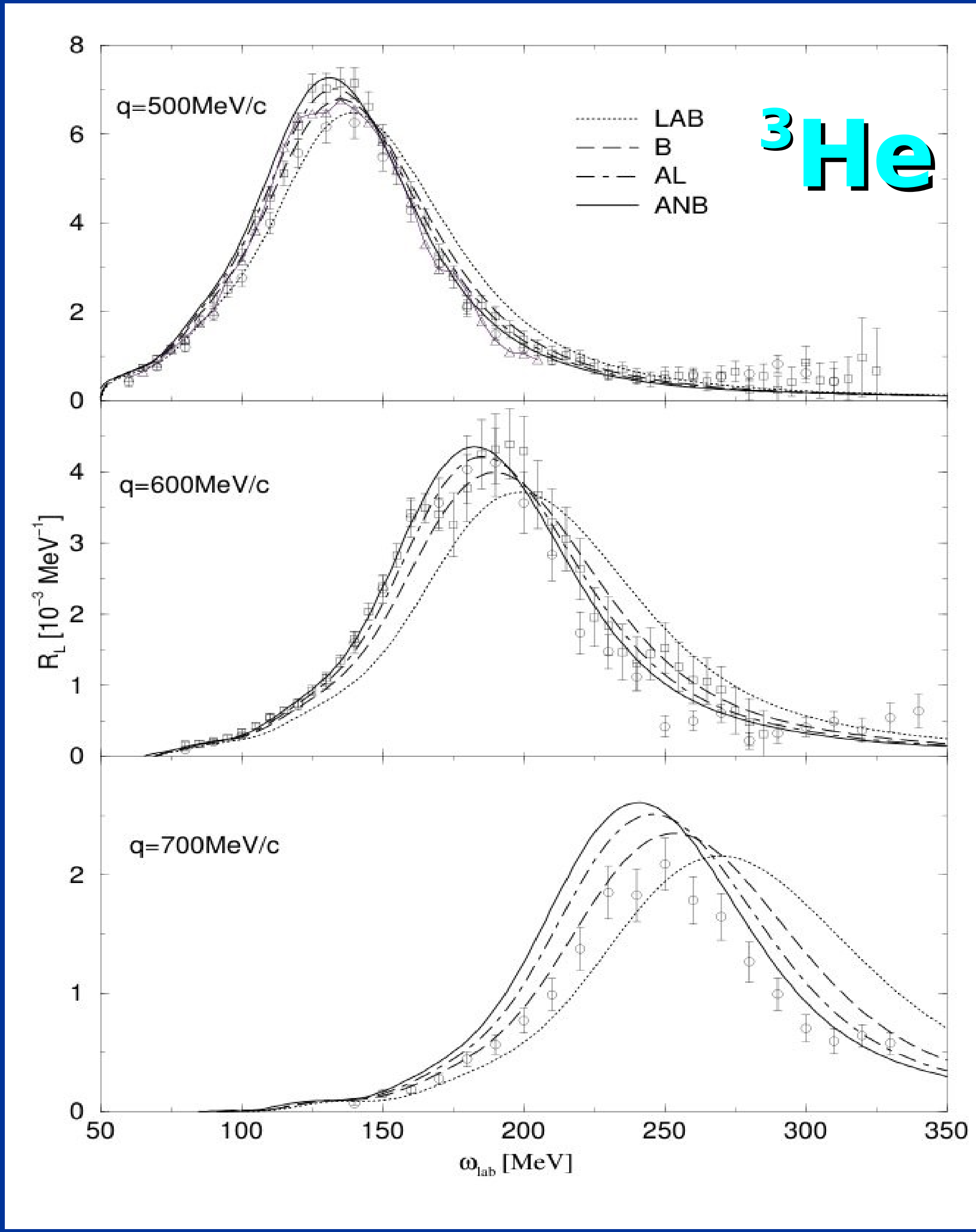
$$R_L^{\mathbf{LAB}}(q,\omega) = \frac{q^2}{q_{fr}^2}\frac{E_i^{fr}}{M_T}R_L^{fr}(q_{fr},\omega_{fr})$$

$$R_T^{\mathbf{LAB}}(q,\omega) = \frac{E_i^{fr}}{M_T}R_T^{fr}(q_{fr},\omega_{fr})$$

Longitudinal response of ^3He

Large frame dependence!!!

V.Efros et al. PRC 72 (2005) 011002



Is there an easy way to cure it?

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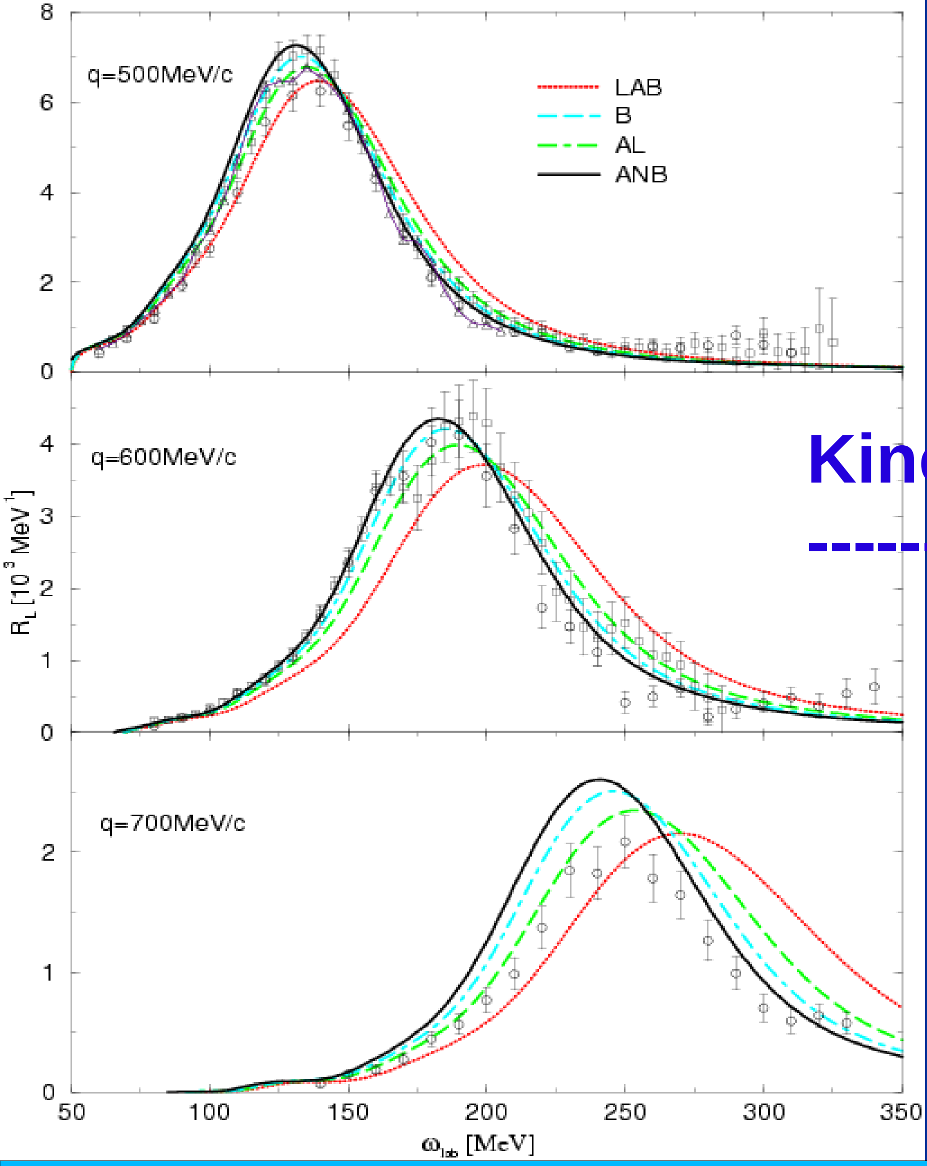
use in each frame the kinematical inputs corresponding to the quasi elastic 2-body assumption i.e. p + (A-1)-system

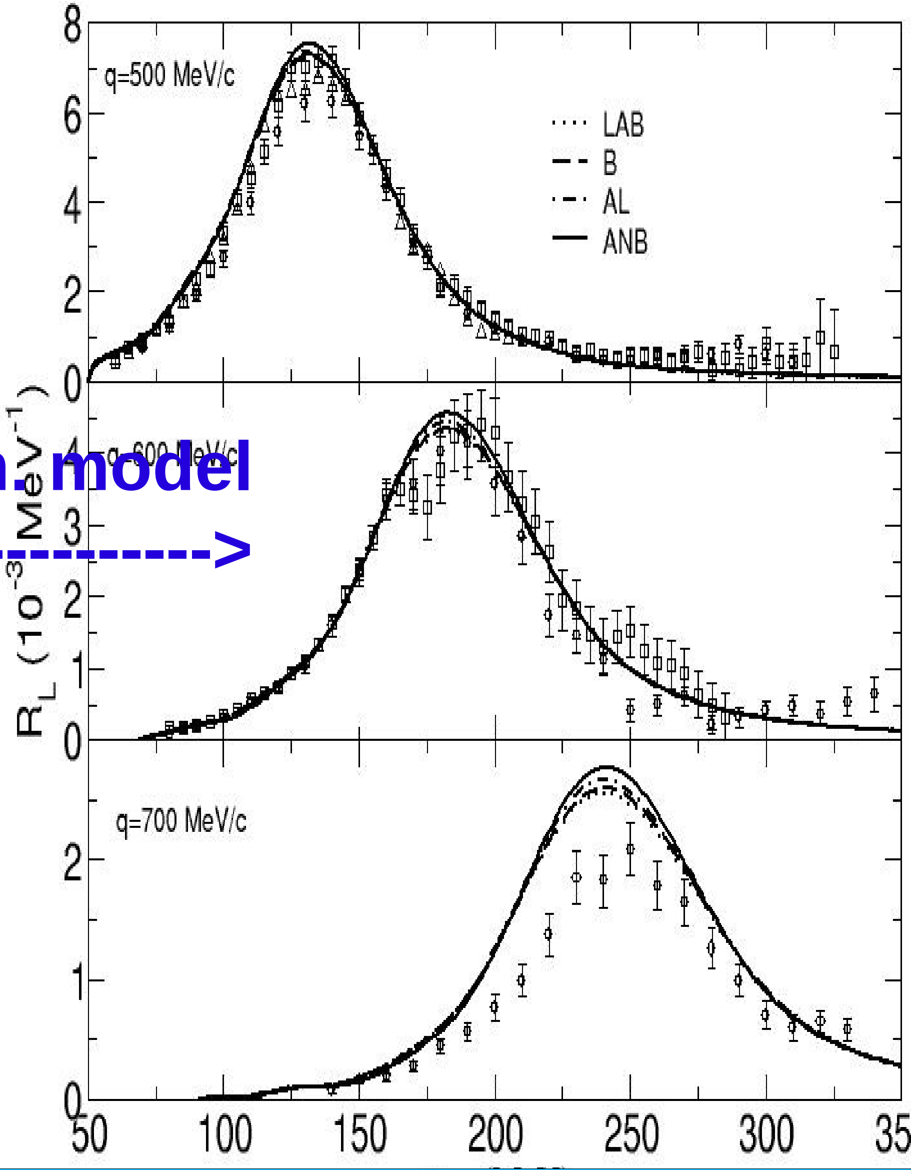
The relative momentum of 2 bodies p + (A-1) can be calculated in each frame in a relativistically correct way.

The relative kinetic energy is then taken in its non relativistic form $p^2_{rel} / 2\mu$ (the input of a non relativistic dynamical calculation)


q=500MeV/c
LAB
B
AL
ANB
q=600MeV/c
q=700MeV/c
R_L [10^3 MeV^{-1}]
ω_{lab} [MeV]

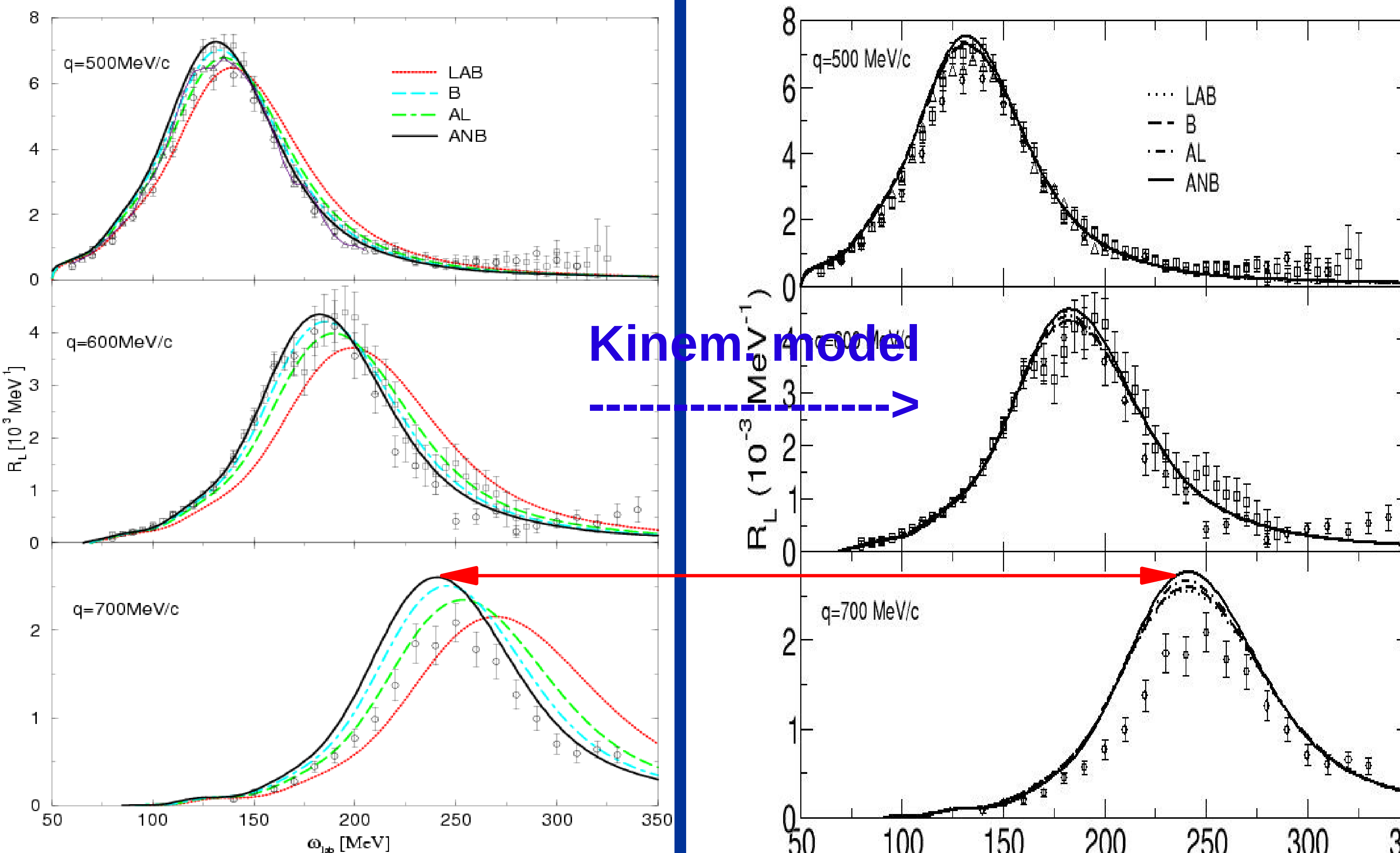
Kinem. model

-------------------->


q=500 MeV/c
LAB
B
AL
ANB
q=700 MeV/c
R_L (10^{-3} MeV^{-1})

remark:

Of the 4 frames the **ANB** result is the less affected by the relativistically correct kinematical model.



q=500MeV/c
LAB
B
AL
ANB
q=600MeV/c
q=700MeV/c
R_L [10^3 MeV^-1]
ω_lab [MeV]
Kinem. model
----------------->
q=500 MeV/c
LAB
B
AL
ANB
q=700 MeV/c
R_L (10^-3 MeV^-1)

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The reason is that in the ANB frame the momenta of the active particle are the smallest (about q/2!).

Therefore the error on the kinetic energy is the smallest:

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$$T \cong \frac{p^2}{2\,m} - \frac{p^4}{8\,m^3} + \ldots \qquad\qquad \frac{\Delta T}{T} \cong \frac{p^2}{4\,m^2}$$

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$$\textbf{LAB}: \quad \frac{\Delta T}{T} \cong \frac{q^2}{4\,m^2} \qquad\qquad \textbf{ANB}: \quad \frac{\Delta T}{T} \qquad \frac{q^2}{16\,m^2} \quad \textbf{!!!}$$

Moreover: the **peak position** in the **ANB** frame is always relativistically correct, in fact in general:

$$\omega_{peak} \cong T(p_f) - T(p_i)$$

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rel. different from **n.r.** !!!

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LAB : $\omega_{peak} \cong T(q) - T(0)$

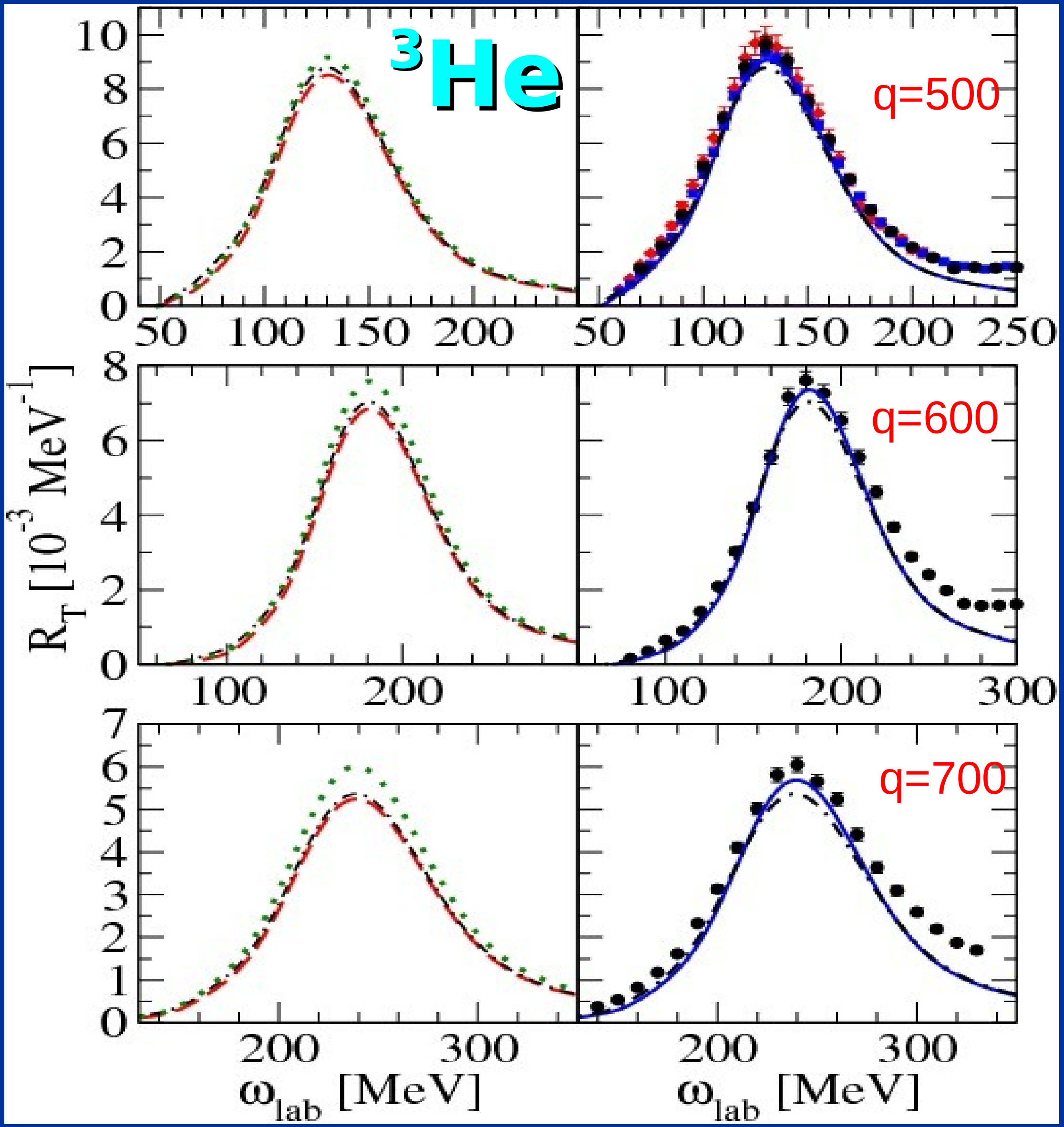
rel. different from **n.r.** !!!

ANB : $\omega_{peak} \cong T(q/2) - T(q/2) = 0$

rel. equal to **n.r.**

always correct **!!!**

$R_T(q,\omega)$ in the "quasi elastic" regime



V.Efros et al .PRC 81 (2010) 034001

PRC 83 (2011) 057001

Conclusion N. 1

- the **LIT** represents an accurate viable method to study reactions to the **“far” continuum** where the many-body scattering problem (all channels!) is not solvable (e.g. **A>3)**

- **only bound state technique is needed**

Conclusion N. 2:

- Perform a **non relativistic** **dynamical** calculation of **S (q, ω)** in the *quasi elastic regime* in the **ANB** frame

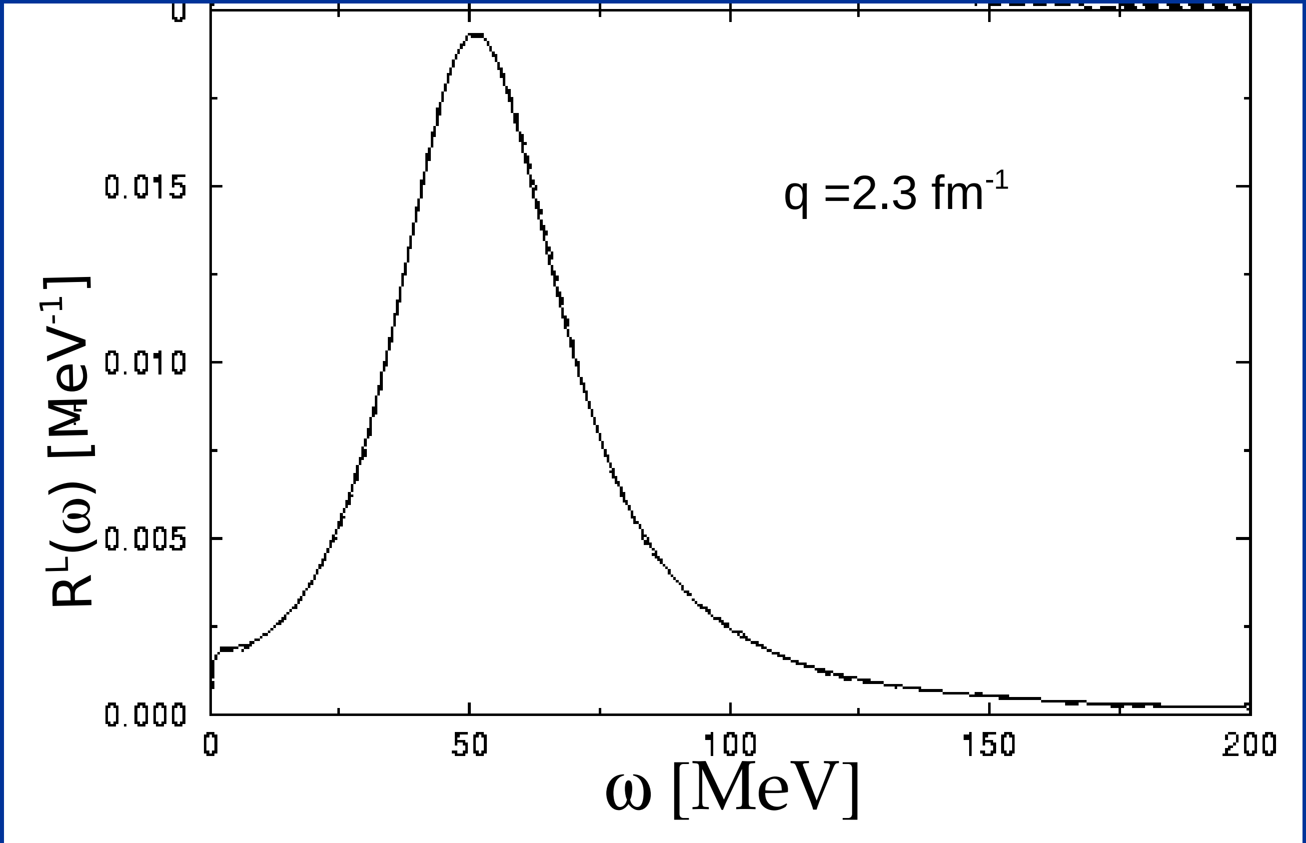
- use the **relativistically** correct “2-body” **kinematics**

the end

The benchmarks for the LIT method

test on the Deuteron:

$S(q,\omega)$ is the longitudinal (e,e') response function $R_L(q,\omega)$



Phys Lett. B338 (1994) 130

test on the Triton:

S(q,ω) is the Dipole Photoabsorption Cross Section $\sigma_\gamma(q = \omega)$

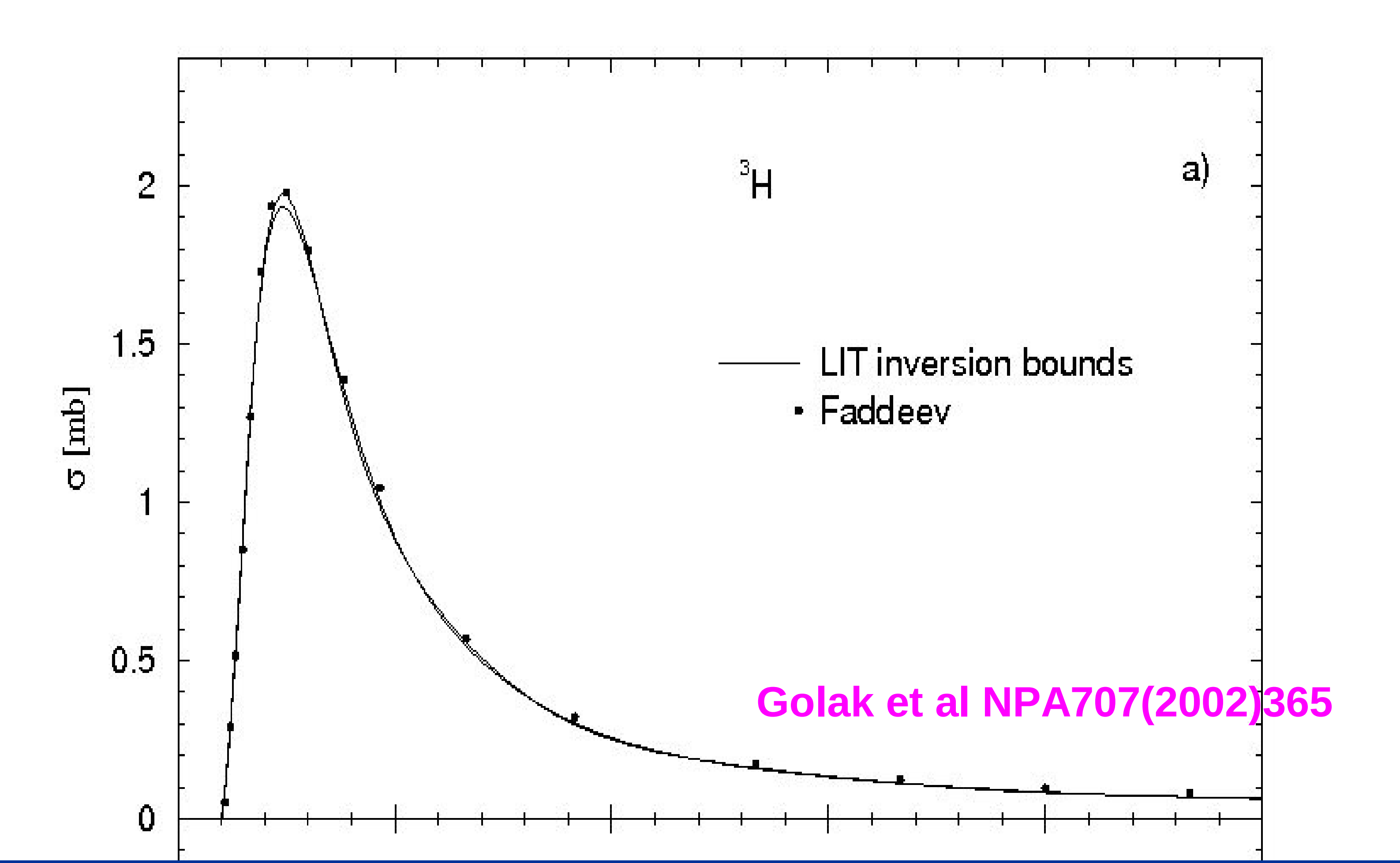


Illustration of requirement N.2:
one can invert the integral transform minimizing instabilities

Inversion of the LIT: the **regularization** method

$$R(\omega) = \sum_{n=1}^{N_{max}} c_n \chi_n(\omega, \alpha_i)$$

The χ_n are given functions with nonlinear parameters α_i. A basis set frequently used for LIT inversions is

$$\chi_n(\omega, \alpha_i) = \omega^{\alpha_1} \exp(-\frac{\alpha_2 \omega}{n}) .$$

Substituting such an expansion in the integral equation

$$\Phi(\omega_0, \Gamma) = \sum_{n=1}^{N_{max}} c_n \tilde{\chi}_n(\omega_0, \alpha_i) ,$$

where

$$\tilde{\chi}_n(\omega_0, \alpha_i) = \int_0^\infty d\omega \frac{\chi_n(\omega, \alpha_i)}{(\omega - \omega_0)^2 + \Gamma^2} .$$

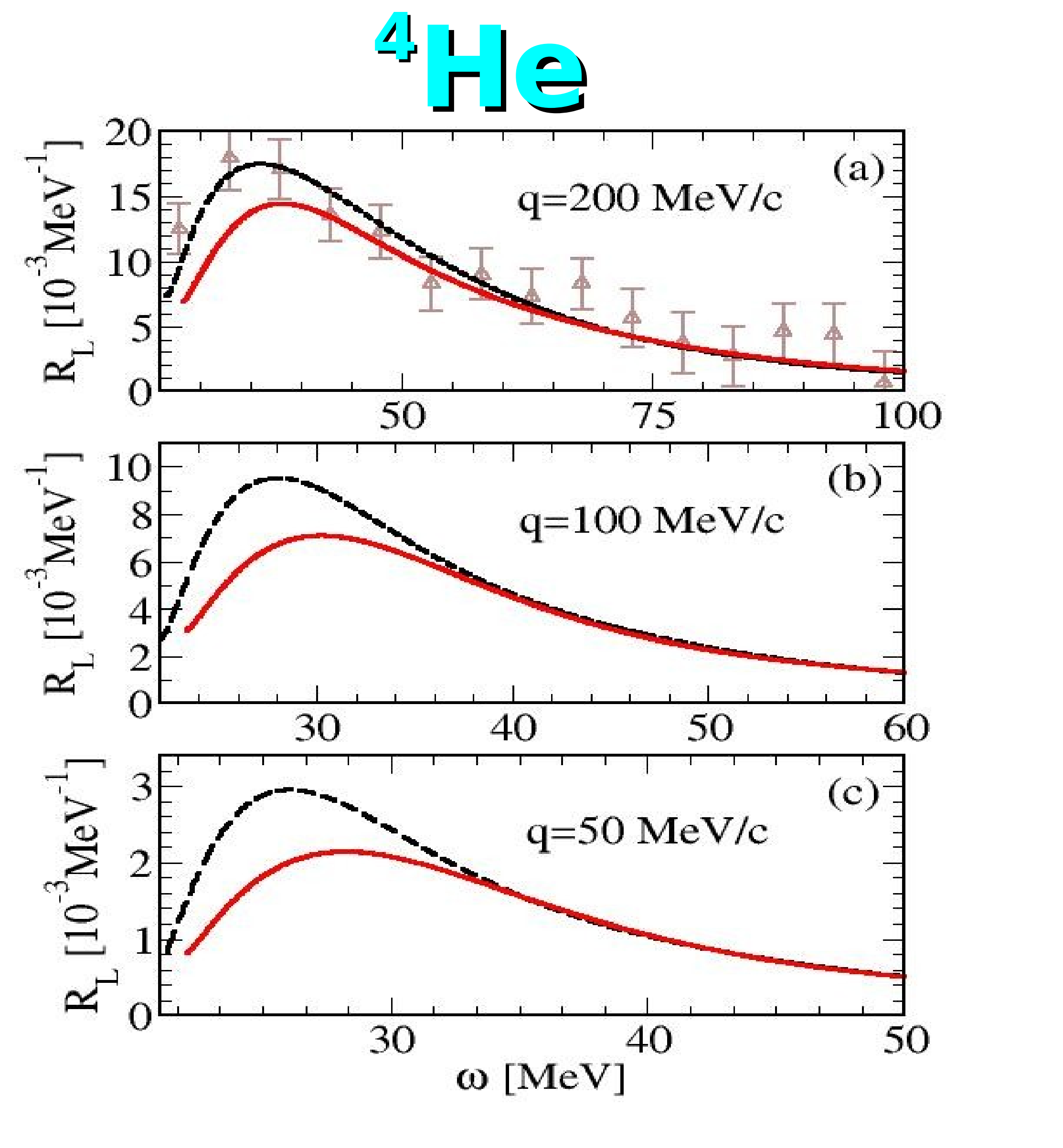
For given α_i the linear parameters c_n are determined from a least–square best fit to the calculated $\Phi(\omega_0, \Gamma)$ for a number of ω_0 points much larger than N_{max}.

Works well with bell shaped kernels (and not too narrow resonances)

A = 4

electron scattering $R_L(q,\omega)$

SURPRISE:
LARGE EFFECT OF
3-BODY FORCE AT LOW Q



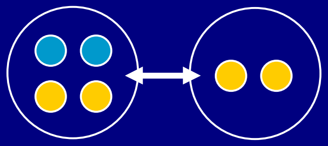
Black curve: AV18
Red curve: AV18+UIX

S.Bacca et al.,
PRL 102 (2009) 162501

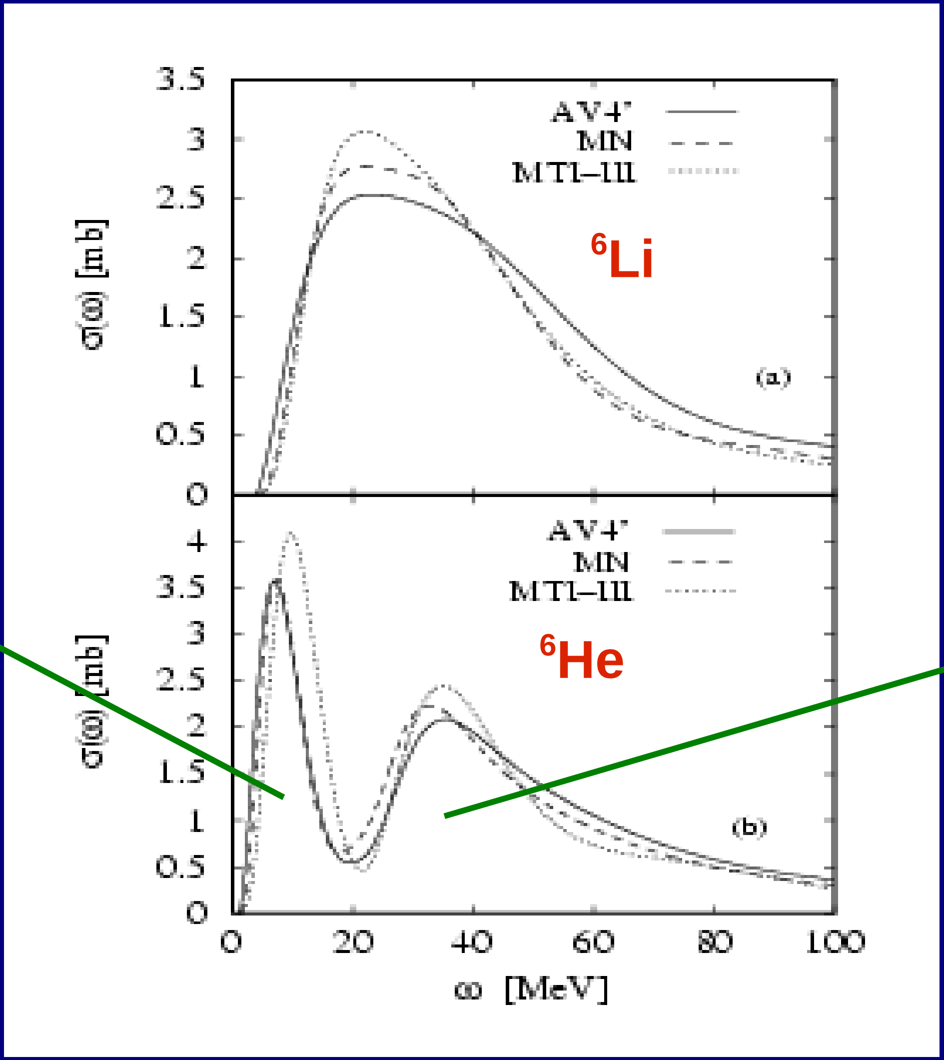
A = 6

6-Body photoabsorption (total photodisintegtration)

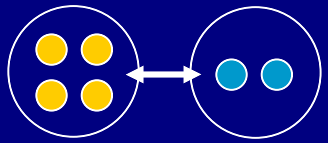
S.Bacca et al. PRL89(2002)052502



soft
mode ???



EIHH
MT

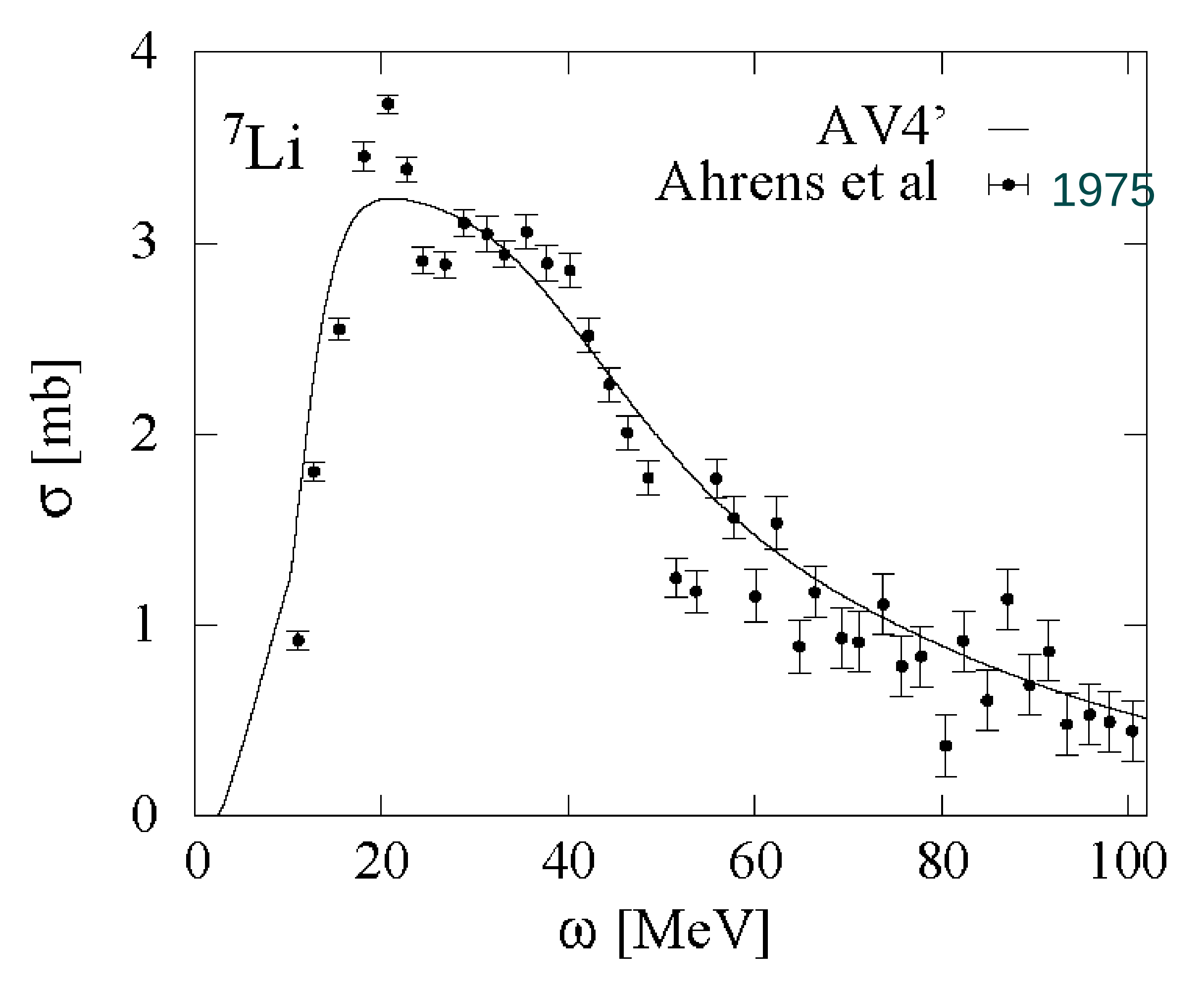


classical GT
mode ???

A = 7

7-Body photoabsorption (total photodisintegration)

S.Bacca et al. PLB 603(2004) 159



A very good method to solve bound states:

the Effective Interaction in Hyperspherical Harmonics method (EIHH)

N.Barnea, W.Leidemann, G.O. PRC61(2000)054001

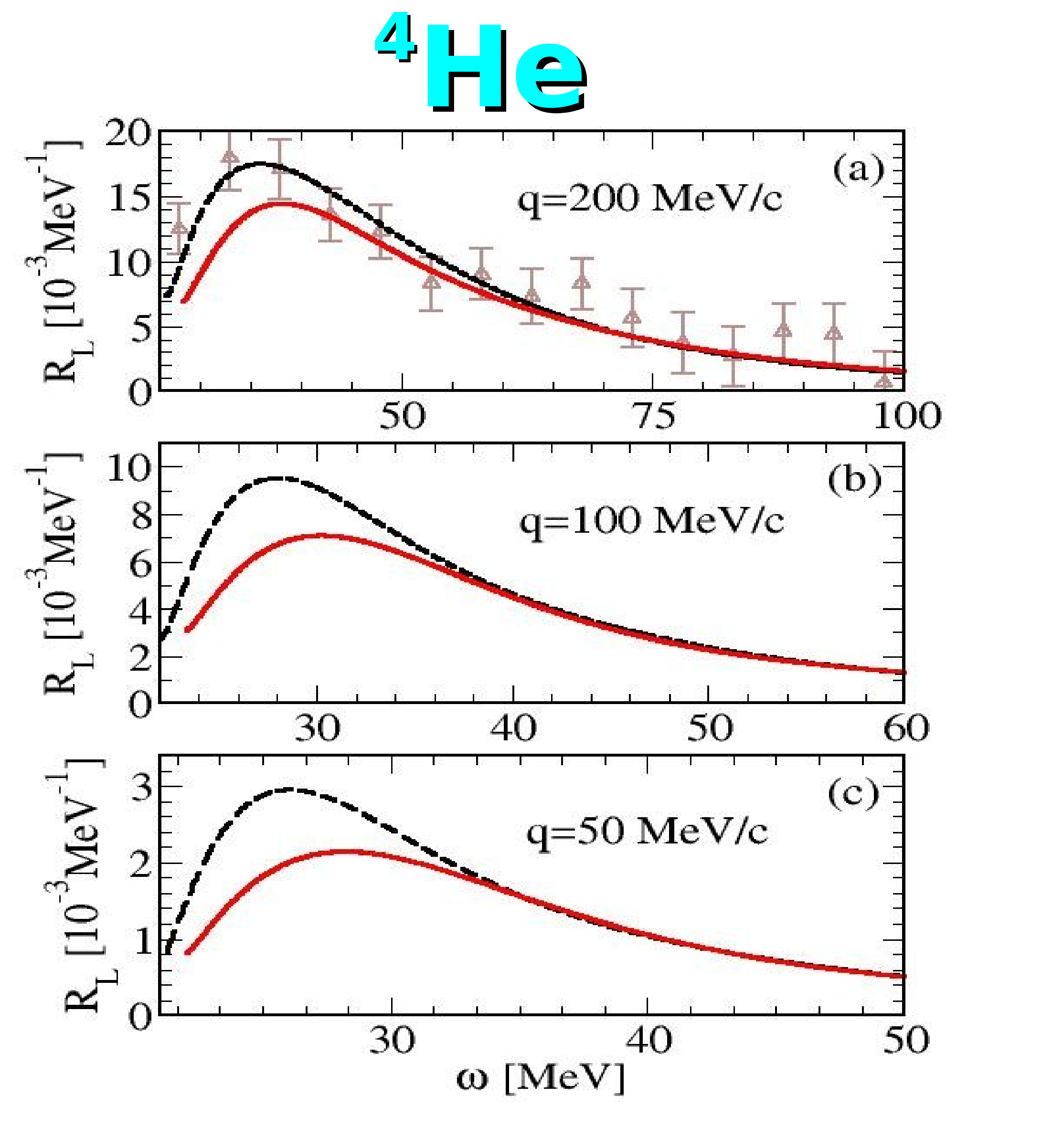
- Expansion on *Hyperspherical Harmonics* basis
- Use of **Lee – Suzuki** unitary transformation to obtain the *effective interaction*
- Fast convergence

Ab initio non relativistic calculations of $S(q,\omega)$: how far in q are they reliable?

electron scattering $R_L(q,\omega)$

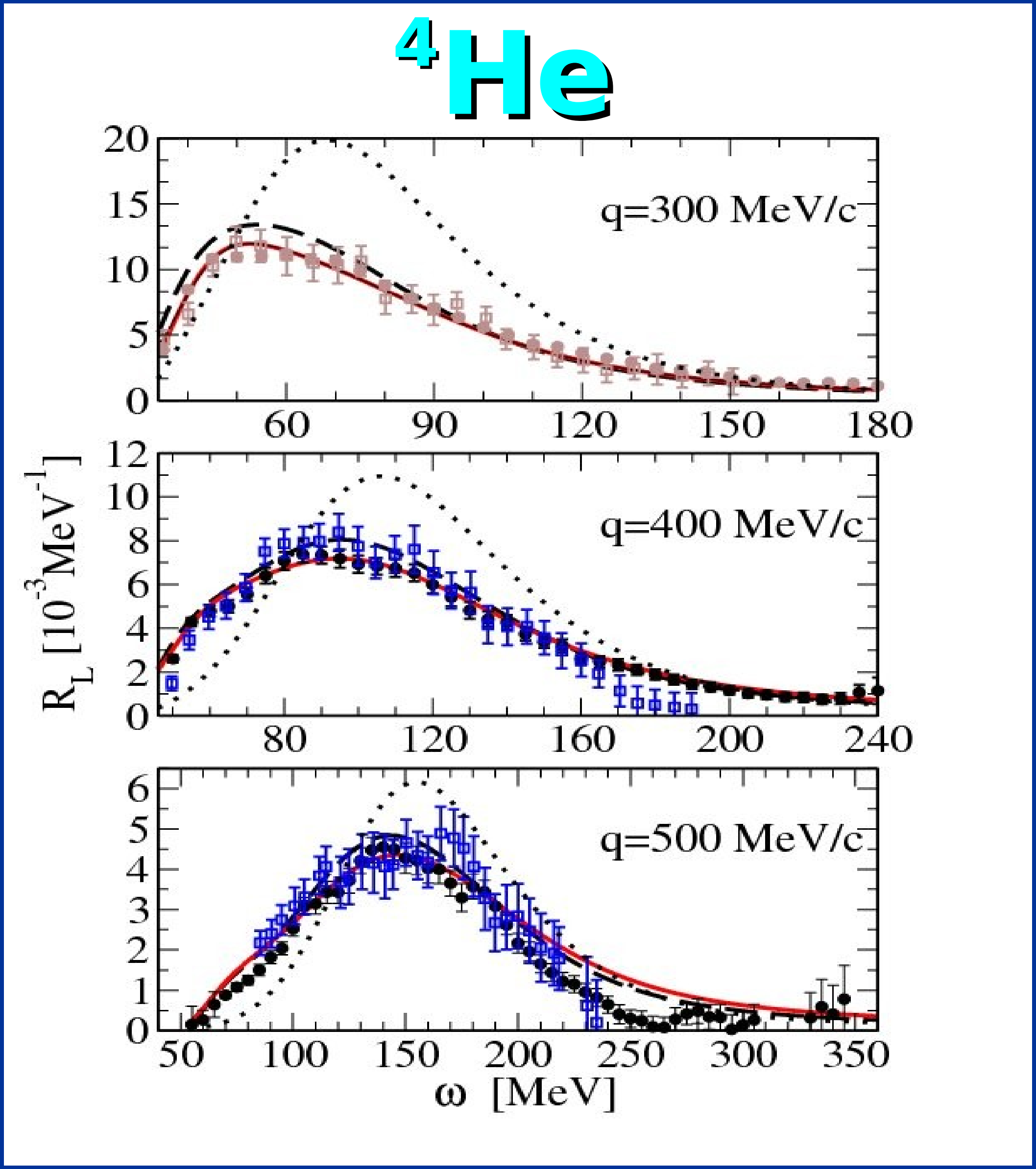
SURPRISE:
LARGE EFFECT OF
3-BODY FORCE AT LOW Q

NO MEASUREMENTS
AT LOW q !!!



S.Bacca et al.,
PRL 102 (2009) 162501

$R_L(q,\omega)$ in the "quasi elastic" regime



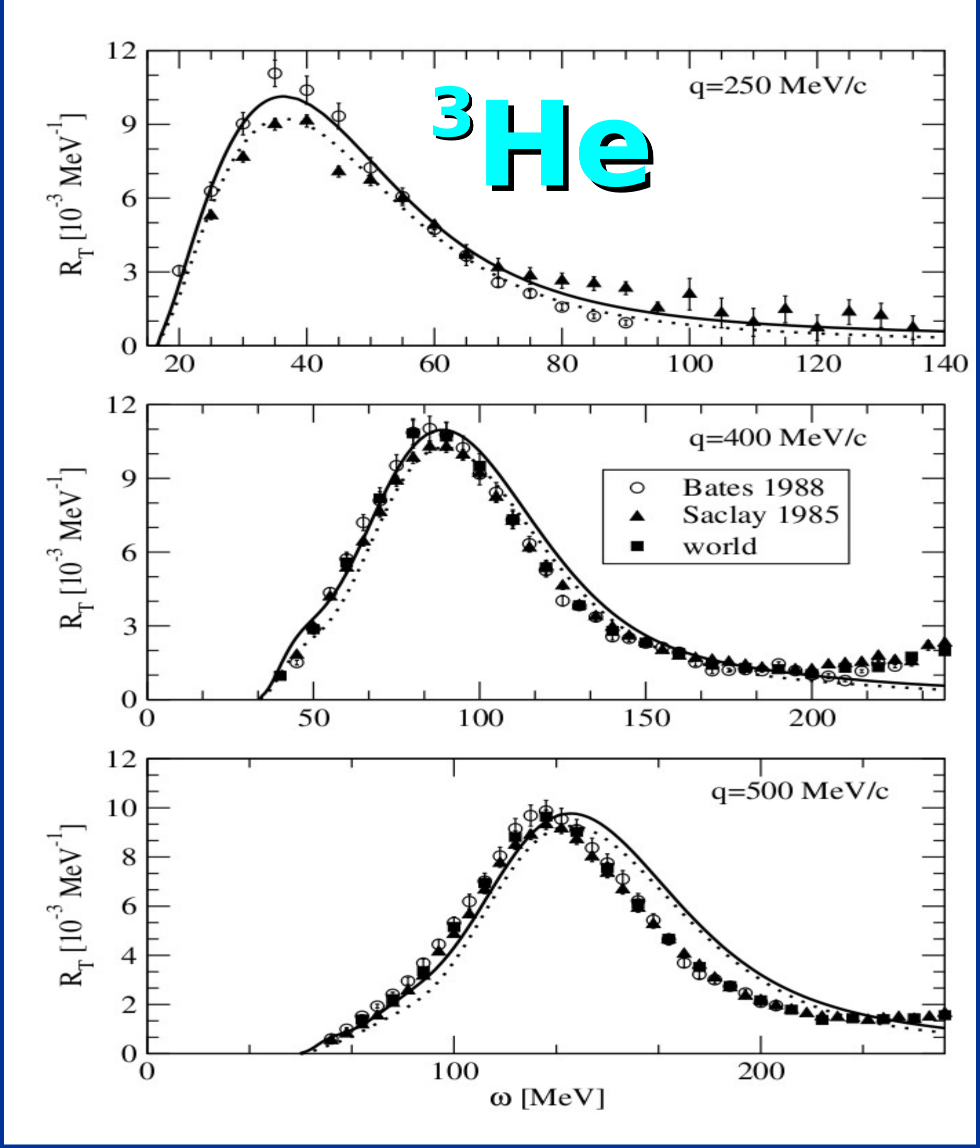
Dotted PWIA

dashed: AV18

full: AV18+UIX

S.Bacca et al.,
PRL 102 (2009) 162501

$R_L(q,\omega)$ in the "quasi elastic" regime



S.Della Monaca et al.
PRC 77(2008) 044007