Recent progress in ab-initio four-body scattering calculations

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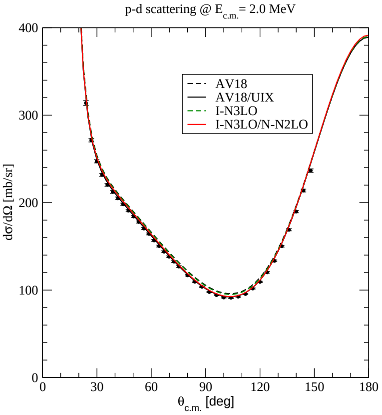
of Quantum Few-Body Systems, Erice, October 13, 2011

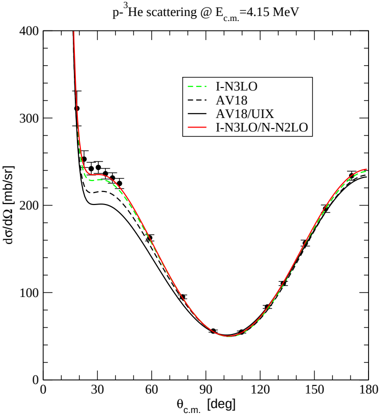
Outline

Collaborators

- A. Kievsky & L.E. Marcucci - *INFN & Pisa University, Pisa (Italy)*
- L. Girlanda *Lecce University, Lecce (Italy)*
- R. Schiavilla *Jefferson Lab. & ODU, Norfolk (VA, USA)*
- A. Baroni PhD student, ODU (USA)

1950-2011: development of “realistic” NN+3N interaction models (see below)





Few-Nucleon Systems: Current interest (2)

- Solution of persistent disagreements in $A = 3, 4$ systems
  - $N - d$ & $p - {}^3\mathrm{He}$ A_y puzzle
  - space star anomaly in $3N$ breakup
  - other anomalies in $3N$ breakup
- Relativistic effects/extension above the pion production threshold
- 3N force & nuclear/neutron matter equation of state
- applications
  - electron scattering - extraction of nucleon properties
  - reactions of astrophysical interest
  - energy production
  - test of fundamental symmetries in few nucleon systems

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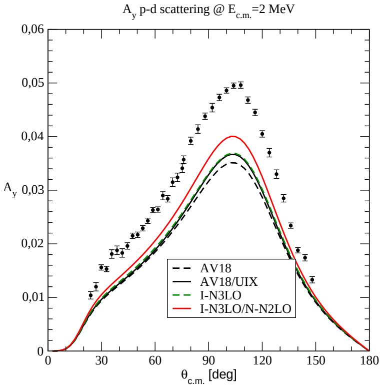
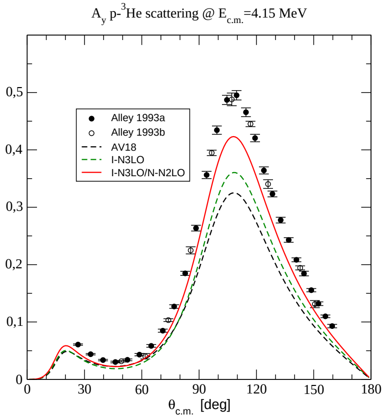
A_y "puzzle"



A_y p-d scattering @ E_c.m.=2 MeV
0,06
0,05
0,04
A_y 0,03
0,02
0,01
0
0
30
60
90
120
150
180
θ_c.m. [deg]
AV18
AV18/UIX
I-N3LO
I-N3LO/N-N2LO



A_y p-3He scattering @ E_c.m.=4.15 MeV
0,5
0,4
0,3
0,2
0,1
0
0
30
60
90
120
150
180
θ_c.m. [deg]
Alley 1993a
Alley 1993b
AV18
I-N3LO
I-N3LO/N-N2LO

NN & 3N interaction

NN potentials

- ”Old models”: Argonne V18, CD-Bonn, Nijmengen ($\chi^2 \approx 1$)
- Fit of 3N data using non-locality in P-waves (INOY [Doleschall, 2008])
- Effective field theory (EFT)
  - [Epelbaum and Coll, 1998-2006]
  - [Entem & Machleidt, 2003]

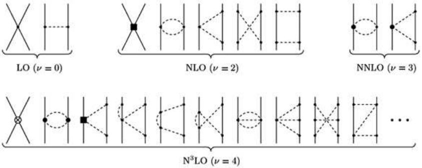
- Low-q interaction [Bogner and Coll., 2001-2007]
- UCOM interaction [Roth and Coll. 2004-2010]
- SRG interaction [Furnstahl and Coll.,2008-2010]

3N potentials

- “Old models”: Tucson-Melbourne [Coon *et al*, 1979, Friar *et al*, 1999]; Brazil [Robilotta & Coelho, 1986]; Urbana [Pudliner *et al*, 1995]
- Effective field theory
  - at N2LO [Epelbaum *et al*, 2002]
- Illinois [Pieper *et al*, 2001]
- Under progress: N3LO, N4LO

NN interaction from EFT

- $N - \pi$ interaction "dictated" by chiral symmetry [Weinberg (1990), Bernard, Kaiser & Meissner, (1995); Ordonéz, Ray & van Kolck (1996), . . .]
- Contributions organized as an expansion over $(Q/\Lambda_\chi)^\nu$ $[\Lambda_\chi \approx 1$ GeV]
- NN interaction:
  - N3LO-Jülich [Epelbaum and Coll, 1998-2006]
  - N3LO-Idaho (I-N3LO) [Entem & Machleidt, 2003]



- 3N interaction
  - J-N2LO [Epelbaum *et al*, 2002], N-N2LO [Navratil, 2007]
  - Under progress: N3L0 [Epelbaum & Coll.], N4LO [Pisa, Entem & Machleidt]

Theoretical methods for bound states

Kamada *et al*, PRC **64**, 044001 (2001)

Faddeev-Yakubovsky (FY) Equations

Solved in configuration or momentum space

- Bochum-Cracow-Jülich, Lisboa, Grenoble, LANL, . . .
- Alt-Grassberger-Sandhas (AGS) Equations

GFMC

- Argonne-Los Alamos

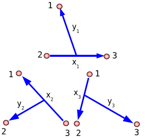
Variational methods

1. Gaussian basis [Kamimura & Hiyama, Varga & Suzuki]
2. HH [Pisa, Barnea, Orlandini & Coll.]
3. HO (NCSM) [Navratil and Coll., Goteborg, . . .]

$A = 4$ scattering

- Accurate solutions with a realistic NN interaction
  - AGS [Deltuva & Fonseca, 2007], FY [Lazauskas & Carbonell, 2009]

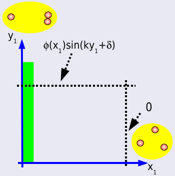
$$(H_0 + V_1 + V_2 + V_3 - E)\Psi = 0 \qquad V_i \equiv V(j,k)$$



Faddeev method ($A = 3$)

$$\Psi = \psi_1 + \psi_2 + \psi_3 \qquad \psi_i \equiv \psi(\mathbf{x}_i, \mathbf{y}_i)$$

$$(H_0 + V_i - E)\psi_i = -V_i(\psi_j + \psi_k)$$



AGS Equations ($A = 3$)

$$U_{ij} = \bar{\delta}_{ij} G_0^{-1}(E) + \sum_k \bar{\delta}_{ik} T_k(E) G_0(E) U_{kj}(E)$$

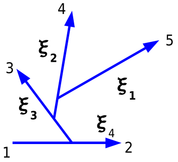
$$G_0(E) = (E - H_0 + i\epsilon)^{-1} \qquad \bar{\delta}_{ij} = 1 - \delta_{ij}$$

- $T_k(E) =$ two-body t-matrix
- U_{ij} transition operators
- Boundary conditions automatically verified

HH method

A. Kievsky, S. Rosati, MV, L.E. Marcucci, and L. Girlanda J. Phys. G, **35**, 063101 (2008)

- $N = A - 1$ Jacobi vectors ($D = 3N$)
- hyperradius $\rho^2 = \sum_{k=1}^{N} (\xi_k)^2 = \frac{2}{A} \sum_{i<j} r_{ij}^2$
- hyperangles $\Omega = D - 1$ angles
- $\cos\varphi_k = \frac{\xi_k}{\sqrt{\xi_1^2 + \ldots + \xi_k^2}}$



The HH functions are the eigenfunctions of $L^2(\Omega)$

$$H_0 = \sum_{k=1}^{n} \nabla_k^2 = \left(\frac{\partial^2}{\partial\rho^2} + \frac{D-1}{\rho} \frac{\partial}{\partial\rho} - \frac{L^2(\Omega)}{\rho^2} \right) \qquad L^2(\Omega)\mathcal{Y}_{[K]}(\Omega) = K(K + D - 2)\mathcal{Y}_{[K]}(\Omega)$$

$$\Phi_\alpha = L_m^{(3A-4)}(\beta\rho) e^{-\beta\rho/2} \sum_{perm.} \mathcal{Y}_{[K]}(\Omega_{ijk\ldots}) \qquad \Psi = \sum_\alpha c_\alpha \Phi_\alpha$$

Scattering calculation (1)

Example: $A - B$ elastic scattering for a given J^π

$$\Omega^{F}_{LS}(A,B) = \sum_{perm.=1}^{N} \left[Y_L(\hat{\mathbf{r}}_{AB})[\phi_A\phi_B]_S \right]_{JJ_z} \frac{F_L(\eta, q_{AB}r_{AB})}{q_{AB}r_{AB}}$$

$$\Omega^{G}_{LS}(A,B) = \sum_{perm.=1}^{N} \left[Y_L(\hat{\mathbf{r}}_{AB})[\phi_A\phi_B]_S \right]_{JJ_z} \frac{G_L(\eta, q_{AB}r_{AB})}{q_{AB}r_{AB}} (1 - e^{-\gamma r_{AB}})^{2L+1}$$

$$\Omega^{\pm}_{LS}(A,B) = \Omega^{G}_{LS}(A,B) \pm i\Omega^{F}_{LS}(A,B)$$

$$|\Psi_{LS}\rangle = \sum_{\alpha} c_{LS,\alpha}\Phi_\alpha + |\Omega^{F}_{LS}(p, {}^3\mathrm{He})\rangle + \sum_{L'S'} T_{LS,L'S'} |\Omega^{+}_{L'S'}(p, {}^3\mathrm{He})\rangle$$

- Sum over LS such that $\vec{L} + \vec{S} = \vec{J}$ and $(-)^L = \pi$
- $T_{LS,L'S'}$ = T-matrix elements
- $c_{LS,n}$ and $T_{LS,L'S'}$ determined using the Kohn variational principle (KVP)

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Scattering state calculation (2)

Kohn Variational Principle

$$\mathcal{F}(c_{LS,\alpha}, T_{LS,L'S'}) = T_{LS,L'S'} - \langle \mathcal{T}\Psi_{L'S'}|H - E|\Psi_{LS}\rangle$$

- $\mathcal{T}\Psi_{L'S'}$ = "time reversed" wave function
- Problem: evaluation of the matrix elements $A^X_{\alpha,LS} = \langle \mathcal{T}\Phi_\alpha|H - E|\Omega^X_{LS}\rangle$ and $B^{XX'}_{LS,L'S'} = \langle \mathcal{T}\Omega^X_{LS}|H - E|\Omega^{X'}_{L'S'}\rangle$ $(X = F, G)$

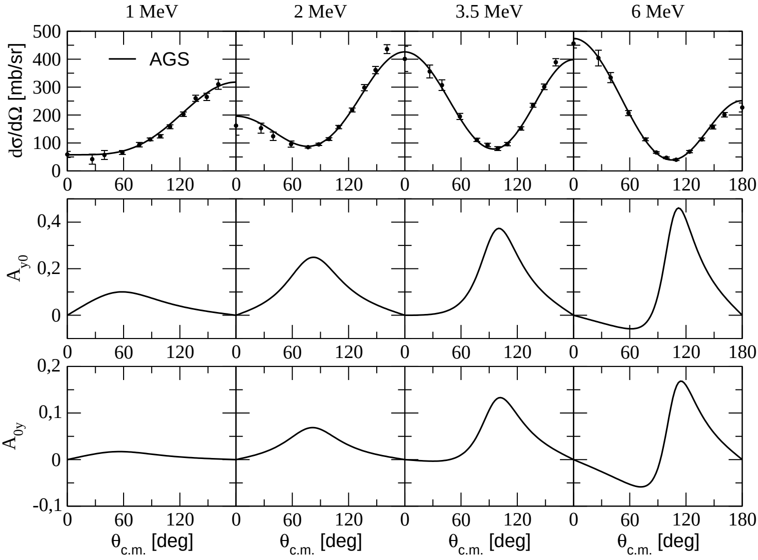
$$\begin{pmatrix} H_{1,1} - E & \cdots & H_{1,\mathcal{N}} & A^G_{1,LS} \\ \cdots & \cdots & \cdots & \cdots \\ H_{\mathcal{N},1} & \cdots & H_{\mathcal{N},\mathcal{N}} - E & A^G_{\mathcal{N},LS} \\ A^G_{1,LS} & \cdots & A^G_{\mathcal{N},LS} & B^{GG}_{LS,LS} \end{pmatrix} \begin{pmatrix} c_{LS,1} \\ \cdots \\ c_{LS,\mathcal{N}} \\ T_{LS,LS} \end{pmatrix} = \begin{pmatrix} -A^X_{LS,1} \\ \cdots \\ -A^X_{LS,\mathcal{N}} \\ 1 - B^{GF}_{LS,LS} - B^{FG}_{LS,LS} \end{pmatrix}$$

A benchmark between FY, AGS & HH results

Defined at the Critical Stability Workshop 2008

- AGS: Deltuva & Fonseca, PRL **98** 162502 (2007)
- FY: Lazauskas & Carbonell, PRC **70**, 044002 (2004)
- $n - {}^3\mathrm{H}$ & $p - {}^3\mathrm{He}$ elastic scattering $0 \leq E_{c.m.} \leq B_3 - B_2 \approx 5.5$ MeV
- NN interaction models:
  - AV18 [Wiringa, Stoks & Schiavilla (1995)]
  - I-N3LO [Entem & Machleidt (2003)]
  - V_{low-q} [Bogner, Kuo & Schwenk, (2003)] (derived from the CD-Bonn potential [Machleidt (2001)])
- preprint [arXiv:1109.3625]

$n - {}^3\mathrm{H}$ scattering (I-N3LO pot.)



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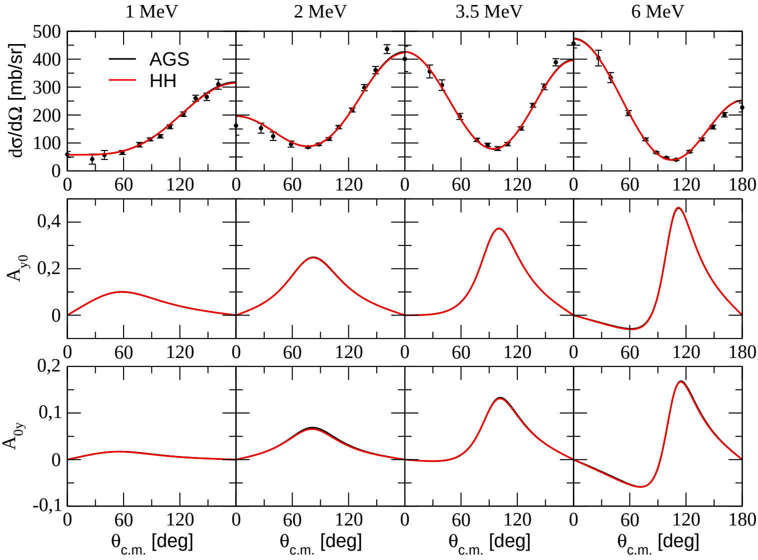

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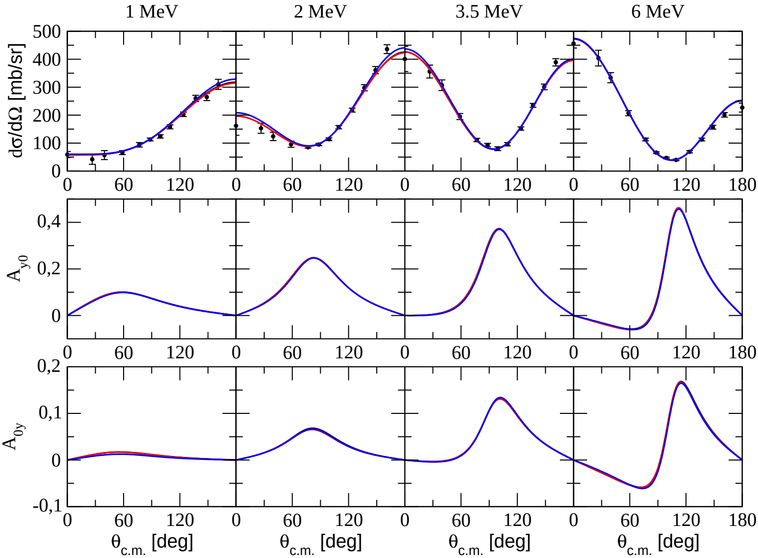

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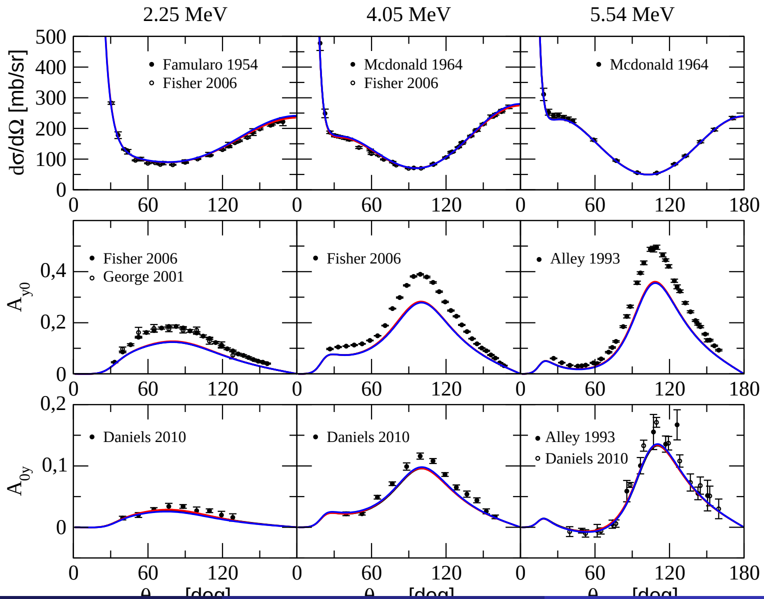
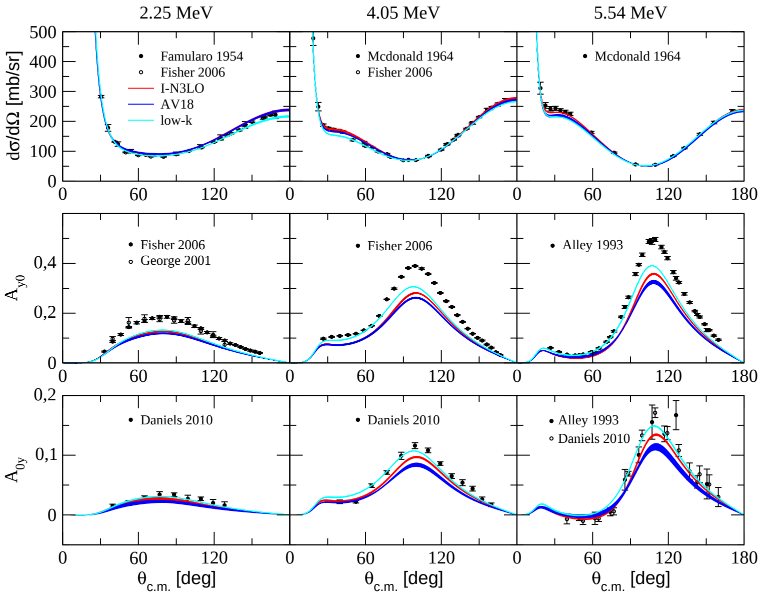
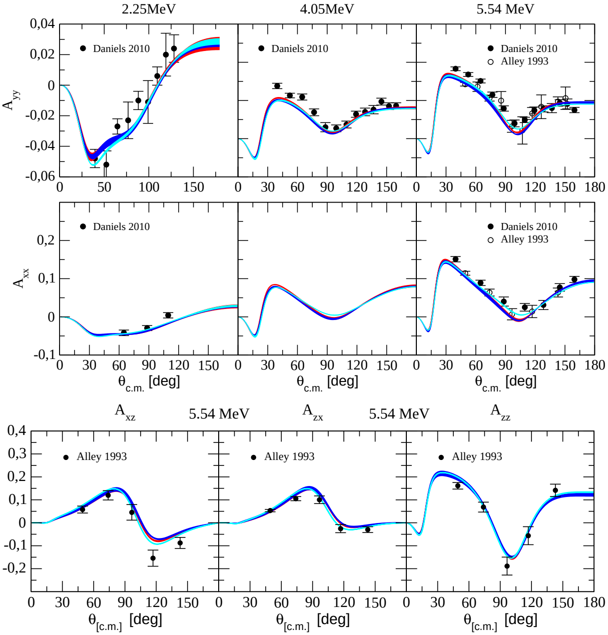
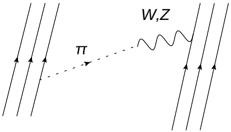


Predictions by different potentials




2.25MeV
4.05MeV
5.54 MeV
Daniels 2010
Alley 1993
A_{yy}
A_{xx}
θ c.m. [deg]
A_{xz}
5.54 MeV
A_{zx}
5.54 MeV
A_{zz}
Alley 1993
θ [c.m.] [deg]

PV Effects in Few-Nucleon Systems (1)



- PV in nuclei: extraction of the fundamental PV coupling constants
  - πNN coupling constants h^1_π
  - [Ramsey-Musolf & Page, 2006]
- $A = 2, \ldots, 4$ (and more): accurate calculation of bound/scattering states
  - Measurements of PV observables for F, Cs, ...: difficult to be analyzed theoretically
  - Few-nucleon systems: precise numerical techniques for solving the Schroedinger equation
  - PV effects $\sim 10^{-7}$: experimental challenge!

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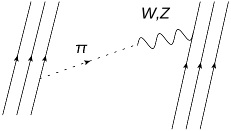

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PV Effects in Few-Nucleon Systems (2)

- Measurement of PV observables in few nucleon systems
  - Performed
    - $\vec{p}p$ [Bonn, PSI, TRIUMF, ...]
    - proton spin rotation $p-\alpha$ [Lang & Coll., (1985)]
  - Under progress/planned
    - $\vec{n}+p\rightarrow d+\gamma$ "NPDgamma" [SNS]
    - neutron spin rotation: $\vec{n}\,H$, $\vec{n}d$, $\vec{n}\,^4\mathrm{He}$ [NIST]
    - $^3\mathrm{He}(\vec{n},p)^3\mathrm{H}$ longitudinal asymmetry [SNS]
  - $\vec{n}-d$ spin rotation [Schiavilla & Coll, 2008-2010] [Song, Lazauskas & Gudkov (2010)]

Questions

- PV potential derived from an EFT (two-pion exchange contribution)
  - [Zhu, Maekawa, Holstein, Ramsey-Musolf, & van Kolck (2005)]
  - [Hyun, Ando, & Desplanques (2007)]
- Planned measurement of the $^3\mathrm{He}(\vec{n},p)^3\mathrm{H}$ longitudinal asymmetry at the SNS facility (ORNL)
  - order of magnitude of the observable?
  - sensitivity to the paramemters of the PV interaction?
- PV pion-nucleon coupling constant

$$\mathcal{L}^{PV}_{\pi NN}(x) = \frac{h^1_\pi}{\sqrt{2}} \overline{N}(x) \left(\vec{\tau} \times \vec{\pi}(x) \right)_z N(x) + \ldots$$

  - Magnitude of h^1_π elusive [Haxton (2008)]
  - First estimate from QCD $h^1_\pi \approx 1 \times 10^{-7}$ [Wasem (2011)]

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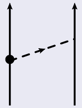
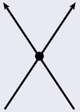
PV potential at order $\mathcal{O}(Q)$

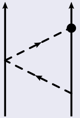
$$\boldsymbol{k} = \boldsymbol{p}_1' - \boldsymbol{p}_1 = -(\boldsymbol{p}_2' - \boldsymbol{p}_2) \qquad \boldsymbol{Q} = (\boldsymbol{p}_1' + \boldsymbol{p}_1 - \boldsymbol{p}_2' - \boldsymbol{p}_2)/2$$



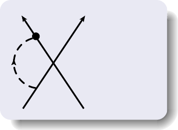
$$-\frac{g_A h_\pi^1}{2\sqrt{2} f_\pi} i(\vec{\tau}_1 \times \vec{\tau}_2)_z \frac{(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot \boldsymbol{k}}{k^2 + m_\pi^2} \qquad (\sim Q^{-1}) + \mathrm{RC} \quad (\sim Q^1)$$



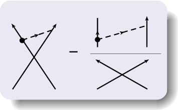
$$\frac{C_1}{4\pi f_\pi m_\pi^2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot \boldsymbol{Q} + \frac{C_2}{4\pi f_\pi m_\pi^2} i(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)\cdot \boldsymbol{k} + \ldots \qquad (\sim Q^1)$$



$$+\frac{g_A h_\pi^1}{2\sqrt{2} f_\pi} \frac{m_\pi^2}{(4\pi f_\pi)^2} i(\vec{\tau}_1 \times \vec{\tau}_2)_z (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot \boldsymbol{k} L(k) \qquad (\sim Q^1)$$



$$\sim \int d^3\boldsymbol{q} f(q^2)\boldsymbol{q}\cdot\sigma \to 0$$



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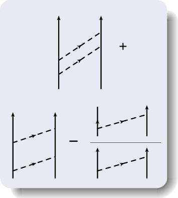


$$+\frac{g_A h_\pi^1}{2\sqrt{2}f_\pi}\frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2}\Big(4(\vec{\tau}_1+\vec{\tau}_2)_z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}L(k)$$
$$+i(\vec{\tau}_1\times\vec{\tau}_2)_z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}\left[H(k)-3L(k)\right]\Big) \qquad (\sim Q^1)$$

$$s = \sqrt{k^2+4m_\pi^2} \qquad L(k) = \frac{1}{2}\frac{s}{k}\log\left(\frac{s+k}{s-k}\right) \qquad H(k) = \frac{s^2-k^2}{s^2}L(k)$$

Summary

$$V_{PV} = \underbrace{V_{\mathrm{OPE-LO}} + V_{\mathrm{OPE-RC}} + V_{\mathrm{TPE-triangle}} + V_{\mathrm{TPE-box}}}_{\sim h^1_\pi} + V_{CT}$$

$$\begin{aligned}
V_{CT} &= \frac{C_1}{4\pi f_\pi m_\pi^2}\mathbf{Q}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) + \frac{C_2}{4\pi f_\pi m_\pi^2} i\mathbf{k}\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2) \\
&+ \frac{C_3}{4\pi f_\pi m_\pi^2}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z i\mathbf{k}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2) + \frac{C_4}{4\pi f_\pi m_\pi^2}(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)_z \mathbf{Q}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) \\
&+ \frac{C_5}{4\pi f_\pi m_\pi^2}\Big[3(\boldsymbol{\tau}_1)_z(\boldsymbol{\tau}_2)_z - \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\Big]\mathbf{Q}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) \\
V_{\mathrm{OPE-RC}} &= \left(\frac{g_A h^1_\pi}{8\sqrt{2} f_\pi M^2}\right)(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z \frac{1}{k^2+m_\pi^2}\times \\
&\Big\{-4iQ^2\mathbf{k}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2) + \mathbf{k}\cdot\boldsymbol{\sigma}_1\,(\mathbf{k}\times\mathbf{Q})\cdot\boldsymbol{\sigma}_2 + \mathbf{k}\cdot\boldsymbol{\sigma}_2\,(\mathbf{k}\times\mathbf{Q})\cdot\boldsymbol{\sigma}_1\Big\}
\end{aligned}$$

additional parameter: cutoff Λ ($= 500-700$ MeV)

Comparing with the DDH potential (ρ and ω exchanges – resonance saturation) [Desplanques, Donogue, & Holstein (1980)]:
$h^1_\pi = 4.56$, $C_1 \approx -1$, $C_2 \approx +10$ $C_3 \approx -0.3$ $C_4 \approx -1$ $C_5 \approx +1$ (Units: 10^{-7})

$p - p$ longitudinal symmetry at energy E

$$A_z^{pp}(\theta,E) = \frac{\sigma_{+1/2}(\theta,E) - \sigma_{-1/2}(\theta,E)}{\sigma_{+1/2}(\theta,E) + \sigma_{-1/2}(\theta,E)} \qquad \overline{A}_z^{pp}(E) = \frac{\int_{\theta_1\le\theta\le\theta_2} d\hat{r}\; A_z^{pp}(\theta,k)\sigma(\theta,E)}{\int_{\theta_1\le\theta\le\theta_2} d\hat{r}\; \sigma(\theta,E)} ,$$

Experiments

- Bonn [Eversheim *et al.*, 1991] $\overline{A}_z(13.6\mathrm{MeV}) = (-0.97 \pm 0.20) \times 10^{-7}$
- PSI [Kistryn *et al.*, 1987] $\overline{A}_z(45\mathrm{MeV}) = (-1.53 \pm 0.21) \times 10^{-7}$
- TRIUMF [Berdozet *al.*, 2003] $\overline{A}_z(221\mathrm{MeV}) = (+0.84 \pm 0.34) \times 10^{-7}$

Isospin matrix elements

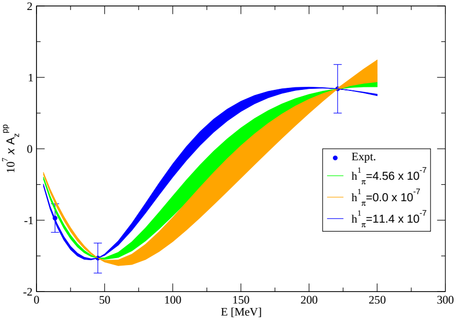
$$\langle T=1, T_z=+1|(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z|T=1, T_z=+1\rangle = 0$$
$$\langle T=1, T_z=+1|(\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2)_z|T=1, T_z=+1\rangle = 2$$
$$\langle T=1, T_z=+1|I_{ij}(\boldsymbol{\tau}_1)_i(\boldsymbol{\tau}_2)_j|T=1, T_z=+1\rangle = 2$$

$$\overline{A}_z^{pp}(E) = a_0(E)h_\pi^1 + a_1(E)C_1' + a_2(E)C_2 \qquad C_1' = C_1 + 2C_4 + 2C_5$$

Note: at low energies the observable is dominated by ${}^1S_0 \leftrightarrow {}^3P_0$, then $a_i(E) \sim \sqrt{E}$

Reasonable range for $h_\pi^1 = 0 \div 11.4 \times 10^{-7}$, "best value" $h_\pi^1 = 4.56 \times 10^{-7}$
[Desplanques, Donogue, & Holstein, 1980]

Λ [MeV]	$C_1' \times 10^7$	$C_2 \times 10^7$	$C_1' \times 10^7$	$C_2 \times 10^7$	$C_1' \times 10^7$	$C_2 \times 10^7$
	$h_\pi^1 = 4.56 \times 10^{-7}$		$h_\pi^1 = 0$		$h_\pi^1 = 11.4 \times 10^{-7}$	
500	-2.15516	9.98171	-1.51765	4.00256	-3.11142	18.9504
600	-2.69957	10.03513	-2.18203	4.42237	-3.47588	18.4543
700	-4.22214	10.66532	-4.68110	5.86730	-3.53372	17.8623



$\vec{p} - p$ longitudinal asymmetry

bands=variation with Λ

$\vec{n} - {}^3\mathrm{He} \rightarrow p - {}^3\mathrm{H}$ longitudinal asymmetry

- initial state ($n - {}^3\mathrm{He}$) $q \approx 0$: ${}^1S_0$, ${}^3S_1$
- final state ($p - {}^3\mathrm{H}$) $q = 0.165$ fm^{-1}:
  - $J = 0$: ${}^1S_0$, ${}^3P_0$
  - $J = 1$: ${}^3S_1 - {}^3D_1$, ${}^1P_1 - {}^3P_1$

Neglecting ${}^3D_1$, we have to compute the transition matrix elements $T^J_{LS,L'S'}$

PC	
${}^1S_0 \rightarrow {}^1S_0$	$T^0_{00,00}$
${}^3S_1 \rightarrow {}^3S_1$	$T^0_{01,01}$

PV	
${}^1S_0 \rightarrow {}^3P_0$	$T^1_{00,11}$
${}^3S_1 \rightarrow {}^1P_1$	$T^1_{01,10}$
${}^3S_1 \rightarrow {}^3P_1$	$T^1_{01,11}$

- PC T-matrix elements + $\Psi^{J\pi}_{LS}$: using the KVP+HH method, starting from a NN+3N interaction model (neglecting the PV potential)
- PV T-matrix elements: $T^J_{0J,1S} = \langle \mathcal{T}\Psi^{J-}_{1S'} | V_{PV} | \Psi^{J+}_{0J} \rangle$ (Monte Carlo code by R. Schiavilla)

Results for the PV EFT potential [I-N3LO/N-N2LO]

$$a_z = h_\pi^1 a_0 + C_1 a_1 + C_2 a_2 + C_3 a_3 + C_4 a_4 + C_5 a_5$$

$C_1 = C_1', C_{2,3,4} = 0$

Table: a_z (in units of 10^{-7})

	$h_\pi^1 = 4.56 \times 10^{-7}$		$h_\pi^1 = 0$		$h_\pi^1 = 11.4 \times 10^{-7}$	
	OPE/LO	FULL	OPE/LO	FULL	OPE/LO	FULL
500	-0.551	-0.544	0.000	$+0.044$	-1.377	-1.425
600	-0.554	-0.578	0.000	$+0.034$	-1.385	-1.497
700	-0.546	-0.584	0.000	$+0.009$	-1.366	-1.473

Dependence on $C_{3,4,5}$: sensitivity study

$C_{3,4,5} = \pm 1$ but keeping fix the combination $C_1' = C_1 + 2C_4 + 2C_5$

$\Delta(a_z) = 20\%$

Conclusions

$A = 4$ scattering

- Completed benchmark for $n - {}^3\mathrm{H}$ & $p - {}^3\mathrm{He}$ scattering
  - useful as a test of other techniques

PV

- Re-derivation of the EFT PV potential at $\mathcal{O}(Q)$
  - TPE in agreement with Desplanques & Coll., 2007
  - number of different LEC: 6 (h^1_π + 5 contact interactions)
- study of $\vec{p} - p$ longitudinal asymmetry (it allows to fix 2 LEC's)
- study of the $\vec{n} + {}^3\mathrm{He} \to p + {}^3\mathrm{H}$ longitudinal asymmetry

- Calculation of $A = 4$ scattering with NN+3N models
- Calculation of other PV observables
  - $\vec{n} + p \to d + \gamma$: measurement in progress ORNL
  - spin rotation in $\vec{n} - \alpha$ ($A > 4$: with Gattobigio, INLN (Nice))

Additional Material

Chiral symmetry $G = SU(2)_L \times SU(2)_R$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad q_{R/L} = \frac{(1 \pm \gamma^5)}{2} q = \begin{pmatrix} u_{R/L} \\ d_{R/L} \end{pmatrix} \qquad \begin{array}{l} q'_R = R q_R = \exp\left(-i\vec{\theta}_R \cdot \vec{\tau}/2\right) q_R \\ q'_L = L q_L = \exp\left(-i\vec{\theta}_L \cdot \vec{\tau}/2\right) q_L \end{array}$$

$\vec{\theta}_R = \vec{\theta}_L = \vec{\theta}_V$: isospin transformation $\quad \vec{\theta}_R = -\vec{\theta}_L = \vec{\theta}_A$: axial transformation

Realization of the chiral symmetry for hadrons

[Weinberg, 1968, 1990],[CCWZ, 1969],[Gasser & Leutwyler, 1984], . . .

"Compensator field" h

- $\xi = \exp(i\vec{\pi} \cdot \vec{\tau}/2f_\pi)$
- $\xi' = L\xi h^\dagger = h\xi R^\dagger$
- $h \equiv h(L, R, \pi)$

Nucleons

- $N' = hN$
- However $(\partial_\mu N)$ does not transform "covariantly"

$$u_\mu = i\left(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi\right) \qquad D_\mu = \partial_\mu + (1/2)\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right)$$

Transformations: $u'_\mu = h u_\mu h^\dagger \qquad (D_\mu N)' = h D_\mu N$

PV potential from EFT

- $\mathcal{L}^{PV}_{\rm Hadrons}$ violates G as the "standard model" Lagrangian $\mathcal{L}^{\rm weak}_q$ ($\mathcal{P}$-odd but $\mathcal{CP}$ even terms)
- Weak Lagrangian for $M_{W,Z} \to \infty$

$$\mathcal{L}_{weak} = \frac{G_F}{\sqrt{2}} \left(J^{\mu\dagger}_W J_{W\mu} + J_{W\mu} J^{\mu\dagger}_W + J^{\mu\dagger}_Z J_{Z\mu} \right)$$

- In terms of the L, R components ($s_w = \sin\theta_W$)

$$J^\mu_W \sim 2\cos\theta_c \overline{q}_L \gamma^\mu \tau_- q_L$$

$$J^\mu_Z \sim -\frac{2}{3} s^2_W \left(\overline{q}_R \gamma^\mu q_R + \overline{q}_L \gamma^\mu q_L \right) + \left(2 - 2s^2_W \right) \overline{q}_L \gamma^\mu \tau_z q_L - 2s^2_W \overline{q}_R \gamma^\mu \tau_z q_R$$

- Expression up to one four-gradient: [Kaplan & Savage, 1992]

$$\mathcal{L}^{PV,-1}_{\Delta T=1} = -\frac{h^1_\pi f_\pi}{2\sqrt{2}} \overline{N} (\xi^\dagger \tau_z \xi - \xi \tau_z \xi^\dagger) N \sim \frac{h^1_\pi}{\sqrt{2}} \overline{N} (\vec{\tau} \times \vec{\pi})_z N$$

+ 5 contact terms ($\overline{N}\Gamma_A N \overline{N}\Gamma_B N$) of order Q [Girlanda, 2008] LEC'c $\sim G_F f^2_\pi \approx 10^{-7}$