Effective Field Theory for light nuclear systems

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Critical Stability 2011 *October 9-15 2011*

nucleon-nucleon interaction(s)

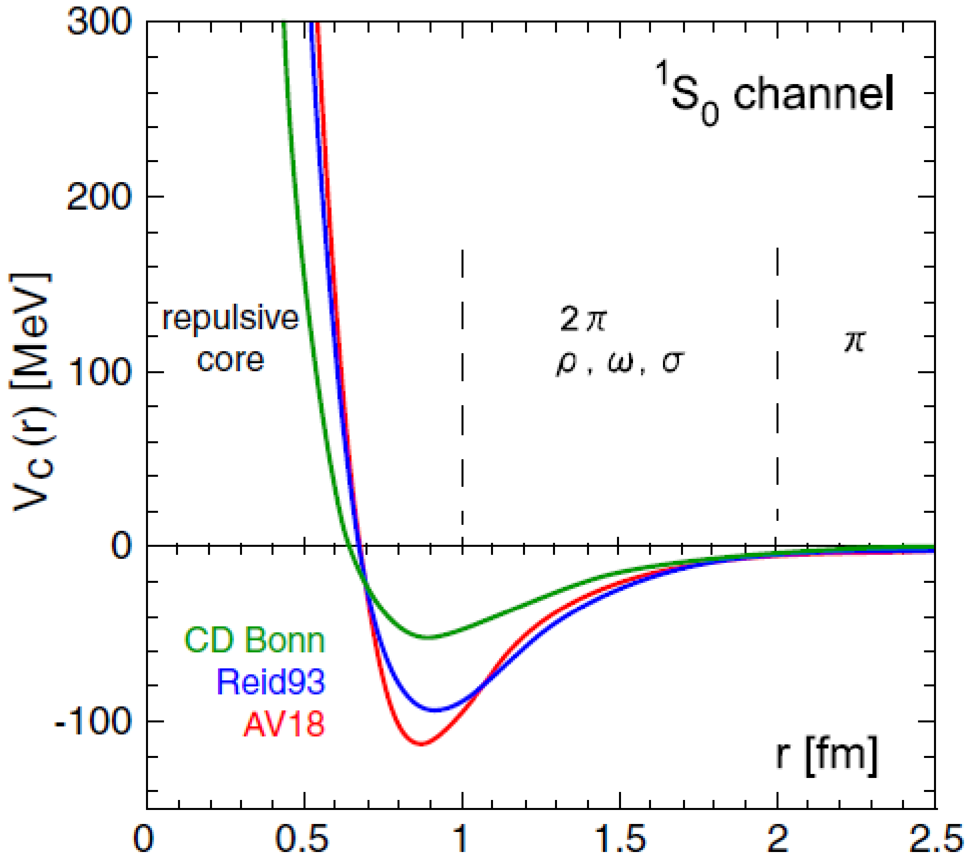


FIG. 1 (color online). Three examples of the modern *NN* potential in the $^1S_0$ (spin singlet and *s*-wave) channel: CD-Bonn [17], Reid93 [18], and AV18 [19] from the top at $r = 0.8$ fm.

(taken from N. Ishii et al, PRL 99, 022001 (2007))

there is an infinite number of two-nucleon potentials that can reproduce two-body physics:

Unitary transformation : $U^\dagger U = 1$

$$E_n = \langle\Psi|H|\Psi\rangle = \langle\Psi|U^\dagger U H U^\dagger U|\Psi\rangle$$

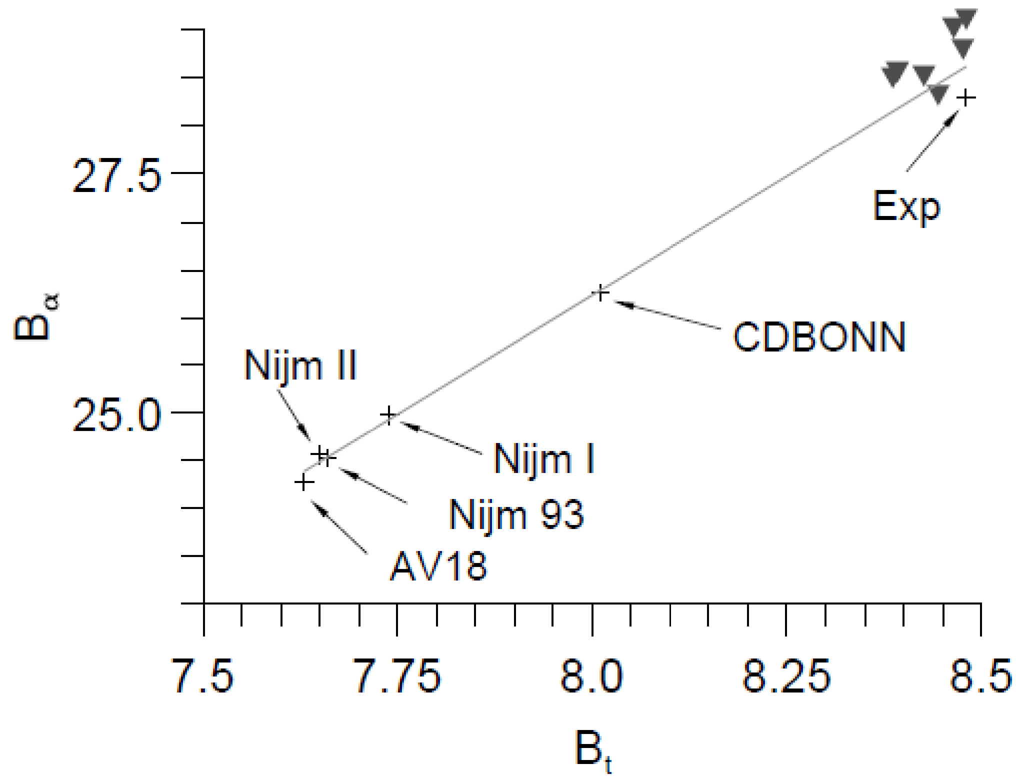
$$E_n = (\langle\Psi|U^\dagger)UHU^\dagger(U|\Psi\rangle) = \langle\tilde{\Psi}|\tilde{H}|\tilde{\Psi}\rangle$$

phenomenological potentials reproduce two-nucleon data (phase shift, deuteron binding energy) BUT ...

Nuclear many-body physics

→ nucleons are not point particles, some degrees of freedom are neglected e.g Δ -resonance, polarization effects.....

→ three (many)-body forces are present !

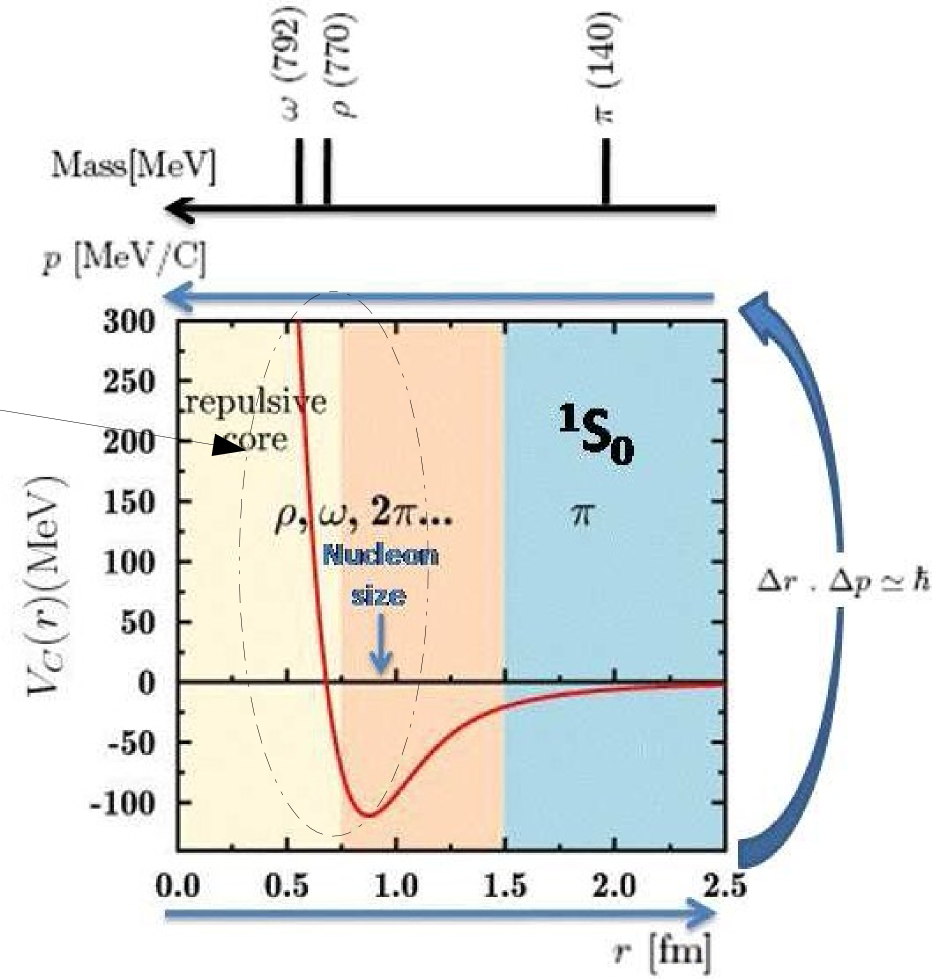


(taken from Nogga et al, Phys. Rev. Lett. 85, 944–947 (2000))

=> different many-body forces for different two-body interactions

Nuclear many-body physics

i) technical difficulty associated with the strong repulsive core at short distance (high energy)



ii) unitary transformations could be used to soften the potential at short distance but then **induced many-body forces** appear.

Effective Field Theory (EFT)

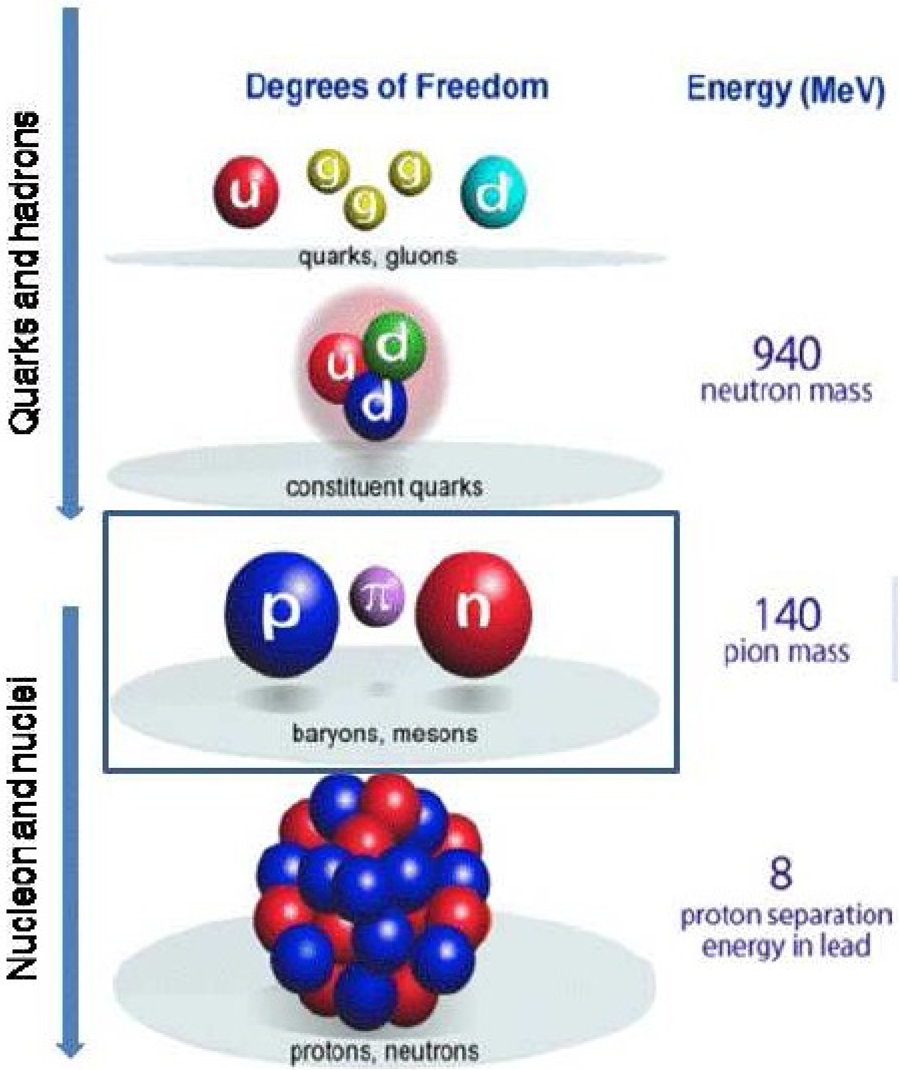
Construction of interactions in the ''philosophy'' of EFT:

-> improvable order by order.

-> many-body and two-body interactions in a same framework.

-> soft interaction.

Construction of an Effective Field Theory



i) Identify the relevant degrees of freedom :

-> details of physics at short distance are irrelevant for low energy physics, high-energy degrees of freedom are integrated out.

ii) Construct the most general potential/Lagrangian consistent with the symmetries of the system

iii) Design an organizational principle (power counting) that can distinguish between more or less important contributions.

nucleon-nucleon system at low energy

$$a(^1S_0) \sim -20 \text{ fm}, \;\; a(^3S_1) \sim 5 \text{ fm}$$

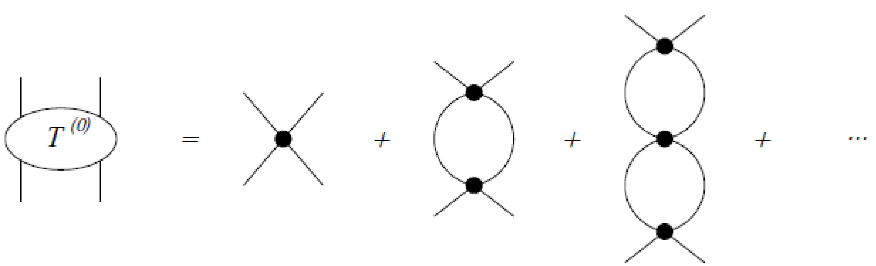
$$a(^1S_0), a(^3S_1) \gg \;\; 1/m_\pi \sim 1.4 \text{fm}$$

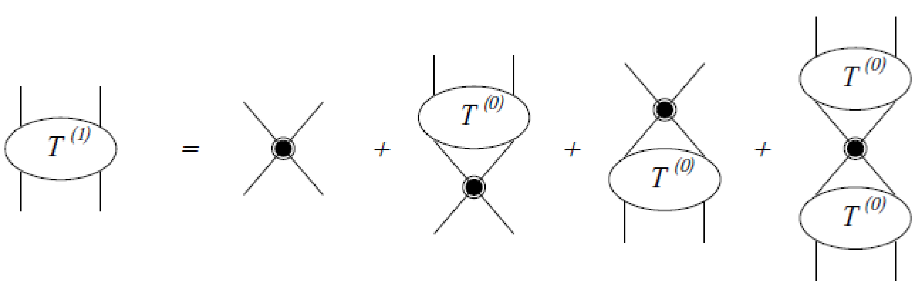
pionless EFT potential :

$$V(\vec{p}\,', \vec{p}, \Lambda) = \sum_{i,j} c_{i,j}(\Lambda)(\vec{p}\,')^i(\vec{p})^j$$

Low-energy constants fixed using the low-energy physics contained in the effective range expansion :

$$k \cot \delta = -\frac{1}{a_2} + \frac{1}{2} r_2 k^2 + \ldots, \qquad V(\vec{p}\,', \vec{p}, \Lambda) = c_{0,0}(\Lambda) + c_{2,2}(\Lambda)\left(\vec{p}\,'^2 + \vec{p}^{\,2}\right) + \ldots$$





Outline

i) No Core Shell Model defined as an EFT

-> trapping of the system to improve on the potential

-> few results for fermions systems

-> three-nucleon systems

ii) Halo-EFT with a (continuum) Shell Model approach

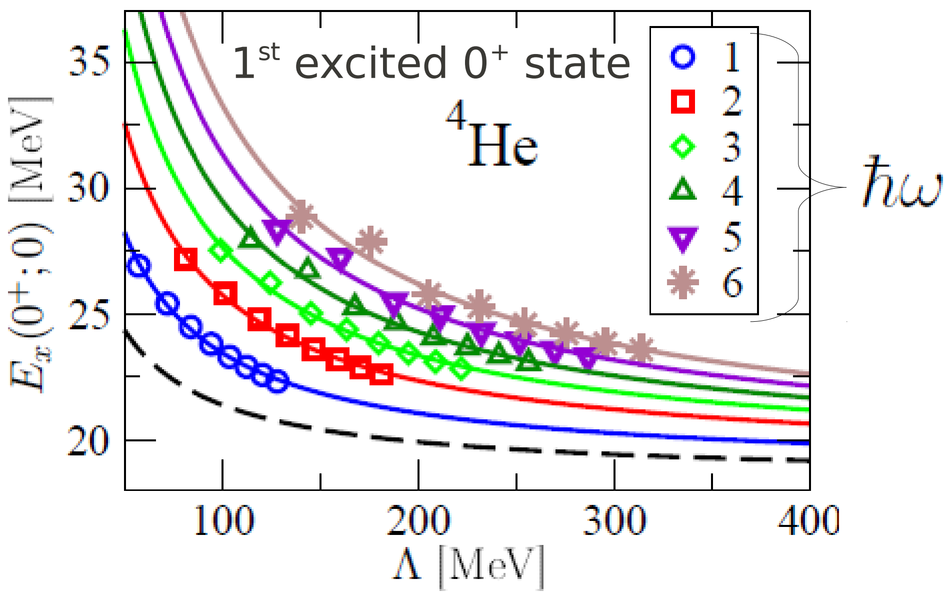
-> few words about the Gamow Shell Model

-> EFT description of the ^{6}He as a 3-body system

No Core Shell Model as an Effective Field Theory

- No Core Shell Model to solve the many-body Schrödinger equation

- low energy constants fixed within the model space characterized by a cutoff $\Lambda = N_{max}\hbar\omega$



Stetcu et al, PLB653, 2007

-> calculation at **Leading order** :

two N-N contact interactions in the $^3S_1$, $^1S_0$ channel and a three-body contact interaction in the 3-nucleon $S_{1/2}$ channel

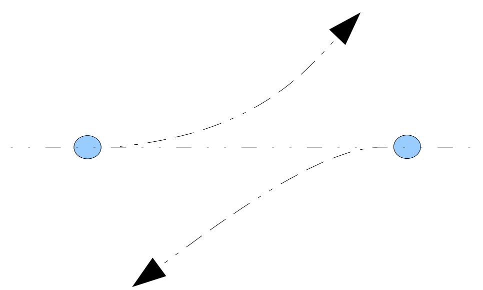
-> low energy constants fitted to the binding energy of the deuteron, triton and ^{4}He.

Question : How to construct an EFT within a bound many-body model space beyond **Leading-Order** ?

Question : How to go beyond Leading-Order ?

We need more data to constraint the two-body coupling constants, but there is only one two-nucleon bound system in nature (deuteron)....

What about the information from scattering physics ?



Effective range expansion (ERE) $$k\,\cot\delta(k) = -\frac{1}{a_2} + \frac{1}{2} r_2 k^2 + \ldots,$$

How to connect scattering physics (given by the ERE) with the bound state
physics of nuclei ?

Answer : by trapping nuclei in an external potential

Spectrum of a two-particle system within a harmonic oscillator trap

$$\left(\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt\right) \qquad \left(b = \sqrt{\frac{\hbar}{\mu\omega}}\right)$$

$$\boxed{\frac{\Gamma\left(\frac{3}{4} - \frac{E}{2\hbar\omega}\right)}{\Gamma\left(\frac{1}{4} - \frac{E}{2\hbar\omega}\right)} = -\frac{bk}{2}\cot\delta}$$

bound state in the trap

$$E = \frac{\hbar^2 k^2}{2\mu}$$

phase shift (scattering physics)

$$k\,\cot\delta(k) = -\frac{1}{a_2} + \frac{1}{2} r_2 k^2 + \ldots,$$

Construction of an Effective Field Theory for two-fermion **in a trap**

Spectrum for two trapped fermions interacting in the S-wave (l=0) at unitarity :

''data'' to constraint the EFT

$$\frac{\Gamma\left(\frac{3}{4}-\frac{E}{2\hbar\omega}\right)}{\Gamma\left(\frac{1}{4}-\frac{E}{2\hbar\omega}\right)}=0$$

$$V(\vec{p}\,',\vec{p})=\sum_{i,j}c_{i,j}(\vec{p}\,')^i(\vec{p})^j$$

At LO, the power counting is such that : $V(\vec{p}\,',\vec{p})=c_{0,0}$

Diagonalization of the trap+potential within a finite H.O basis:

$$\left(\frac{p^2}{2\mu}+\frac{1}{2}\mu r^2\omega^2+C_0\delta(\vec{r})\right)\Psi(\vec{r})=E\Psi(\vec{r})$$

$$\Psi(\vec{r})=\sum_{n=0}^{n_{max}}d_n\phi_n(\vec{r})$$

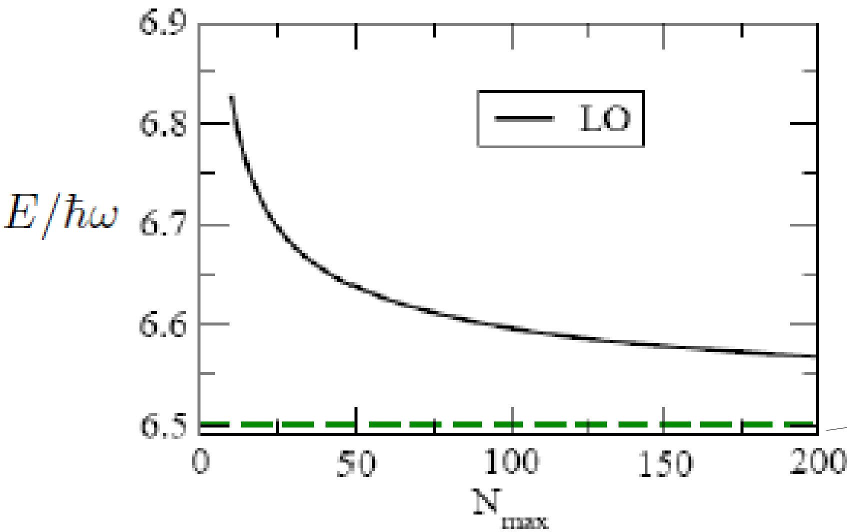
so far C_0 has not been fixed....

For any energy E solution of the Schrodinger equation :

$$\frac{1}{C_0(n_{max})} = -\sum_{n=0}^{n_{max}} \frac{(\phi_n(0))^2}{2n+3/2-E/\hbar\omega}$$

we fit C_0 such that the ground state corresponds to ground state given by the "data"

Energy of the 3rd excited state given by the EFT potential (at unitarity)



$(N_{max} = 2n_{max})$

solution of $\frac{\Gamma\left(\frac{3}{4}-\frac{E}{2\hbar\omega}\right)}{\Gamma\left(\frac{1}{4}-\frac{E}{2\hbar\omega}\right)} = 0$

Beyond LO

$$\frac{\Gamma\left(\frac{3}{4}-\frac{E}{2\hbar\omega}\right)}{\Gamma\left(\frac{1}{4}-\frac{E}{2\hbar\omega}\right)}=0 \longrightarrow$$

excited states to fix the coupling constants appearing at higher order

$$V(\vec{p}\,',\vec{p}) = \sum_{i,j} c_{i,j} (\vec{p}\,')^i (\vec{p})^j$$

-> Next-to-Leading-Order (NLO): first order perturbation theory (as dictated by the power counting in the absence of a trap)

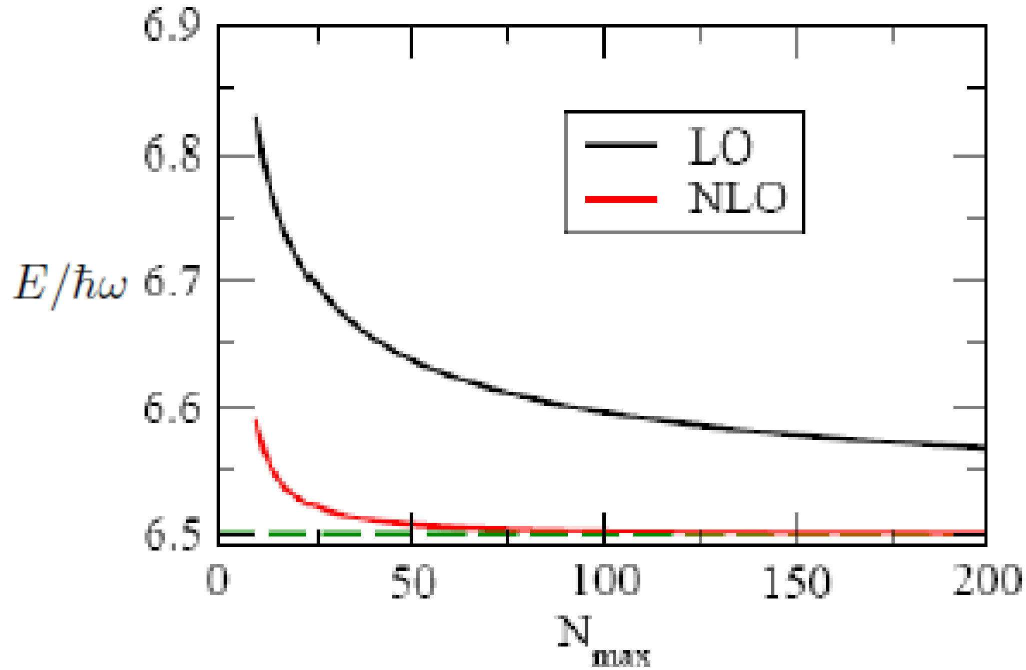
$$V_{NLO}(\vec{p}\,',\vec{p}) = c_2^{(1)}(\vec{p}\,'^2 + p^2) + c_0^{(1)}$$

$$\Delta E_n = \langle \Psi_n | V_{NLO} | \Psi_n \rangle$$

$$\Delta E_0 = 0$$

$$\Delta E_1 = E_1^{data} - E_1^{LO}$$

g.s and 1st excited state fitted to data

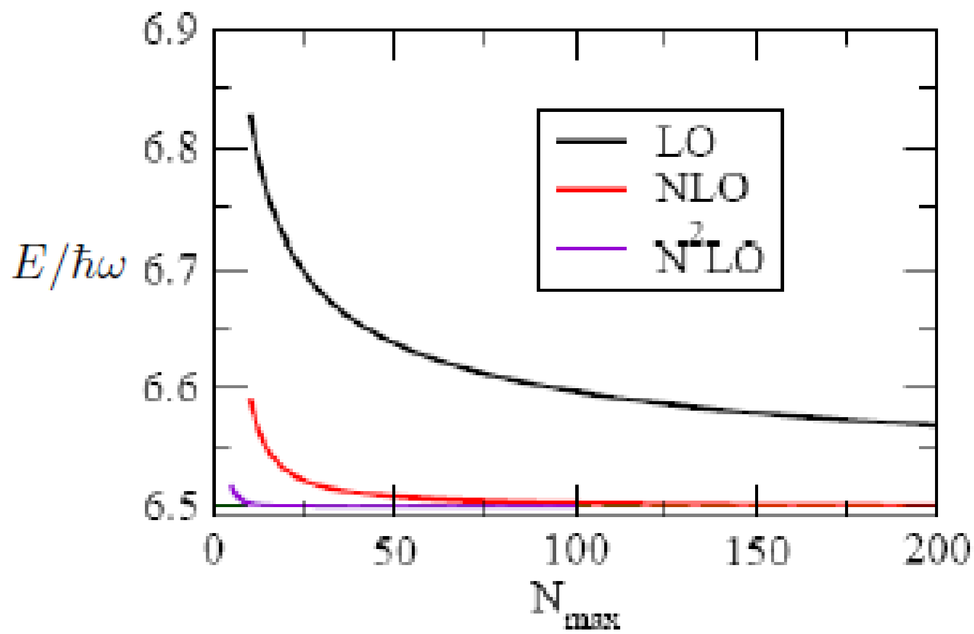


$(N_{max} = 2n_{max})$

Beyond NLO

-> NNLO : 2nd order perturbation theory

$$\Delta E_n = \sum_{i\neq n} \frac{|\langle \Psi_n | c_2^{(1)}(p^2 + p'^2) + c_0^{(1)} | \Psi_i \rangle|^2}{E_i - E_n}$$
$$+ \langle \Psi_n | c_2^{(2)}(p^2 + p'^2) + c_0^{(2)} | \Psi_n \rangle$$
$$+ \langle \Psi_n | c_4^{(2)}(p^2 + p'^2)^2 | \Psi_n \rangle$$



$(N_{max} = 2n_{max})$

i) potential improvable order by order
ii) faster convergence to the data as more corrections are included

Calculations for finite scattering length and finite range show same qualitative behaviour

I. Stetcu, J. R, B.R. Barrett, and U. van Kolck., Ann. Phys. 325 (2010) 1644.

More than 2 fermions in a H.O trap

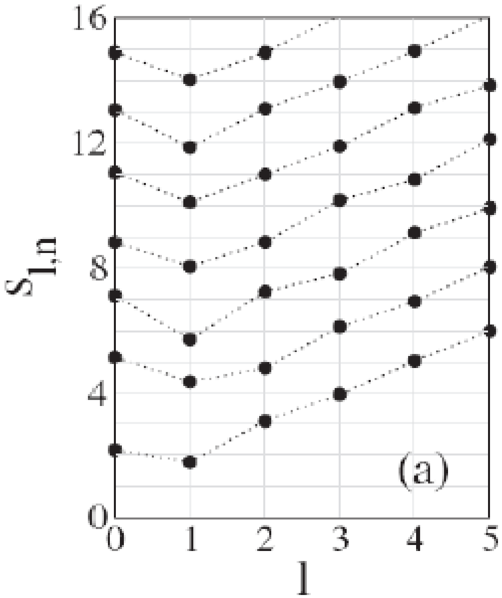
$$\boxed{H = \sum_{i<j} \frac{(\vec{p_i} - \vec{p_j})^2}{2mA} + \frac{1}{2}\frac{m\omega^2}{A}\sum_{i<j}(\vec{r_i} - \vec{r_j})^2 + \sum_{i<j} V_{ij} + ...}$$

-> two-body interaction V_{ij} constructed with EFT with the 2 fermion trapped system
-> resolution of the many-body problem with the No Core Shell Model formalism.

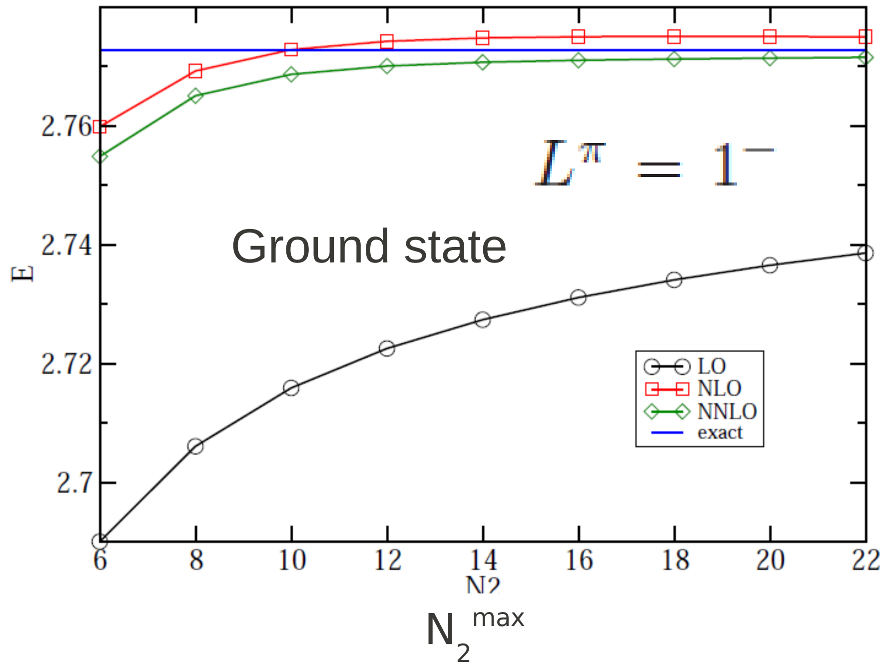
→ The three-fermion system at unitarity has been solved exactly

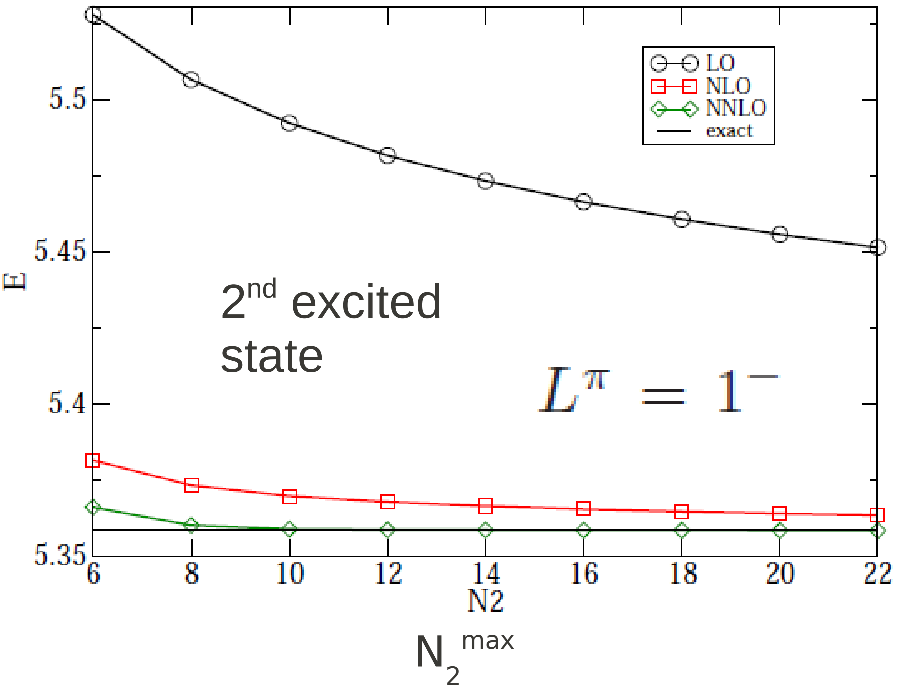
$$E = E_{c.m.} + (s_{l,n} + 1 + 2q)\omega$$

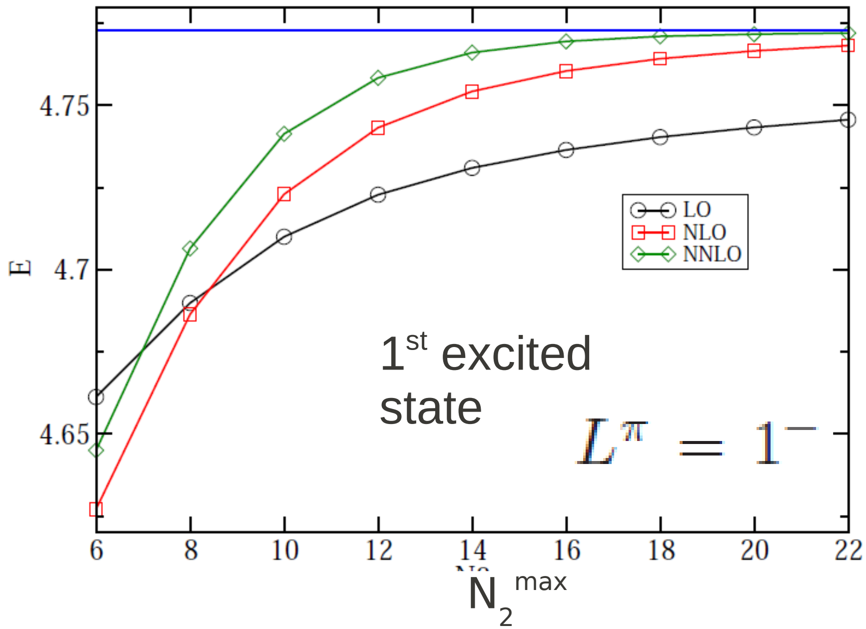
F. Werner and Y. Castin, Phys. Rev. Lett. **97**, 150401 (2006)



Three-fermion system at Unitarity with EFT potential





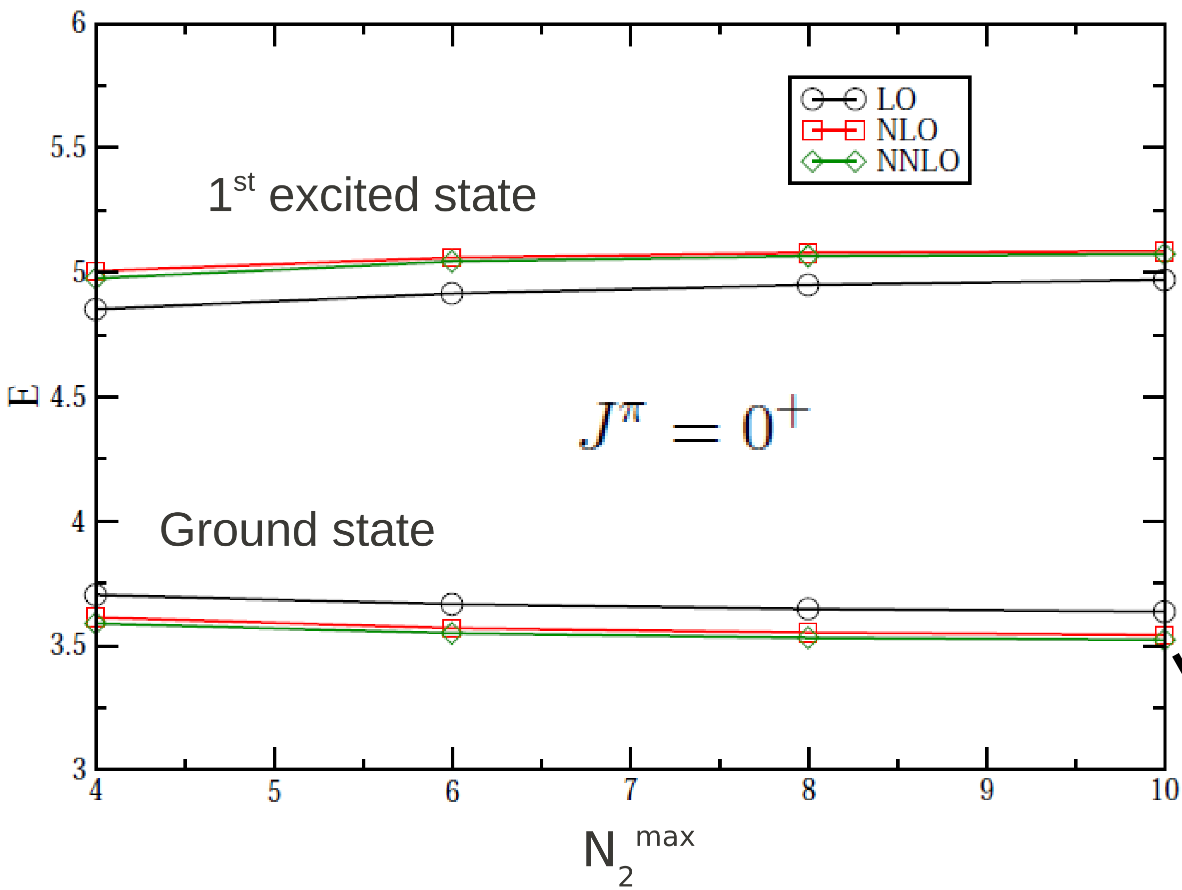


→ convergence is faster as more corrections are considered.

→ same convergence pattern observed for finite a_2 and r_2.

J. R, I. Stetcu, B.R. Barrett , M.C. Birse and U. van Kolck., PRA 82 (2010) 032711.

Four-fermion system at Unitarity



Comparison with other approaches:

-> 3.545 ∓0.003
Alhassid et al, PRL 100, 230401 (2008)

-> 3.6 ∓0.1
Chang et al, PRA 76, 021603 (2007)

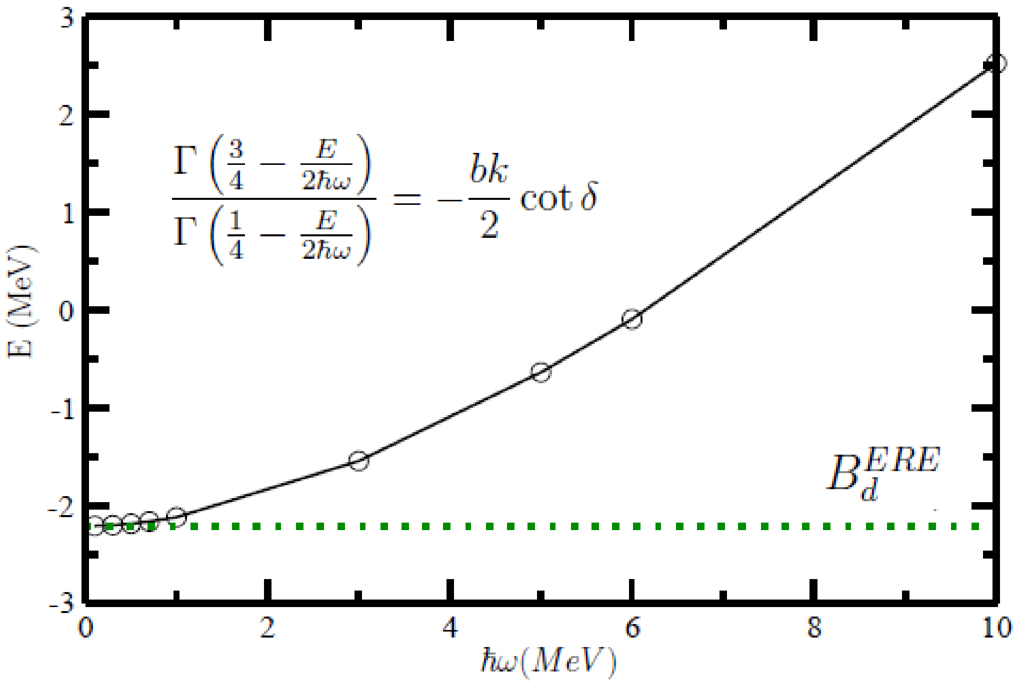
NNLO, N_2^{max}=10

E= 3.52

Back to nuclei....

How far can we go in trapping the system to describe intrinsically untrapped physics *i.e.* free nuclei ?

Energy of a "trapped deuteron"



Binding energy of a free deuteron

$$k\ \cot\delta = -\frac{1}{a_2} + \frac{1}{2}r_2k^2$$

$$ik + k\cot\delta = 0$$

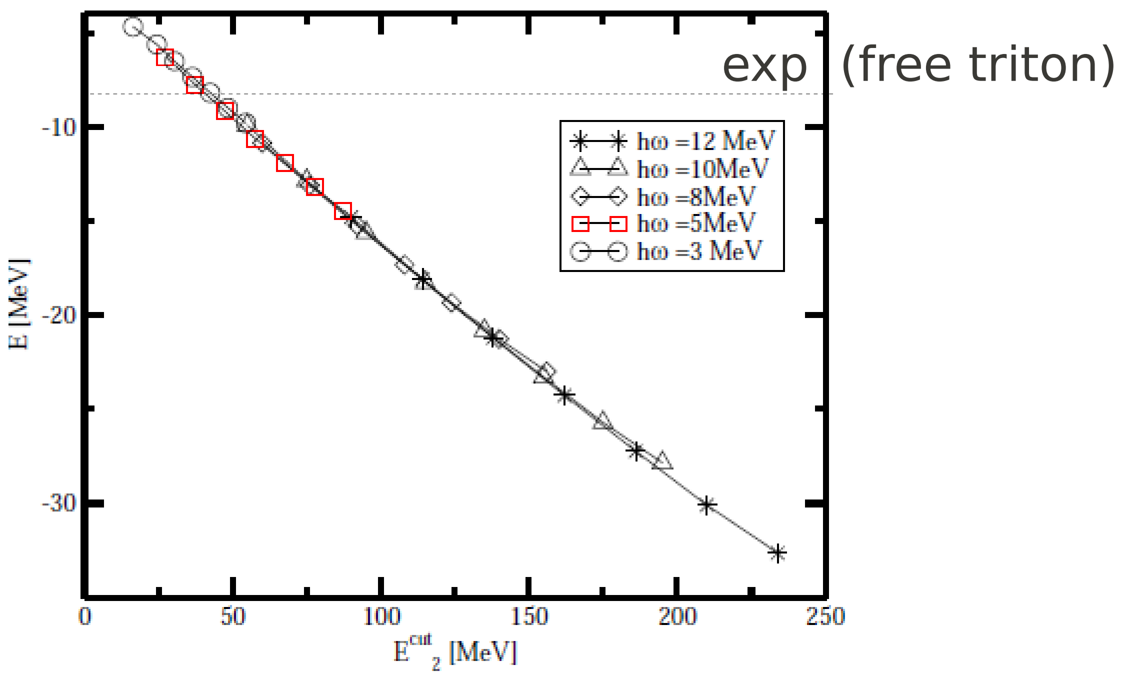
$$a_t = 5,425fm \quad r_t = 1.75fm$$

B_d^{ERE} ~-2,221 MeV

=> ω should be as small as possible

but not too small since EFT cutoff $\Lambda = N^{max}\hbar\omega$

Binding energy of a trapped triton at Leading Order

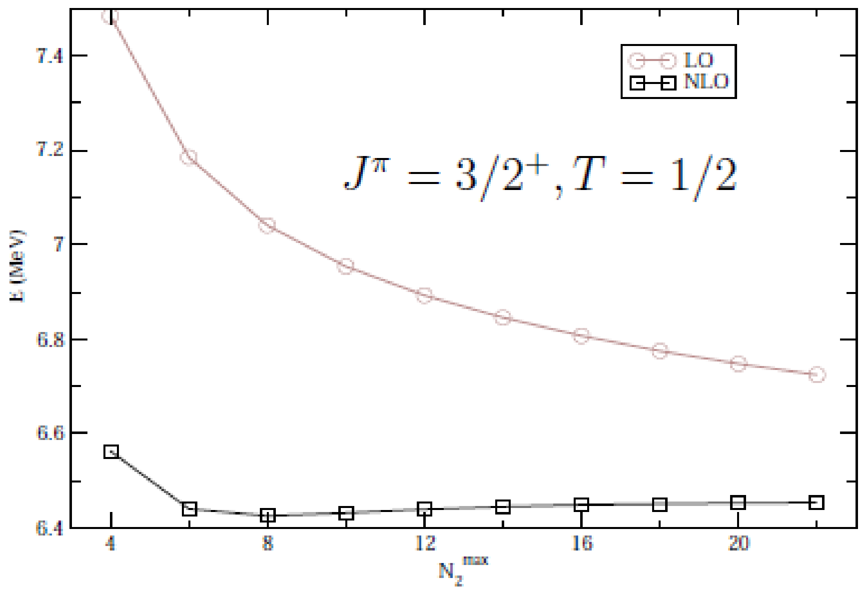


-> the 3-nucleon system collapses as the two-(three)body cutoff is increased (Thomas effect)

-> need for a three-body force at LO (as in the continuum)

3 nucleons at LO/NLO in a "weak" trap

$$\hbar\omega = 3 \text{ MeV}$$



→ convergence of energy as the two-body cutoff N_2^{max} increases

→ NLO converges faster than LO

→ no 3-body force at these orders (as in the untrapped case) due to Pauli principle

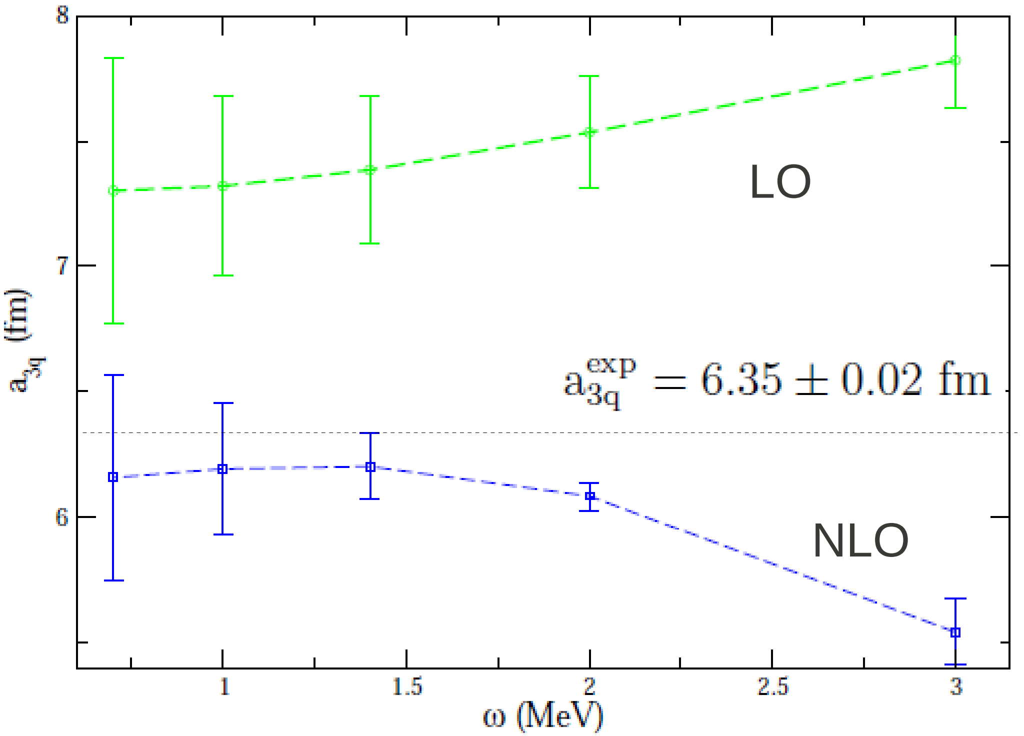
But there is no bound three-nucleon system in this channel !!! What can we learn from that ?

For a weak enough trap, the lowest states coupled to $J^\pi = 3/2^+, T = 1/2$ correspond to n-d scattering

$$\boxed{\frac{\Gamma\left(\frac{3}{4}-\frac{E_3-E_d}{2\hbar\omega}\right)}{\Gamma\left(\frac{1}{4}-\frac{E_3-E_d}{2\hbar\omega}\right)} = \frac{b'}{2\,\mathrm{a_{3q}}} - \frac{r_{n-d}b'k^2}{4} + \ldots.}$$

=> extraction of scattering physics from bound state spectrum

Scattering length $\mathrm{a_{3q}}$ of the n-d (L=0,S=3/2) channel



Agreement with previous EFT results in the continuum at NLO

$$\mathrm{a_{3q}^{EFT\ cont}} = 6.33\ \mathrm{fm}$$

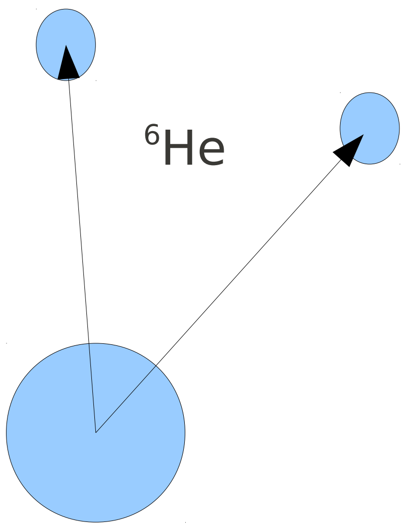
Bedaque, Van Kolck, Phys.Lett. B428 (1998)

J. R, I. Stetcu, B.R. Barrett ,and U. van Kolck., to be submitted

Halo EFT

$B_{valence} << B_{core}$, E_{ex}(core)
valence nucleon(s) in the halo far away from the core

separation of scale



$E(^5He) - E(\alpha) \sim (0.9, -0.3)$ MeV

$E(^6He) - E(\alpha) \sim -1$ MeV

$B(\alpha) \sim 28$ MeV

Partial wave $l_\pm$	$a_{l\pm}$ [fm^{1+2l}]	$r_{l\pm}$ [fm^{1-2l}]	$\mathcal{P}_{l\pm}$ [fm^{3-2l}]
0+	2.4641(37)	1.385(41)	–
1−	−13.821(68)	−0.419(16)	–
1+	−62.951(3)	−0.8819(11)	−3.002(62)

scattering volume and range at LO

- n-α potential at LO for $p_{3/2}$

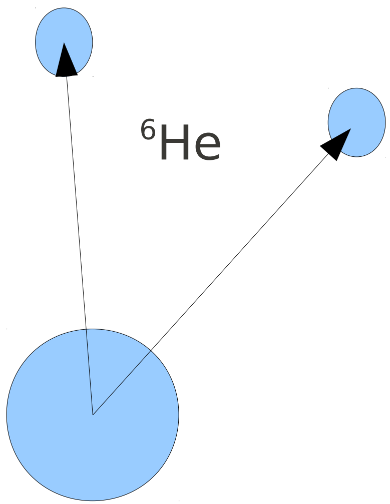
$$V_{n-\alpha}(\vec{k}',\vec{k}) = \frac{\vec{k}'\cdot\vec{k}}{A+BE} e^{-(k'^2+k^2)/\Lambda^2_{n-\alpha}}$$

energy dependent potential

low-energy constants A,B fixed with $k^{2l+1}\cot\delta = -\frac{1}{a_l} + \frac{r_l}{2}k^2$

(Bertulani et al 2002)

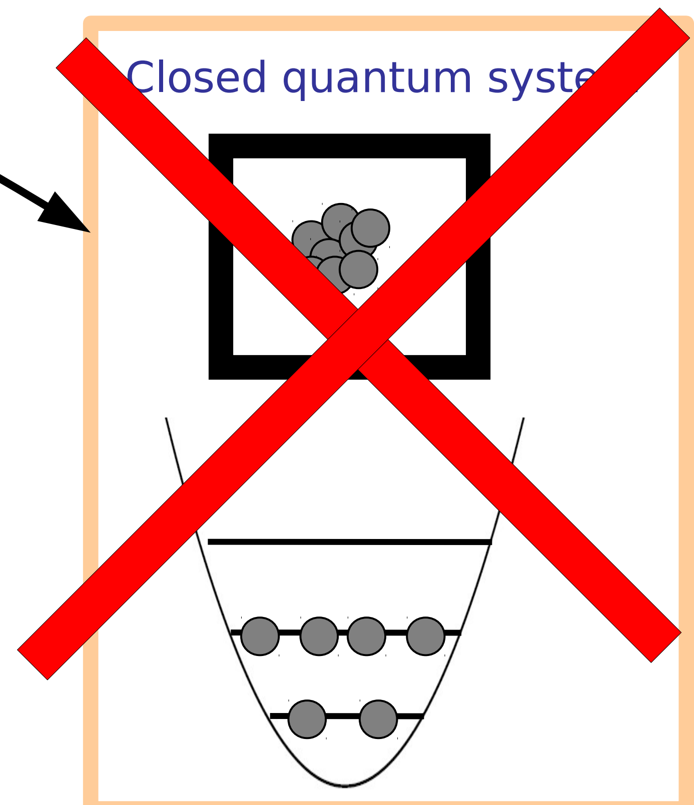
^{6}He as a three-body system within a shell model formalism



halo system, strong coupling with the continuum.....

open quantum system

the standard shell model (Harmonic Oscillator basis) is not well adapted for exotic systems.

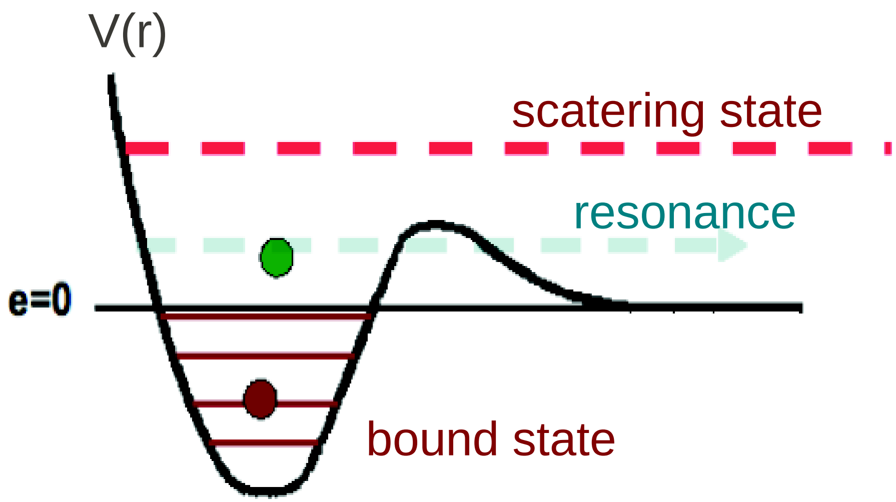


Gamow Shell Model

N. Michel et al., PRL. 89 (2002)
G. Hagen et al., PRC. 71 (2005)
J.R et al., PRL. 97 (2006)
N. Michel et al., JPG. 36 (2009)
G.Hagen et al., PRL. 104 (2010)

- shell model for nuclei far from stability (*open quantum systems)*

$$u''(r) = \left[\frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}V(r) - k^2\right] u(r)$$

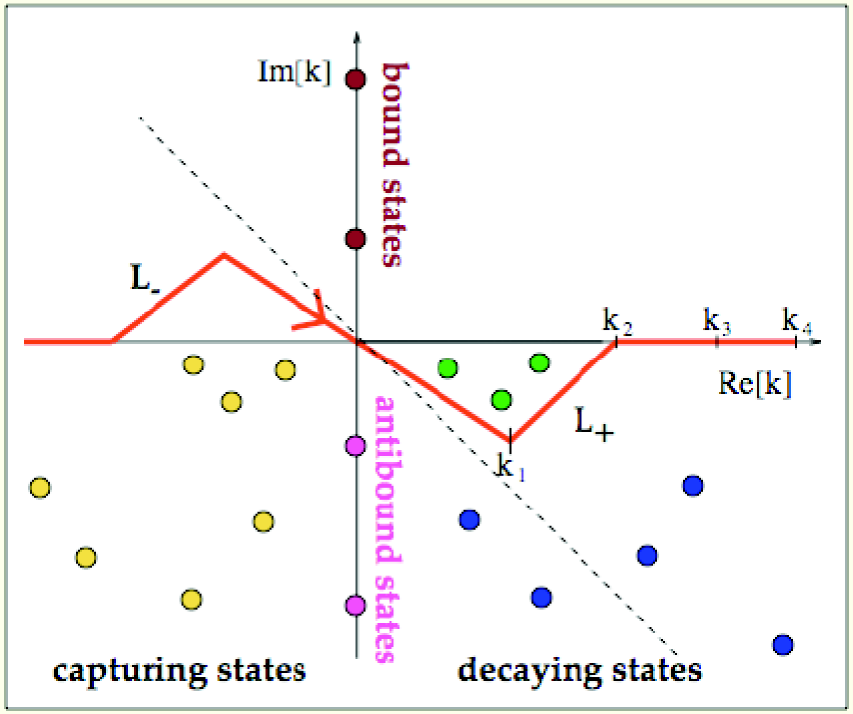


$$u(r) \sim C_+ \; H^+_{l,\eta}(kr) \;\; , \; r \to +\infty \;\; \text{(bound,resonant)}$$

$$u(r) \sim C_+ \; H^+_{l,\eta}(kr) + C_- \; H^-_{l,\eta}(kr) \;\; , \; r \to +\infty \;\; \text{(scattering)}$$

Gamow states and completeness relations

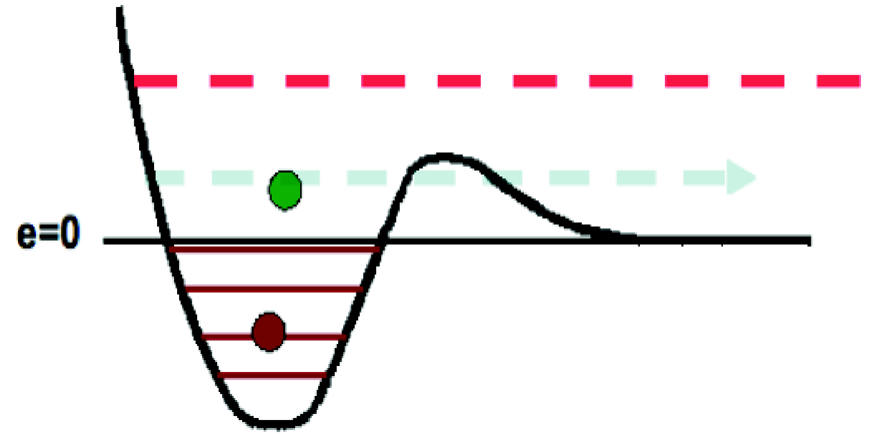
T. Berggren, Nucl. Phys. A109, 265 (1968); Nucl. Phys. A389, 261 (1982)
T. Lind, Phys. Rev. C47, 1903 (1993)



$$\sum_{n=b,r} |u_n\rangle\langle\tilde{u}_n| + \frac{1}{\pi}\int_{L_+} |u(k)\rangle\langle u(k^*)| \; dk = 1$$

particular case: Newton completeness relation

$$\sum_{n=b} |u_n\rangle\langle\tilde{u}_n| + \frac{1}{\pi}\int_{R} |u(k)\rangle\langle u(k^*)| \; dk = 1$$



Many-body basis states are given by Slater determinants built from a discretized Berggren single-particle basis

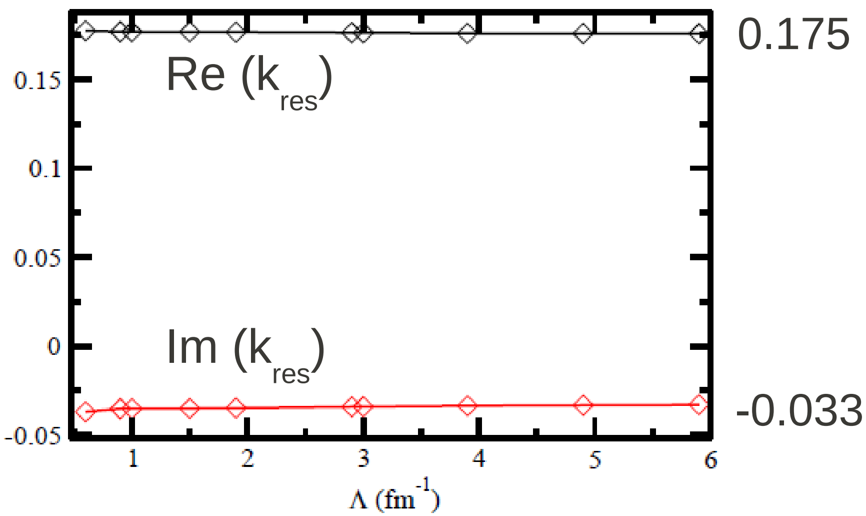
$$\sum_i |SD_i\rangle\langle\widetilde{SD_i}| \simeq 1$$

N-body spectrum

=

- N-body bound state(s)
- N-body resonance(s)
- scattering states

$n-\alpha$ $p_{3/2}$ resonance at LO



$$V_{n-\alpha}(\vec{k}', \vec{k}) = \frac{\vec{k}' \cdot \vec{k}}{A + BE} e^{-(k'^2+k^2)/\Lambda^2_{n-\alpha}}$$

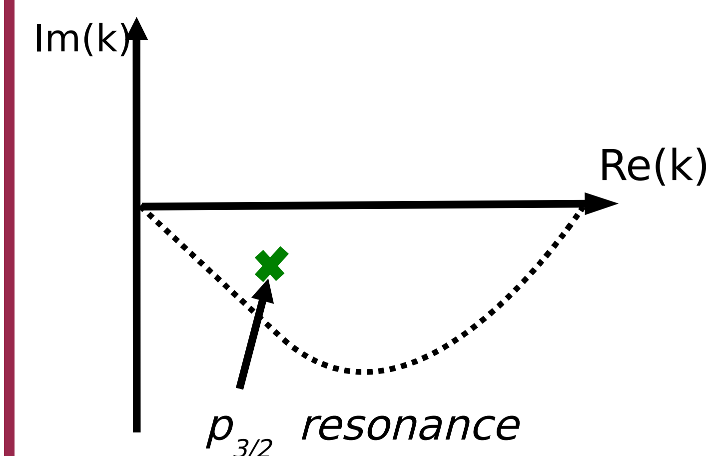
one-body basis Berggren completeness relation

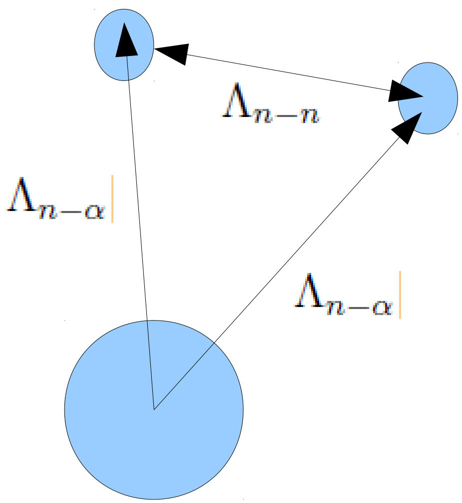
$$|\Phi_r\rangle\langle\tilde{\Phi}_r| + \sum_n |\Phi_n\rangle\langle\tilde{\Phi}_n| = 1$$

bi-orthogonal complement

bi-orthogonal complement

$$\langle\tilde{\Phi}_i|\Phi_j\rangle = \delta_{i,j}$$





^{6}He as a three-body system

$$\boxed{H = \sum_{i=1}^{2} \frac{p_i^2}{2\mu} + V_{n_1-\alpha} + V_{n_2-\alpha} + V_{n-n}}$$

Potentials at Leading Order :

$$\boxed{V_{n-\alpha}(\vec{k'},\vec{k}) = \frac{\vec{k'}\cdot\vec{k}}{A+BE} e^{-(k'^2+k^2)/\Lambda^2_{n-\alpha}}}$$

$$V_{n-n}(\vec{k'},\vec{k}) = C_0 e^{-(k'^2+k^2)/\Lambda^2_{n-n}}$$

one-body basis generated by the eigenstates of the $n-\alpha$ (energy-dependent) potential

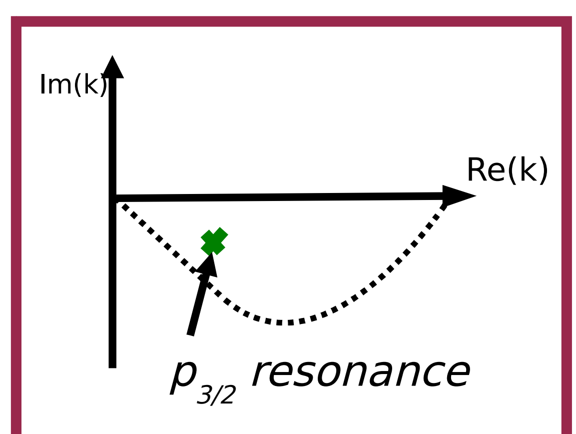
$$\boxed{|\Phi_r\rangle\langle\tilde{\Phi}_r| + \sum_n |\Phi_n\rangle\langle\tilde{\Phi}_n| = 1}$$

A,B,C0 fixed with the effective range expansion parameters at LO

$$a^{n-\alpha}_{p_{3/2}} = -62.951 \text{ fm}^3$$

$$r^{n-\alpha}_{p_{3/2}} = -0.882 \text{ fm}^{-1}$$

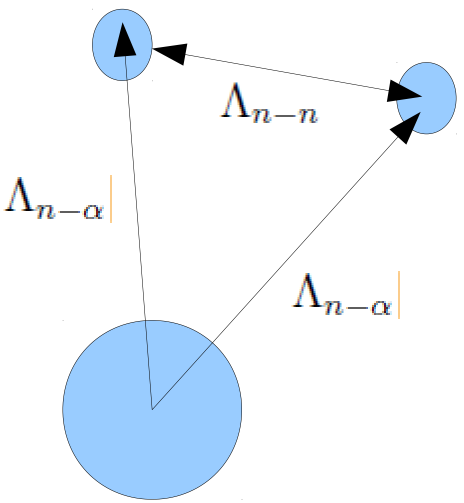
$$a^{n-n}_{^1S_0} = -18.7 \text{ fm}$$



shells included at LO :

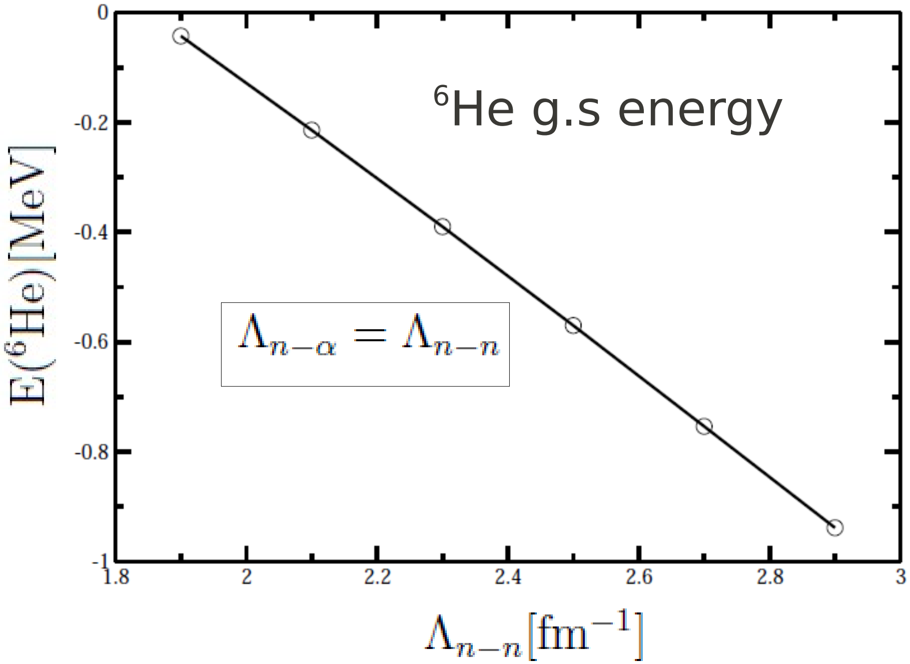
- $p_{3/2}$ resonance
- $p_{3/2}$ complex scattering state

The 2-body subsystems have all been properly renormalized at Leading Order :



- $n - \alpha$ in the $p_{3/2}$ wave
- n-n in $^1S_0$ channel

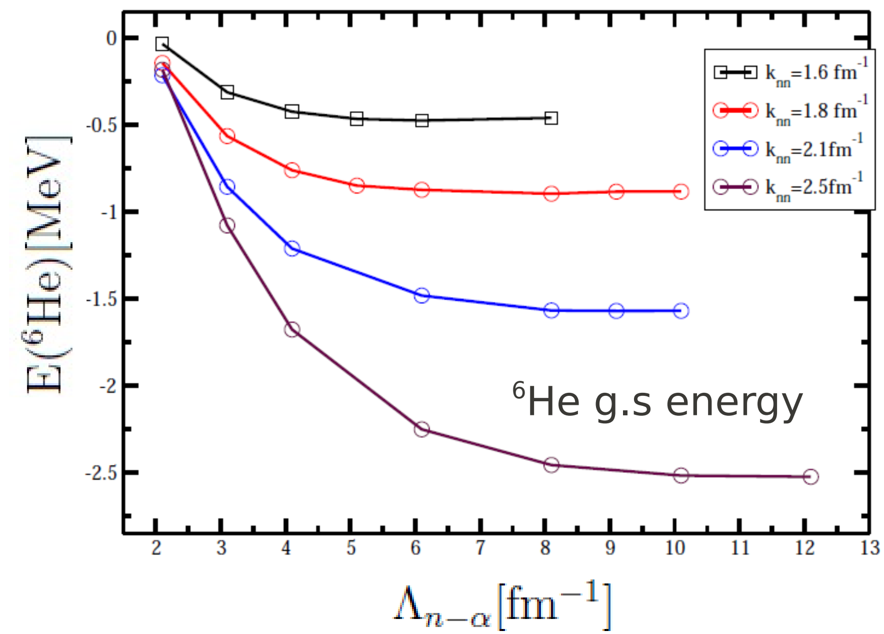
Is the 3-body system ^{6}He properly renormalized at LO with only 2-body forces ?

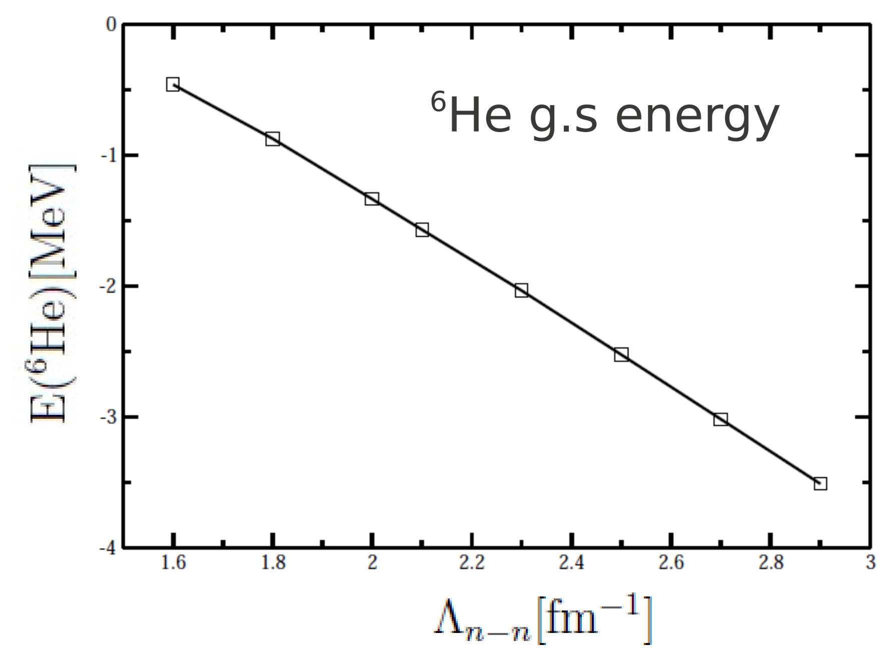


collapse of the system with only two-body forces

3-body force needed at LO for renormalization of ^{6}He

for Λ_{n-n} fixed, $\Lambda_{n-\alpha}$ is increased until convergence.





No Core Shell Model defined as an Effective Field Theory

→ application of Effective Field Theory to trapped systems

→ extraction of the scattering length a_{3q} of the deuteron-neutron scattering

→ perspectives : four,five nucleons systems

Halo-EFT with a (continuum) Shell Model approach

→ 3-body force needed at LO in ^{6}He

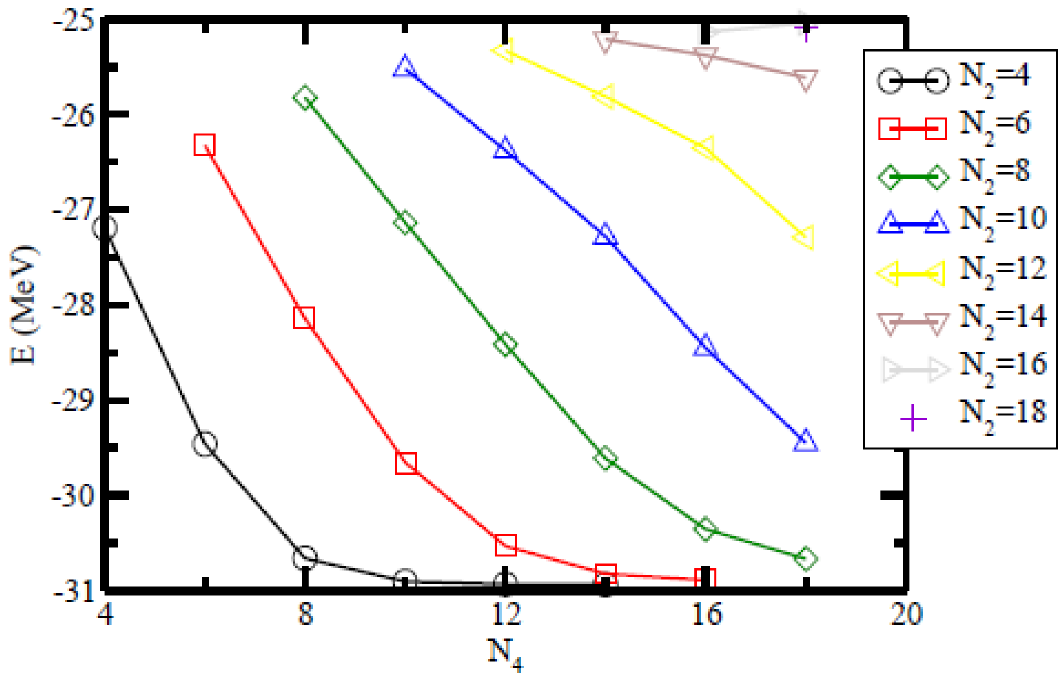
→ perspectives : 3-body weakly bound, resonant systems

^{4}He g.s in a trap

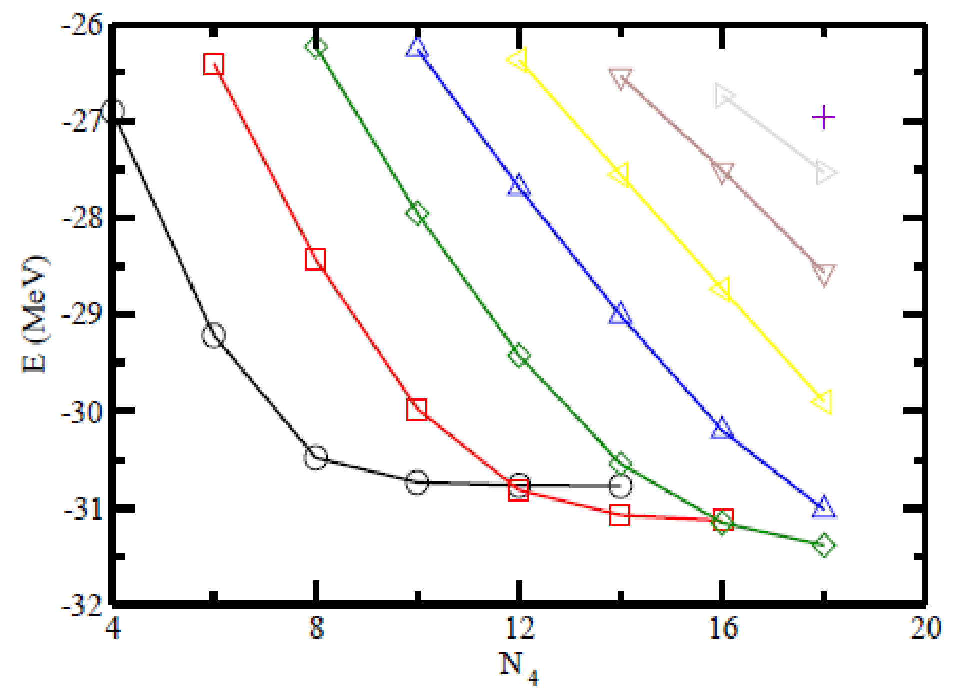
→ 3b force adjusted such that the triton binding energy in the trap is fixed to E_t=-8.482 MeV

$$\boxed{\hbar\omega = 3MeV}$$

Leading-Order

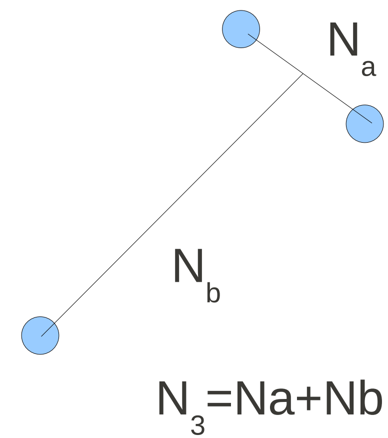


Next-to-Leading-Order

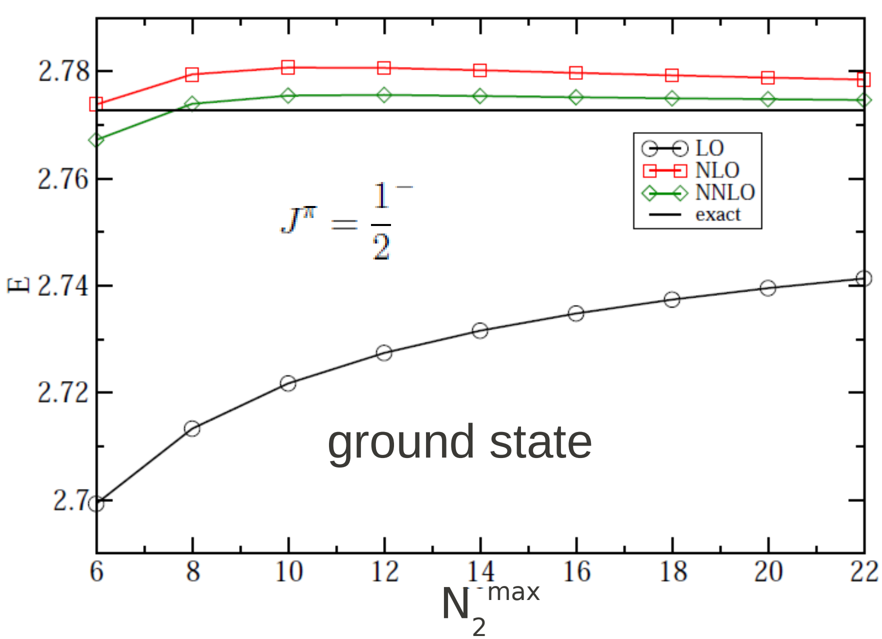


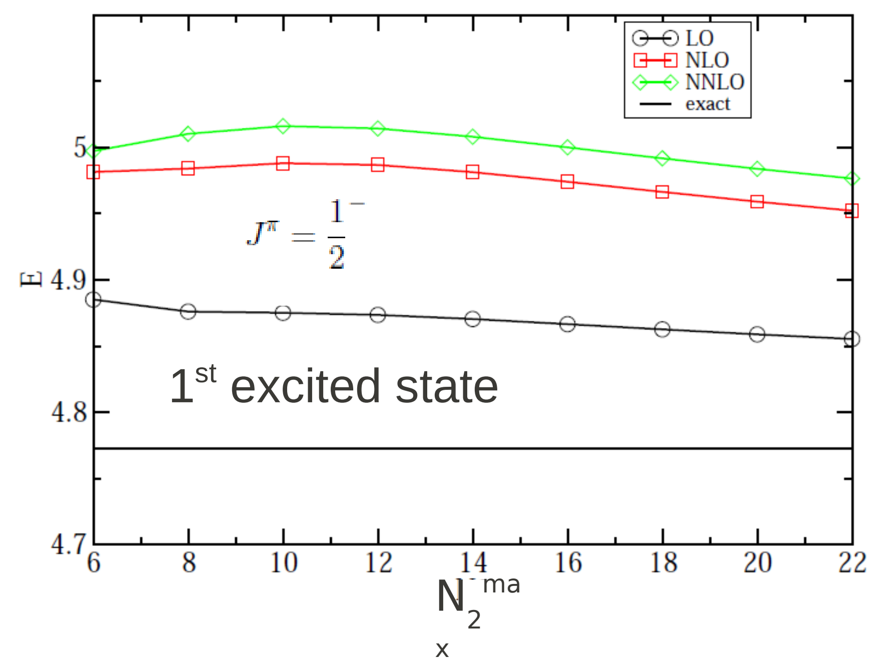
Three fermions in a trap

-> two-body interaction defined by cutoff N_2^{max} and three body model space defined by cutoff N_3^{max}



A) cut off *a la* Shell Model : two body cut-off N2 is fixed by N3.





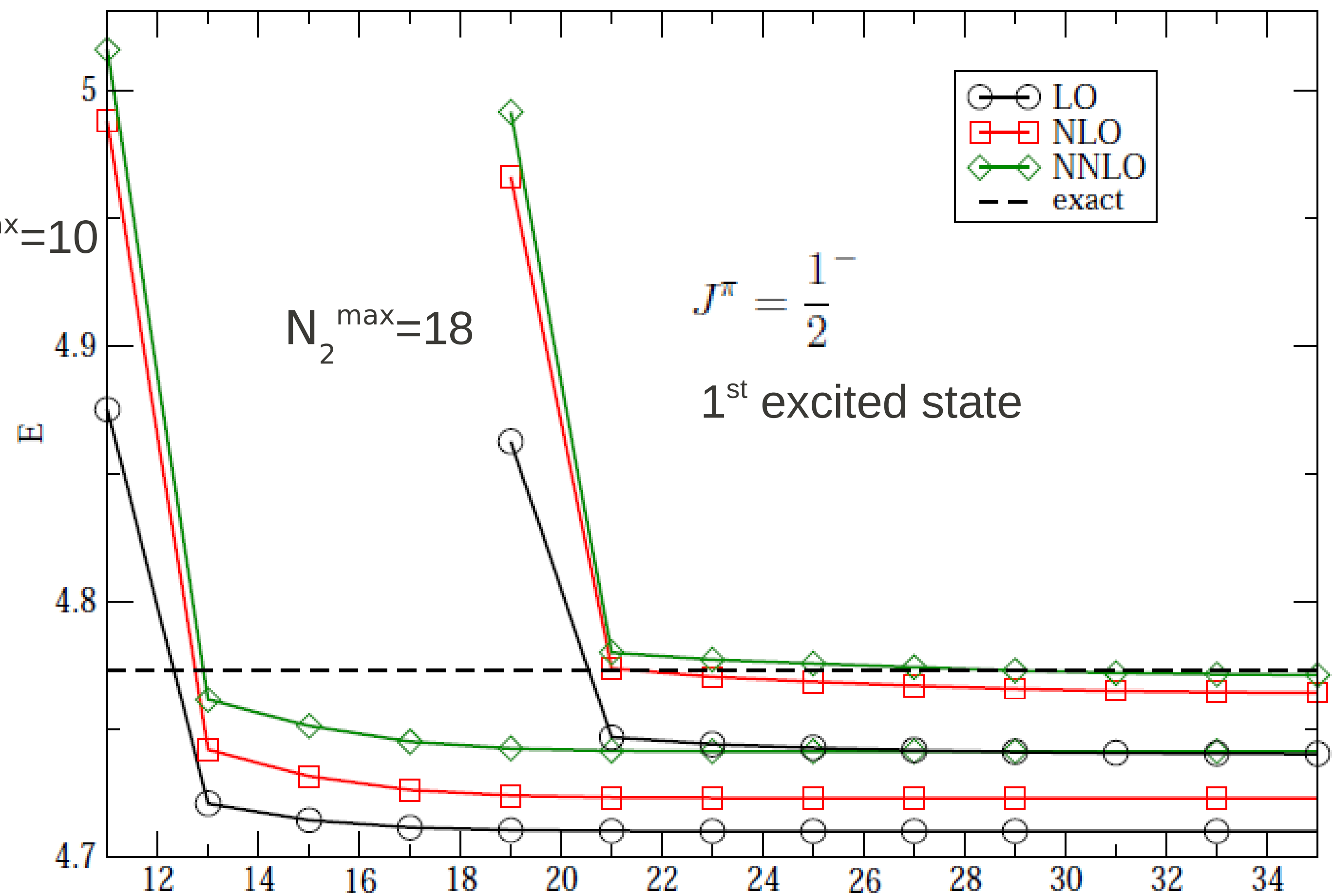
Problem !!! : NLO , NNLO further away from exact value than LO for some states

Three fermions in a trap

B) Solution :

-> for a fixed two-body cut-off N_2^{max} the three body cut-off N_3^{max} is increased until convergence (completeness is r

N_2^{max}=10

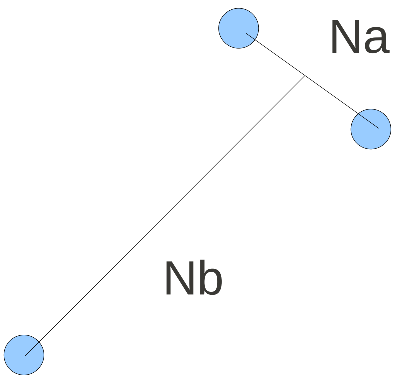


N_2^{max}=18

$J^\pi = \frac{1}{2}^-$

1st excited state

N_3^{max}



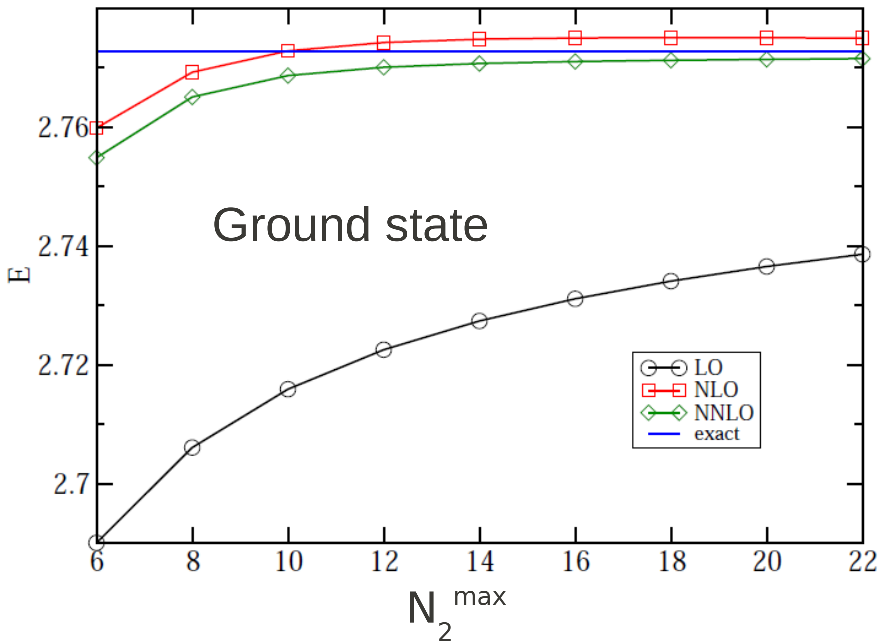
N3=Na+Nb

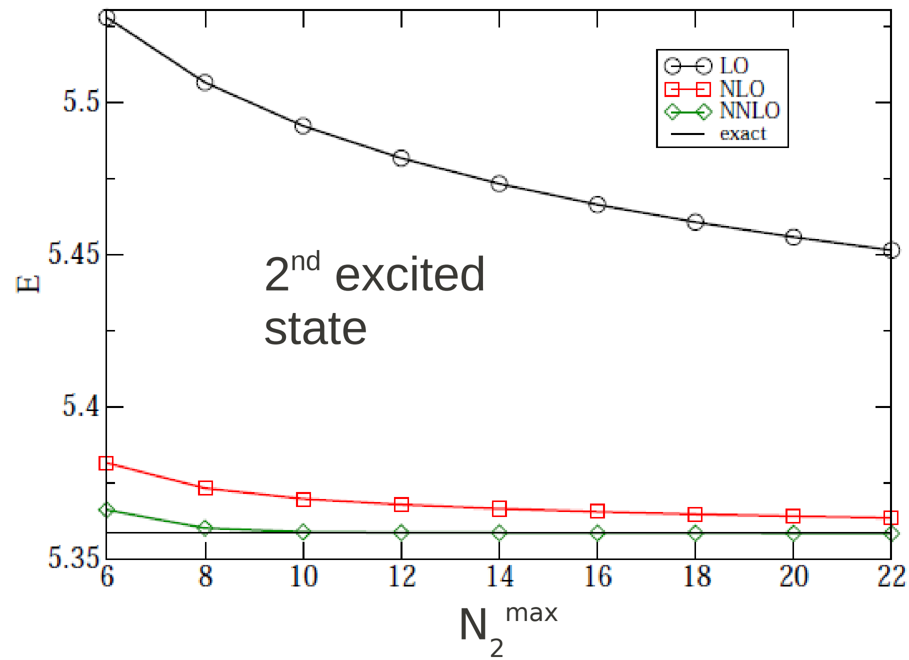
->no restriction on N3

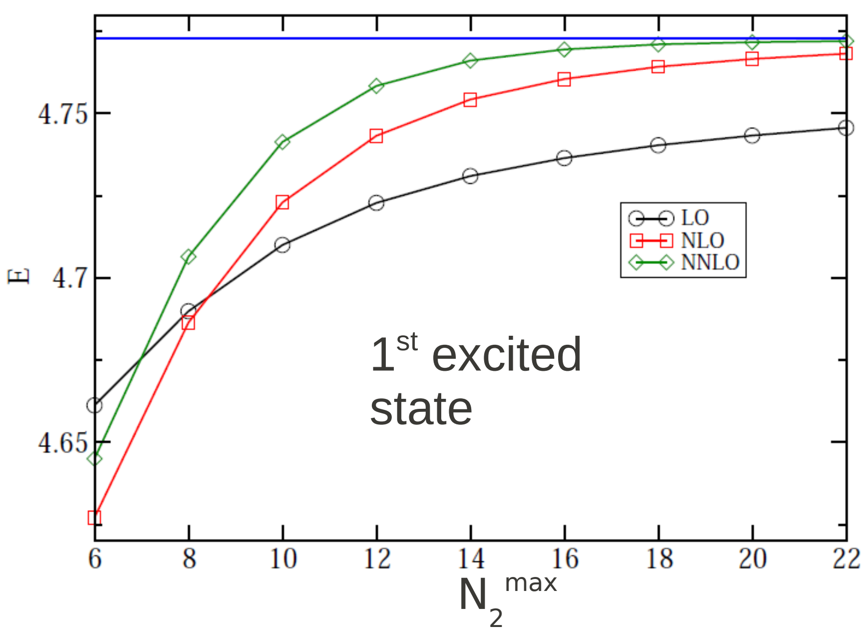
-> for $N_a > N_2^{max}$ the interaction is "switched off"

-> correct ordering of the different orders, faster convergence

J. R, I.Stetcu, B. Barrett, U. Van Kolck, M. Birse, PRA 82, 2010







-> convergence increases as more corrections are considered

Two nucleons in the $^3S_1$ channel at Next-to-Leading order :

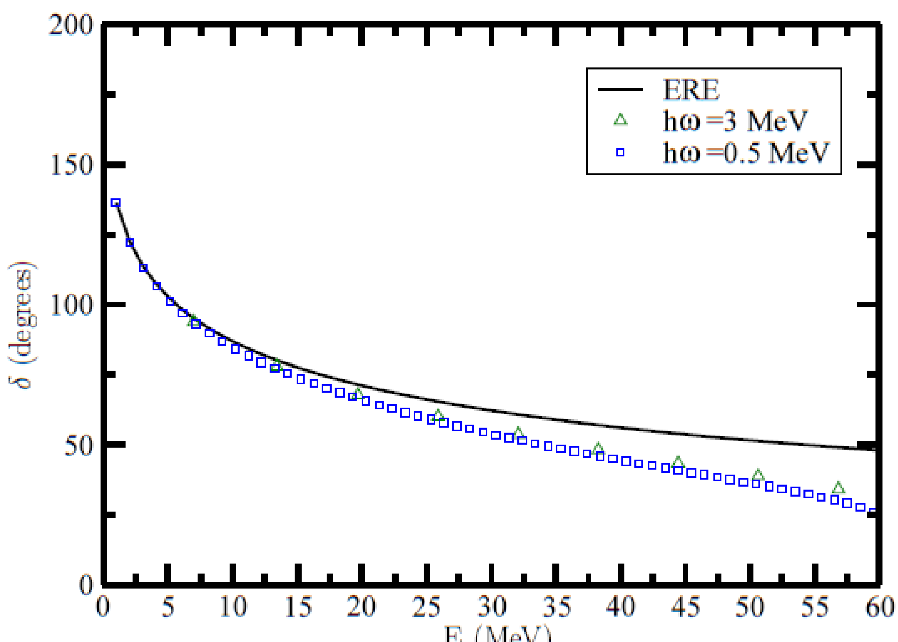
$$\left[\frac{p^2}{2\mu} + \frac{1}{2}\mu\omega r^2 + V(r,r')\right]\Phi_{l,j,m}(r) = E\Phi_{l,j,m}(r)$$

$$\Phi_{l,j,m}(\vec{r}) = \sum_{n}^{n_{max}} a_n \phi_{n,l,j,m}(\vec{r})$$

$$V(\vec{p}\,',\vec{p}) = \underbrace{c_0(\Lambda(n_{max},\omega))}_{\text{LO}} + \underbrace{c_2(\Lambda(n_{max},\omega))}_{\text{NLO}}\left(\vec{p}\,'^2 + \vec{p}\,^2\right)$$

Coupling constant c_0,c_2 fitted to the ground state (deuteron) and the first excited state in the trap.

$^3S_1$ N-N phase shift in the trap



Cutoff defined as

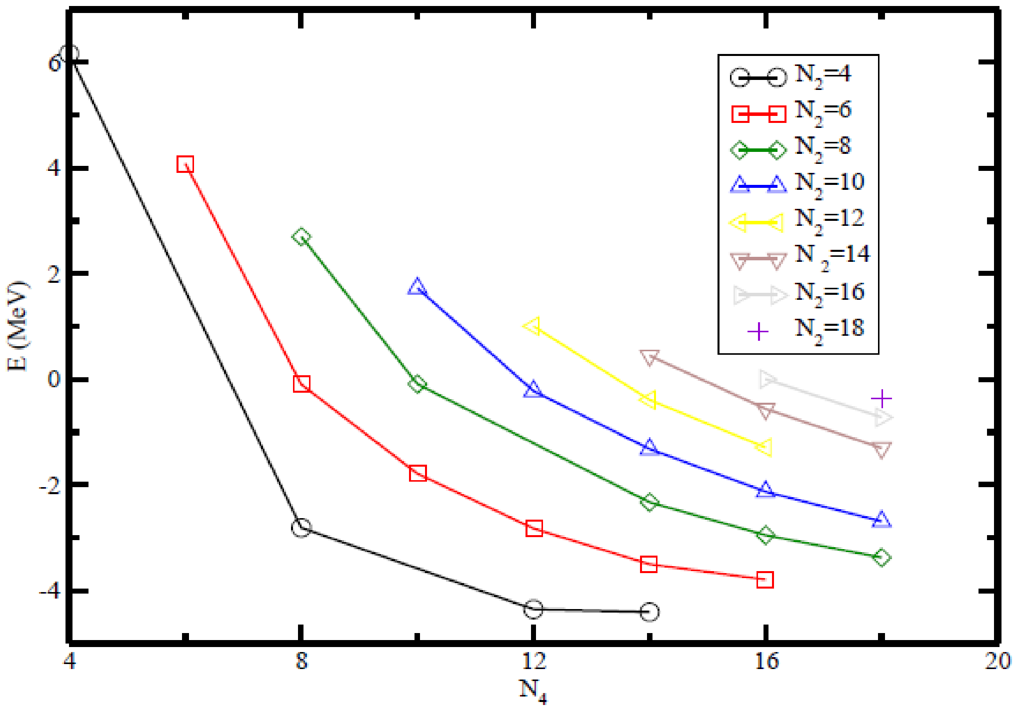
i) n_{max}

ii) $\Lambda = (2n_{max} + l + 3/2)\hbar\omega$

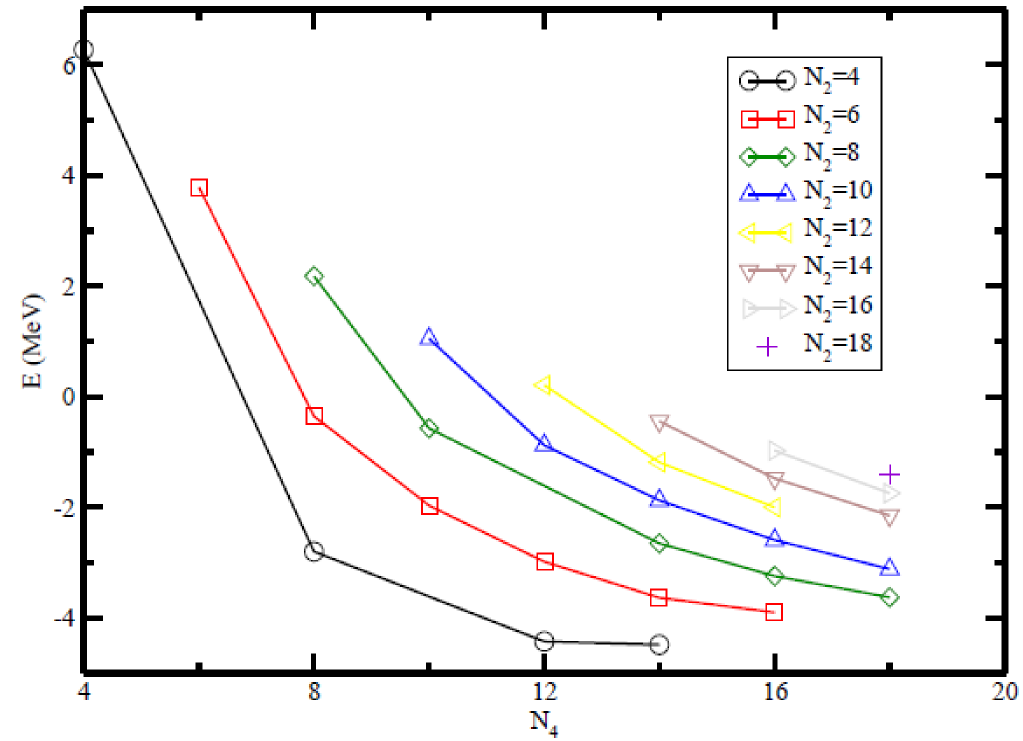
1st excited state in ^{4}He in a trap

$$\boxed{\hbar\omega = 3MeV}$$

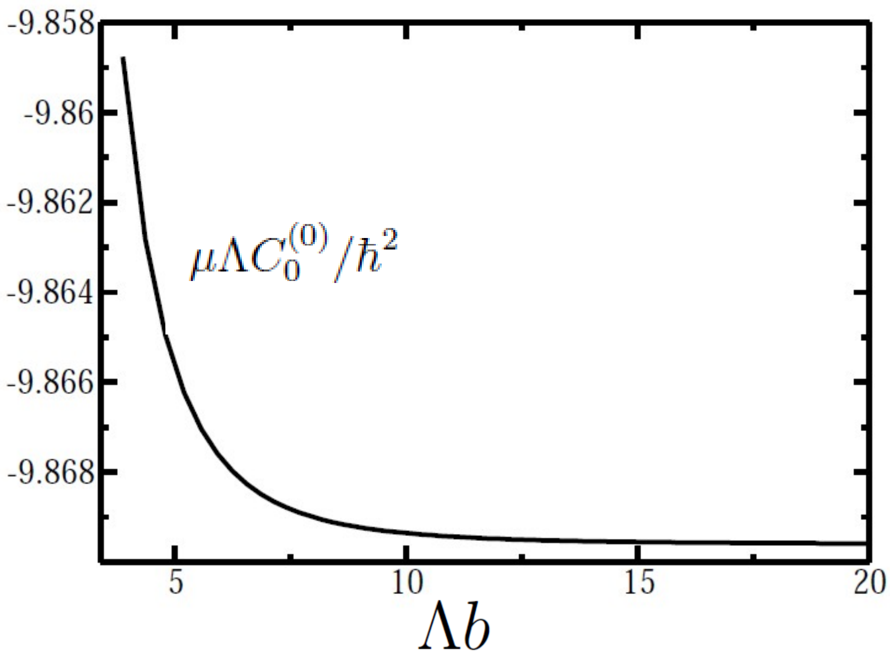
Leading-Order

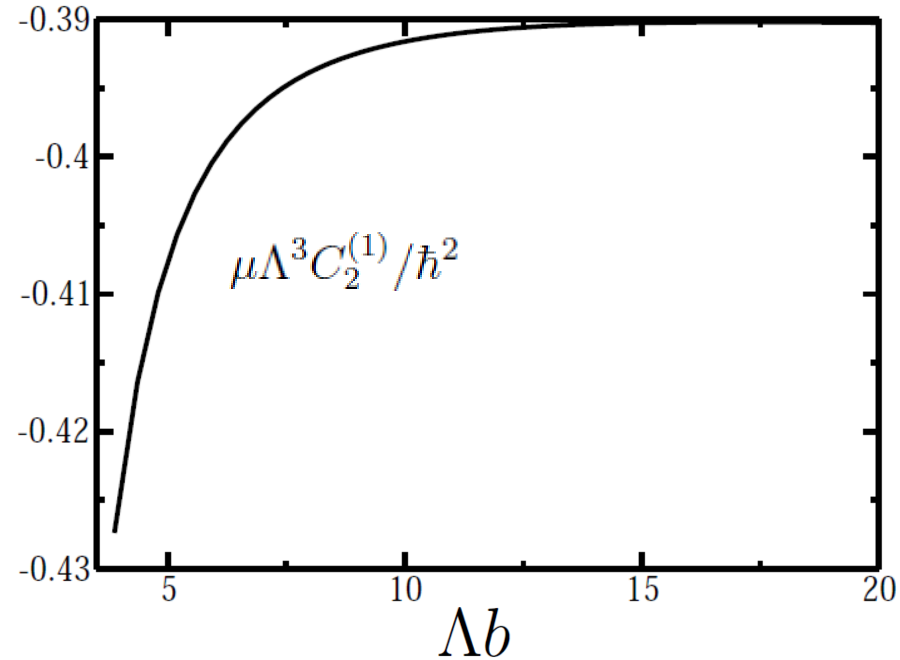


Next-to-Leading-Order

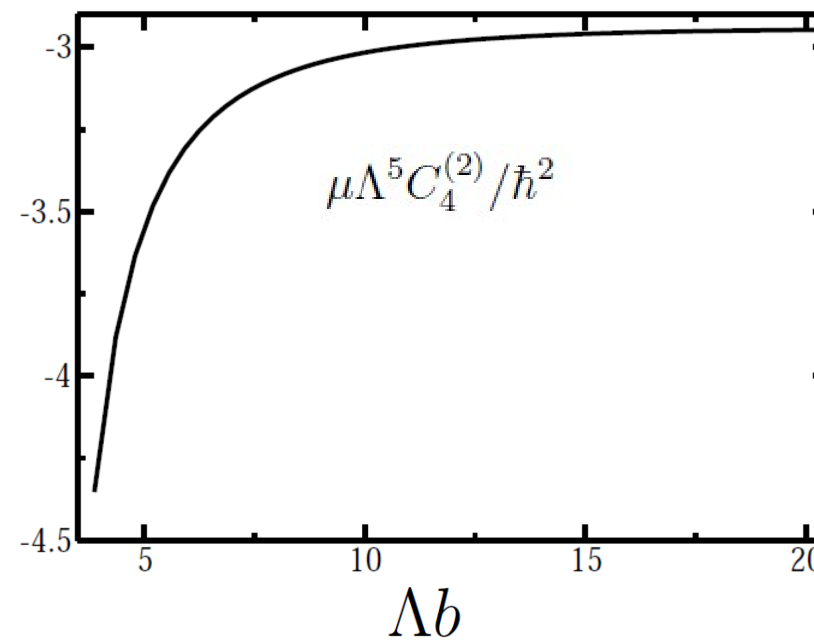


Running of the coupling constants at unitarity.

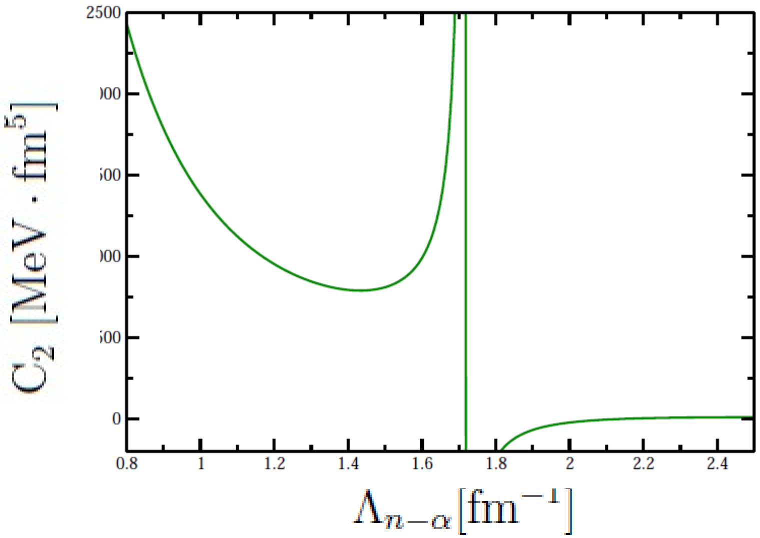


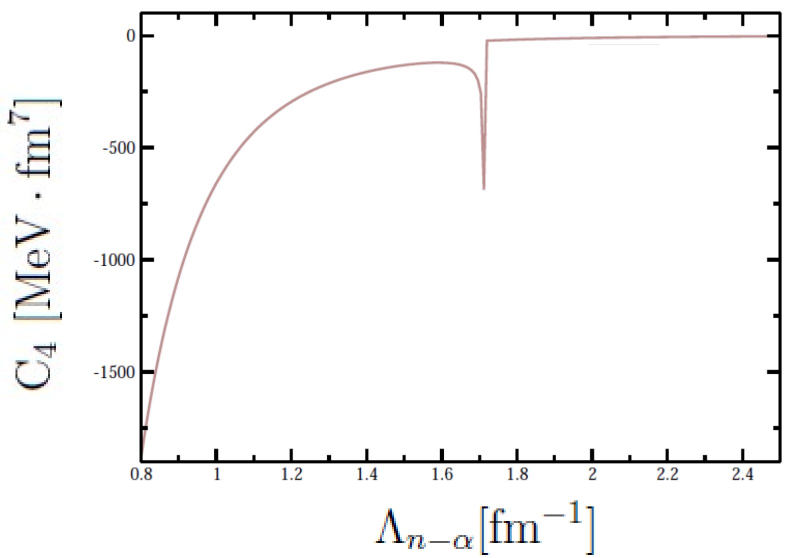


$$
\begin{aligned}
V(\vec{p}\,',\vec{p}) &= \frac{1}{\Lambda}(\Lambda c_0(\Lambda)) \\
&+ \frac{1}{\Lambda}(\Lambda^3 c_2(\Lambda))\left[\left(\frac{\vec{p}\,'}{\Lambda}\right)^2 + \left(\frac{\vec{p}}{\Lambda}\right)^2\right] \\
&+ \frac{1}{\Lambda}(\Lambda^5 c_4(\Lambda))\left[\left(\frac{\vec{p}\,'}{\Lambda}\right)^4 + \left(\frac{\vec{p}}{\Lambda}\right)^4\right]
\end{aligned}
$$

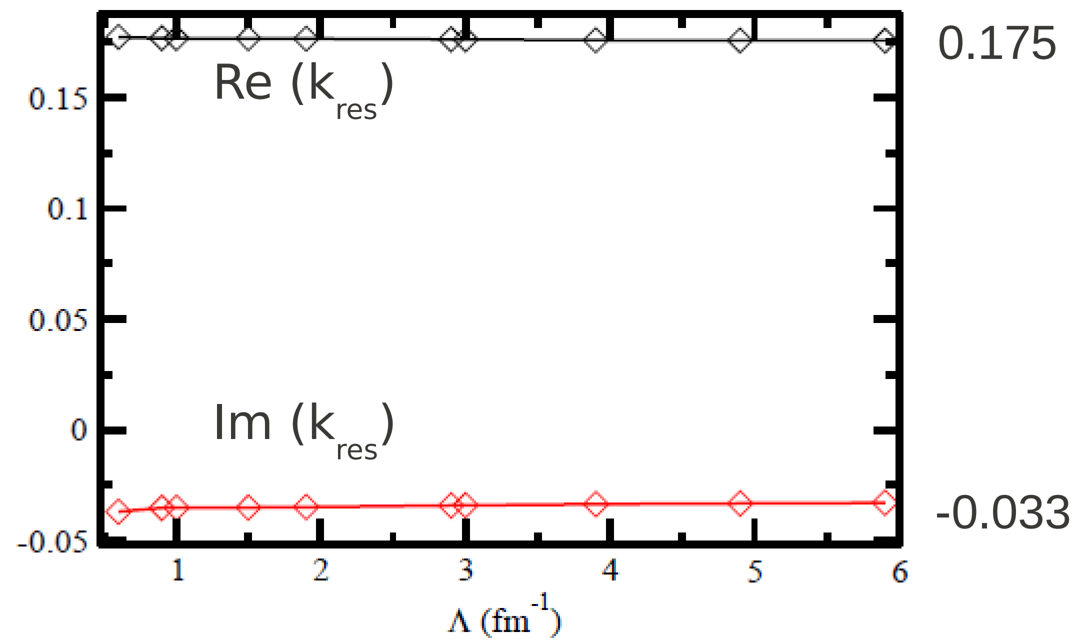


-> good power counting !





$n-\alpha$ $p_{3/2}$ resonance at LO



at $\Lambda_{n-\alpha}$ ~ 1.7 fm^{-1} :

- discontinuity of C_2 , C_4
- C_2 , C_4 become complex with large imaginary part

→ complex potential are "tricky" to use in many-body physics