Resonances their relations to Spectral Densities and Scattering Cross Sections.

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Outline of Talk.

I will discuss the influence of resonances, in a wide sense, to the spectrum of a Schrödinger problem using two related approaches.

- Titchmarsh-Weyl theory and its relations to spectral densities.
- The influence of resonances on scattering cross sections.

The Schrödinger Eq. and the Green function

See : J.Math. Phys. **27**, 2629 (1986)

- The Schrödinger partial wave eigenvalue problem

$$L_\ell u(r) = \lambda u(r) \text{ with } L_\ell = -\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} + V_0(r) \tag{1}$$

- Green function

$$(\lambda - L_\ell)G_\ell^+(r,r') = \delta(r - r') \tag{2}$$

- Let $\psi_\ell(r)$ be the regular Schödinger solution as $r \to 0$ and $\chi_\ell(r)$ be the be the regular Schödinger solution as $r \to \infty$. Since for fixed r' $G_\ell^+(r,r')$ is regular as $r \to 0$ and $r \to \infty$

$$\begin{aligned} G_\ell^+(r,r') &\propto \psi_\ell(r); \quad r \to 0 \\ \text{and } G_\ell^+(r,r') &\propto \chi_\ell(r); \quad r \to \infty \end{aligned} \tag{3}$$

$$\Rightarrow \quad G_\ell^+(r,r') = \psi_\ell(r_<)\chi_\ell(r_>)/W[\psi_\ell,\chi_\ell]. \tag{4}$$

The Classical Titchmarsh-Weyl theory

- Consider the one-dimensional partial wave Schrödinger eq. (1) on a finite interval $[0, b]$ and let ϕ_ℓ and ψ_ℓ be two linearly independent solutions at a given eigenvalue λ. They are then defined though

$$\begin{pmatrix} \phi_\ell & \psi_\ell \\ \phi'_\ell & \psi'_\ell \end{pmatrix}_{r=a} = \begin{pmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{pmatrix} \text{ where } a \in (0, b) \qquad (5)$$

- Any Schrödinger solution, except ψ_ℓ can be written

$$\chi_\ell(r) = \phi_\ell(r) + \psi_\ell(r)m(E) \qquad (6)$$

This defines the Titchmarsh-Weyl $m-$function.

- Now impose the boundary condition at $r = b > a$

$$\cos(\beta)\chi_\ell(r) + \sin(\beta)\chi'_\ell(r) = 0 \quad -\pi \le \beta \le \pi \qquad (7)$$

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$$\beta \in \mathrm{R} \quad \Rightarrow \quad \mathcal{I}m(\ \chi'/\chi\ ) = 0 \qquad (8)$$

The Classical Titchmarsh-Weyl theory II.

- The Titchmarsh-Weyl $m-$function can be shown to be represented by a circle with an center C_b and a radius R_b as

$$C_b = -[\phi_\ell, \psi_\ell](b)/[\psi_\ell, \psi_\ell](b),$$
$$R_b = 1/[\psi_\ell, \psi_\ell](b)$$
$$\text{with } [f, g](t) = f(t)\bar{g}'(t) - f'(t)\bar{g}(t)$$

- Let $b \to \infty$ along a path with a positive small imaginary part $\imath\epsilon$ and then let $\epsilon \to 0$

$$\Rightarrow R_b \to 0 \text{ (Limit point case) or } R_b \to R_\infty > 0 \text{ (Limit circle case).} \quad (9)$$

Physics application are of limit point type.

The Classical Titchmarsh-Weyl theory III.

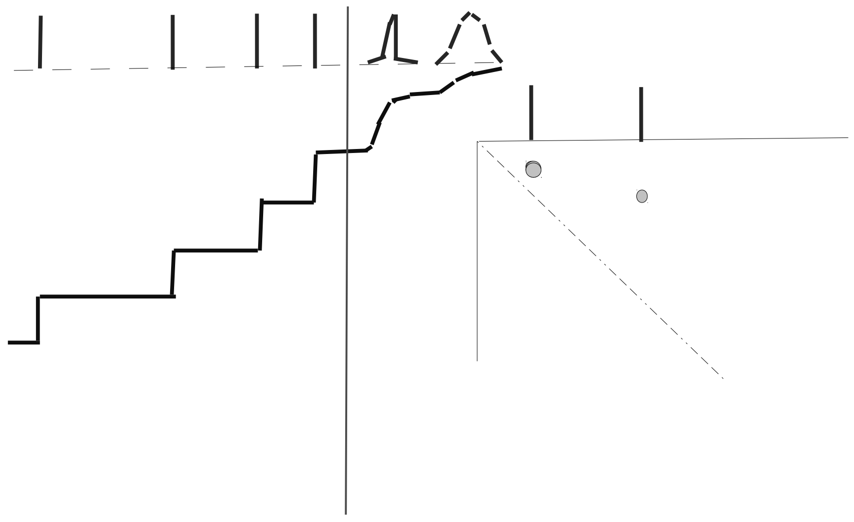

Figure 1: Naive picture of spectral function and spectral density.

The Classical Titchmarsh-Weyl theory IV.

- It can be shown that $m(E)$ is an analytic function for which relates to

$$\text{completeness} : \delta(r - r') = \int_{-\infty}^{+\infty} \psi_\ell(\omega, r)\psi_\ell(\omega, r')d\rho(\omega)$$

$$\text{where} : \rho(\omega_1) - \rho(\omega_2) = \lim_{\epsilon \to 0^+} \frac{1}{\pi} \int_{\omega_1}^{\omega_2} \mathcal{I}m(\ m(\lambda + \imath\epsilon)\ )d\lambda$$

$$\Rightarrow \text{Spectral density} : \mathcal{I}m(\ m(E)\ ) = \pi \left(\frac{d\rho}{d\omega} \right)_{\omega = E} \tag{10}$$

- Using eq. (6) : $\chi_\ell(r) = \phi_\ell(r) + \psi_\ell(r)m(E)$ we can write the Titchmarsh-Weyl $m-$function as

$$m_{TW}(E) = \frac{W[\phi_\ell, \chi_\ell]}{W[\chi_\ell, \psi_\ell]} \tag{11}$$

- which implies that $m_{TW}(E)$ can be evaluated from the logarithmic derivative χ'/χ.

The Classical Titchmarsh-Weyl theory V.

We also need to define :

$$m(\lambda) = \begin{cases} m^+(\lambda); & \Im(\lambda) > 0 \\ m^-(\lambda); & \Im(\lambda) < 0 \end{cases} \text{ as well as } \chi(\lambda) = \begin{cases} \chi^+(\lambda); & \Im(\lambda) > 0 \\ \chi^-(\lambda); & \Im(\lambda) < 0 \end{cases} \tag{12}$$

Appearing in

$$\chi_\ell^\pm(r) = \phi_\ell(r) + \psi_\ell(r) m^\pm(E) \tag{13}$$

Complex scaling - Brief reminder.

$$\xi(r) = \begin{cases} r, & r < r_0, \\ r + f(r, r_0, \theta, \rho), & r \geq r_0, \end{cases} \tag{14}$$

where

$$f(r, r_0, \theta, \rho) = (e^{i\theta} - 1)(r - r_0)(1 - \exp(-\rho(r - r_0)^2)). \tag{15}$$

We define $\eta = \exp(\imath\theta)$

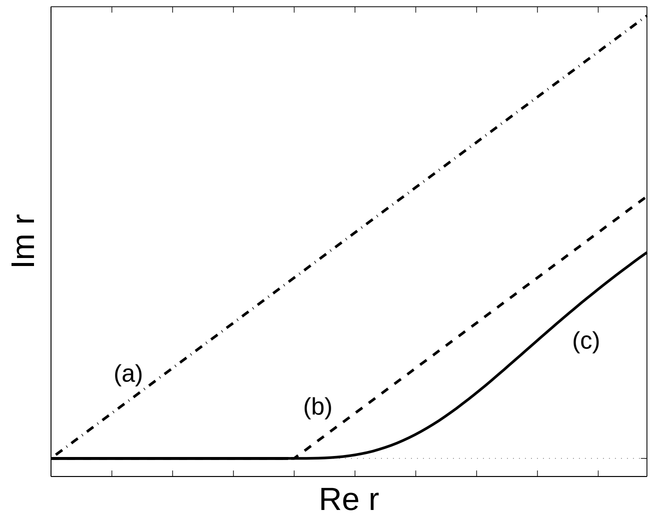

Figure 2: a) uniform complex scaling, b) sharp exterior scaling and c) smooth exterior scaling.

Titchmarsh-Weyl resonances

- The ABC theory can be applied to the Titchamarsh-Weyl theory by starting with

$$\left(-\frac{d^2}{dr^2}+\frac{\ell(\ell+1)}{r^2}+\eta^2V_0(\eta r)-\eta^2\lambda\right)=0 \tag{16}$$

- The boundary conditions now take the form

$$\begin{pmatrix} \phi_{\ell,\eta} & \psi_{\ell,\eta} \\ \phi'_{\ell,\eta} & \psi'_{\ell,\eta} \end{pmatrix}_{r=a} = \begin{pmatrix} \sin\alpha & \cos\alpha \\ -\eta\cos\alpha & \eta\sin\alpha \end{pmatrix} \tag{17}$$

- Provided the energy parameter is within an rotated, uncovered sector the dilated TW-circle still converges to a point(Limit point case)

- Rewriting the dilated Schrödinger eq. in terms of the logarithmic derivative $z_{\ell,\eta}=\chi'_{\ell,\eta}/\chi_{\ell,\eta}$ we get a complex dilated Riccati eq.

$$z'_{\ell,\eta}(r)=2\eta^2(V_0(\eta r)-\lambda)-z^2_{\ell,\eta} \tag{18}$$

From which the $m_{TW}(E)$ can be computed by analytical continuation.

Generalize spectral expansion of the Green function I.

- Use the resolvent formulation of the Green function

$$G^+(\lambda; r, r') = \lim_{\epsilon \to 0} \int_{-\infty}^{+\infty} \frac{d\hat{\tau}(\omega)}{\lambda + \imath\epsilon - \omega} = \tag{19}$$

$$\lim_{\epsilon \to 0} \left\{ \sum_j \frac{\mathcal{R}es\ [\ G^+(\lambda_j; r, r')\ ]}{\lambda + \imath\epsilon - \lambda_j} + \int_0^{+\infty} \frac{[d\hat{\tau}/d\omega] d\omega}{\lambda + \imath\epsilon - \omega} \right\}_{\lambda \neq \lambda_j} \tag{20}$$

- We also need to specify the complex dilated forms of χ and m. First note that the rotated cut is $\eta^{-2} R^+ \quad \Rightarrow$

$$m_\eta(\lambda) = \begin{cases} m_\eta^+(\lambda); & \Im(\eta^2\lambda) > 0 \\ m_\eta^-(\lambda); & \Im(\eta^2\lambda) < 0 \end{cases} \;;\; \chi_\eta(\lambda) = \begin{cases} \chi_\eta^+(\lambda); & \Im(\eta^2\lambda) > 0 \\ \chi_\eta^-(\lambda); & \Im(\eta^2\lambda) < 0 \end{cases} \tag{21}$$

Generalize spectral expansion of the Green function II.

- Extracting the imaginary part we find

$$\mathcal{I}m(\ G_\ell^+(\lambda; r, r')\ ) = -\psi_\ell(r)\psi_\ell(r')\mathcal{I}m(\ m^+\ ) \tag{22}$$

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$$G_\ell^+(\lambda; r, r') = \lim_{\epsilon\to 0} \int_{-\infty}^{+\infty} ds \frac{\psi_\ell(\omega, r)\psi_\ell(\omega, r')d\rho(\omega)}{\lambda + \imath\epsilon - \omega} \tag{23}$$

- with

$$\frac{d\rho(\omega)}{d\omega} = \begin{cases} \sum_j \frac{\delta(\omega - \lambda_j)}{< \psi_\ell(\lambda_j)|\psi_\ell(\lambda_j) >}; & \omega < 0 \\ \frac{1}{\pi}\mathcal{I}m(\ m^+(\omega)\ ); & \omega > 0 \end{cases} \tag{24}$$

- Letting the resolvent operator work on eq. (23) one can show that the spectral densities (24) and (10) are the same.
- The Green function above is thus uniquely defined by m_{TW}

The Nevanlinna repr. of $m(E)$ and its extension I.

- A Cauchy analytic function $f(z)$ is said to be of Nevanlinna type if it maps the upper (lower) complex half plane onto itself.
- It can be shown that $m(\lambda)$ is a function of Nevanlinna type.
- Nevanlinna functions posses a uniquely defined spectral functions $\sigma(\omega)$such that

$$f(z) = \int_{-\infty}^{+\infty} \frac{d\sigma(\omega)}{\omega - z}; \frac{d\sigma(\omega)}{d\omega} = \begin{cases} \sum_j - \mathcal{R}es \, [\; f(z_j) \;] \delta(\omega - z_j); \omega < 0 \\ \frac{1}{\pi} \mathcal{I}m(\; f(\omega + \imath 0) \;); \omega > 0 \end{cases} \tag{25}$$

- Consider $\ell = 0$ only and let $m_{\mathrm{free}} = \imath\sqrt{\lambda}$
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$$\Rightarrow \; m(\lambda) - \imath\sqrt{\lambda} = \int_{-\infty}^{+\infty} \frac{d\sigma(\omega)}{\omega - z}; \;\; d\sigma(\omega) = d\rho(\omega) - d\rho_{\mathrm{free}}(\omega) \tag{26}$$

The Nevanlinna repr. of $m(E)$ and its extention II.

- Above we defined $m^+(\lambda)$ and $m^-(\lambda)$ to have analytic continuation onto higher Riemann Sheets. It can be shown that for real λ the differ only by the signs of their imaginary parts

$$\mathcal{I}m(\ m^+(\lambda)\ ) = (m^+(\lambda) - m^-(\lambda))/2\imath \equiv \mathcal{I}mg(\ m(\lambda)\ ) \tag{27}$$

thereby defining $\mathcal{I}mg(\ m(\lambda)\ )$.

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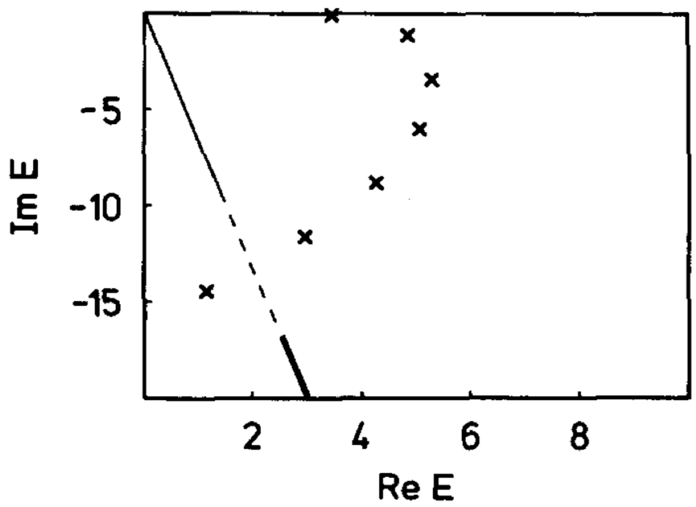
$$\Rightarrow m^+(\lambda) - \imath\sqrt{\lambda} = \sum_j \frac{\mathcal{R}es\ [\ m^+(\lambda_j)\ ]}{\lambda - \lambda_j} + \int_C \frac{\mathcal{I}mg(\ m^+(\omega) - \imath\sqrt{\omega}\ )d\omega}{\omega - \lambda} \tag{28}$$

The Nevanlinna repr. of $m(E)$ and its extention III.

- We thus find

$$\frac{d\rho(\omega)}{d\omega} = \frac{1}{\pi}\mathcal{I}mg(\ m^{+}(\omega)\ ) \tag{29}$$



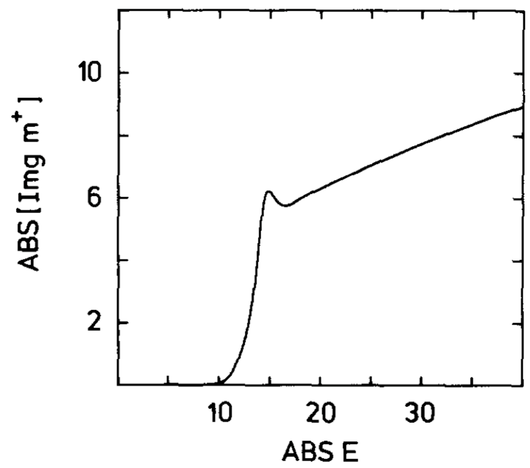


Figure 3: **Left figure :** Pole string for $m^{+}(\lambda), \lambda = 2E$. The ray $\eta^{-2}R^{+}$ displays the integration contour for the Nevanlinna representation. **Right figure** The generalized spectral density is vanishingly small along the weakly drawn line. Outside, along the thick solid line the generalized spectral density is that of a free particle. The broken line shows the transition region.

Two-Channel scattering problem I.

`Int. J. Quantum Chem.` **109**`,414 ( 2009 ), J. Phys. B.` **42**`,044011 (2009)`

- Consider a scattering process that can be described as a two-channel Schrödinger problem

$$\left(-\frac{d^2}{dr^2}+U_{11}+\frac{\ell(\ell+1)}{r^2}-k_1^2\right)\phi_1(r)+U_{12}\phi_2(r)=0,$$
$$\left(-\frac{d^2}{dr^2}+U_{22}+\frac{\ell(\ell+1)}{r^2}-k_2^2\right)\phi_2(r)+U_{21}\phi_1(r)=0, \quad (30)$$

where ℓ is the angular momentum quantum number and

$$k_n=\sqrt{2m(E-E_n)}. \quad (31)$$

E_n denotes the channel threshold and m is the reduced mass of the studied system.

- The scattering is described by the Scattering Operator $\hat{S}$ acting on the incoming wave Ψ_{in} to give the outgoing wave Ψ_{out}

$$\Psi_{out}=\hat{S}\Psi_{in} \quad (32)$$

Two-Channel scattering problem II.

- For a central potentials $U(\mathbf{r} = U(r)$ we formulate the problem in terms of an expectation value problem : the partial wave scattering matrix $S_{\ell,ij}(E)$ yielding the total cross section in terms of the partial wave scattering cross sections

$$\sigma_{ij}(E) = \sum_{\ell=0}^{\infty} \sigma_{\ell,ij}(E); \quad \sigma_{\ell,ij}(E) = 4\pi(2\ell+1)\left|\frac{S_{\ell,ij}(E) - \delta_{ij}}{2\imath k_i}\right|^2 \tag{33}$$

- The Mittag-Leffler (ML) expansion of the one channel partial wave S-matrix is given by:

$$\begin{aligned} S_\ell(k) = S_\ell(a) \quad &+ \quad \sum_R Res\left[S_\ell(z_R)\right]\left\{\frac{1}{k - z_R} + \frac{1}{z_R - a}\right\} \\ &+ \quad \frac{k-a}{2\pi i}\oint_C \frac{S_\ell(z)\,dz}{(z-k)(z-a)}. \end{aligned} \tag{34}$$

Two-Channel scattering problem III.

- To generalize the method described in to many channels, the contribution to the partial wave S-matrix from one single resonance is defined as the residue term in the ML expansion:

$$Res\left[\mathbf{S}_\ell(\mathbf{K}_R)\right]\left(\mathbf{K}-\mathbf{K}_R\right)^{-1}.$$

- For the two-channel case

$$\mathbf{K} = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}. \qquad (35)$$

$\mathbf{K}_R$ is defined in the same way and is the momentum matrix at the resonance energy. Here and below matrices are labeled with fat symbols. The residues of the S-matrix can then be calculated and used to build the reduced S-matrix, $\tilde{\mathbf{S}}_\ell(\mathbf{K},\mathbf{K}_R)$:

$$\tilde{\mathbf{S}}_\ell(\mathbf{K},\mathbf{K}_R) = \mathbf{S}_\ell(\mathbf{K}) - Res\left[\mathbf{S}_\ell(\mathbf{K}_R)\right]\left(\mathbf{K}-\mathbf{K}_R\right)^{-1}. \qquad (36)$$

Mittag- Leffler expansion IV.

- The reduced partial cross section is then given by

$$\tilde{\sigma}_{\ell,ij}(E, E_R) = 4\pi(2\ell+1)\left|\frac{\tilde{S}_{\ell,ij} - \delta_{ij}}{2\imath k_i}\right|^2. \quad (37)$$

The difference between σ_ℓ and $\tilde{\sigma}_\ell$ can then be used to identify a possible feature in the cross section.

The Residues of the S-matrix at resonant energies.

- The many-channel partial wave S-matrix can be expressed as

$$\mathbf{S}_\ell = \mathbf{1} + 2i\mathbf{K}^{1/2}\mathbf{F}_\ell\mathbf{K}^{1/2}, \tag{38}$$

where $\mathbf{K}$ is defined in (35) and $\mathbf{F}_\ell$ is the scattering amplitude (proportional to the T-matrix elements)

$$\mathbf{F}_\ell = -\mathbf{K}^{-1}\int_0^\infty dr\, \hat{\mathbf{j}}_\ell(\mathbf{K}r)\mathbf{U}(r)\mathbf{\Psi}_\ell(r)\mathbf{K}^{-1}. \tag{39}$$

Here $\hat{\mathbf{j}}_\ell(\mathbf{K}r)$ is a diagonal matrix of the Riccati-Bessel functions, $\mathbf{U}(r)$ - potential matrix and $\mathbf{\Psi}_\ell(r)$ is a physical solution of SE (30) which also satisfies the integral equation

$$\mathbf{\Psi}_\ell(r) = \hat{\mathbf{j}}_\ell(\mathbf{K}r) + \int_0^\infty dr'\, \mathbf{G}(r,r')\mathbf{U}(r')\hat{\mathbf{j}}_\ell(\mathbf{K}r'). \tag{40}$$

Each column in the matrix $\mathbf{\Psi}_\ell(r)$ is a solution to the Schrödinger eq. with appropriate boundary conditions. The number of the column corresponds to that of the incoming channel.

Two-Channel Green function and the S-matrix.

- The two-channel partial wave Green function is expressed in its eigenfunctions. Then, by applying complex dilation to the Green function and discretizing the contributions from the continuous spectrum, the complex dilated two-channel partial Green function is obtained as a finite sum over the eigenfunctions

$$\mathbf{G}(r,r') = \sum_{R=1}^{N} \frac{1}{E-E_R} \left[\begin{array}{c} \phi_1^R(r) \\ \phi_2^R(r) \end{array} \right] \left[\begin{array}{cc} \phi_1^R(r'), & \phi_2^R(r') \end{array} \right]. \qquad (41)$$

- Substituting (39)-(41) into eq. (38), the following expression for the S-matrix is obtained

$$\mathbf{S}_\ell = 1 - 2i\mathbf{K}^{-1/2} \int_0^\infty dr\, \hat{\mathbf{j}}_\ell(\mathbf{K}r)\mathbf{U}(r)\hat{\mathbf{j}}_\ell(\mathbf{K}r)\mathbf{K}^{-1/2} \qquad (42)$$

$$-\mathbf{K}^{-1/2} \sum_{R=1}^{N} \frac{2i}{E-E_R} \int_0^{\xi(\infty)} d\xi(r)\, \hat{\mathbf{j}}_\ell(\mathbf{K}\xi(r))\mathbf{U}(\xi(r))\phi_R(\xi(r)) * \qquad (43)$$

$$\int_0^{\xi(\infty)} d\xi(r)\, \phi_R^T(\xi(r))\mathbf{U}(\xi(r))\hat{\mathbf{j}}_\ell(\mathbf{K}\xi(r))\mathbf{K}^{-1/2}. \qquad (44)$$

The residues of the S-matrix at resonant energies.

- Yielding the residue of the S-matrix at a resonance momentum

$$Res\left[\mathbf{S}_\ell(\mathbf{K}_R)\right] = -i\mathbf{K}_R^{-1}\int_0^{\xi(\infty)} d\xi(r)\,\mathbf{K}_R^{-1/2}\hat{\mathbf{j}}_\ell(\mathbf{K}_R\xi(r))\mathbf{U}(\xi(r))\phi_R(\xi(r)) * \quad (45)$$

$$\int_0^{\xi(\infty)} d\xi(r)\,\phi_R^T(\xi(r))\mathbf{U}(\xi(r))\hat{\mathbf{j}}_\ell(\mathbf{K}_R\xi(r))\mathbf{K}_R^{-1/2}. \quad (46)$$

- The potential in our example is

$$\mathbf{V}(r) = \begin{bmatrix} -1.0 & -7.5 \\ -7.5 & 7.5 \end{bmatrix} r^2 e^{-r}, \quad (47)$$

where the threshold energies are $E_1 = 0$ and $E_2 = 0.1$, respectively. As in the previous sections, the notation $U(r) = 2mV(r),\ m = 1$ is used.

Resonance Energies

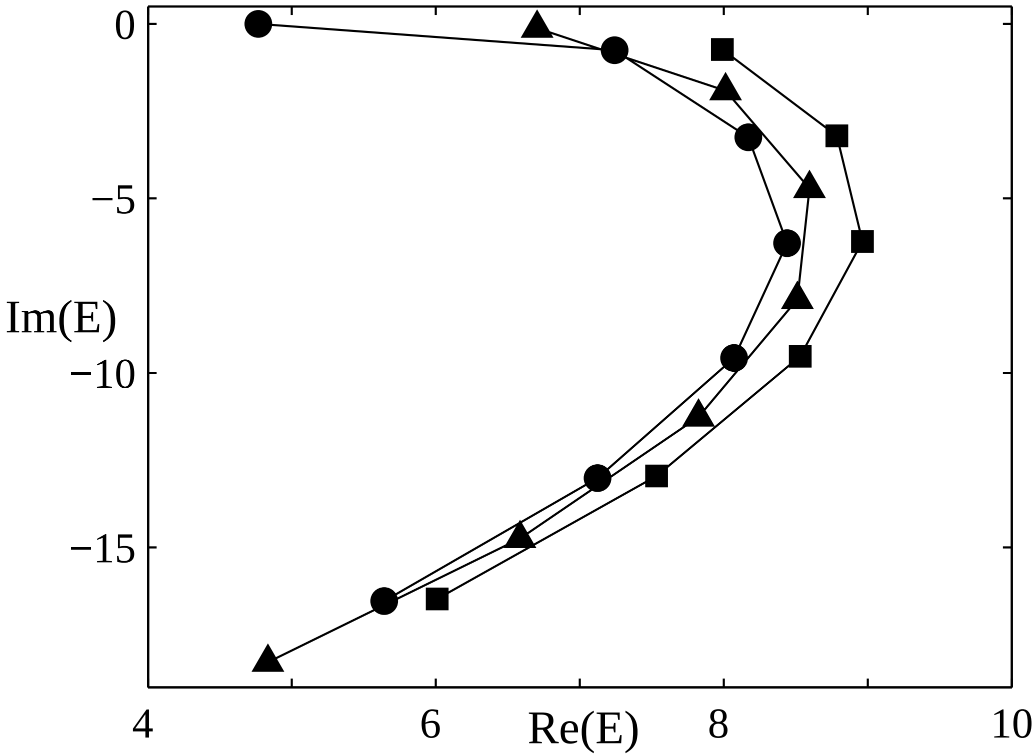


Figure 4: Complex eigenvalues for the Noro-Taylor potential. Energies for $\ell = 0$ are marked with circles, for $\ell = 1$ and $\ell = 2$ with triangles and squares, respectively.

Resonance Residues

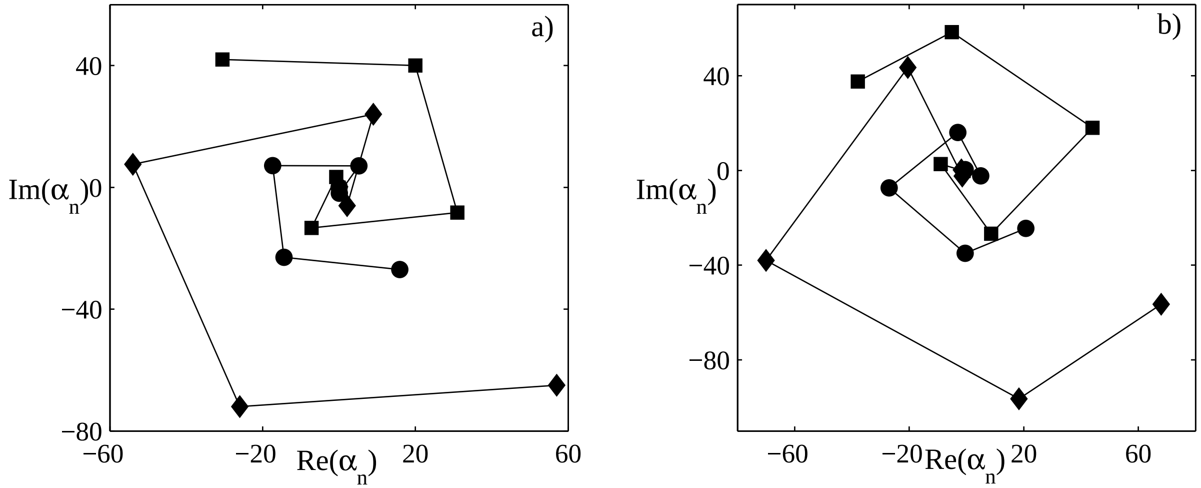


Figure 5: Residues of the partial wave S-matrix elements a) $\ell = 1$ and b) $\ell = 2$. Circles corresponds to the residues of S_{11}, squares and diamonds to the residues of S_{12} and S_{22} respectively. $\alpha_n = Res\,[S_{ij}(E_n)]$

- The residues " rotate anti-clock vice as n increases
- $|Res(S_{\ell,ij}(E_R)|$ Grows along the string of resonances $\Rightarrow$ The common statement **" A wide resonance does not influence the cross section since it is so far out in the complex plane" is wrong.**

Contribution to cross section

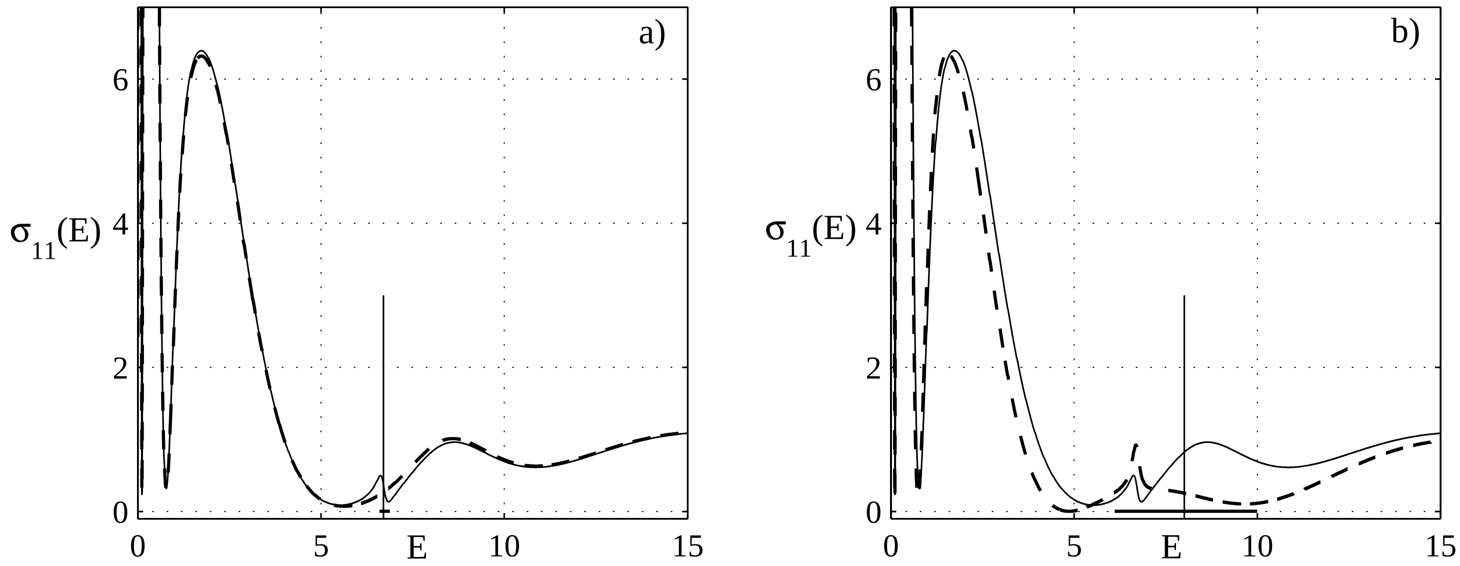


Figure 6: σ_{11} cross sections for $\ell = 1$. Solid line corresponds to the partial wave cross section, dashed line corresponds to the reduced cross section based on eq. (37) without contribution from a) the first pole b) the second pole. Vertical lines indicate positions of the excluded resonances, horizontal lines show intervals corresponding to the resonance width.

Breit-Wigner approximation I.

- The Breit-Wigner (BW) formula for the partial wave S-matrix in the neighborhood of an isolated multichannel resonance, with a complex energy E_R, is given by

$$S_{ij}^{BW}(E, E_R) = \exp(2i\gamma_i)\delta_{ij} - i\exp(i(\gamma_i + \gamma_j))\frac{\sqrt{\Gamma_i\Gamma_j}}{E - E_R}, \qquad (48)$$

yielding the BW residue as

$$-i\exp(i(\gamma_i + \gamma_j))\sqrt{\Gamma_i\Gamma_j}.$$

Here γ_i is a background phase shift and Γ_i is the corresponding partial width.

- One obvious problem with Breit-Wigner partial wave S-matrix is that it includes unknown values, namely the background phase shifts γ_i. It is unclear how this phase shifts are to be determined.

Breit-Wigner approximation II.

- We have here chosen to calculate them such that in a close vicinity of a resonance (Re(E_R)) a partial wave S-matrix element $S_{ij}(E)$, obtained with the logarithmic derivative method, is equal to $S_{ij}^{BW}(E, E_R)$ in eq. (48). We may now define a reduced BW partial-wave S-matrix, as

$$\tilde{S}_{ij}^{BW}(E, E_R) = S_{ij}(E) + i\exp(i(\gamma_i + \gamma_j))\frac{\sqrt{\Gamma_i\Gamma_j}}{E - E_R}. \qquad (49)$$

- Then we can define a BW reduced cross section as

$$\tilde{\sigma}_{ij}^{BW}(E, E_R) = 4\pi(2\ell + 1)\left|\frac{\tilde{S}_{ij}^{BW}(E, E_R) - \delta_{ij}}{2\imath k_i}\right|^2. \qquad (50)$$

Breit-Wigner approximation III.

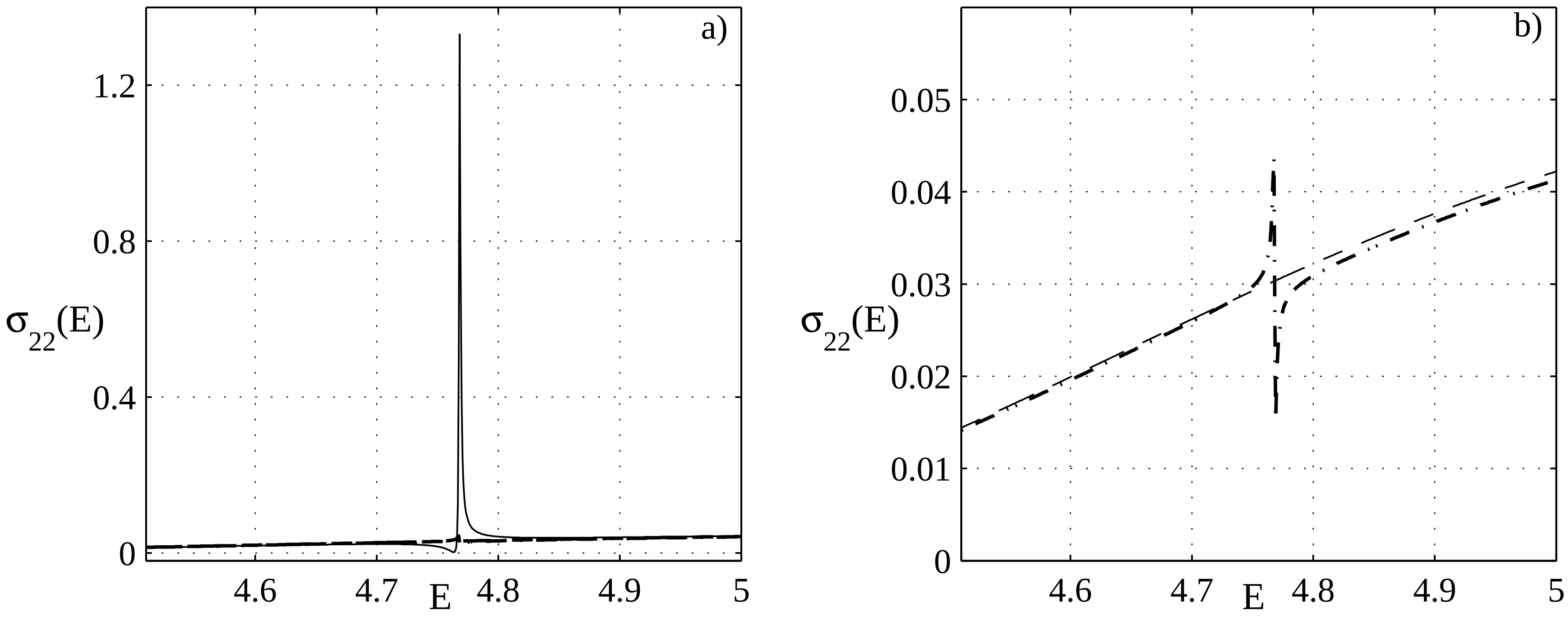


Figure 7: σ_{22} cross sections for $\ell = 0$. In a) solid line corresponds to the partial wave cross section, the dashed line corresponds to the reduced cross section based on subtracting S-matrix residue from the first pole (eq. (37)). In b) the dash-dotted line reveres to the Breit-Wigner based reduced cross section (eq. (50)) without contribution from the first pole while the dashed line has the same meaning as in a).

Breit-Wigner approximation IV.

• The Mittag-Leffler based S-matrix residue description of the resonance contribution to the studied cross sections is successful.

•The background phase in the Breit-Wigner varies even over a narrow resonance leaving a contribution from the resonance pole.

Summary.

• The resonance poles associated with a potential form a string in the complex plane.

•The spectral density can be partitioned into contributions from the resonances and the free particle spectral density

•The contributions form resonances to the scattering cross section can be formulated in terms of the S-matrix residues of resonances energies

•The S-matrix residues " rotate " anti-clock vice as n increases

•$|Res(S_{\ell,ij}(E_R)|$ Grows along the string of resonances $\Rightarrow$ The common statement **" A wide resonance does not influence the cross section since it is so far out in the complex plane" is wrong.**

•The Mittag-Leffler based S-matrix residue description of the resonance contribution to the studied cross sections is successful.

•The background phase in the Breit-Wigner varies even over a narrow resonance leaving a contribution from the resonance pole.

Thank you - For your attention !

Thanks for inviting me !

I feel honored !

I has been a pleasure to come here !