Correlation properties of few charged bosons in anisotropic traps

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Critical Stability 2011

Outline

Experimentally accessible quantum systems

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contact interaction

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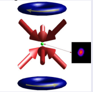
ultracold atoms trapped in optical lattices or optical microtraps



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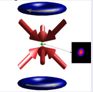

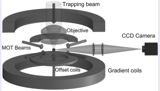

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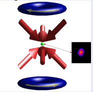

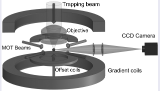

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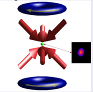

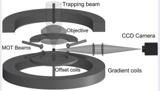

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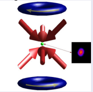

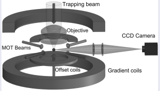

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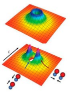

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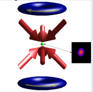

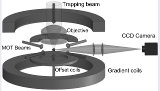

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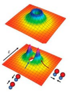

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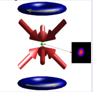

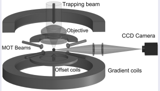

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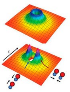

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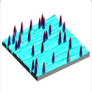

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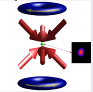

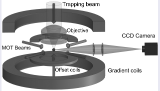

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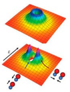

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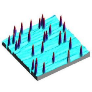

ions in EM traps

System of N Coulombically interacting bodies

Schrödinger equation

Hamiltonian:

$$\rho_i = \sqrt{y_i^2 + z_i^2}$$

$$H = \sum_{i=1}^{N}[-\frac{\hbar^2\nabla_i^2}{2m} + \frac{m}{2}(\omega_x^2 x_i^2 + \omega_\perp^2 \rho_i^2)] + \sum_{i<j} \frac{\gamma}{|\mathbf{r}_i - \mathbf{r}_j|},$$

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Correlation measures

Correlation energy

$E_{corr} = E_{MF} - E_{exact}$ (Lövdin 1955)

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Statistical correlation coefficient

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expectation values weighted with the probability density $|\psi(\mathbf{r}_1, ..., \mathbf{r}_N)|^2$ and integrated over the remaining variables.

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Measures based on 1-RDM

entanglement entropies - global characteristics for pure states

1-RDM admits a Schmidt decomposition

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Rényi entropies

$S(q) = \frac{1}{1-q} \ln \sum \lambda_k^q$

We consider strong anisotropy case $\epsilon \gg 1$

single transverse-mode approximation

The particles assumed to stay in the lowest state of $H_{\perp}$

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$N-$body wave function approximated by:
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Integration over $\perp$ gives quasi-1D Schrödinger equation

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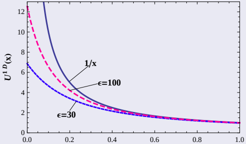

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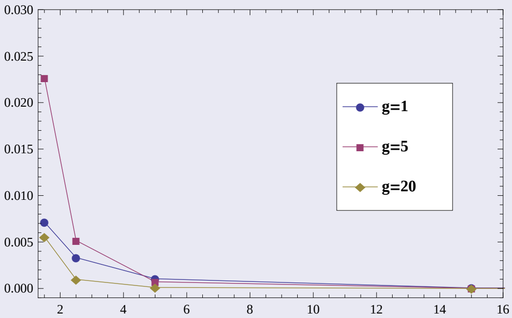
Applicability of the single-mode approximation: $N = 2$

Comparison of E_0^{1D} with exact E_0 from 3D Hamiltonian

relative error

$$\Delta E = \frac{|E_0 - E_0^{1D}|}{E_0}$$



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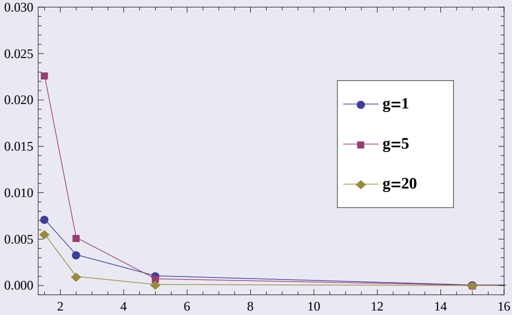

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Anisotropies $\epsilon \gtrsim 5$ are sufficiently large for employing the single mode approximation in calculating GS energy.

1-RDM in the single-mode approximation

single-mode approximation

1-RDM factorizes to the form

$$\rho(\mathbf{r},\mathbf{r}') = \varphi(y)\varphi(y')\varphi(z)\varphi(z')\rho_{1D}(x,x'),$$

with effective 1D RDM

$$\rho_{1D}(x,x') = \\ = \int \psi(x,x_2,...,x_N)\psi(x',x_2,...,x_N)dx_2...dx_N = \sum \lambda_l v_l(x) v_l(x')$$

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single-mode approximation

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We calculate the GS wave function of the quasi-1D Hamiltonian $\psi(x_1,x_2,...,x_N)$ with the quantum diffusion algorithm and use it to determine $\rho_{1D}(x,x^{'})$ and L.

Strictly 1D limit

divergencies

Interaction potential $\frac{g}{|x_i - x_j|}$ diverges at $x_i = x_j$ for $g \neq 0$.

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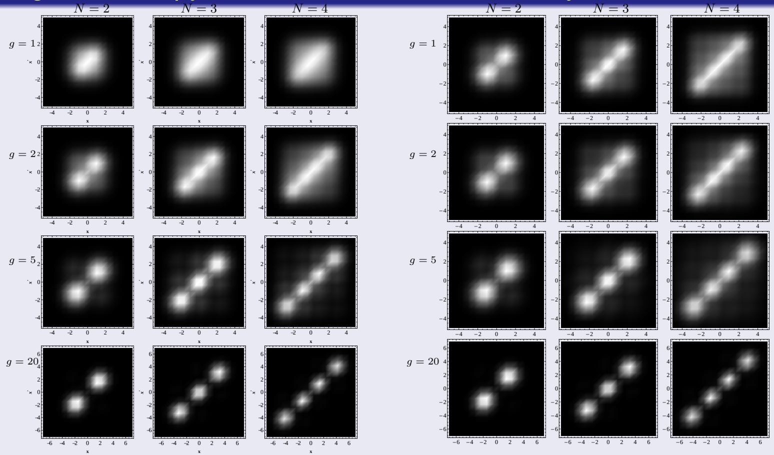
We determine ψ_F using CI with harmonic oscillator basis $\{\varphi_n^{ho}\}$.

Plot of the effective 1-RDM $\rho_{1D}(x, x')$

large anisotropy, $\epsilon = 30$ — strictly 1D



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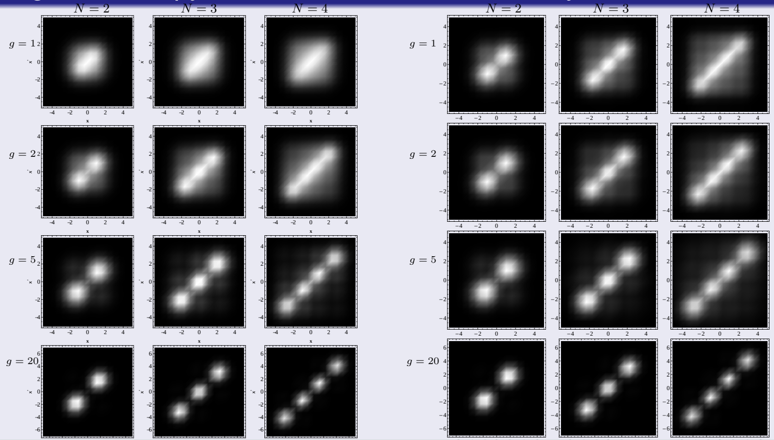

At $\epsilon = 30$ the quasi-1D RDM differs heavily from that of a genuinely 1D system. Big differences at $g \lesssim 5$.

Ground-state linear entropy

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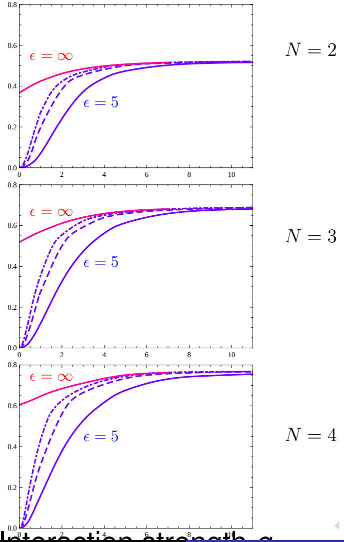
Linear entropy L

discontinuity at $g = 0$



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Jan Kochanowski University

Asymptotics in the strictly 1D system

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numerical results:

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Comparison of the linear and von Neumann entropies

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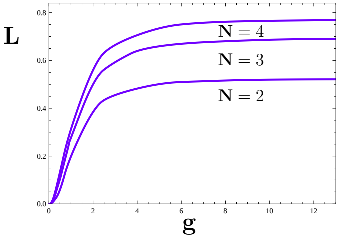
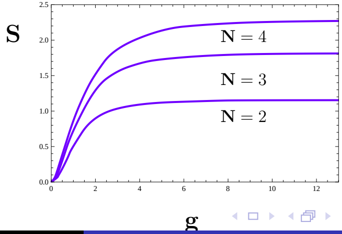
$\epsilon = 30$

linear entropy



von Neumann entropy

P. Kościk, Few-Body Syst.(2011)



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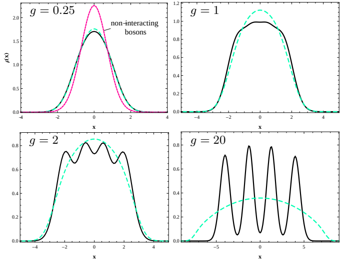

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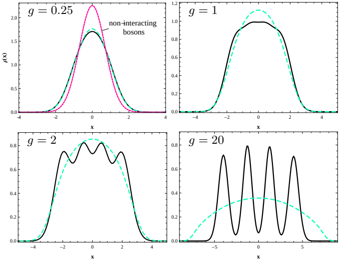

MF fails to describe the internal structure of the system if $g \gtrsim g_{cr}$

Summary and outlook

Systems composed of N=2,3, and 4 Coulombically interacting particles in a harmonic trap of anisotropy ϵ are investigated.

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- Dependence on ϵ is significant only at weak interaction $g \lesssim g_{cr}$. In this region the entanglement properties of the quasi-1D system differ heavily from a genuinely 1D one.