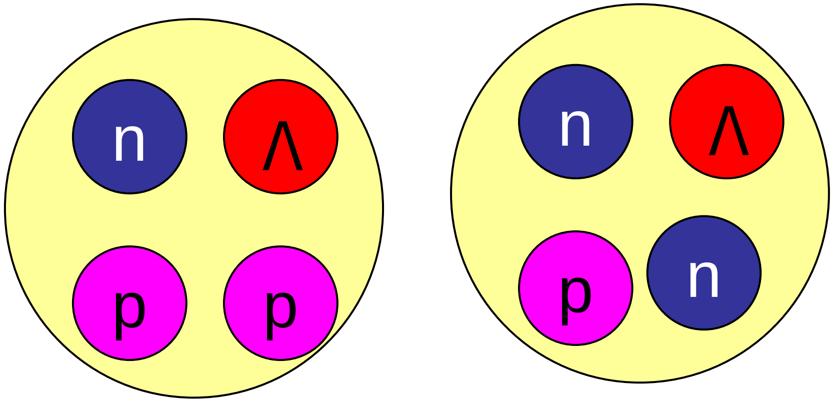
Four-body calculations of ^{4}He tetramer and light hypernuclei using realistic two-body potentials

E. Hiyama (RIKEN)

Outline of my talk

(1) Introduction

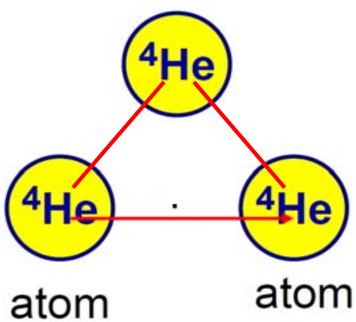
(2)

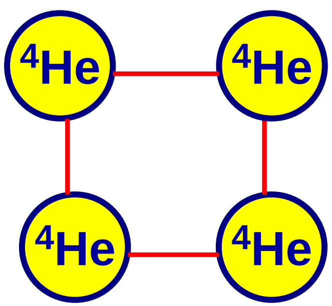


$^{4}_{\Lambda}He$ $^{4}_{\Lambda}H$

These systems are important from view points of critical stability.

(2)





LM2M2 potential

Introduction

My research purpose:

To apply our own method (Gaussian Expansion Method) to N-body problem.

10-body problems

To establish the following framework

・To calculate any interactions such as central force, spin-orbit force, tensor force, momentum dependent force, quadratic spin-orbit force etc.

・To calculate particle conversion interaction such as ΛN-ΣN, ΛΛ—ΞN—ΣΣ etc.

・To calculate bound states, resonant states and to treat continuum states

Present status

	3-body	4-body	5-body
▪ any interactions	done	done	partly
▪ particle-conversion	done	done	Not yet
▪ bound state	done	done	done
▪ resonant state	done	partly	partly
▪ continuum state	partly	partly	partly

➡ to 6-body problem next year

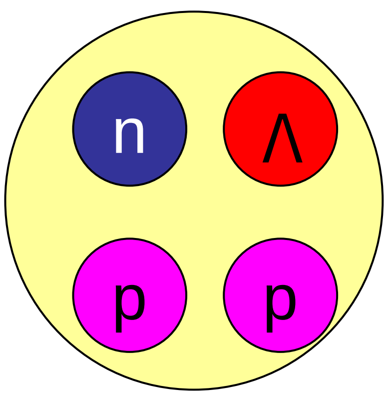
Few-nucleon systems and hypernuclear physics has been encouraging my method to develop to the above treatments.

Especially, to treat potential to have high repulsive core and long range tail is interesting subject for me.

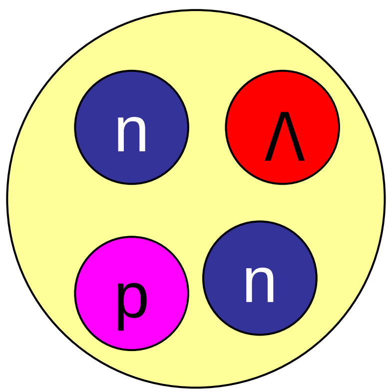
For this purpose, hypernuclear physics provide us many challenging subjects.

In hypernuclear physics, we have realistic interactions such as Nijmegen model (Nijmegen soft core 97, Extended soft core 08, etc)

- To have high repulsive core
- particle conversion interaction such as ΛN-ΣN coupling.



${}^{4}_{\Lambda}He$

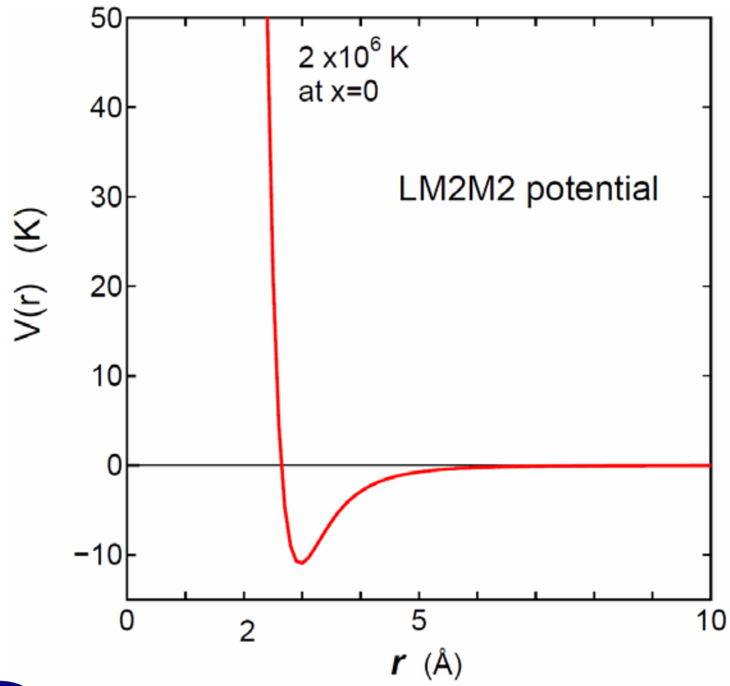


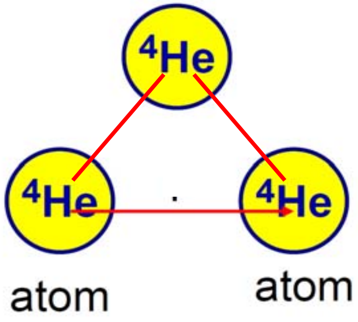
${}^{4}_{\Lambda}H$

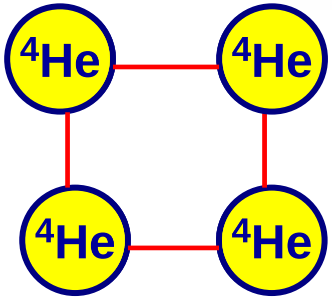
Another interesting subject is to solve bound states in ^{4}He trimer and tetramer systems.

The potential between two ^{4}He has high repulsive core and long-ranged tail.

To solve these systems encourages us to develop our method, Gaussian Expansion Method.







Next, I shall explain our method, Gaussian Expansion Method.

Our few-body caluclational method

Gaussian Expansion Method (GEM) , since 1987

- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

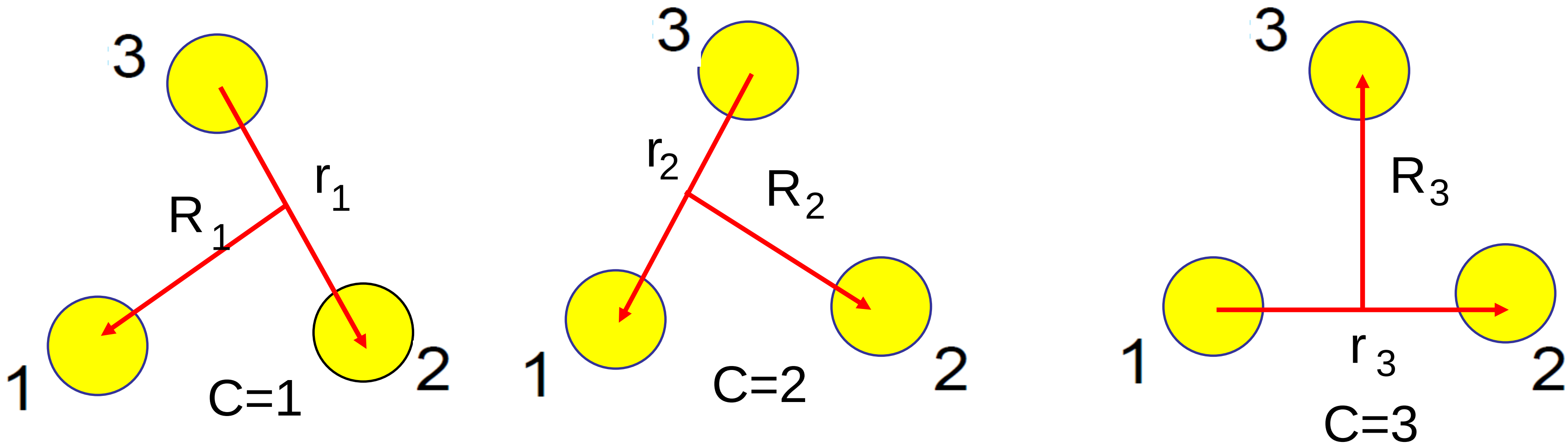
Developed by Kyushu Univ. Group,
Kamimura and his collaborators.

Review article :
E. Hiyama, M. Kamimura and Y. Kino,
Prog. Part. Nucl. Phys. 51 (2003), 223.

High-precision calculations of various 3- and 4-body systems:

Exotic atoms / molecules , Light hypernuclei,
3- and 4-nucleon systems, 3-quark systems,
multi-cluster structure of light nuclei,

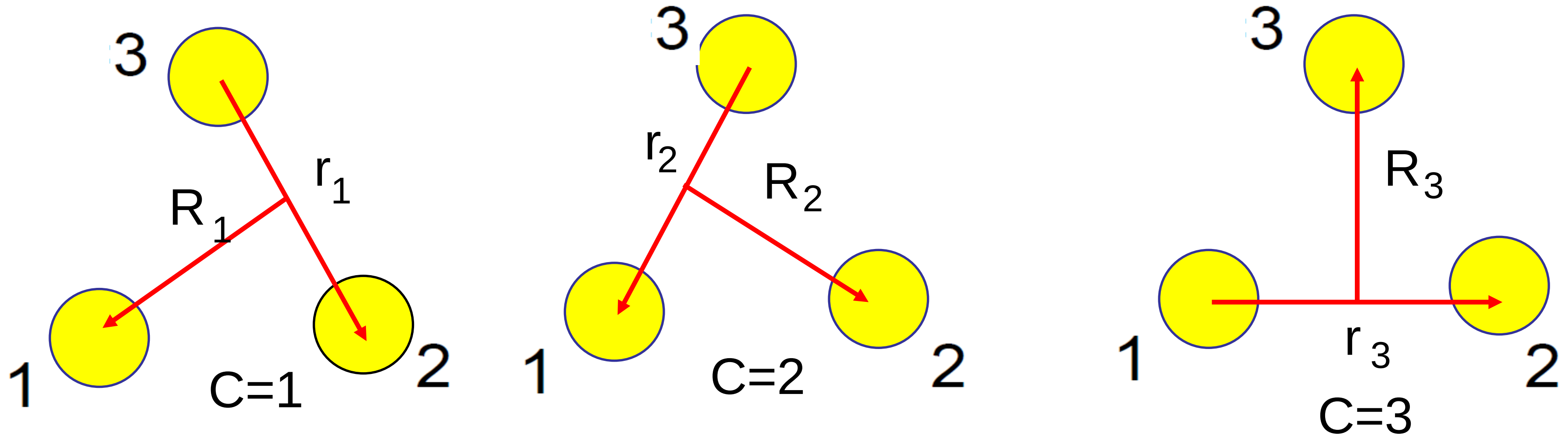
Gaussian Expansion Method (GEM)



$$H = -\frac{\hbar^2}{2\mu_{r_c}}\nabla^2_{\mathbf{r}_c} - \frac{\hbar^2}{2\mu_R}\nabla^2_{\mathbf{R}_c} + V^{(1)}(r_1) + V^{(2)}(r_2) + V^{(3)}(r_3)$$

$$[\, H - E \,]\, \Psi_{JM} = 0$$

$$\Psi_{JM} = \Phi^{(1)}_{JM}(\mathbf{r}_1, \mathbf{R}_1) + \Phi^{(2)}_{JM}(\mathbf{r}_2, \mathbf{R}_2) + \Phi^{(3)}_{JM}(\mathbf{r}_3, \mathbf{R}_3)$$



$$\Psi_{JM} = \Phi^{(1)}_{JM}(\mathbf{r}_1, \mathbf{R}_1) + \Phi^{(2)}_{JM}(\mathbf{r}_2, \mathbf{R}_2) + \Phi^{(3)}_{JM}(\mathbf{r}_3, \mathbf{R}_3)$$

Basis functions of each Jacobi coordinate

$(c = 1 - 3)$

$$\phi^{(c)}_{\underline{nl}}(r_c)\, Y_{\underline{lm}}(\widehat{\mathbf{r}}_c), \quad \psi^{(c)}_{NL}(R_c)\, Y_{LM}(\widehat{\mathbf{R}}_c)$$

$$\Phi^{(c)}_{JM}(\mathbf{r}_c, \mathbf{R}_c) = \sum_{nl,NL} \underline{C_{NL,lm}}\, \phi^{(c)}_{nl}(r_c)\, \psi^{(c)}_{NL}(R_c)\, [Y_l(\widehat{\mathbf{r}}_c) \otimes Y_L(\widehat{\mathbf{r}}_c)]_{JM}$$

$(c = 1 - 3)$

Determined by diagonalizing H

Radial part : Gaussian function

$$\phi_{nl}(r) = r^l \, e^{-(r/r_n)^2}$$

$$\psi_{NL}(R) = R^L \, e^{-(R/R_N)^2}$$

Gaussian ranges in geometric progression

$$r_n = r_1 \, a^{n-1} \qquad (n = 1 - n_{\max}) \; ,$$

$$R_N = R_1 \, A^{N-1} \qquad (N = 1 - N_{\max})$$

Both the short-range correlations and the exponentially-damped tail are simultaneously reproduced accurately.

Next, by solving eigenstate problem, we get eigenenergy E and unknown coefficients C_n .

$$\left[(H_{in}) - E\,(N_{in}) \right] \left[\mathbf{C_n} \right] = 0$$

In principle, we can apply this method to N-body problem.

However,…

By solving eigenstate problem, we get eigenenergy E and unknown coefficients C_n .

$$\left[(H_{in}) - E (N_{in}) \right] \left[\mathbf{C_n} \right] = 0$$

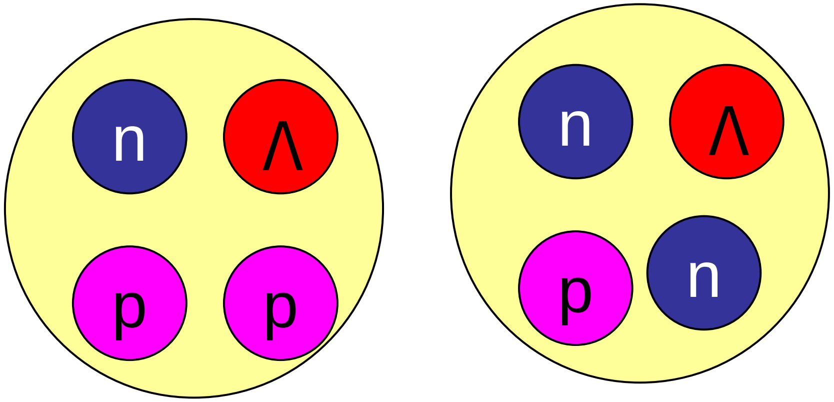
The problem:
we need huge memory to calculate N-body systems.

In September in 2011, in KEK, they provide powerful super computer with 256 GB (HITACHI-SR16000).
I enjoy using this computer.

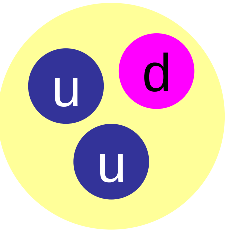
Outline of my talk

(1) Introduction

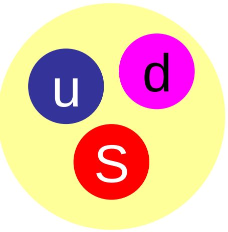
(2)



$^{4}_{\Lambda}He$ $^{4}_{\Lambda}H$



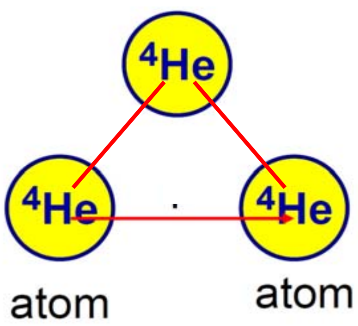
neutron

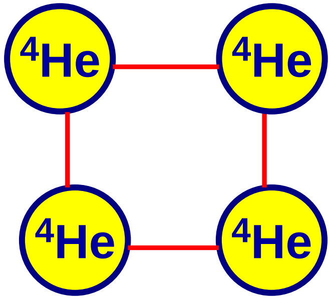


Λ

These systems are important from view points of critical stability.

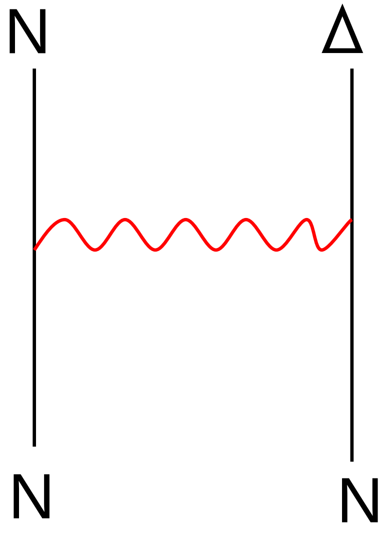
(3)





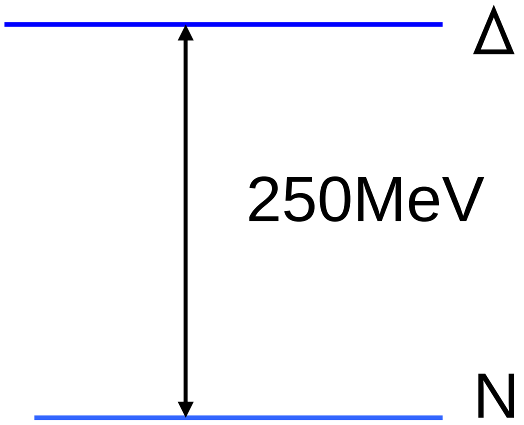
LM2M2 potential

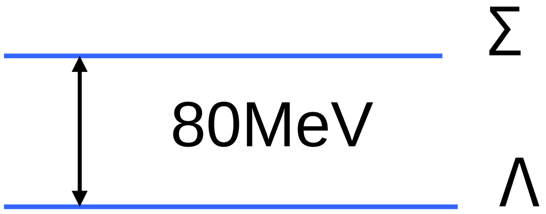
Non-strangeness nuclei



Nucleon can be converted into Δ. However, since mass difference between nucleon and Δ is large, then probability of Δ in nucleus is not large.

On the other hand, the mass difference between Λ and Σ is much smaller, then there is significant probability of Σ in Λ hypernuclei.



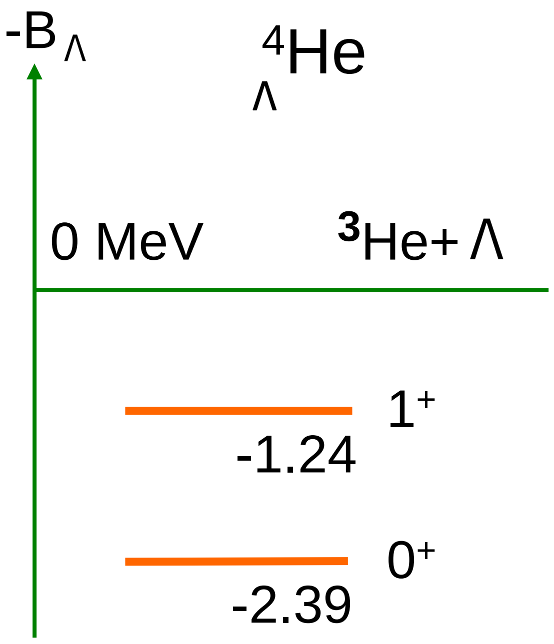


Interesting Issues for the ΛN-ΣN particle conversion in hypernuclei

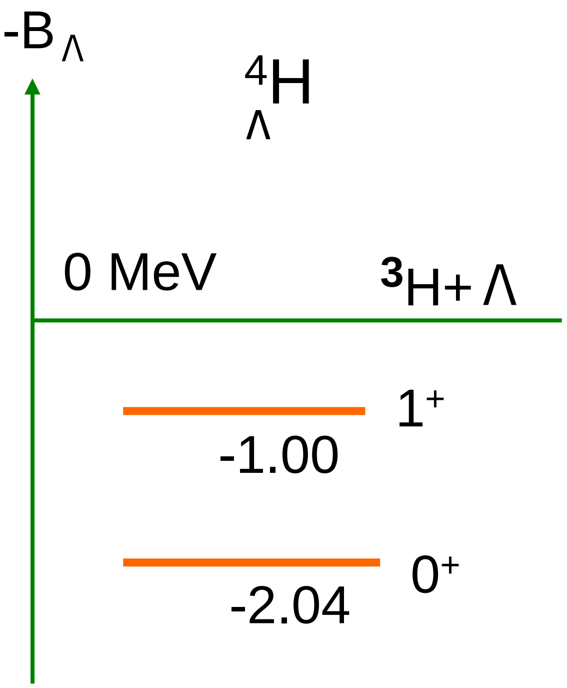
(1)How large is the mixing probability of the Σ particle in the hypernuclei?

(2) How important is the ΛN—ΣN coupling in the binding energy of the Λ hypernuclei?

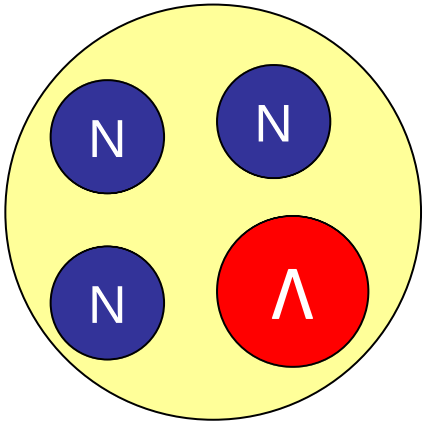
study of $^{4}_{\Lambda}He$ and $^{4}_{\Lambda}H$ is the most useful because both of the spin-doublet states are observed.



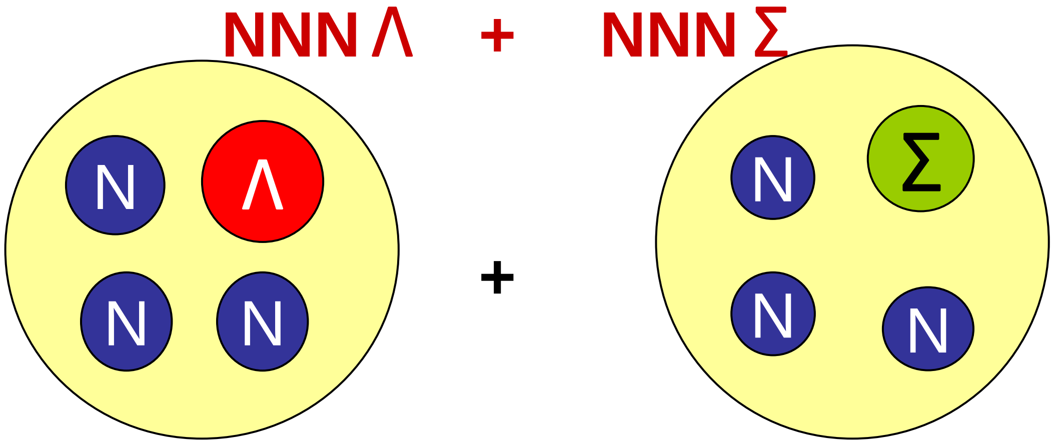
Exp.



Exp.



For precise studies of ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H , it is highly desirable to perform full 4-body calculations taking both the NNNΛ and NNNΣ channels explicitly.



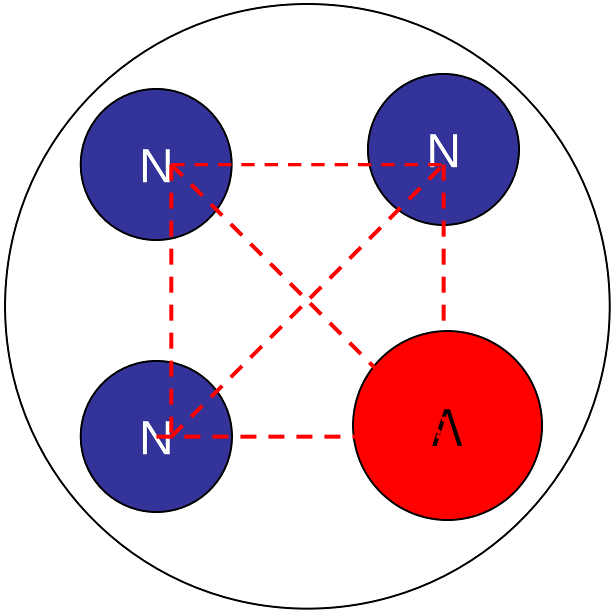
So far, the following authors succeeded in performing this type of difficult 4-body calculation and pointed out that the ΛN−ΣN particle conversion is very important to make these A=4 hypernuclei bound.

Full 4-body clculations :

1) E. Hiyama et al., Phys. Rev. C65, 011301 (R) (2001).

2) A. Nogga et al., Phys. Rev. Lett. 88, 172501 (2002).

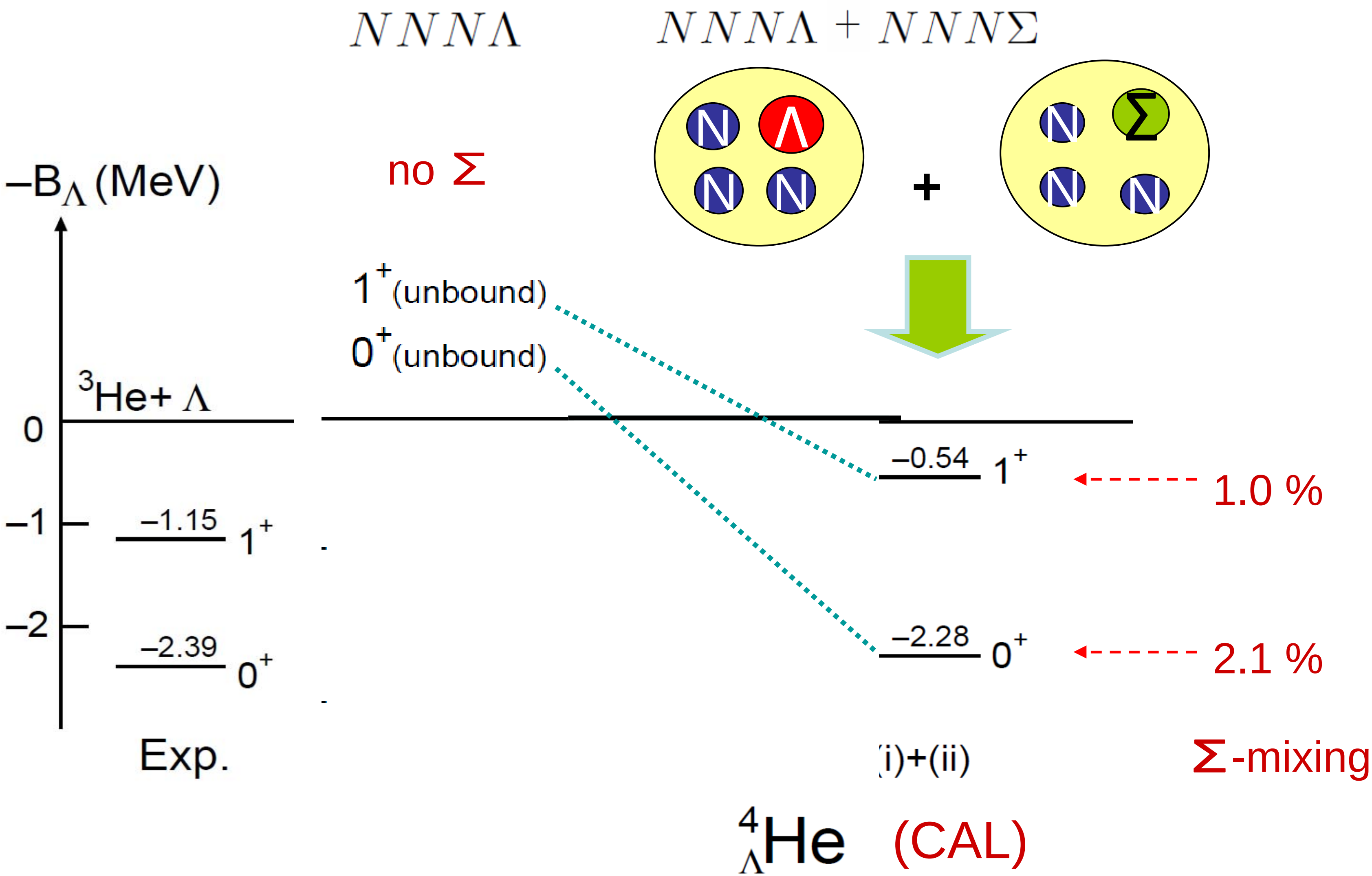
3) H. Nemura et al., Phys. Rev. Lett. 89, 142502 (2002).



$^{4}_{\Lambda}He$, $^{4}_{\Lambda}H$

V_{NN} : AV8 potential

V_{YN} : Nijmegen soft-core ’97f potential



Although the Σ-mixing probability is small,
we find that the Σ-mixing plays an essential role
to make critical stability in these A-4 hypernuclei.

Outline of my talk

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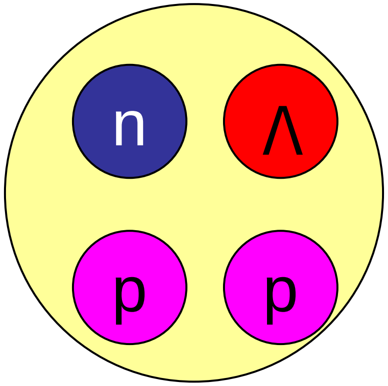
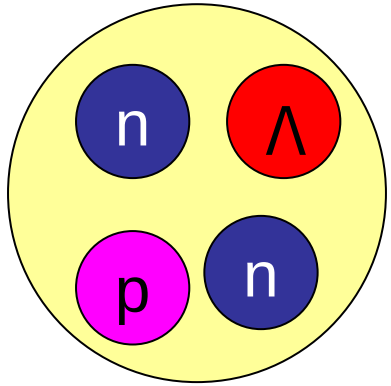
(2)



$^{4}_{\Lambda}$He



$^{4}_{\Lambda}$H

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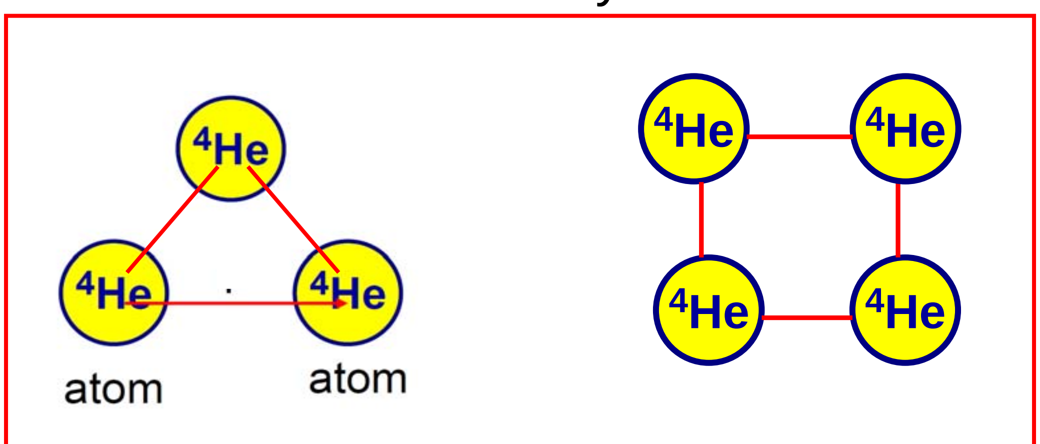
(3)

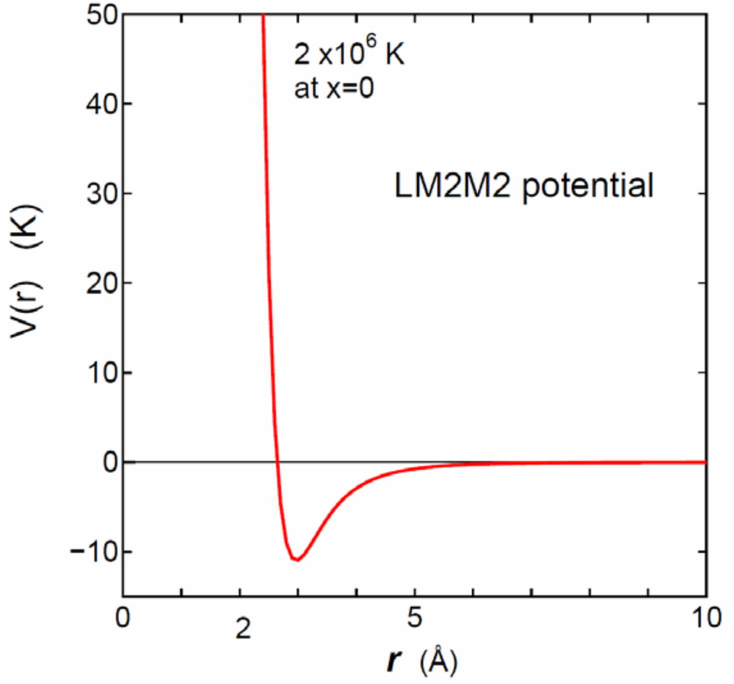


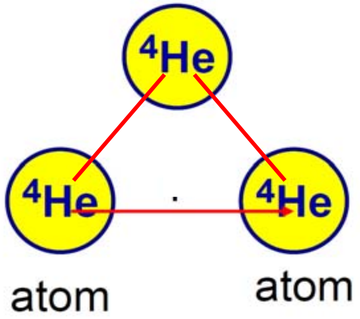
LM2M2 potential

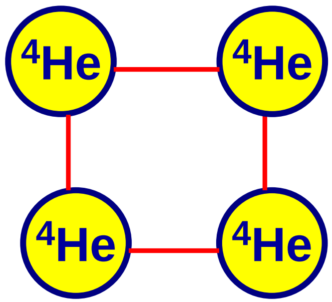
Another interesting subject is to solve bound states in ^{4}He trimer and tetramer systems.

The potential between two ^{4}He such as LM2M2 potential has high repulsive core and long-ranged tail.

To solve these systems encourages us to develop our method, Gaussian Expansion Method.



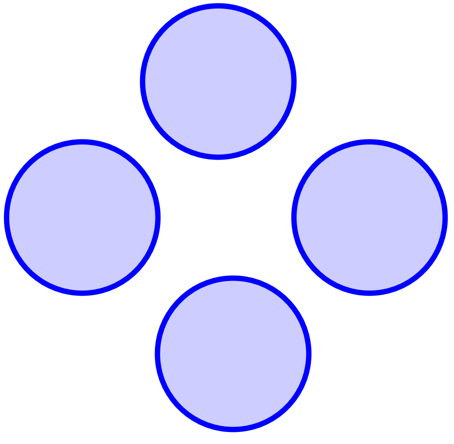




^{4}He Tetramer

ground and excited states

using LM2M2 potential



arXiv : at the end of this month

Variational calculation of ^{4}He tetramer ground and excited states using a realistic pair potential

LM2M2 potential
with a strong
short-range
repulsion.

E. Hiyama
RIKEN Nishina Center, RIKEN, Wako 351-0198, Japan

M. Kamimura
Department of Physics, Kyushu University, Fukuoka 812-8581, Japan,

Outline

1) We take our Gaussian expansion method for few-body systems that was used in the study of hypernuclear physics reported in the 1-st part of this talk.

2) We shall show that the binding energy of the ground state is 558.98 mK and that of excited state is 127.33 mK (only 0.93 mK below the trimer).

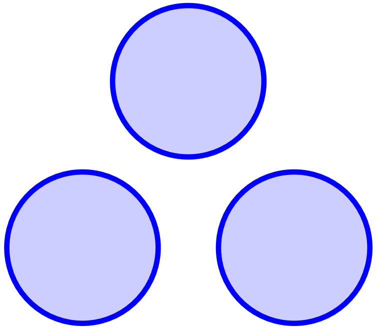
3) We shall precisely discuss about the short-range structure of the tetramer ground and excited states and their asymptotic behavior up to 1000 A.

4) Before presenting the tetramer calculation, we report a trimer calculation in comparison with literature results in order to show reliability of our method.

^{4}He Trimer

ground and excited states

using LM2M2 potential

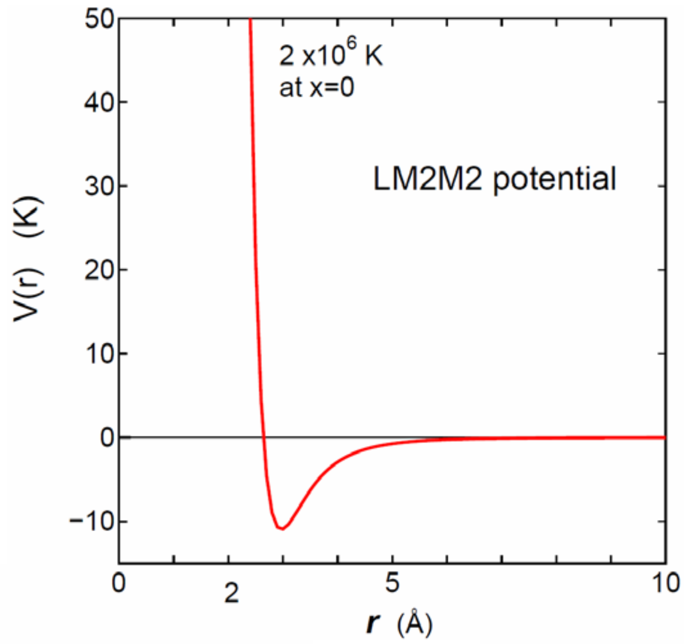


Gaussian expansion method for

1) three identical spinless particles

2) very weakly bound states,

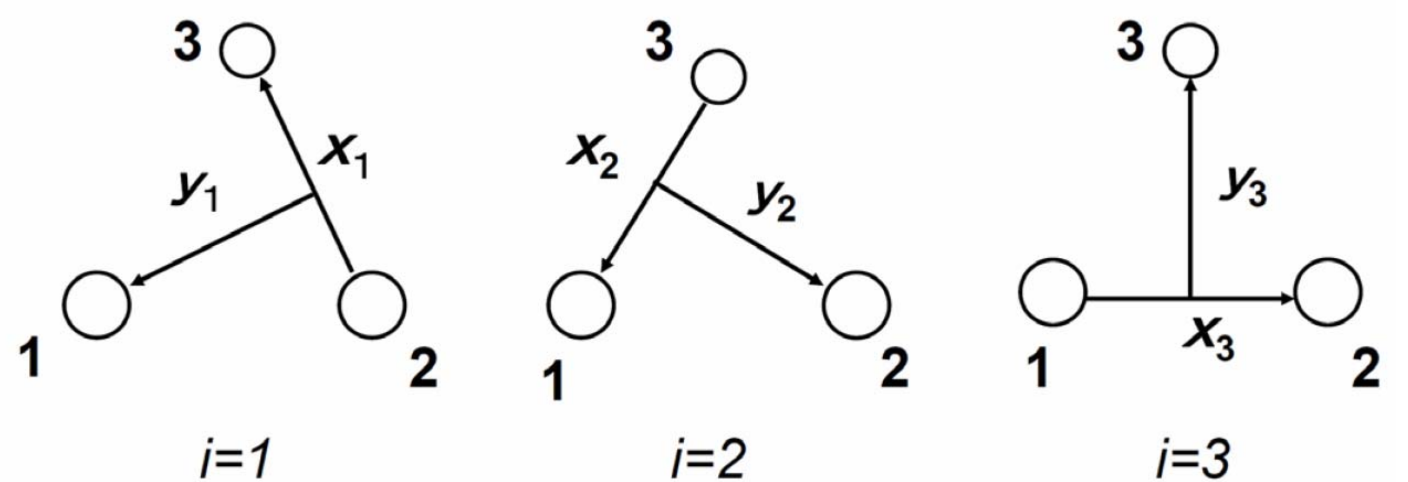
3) using realistic potential (LM2M2)



$$(H - E)\Psi^{(3)} = 0.$$

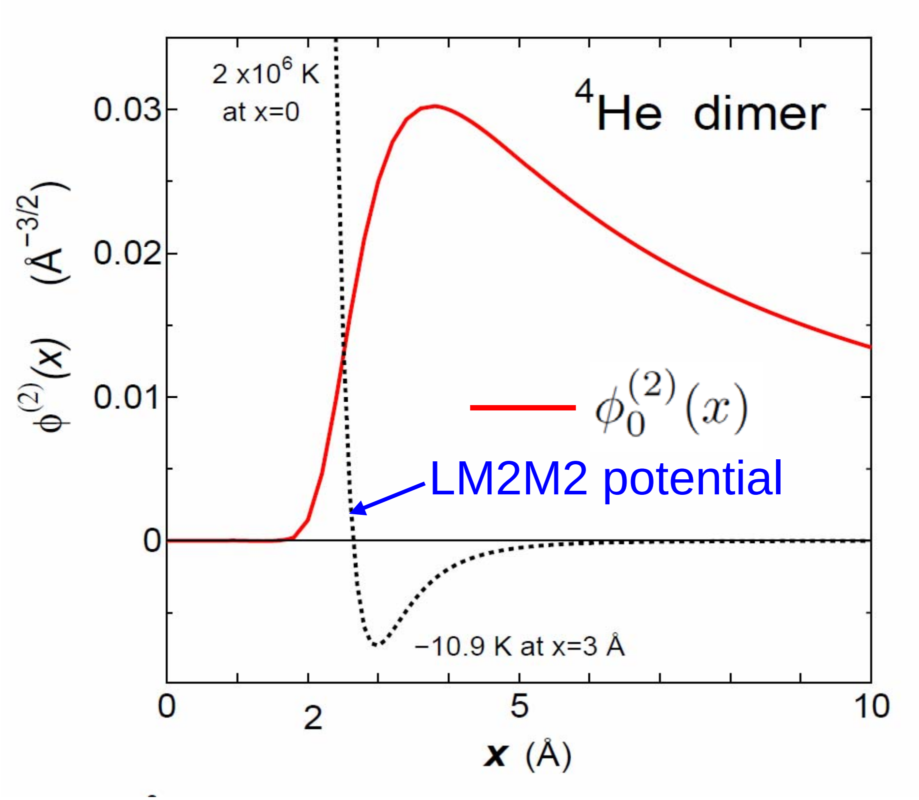
$$H = -\frac{\hbar^2}{2\mu_x}\nabla_x^2 - \frac{\hbar^2}{2\mu_y}\nabla_y^2 + \sum_{1=i<j}^{3} V(r_{ij}), \qquad)$$

We take all the three sets of Jacobi coordinates:



^{4}He Dimer

$B^{(2)} = 1.30348$ mK and $\sqrt{\langle x^2 \rangle} = 70.93$ Å,
the same as those obtained in literature.



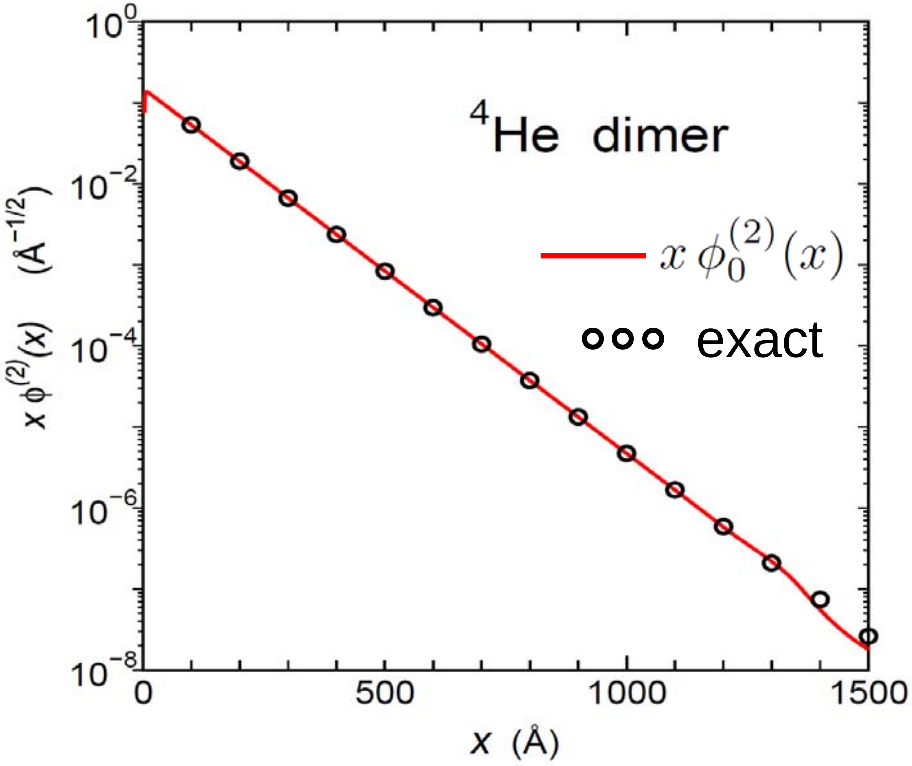


Fig.2

Trimer ground state

(LM2M2 potential)

Present result excellently agree with literature.

trimer	ground state			
	present	Ref.[7]	Ref.[8]	Ref.[9]
binding energy $B_0^{(3)}$ (mK)	126.40	126.39	126.4	126.40
$\langle T \rangle$ (mK)	1660.4	1658	1660	
$\langle V \rangle$ (mK)	−1786.8	−1785	−1787	
$\sqrt{\langle r_{ij}^2 \rangle}$ (Å)	10.96	10.95	10.96	
$\langle r_{ij} \rangle$ (Å)	9.616	9.612	9.636	
$\langle r_{ij}^{-1} \rangle$ (Å^{-1})	0.134	0.135		
$\langle r_{ij}^{-2} \rangle$ (Å^{-2})	0.0228	0.0230		
$\sqrt{\langle r_{iG}^2 \rangle}$ (Å)	6.326		6.49	6.32

[7] R. Lazauskas and J. Carbonell, Phys. Rev. A **73** (2006) 062717.

[8] P. Barletta and A. Kievsky, Phys. Rev. A **64** (2001) 042514.

[9] V.A. Roudnev, S.L. Yakovlev and S.A. Sofianos, Few-Body Syst. **37** (2005) 179.

Trimer **excited** state

(LM2M2 potential)

Present result excellently agree with literature.

trimer	excited state			
	present	Ref.[7]	Ref.[8]	Ref.[9]
binding energy $B_1^{(3)}$ (mK)	2.2706	2.268	2.265	2.2707
$\langle T \rangle$ (mK)	122.15	122.1	121.9	
$\langle V \rangle$ (mK)	-124.42	-124.5	-124.2	
$\sqrt{\langle r_{ij}^2 \rangle}$ (Å)	104.5	104.3		
$\langle r_{ij} \rangle$ (Å)	84.51	83.53	83.08	
$\langle r_{ij}^{-1} \rangle$ (Å^{-1})	0.0265	0.0267		
$\langle r_{ij}^{-2} \rangle$ (Å^{-2})	0.00216	0.00218		
$\sqrt{\langle r_{iG}^2 \rangle}$ (Å)	60.33			59.3

[7] R. Lazauskas and J. Carbonell, Phys. Rev. A **73** (2006) 062717.

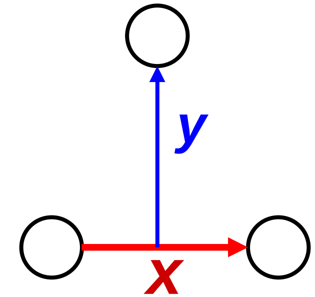
[8] P. Barletta and A. Kievsky, Phys. Rev. A **64** (2001) 042514.

[9] V.A. Roudnev, S.L. Yakovlev and S.A. Sofianos, Few-Body Syst. **37** (2005) 179.

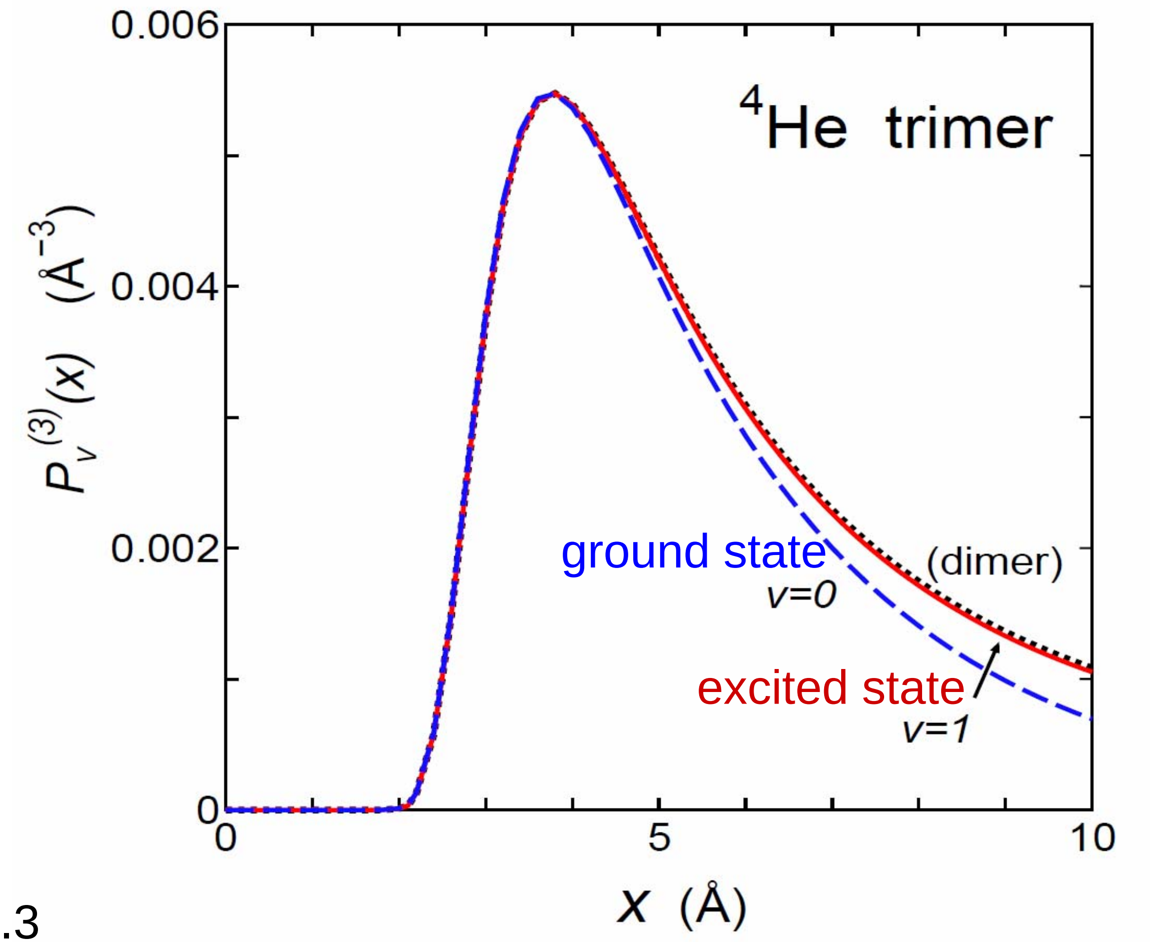
Strong short-range correlation

Pair correlation (distribution) function

$$P_v^{(3)}(\boldsymbol{x}) = \int |\Psi_v^{(3)}|^2 d\mathbf{y}$$



probability of finding two particles at x .



Already multiplied by

—— x 14.5

.... x 6.0

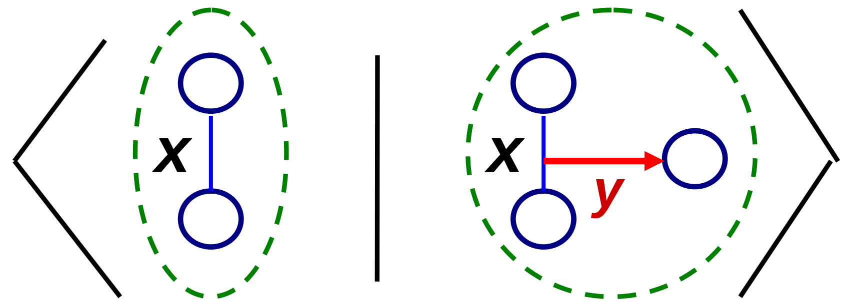
to be normalized
at the peak

Precisely the same shape of the short-range correlations (x<4 A) appear in all the states.

Fig.3

Overlap function

$$\mathcal{O}_v^{(3)}(y) = \int \phi_{00}^{(2)*}(\mathbf{x}_1)\, \Psi_v^{(3)}\, d\mathbf{x}_1 =$$



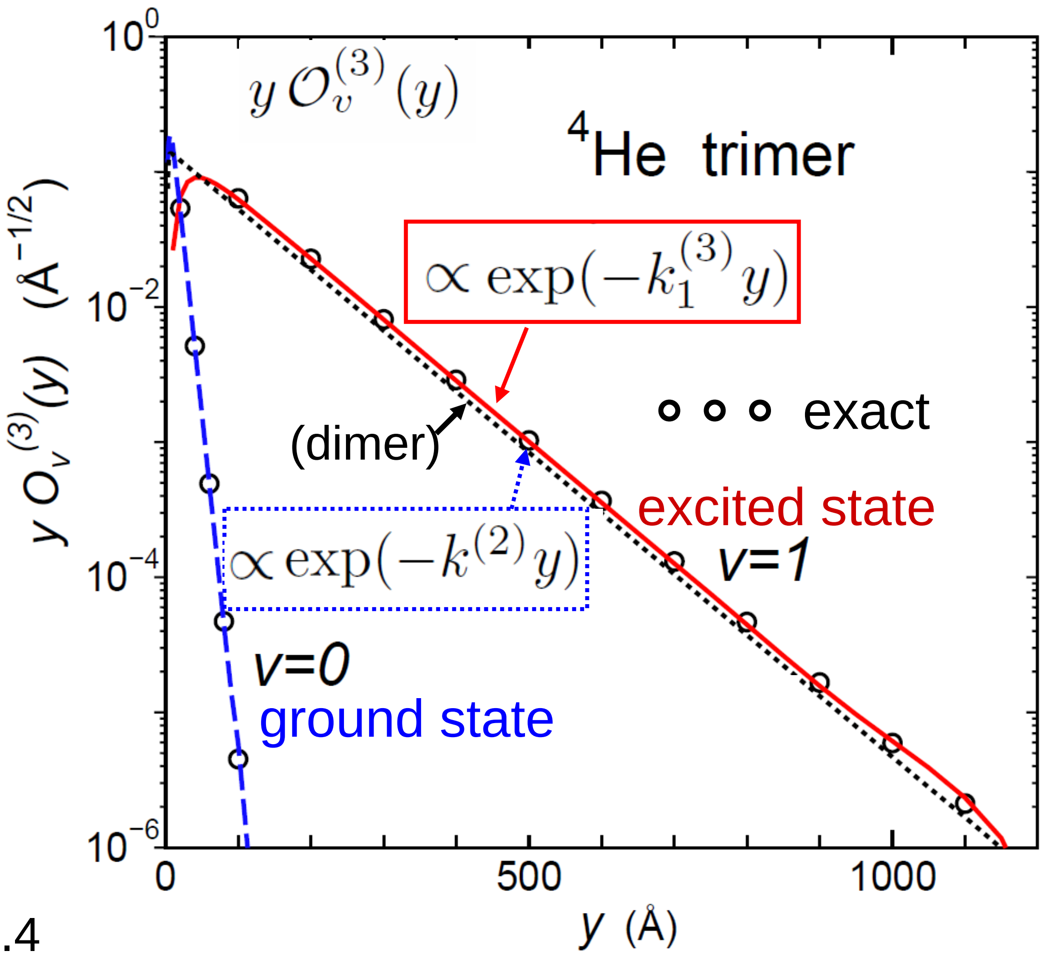


Fig.4

1) Asymptotic behavior of the trimer excited state is exactly decaying up to ∼1000 Å.

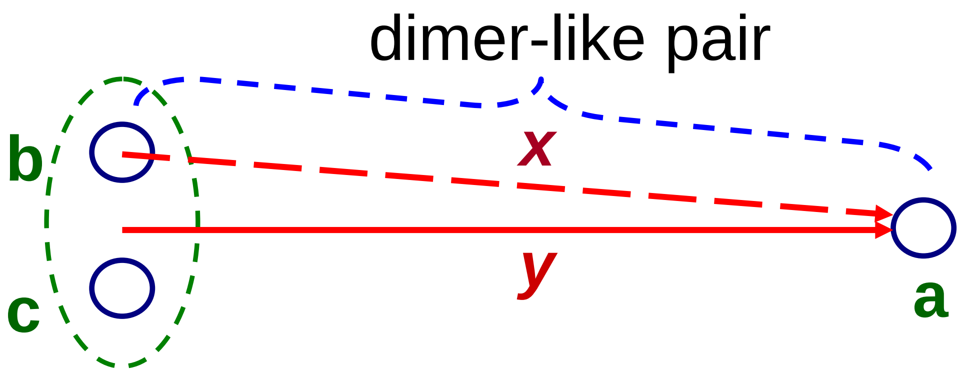
2) Two lines are parallel. Decaying constants are the same to each other.

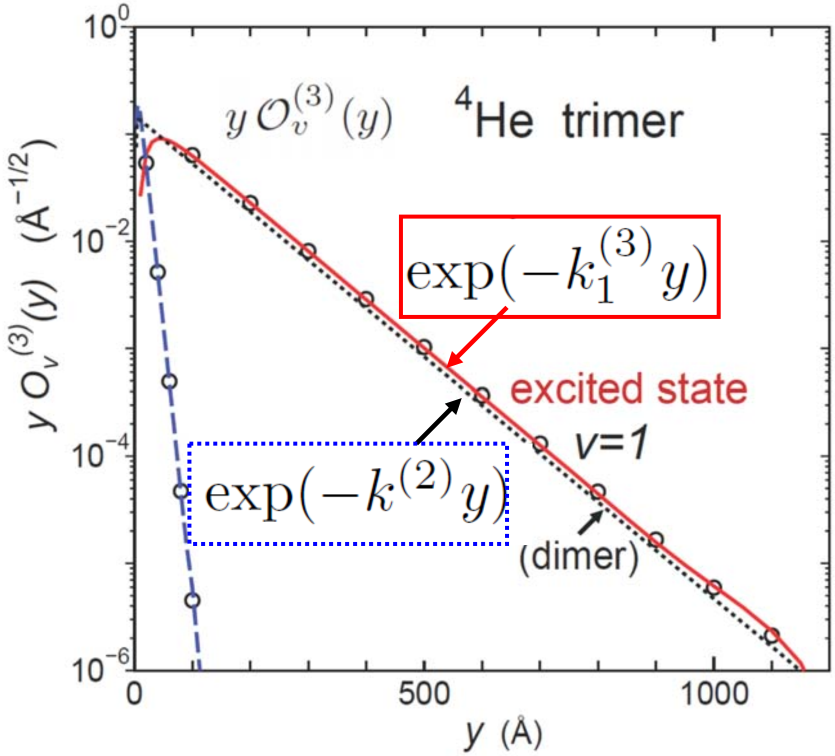
$$k_1^{(3)} = k^{(2)}$$ decaying constant

This is unexpected, but, understandable from the following model:

Dimer-like pair model

for asymptotic behavior of trimer excired state





1) Particle **a** (located far from loosely-bound **b** and **c**) is not affected by the interaction between **b** and **c**,

2) Therefore, the pair **a**-**b** at a relative distance x is asymptotically dimer-like.

3) Since $x \approx y$ asymptitically, the amplitude of particle **a** along y is dimerlike, $k_1^{(3)} = k^{(2)}$

If we accept this model, we can estimate

$$\Delta B_1^{(3)} (= B_1^{(3)} - B^{(2)})$$ binding energy of trimer excited state measured from dimer.

by using $B^{(2)}$ only.

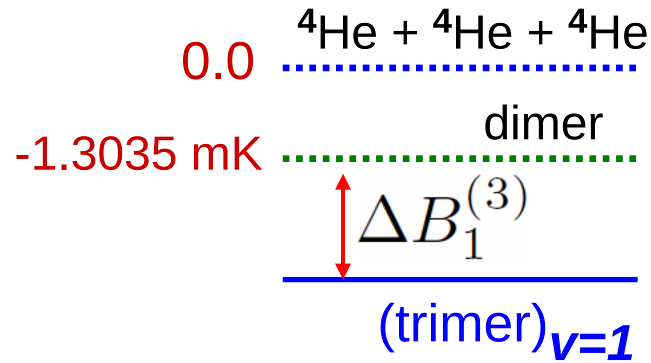
The binding energies are written as

$$B^{(2)} = \frac{\hbar^2}{2\mu_x} (k^{(2)})^2$$

$$\parallel$$

$$\Delta B_1^{(3)} = \frac{\hbar^2}{2\mu_y} (k_1^{(3)})^2$$

where $\mu_x = \frac{1}{2} m$ and $\mu_y = \frac{2}{3} m$,



If we take this relation $k_1^{(3)} = k^{(2)}$, then we have

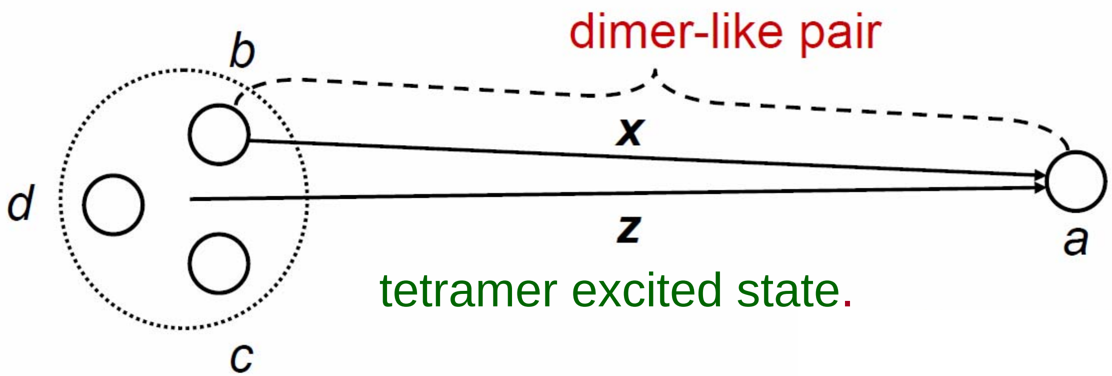
$$\Delta B_1^{(3)} = \frac{3}{4} B^{(2)} = 0.978 \text{mK}$$

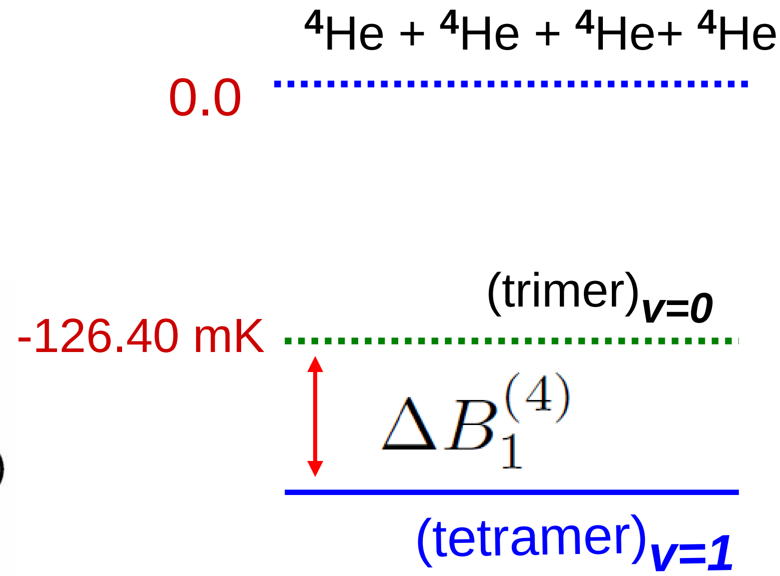
$$B_1^{(3)} = 2.281 \text{ mK}$$

Good model !

3-body calculation
0.967 mK
2.2706 mK

Also, if we accept this
dimer-like pair model
for the asymptotic behavior of
the tetramer excited state.





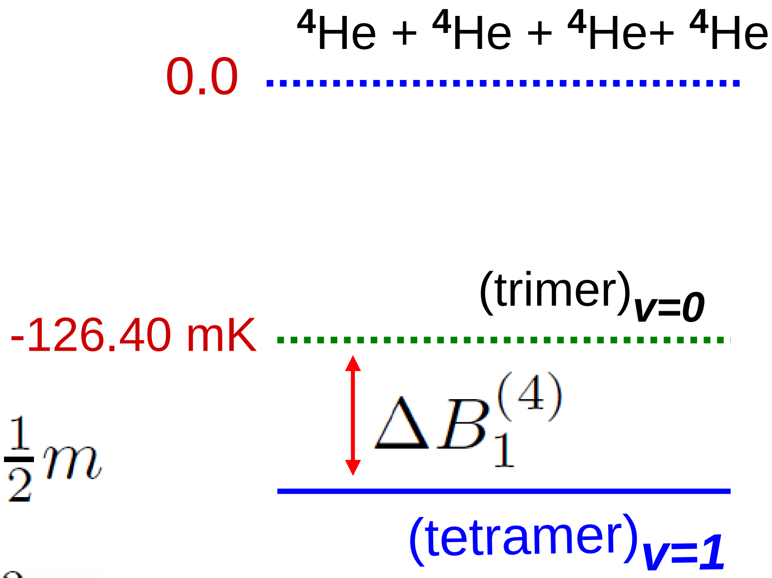
then, we can predict the binding energy

$$\Delta B_1^{(4)} (= B_1^{(4)} - B_0^{(3)})$$

of the tetramer excited state with respect to the trimer ground state as follows:

We can predict this binding energy

$$\Delta B_1^{(4)} (= B_1^{(4)} - B_0^{(3)})$$

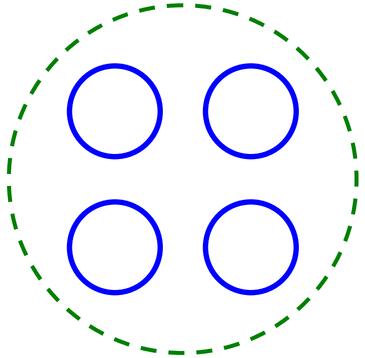


using the relation

$$B^{(2)} = \frac{\hbar^2}{2\mu_x}(k^{(2)})^2 \qquad \mu_x = \frac{1}{2}m$$

$$\|$$

$$\Delta B_1^{(4)} = \frac{\hbar^2}{2\mu_z}(k_1^{(4)})^2 \qquad \mu_z = \frac{3}{4}m$$



and the dimer-like model ($k_1^{(4)} = k^{(2)}$) .

We have

$$\Delta B_1^{(4)} = \frac{2}{3} B^{(2)} = 0.87 \text{ mK}$$

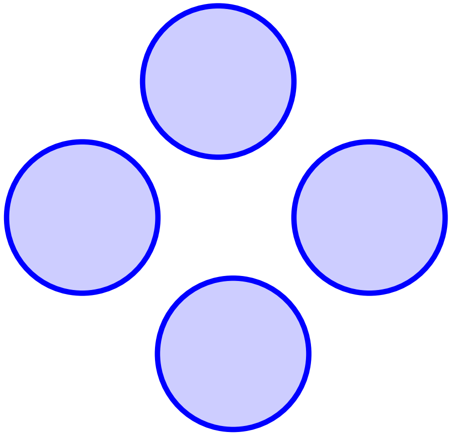
hence $B^{(4)} = 127.27 \text{ mK}$

We shall check this by the four-body calculation.

^4He Tetramer

ground and excited states

using LM2M2 potential



There are 5 calculations of tetramer using realistic pair potentials **(LM2M2, TTY)** .

^{4}He tetramer binding energies

Method	Reference	potential	ground state (mK)	excited state (mK)
Monte Carlo	Lewerenz (1977)	**TTY**	558	
Monte Carlo	Bressanini *et al.* (2000)	**TTY**	559.1	
Monte Carlo	Blume and Greene (2000)	**LM2M2**	557	133
Faddeev	Lazauskas and Carbonell (2006)	**LM2M2**	557.5	127.5
Correlated potential harmonic expansion	Das *et al.* (2011)	**TTY**	558	178

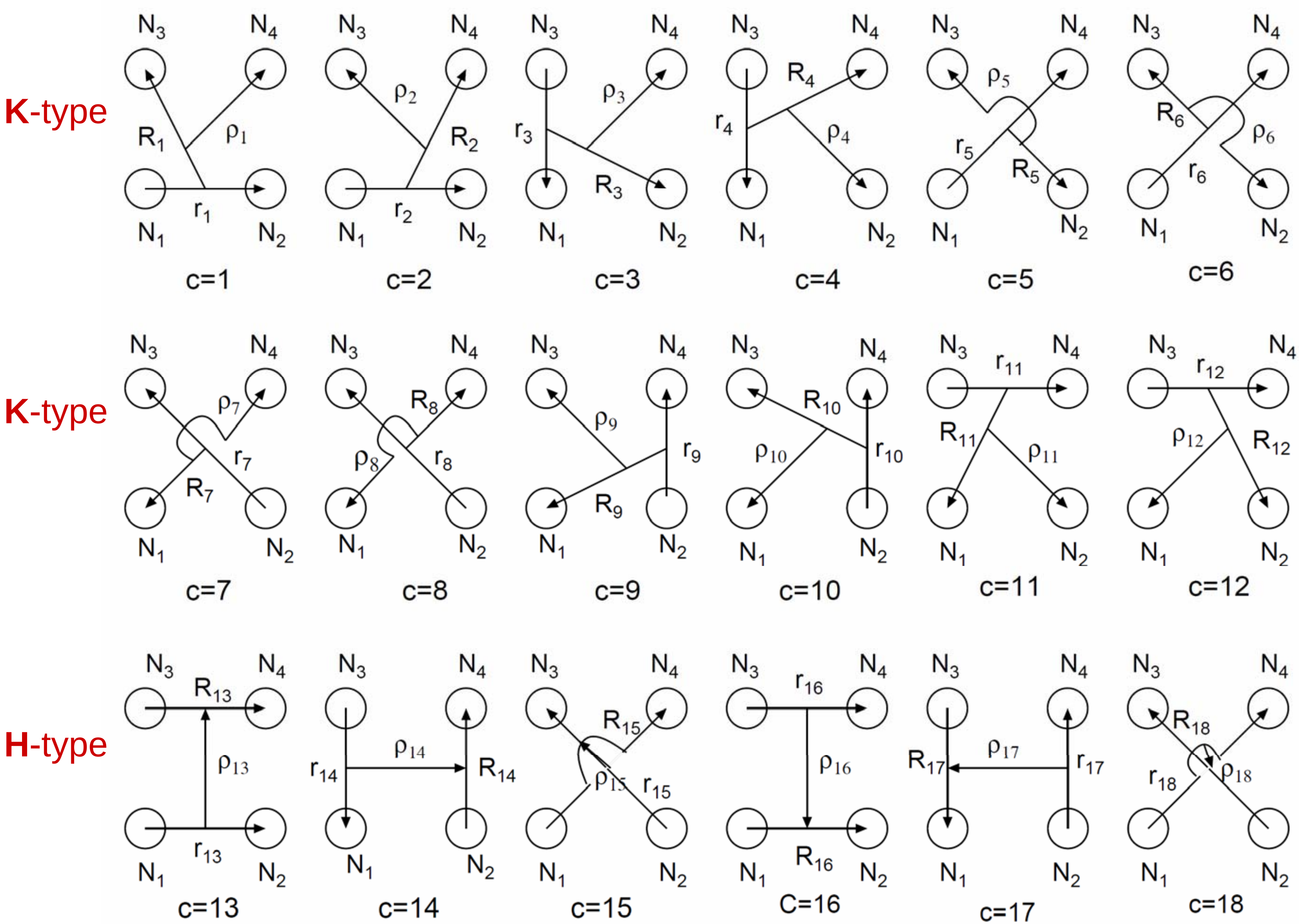
cf. Trimer g.s. = 126.40

This value was not obtained by bound-state calculation,

but was extrapolated from atom-trimer scattering calculation.

So, we intended to confirm this value by our bound-state calculation.

Full 18 sets of Jacobi coordinates for 4-body systems



Tetramer : Convergence of the binding energy with respect to maximum ang.-momentum (l_{max})

tetramer	present				Faddeev [7]
	$B_0^{(4)}$ (mK)		$B_1^{(4)}$ (mK)		$B_0^{(4)}$ (mK)
l_{max}	K+H	(K)	K+H	(K)	
	ground		excited		ground
0	500.71	(185.96)	–	(–)	348.8
2	558.29	(508.62)	127.24	(–)	505.9
4	558.98	(532.56)	127.33	(–)	548.6
6					556.0
8					557.7

Only 0.93 mK (=127.33 - 126.40) below the trimer ground state

In our calculation, l_max =4 is sufficient with totally 29000 symmetric 4-body basis functions.

[7] R. Lazauskas and J. Carbonell, Phys. Rev. A **73** (2006) 062717.

There are 5 calculations of **tetramer** using realistic pair potentials **(LM2M2, TTY)** .

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Correlated ←	Das *et al.* (2011)	**TTY**	558	178
GEM	Present (2011)	**LM2M2**	558.98	127.33

We confirmed that there is a very shallow excited bound state of ^{4}He tetramer.

(a) Tetramer --- binding energies and mean values

tetramer	ground state		excited state	
	present	Faddeev [7]	present	Faddeev [7]
$B^{(4)}$ (mK)	558.98	557.7	127.33	127.5*)
$\langle T \rangle$ (mK)	4282.2	4107	1639.2	
$\langle V \rangle$ (mK)	-4841.2	-4665	-1766.5	
$\sqrt{\langle r_{ij}^2 \rangle}$ (Å)	8.43	8.40	54.5	34.4*)
$\langle r_{ij} \rangle$ (Å)	7.70		35.8	
$\langle r_{ij}^{-1} \rangle$ (Å^{-1})	0.155		0.0792	
$\langle r_{ij}^{-2} \rangle$ (Å^{-2})	0.0285		0.0117	
$\sqrt{\langle r_{iG}^2 \rangle}$ (Å)	5.16		33.3	

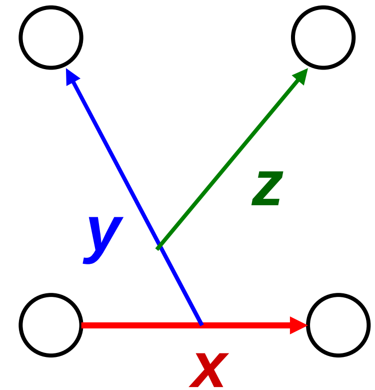
*) As for the excited state, Faddeev bound-state calculation was not performed, but these results were extrapolated from the low-enegy scattering calculation.

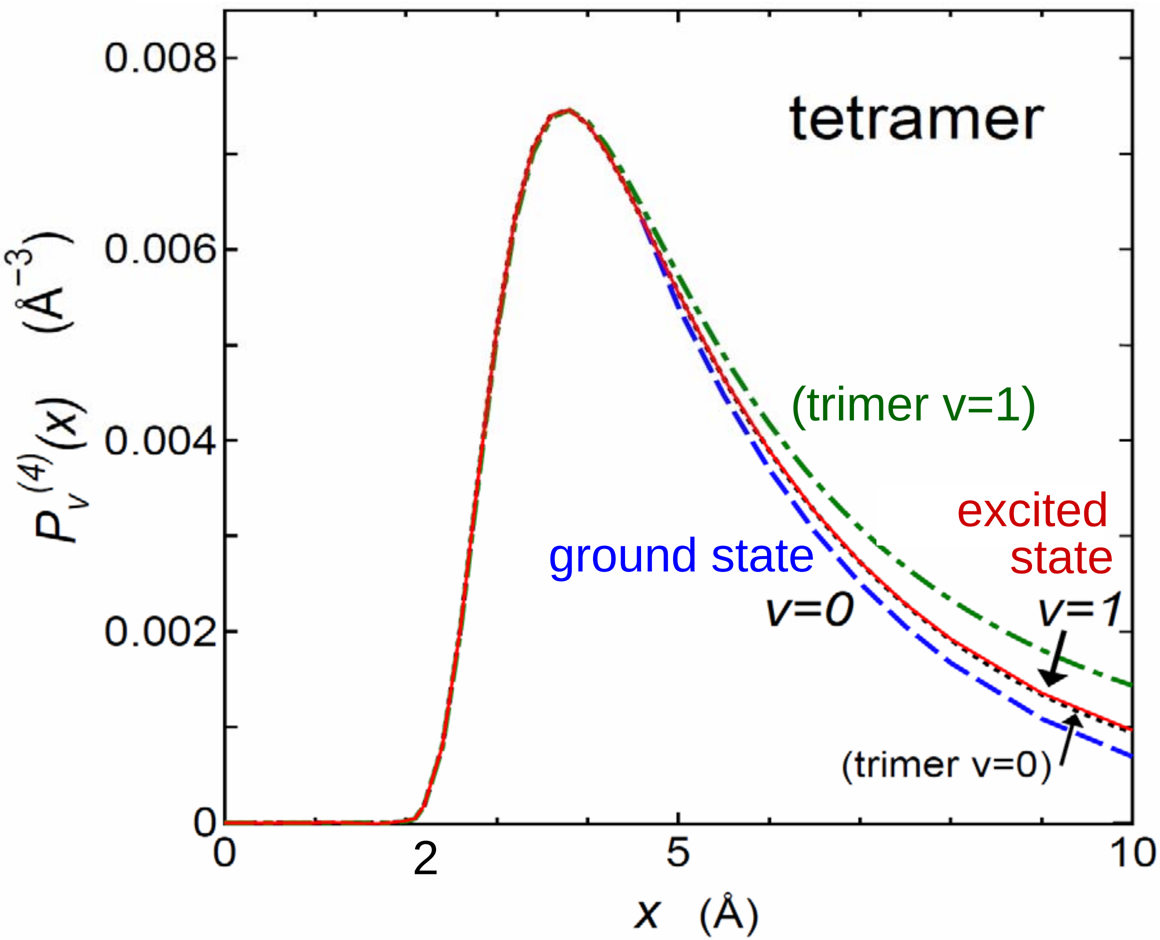
[7] R. Lazauskas and J. Carbonell, Phys. Rev. A **73** (2006)

Strong short-range correlation

Pair correlation function along x :

$$P_v^{(4)}(\mathbf{x}) = \int |\Psi_v^{(4)}|^2 \mathrm{d}\mathbf{y}\mathrm{d}\mathbf{z} \quad (v=0,1)$$





Already multiplied by

- x 2.76
- x 1.36
- x 19.8

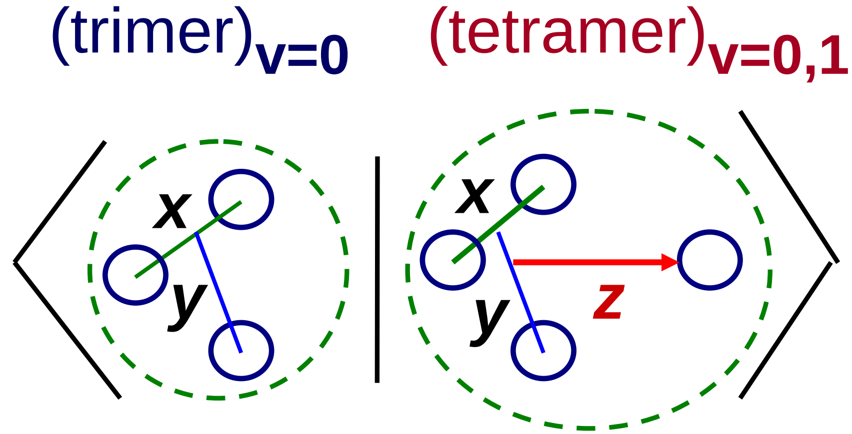
to be normalized at the peak.

Precisely the same shape of the short-range correlations (x< 4 A) appear in all the states.

g.8

Overlap function

$$\mathcal{O}_v^{(4)}(z) = \int \Psi_0^{(3)*}\,\Psi_v^{(4)}\,\mathrm{d}\mathbf{x}\,\mathrm{d}\mathbf{y} =$$



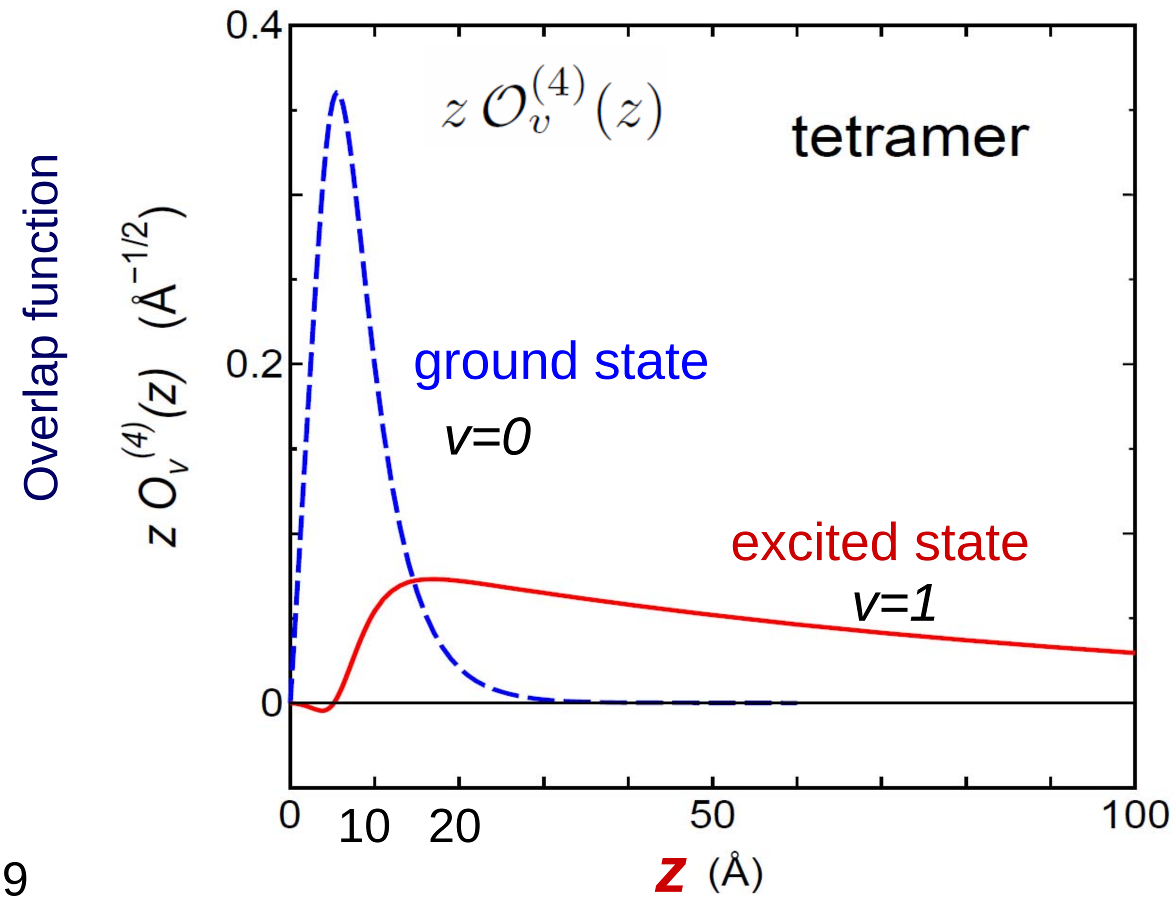


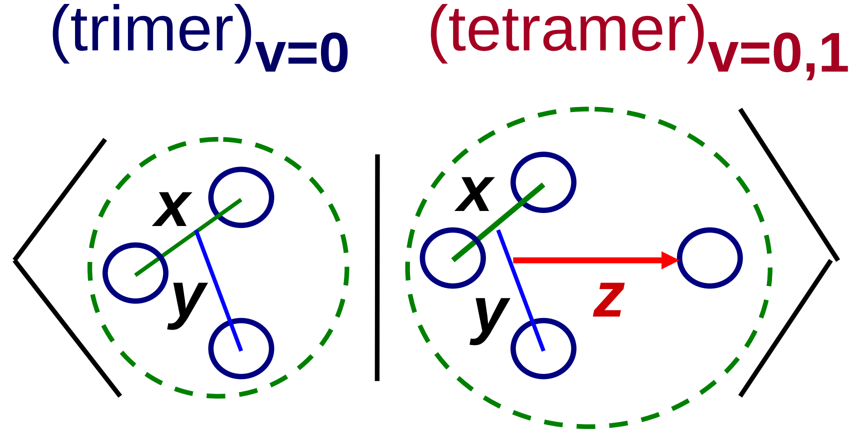
Fig.9

The excited state is much more dilute state than the ground state.

Overlap function

asymptotic region

$$\mathcal{O}_v^{(4)}(z) = \int \Psi_0^{(3)*}\, \Psi_v^{(4)}\, \mathrm{d}\mathbf{x}\, \mathrm{d}\mathbf{y} =$$



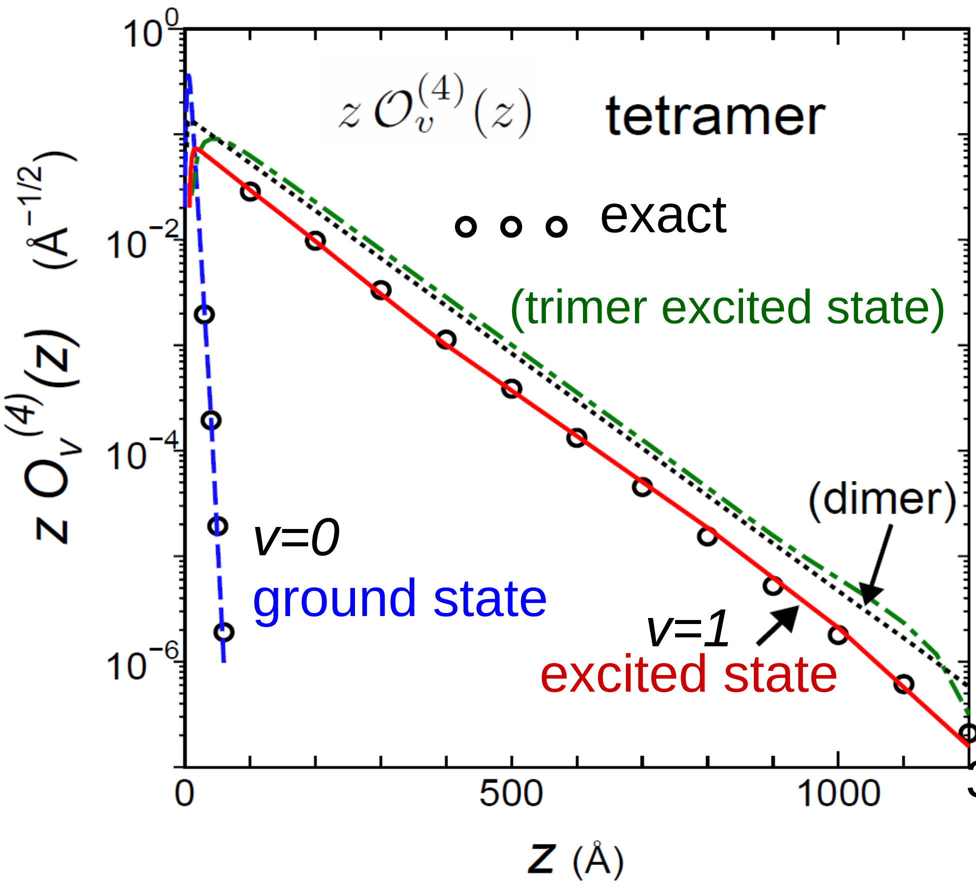


Fig.10

1) Asymptotic behavior of the **tetramer excited state** is almost exactly decaying up to ∼1000 Å.

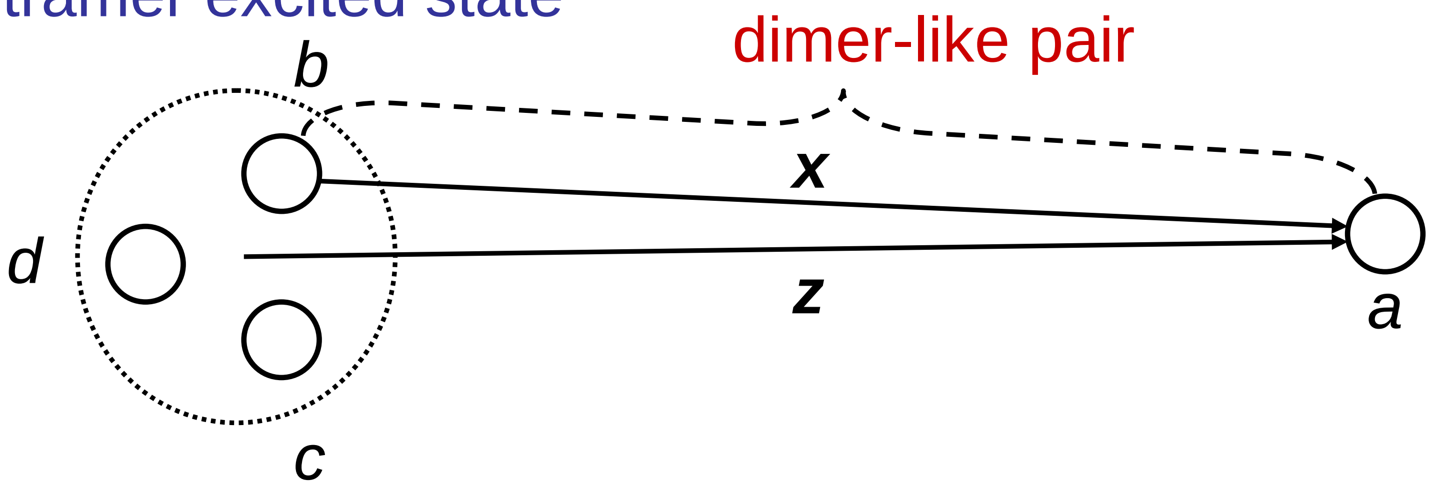
2) Three lines are parallel. Decaying constants are the same to each other.

$$k_1^{(4)} = k_1^{(3)} = k^{(2)}$$

3) Dimer-like pair model in asymptotic region works well.

Dimer-like pair model in the asymptotic region

tetramer excited state



As I mentioned before, the dimer-like pair model predicts

$$\Delta B_1^{(4)} = \frac{2}{3} B^{(2)} = 0.87\ \mathrm{mK}$$

$$B_1^{(4)} = 127.27\ \mathrm{mK}$$

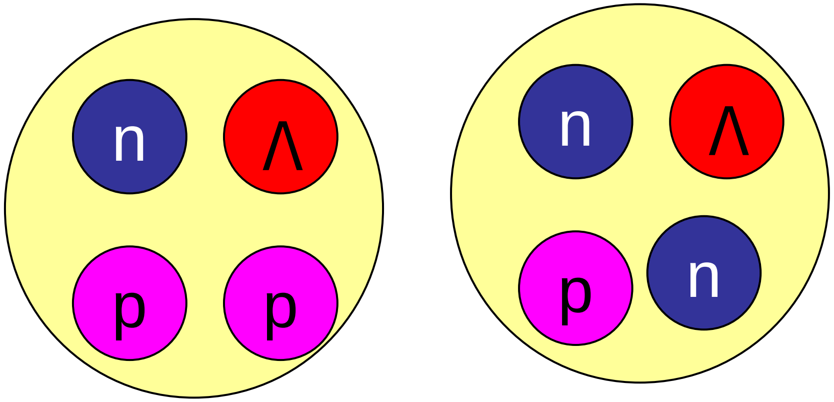
Our 4-body calculation
0.93 mK
127.33 mK

Our 4-body calculation

Very good!

Summary

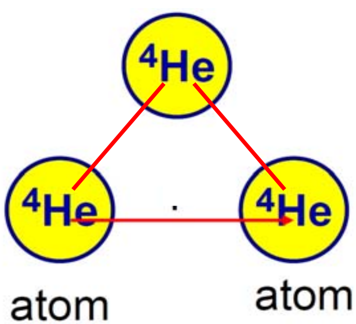
(1)

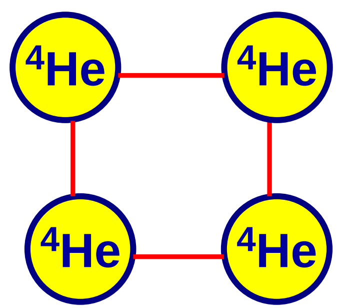


${}^{4}_{\Lambda}He$ ${}^{4}_{\Lambda}H$

These systems are important from view points of critical stability.

(2)





LM2M2 potential

Summary (tetramer)

・ As for the ^{4}He trimer ground and excited states, our results are in excellent agreement with those by literature calculations.

・ We then obtained binding energies of the **tetramer** ground and excited states to be 558.98 mK and 127.33 mK (0.93 mK below the atom-trimer threshold), respectively.

・We illustrate the short-range structure and accurate asymptotic behavior (up to ～1000 A) of the trimer and tetramer wave functions.

・ Precisely the same shape of the short-range correlations in the dimer appear in the ground and excited states of trimer and tetramer.

- The analysis of the asymptotic behavior of the trimer excited state generates a simple model to predict the binding energy of the tetramer excited bound state

Thank you!