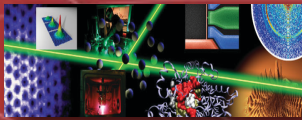


The Persistence of Attraction: the Dipolar Efimov Effect

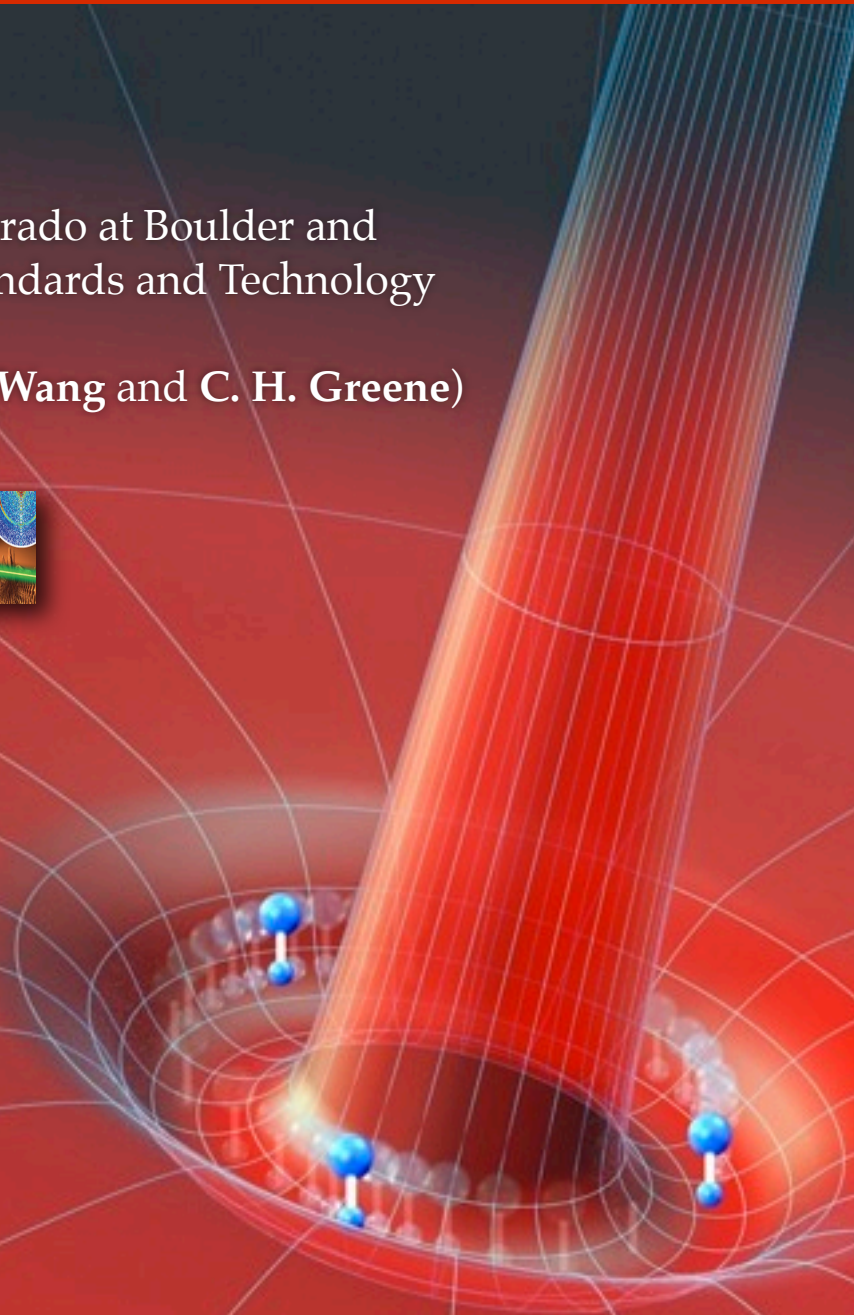
Jose P. D'Incao

JILA, University of Colorado at Boulder and
National Institute of Standards and Technology

(in collaboration with **Y. Wang** and **C. H. Greene**)



NIST
University of Colorado



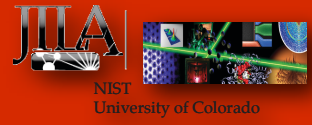
Air Force Office of
Scientific Research
MURI



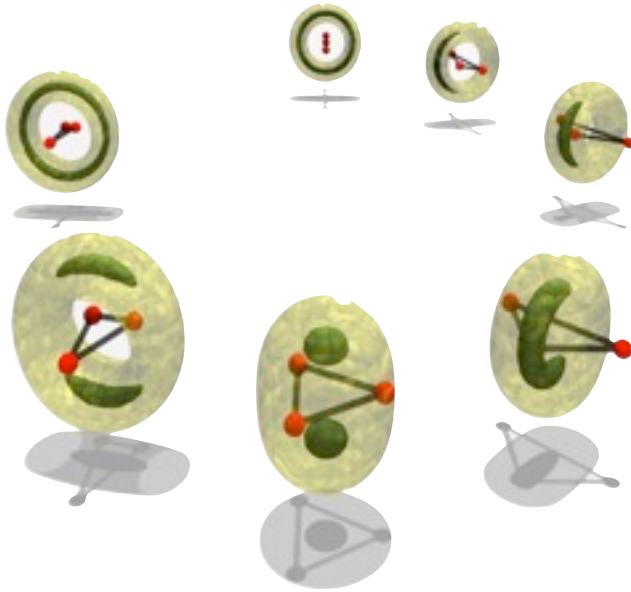
National Science
Foundation

(Figure credit:
Brad Baxley)

Overview



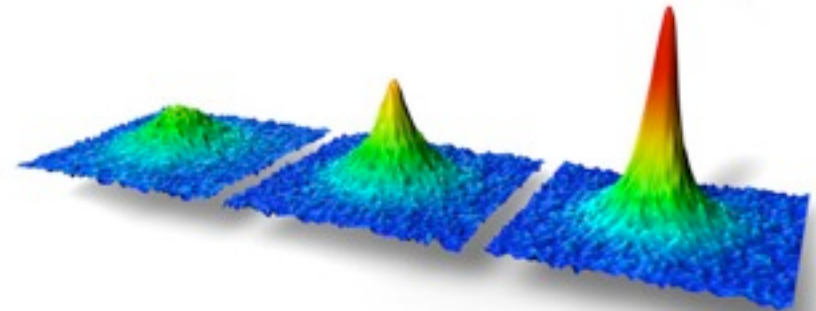
Universal Few-Body Physics



From the theoretical side:

- ✓ Signatures of Efimov Physics,
- ✓ Four- and More-bodies universal states,
- ✓ New families of universal states,

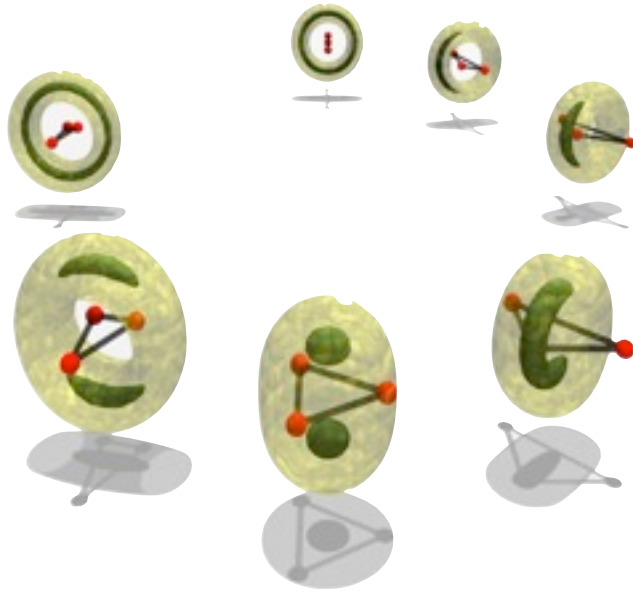
Ultracold Quantum Gases ($<peV$)



From the experimental side:

- ✓ clean and accurate experiments
- ✓ **CONTROL** of interactions (B -field)
- ✓ can explore the universal regime (low T and strong interactions)

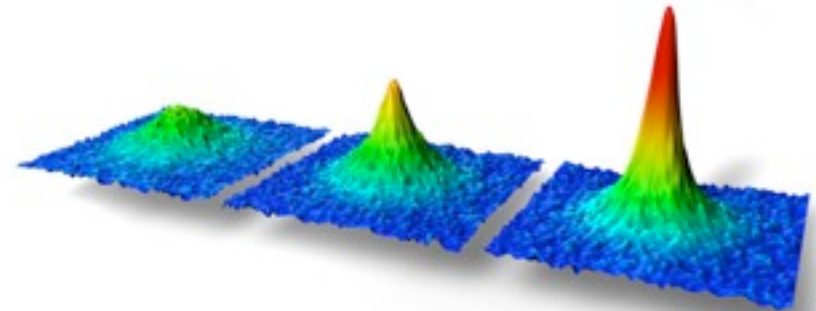
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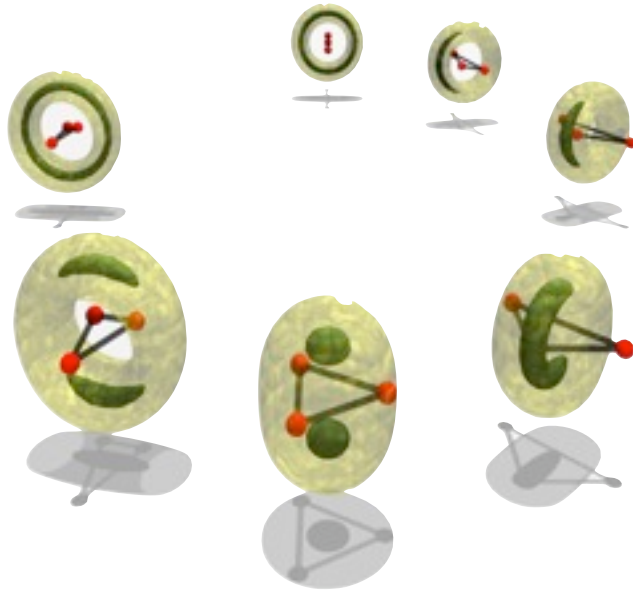
From the experimental side:

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- ✓ **CONTROL** of interactions (B -field)
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Allows for the use of simple models !!!

Universal Few-Body Physics



From the theoretical side:

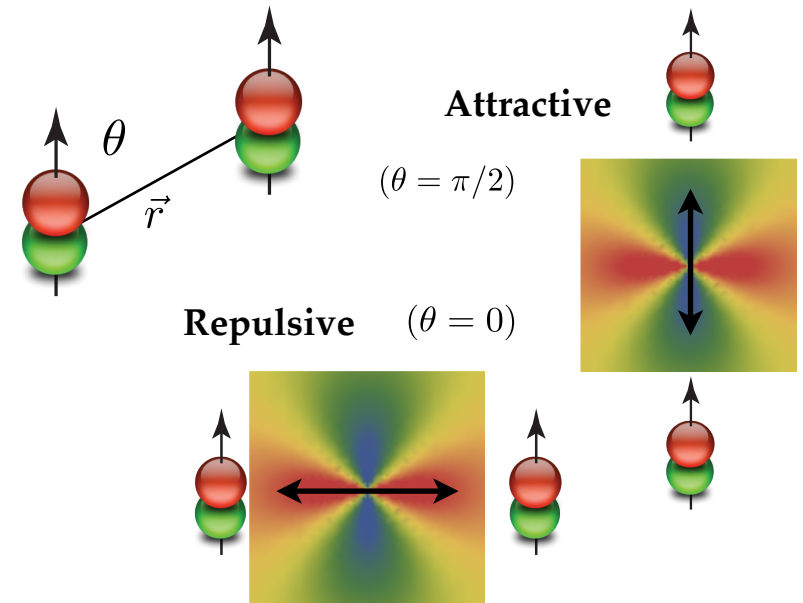
- ✓ Signatures of Efimov Physics,
- ✓ Four- and More-bodies universal states,
- ✓ New families of universal states,

Ultracold Dipolar Quantum Gases

- ✓ **CONTROL** of interactions (E -field)
- ✓ new phases, quantum computing, etc ...
- ✓ long range anisotropic interactions



- ✓ **The Efimov effect persist !!!**
- ✓ **Dipolar interaction is extremely beneficial !!!**
- ✓ **Possible *new* few-body physics !!!**



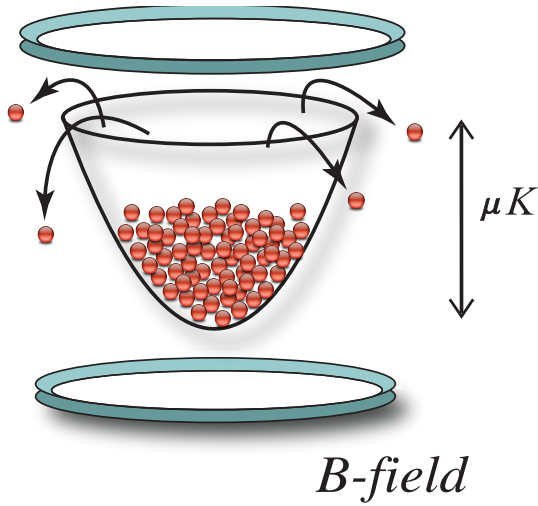
Few-body physics in Ultracold Gases (Why do Experiments care about it?)

Atomic/Molecular Losses

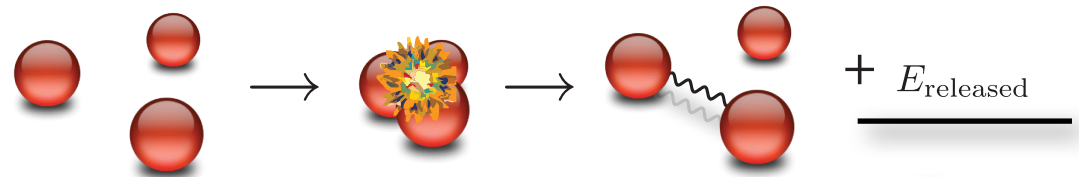
(three-body recombination, ...)

Atomic/Molecular Losses (three-body recombination, ...)

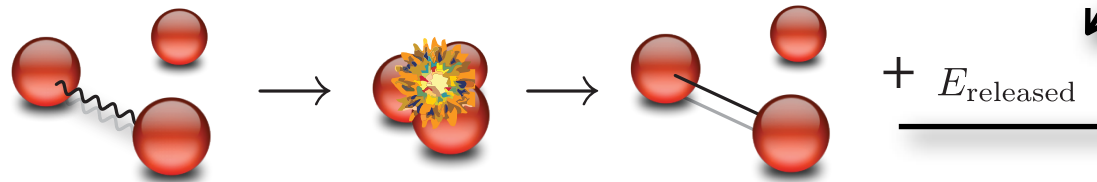
Stability and Lifetime of condensates !!!



Three-body Recombination ($B + B + B \rightarrow B_2^* + B$)



Vibrational Relaxation ($B_2^* + B \rightarrow B_2 + B$)



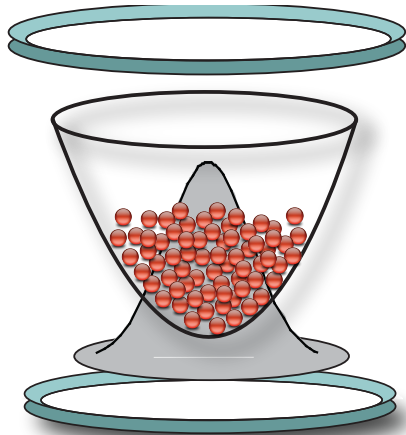
... loss !!!

Elastic parameters

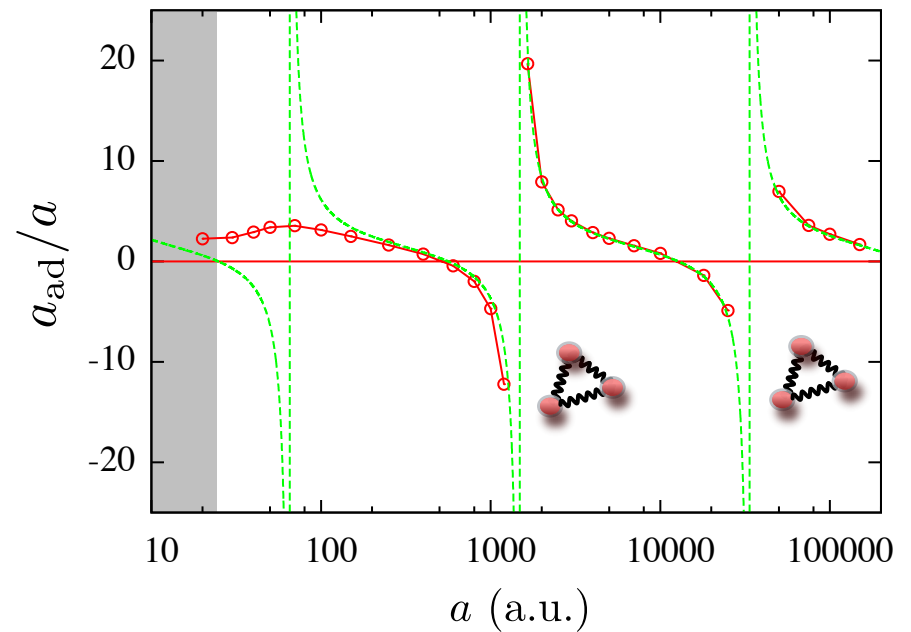
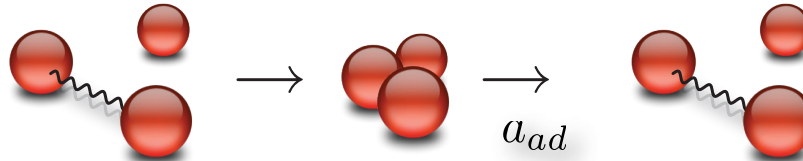
(strength of interactions, equation of state, ...)



Control of few-body correlations !!!



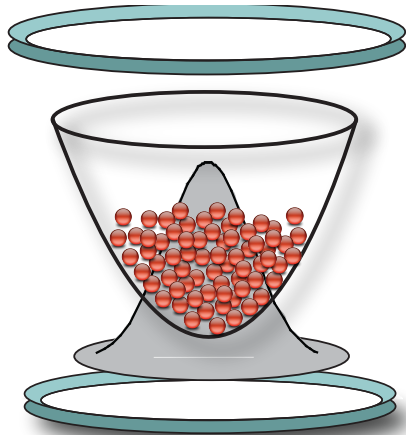
Atom-dimer scattering $(B_2^* + B \rightarrow B_2^* + B)$



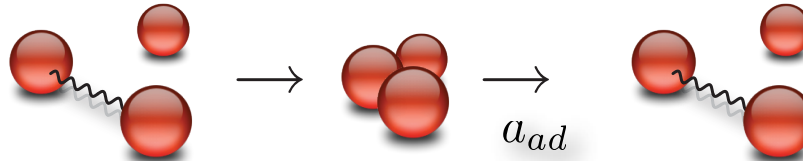
Elastic parameters

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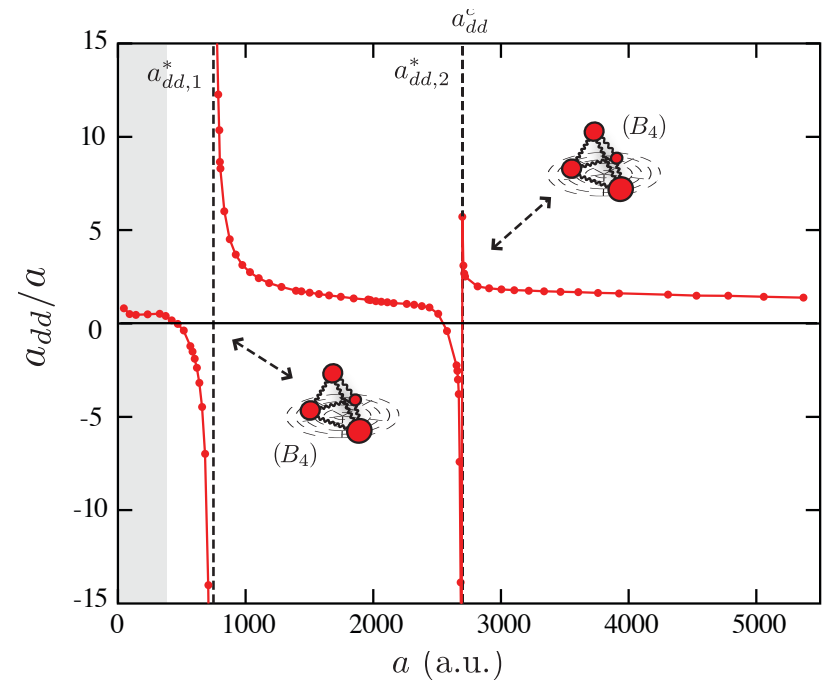
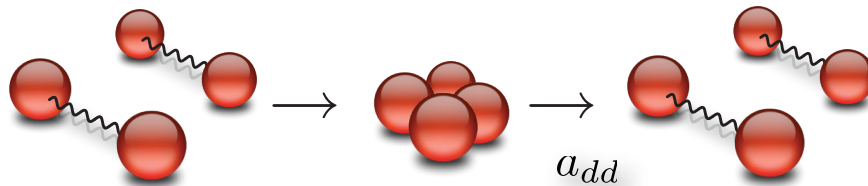
Control of few-body correlations !!!



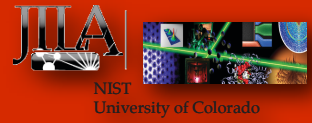
Atom-dimer scattering $(B_2^* + B \rightarrow B_2^* + B)$

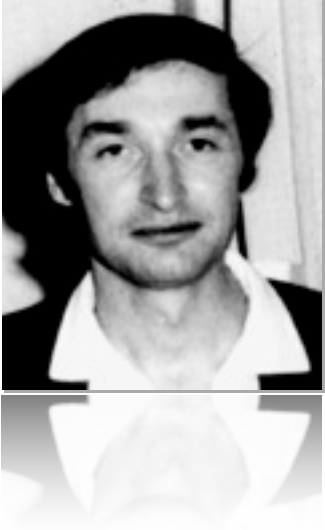


Dimer-dimer scattering $(B_2^* + B_2^* \rightarrow B_2^* + B_2^*)$



Few-body Physics in Ultracold Gases

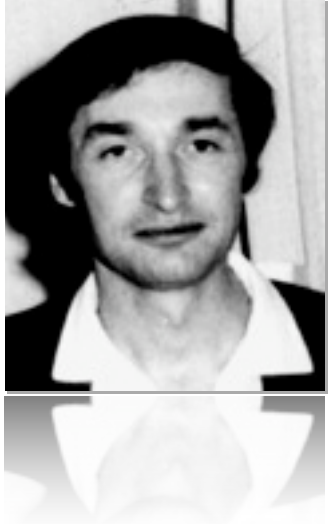




Efimov Physics

appearance of an *attractive* or *repulsive* three-body effective interaction ... in the strongly interacting regime ($|a| \gg r_0$)

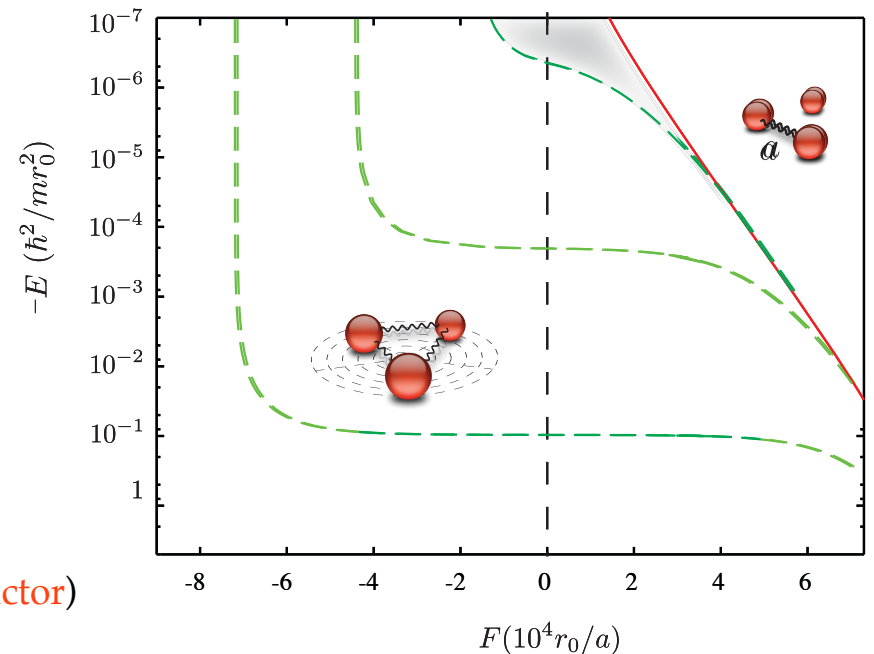
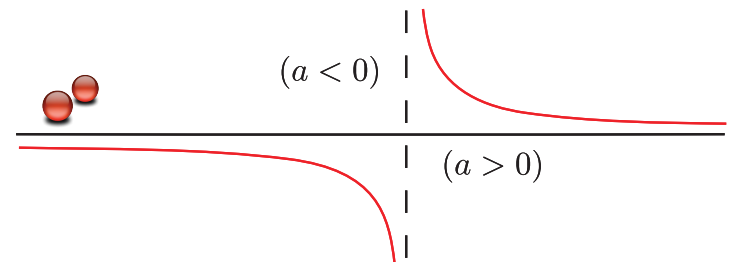
- ☑ Control of the few-body interactions
- ☑ Scattering length dependence on 3-body collision rates
- ☑ Losses = Probe of universal states



Efimov Physics

appearance of an *attractive* or *repulsive* three-body effective interaction ... in the strongly interacting regime ($|a| \gg r_0$)

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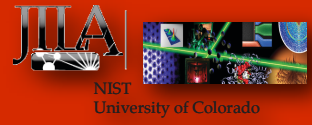


In ultracold gases a is a tunable parameter, which allows to the exploration of the full **Efimov energy spectrum**

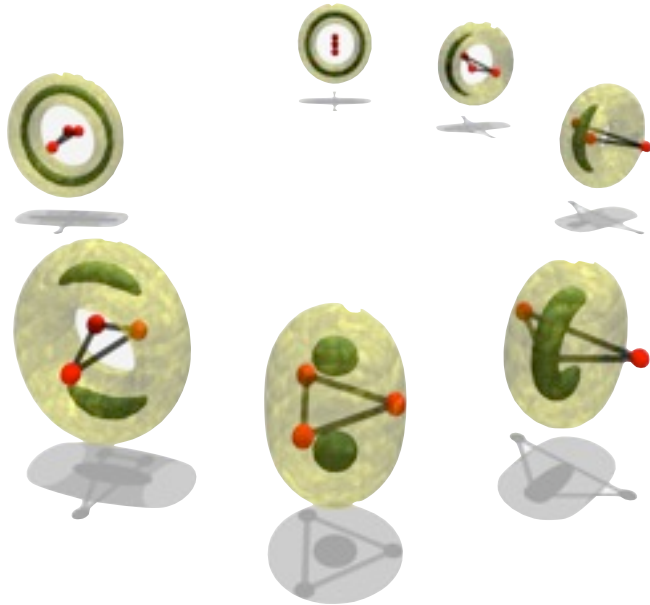
$$E_n = \frac{E_0}{[e^{\pi/s_0}]^{2n}}, \quad n = 1, 2, 3, \dots$$

$$a_n = [e^{\pi/s_0}]^n a_0 \quad e^{\pi/s_0} \approx 22.7 \quad (\text{geometric factor})$$

Universal Four-boson States



- Universal properties of the properties of the four-body system with large scattering lengths, Hammer & Platter, *EPJA* **32**, 113 (2007)
- Signatures of universal four-body phenomena and its relation to the Efimov effect von Stecher, D’Incao, and Greene, *Nat. Phys.* (2009)



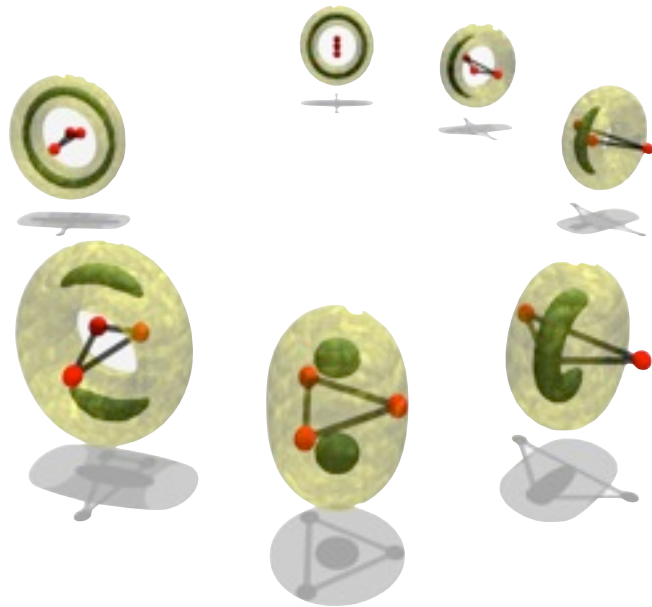
- Two four-boson states for each Efimov trimer [... see also Deltuva, *FBS* **50**, 391 (2011)]

$$E_{4b}^{(n,m)} = c_m E_{3b}^{(n)} \quad \begin{array}{l} m = 1, 2 \\ n = 1, 2, \dots, \infty \end{array}$$

($c_1 \approx 4.58$, $c_2 \approx 1.01$)

Universal Four-boson States

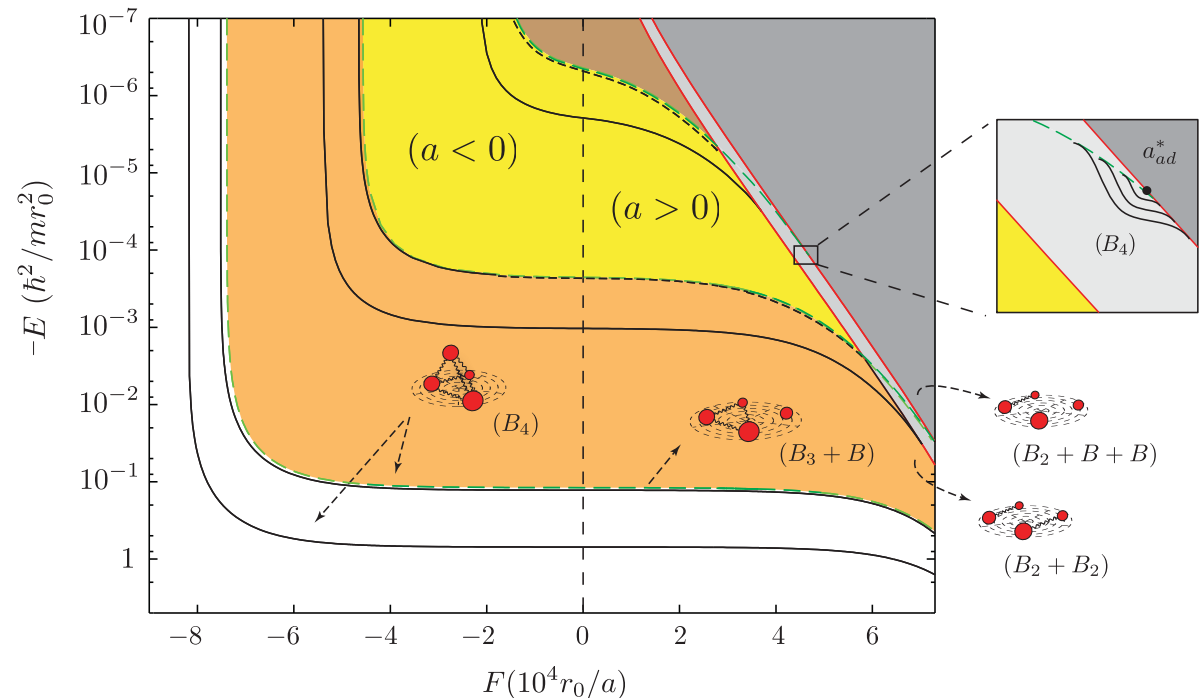
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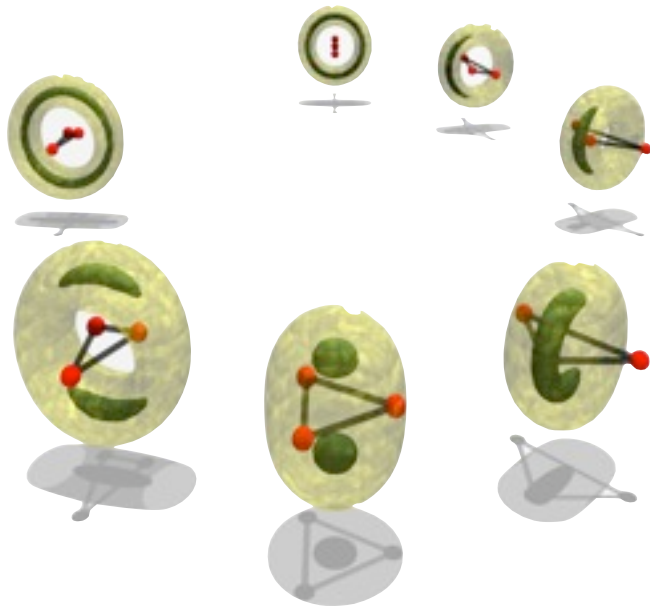
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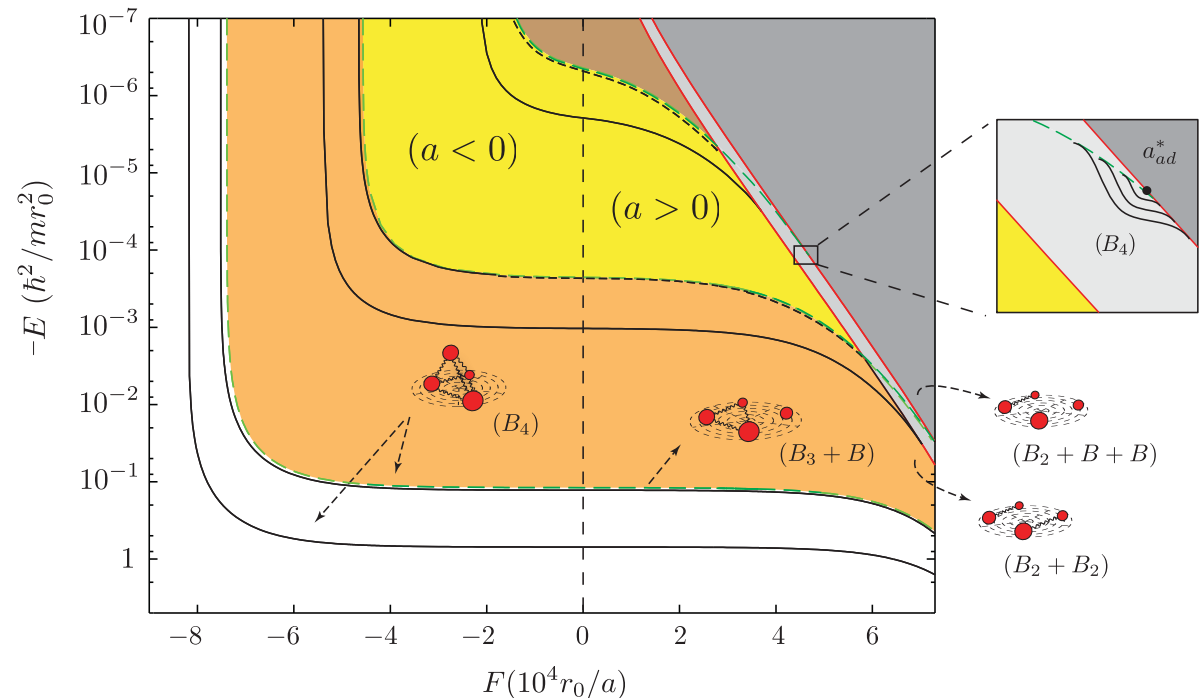


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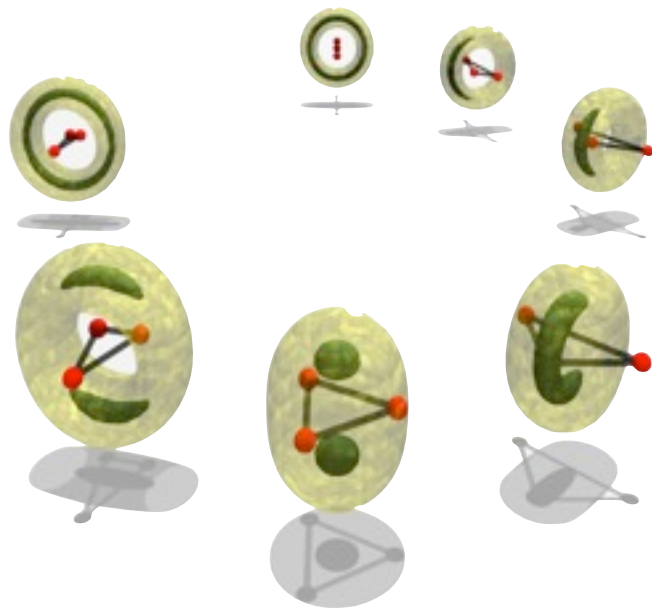
Controversy : (Four-body parameter ?)

- [Yes?] Yamashita, Tomio, Delfino, & Frederico, *EPL* **75**, 555 (2006); Hadizadeh, Yamashita, Tomio, Delfino, & Frederico, *PRL* **107**, 135304 (2011)

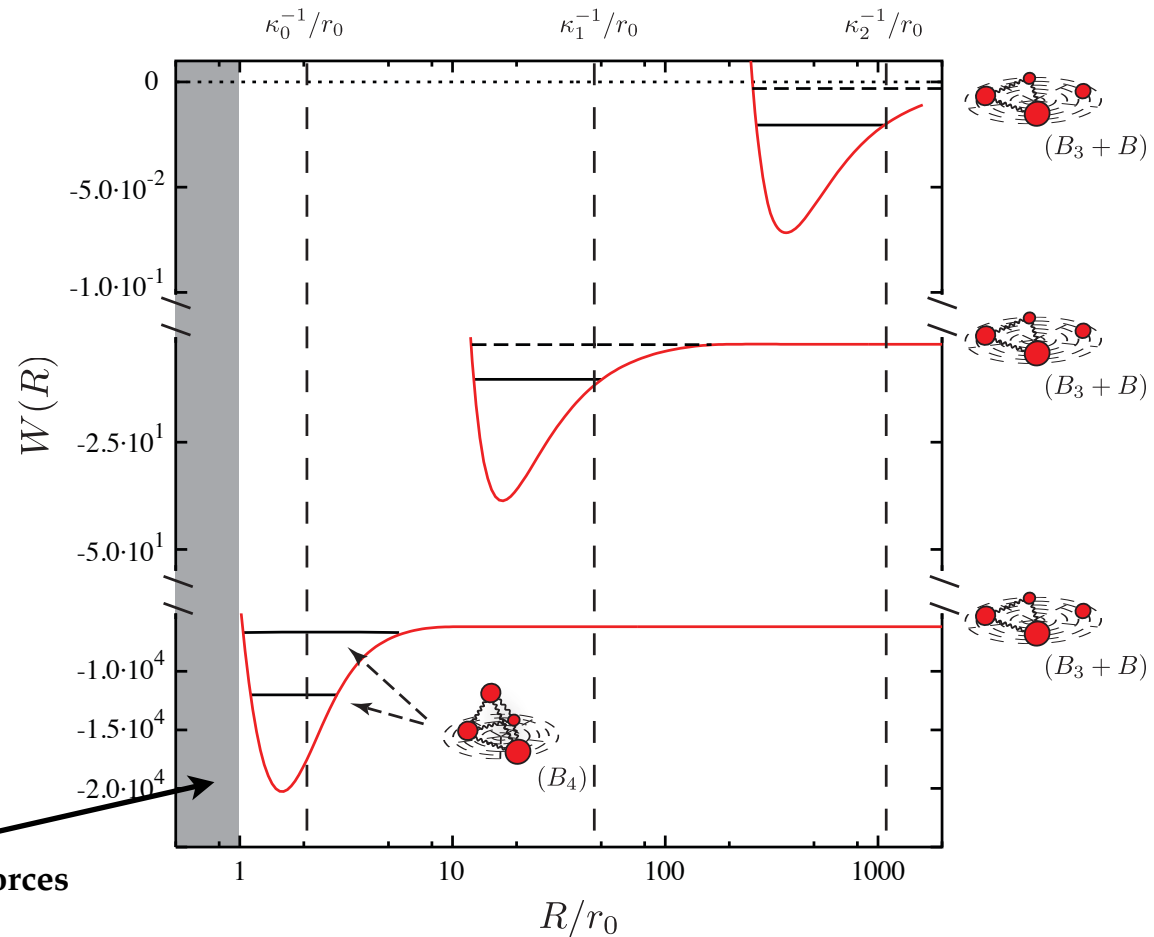


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Four-bosons hyperspherical potentials at $|a| \rightarrow \infty$



Four-body forces

Universal Four-boson States

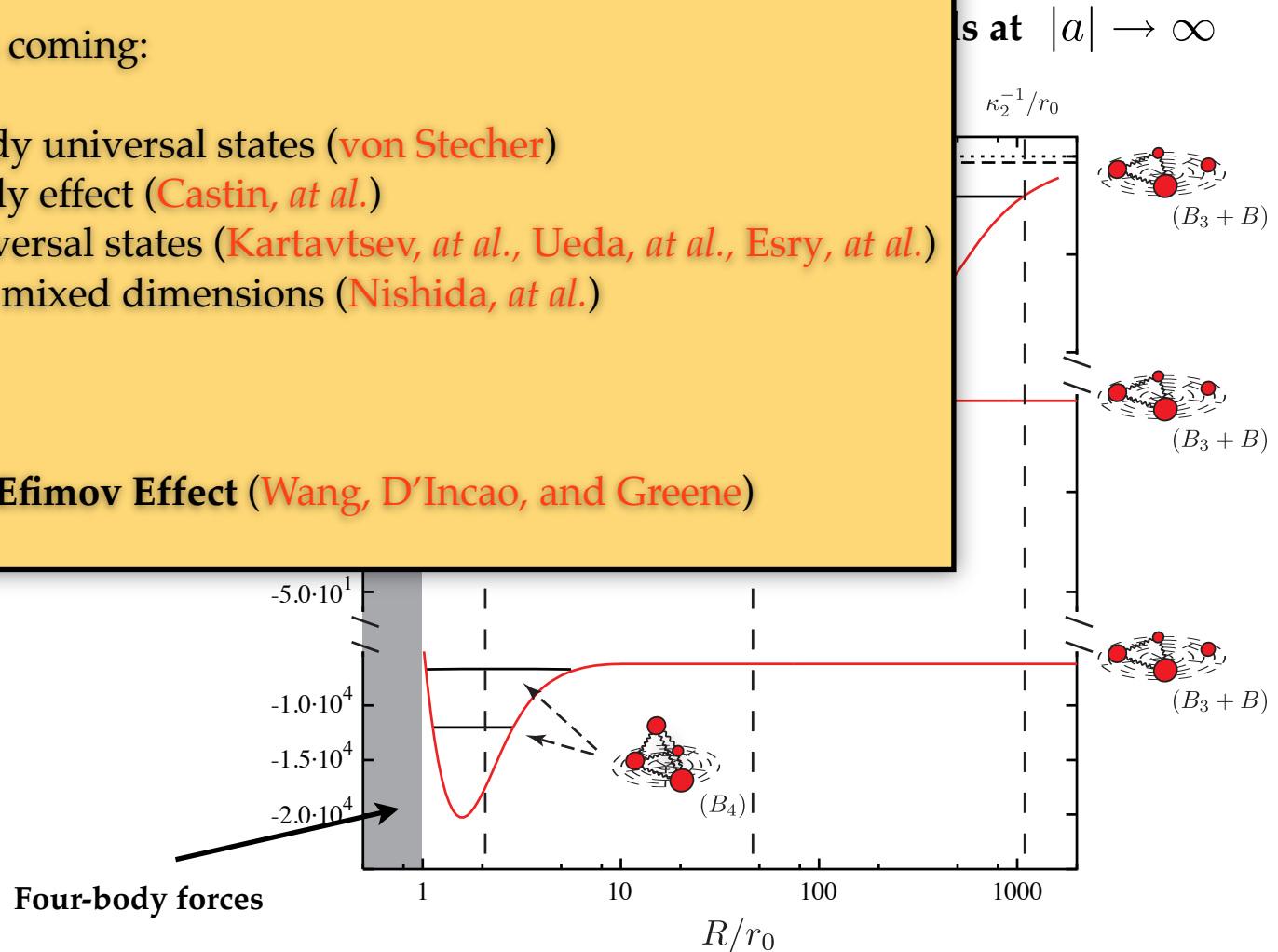
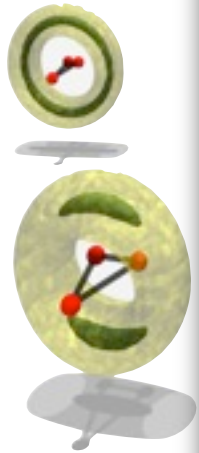
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... and much more is coming:

- five- and six-body universal states (von Stecher)
- Efimov four-body effect (Castin, *at al.*)
- Three-body Universal states (Kartavtsev, *at al.*, Ueda, *at al.*, Esry, *at al.*)
- Efimov effect in mixed dimensions (Nishida, *at al.*)

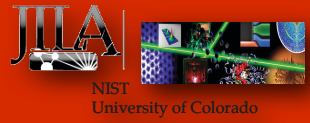
... and of course:

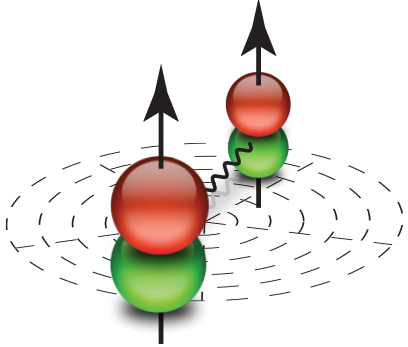
Dipolar Efimov Effect (Wang, D’Incao, and Greene)



The arrival of dipolar gases

Ultracold Dipolar Few-body Physics



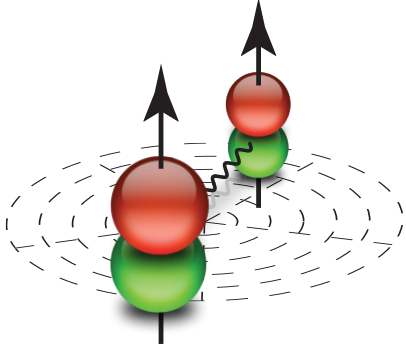


Ultracold Dipolar gases


Atomic (magnetic) dipoles: weak interactions !!!

Molecular (electric) dipoles: strong interactions !!!

(challenging experiments)



SCIENCE VOL 327 12 FEBRUARY 2010



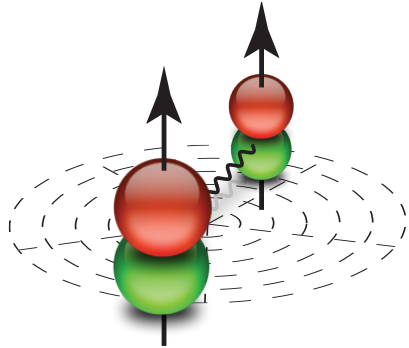
Quantum-State Controlled Chemical Reactions of Ultracold Potassium-Rubidium Molecules

S. Ospelkaus,^{1*} K.-K. Ni,^{1*} D. Wang,¹ M. H. G. de Miranda,¹ B. Neyenhuis,¹ G. Quémener,¹ P. S. Julienne,² J. L. Bohn,¹ D. S. Jin,^{1†} J. Ye^{1†}

JILA, NIST and University of Colorado, Department of Physics, University of Colorado, Boulder, CO 80309, USA.
Institute, NIST and University of Maryland, Gaithersburg, MD 20899, USA.

... also : Innsbruck, Yale, Heidelberg, Hanover, ...

| | d_ℓ^{\max} | $kd_\ell^{\max} (T = 100\text{nK})$ |
|------|-------------------------------|-------------------------------------|
| RbK | $\approx 6 \times 10^3 a_0$ | 2.4 |
| RbCs | $\approx 5 \times 10^4 a_0$ | 23 |
| LiCs | $\approx 6 \times 10^5 a_0$ | 238 |
| SrO | $\approx 1.1 \times 10^6 a_0$ | 467 |
| IK | $\approx 2.8 \times 10^6 a_0$ | 1100 |



SCIENCE VOL 327 12 FEBRUARY 2010



Quantum-State Controlled Chemical Reactions of Ultracold Potassium-Rubidium Molecules

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University of Colorado, Boulder, CO 80309, USA.
Institute, NIST and University of Maryland, Gaithersburg, MD
20899, USA.

... also : Innsbruck, Yale, Heidelberg,
Hanover, ...

“Universal” dipolar molecules:

RbK, LiNa, LiK, LiRb, LiCs

(unit probability of reactive collisions)

... reaction dynamics (ultracold controllable chemistry), ...

[see: Idziaszek and Julienne
PRL 104, 113202 (2010); Zuchwski
and Hutson, arXiv:1003.1418]

“Non-universal” dipolar molecules:

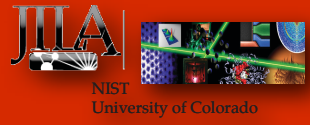
RbCs, NaK, NaRb, NaCs, KCs

(no reactive collisions)

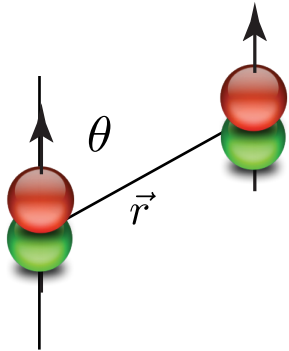
..., dipolar quantum phases, ...

Dipolar Few-body Physics (... **what changes ?**)

Ultracold Dipolar Few-body Physics



Dipole-Dipole Interaction:



$$v_{dip}(\vec{r}) = \frac{d_l}{m} \frac{1 - 3 \cos^2 \theta}{r^3}$$

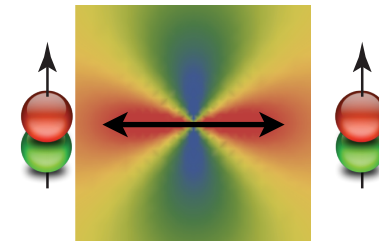
d_l : dipole length

(experimentally controllable parameter)

(**anisotropic** and
long-ranged interactions)

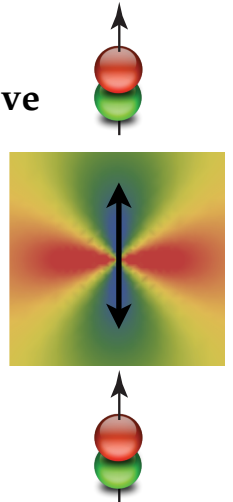


Repulsive ($\theta = 0$)

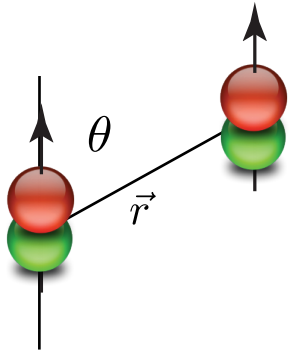


Attractive

($\theta = \pi/2$)



Dipole-Dipole Interaction:



$$v_{dip}(\vec{r}) = \frac{d_l}{m} \frac{1 - 3 \cos^2 \theta}{r^3}$$

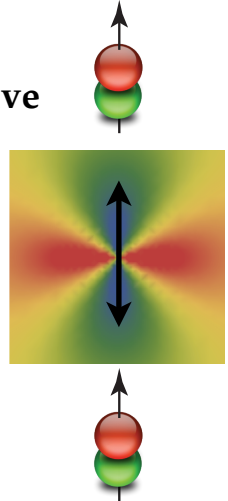
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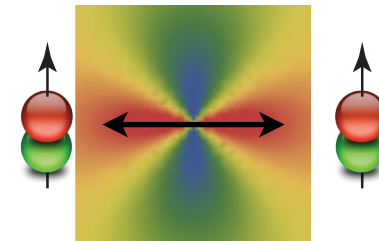
(anisotropic and long-ranged interactions)

Attractive

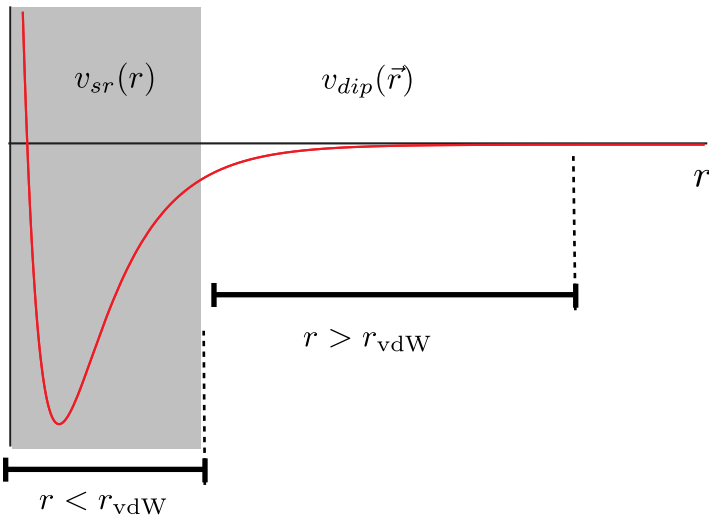
($\theta = \pi/2$)



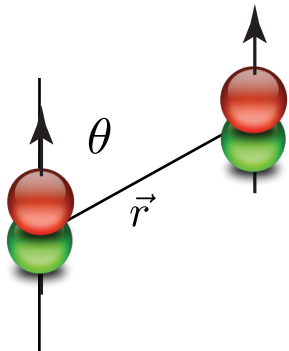
Repulsive ($\theta = 0$)



Dipole-Dipole Model Interaction



Dipole-Dipole Interaction:



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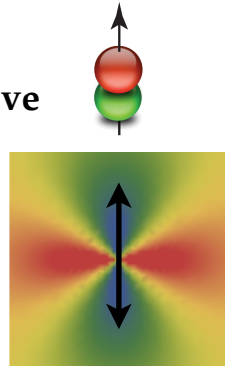
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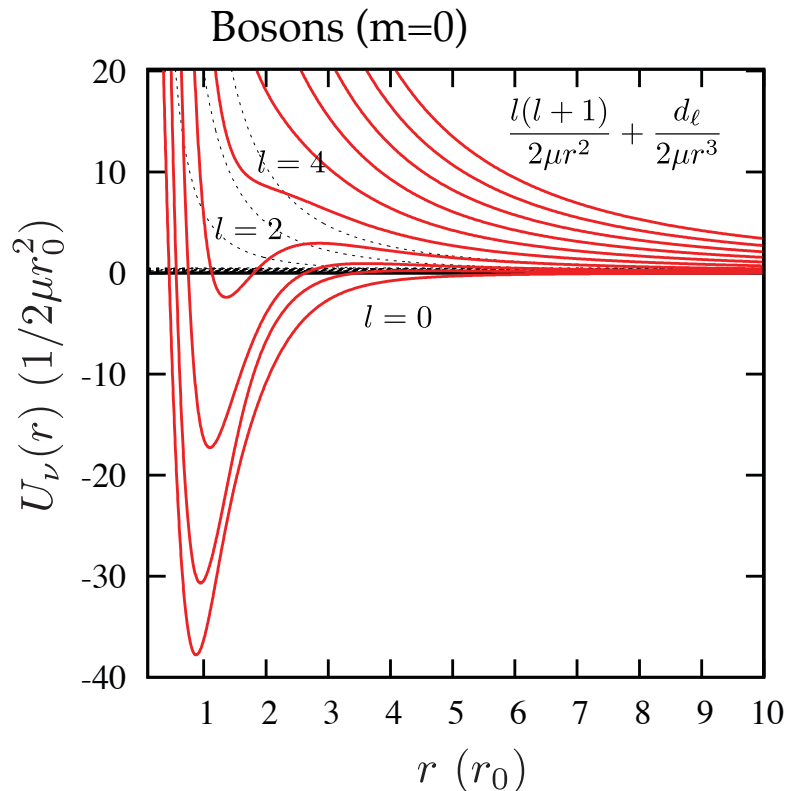
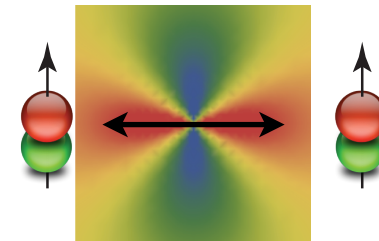
(anisotropic and long-ranged interactions)

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Repulsive ($\theta = 0$)

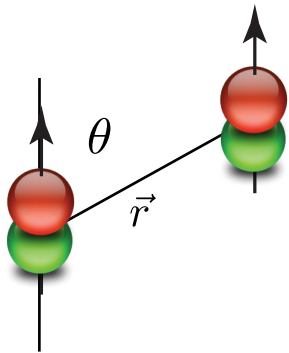


phase-shift : $\delta_l \propto k \ (\forall l)$
(not k^{2l+1})

Scattering length matrix:

$$a_{ll'} = -\frac{K_{ll'}}{k} \neq 0 \quad (k \rightarrow 0)$$

Dipole-Dipole Interaction:



$$v_{dip}(\vec{r}) = \frac{d_{\ell}}{m} \frac{1 - 3 \cos^2 \theta}{r^3}$$

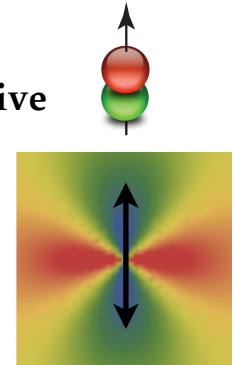
d_{ℓ} : dipole length

(experimentally controllable parameter)

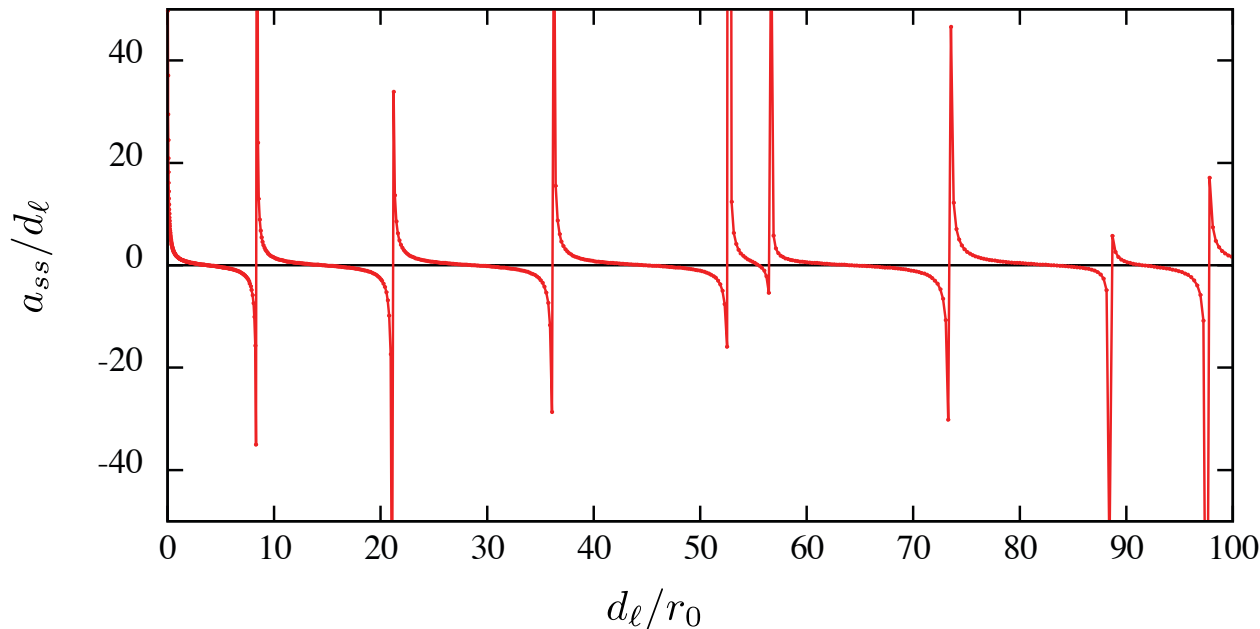
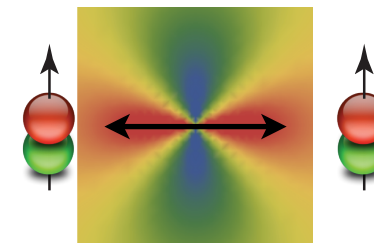
(anisotropic and long-ranged interactions)

Attractive

($\theta = \pi/2$)



Repulsive ($\theta = 0$)

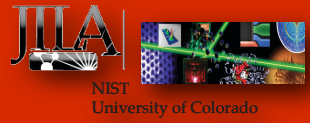


$$\underline{a_{ss}/d_{\ell} \gg 1 \quad (d_{\ell} \gg r_0)}$$

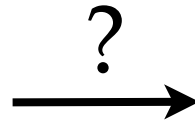
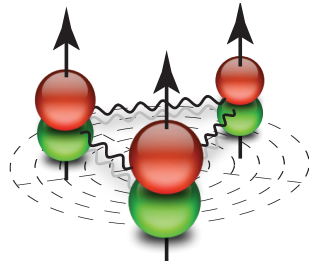
- control of interactions ...
- new phases of the matter ...
- losses (?)

How about Few-body Physics ?

Ultracold Dipolar Few-body Physics

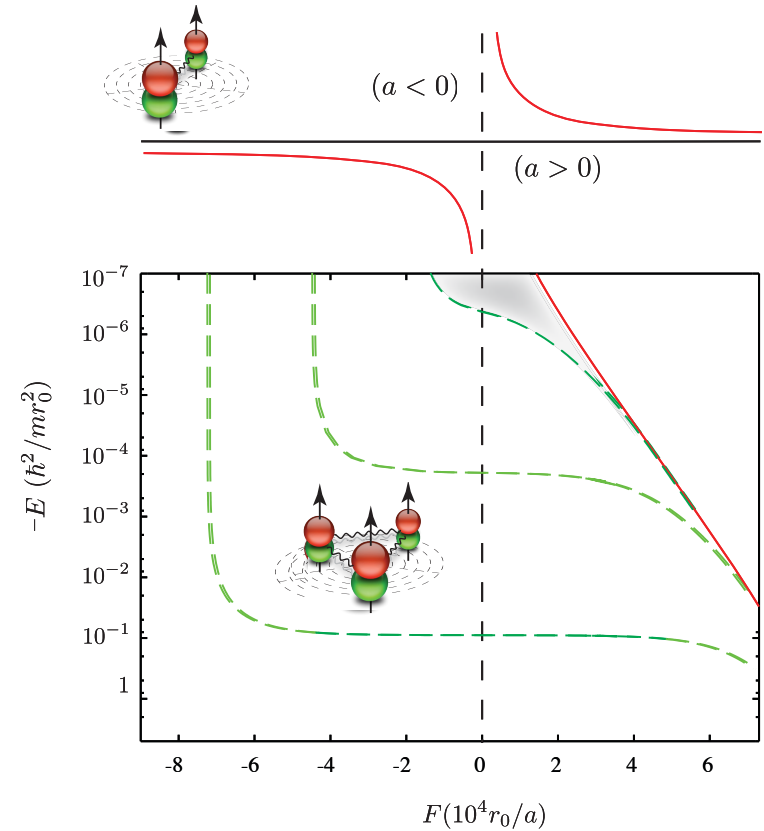


Three-body Dipolar Physics



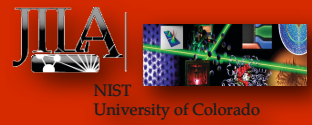
Efimov Physics for dipoles ?

| non-dipolar | dipolar |
|-----------------------------------|---|
| large s -wave scattering length | l is not conserved (multiple partial waves contributes) |
| ZRM offers an analytical solution | ZRM for dipoles (?) (no effective-range) |
| Identical Bosons: $J=0$ | J is also not conserved !!! |



Strong Losses: Tichnor & Rittenhouse PRL (2010)
[perturbative treatment]

Toolkit for Exploring Few-body Physics ...



For N particles ...

$$\hat{H} = -\frac{1}{2\mu} \nabla_T^2 + \sum_{i < j} V(r_{ij})$$

... angles + set of non-compact
coordinates $r_{ij} \rightarrow [0, \infty]$

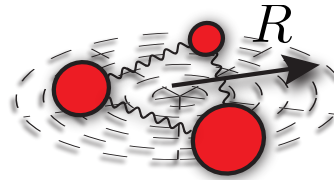
For N particles ...

$$\hat{H} = -\frac{1}{2\mu} \nabla_T^2 + \sum_{i < j} V(r_{ij})$$

... angles + set of non-compact coordinates $r_{ij} \rightarrow [0, \infty]$

... the hyperspherical way !!!

$$\hat{H} = -\frac{1}{2\mu} \frac{d^2}{d^2 R} + \frac{\Lambda^2(\Omega)}{2\mu R^2} + V(R, \Omega)$$



hyperradius R : overall size
(collective motion)

$$R \rightarrow [0, \infty]$$

hyperangles $\{\Omega\}$: internal motion
 $\{\Omega\} \rightarrow [0, \infty \pi]$

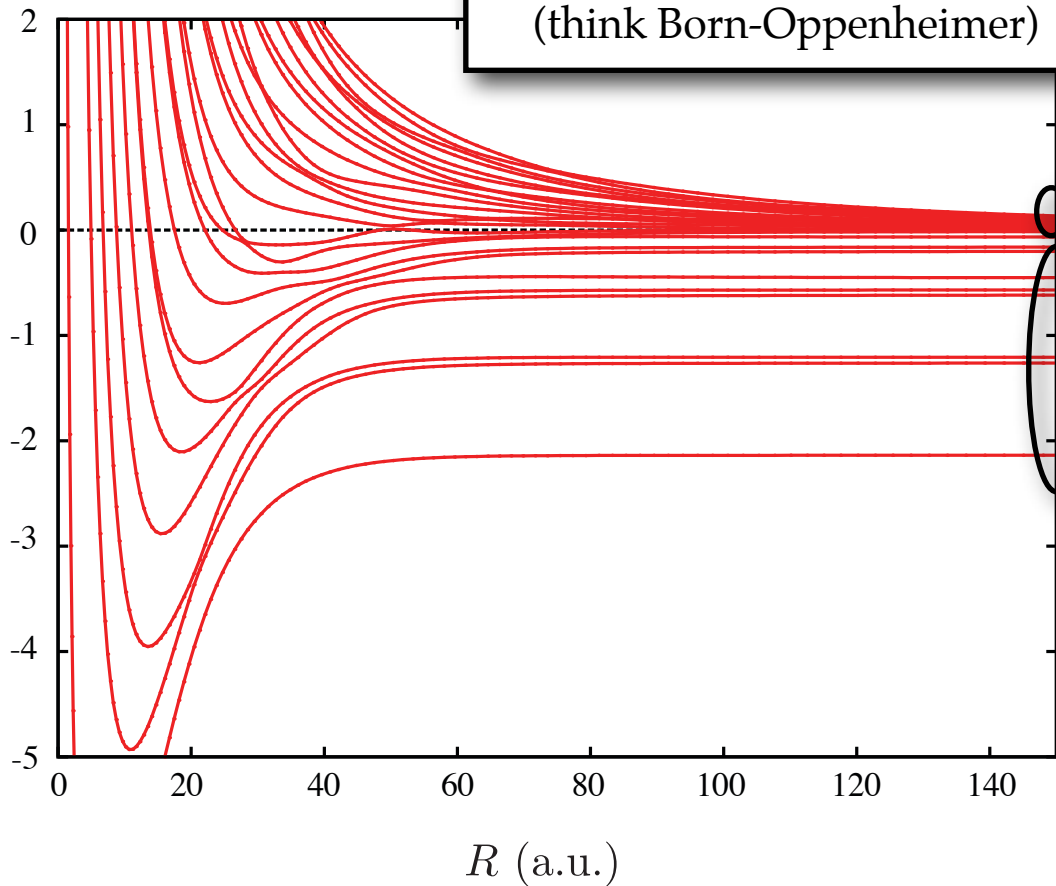
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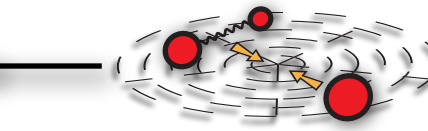
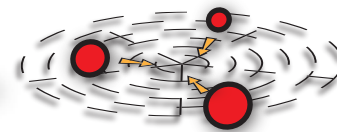
... the hyperspherical way !!!

$$\hat{H} = -\frac{1}{2\mu} \frac{d^2}{d^2 R} + \frac{\Lambda^2(\Omega)}{2\mu R^2} + V(R, \Omega)$$

Hyperspherical Potentials
(think Born-Oppenheimer)



(three-body continuum)



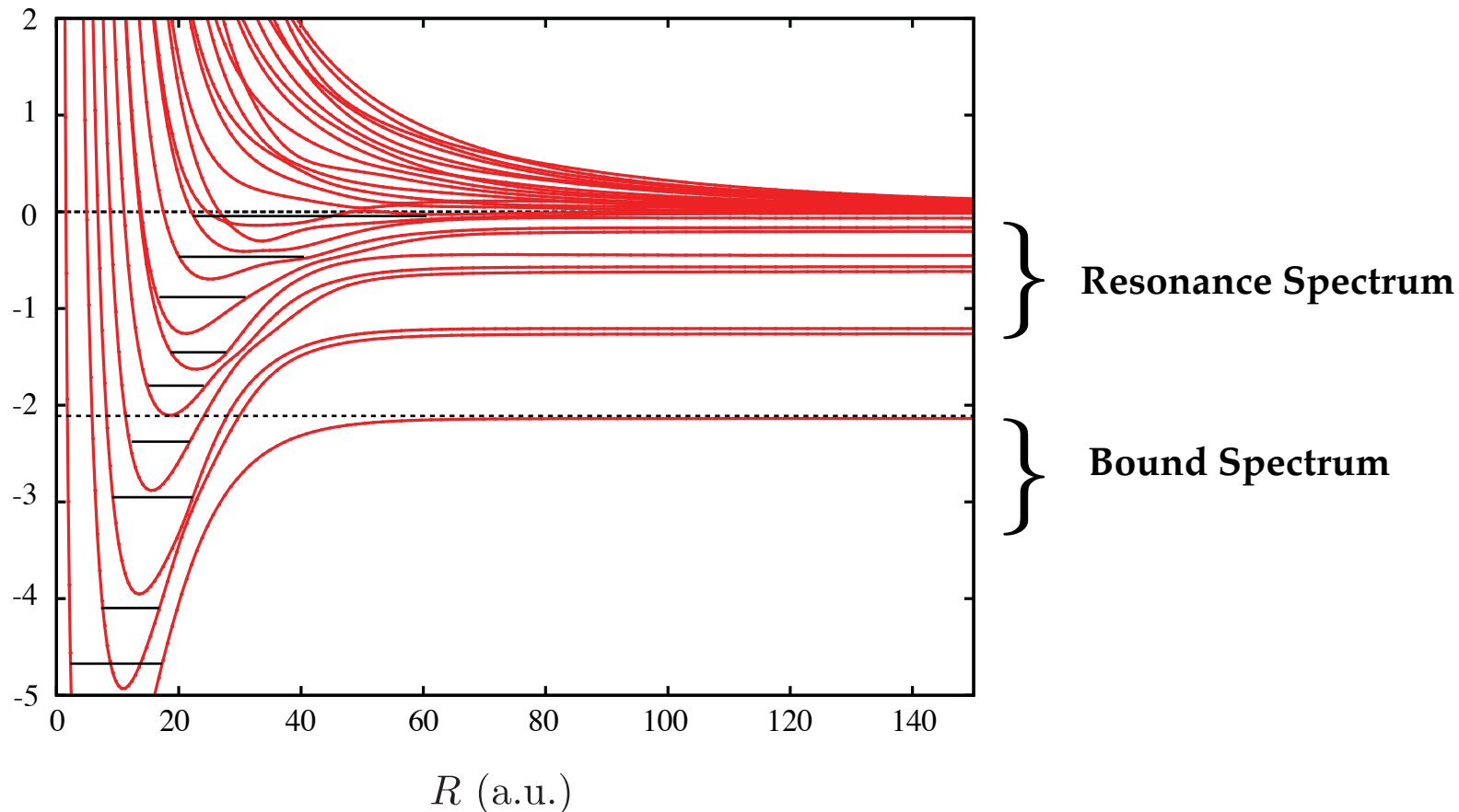
(bound channels)

... fixing R , solving Ω

Bound and Scattering Properties

$$\left[-\frac{1}{2\mu} \frac{d^2}{dR^2} + U_\nu(R) - E \right] F_\nu(R) + \sum_{\nu'} W_{\nu\nu'}(R) F_{\nu'}(R) = 0$$

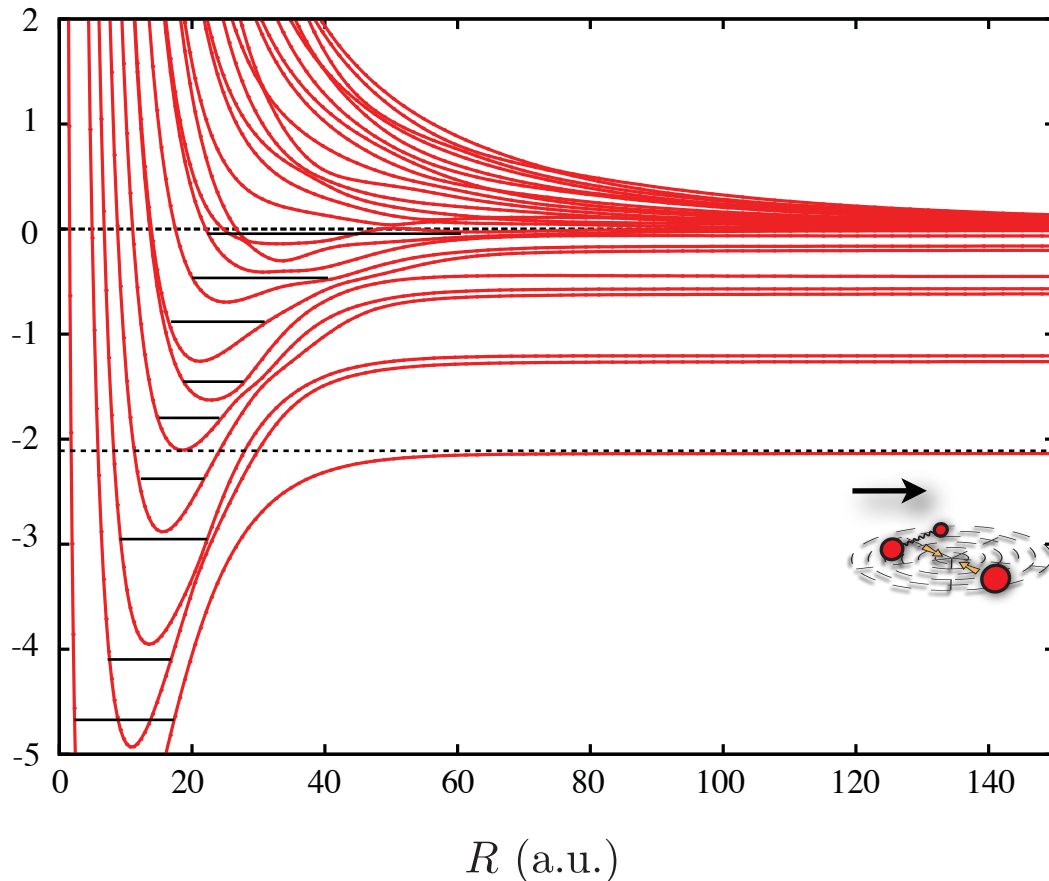
(Hyperradial Schrodinger Equation)



Bound and Scattering Properties

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(Hyperradial Schrodinger Equation)



Ultracold Few-body Collisions

$W_{\nu\nu'}(R)$: non-adiabatic couplings
(drive inelastic transitions)

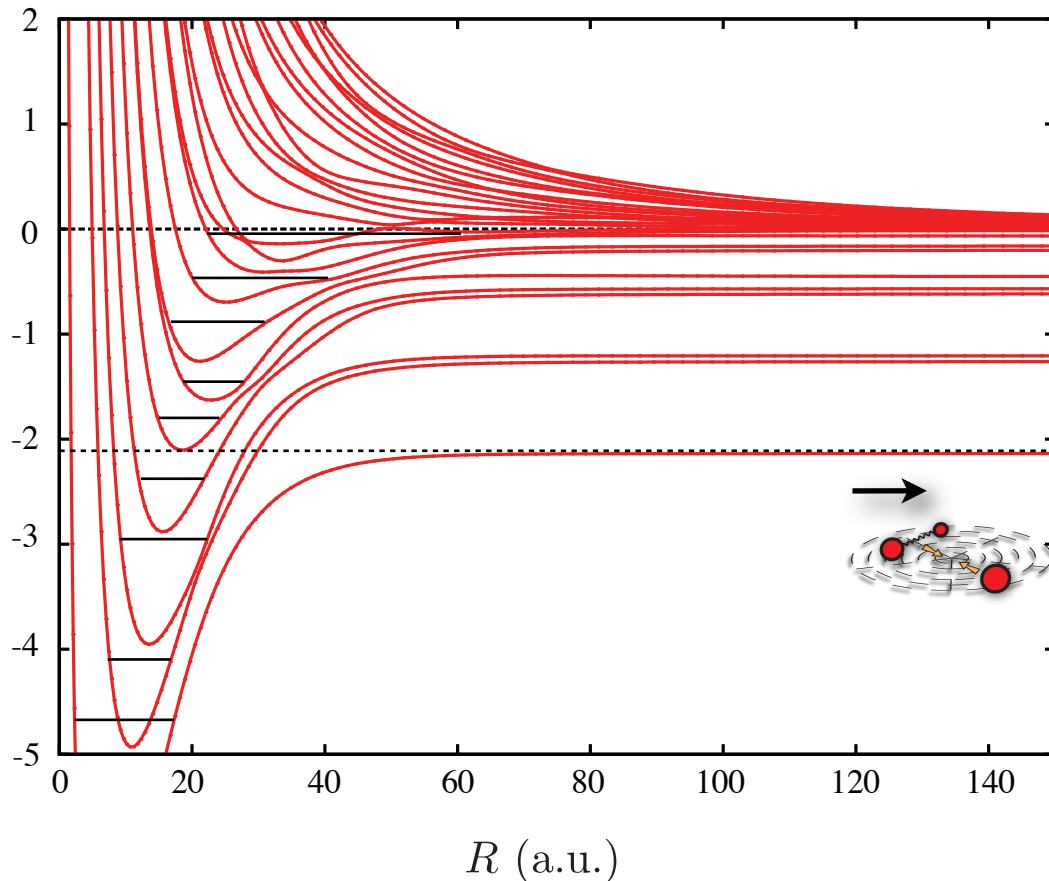
Typical length & energy scales:

- Van der Waals length: $r_0 \approx 100a_0$
- Temperature: $T \approx 100\text{nK}$

Bound and Scattering Properties

$$\left[-\frac{1}{2\mu} \frac{d^2}{dR^2} + U_\nu(R) - E \right] F_\nu(R) + \sum_{\nu'} W_{\nu\nu'}(R) F_{\nu'}(R) = 0$$

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Ultracold Few-body Collisions

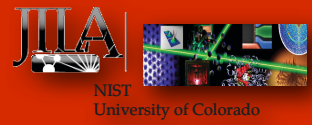
$W_{\nu\nu'}(R)$: non-adiabatic couplings
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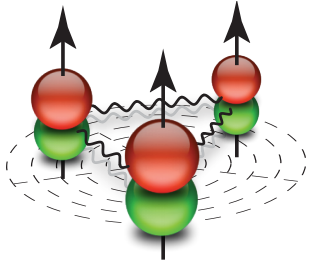
- Van der Waals length: $r_0 \approx 100a_0$
- Temperature: $T \approx 100\text{nK}$

→ Solve Schrödinger equation for $R \approx 10^6 a_0$
(0.05mm!!!)

Toolkit for Exploring Few-body Physics ...



Hyperspherical representation for dipoles ...



$$\Psi_M(R, \Omega) = \sum_{\nu} F_{\nu}^M(R) \Phi_{\nu}^M(R; \Omega)$$

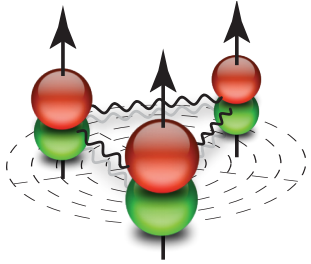
(J is **not** conserved)

$$\Phi_{\nu}^M(R; \Omega) = \sum_J \sum_K \phi_{\nu K}^J(R; \theta, \varphi) D_{KM}^J(\alpha, \beta, \gamma)$$

$$\sum_{J'K'} \left[\left\langle JMK \mid \frac{\hat{\Lambda}(\Omega)}{2\mu R^2} \mid J'MK' \right\rangle \delta_{JJ'} \right.$$

$$\left. + \left\langle JMK \mid \sum_{i>j} v_{dip}(\vec{r}_{ij}) \mid J'MK' \right\rangle - U_{\nu}(R) \right] \phi_{\nu K'}^{J'}(R; \theta, \varphi) = 0$$

Hyperspherical representation for dipoles ...



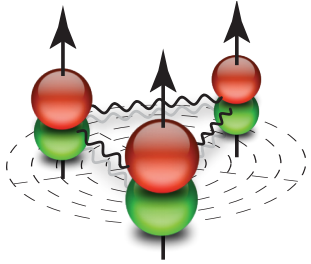
$$\Psi_M(R, \Omega) = \sum_{\nu} F_{\nu}^M(R) \Phi_{\nu}^M(R; \Omega) \quad (J \text{ is not conserved})$$

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$$\begin{aligned} \langle J'K'M' | v_{dd}(\vec{r}_{ij}) | JKM \rangle &= \frac{d_{\ell}}{\mu_{2b} r_{ij}^3} (-1)^{K+M} \delta_{MM'} \sqrt{(2J+1)(2J'+1)} \times \\ &\left[\delta_{KK'} \begin{pmatrix} J & 2 & J' \\ K & 0 & -K' \end{pmatrix} \begin{pmatrix} J & 2 & J' \\ M & 2 & -M' \end{pmatrix} \right. \\ &- \delta_{K-2, K'} \frac{3}{\sqrt{6}} \begin{pmatrix} J & 2 & J' \\ -K & 2 & K' \end{pmatrix} \begin{pmatrix} J & 2 & J' \\ -M & 0 & M' \end{pmatrix} \left(\frac{r_{ij}^x - i r_{ij}^y}{2} \right)^2 \\ &\left. - \delta_{K+2, K'} \frac{3}{\sqrt{6}} \begin{pmatrix} J & 2 & J' \\ K & 2 & -K' \end{pmatrix} \begin{pmatrix} J & 2 & J' \\ M & 0 & -M' \end{pmatrix} \left(\frac{r_{ij}^x + i r_{ij}^y}{2} \right)^2 \right] \end{aligned}$$

Hyperspherical representation for dipoles ...



$$\Psi_M(R, \Omega) = \sum_{\nu} F_{\nu}^M(R) \Phi_{\nu}^M(R; \Omega)$$

(J is **not** conserved)

$$\Phi_{\nu}^M(R; \Omega) = \sum_J \sum_K \phi_{\nu K}^J(R; \theta, \varphi) D_{KM}^J(\alpha, \beta, \gamma)$$

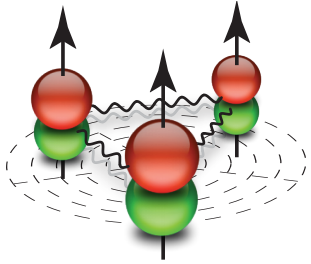
$$\sum_{J'K'} \left[\left\langle JMK \mid \frac{\hat{\Lambda}(\Omega)}{2\mu R^2} \mid J'MK' \right\rangle \delta_{JJ'} \right.$$

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$U_{\nu}(R)$: Adiabatic Potentials

$P_{\nu}(R)$ and $Q_{\nu\nu'}(R)$: nonadiabatic couplings

Hyperspherical representation for dipoles ...



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$U_{\nu}(R)$: Adiabatic Potentials

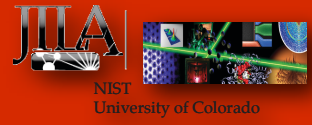
$P_{\nu}(R)$ and $Q_{\nu\nu'}(R)$: nonadiabatic couplings

$$J_{\max} = 14$$

50x50 system of 2D-PDE !!!

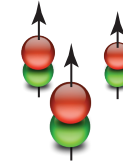
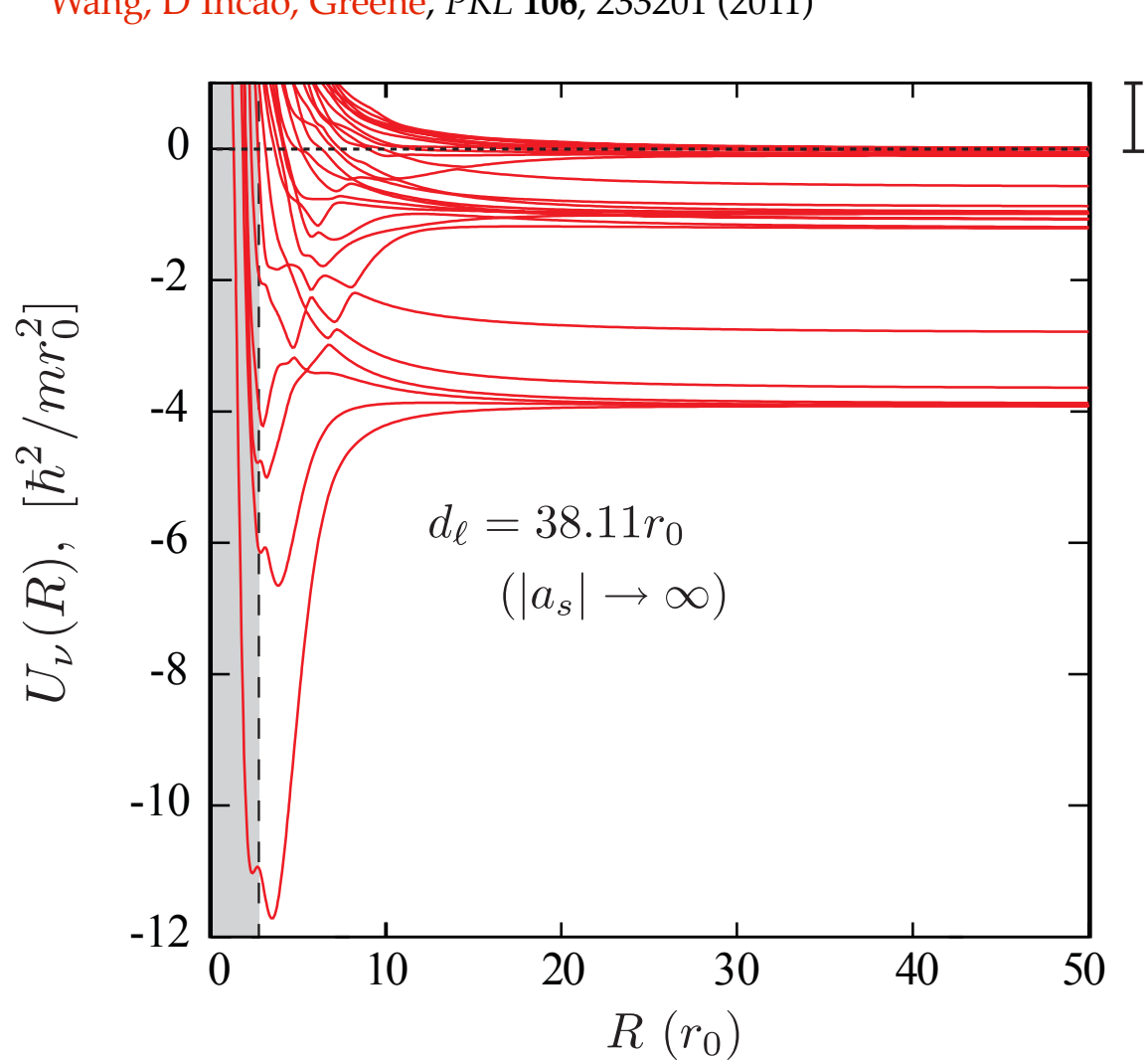
(very challenging !!!)

Dipolar Efimov Effect



Three-dipoles effective potentials

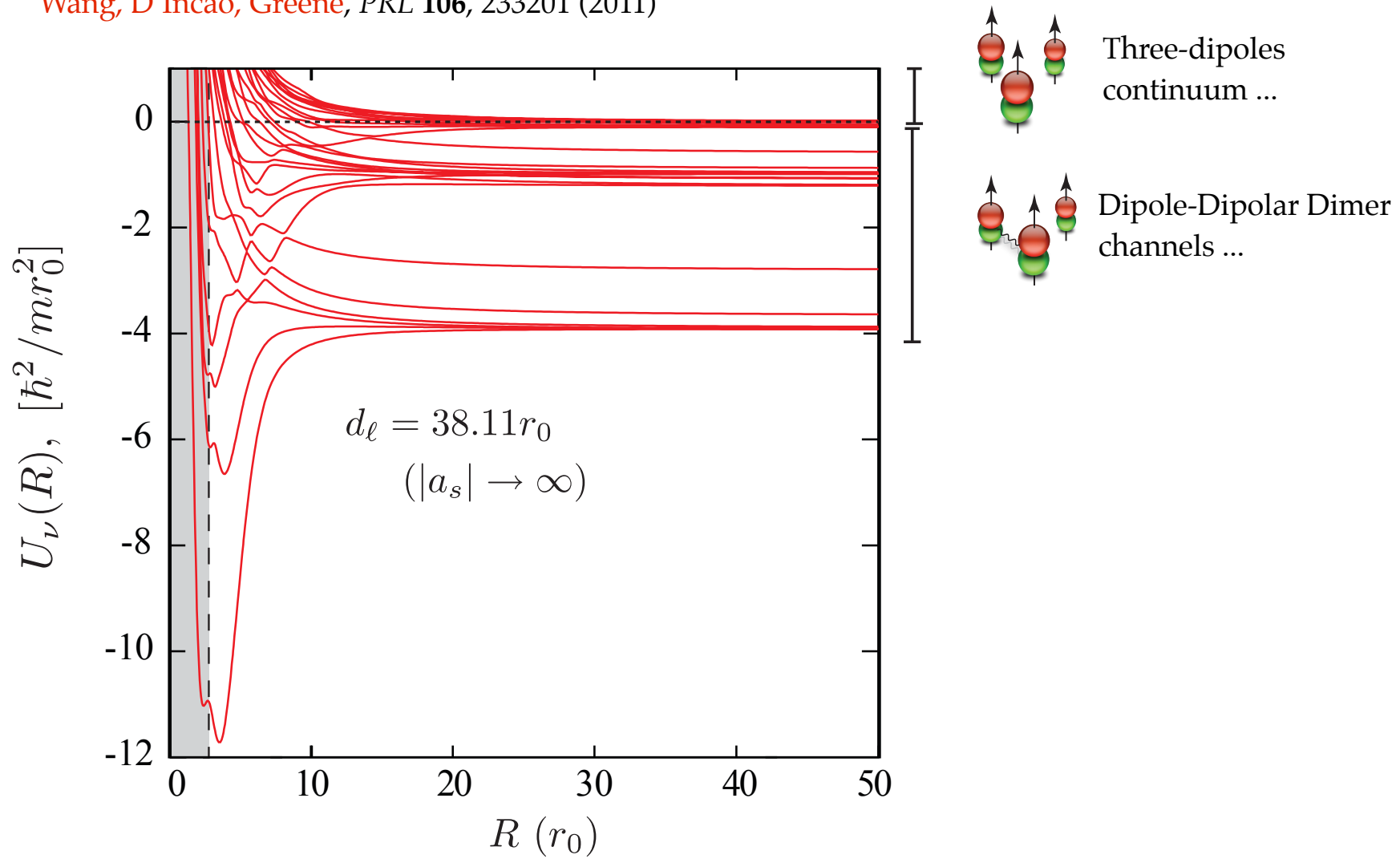
Wang, D'Incao, Greene, *PRL* 106, 233201 (2011)



Three-dipoles
continuum ...

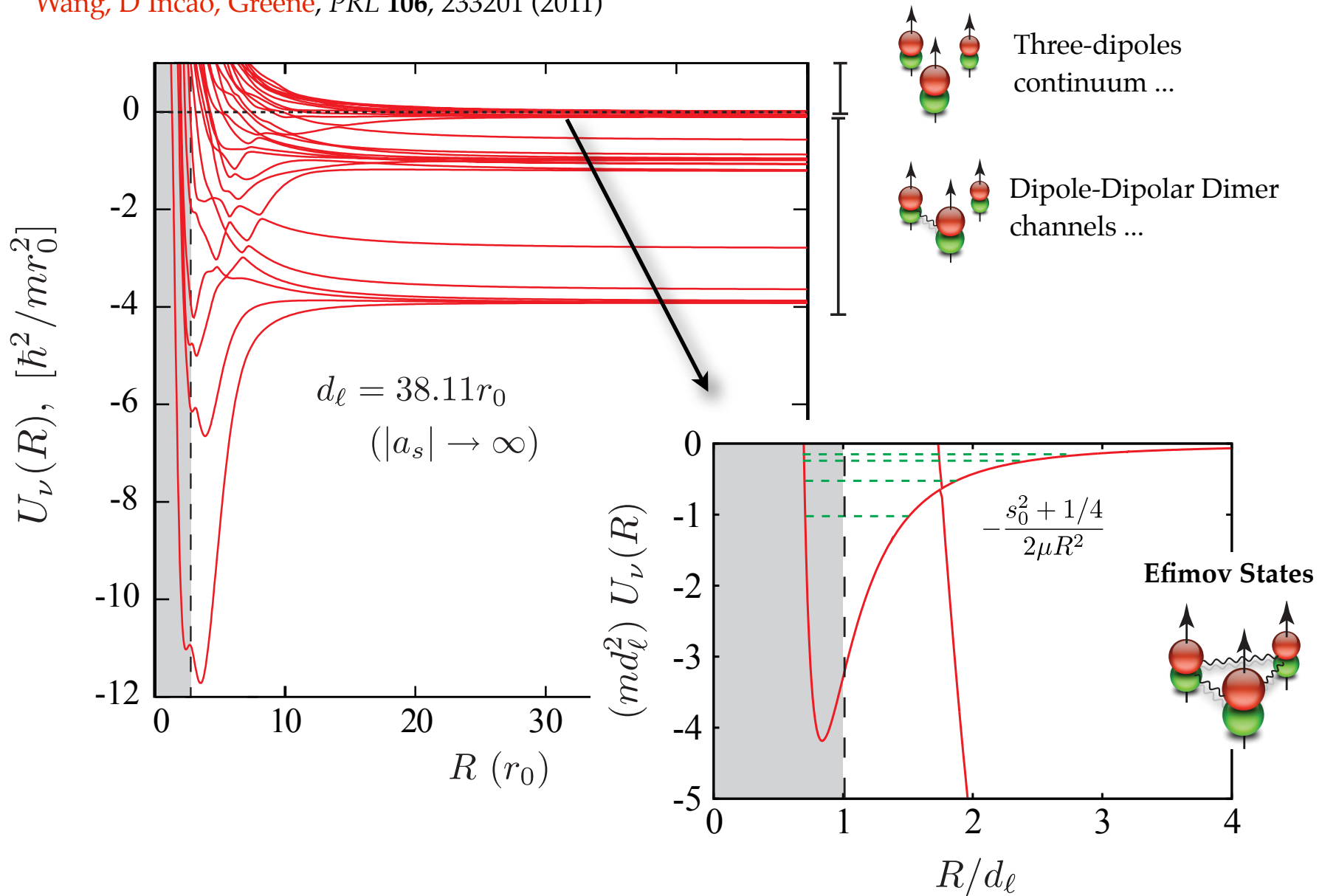
Three-dipoles effective potentials

Wang, D'Incao, Greene, *PRL* 106, 233201 (2011)



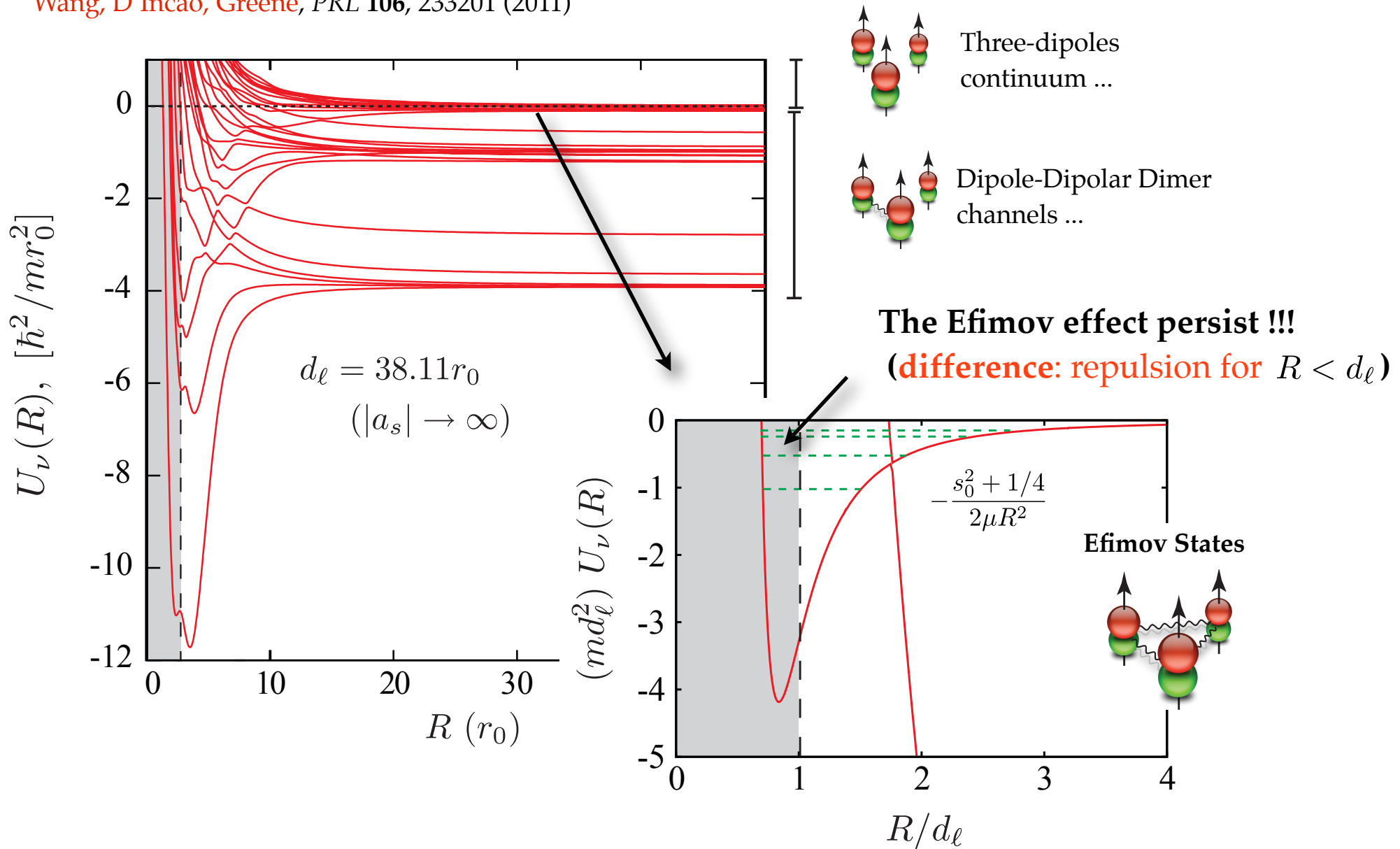
Three-dipoles effective potentials

Wang, D'Incao, Greene, *PRL* 106, 233201 (2011)



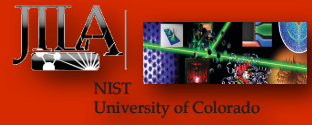
Three-dipoles effective potentials

Wang, D'Incao, Greene, *PRL* 106, 233201 (2011)



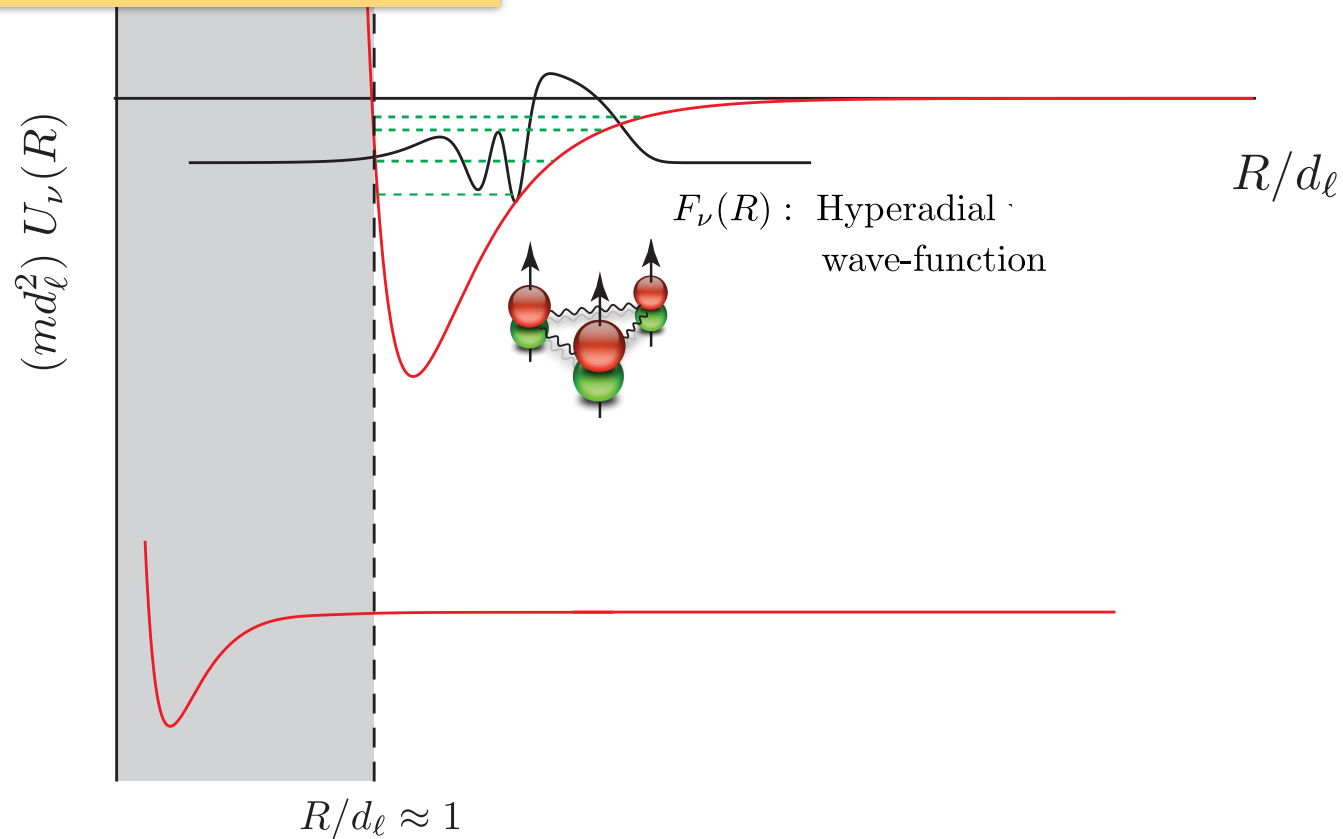
So what ?

Dipolar Efimov Effect



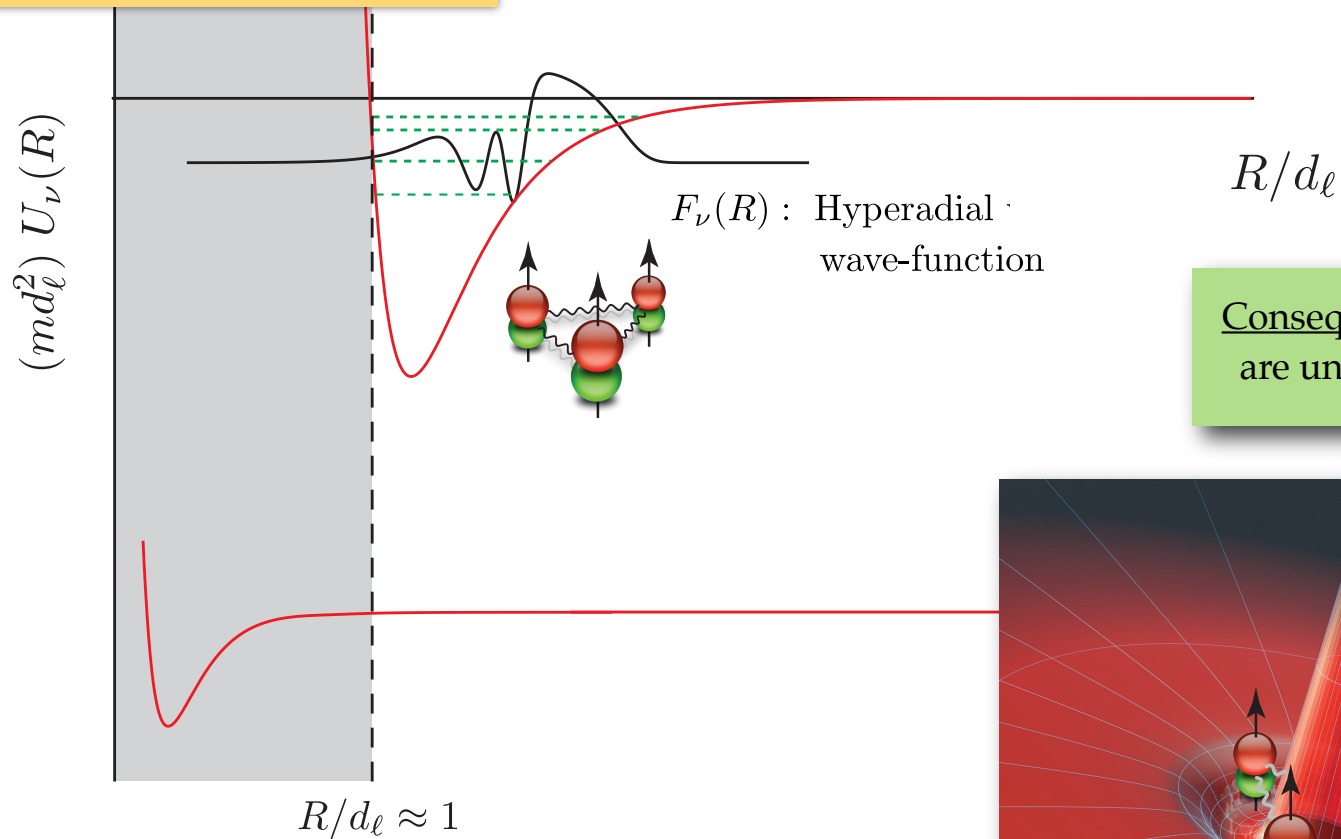
Three-dipoles effective potentials (schematic representation)

The barrier prevents two dipoles to be at distances smaller than d_ℓ !!!

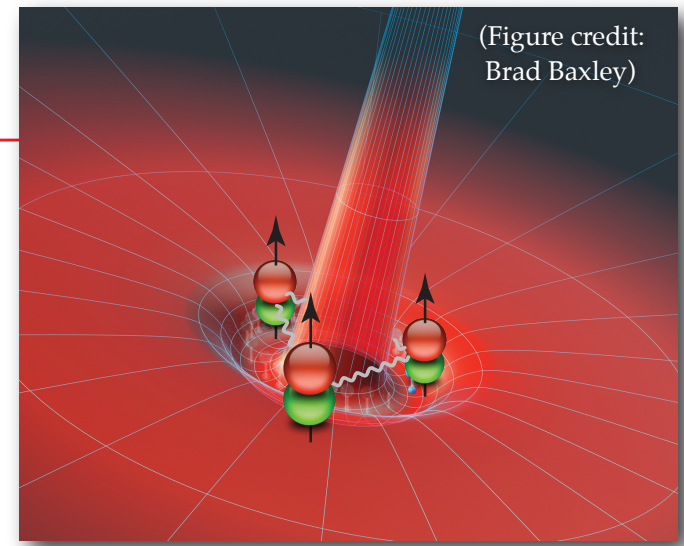


Three-dipoles effective potentials (schematic representation)

The barrier prevents two dipoles to be at distances smaller than d_ℓ !!!



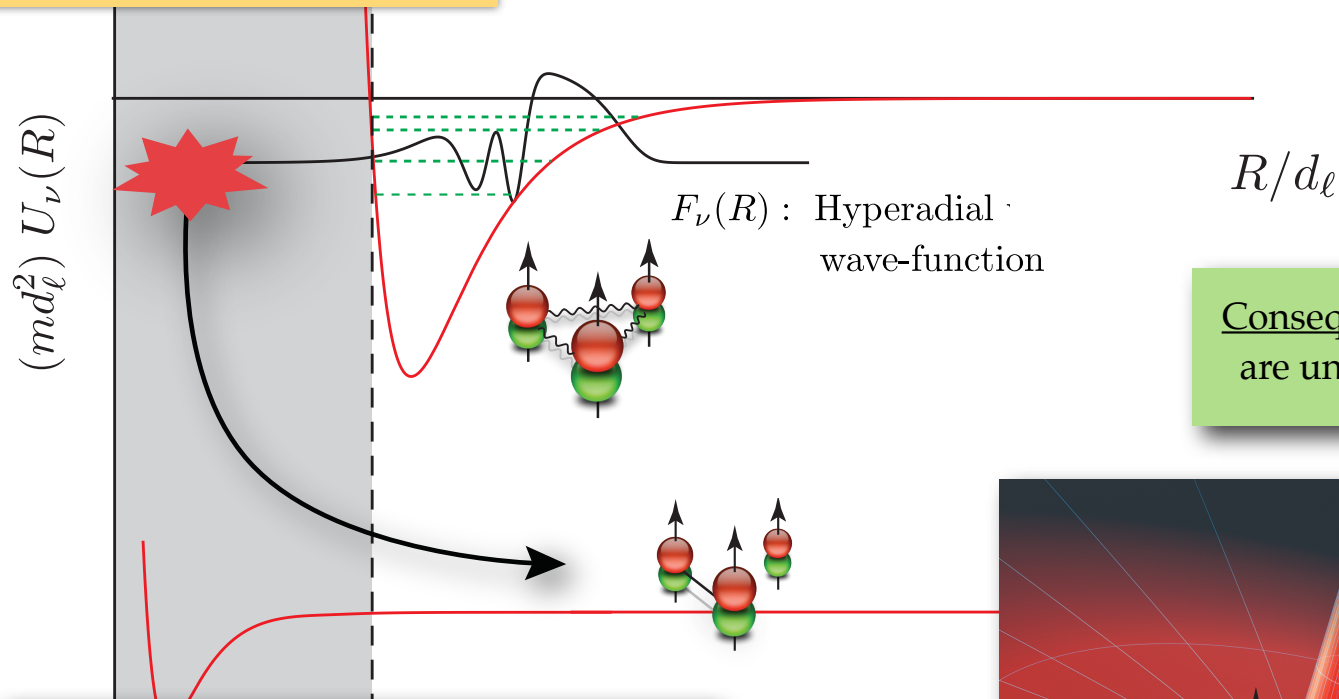
Consequence #1: Energies, geometry, ... are universal !!! (depend only on d_ℓ)



(Figure credit: Brad Baxley)

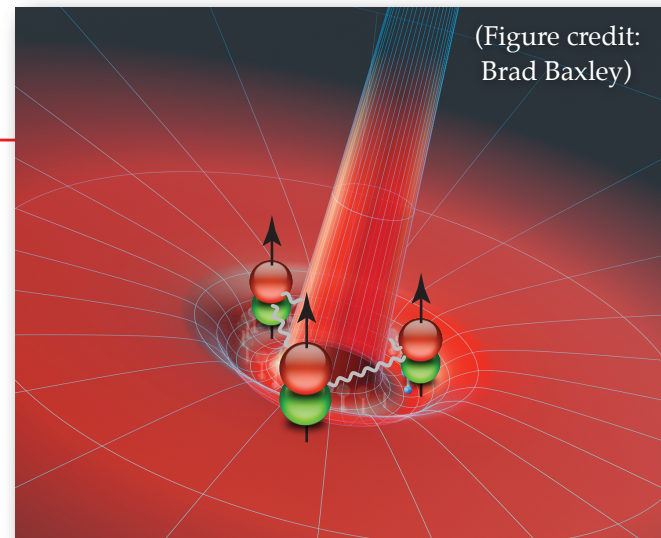
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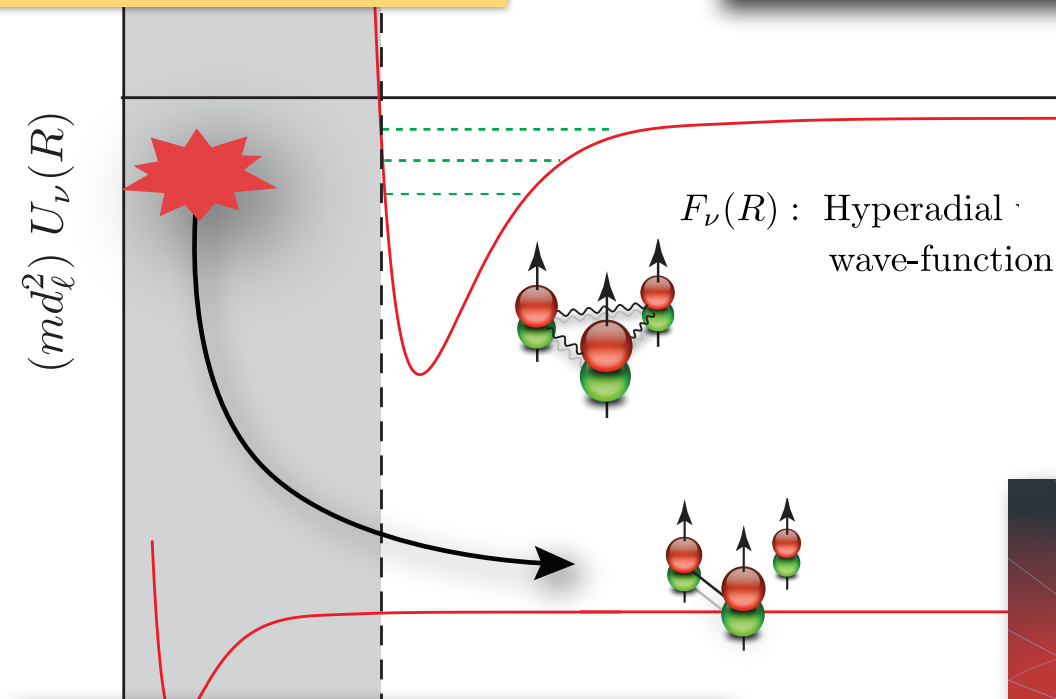
Consequence #2: suppression of decay !!! (long-lived states !!!)



Three-dipoles effective potentials (schematic representation)

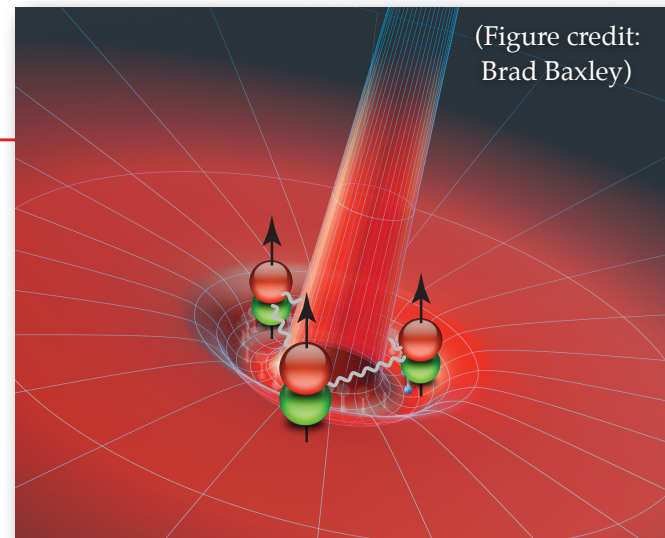
The barrier prevents two dipoles to be at distances smaller than d_ℓ !!!

Consequence #3: universal collisional properties (resonances)

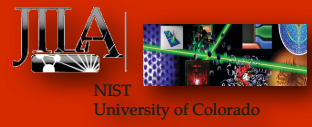


Consequence #1: Energies, geometry, ... are universal !!! (depend only on d_ℓ)

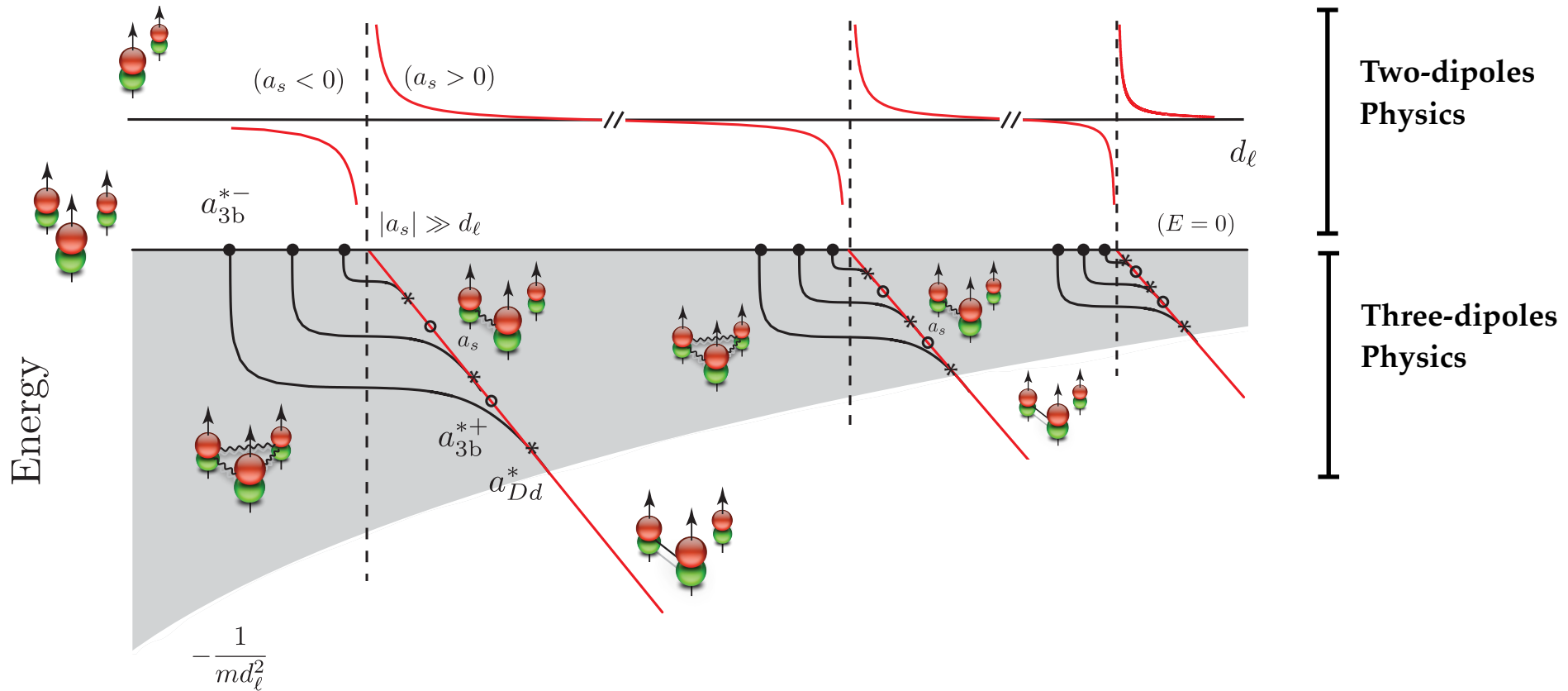
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Dipolar Efimov Effect



Dipolar Efimov Spectrum (schematic representation)

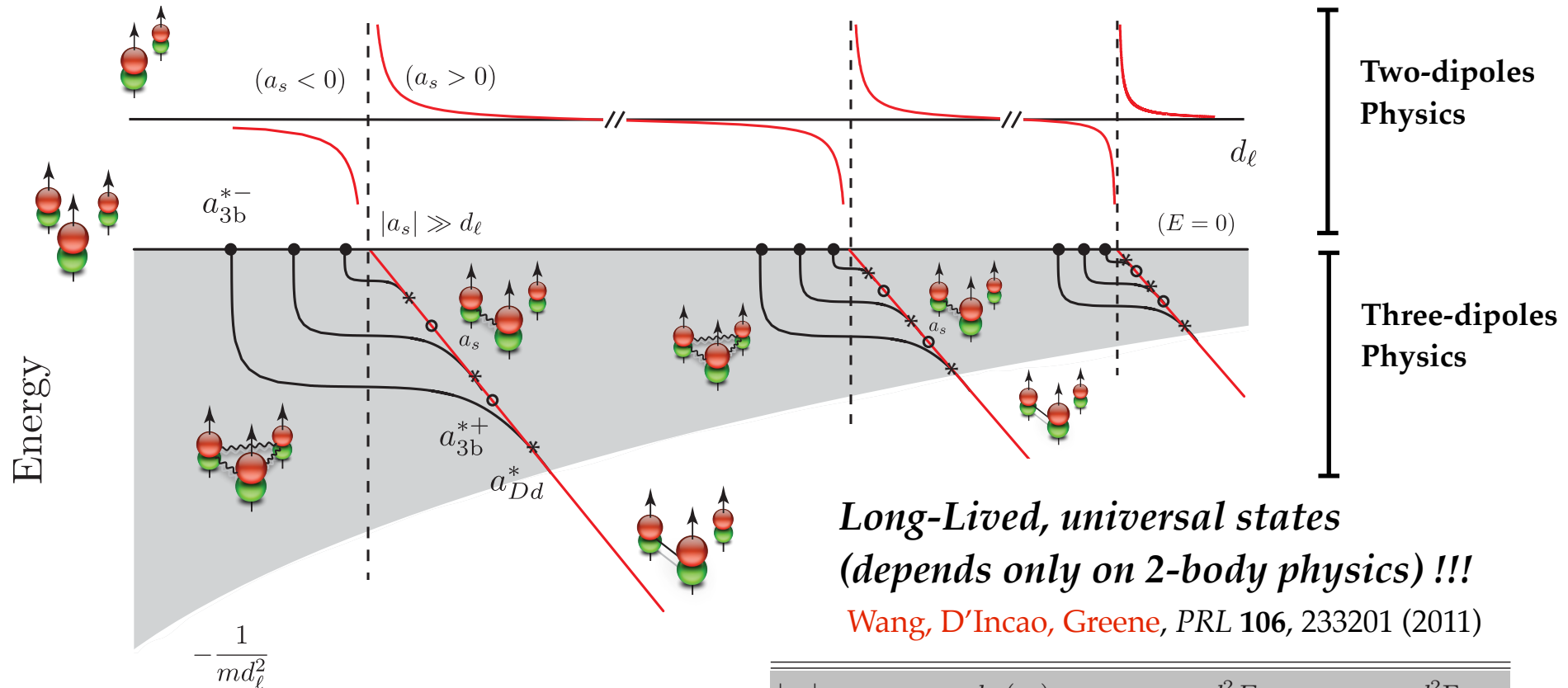


a_{3b}^{*-} : resonances in three-dipoles recombination

a_{3b}^{*+} : interference in three-dipoles recombination

a_{Dd}^* : resonances in dipole-dipolar dimer collisions

Dipolar Efimov Spectrum (schematic representation)

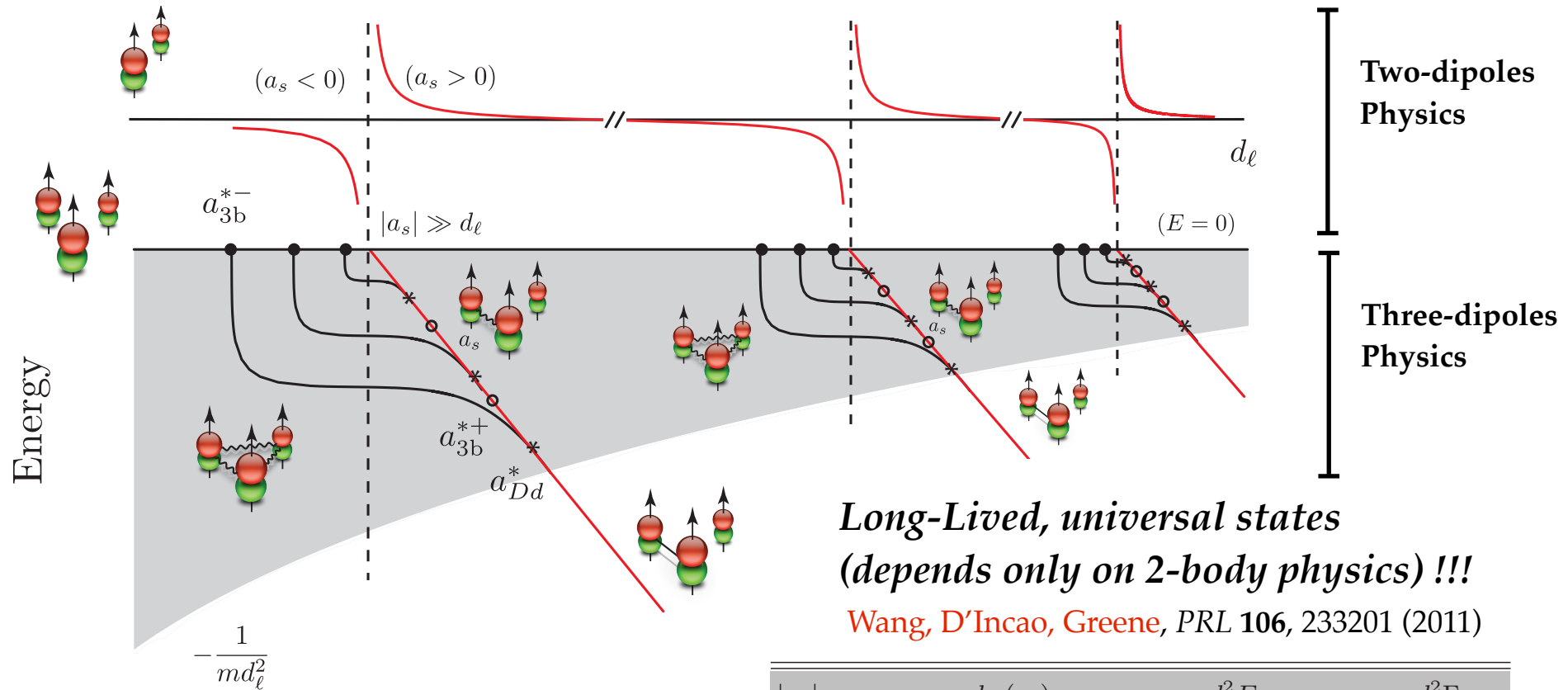


**Long-Lived, universal states
(depends only on 2-body physics) !!!**

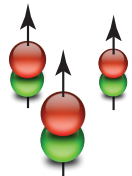
Wang, D'Incao, Greene, PRL 106, 233201 (2011)

| $ a_s \rightarrow \infty$ | d_ℓ (r_0) | $md_\ell^2 E_0$ | $md_\ell^2 \Gamma$ |
|----------------------------|-------------------------------------|-------------------------------------|-------------------------------|
| | 14.534 | 3.06×10^{-2} | 5.2×10^{-3} |
| | 25.498 | 3.03×10^{-2} | 6.6×10^{-3} |
| | 38.110 | 2.95×10^{-2} | 3.2×10^{-3} |
| (3-body) | $a_{3b}^{*-}/d_\ell \approx -8.1$ | $a_{3b}^{*+}/d_\ell \approx 1.8$ | $a_{Dd}^*/d_\ell \approx 8.6$ |
| (4-body) | $a_{4b,1}^{*-}/d_\ell \approx -3.5$ | $a_{4b,2}^{*-}/d_\ell \approx -7.3$ | |
| | $a_{dd,1}^*/d_\ell \approx 20.$ | $a_{dd,2}^*/d_\ell \approx 57.$ | $a_{Dd}^c/d_\ell \approx 58.$ |

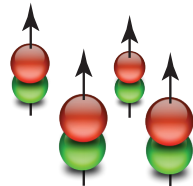
Dipolar Efimov Spectrum (schematic representation)



✓ Three-dipoles resonances

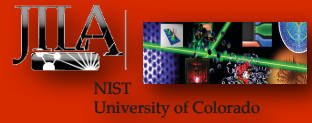


✓ Four-dipoles resonances



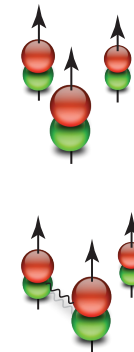
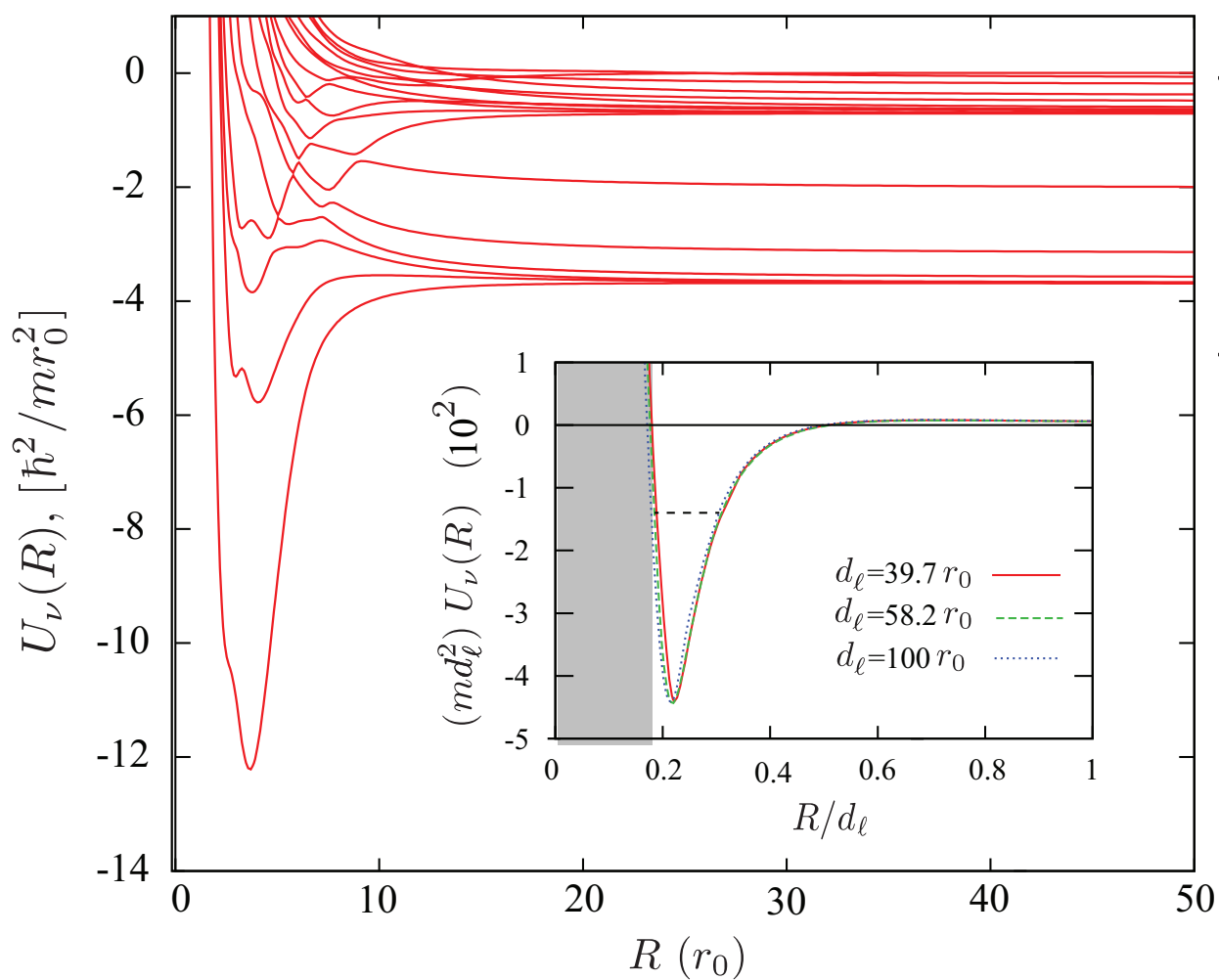
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Fermionic Dipoles - a new class of few-body states !?



Three-fermionic dipoles effective potentials

• “Universal three-body physics for fermionic dipoles”,
 Wang, D’Incao, Greene, arXiv:1106.6133 (2011)



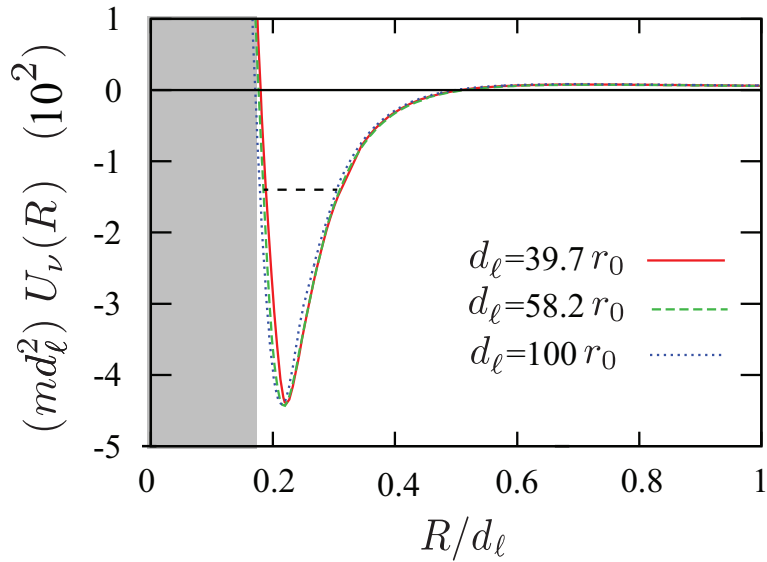
Three-dipoles
continuum ...

Dipole-Dipolar Dimer
channels ...

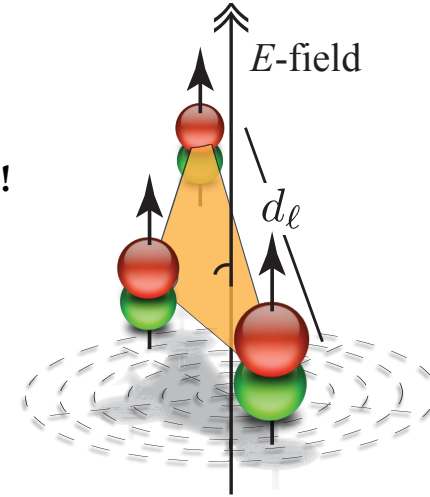
Repulsion for $R < d_\ell$!!!

Three-fermionic dipoles effective potentials

- “Universal three-body physics for fermionic dipoles”,
Wang, D’Incao, Greene, arXiv:1106.6133 (2011)

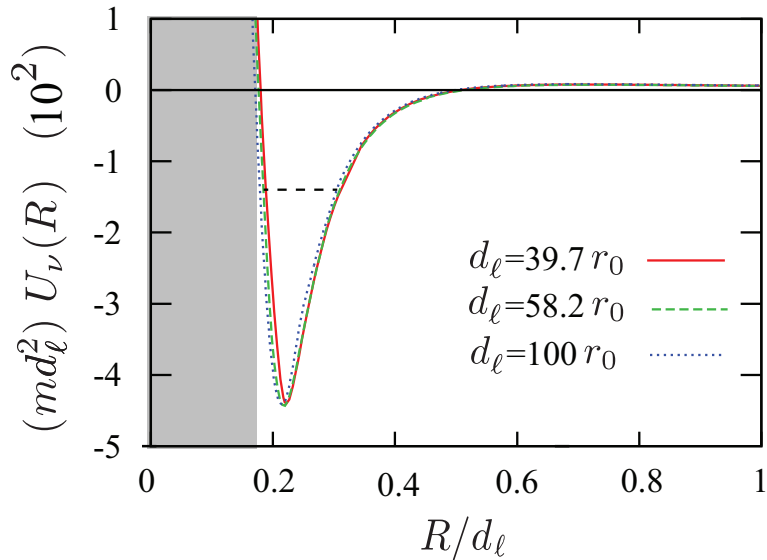


Single Universal State !!!

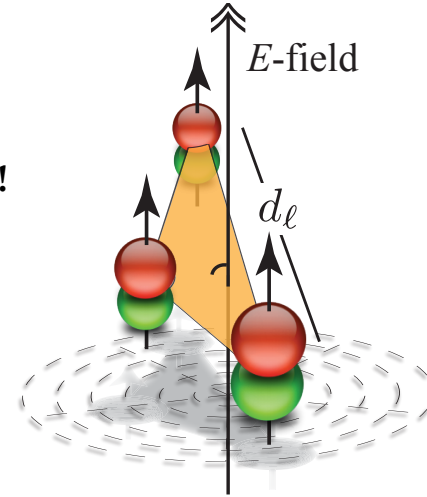


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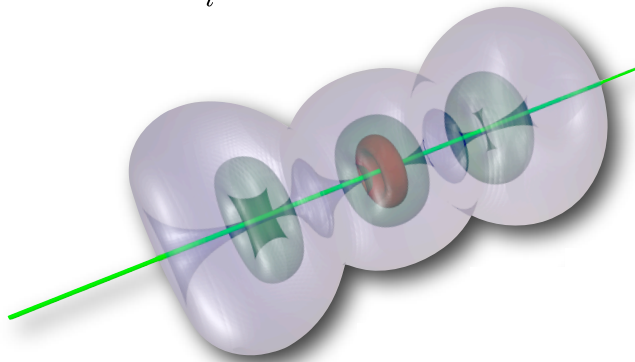


Single Universal State !!!



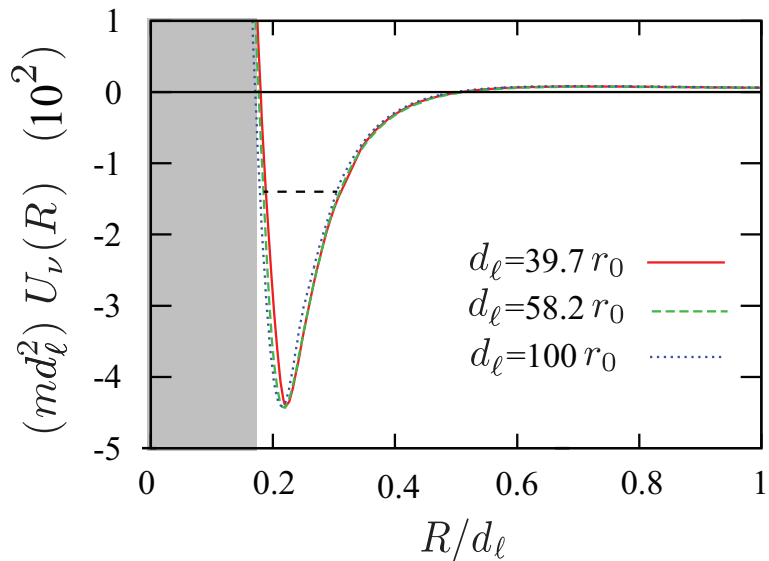
Probability Density:

$$\rho(\vec{r}) = \frac{1}{3} \langle \Psi | \sum_i \delta(\vec{r} - \vec{r}_i) | \Psi \rangle$$

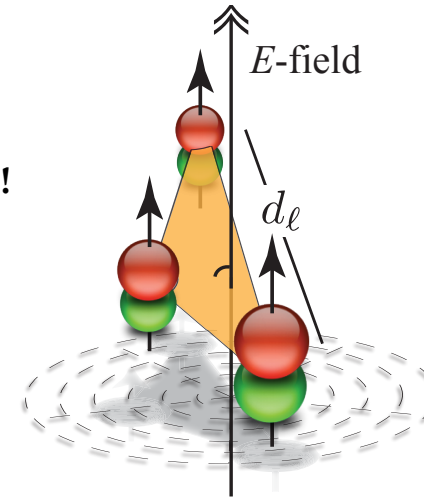


Three-fermionic dipoles effective potentials

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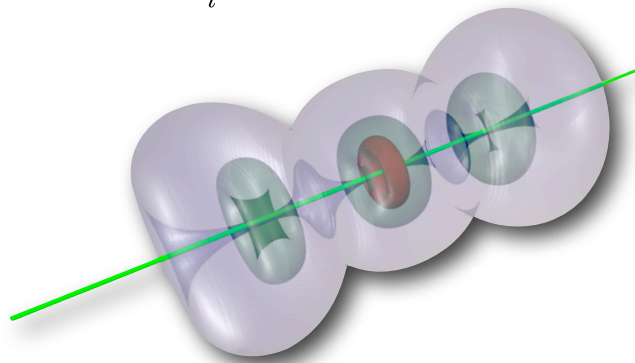


Single Universal State !!!



Probability Density:

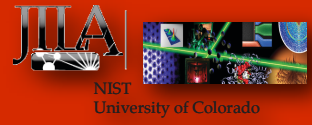
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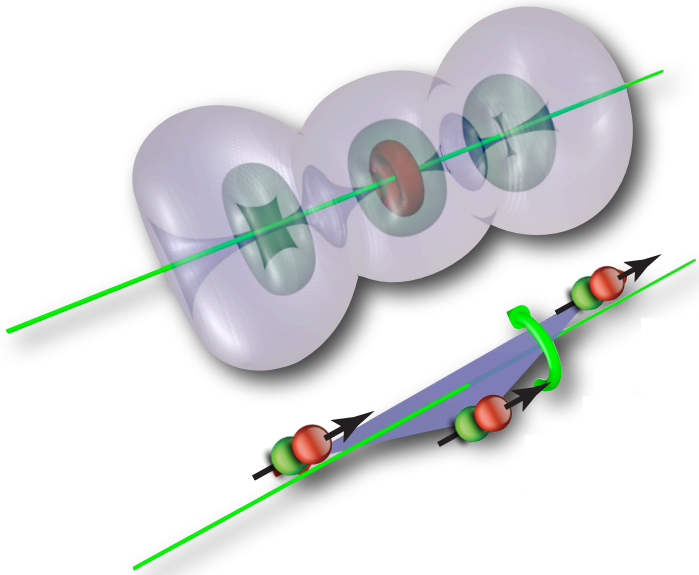
Long-Lived, universal states !!!

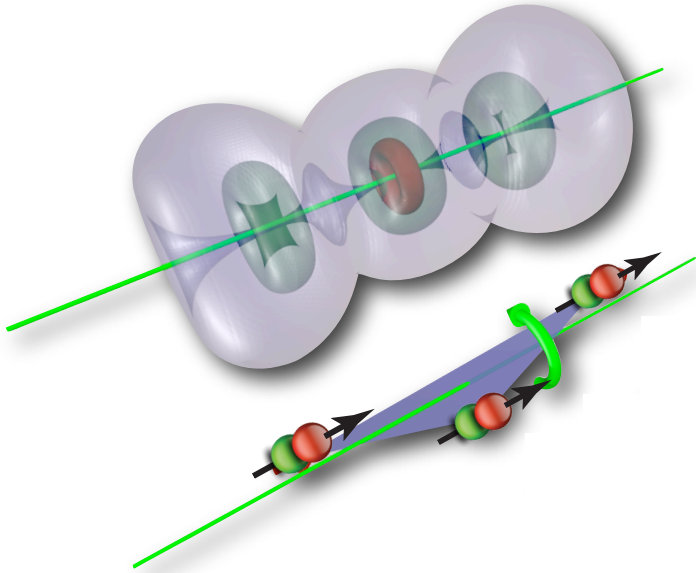
| $E_{2d} \rightarrow 0$ | d_ℓ (r_0) | $m d_\ell^2 E_{3d}$ | $m d_\ell^2 \Gamma$ |
|---------------------------|---------------------------|---------------------------|---------------------------|
| | 39.7 | 171 | 42 |
| | 58.2 | 135 | 43 |
| | 100 | 139 | 17 |
| $\Theta \approx 16^\circ$ | $b_l \approx 0.26 d_\ell$ | $b_s \approx 0.14 d_\ell$ | $\Delta \approx 15^\circ$ |

Summary

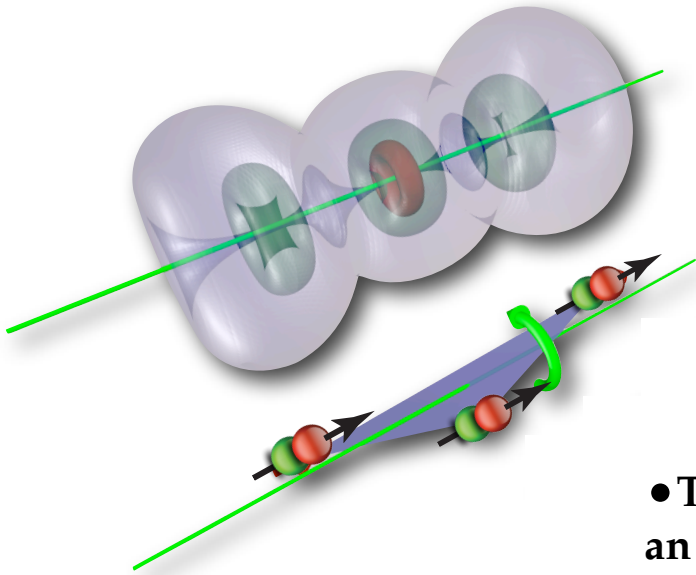


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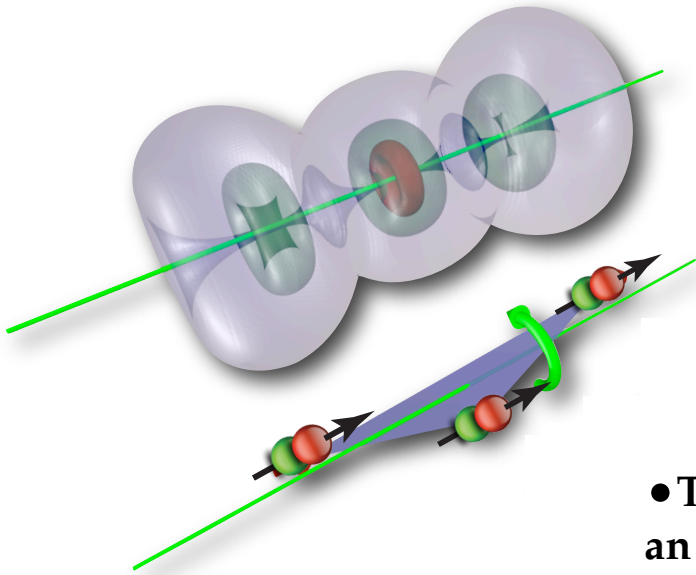


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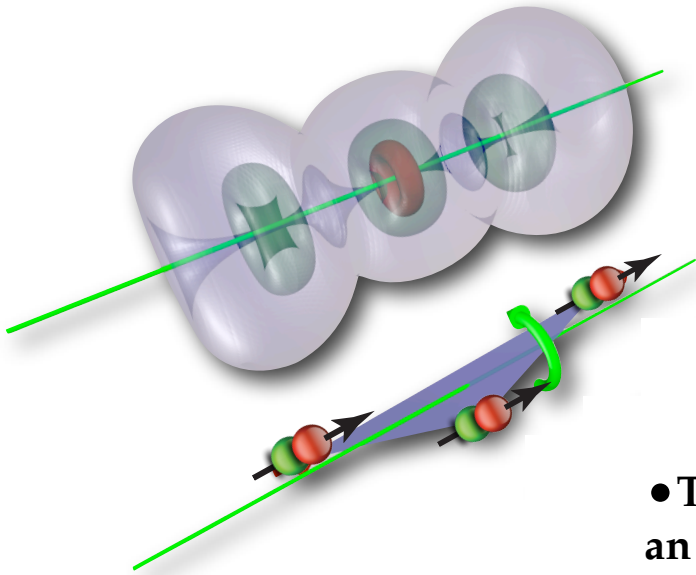
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- Future experiments in ultracold dipolar gases are expected to offer a much better scenario to probe few-body universal states.