“Photodisintegration” of 3-bosons at threshold

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Outline

The 3-body problem

The Hamiltonian

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- Where $x = \mu r$, $g = \frac{V_0 m}{\hbar^2\mu^2}$, $\epsilon = \frac{Em}{\hbar^2\mu^2}$ and $U(x)$ is a Gaussian or a Yukawa potential.

Solving the Schroedinger Equation

The HH expansion in 4 steps

1. Remove the center of mass and introduce hyperspherical coordinates

$$\vec{r}_1, \vec{r}_2, \ldots \vec{r}_A \longrightarrow \vec{R}_{c.m.}, \vec{\eta}_1 \vec{\eta}_2 \ldots \vec{\eta}_{A-1} \longrightarrow \rho = \sqrt{\eta_1^2 + \eta_2^2 + \ldots + \eta_{A-1}^2}, \Omega$$

2. Expand the wave function using hyperspherical harmonics

$$\Psi(\rho, \Omega) = \sum_{K \leq K_{max}} R_{[K]}(\rho) \mathcal{Y}_{[K]}(\Omega)$$

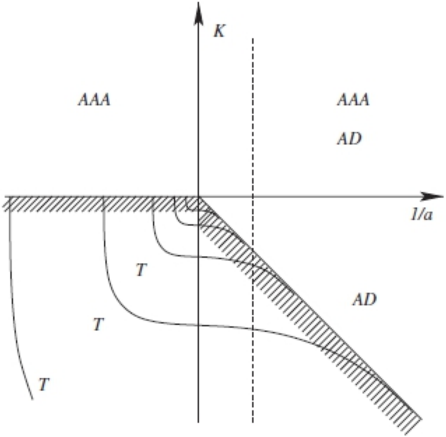
3. Calculate the matrix elements

$$V_{[K][K']}(\rho) = \langle [K] | \sum V_{ij} | [K'] \rangle$$

4. Solve the eigenvalue problem

$$\left[-\frac{1}{2} \left(\frac{\partial^2}{\partial \rho^2} + \frac{3A-4}{\rho} \frac{\partial}{\partial \rho} - \frac{\hat{K}^2}{\rho^2} \right) + \hat{V}(\rho) \right] \Psi = E \Psi$$

Orientation



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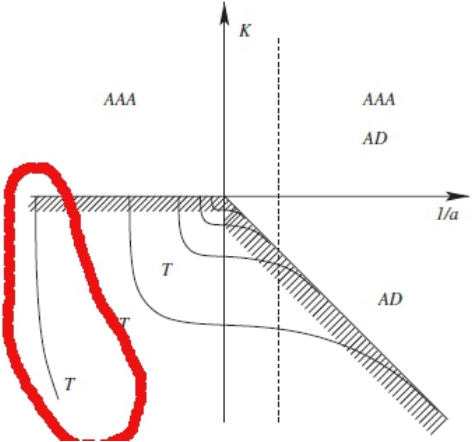
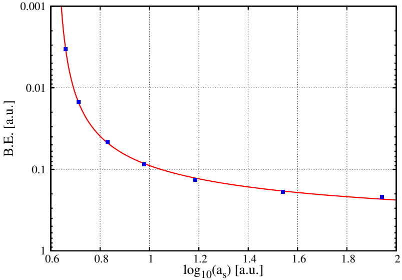
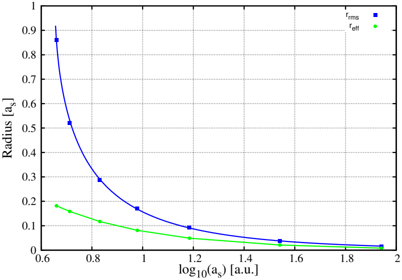


The Ground state

Binding energy



Effective range and RMS matter radius

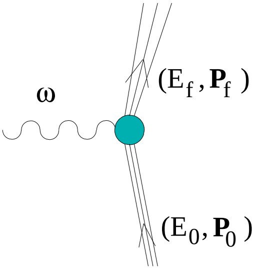


Photoabsorption

Real Photon

$$|\mathbf{q}| = \omega$$

$$\boxed{\sigma(\omega) = 4\pi^2 \alpha \omega R(\omega)}$$



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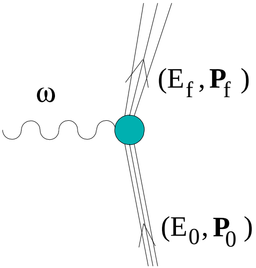

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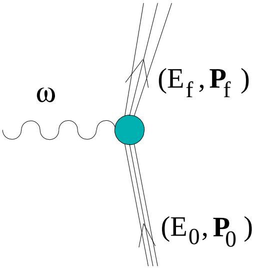

Where

$$T_\lambda(q) = (-)^\lambda \sqrt{2\pi} \sum_J \sqrt{2J+1} \left[E_{J\lambda}(q) + \lambda M_{J\lambda}(q) \right]$$

and

$$E_{J\lambda}(q) = \frac{i}{4\pi} \int d\hat{\boldsymbol{q}} \, (\hat{\boldsymbol{q}} \times \boldsymbol{Y}^{\lambda}_{JJ1}(\hat{\boldsymbol{q}})) \cdot \boldsymbol{J}(\boldsymbol{q})$$

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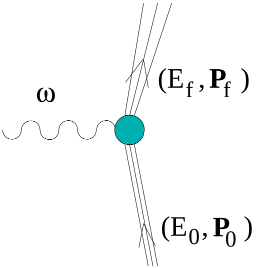

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- Conserved current must fulfill

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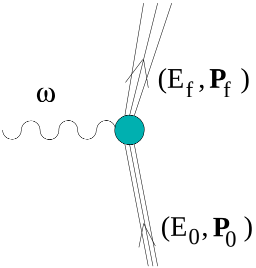

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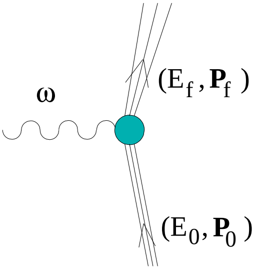

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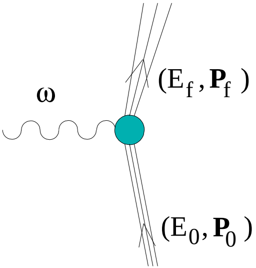

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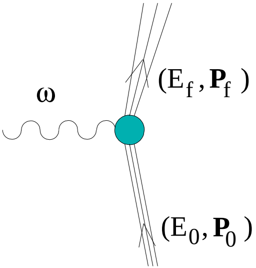
$$E^{K}_{J\lambda}(q) = -\frac{1}{4\pi}\sqrt{\frac{2J+1}{J}}\int d\hat{\boldsymbol{q}}\,\boldsymbol{Y}^{\lambda}_{JJ+11}(\hat{\boldsymbol{q}})\cdot\boldsymbol{J}(\boldsymbol{q})$$



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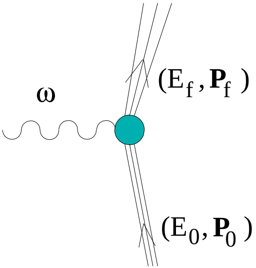
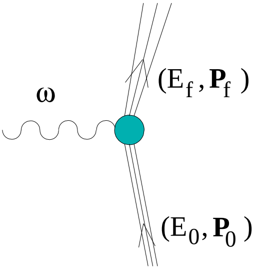
- For low energy photons $qR \ll 1$ the Siegert term is dominant.



A Model for “Photodisintegration”

- We consider a system of nutral particles.



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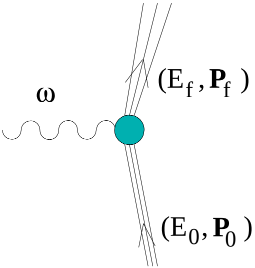

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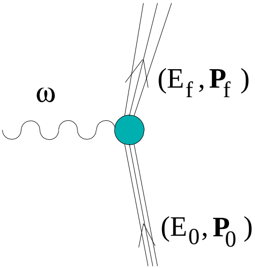

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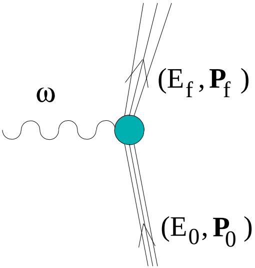

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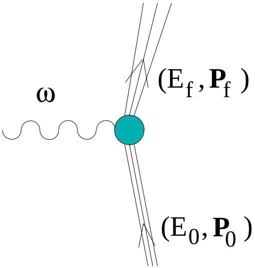

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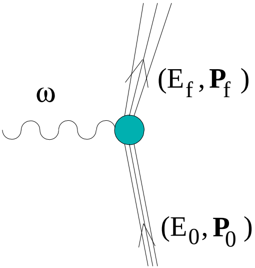
- the **quadrupole** opertor is the leading term



$$R(\omega) = \sum_f |\langle f|\hat{Q}|0\rangle|^2 \delta(E_f - E_0 - \omega)$$

where

$$\hat{Q} = \alpha \sum_i r_i^2 Y_{20}(\hat{r}_i)$$

Calculating the Response

The response function

$$\boxed{R(\omega) = \int d\psi_f |\langle \psi_i \mid \hat{O} \mid \psi_f \rangle|^2 \delta(E_f - E_i - \omega)}$$

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$$L(\sigma) = \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2}$$

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Or

$$L(\sigma) = \int d\psi_f \langle \psi_i \mid \hat{O} \frac{1}{H - E_i - \sigma - i\Gamma} \mid \psi_f \rangle \langle \psi_f \mid \frac{1}{H - E_i - \sigma + i\Gamma} \hat{O} \mid \psi_i \rangle$$

The Lorentz Integral transform (LIT) method

Using closure

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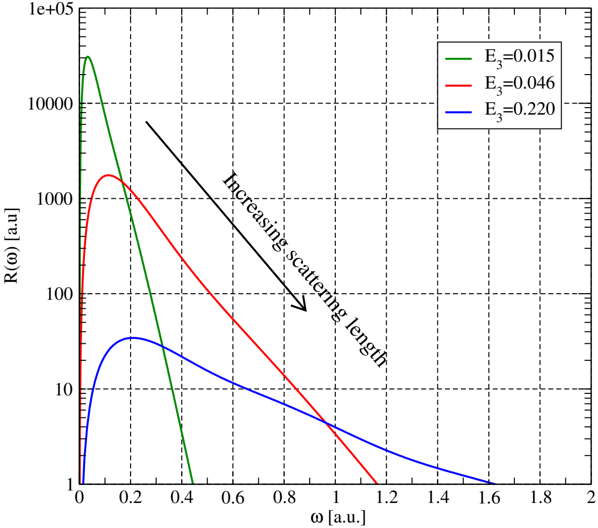
V. Efros, W. Leidemann, and G .Orlandini, PLB **238**, 130 (1994).

The Quadrupole response



Photodisintegration Sum Rules

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$$\begin{aligned} S_1 &= \langle 0|\,[\mathbf{O},[H,\mathbf{O}]]\,|0\rangle = \langle 0|\mathbf{O}\,(H-E_0)\,\mathbf{O}|0\rangle \\ S_0 &= \langle 0|\mathbf{O}\mathbf{O}|0\rangle \\ S_{-1} &= \langle 0|\mathbf{O}\frac{1}{H-E_0}\mathbf{O}|0\rangle \end{aligned}$$

Naive Scaling

- Using simple dimensional arguments we expect that

$$r \sim 1/\sqrt{E}$$

- The Quadrupole operator behaves as r^2 so

$$R(\omega) \sim r^4/E \sim 1/E^3$$

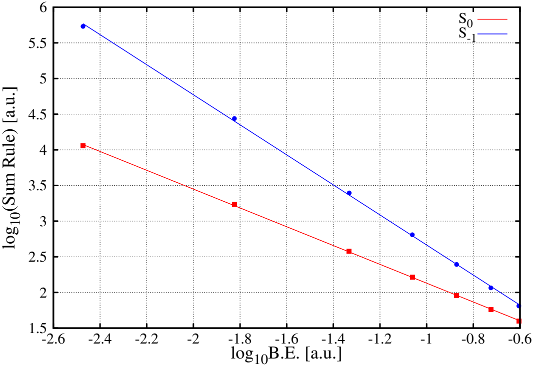
- It follows that the sum rules should fulfill

$$S_n \sim 1/E^{2-n}$$

- or

$$S_0 \sim 1/E^2$$
$$S_{-1} \sim 1/E^3$$
$$S_0/S_{-1} \sim E$$

Calculated Sum Rules

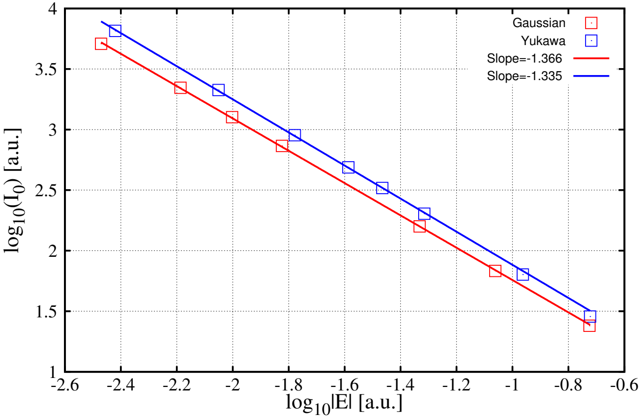


Fitted lines

$$S_0 = A_0 E^{-1.34}$$

$$S_{-1} = A_{-1} E^{-2.13}$$

Potential dependence

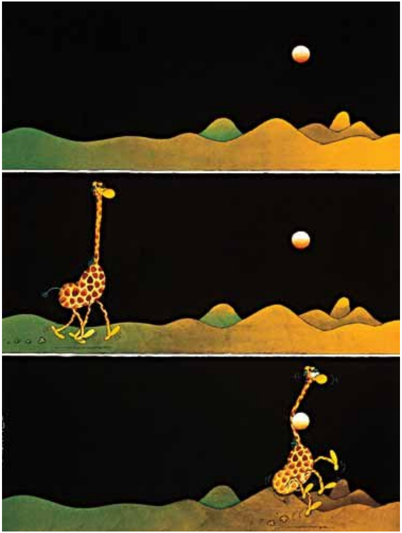


Naive scaling doesn't work !!!

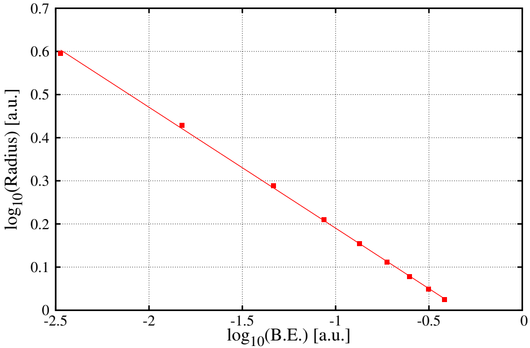
A For S_0 we got a power of 1.33 instead of 2.

B For S_{-1} we got a power of 2.13 instead of 3.

C The results seems to be independent of the short range specifications of the potential.



Last comment - The Matter Radii



The Radii behaves as

$$r_{rms} = CE^{-0.28}$$

Utilizing this result we see that

$$S_0 = A_0 E^{-1.34} \sim r_{rms}^4$$

$$S_{-1} = A_{-1} E^{-2.13} \sim r_{rms}^7$$

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Thanks, It's a great workshop !!!