Critical stability of a dipolar Bose-Einstein condensate: Bright and vortex solitons

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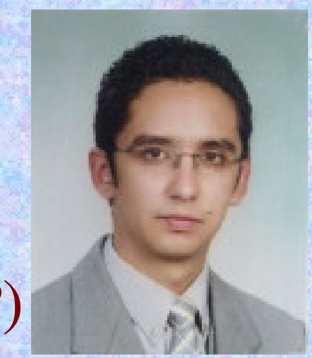
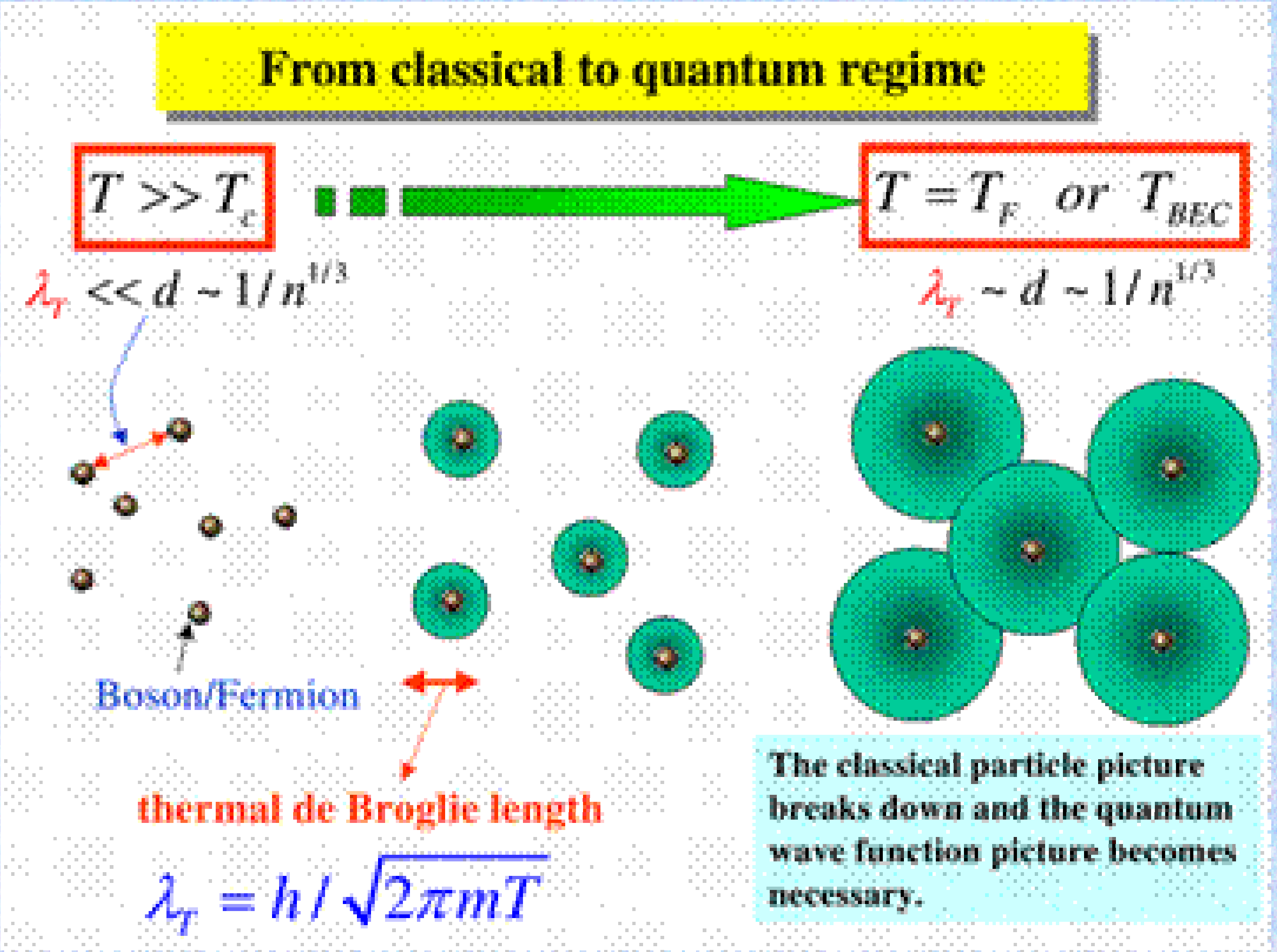


Paulsamy Muruganandam (India)

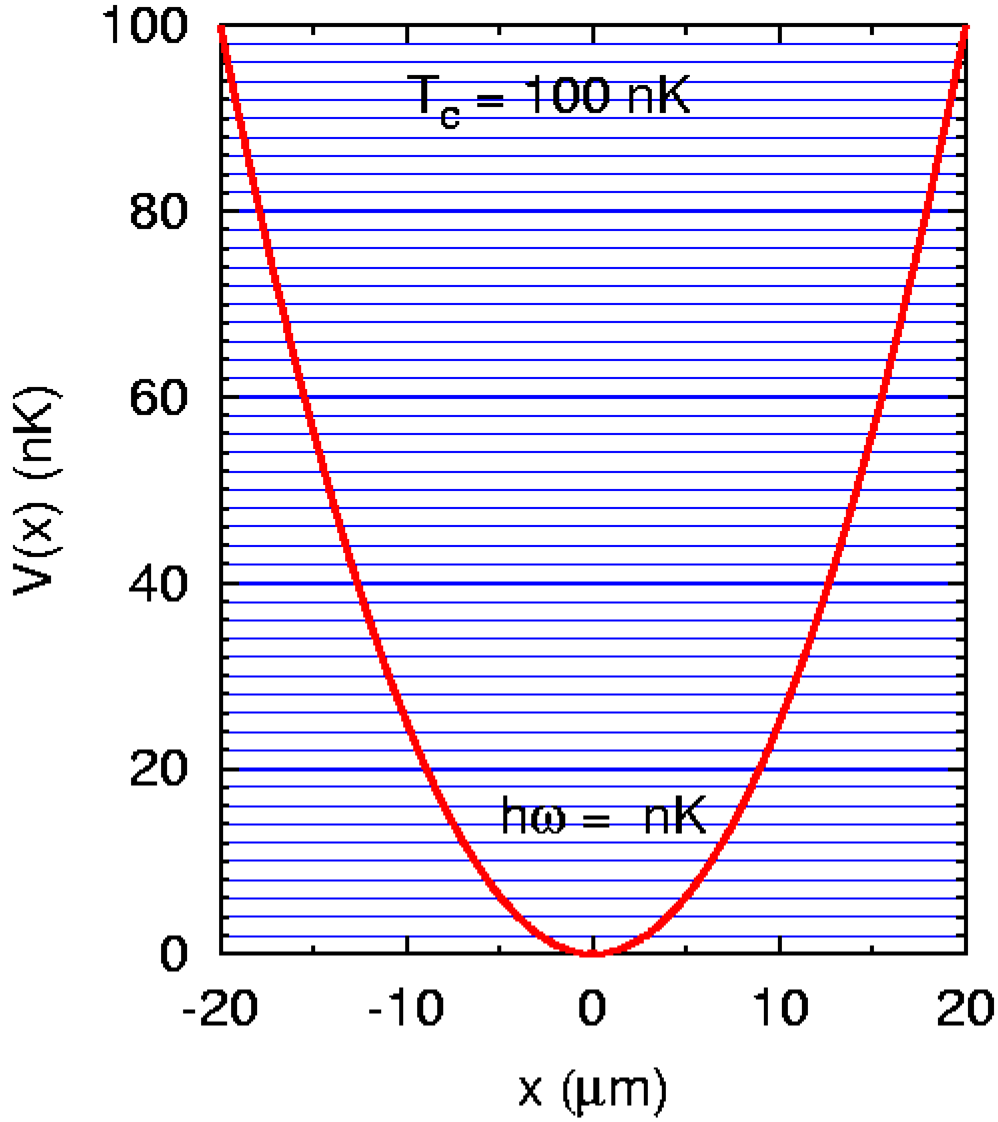
PLAN

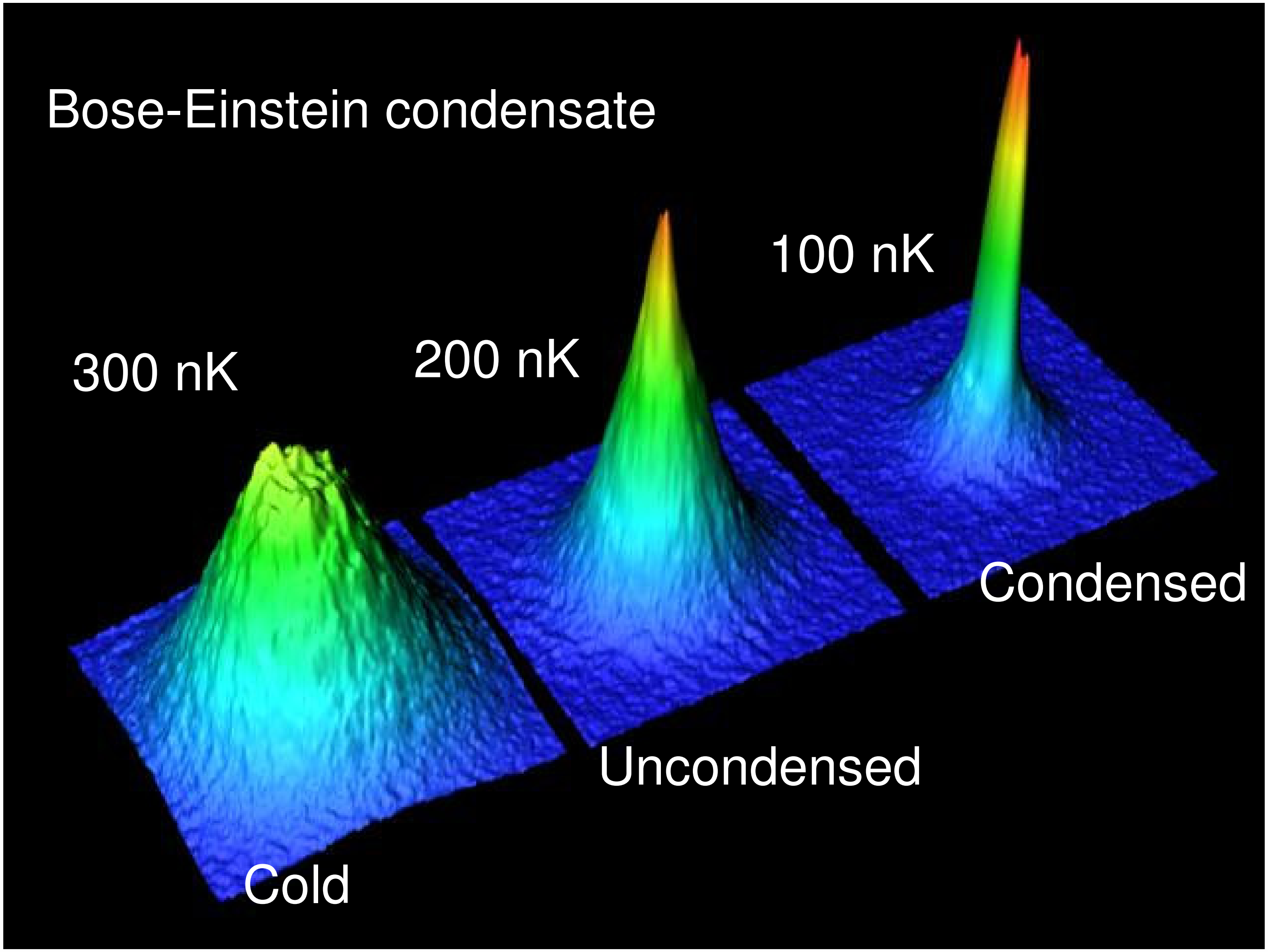
- Bose-Einstein Condensate
- Dipolar atoms
- Solitons, critical stability
- Collision of solitons
- Soliton molecule formation

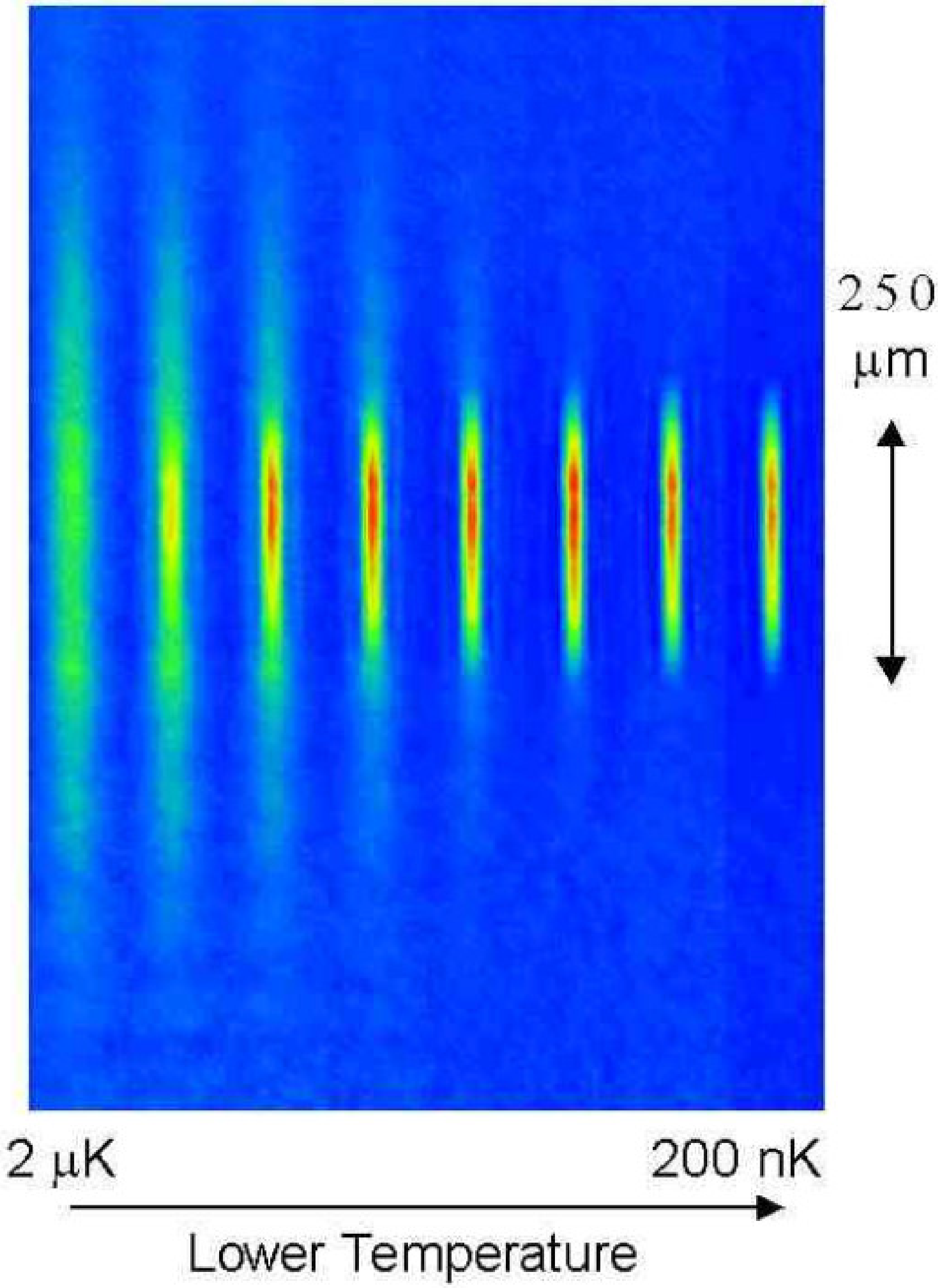
Bose-Einstein Condensate



Harmonic trap and quantum statistics



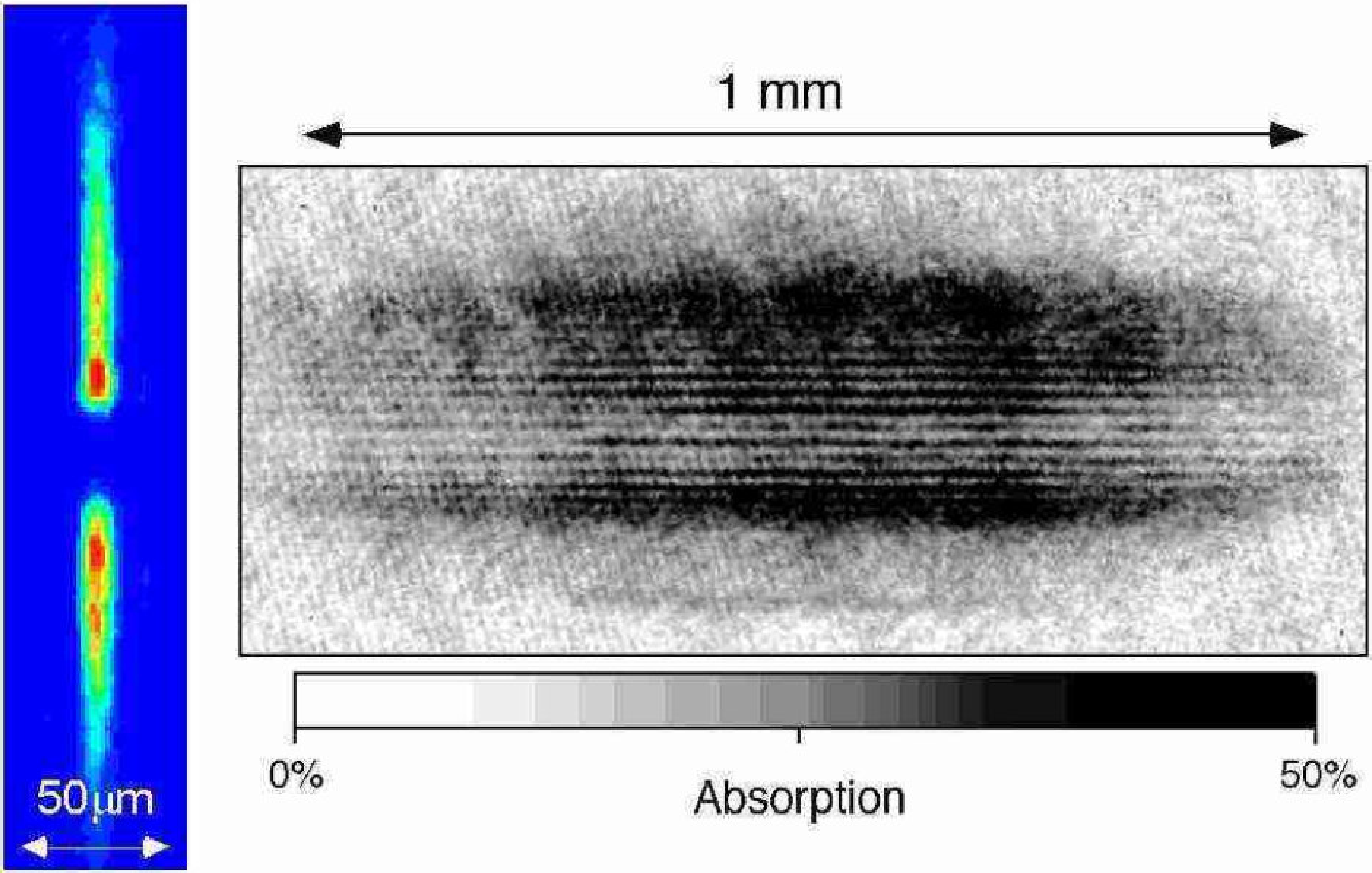

Bose-Einstein condensate
300 nK
200 nK
100 nK
Cold
Uncondensed
Condensed



BEC =
"Droplet" formation
in a
quantum saturated
vapor

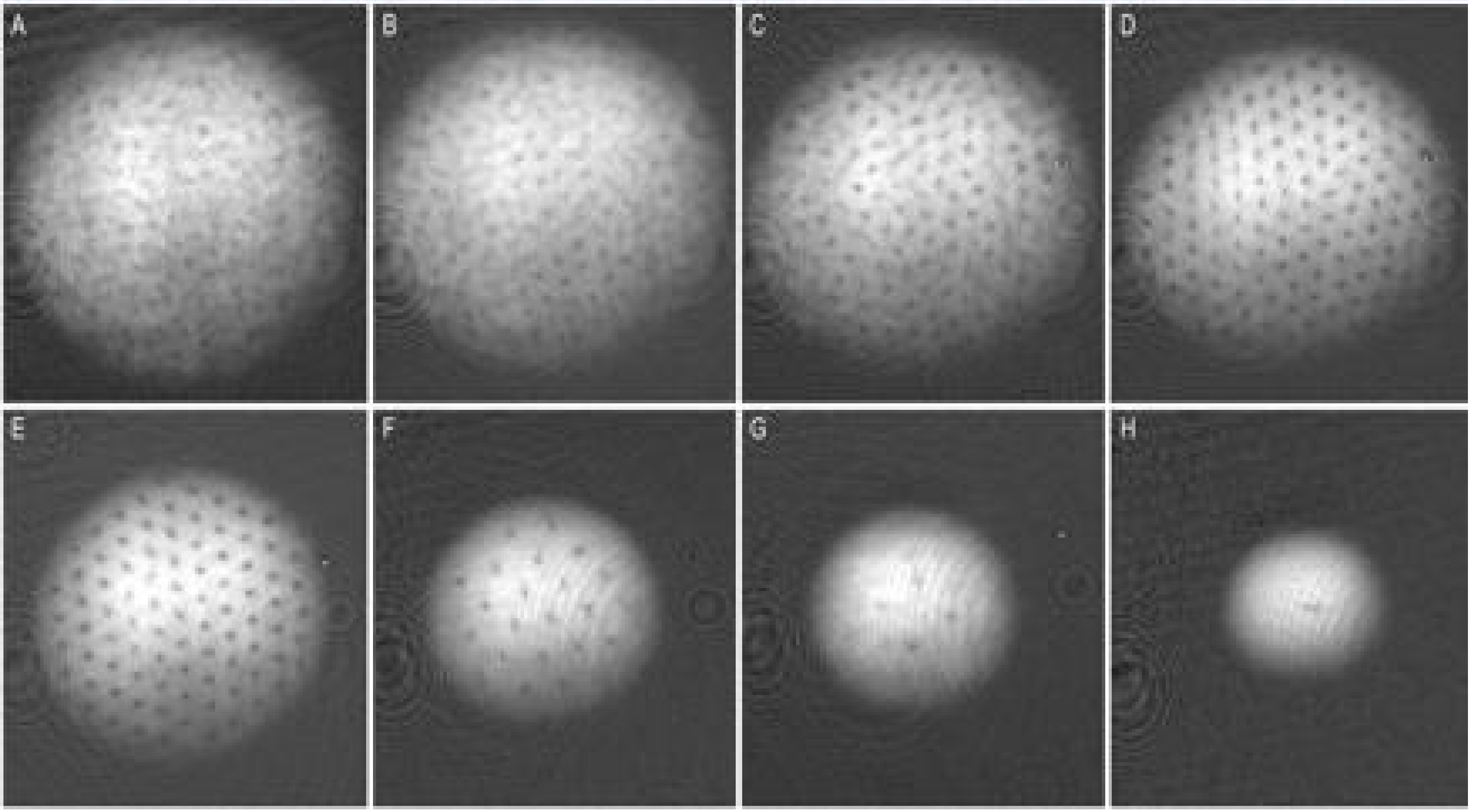
Trapped cloud
during cool-down
(Phase-contrast images)

Two condensates ... interfere!



Andrews, Townsend, Miesner, Durfee, Kurn, Ketterle, Science **275**, 589 (1997)

Vortex-lattice formation: times 25, 100, 200, 500 ms, 1, 5, 10, 40 s on 1 mmX1.2mm view. Ketterle Science 292, 476 (2001)



Bose-Einstein Condensate (BEC)

Uncertainty relation:

$$\Delta x \Delta p \approx h,$$

$$\Delta p = m \Delta \mathrm{v}$$

$$\Delta x \Delta \mathrm{v} \approx h / m$$

Mass (BEC) = 10^8 ~10^{10} X Mass (electron)

Makes the experimetal realization much easier

Mean-field Gross-Pitaevskii Equation

Full quantum many-body Hamiltonian:

$$H = \int \psi^\dagger(\mathbf{r}) \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{ex}(\mathbf{r}) \right] \psi(\mathbf{r})$$

$$+ \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \psi^\dagger(\mathbf{r}) \psi^\dagger(\mathbf{r}') V(\mathbf{r} - \mathbf{r}') \psi(\mathbf{r}) \psi(\mathbf{r}'),$$

$\psi^\dagger(\mathbf{r})$ and $\psi(\mathbf{r}')$ are boson creation and destruction operators ar $\mathbf{r}$ and $\mathbf{r}'$, and V the interaction.

Simple mean-field description:

$$\psi(\mathbf{r}, t) = \phi(\mathbf{r}, t) + \chi(\mathbf{r}, t), \qquad \phi(\mathbf{r}, t) = \langle \psi(\mathbf{r}, t) \rangle,$$

$\chi(\mathbf{r}, t)$ contains quantum fluctuations and $\phi(\mathbf{r}, t)$ is a classical field, often called an order parameter or wave function of the condensate.

Heisenberg equation for field ψ:

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = [\psi, H]$$

$$= [T + V_{ex}(\mathbf{r}) + \int d\mathbf{r}'\psi^\dagger(\mathbf{r}',t)V(\mathbf{r}-\mathbf{r}')\psi(\mathbf{r}',t)]\psi(\mathbf{r},t),$$

In mean-field theory replace field ψ by wave function ϕ. At zero energy in a dilute gas binary collision is important and controlled by the S-wave scattering length. It is proper to replace

$$V(\mathbf{r}-\mathbf{r}') = g\delta(\mathbf{r}-\mathbf{r}') \equiv \frac{4\pi\hbar^2 a}{m}\delta(\mathbf{r}-\mathbf{r}'),$$

Mean-field Gross-Pitaevskii equation

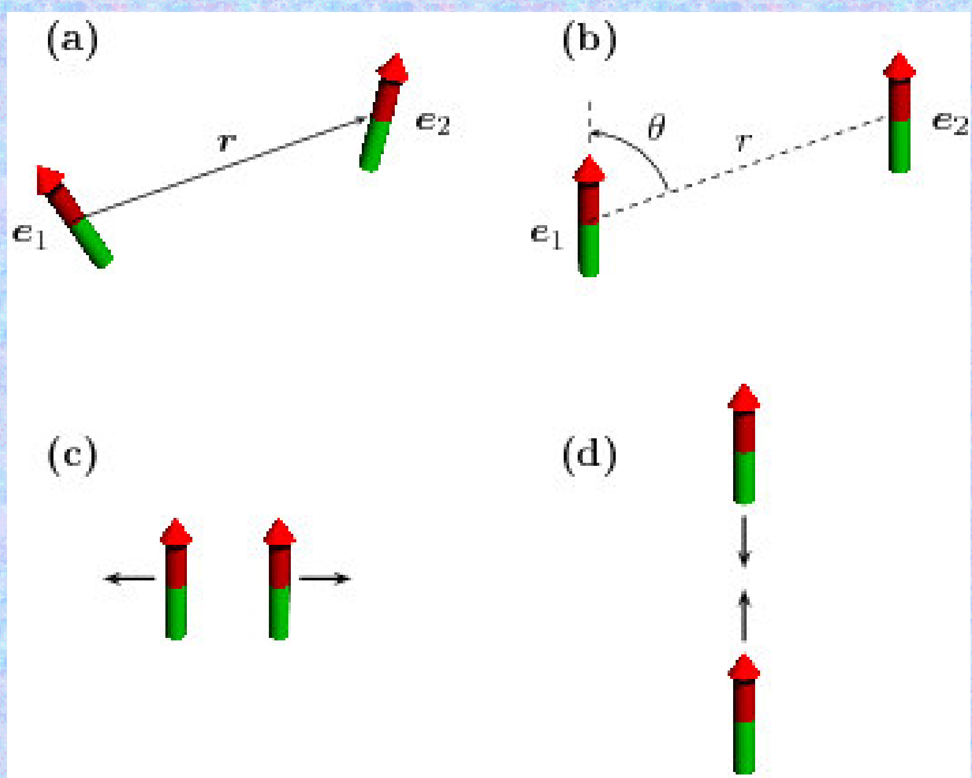
$$i\hbar\frac{\partial}{\partial t}\phi(\mathbf{r},t) = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{ex}(\mathbf{r}) + g|\phi(\mathbf{r},t)|^2\right)\phi(\mathbf{r},t),$$

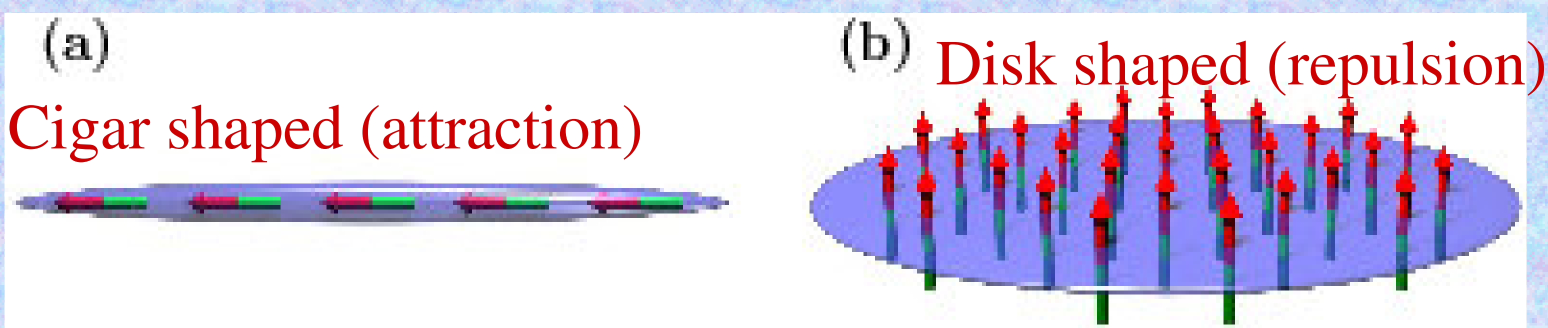
$$\int d\mathbf{r}|\phi|^2 = N.$$

- Application in nonlinear optics

- This potential gives scattering length a in the Born approximation

Dipolar interaction: Atoms and molecules





Dipolar BEC

- Matter in bulk is stable because of short-range repulsion of fundamental interactions in nuclear, atomic, and molecular physics.
- Nonlinear long-range anisotropic interaction (partially attractive and partially repulsive) and short-range repulsion make the BEC robust.
- This makes dipolar BEC solitons more robust than normal BEC solitons.

Static Dipole-Dipole Interactions

Magnetic dipole-dipole interaction: the magnetic moments of the atoms are aligned with a strong magnetic field [Goral, Rzazewski, and Pfau, 2000]

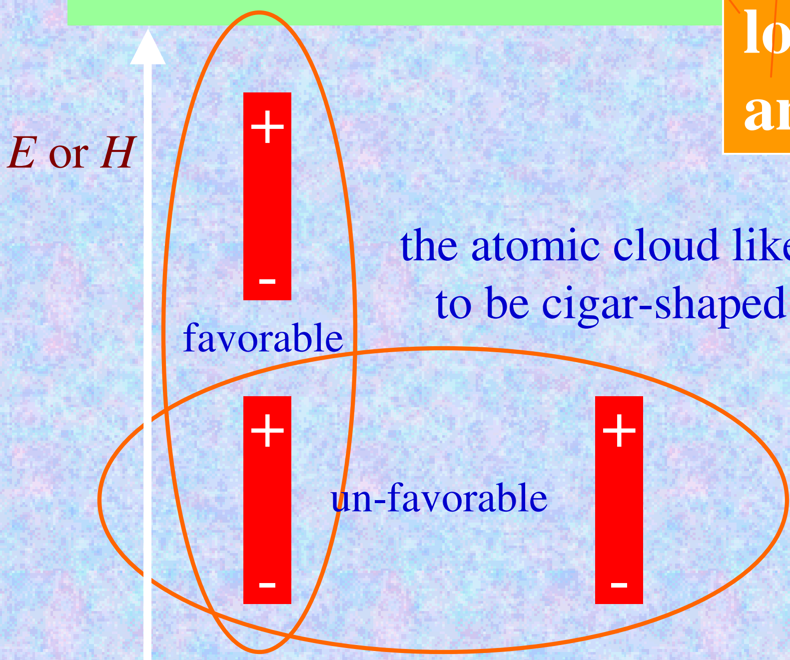
$$U_{dd}(r)=\frac{\beta\mu_0\mu^2}{4\pi}\left[\frac{1-3\cos^2\theta}{r^3}\right]$$

Electrostatic dipole-dipole interaction: (i) permanent electric moments (polar molecules); (ii) electric moments induced by a strong electric field E [Yi and You 2000; Santos, Shlyapnikov, Zoller and Lewenstein 2000]

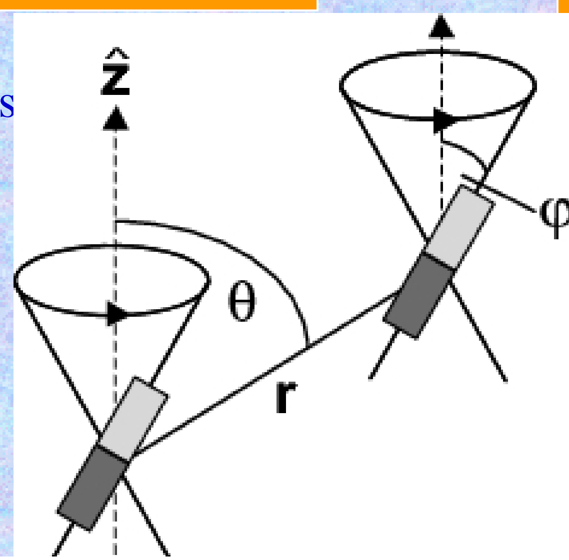
$$U_{dd}(r)=\frac{\beta\alpha^2}{4\pi\varepsilon_0}\left[\frac{1-3\cos^2\theta}{r^3}\right]$$

long-range + anisotropic

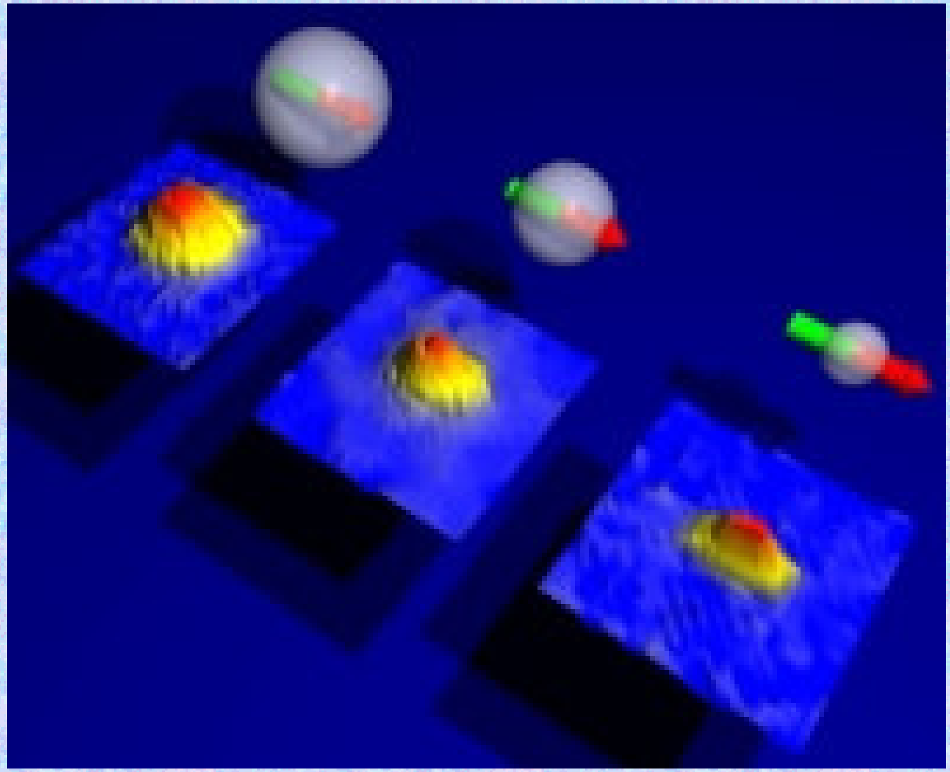
tunability



the atomic cloud likes to be cigar-shaped



Change of shape of BEC as the atomic interaction is reduced in a dipolar BEC

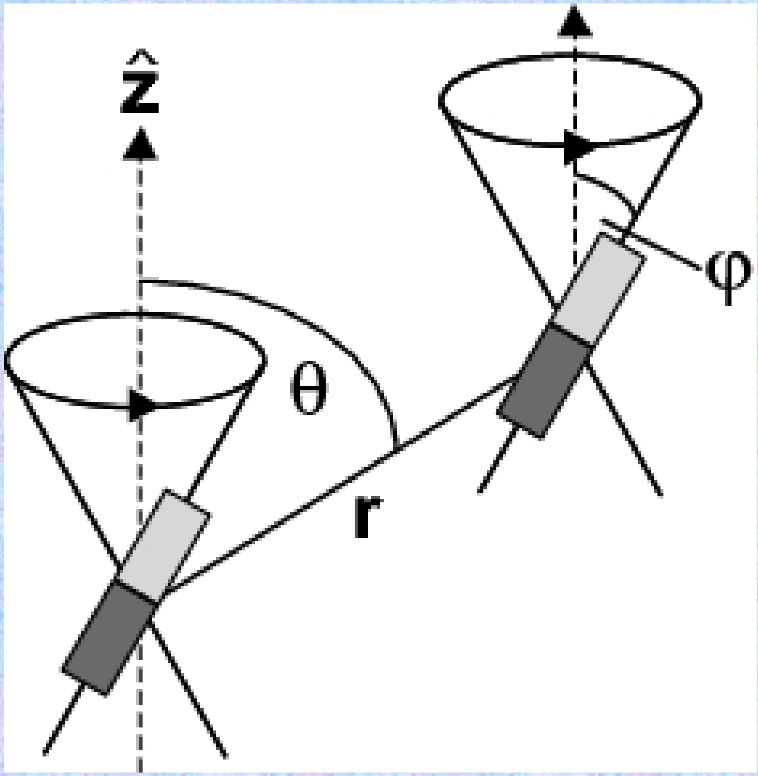


Tuning of dipolar interaction by rotating orienting field

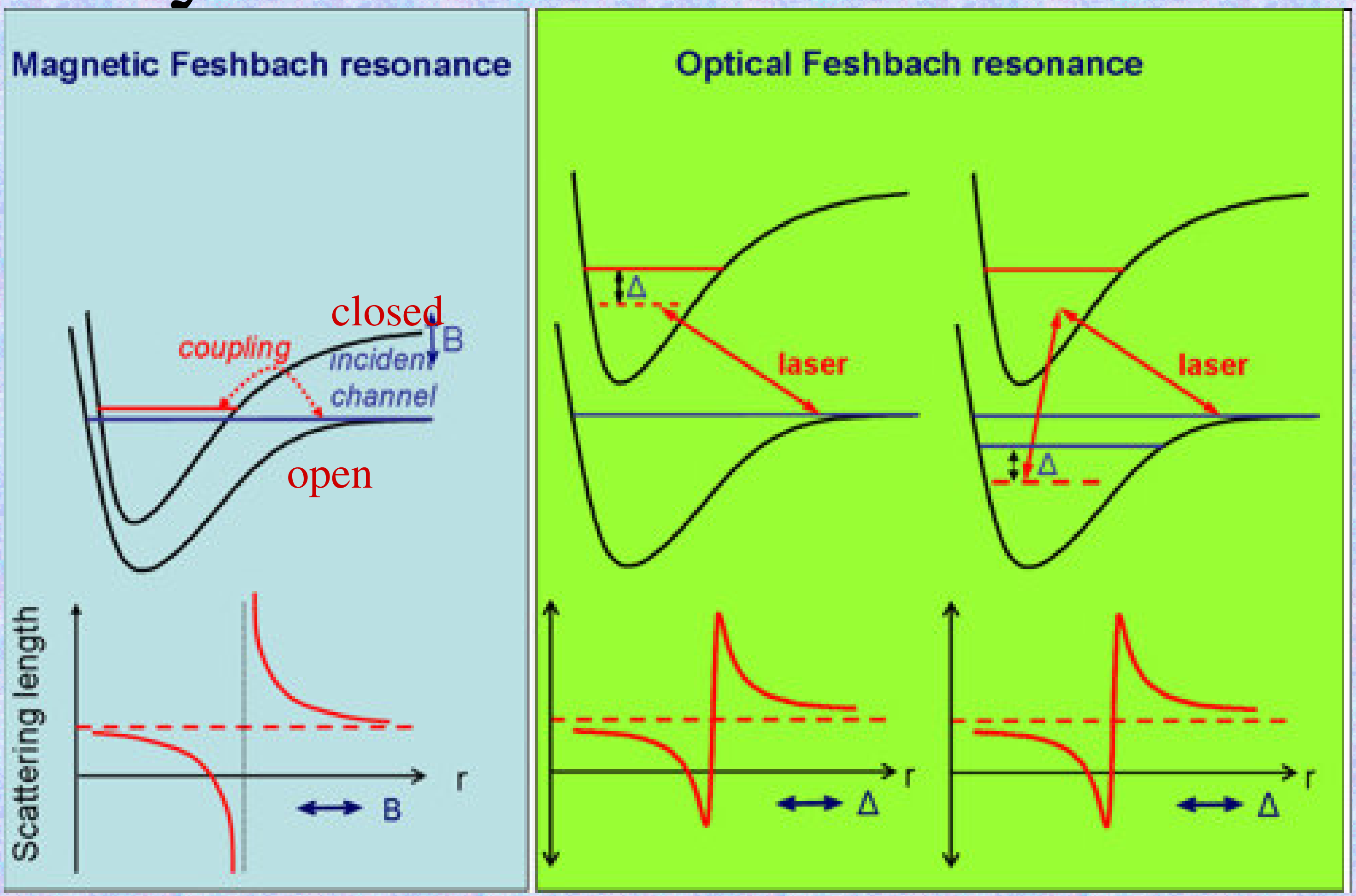
$$U_{dd}(R) = 3a_{dd}\frac{(1-3\cos^2\theta)}{R^3}\frac{(3\cos^2\varphi-1)}{2};\ a_{dd} = \frac{\mu_0\mu^2 m}{12\pi\hbar^2}$$

Strongly anisotropic Magnetic/Electric Dipole-Dipole Interactions

$$\mathrm{B(t)} = \mathrm{B}[\cos(\varphi)\mathbf{z} + \sin(\varphi)(\cos(\Omega \mathrm{t})\mathbf{x} + \sin(\Omega \mathrm{t})\mathbf{y})]$$



Tuning of short-range interaction by a Feshbach resonance



Generalized Gross-Pitaevskii Equation
(mean-field equation for the BEC)

$$i\hbar \frac{\partial \psi(r,t)}{\partial t} = -\left[\frac{\hbar^2}{2m} \nabla^2 + V_{trap} + \frac{4\pi\hbar^2 aN}{m} |\psi(r,t)|^2 \right] \psi(r,t)$$

$$+ \frac{3\hbar^2 a_{dd} N}{m} \int U_{dd}(r - r') |\psi(r',t)|^2 dr' \psi(r,t)$$

$$= \mu \psi(r,t) \qquad \text{stationary state}$$

$$V_{trap} = \frac{1}{2} m\omega^2 (x^2 + y^2) \qquad \text{bright soliton}$$

$$V_{trap} = \frac{1}{2} m\omega^2 (x^2 + y^2) + \frac{\hbar^2}{2m(x^2 + y^2)} \qquad \text{vor tex soliton}$$

$$V_{trap} = -a[\cos(2x) + \cos(2y) + \cos(2z)] \qquad \text{gap soliton in OL}$$

Variational Equations

Lagrangian density

$$\mathcal{L} = \frac{i}{2}\left(\phi\phi_t^\star - \phi^\star\phi_t\right) + \frac{1}{2}|\nabla\phi|^2 + \frac{\rho^2}{2}|\phi|^2 + 2\pi a N|\phi|^4 + \frac{N}{2}|\phi|^2\int U_{dd}(\mathbf{r}-\mathbf{r}')|\phi(\mathbf{r}')|^2 d\mathbf{r}'. \quad (1)$$

Gaussian ansatz

$$\phi(\mathbf{r},t) = \exp(-\rho^2/2w_\rho^2 - z^2/2w_z^2 + i\alpha\rho^2 + i\beta z^2)/(w_\rho\sqrt{w_z}\pi^{3/4})$$

Effective Lagrangian L (per particle)

$$L \equiv \int \mathcal{L}\, d\mathbf{r} = \left(w_\rho^2\dot\alpha + \frac{1}{2}w_z^2\dot\beta + 2w_\rho^2\alpha^2 + w_z^2\beta^2\right) + \frac{1}{2}\left(\frac{1}{w_\rho^2} + \frac{1}{2w_z^2} + w_\rho^2\right) + \mathcal{E}_{\text{dip}}, \quad (2)$$

with $\mathcal{E}_{\text{dip}} = N[a - a_{dd}f(\kappa)]/(\sqrt{2\pi}w_\rho^2 w_z)$, $f(\kappa) = [1+2\kappa^2 - 3\kappa^2 d(\kappa)]/(1-\kappa^2)$, $d(\kappa) = (\text{atanh}\sqrt{1-\kappa^2})/\sqrt{1-\kappa^2}$, $\kappa = w_\rho/w_z$. Equations for the widths

$$\ddot w_\rho + w_\rho = \frac{1}{w_\rho^3} + \frac{1}{\sqrt{2\pi}}\frac{N}{w_\rho^3 w_z}\left[2a - a_{dd}g(\kappa)\right], \quad (3)$$

$$\ddot w_z = \frac{1}{w_z^3} + \frac{1}{\sqrt{2\pi}}\frac{2N}{w_\rho^2 w_z^2}\left[a - a_{dd}h(\kappa)\right], \quad (4)$$

with $g(\kappa) = [2-7\kappa^2-4\kappa^4+9\kappa^4 d(\kappa)]/(1-\kappa^2)^2$, $h(\kappa) = [1+10\kappa^2-2\kappa^4-9\kappa^2 d(\kappa)]/(1-\kappa^2)^2$.

BECs of ^{52}Cr (Griesmaier/Pfau 2005) and ^{164}Dy (Lu/Lev, 2011)

Dipole moment μ of $^{52}Cr = 6\mu_{B,}$ $a_{dd} = 15\ a_0$

Dipole moment μ of $^{164}Dy = 10\mu_B$ $a_{dd} = 131\ a_0$

Dipole moment μ of $^{87}Rb = 1\mu_B$ $a_{dd} = 0.69\ a_0$

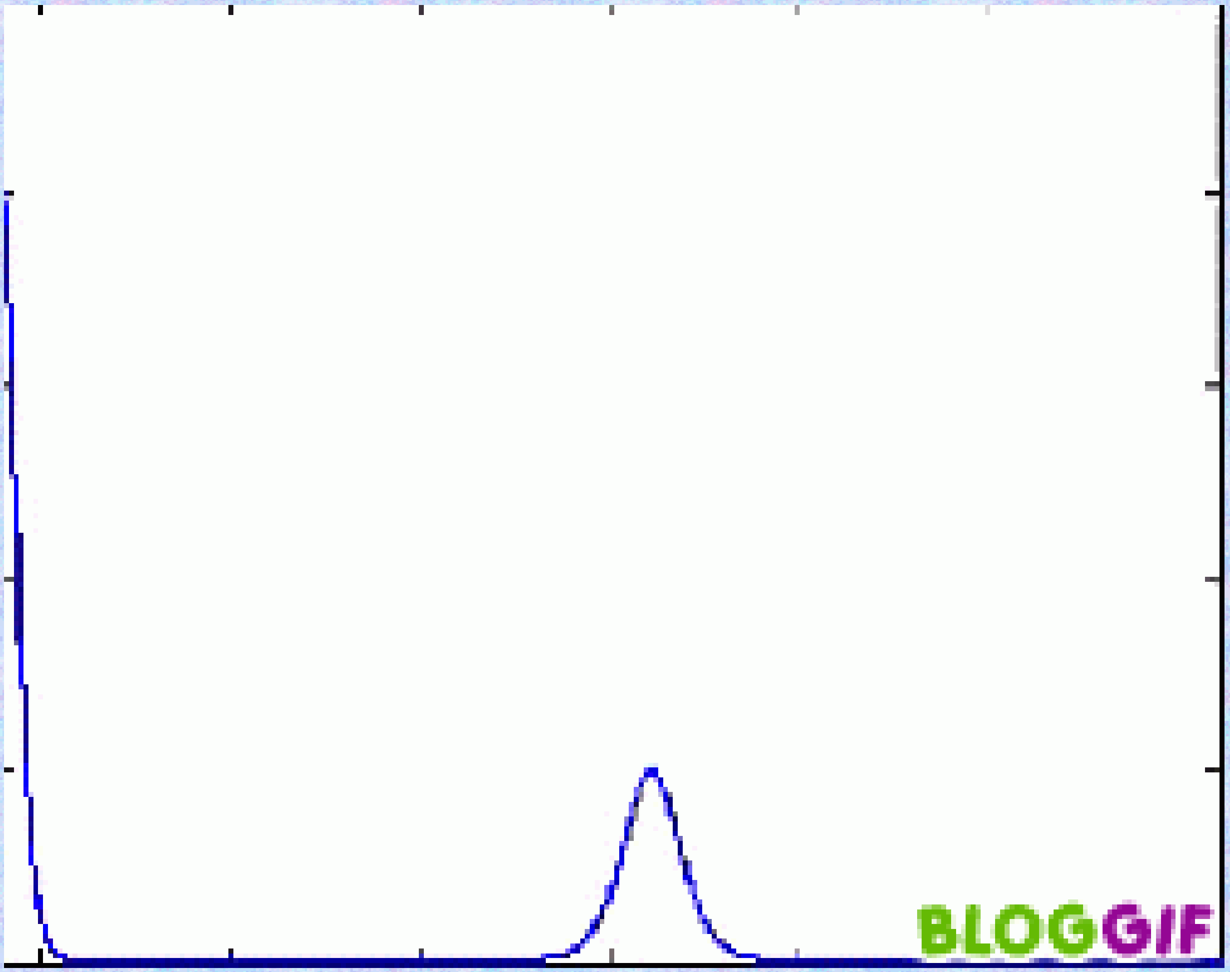
μ_B = Bohr Magneton a_0 = Bohr radius

$$a_{dd} = \frac{\mu_0 \mu^2 m}{12\pi\hbar^2}$$

Soliton in one dimension (1D)

- An 1D **soliton** is a self-reinforcing solitary wave that maintains its shape while it travels at constant speed.
- It is generated from a balance between repulsive kinetic energy and attractive nonlinear interaction.
- It is robust and two 1D solitons can penetrate through each other in collision.
- They appear in many physical systems as well as many mathematical models.

Soliton-soliton collision



Solitons in three dimensions (3D)

- 3D solitons can be formed with supporting harmonic traps (potential) in transverse directions and no trap in axial direction.
- 3D solitons are more fragile than 1D solitons.
- Dipolar BEC soliton in 3D is more stable than normal soliton.

Experiment and 1D model of BEC soliton

- Use harmonic traps in transverse directions and assume that BEC is in the ground state of transverse traps $\Psi(x,y,z)=\psi(x)\phi_{ground}(y,z)$
- Substitute in the Gross-Pitaevskii (GP) equation and integrate out the transverse coordinates.
- Hence obtain a 1D GP equation, commonly used in the study of solitons.

Gross-Pitaevskii Equation: BEC Soliton in 1D

One – dimension (1D)

$$-\frac{1}{2}\frac{\partial^2\Psi}{\partial z^2} + g_1 |\Psi|^2 \Psi = i\frac{\partial\Psi}{\partial t}$$

$$\int |\Psi|^2 \, dz = 1, \qquad g_1 = 2aN/l^2, \qquad l = \sqrt{\hbar/(m\omega)}$$

» However, there is no collapse in 1D, and this model may yield qualitatively incorrect result.

» For nondipolar BEC, Salasnich suggested modification of 1D model to include collapse

» The present calculation done in 3D.

Dipolar soliton in 3D

- Can be more stable due to short-range atomic repulsion and long-range dipolar attraction as in matter in bulk.

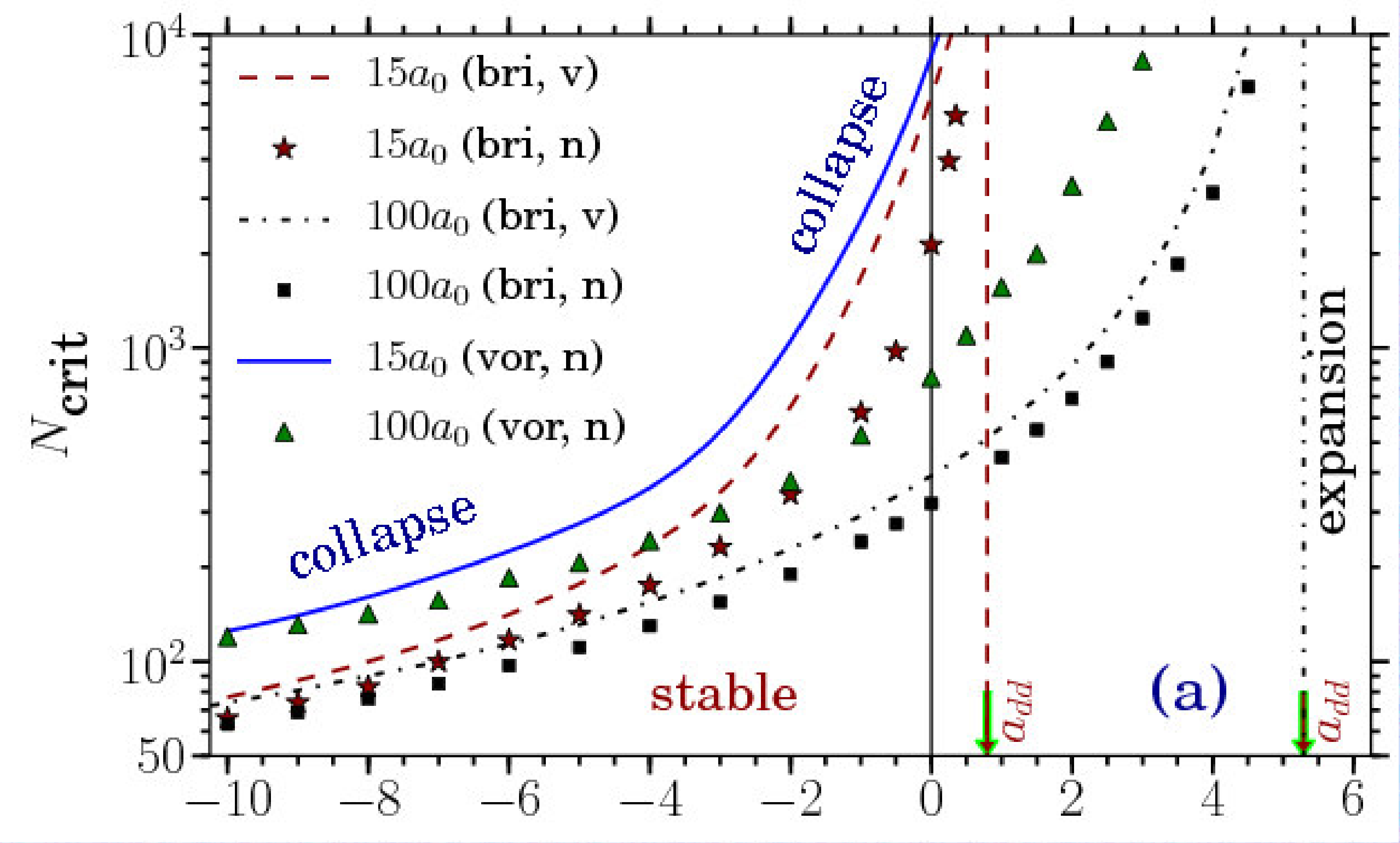
- When a, a_{dd} = positive, and a transverse trap is placed perpendicular to polarization direction.

- Bright and vortex solitons can be formed for repulsive atomic interactions.

Results of calculations

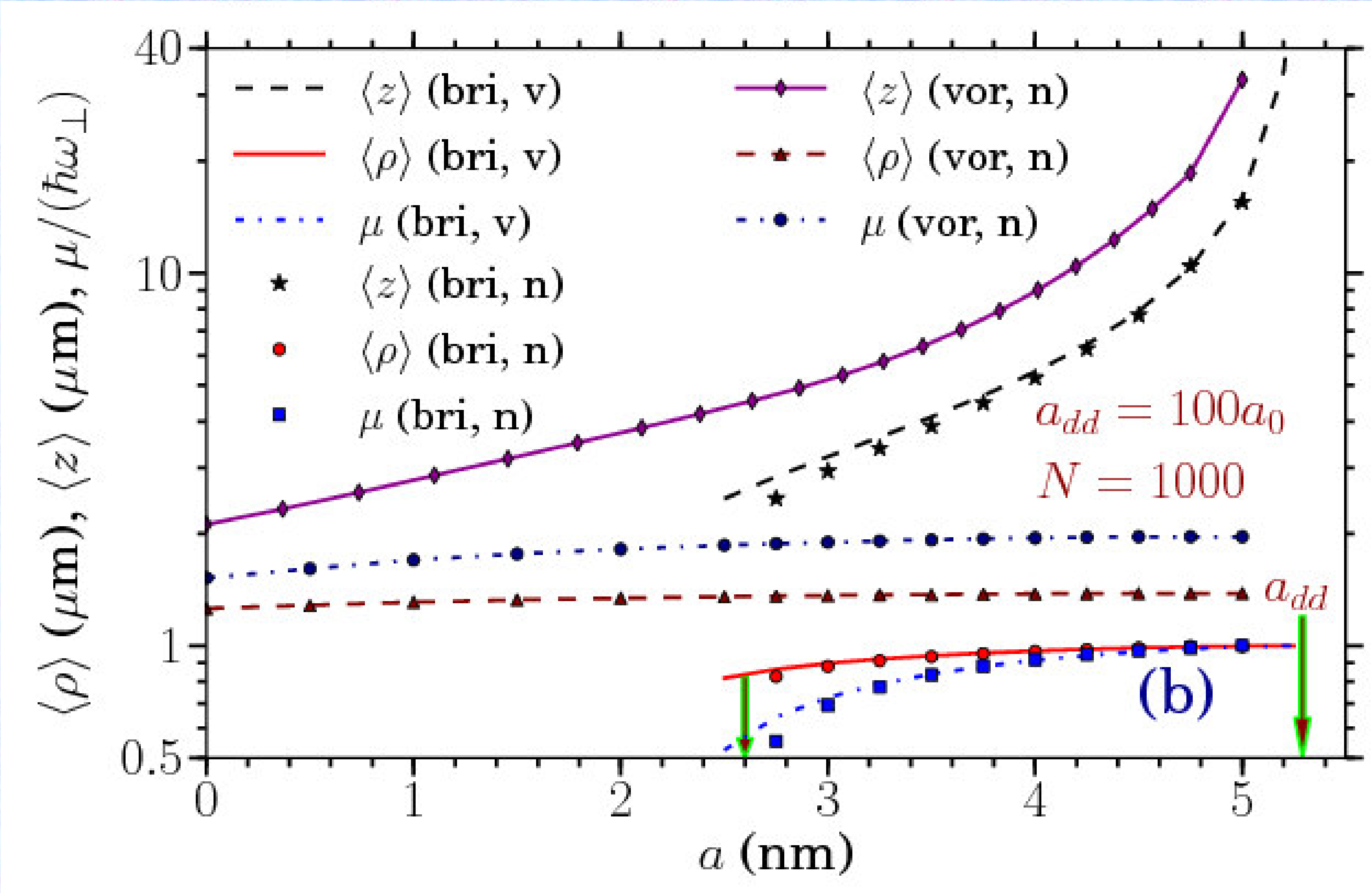
- The three-dimensional GP equation is solved numerically by split-step Crank-Nicolson method without further approximation.
- Fortran programs for GP Eq. published in Comput. Phys. Commun. 180 (2009) 1888-1912
- Results are compared with Gaussian variational approximation.

Stability of a dipolar bright and vortex BEC soliton

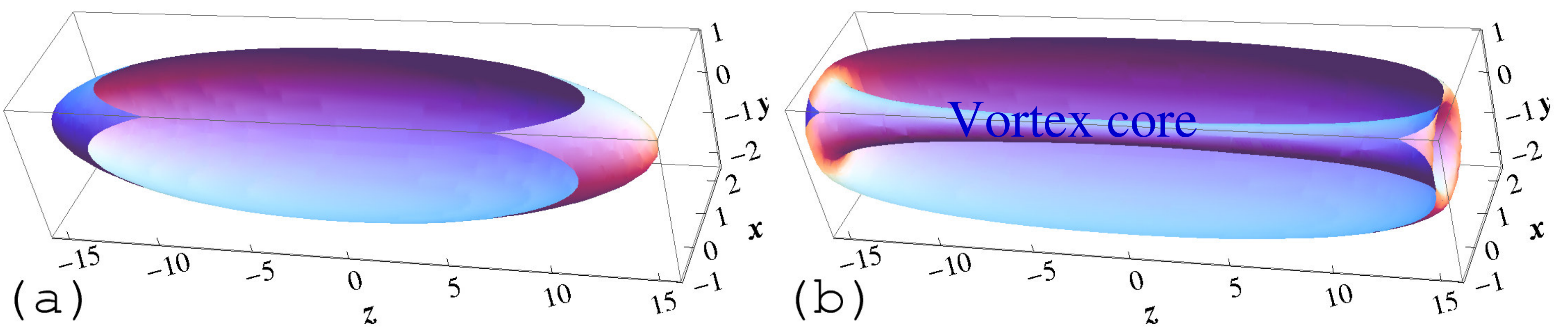


Scattering length a (nm)

RMS sizes and Chemical potential



Soliton profile

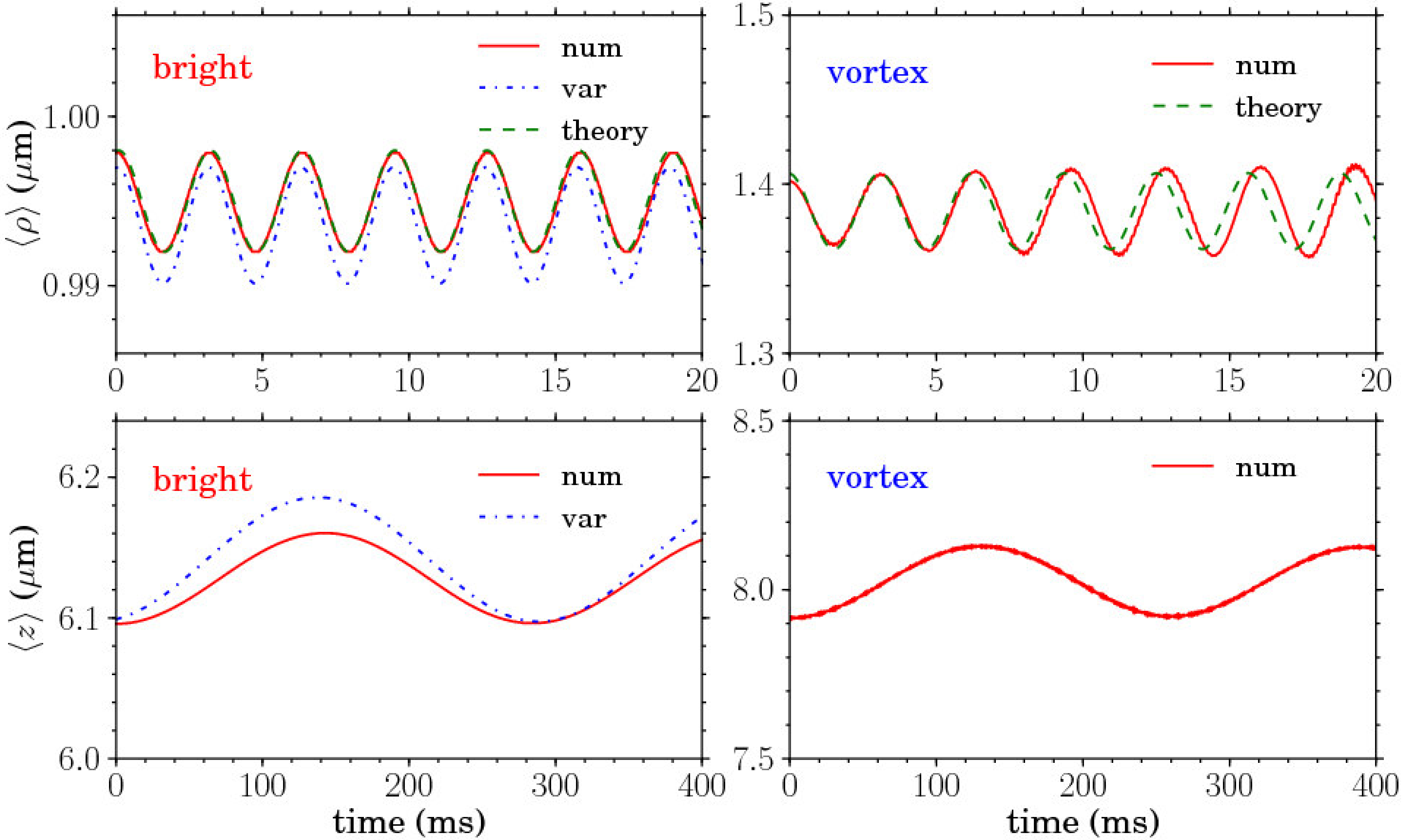


(a) Bright soliton of 1000 atoms with $a_{dd} = 15a_0$ and $a = 0.5$ nm

(b) Vortex soliton of 1000 atoms with $a_{dd} = 100a_0$ and $a = 4$ nm

(Compare $a_{dd} = 15a_0$ for Cr and $a_{dd} = 130a_0$ for Dy.)

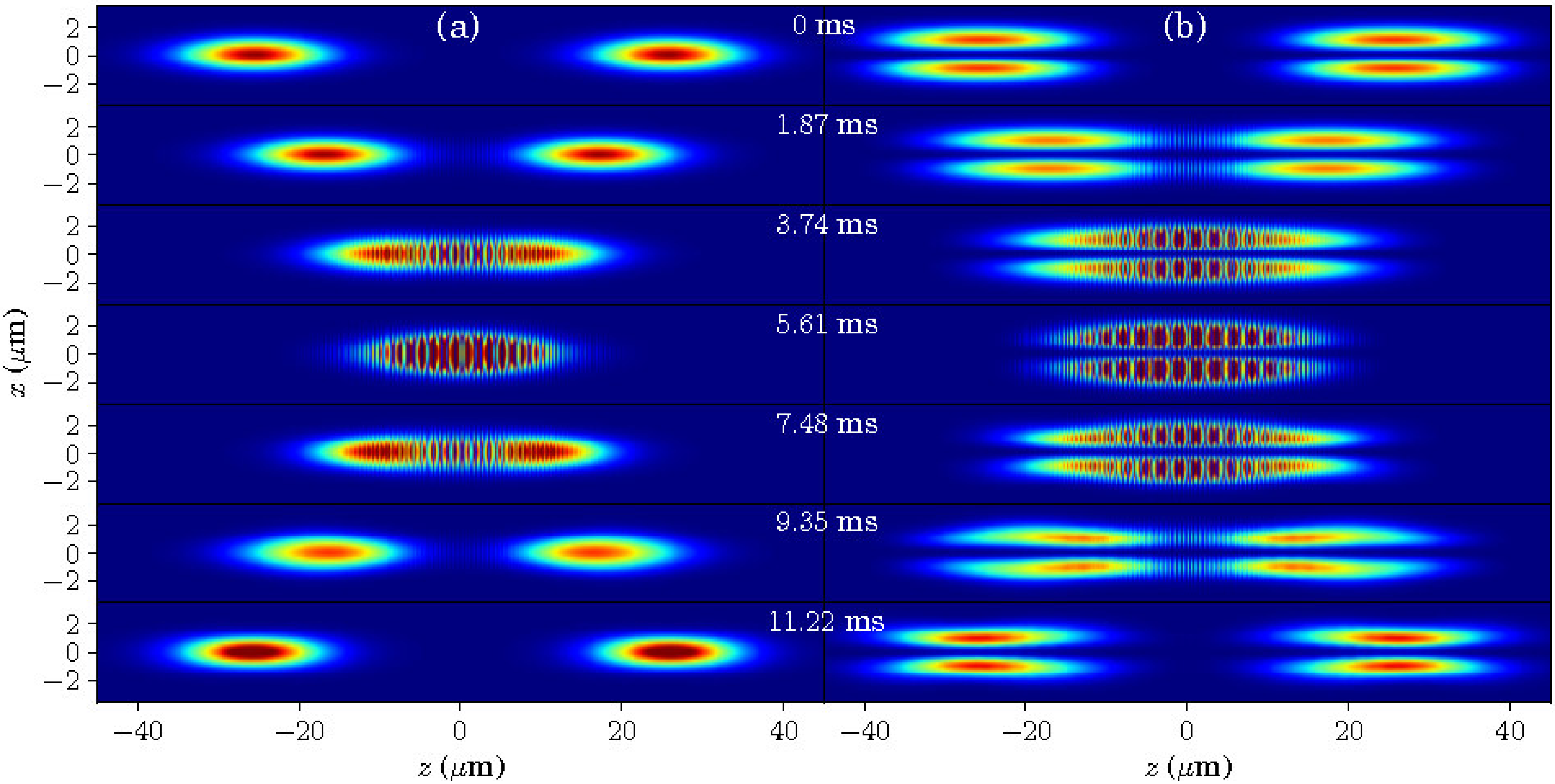
Soliton stability



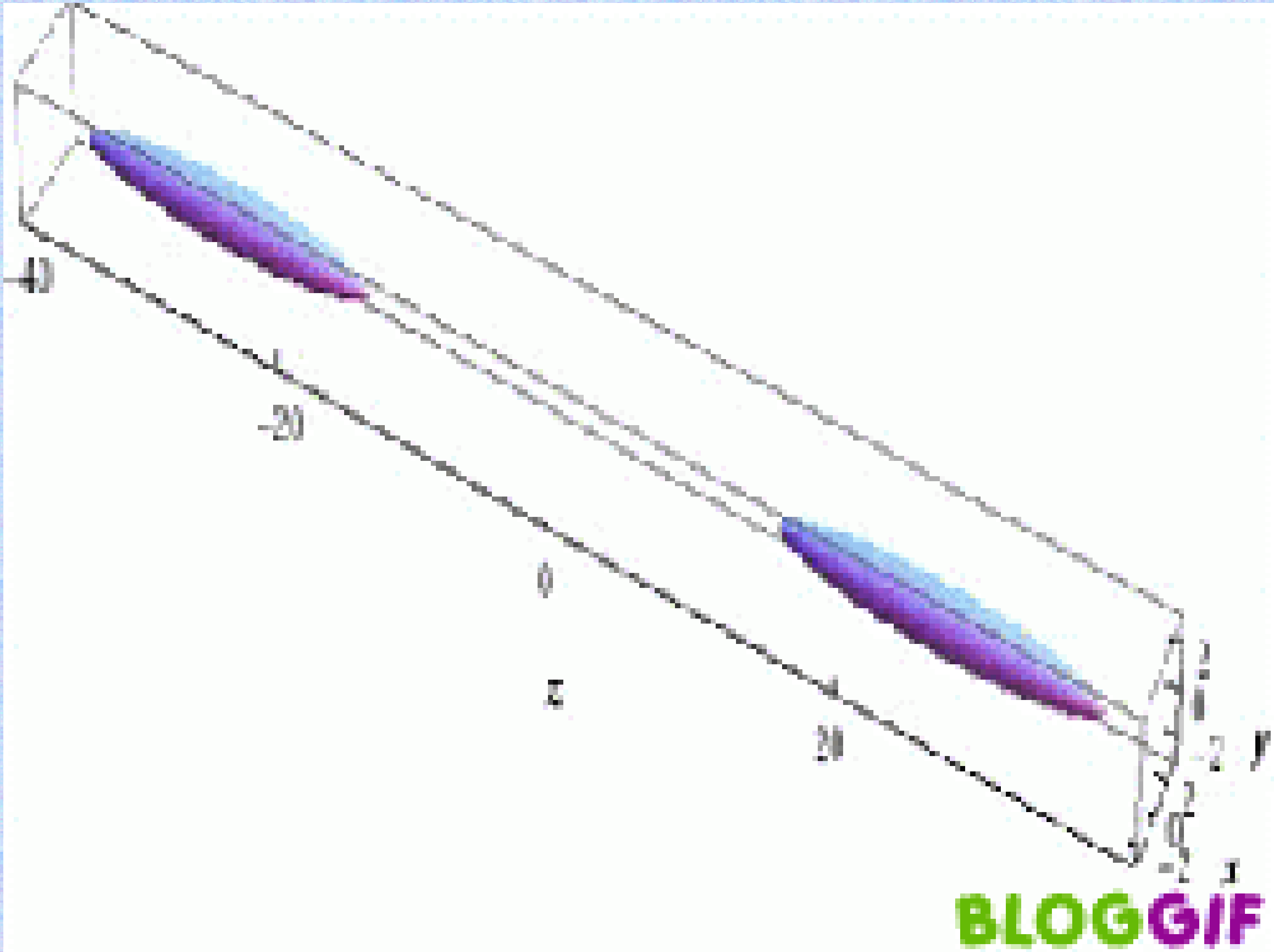
Soliton-soliton Interaction

- Frontal collision at medium to high velocities.
- Numerical simulation in 3D shows that the two dipolar solitons pass through each other.
- Molecule formation at low velocities. Soliton$_2$
- If two solitons in 3D are kept side-by-side at rest, due to long range dipolar interaction they attract and slowly move towards each other. They penetrate, coalese and never come out and form a soliton molecule.

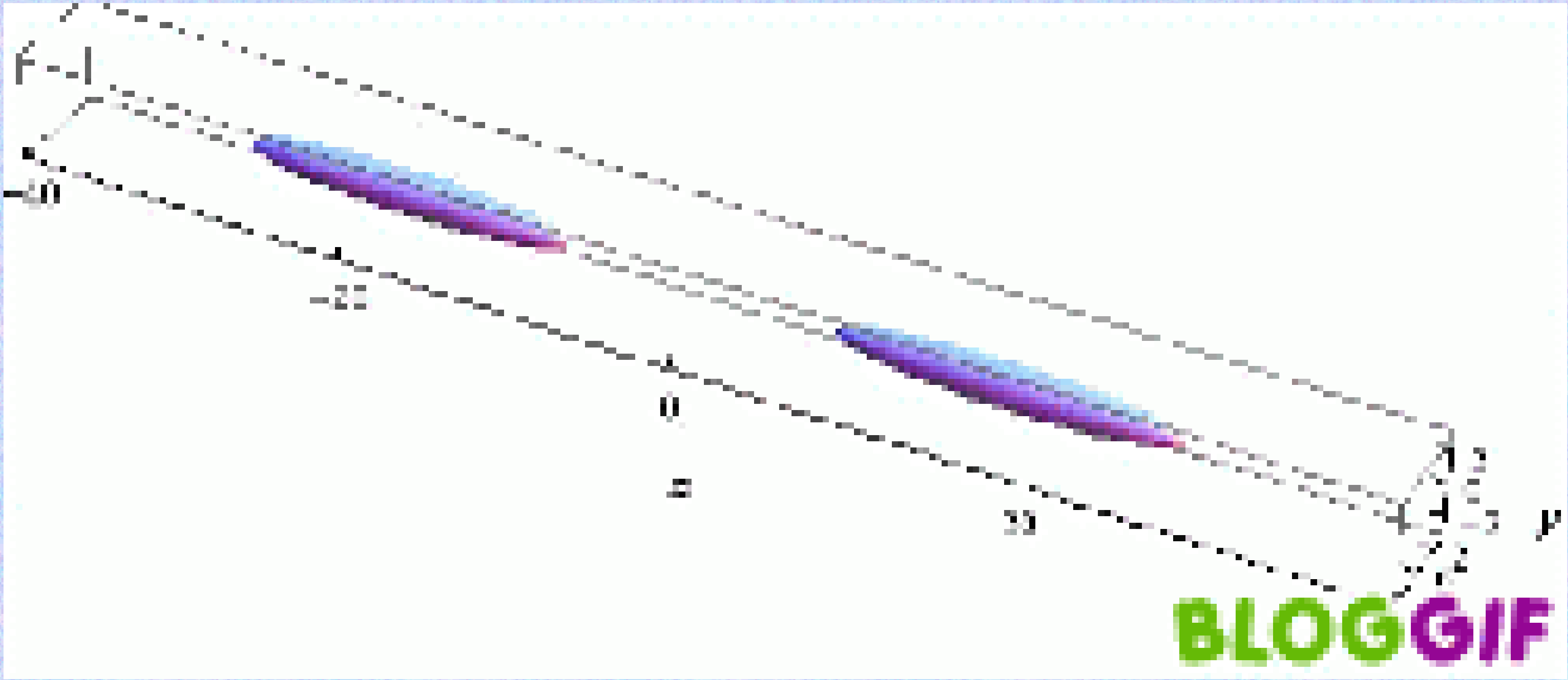
Elastic collision of bright and vortex solitons



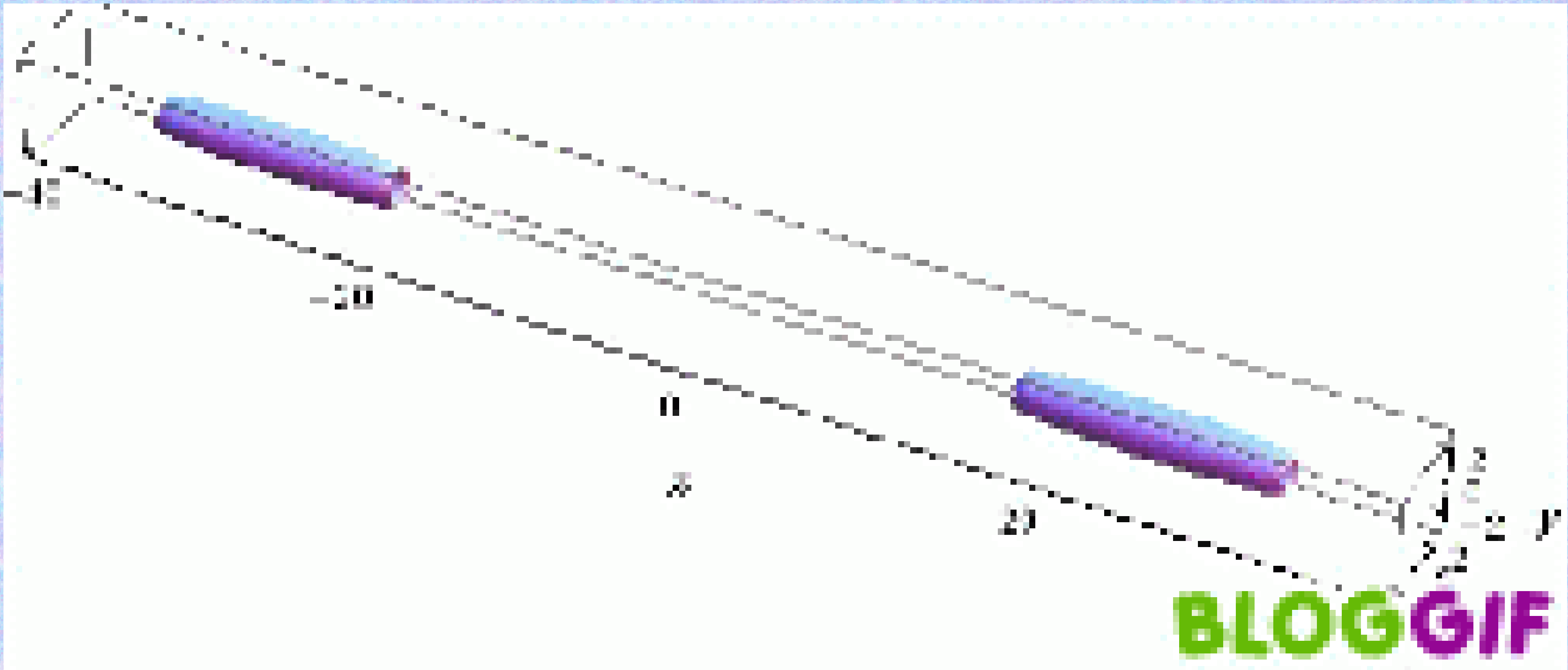
Elastic collision of bright solitons



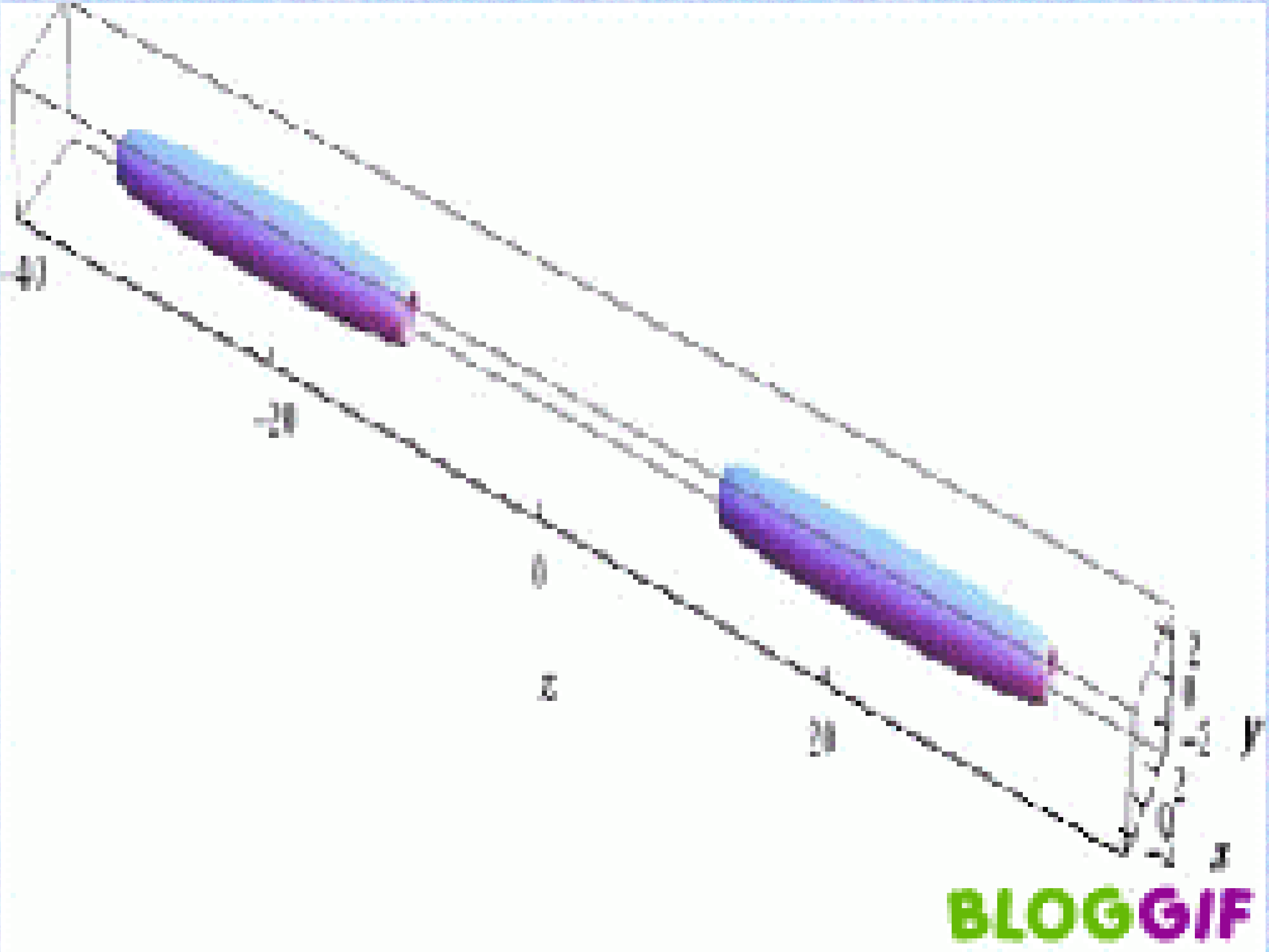
Molecule formation from bright solitons



Elastic collision of vortex solitons



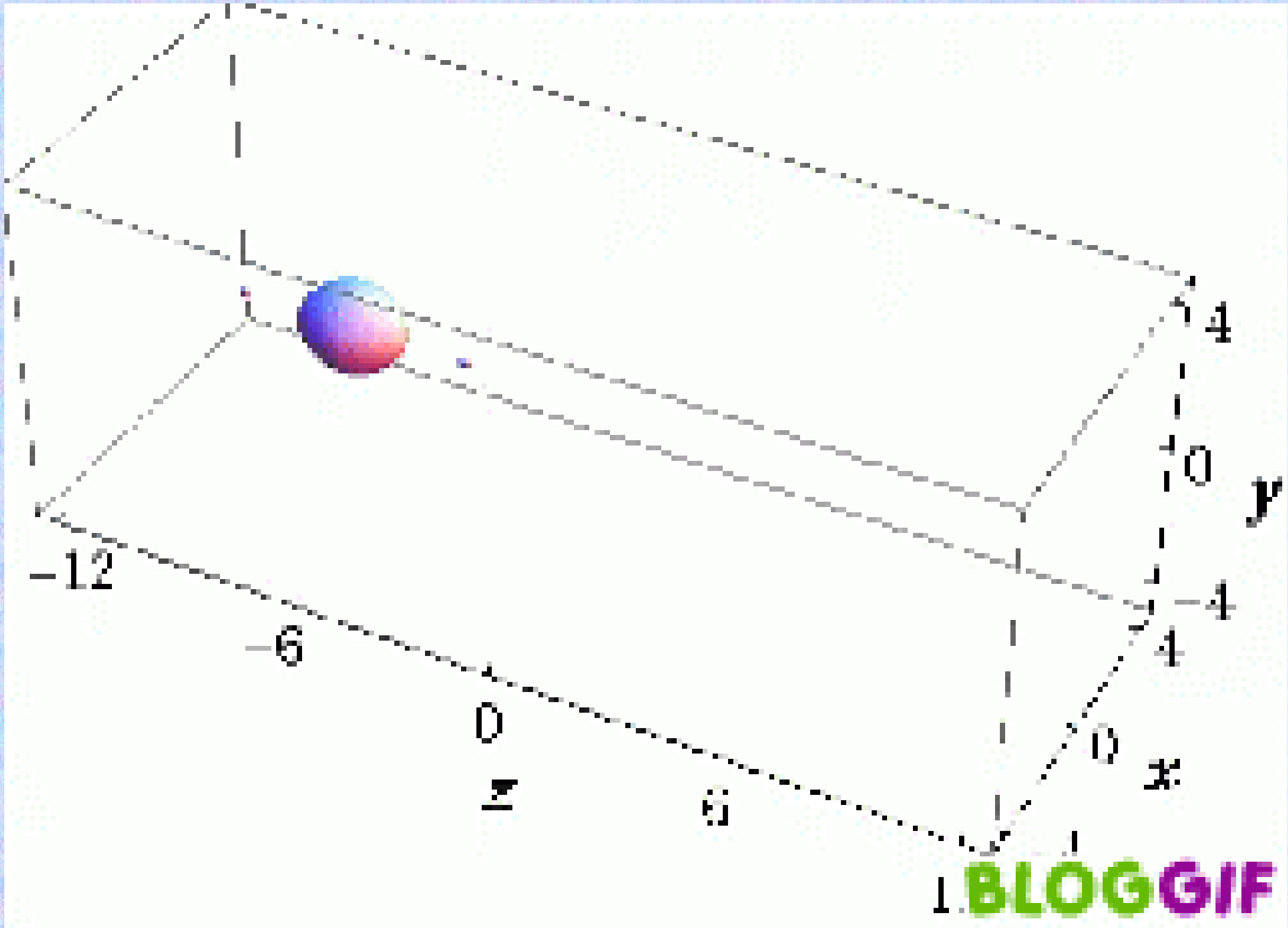
Molecule formation from vortex solitons



Gap soliton

- They lie in the band gap of the periodic lattice.
- Appear for repulsive atomic interaction.
- They are stable and can be dragged with the lattice – OL potential.

Motion of Gap soliton with lattice



Summary & Conclusion

- A new type of robust bright and vortex solitons encountered in repulsive dipolar BEC
- Frontal collision and molecule formation.
- Dipolar interaction is responsible for the robust nature.
- Experiments needed to verify theory.

Gross-Pitaevskii Equation

$$\text{Two} - \text{dimension}$$

$$-\frac{1}{2}\nabla_{x,y}{}^{2}\Psi + U\Psi + g_{2}\,|\,\Psi\,|^{2}\,\Psi = i\frac{\partial\Psi}{\partial t};$$

$$\int |\,\Psi\,|^{2}\, d^{2}r = N,$$

$$U = V(x) + V(y)$$

$$V(x) = \sum_{i=1}^{2}\frac{4\pi^{2}s_{i}}{\lambda_{i}{}^{2}}\sin^{2}\left(\frac{2\pi x}{\lambda_{i}}\right)$$

$$V(x) = \sum_{i=1}^{2}\frac{4\pi^{2}s_{i}}{\lambda_{i}{}^{2}}\cos^{2}\left(\frac{2\pi x}{\lambda_{i}}\right)$$

$$\lambda_{1} = 5, \quad \lambda_{2} = 0.86\lambda_{1}, \quad s_{1} = s_{2} = 2$$

Gross-Pitaevskii Equation: Dipolar atom

$$\left[-\frac{1}{2}\nabla^2 + U + g_3 |\Psi(\mathrm{r})|^2 + \int d\mathrm{r}' \mathrm{U}_{\mathrm{dd}}(\mathrm{r} - \mathrm{r}')\Psi(\mathrm{r}')\right]\Psi(\mathrm{r}) = \mu\Psi(\mathrm{r})$$

$$\mathrm{U}_{\mathrm{dd}}(\mathrm{r}) = 3Na_{dd}(1-3\cos^2\vartheta)/r^3, \text{ For Cr } a_{dd} = 15a_0$$

$$\int |\Psi|^2 d^3r = N, \quad g_3 = 4\pi aN,$$

$$U = V(x) + V(y) + V(z)$$

$$V(x) = \sum_{i=1}^{2} \frac{4\pi^2 s_i}{\lambda_i^2} \sin^2\left(\frac{2\pi x}{\lambda_i}\right)$$

$s_1 = s_2 = 2$, $\lambda_1 = 5$, $\lambda_2 = 0.862\lambda_1$

Gross-Pitaevskii Equation

$$\mathrm{Two-dimension}$$

$$-\frac{1}{2}\nabla_{x,y}{}^{2}\Psi + U\Psi + g_2 |\Psi|^2 \Psi = \mu\Psi; \qquad \hbar = m = \omega = 1$$

$$\int |\Psi|^2 d^2 r = N,$$

$$U = \sum_{i=1}^{s} A \exp[-c(x-\beta_i)^2 - c(y-\gamma_i)^2]$$

$$A = 8,\ \mathrm{c} = 20$$