Universal and non universal Efimov physics in ultracold gases

11-10-11, EMFCSC, Erice

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Where innovation starts

Outline

Introduction - The Efimov effect

- Universality in few-body physics
- Tune the two-body interaction

Non-universal effects?

- Role of the three-body parameter
- Direct association of Efimov trimers

Observability of Efimov physics in a lattice

- Three-body bound states

Efimov Physics vs interaction strength

Efimov effect:

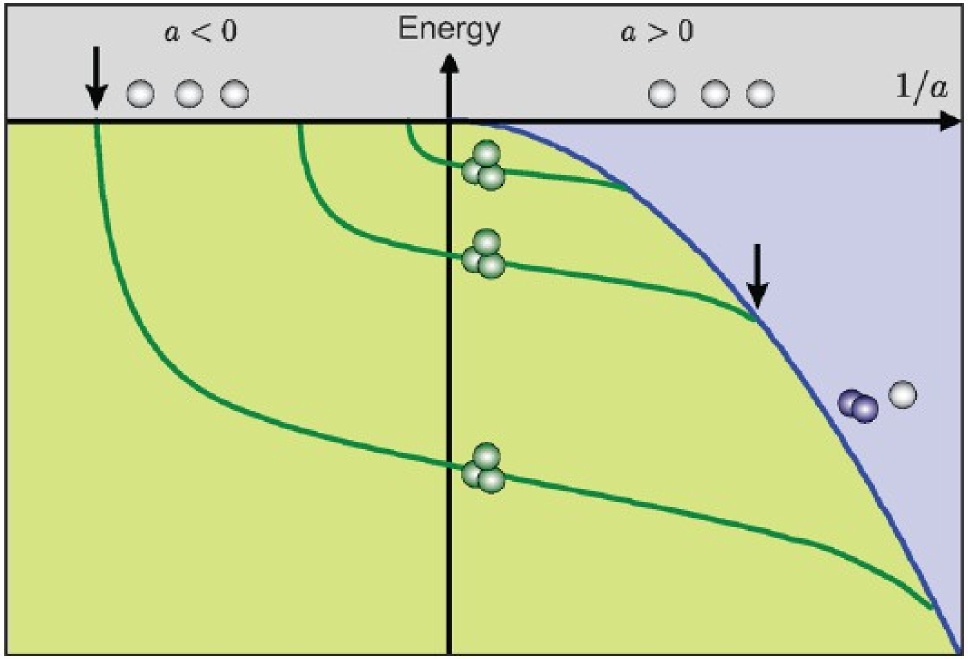
- **accumulation of trimer bound states near zero energy and diverging scattering length**

difference in scattering length:

$$e^{\pi/s_0} \approx 22.7$$

spacing bound states:

$$E_{n+1}/E_n = e^{-2\pi/s_0}$$



$$E_b = \frac{-\hbar^2}{m a^2}$$

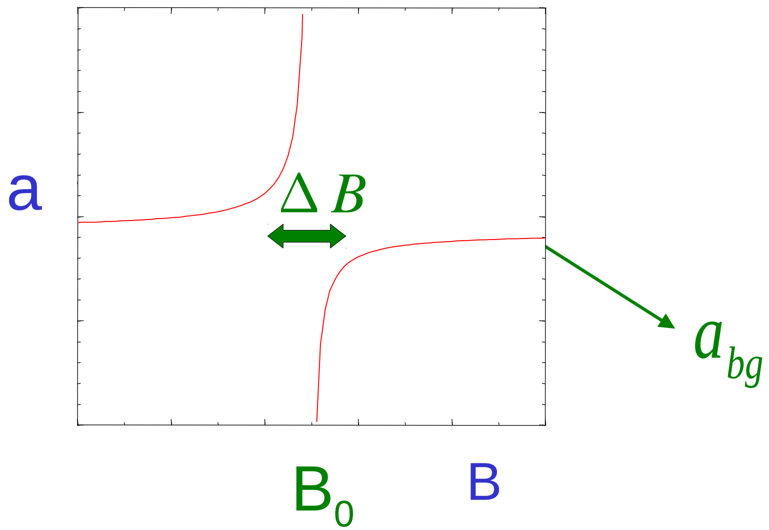
Overview: [F. Ferlaino and R. Grimm, Physics 3, 9 (2010); C. Greene, Physics Today, march 2010]

Control over interactions

Feshbach resonances:

- Control over interaction
- Tunable bound state: create cold dimers and trimers

Dispersive shape scattering length



$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

Prediction of Fesbach resonances

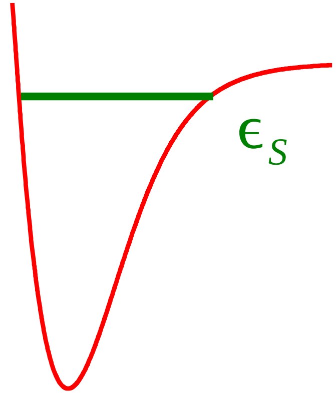
- **Coupled-channels scattering Model**
  - **Numerical calculation with potentials and spin structure**
  - **States coupled via Hyperfine and Zeeman interactions**

$$H_{hf}=\frac{a_{hf}}{\hbar^2}\vec{s}\cdot\vec{i} \qquad H_Z=\gamma_e\vec{s}\cdot\vec{B}+\gamma_i\vec{i}\cdot\vec{B}$$

Total electron/nuclear spin: $\vec{S}=\vec{s}_1+\vec{s}_2;\vec{I}=\vec{i}_1+\vec{i}_2$

  - **Find resonances in scattering length**

- **Simple model: based on highest bound state**



Replace whole potential
by only a number

ϵ_S

Asymptotic bound-state model

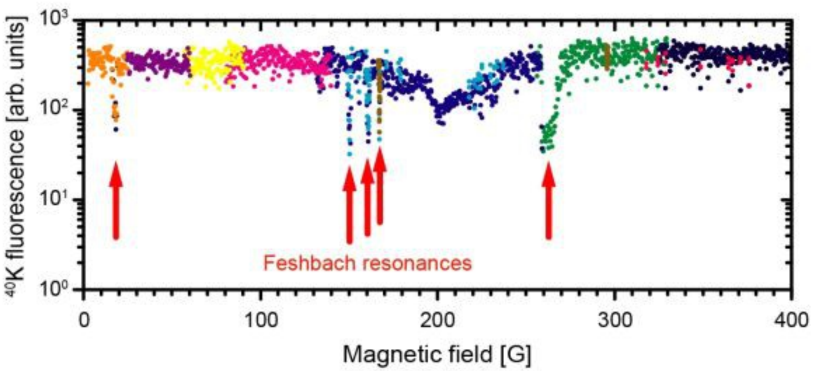
- **~~Complicated coupled radial equations~~**

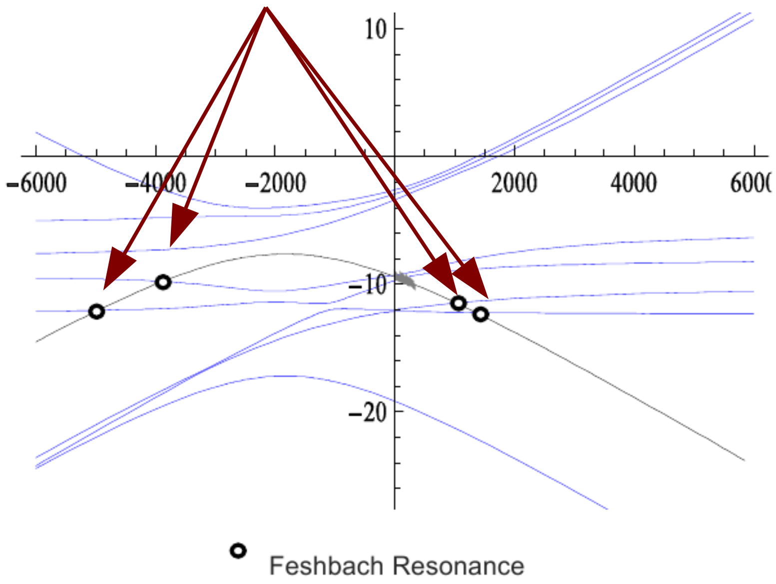
  → simple matrix diagonalization

  gives rise to coupled field-dependent dimer states

  Developed in collaboration with Tobias Tiecke and Jook Walraven, UvA
  First applied to Innsbruck data (Grimm)

$a \rightarrow \pm\infty$:Transition from bound to unbound
Feshbach resonances!





See e.g.: [T. G. Tiecke et al, Phys. Rev. Lett. 104, 053202 (2010)]

Change the ABM parameters

Universality

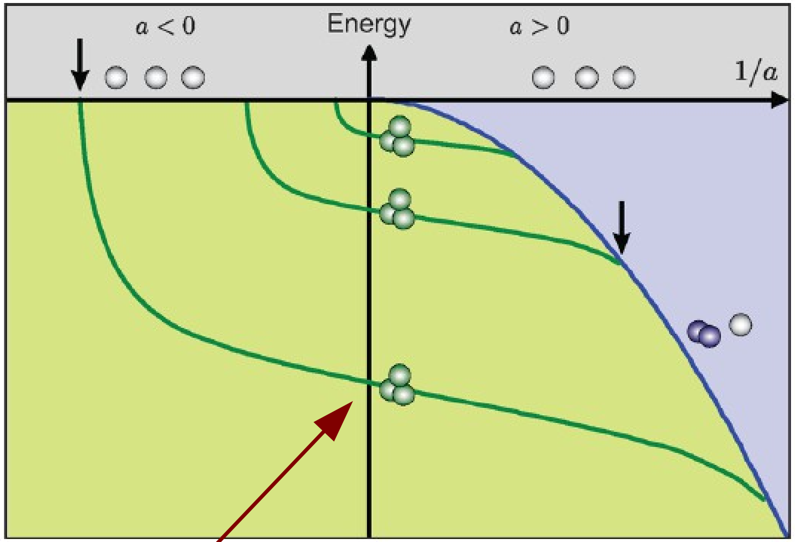
Few body physics: universal when interactions insensitive to microscopic details interaction

Depend only on one (or few) universal parameters

- **Scattering length**
- **Three body parameter** κ
- **Width of resonance**

$$k \cot \delta(k) = \frac{-1}{a} + \frac{1}{2} R^{eff} k^2$$

Inversely proportional to width ΔB



$$\frac{\hbar^2 \kappa^2}{2\mu}$$

Energy lowest Efimov state

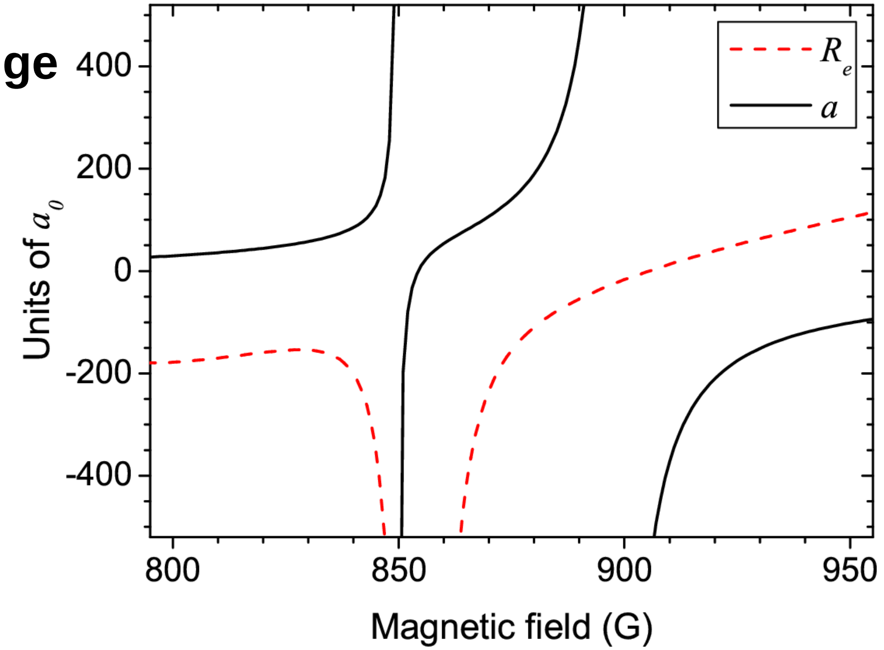
Three-body physics in ^{7}Li

Experiment performed in group Lev Khaykovic (Bar Ilan Univ.)

^7Li three-body recombination loss in the vicinity of a Feshbach resonance.

- **Feshbach resonances for $|f=1, m_f=0\rangle$ state (coupled channels calculation)**

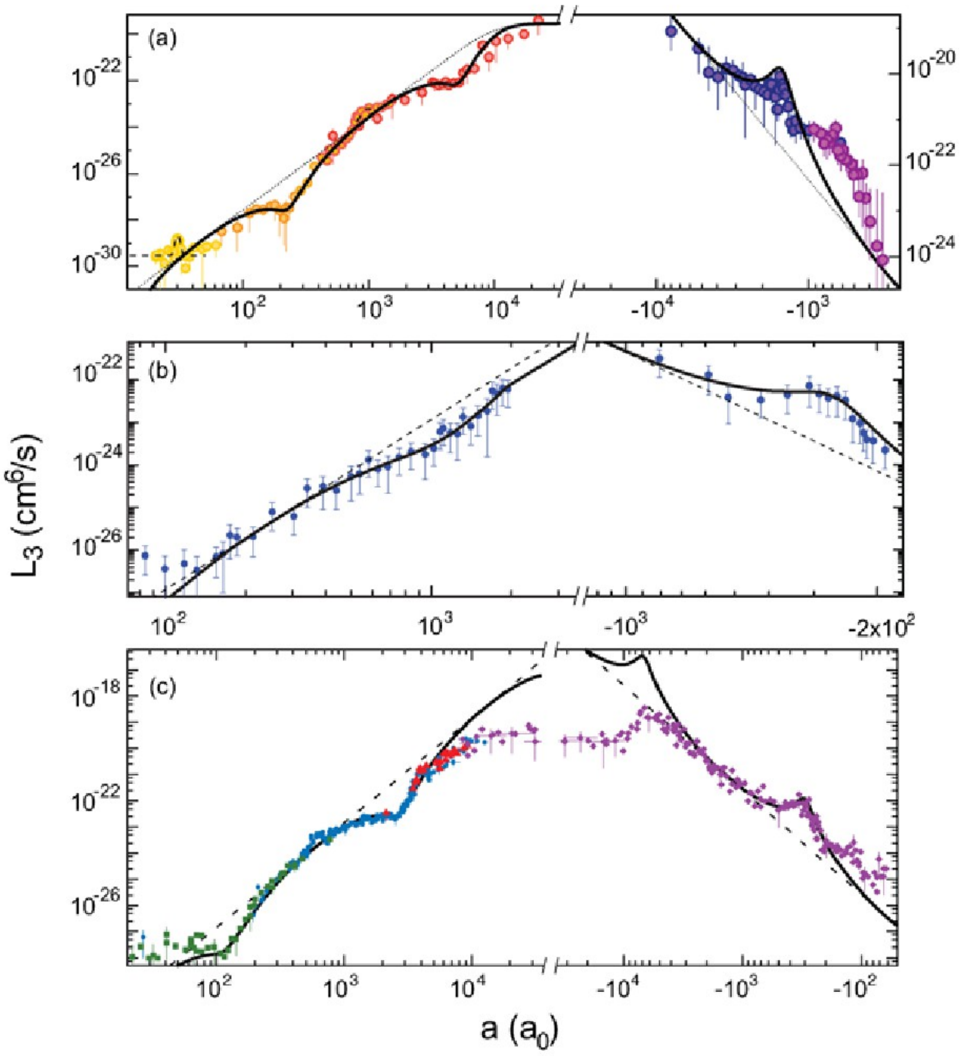
Scattering length and Effective Range



Universality depends strongly on width of resonance.

Captured by Effective Range R^{eff}

3-body recombination: more features



University of Florence: broad Feshbach resonance ^{39}K

→ non-universal

[Zaccanti et al, Nature Phys.5, 586 (2009)]

^{7}Li from Bar Ilan University

$$|f=1, m_f=0\rangle$$

→ universal

[PRL 103, 163202 (2009)]

^{7}Li from Rice University

$$|f=1, m_f=1\rangle$$

→ non-universal

[S.E. Pollack, D. Dries and R.G. Hulet, Science 326, 1683 (2009)]

Universality mostly at one side of resonance, not across

Check of *a(B)*

Address problem carefully:

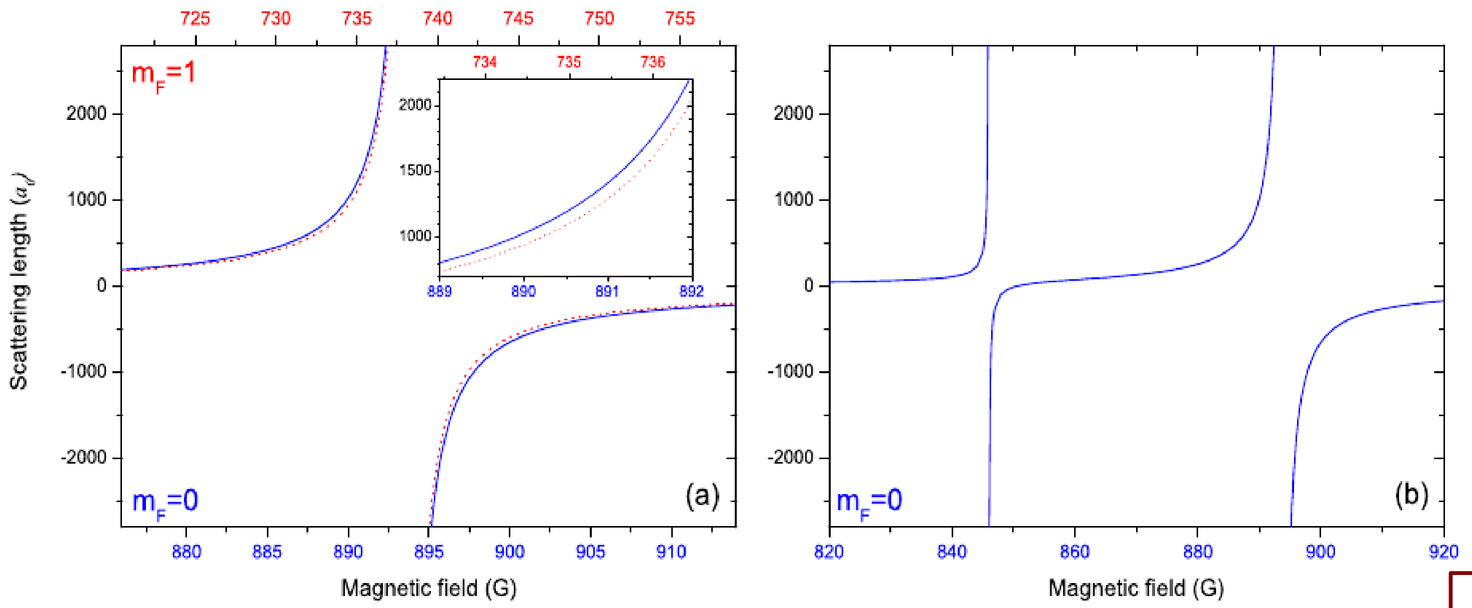
- **Underlying assumption: precise knowledge *a(B)***

Coupled channels calculations:

- **Best available singlet-triplet potentials**
- **Improved via accumulated phase method using available $^{6}Li/^{7}Li$ data**

Characterize the resonances

- **But how to account for experimental uncertainties of particular experiment?**
  - **Match potentials/parameters to local data, to be consistent with magnetic field scaling**

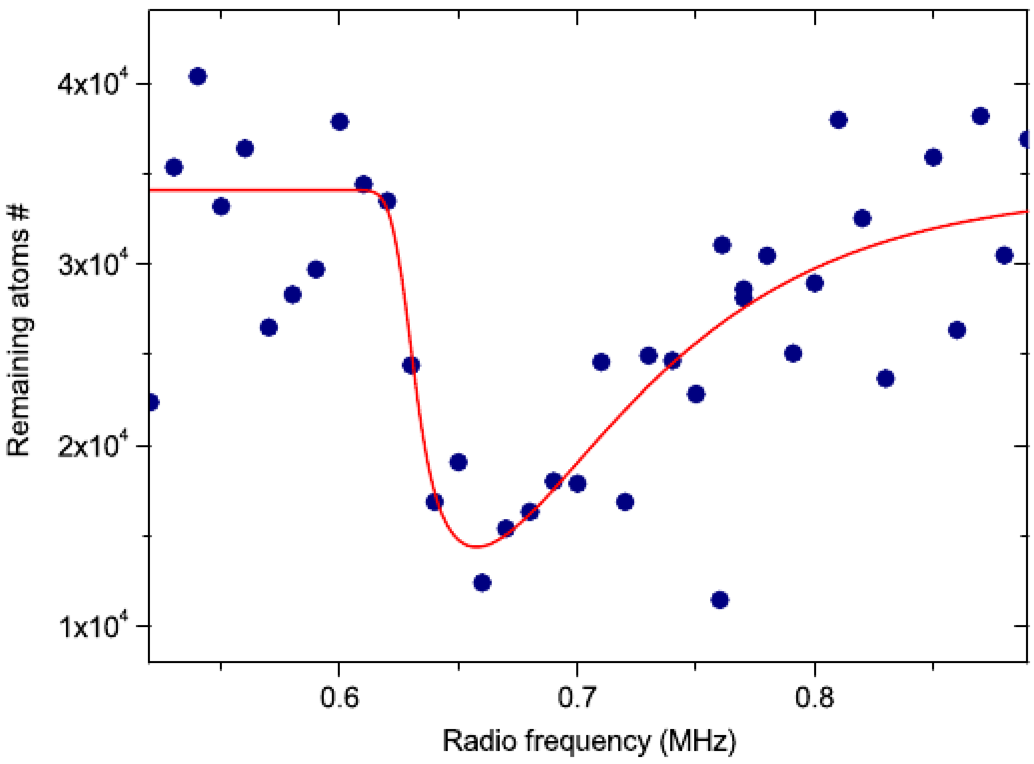


- **3-body recombination in both ^7Li spin states**
  - **Coupled channels calculations**

$$|f=1, m_f=0\rangle$$
$$|f=1, m_f=1\rangle$$

Measurement of dimer binding energies

- **Needed for direct link between scattering length and field-dependent 3-body recombination spectrum**
  - **RF association of dimers**
  - **Determine resonance position B_0, binding energy E_B**

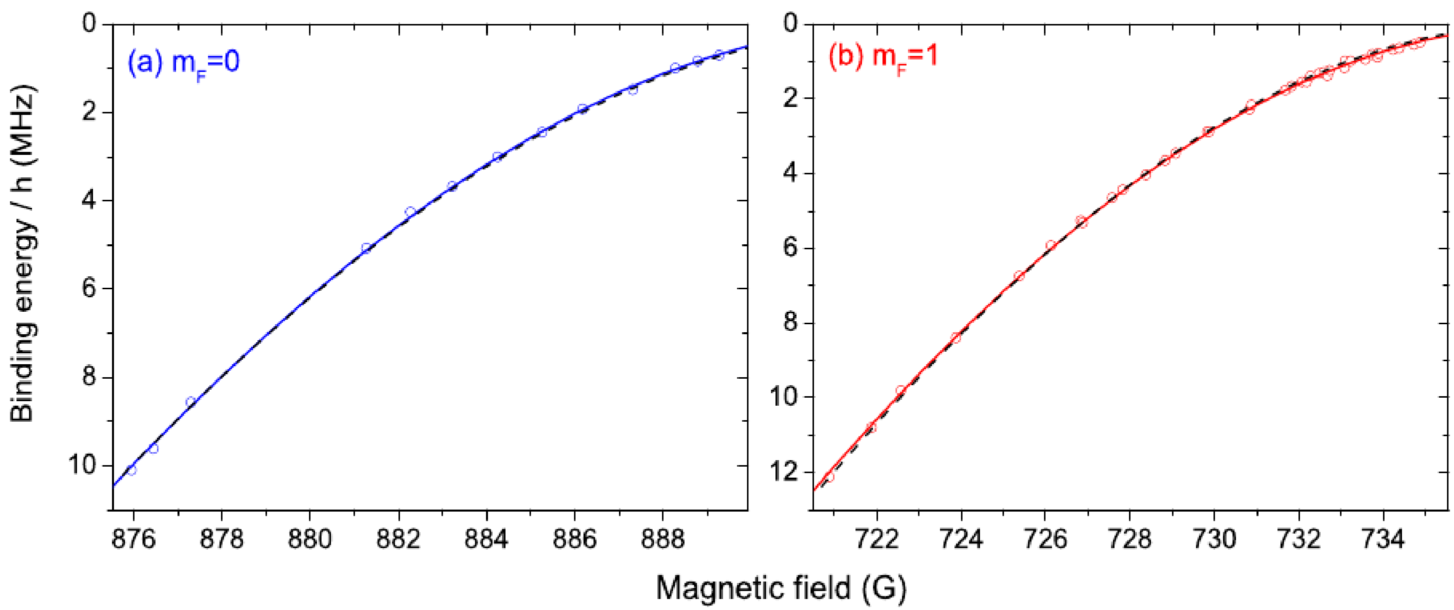


Analyze binding energies

- **Same coupled channels model for two different spin states $\left|f=1, m_f=0\right\rangle$ and $\left|f=1, m_f=1\right\rangle$**
  - **Excellent fit of data**
  - **Yields very accurate mapping of *a(B)* curves!**

state	type	B_0 (G)	
		Combined fit	Experimental
$\|m_F=0\rangle$	narrow	845.54	844.9(8)
$\|m_F=0\rangle$	wide	893.95(5)	893.7(4)
$\|m_F=1\rangle$	wide	737.88(2)	738.2(4)

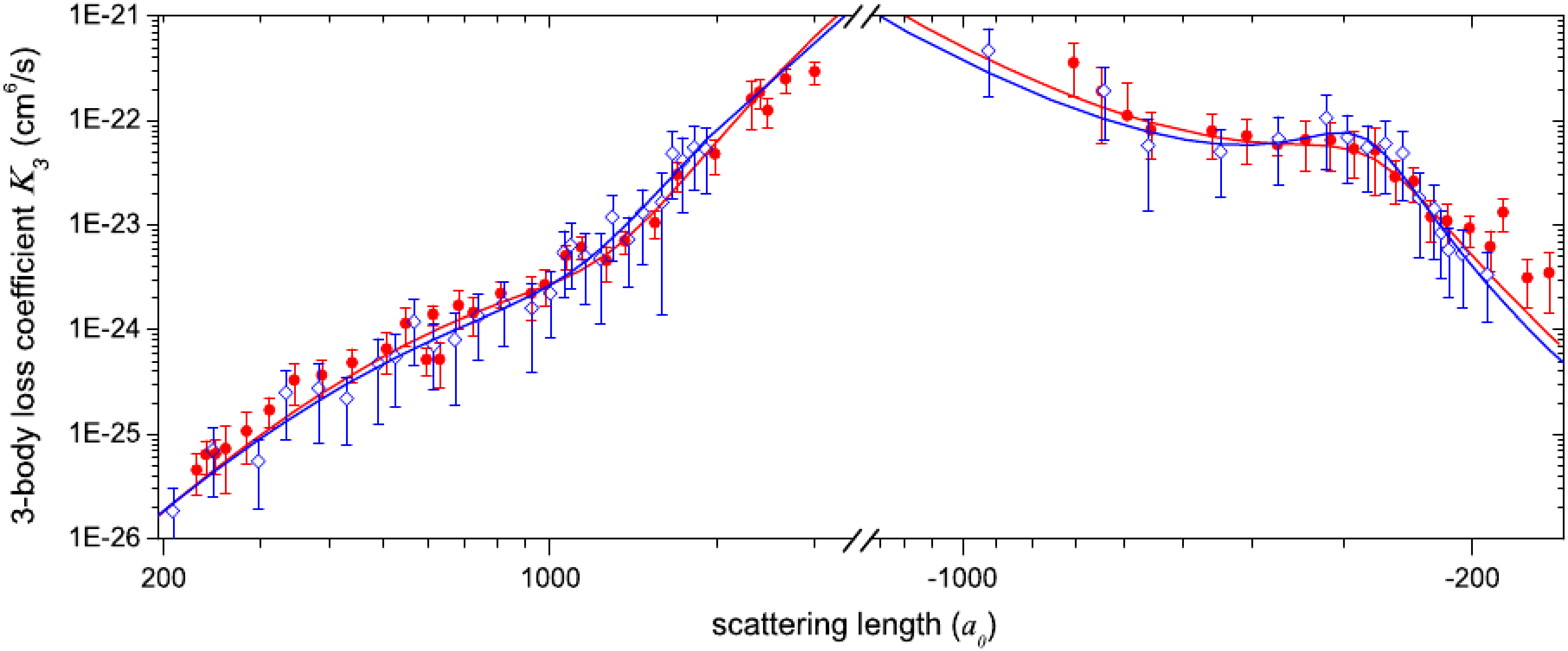
- **Better ^{7}*Li* potentials: a_S =34.33(2) a_0, a_T =-26.87(8) a_0**



[N. Gross , Z. Shotan , O. Machtey , S. Kokkelmans and L. Khaykovich, Comp.Rend.Phys (2011)]

Recomination rate for two spin states

- **Very similar results:**



Also the same three-body Parameter! Potentials very similar at high fields

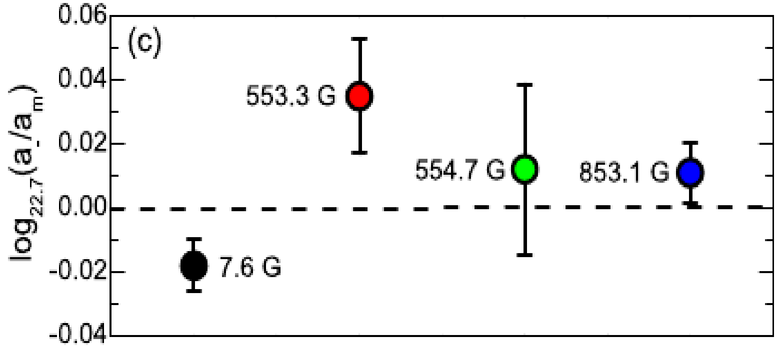
state	η_+	η_-	a_+/a_0	a_-/a_0	$a_+/\|a_-\|$
$\|m_F = 0\rangle$	0.213(79)	0.180(48)	238(25)	-280(12)	0.85(11)
$\|m_F = 1\rangle$	0.170(41)	0.253(62)	265(16)	-274(12)	0.97(8)

Ref.: [N. Gross, Z. Shotan, S. Kokkelmans, L. Khaykovich, PRL **105**, 103203 (2010)]

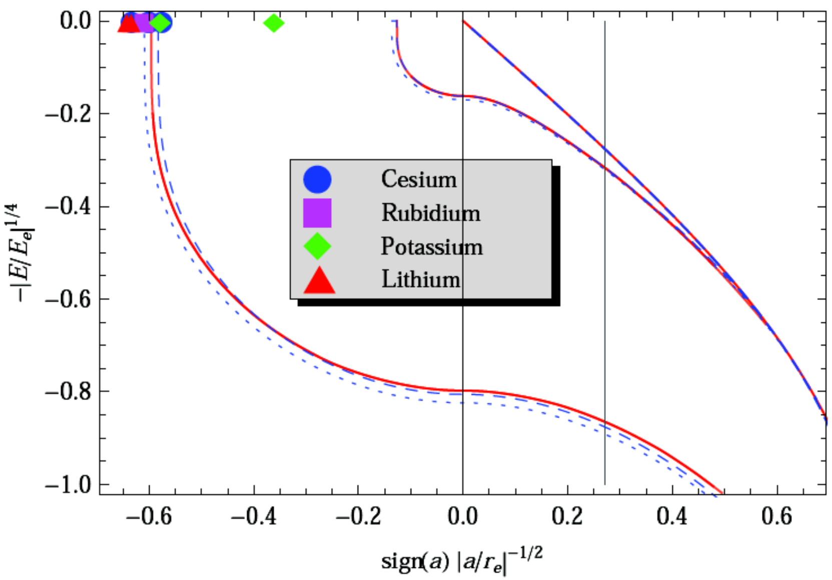
Three-body parameter

- **Cs: Multiple FB resonances**
  - **Same three-body parameter**
  - **Measurements only for *a<0***

[Berninger at al. arXiv/1106.3933]



- **Combine several experiments**



[P. Naidon, E. Hiyama and M. Ueda, arXiv/1109.5807]

Is there a universal scaling of κ with range potential?

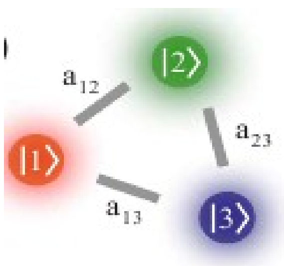
Few body physics with fermions

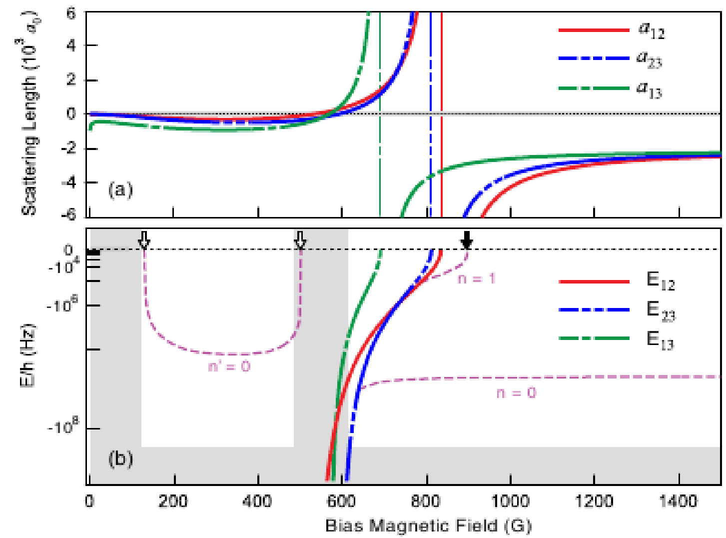
- **Efimov effect for fermions: three different spin states**

[A.N. Wenz et al, PRA **80**, 040702 2009]

[J. R. Williams et al, PRL **103**, 130404 (2009)]

[Nakajima et al, PRL **105**, 023201 (2010)]

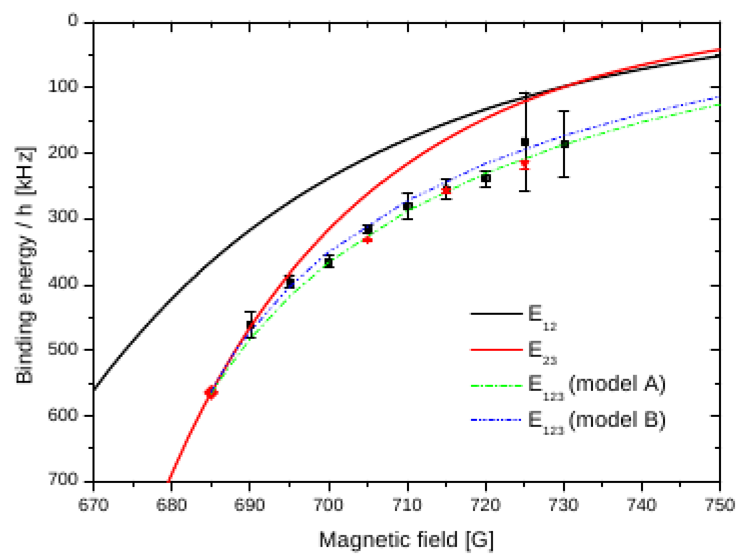




- **Measurement binding energy Efimov states: RF association**

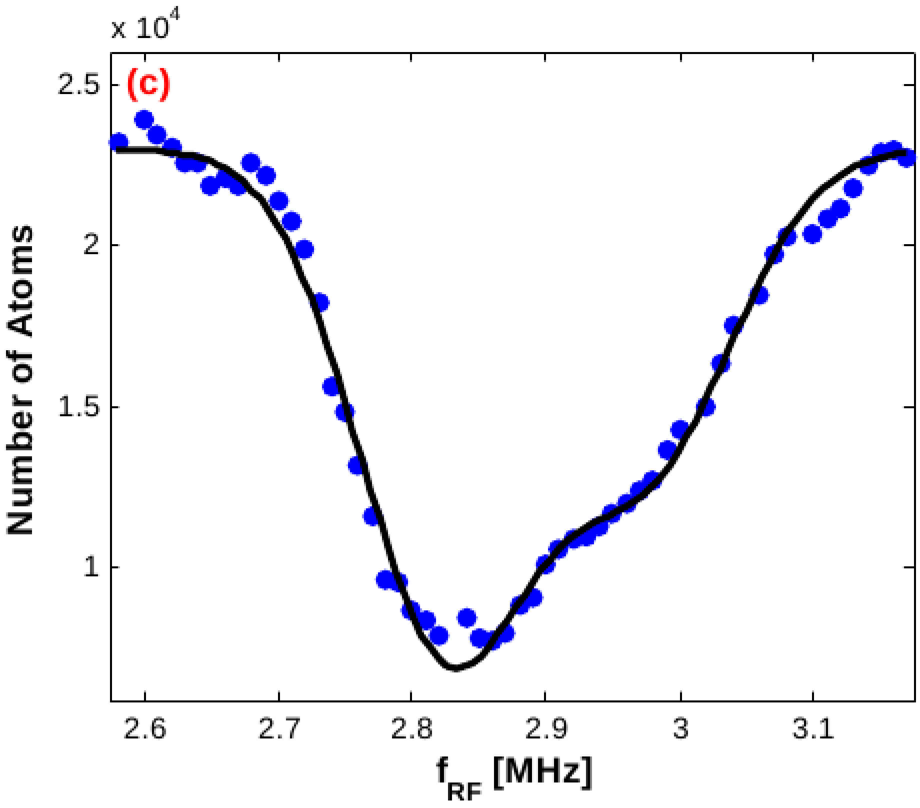
[T. Lompe et al, Science **330**, 940 (2010).]

[S. Nakajima et al, Phys. Rev. Lett. **106**, 143201 (2011)]



RF association of trimers with bosons

- **Performed at Bar-Ilan with ^{7}Li**

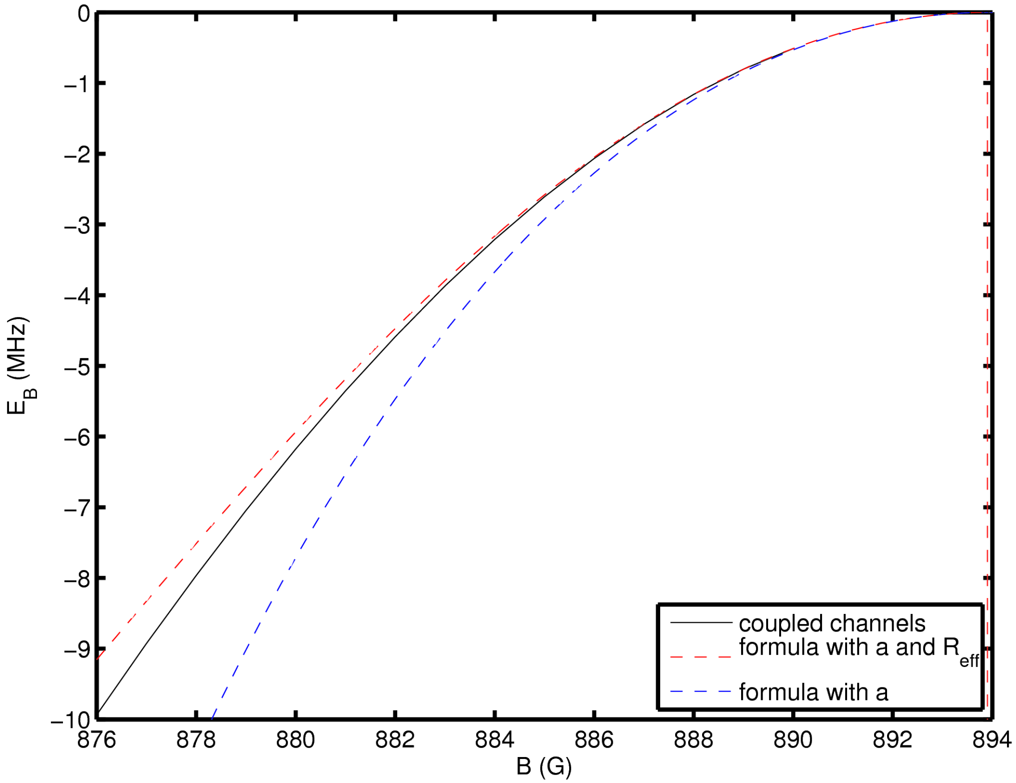


[O. Machtey, Z. Shotan, N. Gross, L. Khaykovich, to be submitted]

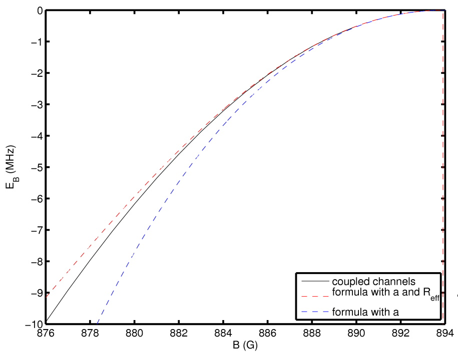
How universal is the two-body physics?

- **Deviations from** $E_d = -\hbar^2 / m a^2$
- **Even deviations from Effective range**

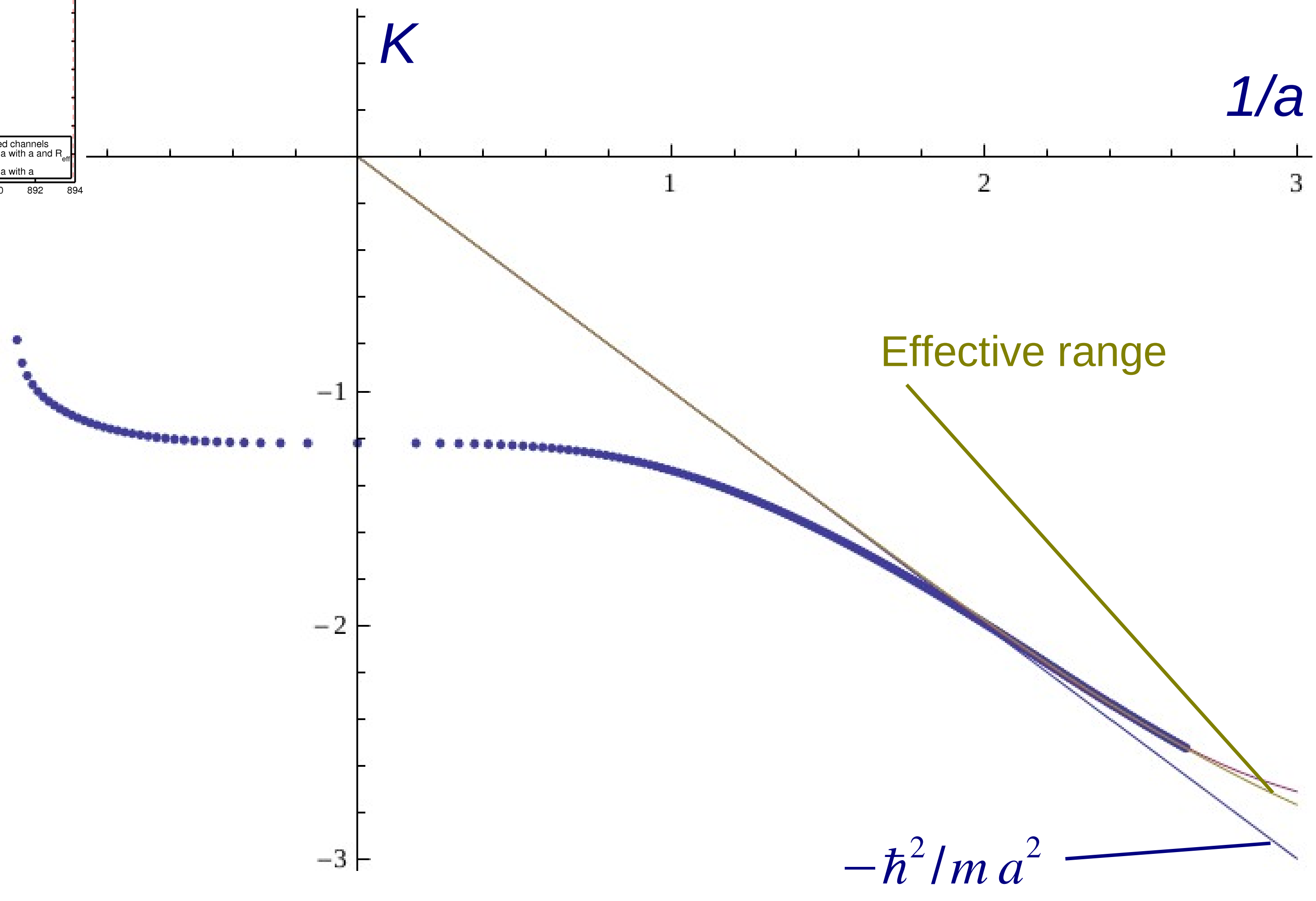
$$k \cot \delta(k) = \frac{-1}{a} + \frac{1}{2} R^{eff} k^2$$



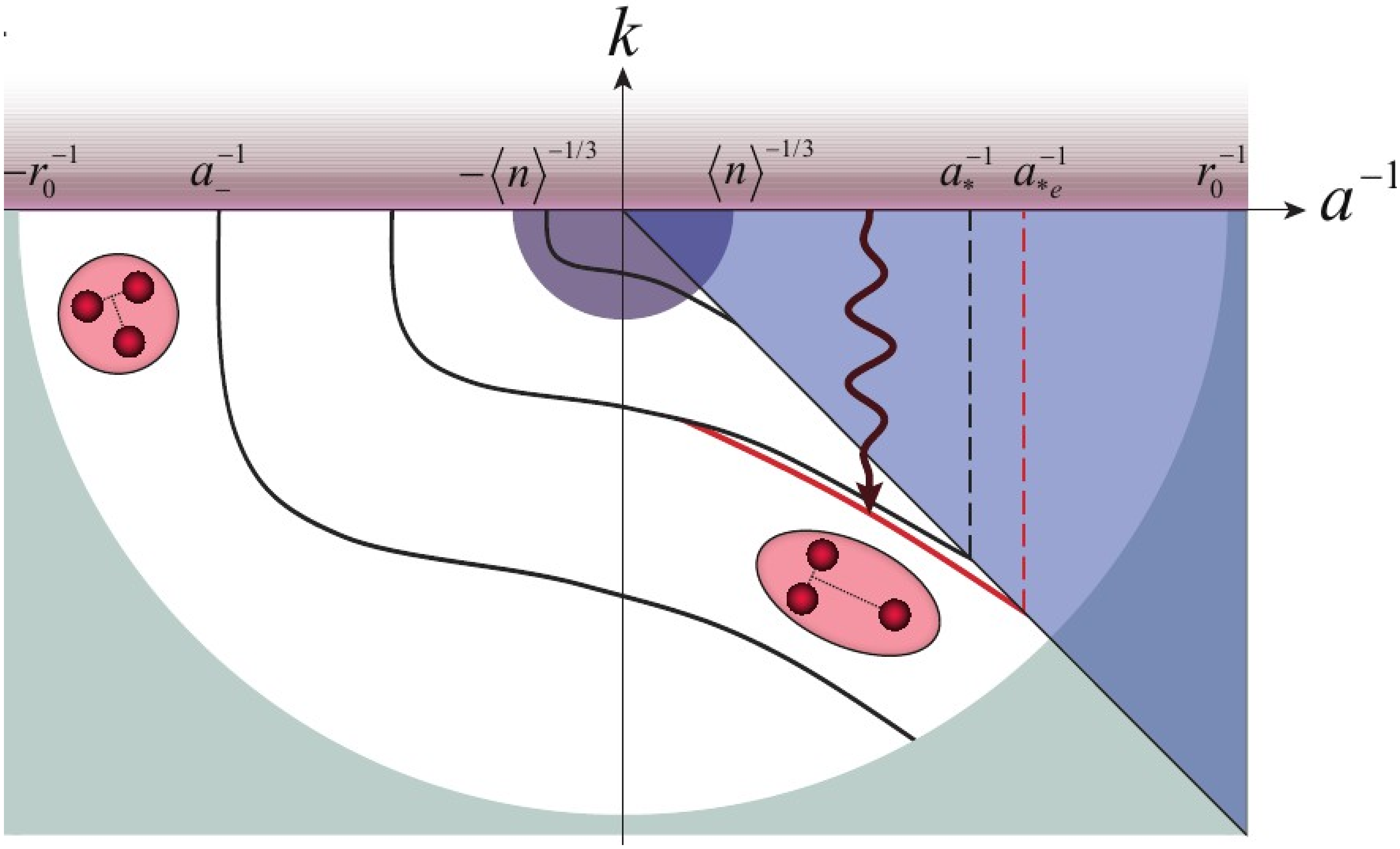
How universal is the three-body physics?



- **Efimov state incl. non-universal corrections**



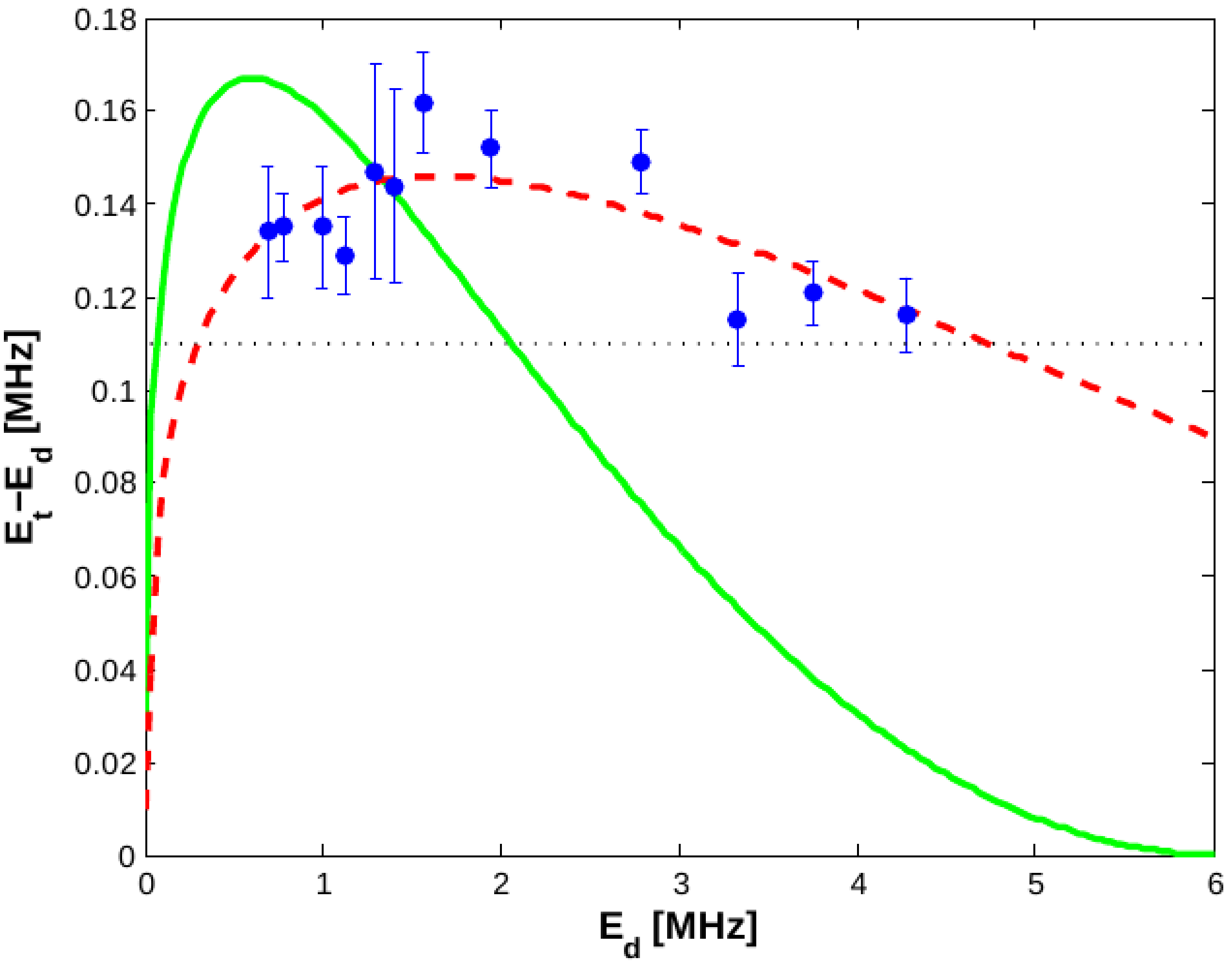
Deviations from universality



Non-universal trimer energy

- **Shift $a_* = 288\, a_0$ → $a_{*e} = 180 a_0$**

$|f=1, m_f=0\rangle$ spin state

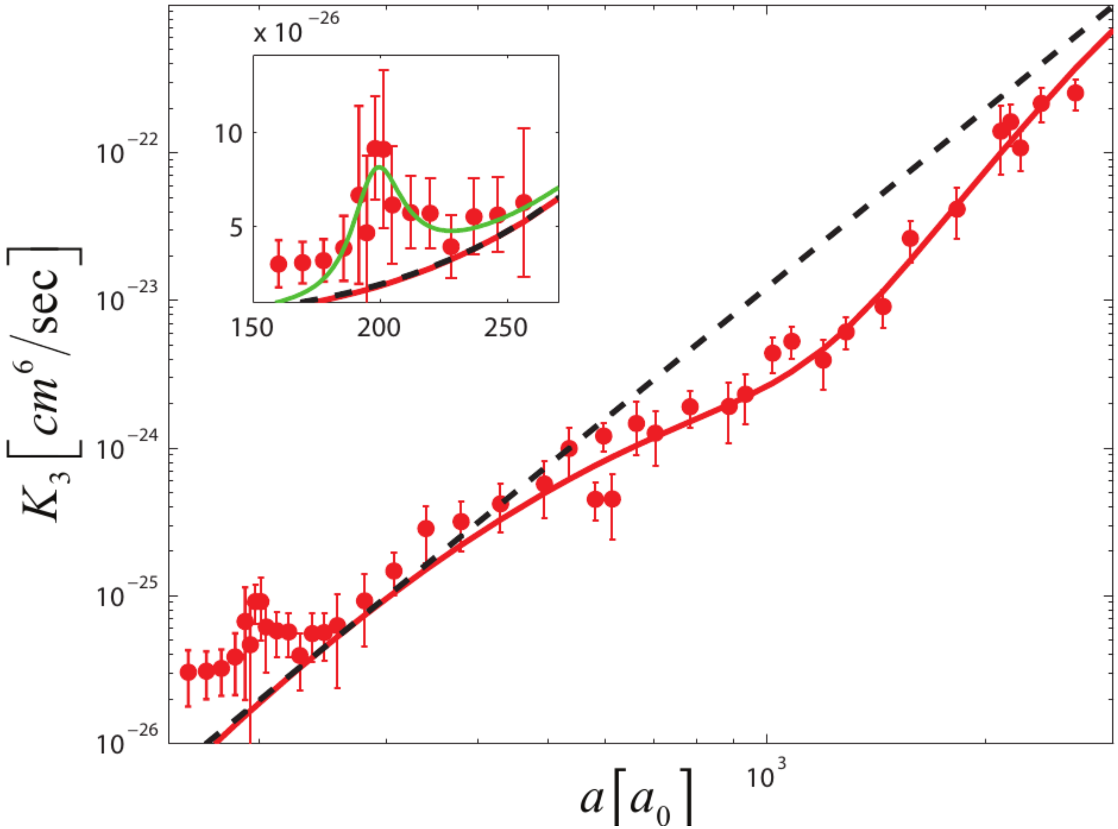


Avalanche resonance

- **Another signature of non-universal behavior**
- **At consistent value scattering length**

$|f=1, m_f=1\rangle$

spin state

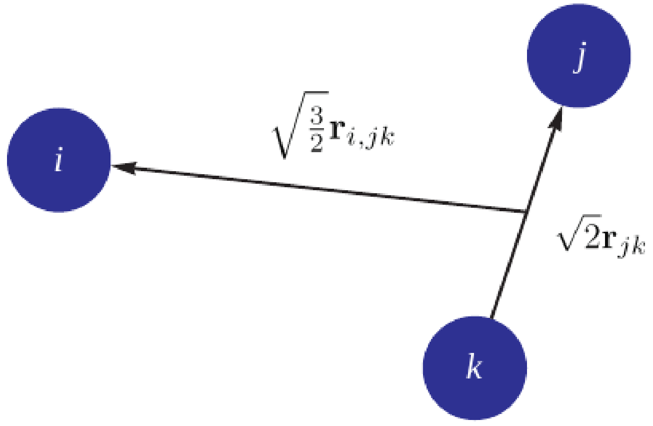


Trimers in a harmonic potential

- **Is it possible to stabilize the trimers?**
  - **Efimov trimer already instable by itself**
  - **Deep optical lattice: will it help?**

- **Three particles in harmonic potential with a Feshbach resonance:**

$$H = -\frac{1}{2}\sum_i \Delta_i + \frac{1}{2a_{ho}^4}\sum_i r_i^2 + \sum_i V_i$$

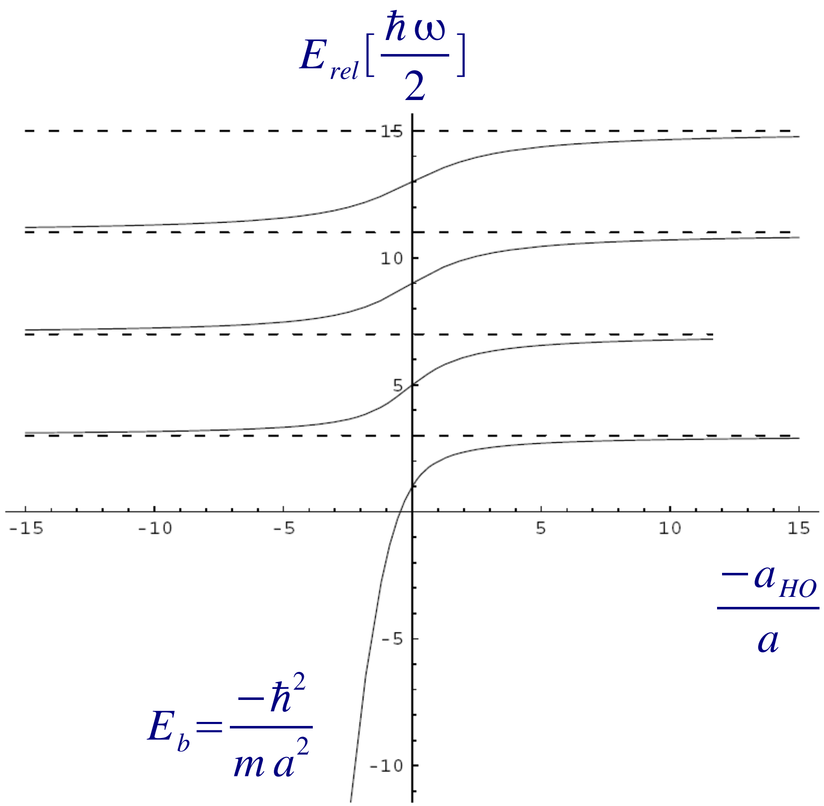


a_{ho} : harmonic oscillator length
r_i : position particle i
V_i : int. between particles j and k

Two particles in a h.o. potential

Feshbach resonance modeled by scattering length *a*

- **Resonance couples two consecutive HO levels**



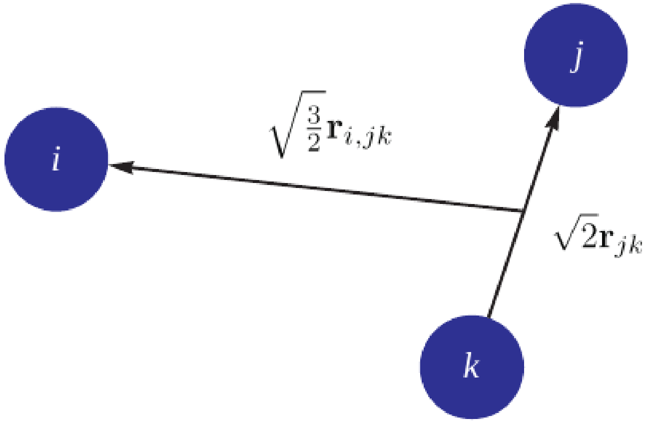
$$\frac{1}{2a} = \frac{\Gamma\left(\frac{3-E_s}{4}\right)}{\Gamma\left(\frac{1-E_s}{4}\right)}$$

[T. Busch et al., Found. Phys. **28**, 549 (1998).]

Three particles: hyperspherical approach

- **Adiabatic expansion** $\psi(R,\alpha) = \frac{1}{2\pi R^{5/2}\sin(2\alpha)} \sum_n f_n(R)\phi_n(R,\alpha)$

Hyperradius $R > 0$:
$R^2 = r^2_{i,jk} + r^2_{jk}$
Hyperangle $\alpha_i \in [0, \pi/2]$.
$r_{i,jk} = R\cos\alpha_i$,
$r_{jk} = R\sin\alpha_i$,
(i,j,k) cyclic permutation (1,2,3).



- **Faddeev eqn:** [Fedorov and Jensen, PRL **71**, 4103 (1993)]
  - **Hyperangular problem:** $F^{(R)}\phi_n(R,\alpha) = \lambda_n(R;V)\phi_n(R,\alpha)$
  - **Find eigenvalues** $\lambda_n(R;V)$ **from:**

$$\cos\left(\lambda_n^{1/2}\frac{\pi}{2}\right) - \frac{8}{\sqrt{3}}\lambda_n^{-1/2}\sin\left(\lambda_n^{1/2}\frac{\pi}{6}\right) = -\sqrt{2}\lambda_n^{-1/2}\sin\left(\lambda_n^{1/2}\frac{\pi}{2}\right) R k_R \cot(\delta(k_R))$$

Hyperspherical approach (2)

- **Solve hyperradial problem:**

$$\left[-\frac{\partial^2}{\partial R^2}+\frac{\lambda_n(R/a)-1/4}{R^2}+\frac{R^2}{a_{ho}^4}\right]f_n(R)=2Ef_n(R)$$

- **Write solution as:**

$$T=\kappa\exp\left[\phi(TR_0;\xi,\theta)\right]\exp\left(\frac{n\pi}{s_0}\right)$$

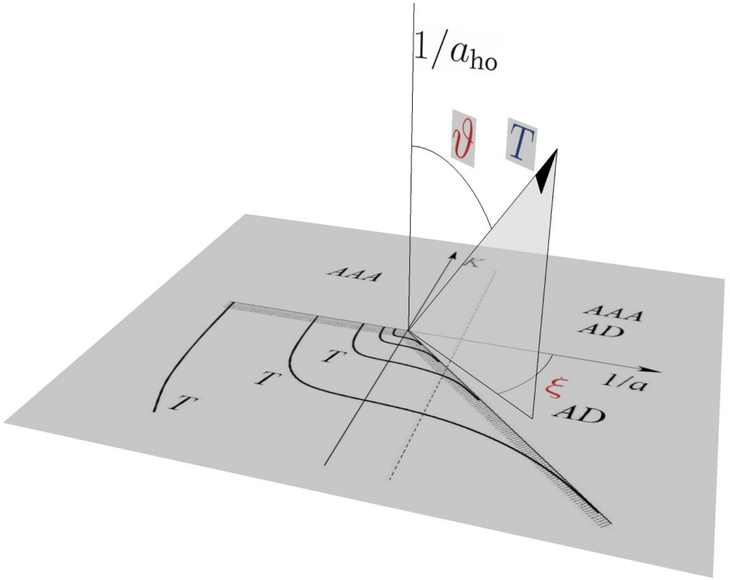
- **Define total energy as**

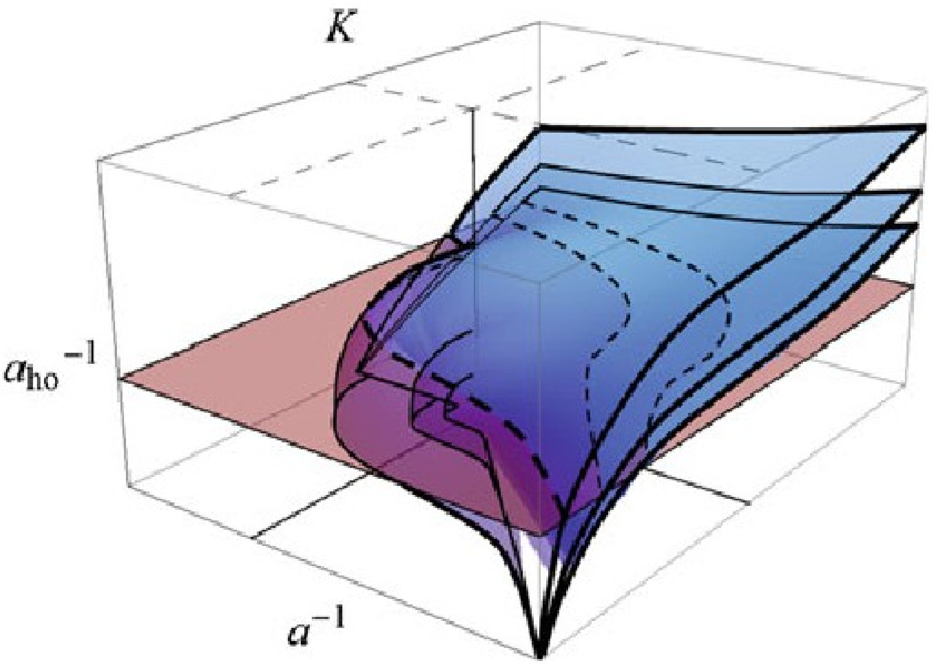
$$T^2=K^2+\frac{1}{a^2}+\frac{1}{a_{ho}^2}$$

T = distance to origin,
: shape of surfaces ,
κ: crossing surface with K − axis
ϕ : characteristic range potential
n : various surfaces

Solutions

Represent solutions in 3D parameter space



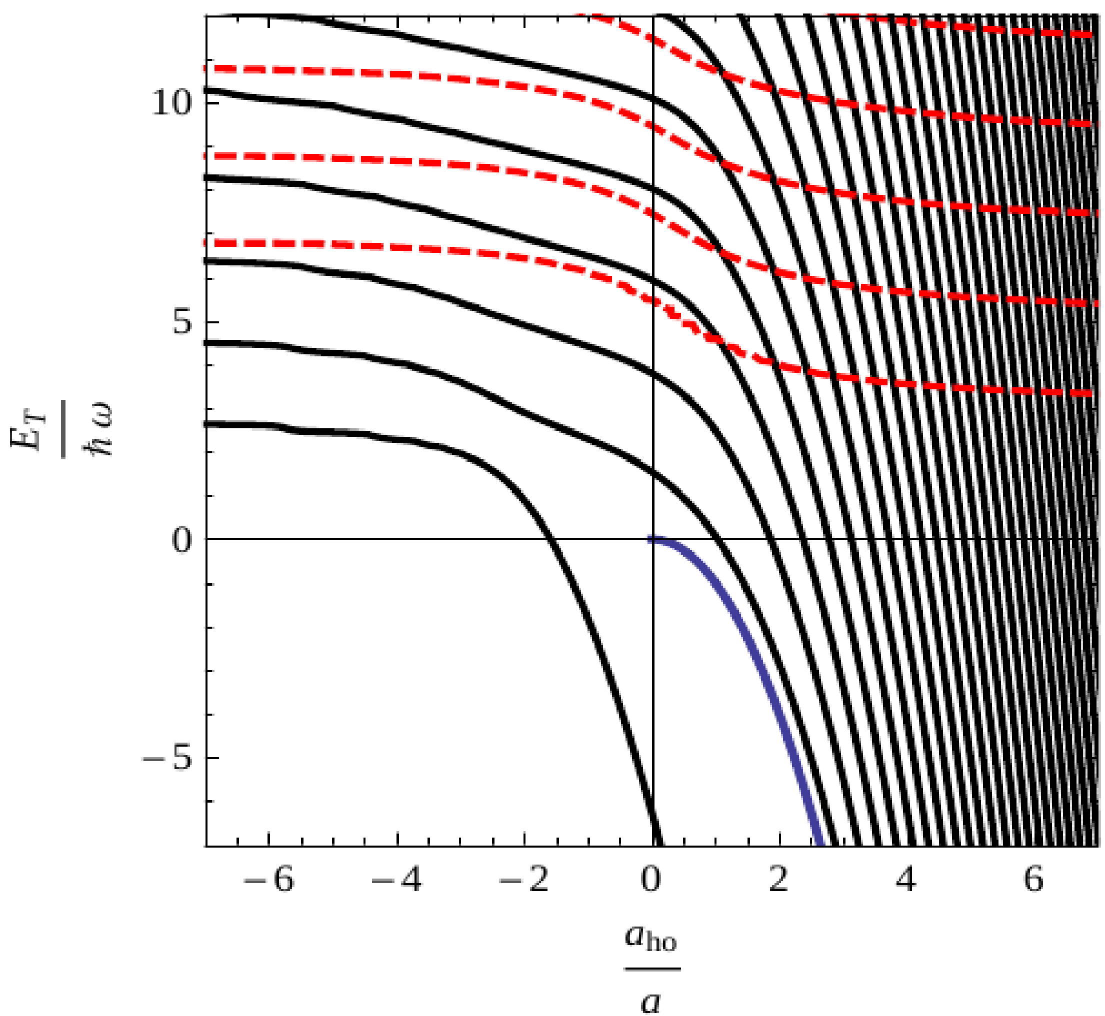


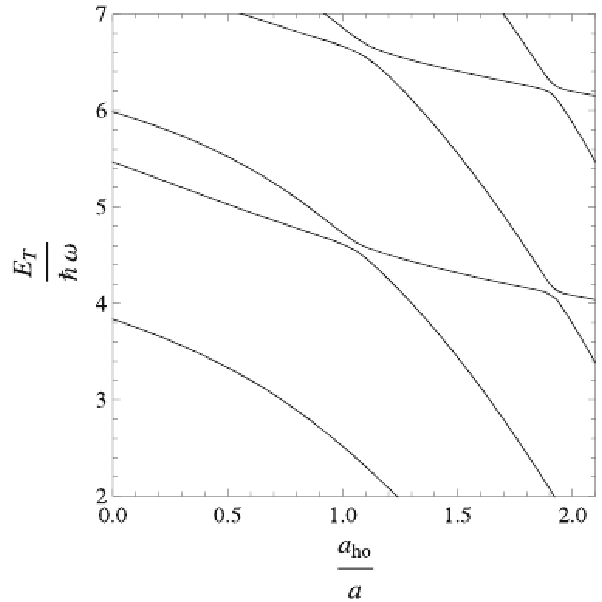
$$\frac{1}{a} = T \sin\theta \cos\xi$$
$$K = T \sin\theta \cos\xi$$
$$\frac{1}{a_{ho}} = T \cos\theta$$

[Jim Portegies and Servaas Kokkelmans, Few Body Sys. (2011)]

Trimer solutions for fixed oscillator strength

- **Energy versus scattering length**





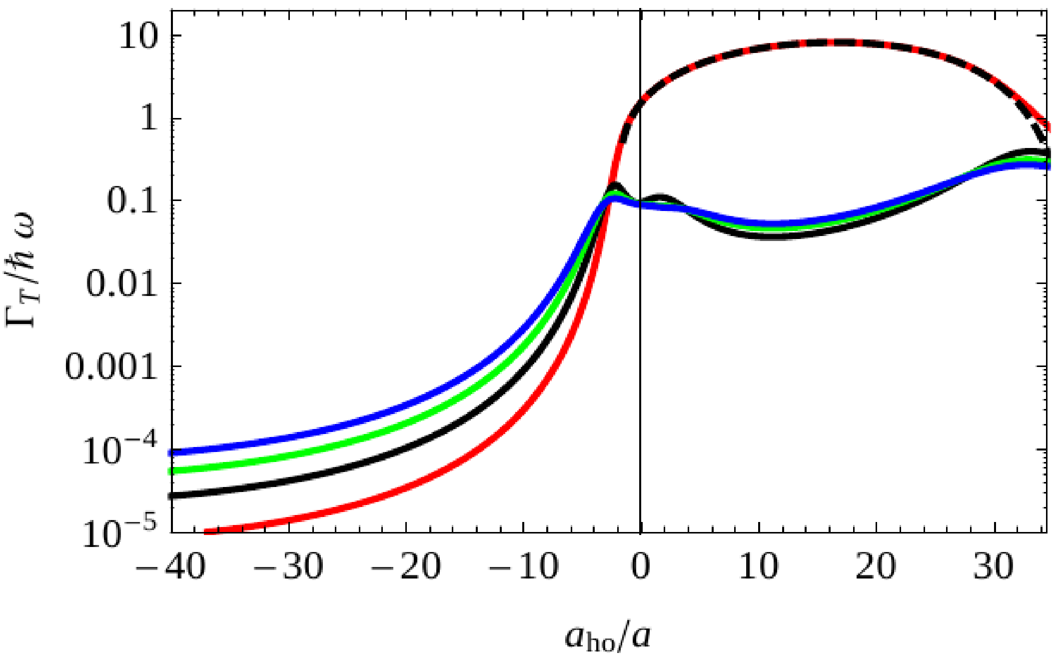
Avoided crossings between different hyperspherical channels

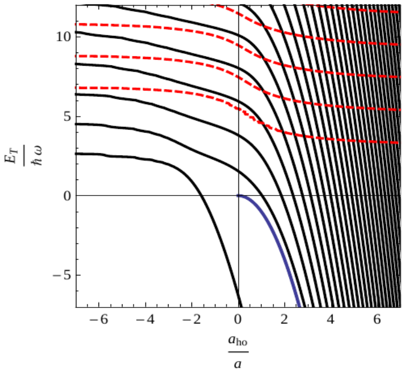
Decay of trimer state

- **Introduce in-elasticity parameter η for short range**

  (complex three-body parameter)

➡ **Find complex eigenvalues** $E = E_T - i\,\Gamma_T/2$



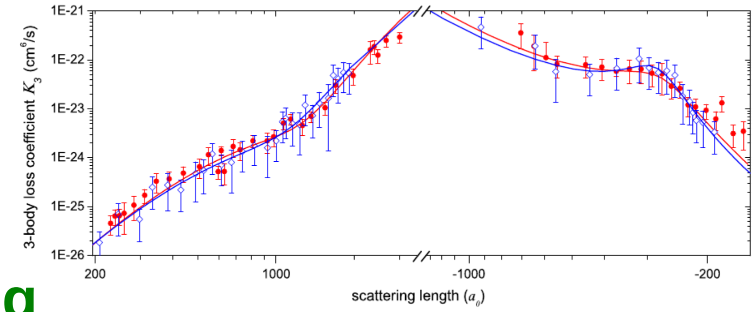


Mixed harm. oscillator - Efimov properties

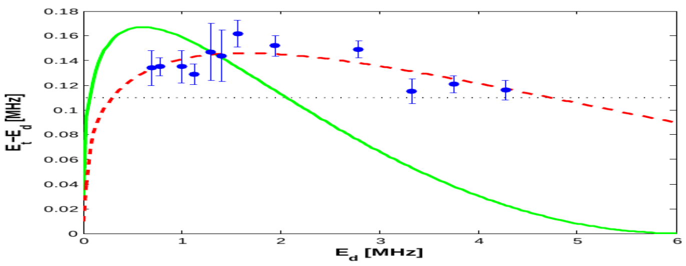
- **Allow for regime with small losses!**

Summary

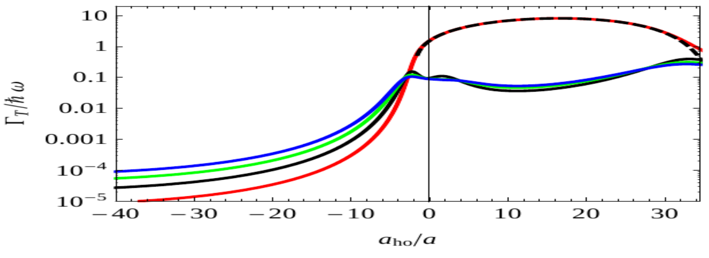
- **Universal Efimov physics observed in ^{7}*Li* 3-body recombination exp.**
  - **Demonstrated in two different hyperfine states**
  - **Spend great effort at mapping out *a(B)* around resonance**



- **Observations of non-universal corrections trimer energy**
  - **Needs better understanding**



- **Trimers in optical lattices**
  - **Study trimers with mixed Efimov – h.o. properties**

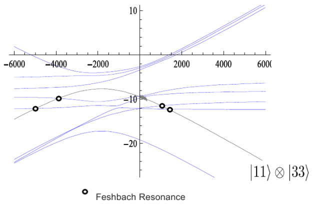


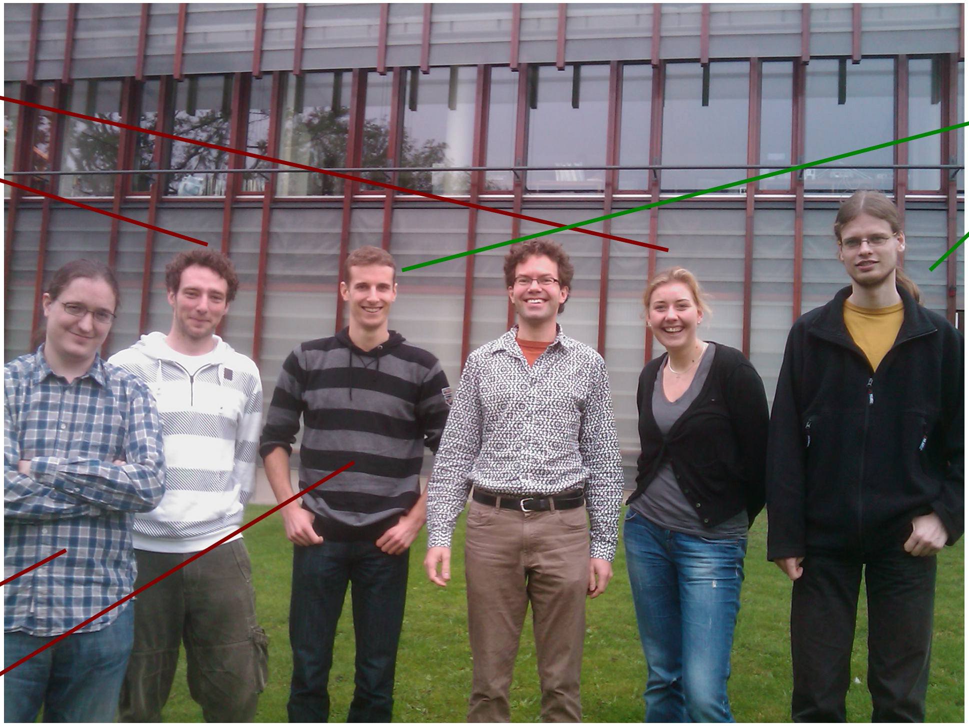
Acknowledgments

Efimov physics in ^{7}Li

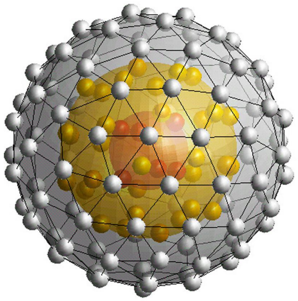
Noam Gross, Zav Shotan, Olga Machtey, Lev Khaykovich

Feshbach res.
Marije Haverhals
Maikel Goosen

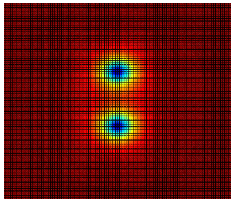




Rydberg lattices
Rick van Bijnen
Sjoerd Smit



Dipolar BEC
Elmer doggen
Rick van Bijnen



Trimers in optical lattices
Jim Portegies
(now Courrant inst, NY)

NWO Nederlandse Organisatie voor Wetenschappelijk Onderzoek

COQT Coherence and Quantum Technology

TU/e Technische Universiteit Eindhoven University of Technology