Universality of unstable bosonic tetramers

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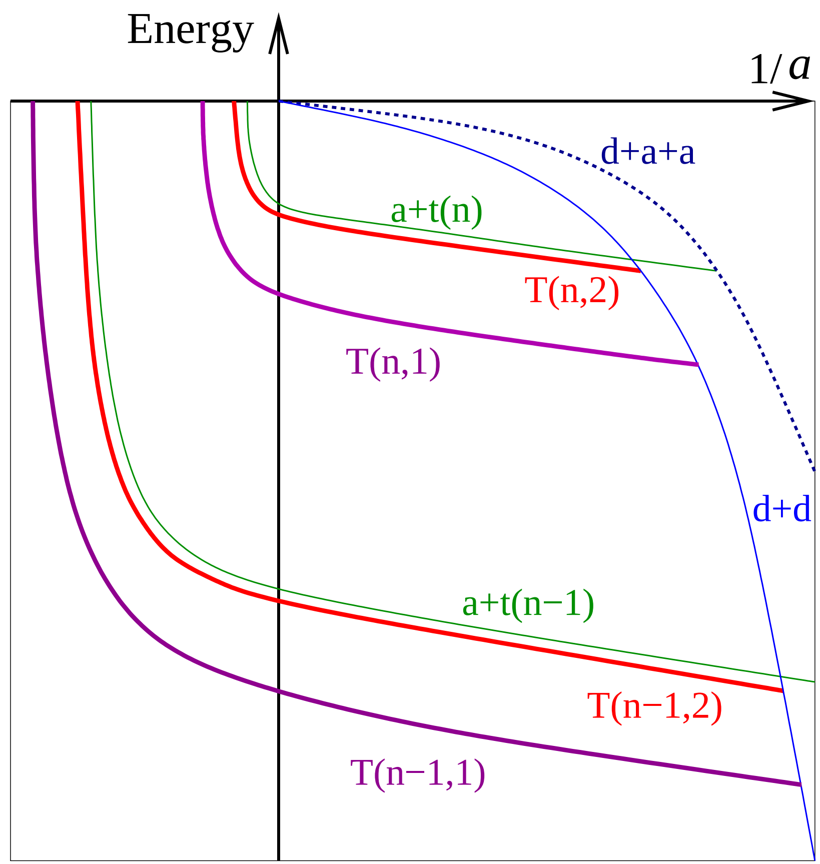
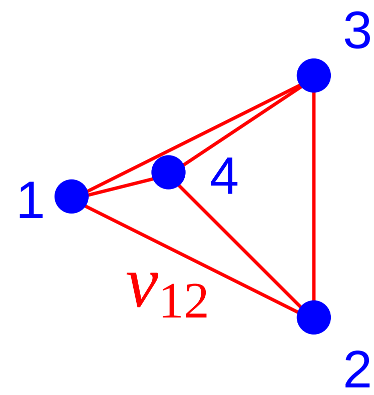
Four-boson Efimov physics



- momentum-space scattering equations
- atom-trimer and dimer-dimer scattering
- four-atom recombination

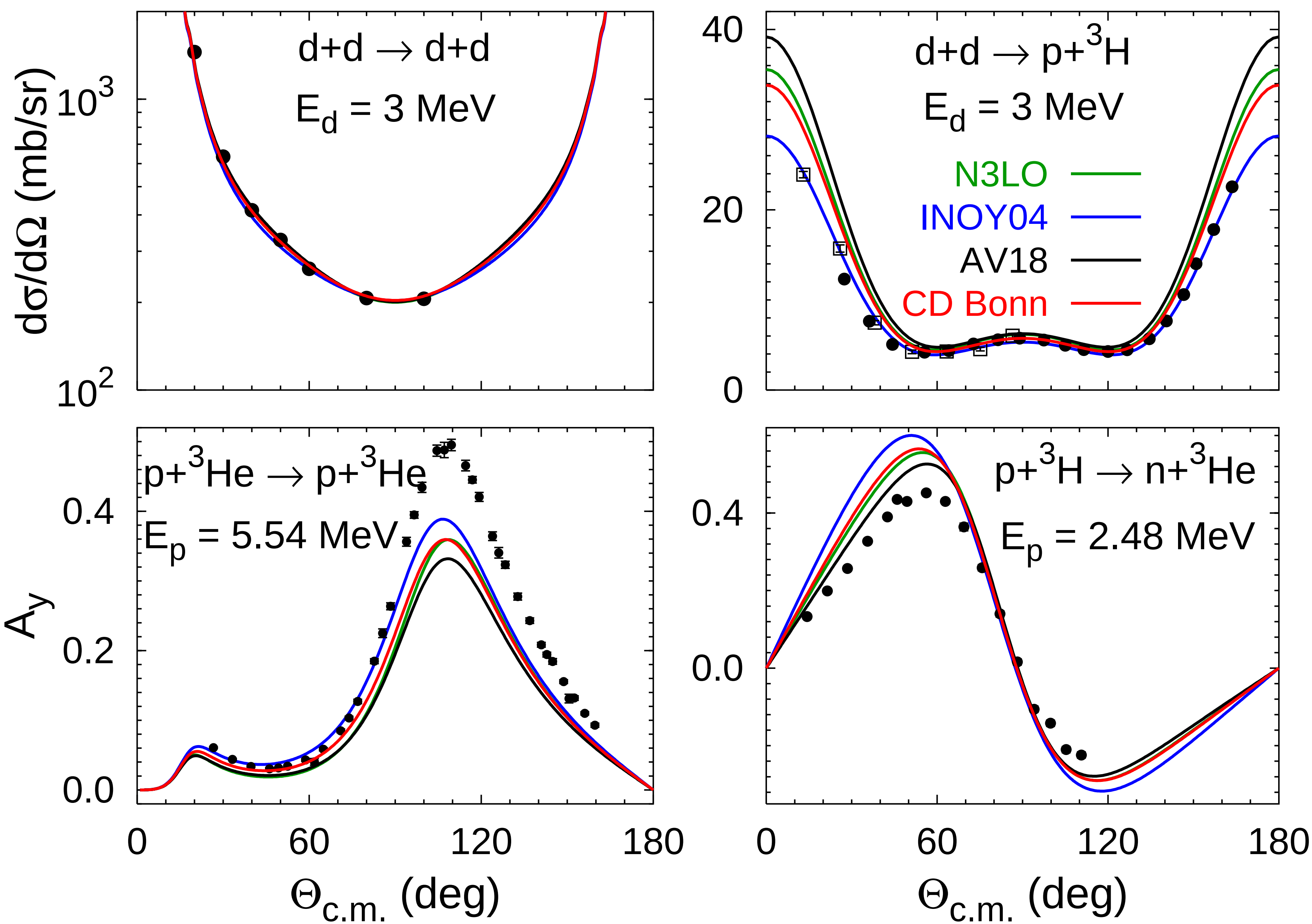
Four-particle scattering



Hamiltonian $H_0 + \sum_{i>j} v_{ij}$

- Wave function:
  Schrödinger equation
- Wave function components:
  Faddeev-Yakubovsky equations
- Transition operators:
  Alt-Grassberger-Sandhas equations

Four-nucleon reactions



[PRC 75, 014005; PRL 98, 162502; PRC 76, 021001 (2007)]

Symmetrized bosonic AGS equations

$$t = v + vG_0 t$$

$$G_0 = (E + i0 - H_0)^{-1}$$

$$u_j = P_j G_0^{-1} + P_j t G_0 u_j$$

$$3 + 1: \quad P_1 = P_{12} P_{23} + P_{13} P_{23}$$

$$2 + 2: \quad P_2 = P_{13} P_{24}$$

$$U_{11} = (G_0 t G_0)^{-1} P_{34} + P_{34} u_1 G_0 t G_0 U_{11} + u_2 G_0 t G_0 U_{21}$$

$$U_{21} = (G_0 t G_0)^{-1} (1 + P_{34}) + (1 + P_{34}) u_1 G_0 t G_0 U_{11}$$

$$U_{12} = (G_0 t G_0)^{-1} + P_{34} u_1 G_0 t G_0 U_{12} + u_2 G_0 t G_0 U_{22}$$

$$U_{22} = (1 + P_{34}) u_1 G_0 t G_0 U_{12}$$

basis states partially symmetrized

Scattering amplitudes

Two-cluster reactions:

$$T_{fi} = s_{fi} \langle \phi_f | U_{fi} | \phi_i \rangle$$

$$|\phi_j\rangle = G_0 t P_j |\phi_j\rangle$$

$$|\Phi_j\rangle = (1 + P_j) |\phi_j\rangle$$

four-body breakup/recombination:

$$T_{0i} = s_{0i} \{ \langle \phi_0 | [1 + (1 + P_1) P_{34}] (1 + P_1) t G_0 u_1 G_0 t G_0 U_{1i} | \phi_i \rangle$$

$$+ \langle \phi_0 | (1 + P_1)(1 + P_2) t G_0 u_2 G_0 t G_0 U_{2i} | \phi_i \rangle \}$$

Wave function

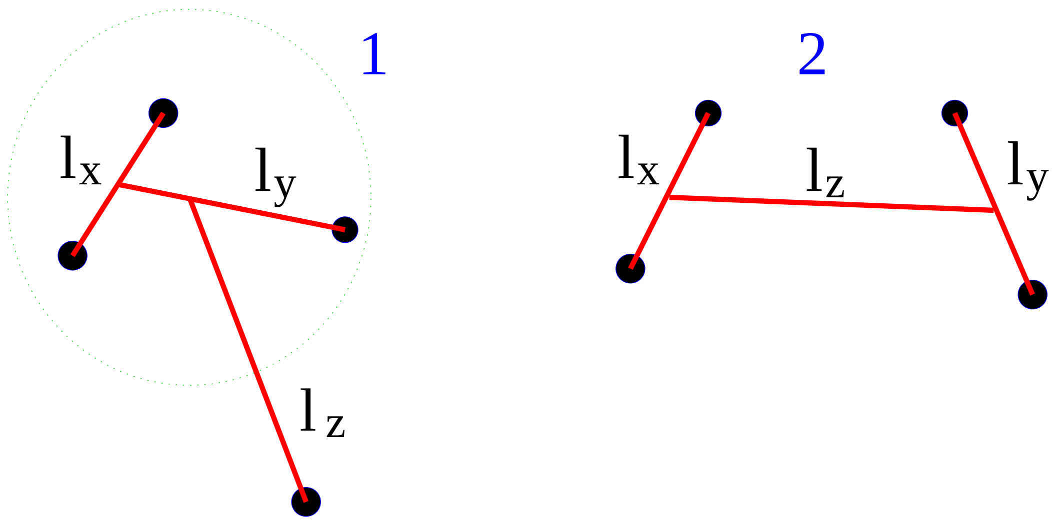
$$|\Psi_i\rangle = s_i\{[1+(1+P_1)P_{34}](1+P_1)|\psi_{1,i}\rangle + (1+P_1)(1+P_2)|\psi_{2,i}\rangle\}$$

with Faddeev-Yakubovsky components

$$|\psi_{j,i}\rangle = \delta_{ji}|\phi_i\rangle + G_0 t G_0 u_j G_0 t G_0 U_{ji}|\phi_i\rangle$$

Solution of AGS equations

$$U_{11}|\phi_1\rangle = G_0^{-1} P_{34} P_1 |\phi_1\rangle + P_{34} u_1 G_0 t G_0 U_{11} |\phi_1\rangle + u_2 G_0 t G_0 U_{21} |\phi_1\rangle$$



- momentum-space partial-wave basis, two types, partially symmetrized
  $|k_x k_y k_z [(l_x l_y) S l_z] J\rangle_1$ & $|k_x k_y k_z [(l_x l_y) S l_z] J\rangle_2$
- set of coupled integral equations in 3 variables
  $_j\langle k_x k_y k_z [(l_x l_y) S l_z] J | U_{ji} | \phi_i \rangle$

Solution of AGS equations

$$U_{11}|\phi_1\rangle = G_0^{-1} P_{34} P_1 |\phi_1\rangle + P_{34} u_1 G_0 t G_0 U_{11} |\phi_1\rangle + u_2 G_0 t G_0 U_{21} |\phi_1\rangle$$

subsystem bound state poles at $E - k_z^2/2\mu_j \to -b_j^n$

$$G_0 u_j G_0 \to \frac{P_j |\phi_j^n\rangle s_{jj} \langle\phi_j^n| P_j}{E - k_z^2/2\mu + i0 + b_j^n}$$

Solution of AGS equations

$$U_{11}|\phi_1\rangle = G_0^{-1}P_{34}P_1|\phi_1\rangle + P_{34}u_1G_0\,t\,G_0\,U_{11}|\phi_1\rangle + u_2G_0\,t\,G_0\,U_{21}|\phi_1\rangle$$

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$$\sum_n \int_{p_n}^{q_n} k_z^2\,dk_z \frac{F_n(k_z)}{k_n^2 - k_z^2 + i0} = \sum_n \left\{ \mathcal{P}\int_{p_n}^{q_n} k_z^2\,dk_z \frac{F_n(k_z)}{k_n^2 - k_z^2} - \frac{1}{2} i\pi k_n F_n(k_n) \right\}$$

$$= \sum_n \left\{ \int_{p_n}^{q_n} k_z^2\,dk_z \frac{F_n(k_z) - F_n(k_n)}{k_n^2 - k_z^2} - \frac{1}{2} k_n F_n(k_n) \left[i\pi + \ln \frac{(k_n + p_n)(q_n - k_n)}{(k_n - p_n)(k_n + q_n)} \right] \right\}$$

Solution of AGS equations

$$U_{11}|\phi_1\rangle = G_0^{-1} P_{34} P_1 |\phi_1\rangle + P_{34} u_1 G_0 t G_0 U_{11} |\phi_1\rangle + u_2 G_0 t G_0 U_{21} |\phi_1\rangle$$

subsystem bound state poles at $E - k_z^2/2\mu_j \to -b_j^n$

$$G_0 u_j G_0 \to \frac{P_j |\phi_j^n\rangle s_{jj} \langle \phi_j^n | P_j}{E - k_z^2/2\mu + i0 + b_j^n}$$

$$\sum_n \int_{p_n}^{q_n} k_z^2 \, dk_z \frac{F_n(k_z)}{k_n^2 - k_z^2 + i0} = \sum_n \left\{ \mathcal{P} \int_{p_n}^{q_n} k_z^2 \, dk_z \frac{F_n(k_z)}{k_n^2 - k_z^2} - \frac{1}{2} i\pi k_n F_n(k_n) \right\}$$

$$= \sum_n \left\{ \int_{p_n}^{q_n} k_z^2 \, dk_z \frac{F_n(k_z) - F_n(k_n)}{k_n^2 - k_z^2} - \frac{1}{2} k_n F_n(k_n) \left[i\pi + \ln \frac{(k_n + p_n)(q_n - k_n)}{(k_n - p_n)(k_n + q_n)} \right] \right\}$$

real equations below three-cluster breakup threshold by

$u_j \to \mathcal{P} u_j$ and $U_{ji} \to K_{ji}$ (K-matrix)

Separable potential

- universal — independent of short-range details

$$v = |g\rangle\,\lambda\,\langle g|\;\delta_{l_x 0}$$

$$t = |g\rangle\,\tau\,\langle g|\;\delta_{l_x 0}$$

$$\langle k_x|g\rangle = [1 + c_2(k_x/\Lambda)^2]e^{-(k_x/\Lambda)^2}$$

- rank 1: at most 1 (shallow) dimer
- small system of 2-variable integral equations for $_j\langle g k_y k_z[(l_x l_y)S l_z]J|G_0 K_{ji}|\phi_i\rangle$:

$$\begin{aligned}\langle g|G_0K_{11}|\phi_1\rangle = {} & P_{34}\langle g|P_1|\phi_1\rangle \\ & + P_{34}\mathcal{P}\,\langle g|G_0u_1G_0|g\rangle\tau\langle g|G_0K_{11}|\phi_1\rangle \\ & + \mathcal{P}\,\langle g|G_0u_2G_0|g\rangle\tau\langle g|G_0K_{21}|\phi_1\rangle\end{aligned}$$

Separable potential

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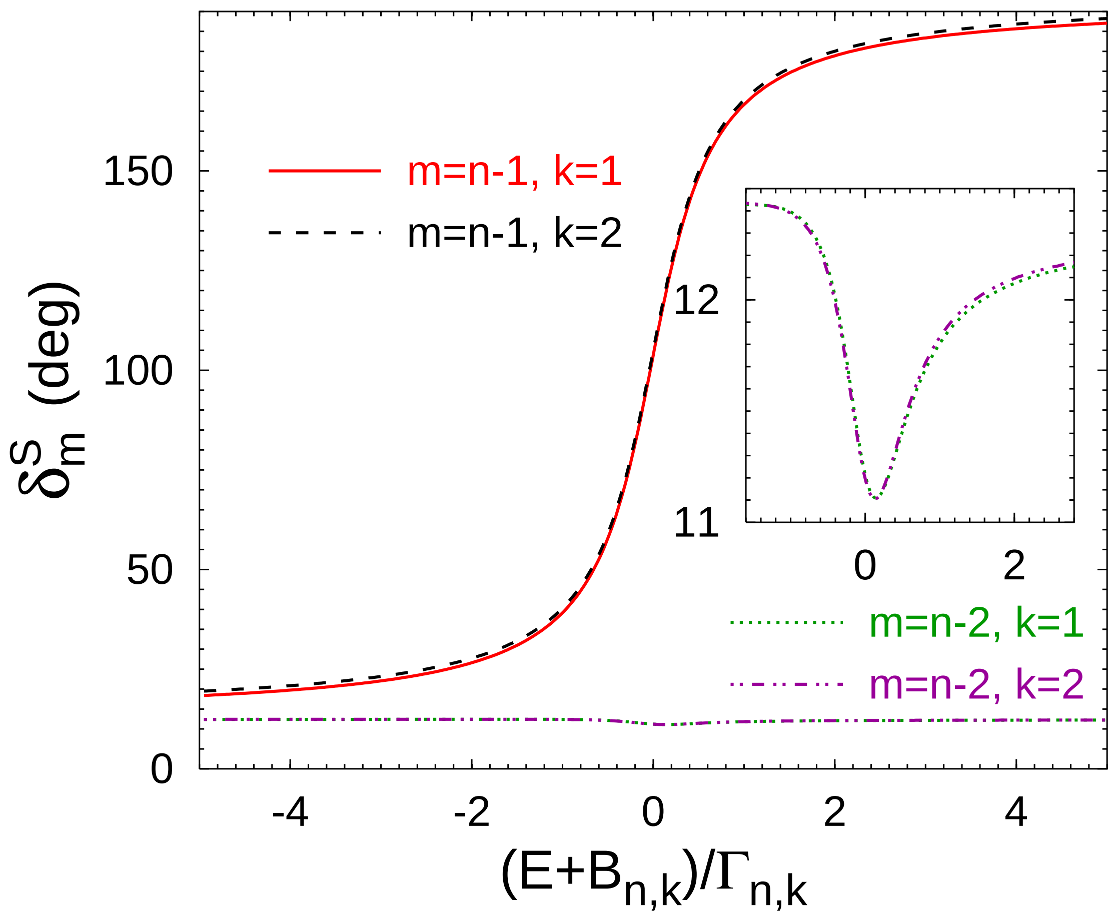
$l_x = 0$ but $l_y, l_z \leq 3$ needed for convergence!

Efimov ratio b_n/b_{n-1} for trimers $(a \to \infty)$

$$\langle k_x|g\rangle = [1 + c_2(k_x/\Lambda)^2]e^{-(k_x/\Lambda)^2}$$

n	I. $c_2 = 0$	II. $c_2 = -9.17$
1	548.114	2126.360
2	515.214	518.570
3	515.036	515.042
4	515.035	515.035
5	515.035	515.035

Unstable tetramers in atom-trimer scattering



$$U_{ji} \approx \frac{\hat{U}_{ji}^{(-1)}}{E - E_T} + \hat{U}_{ji}^{(0)} + \hat{U}_{ji}^{(1)}(E - E_T) + \ldots$$

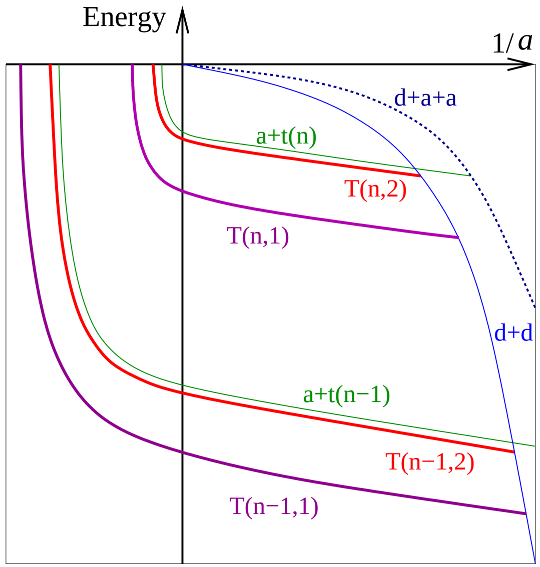
$$E_T = -B_{n,k} - i\Gamma_{n,k}/2$$

Unstable tetramers $(a \to \infty)$

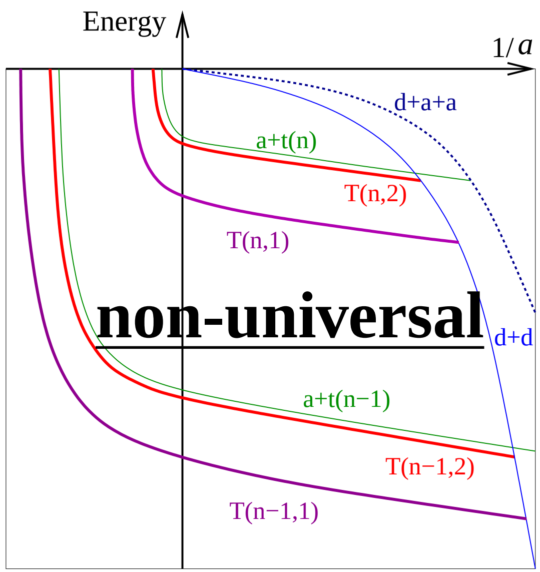
n	$B_{n,1}/b_n$	$\Gamma_{n,1}/2b_n$	$B_{n,2}/b_n$	$\Gamma_{n,2}/2b_n$
I. 0	5.6402		1.04185	
1	4.5169	0.03363	1.00105	3.82×10^{-4}
2	4.6035	0.01366	1.00216	2.14×10^{-4}
3	4.6098	0.01471	1.00226	2.36×10^{-4}
4	4.6102	0.01484	1.00227	2.38×10^{-4}
5	4.6102	0.01483	1.00227	2.38×10^{-4}
II. 0	3.2192			
1	4.9923	0.01360	1.00996	4.18×10^{-4}
2	4.6108	0.02084	1.00227	3.34×10^{-4}
3	4.6098	0.01493	1.00226	2.39×10^{-4}
4	4.6102	0.01483	1.00227	2.38×10^{-4}
5	4.6102	0.01483	1.00227	2.38×10^{-4}
[UC, $n \leq 1$]	4.58		1.01	

[UC: J. von Stecher et al., Nature Phys. 5, 417 (2009)]

Unstable tetramers



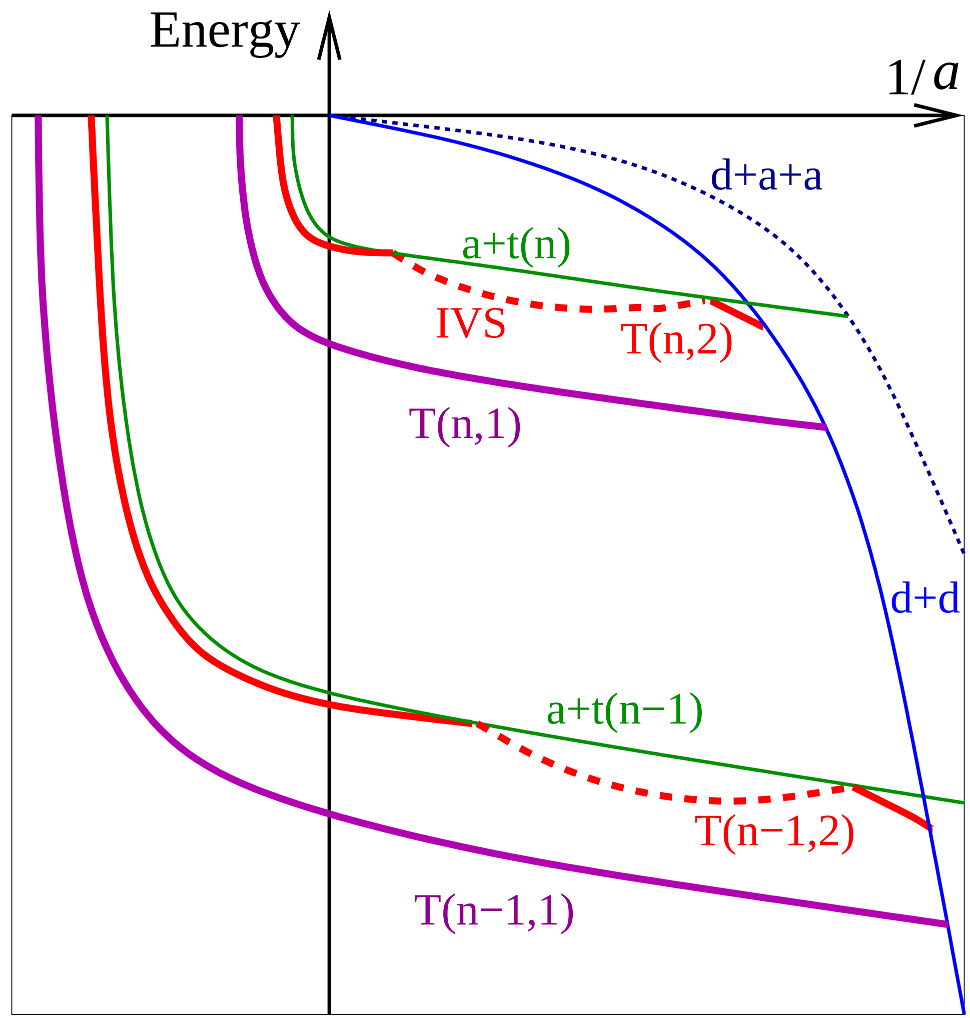
Unstable tetramers



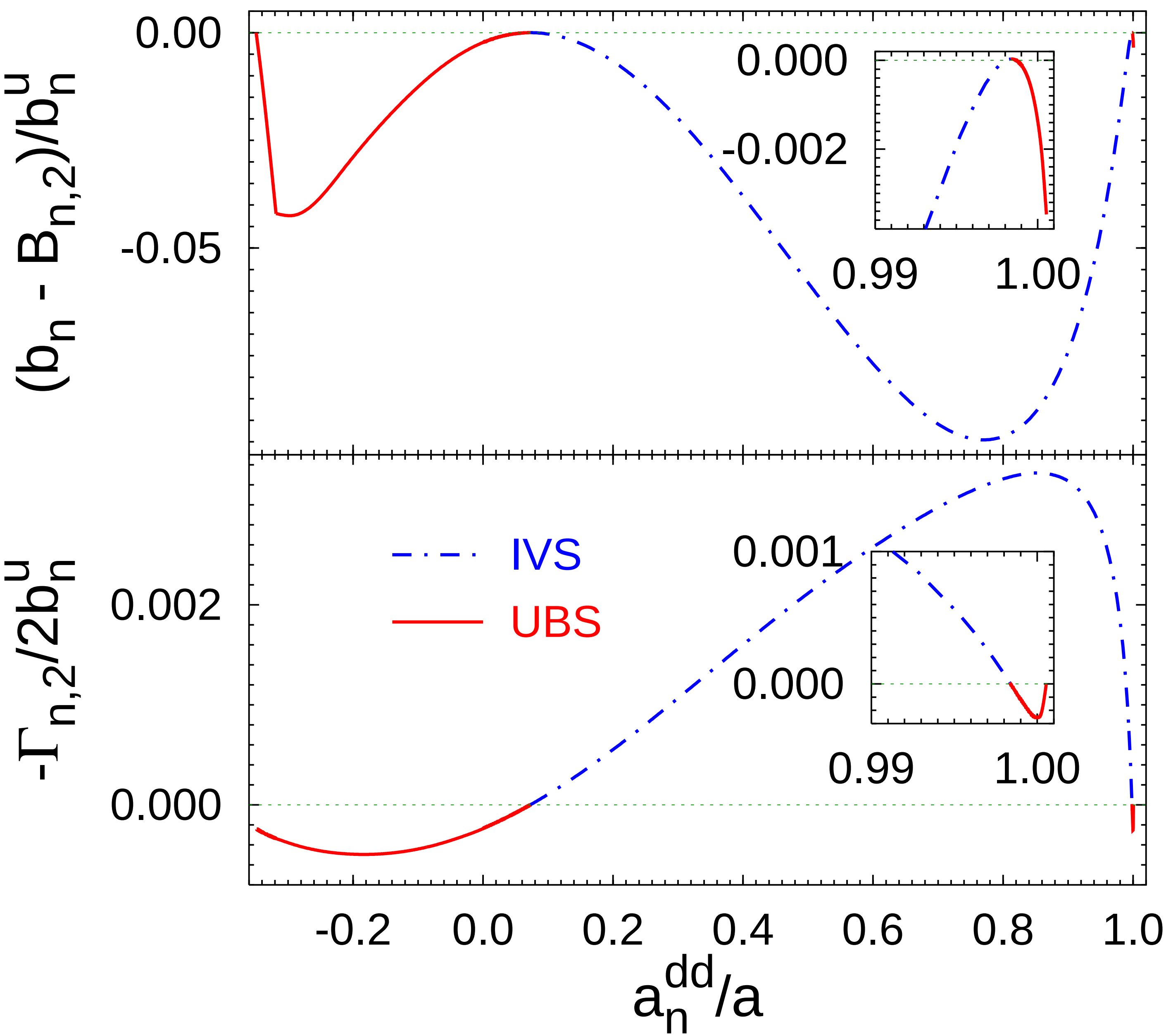
unstable bound state

↑↓

inelastic virtual state



Tetramer: UBS $\rightleftharpoons$ IVS



$$b_n^u = b_n|_{a\to\infty}$$

$$a_n^{dd} : b_n = 2b_d$$

$$a_n^{v,j} : B_{n,2} \approx b_n$$

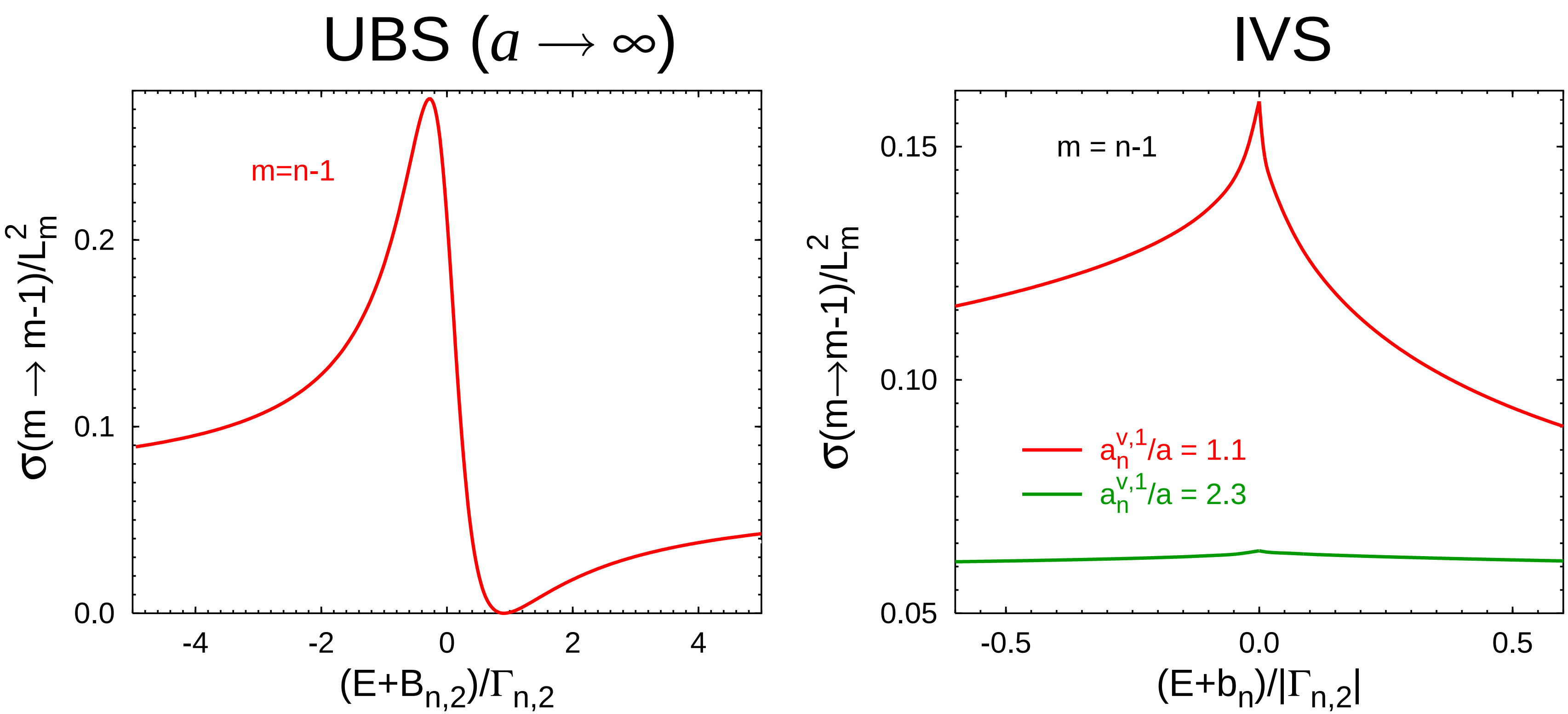
$$\Gamma_{n,2} = 0$$

$$a_n^{dd}/a_n^{v,1} = 0.0729$$

$$a_n^{dd}/a_n^{v,2} = 0.9984$$

(EPL 95, 43002)

Atom-trimer scattering: tetramer UBS vs IVS



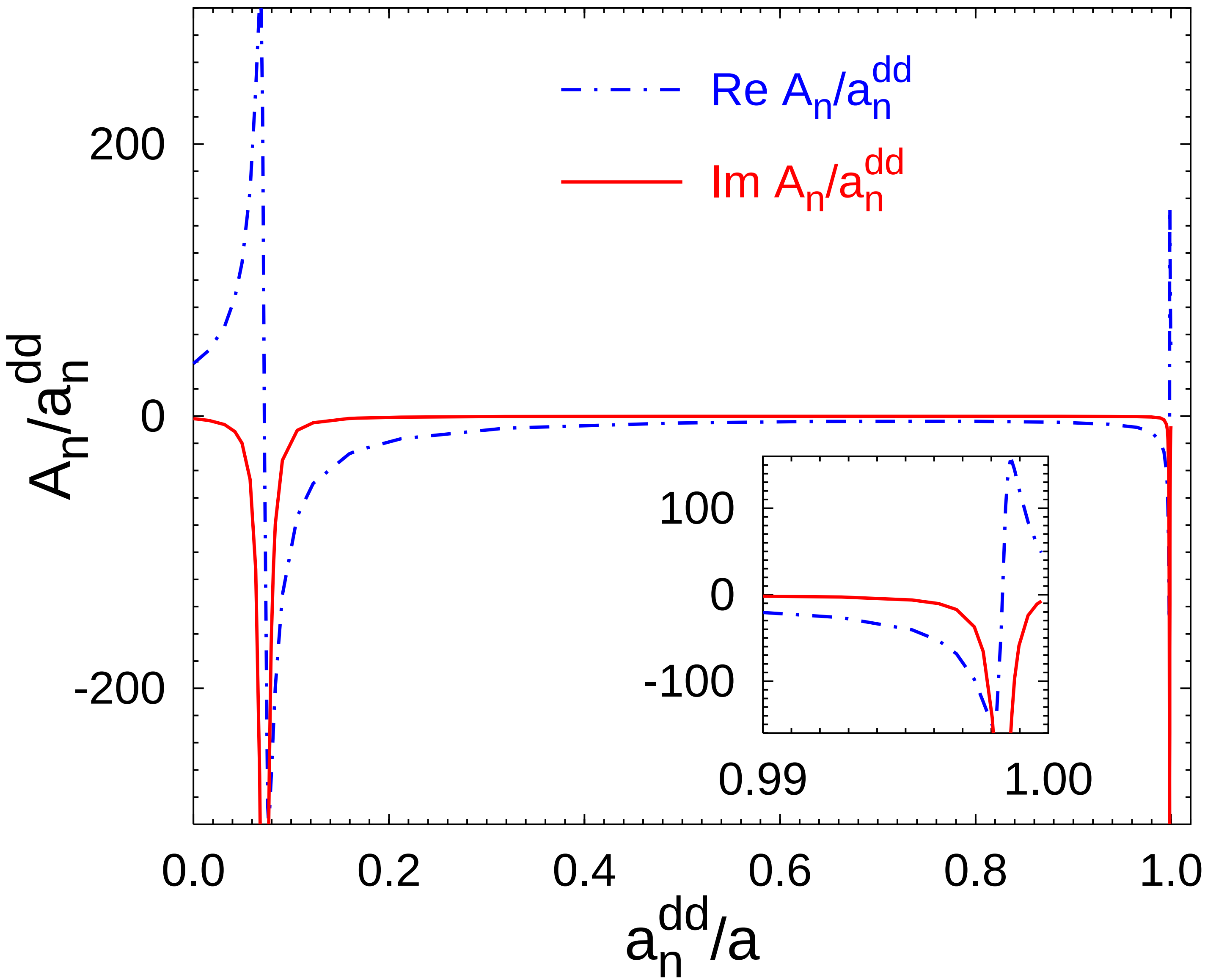
$$\sigma(m \to m') \equiv \sigma[a + t(m) \to a + t(m')], \quad L_m = \frac{\hbar}{\sqrt{2\mu_1 b_m}}$$

IVS properties from effective-range expansion:

$$-1/A_n + \tfrac{1}{2} r_n K_n^2 - iK_n = 0$$

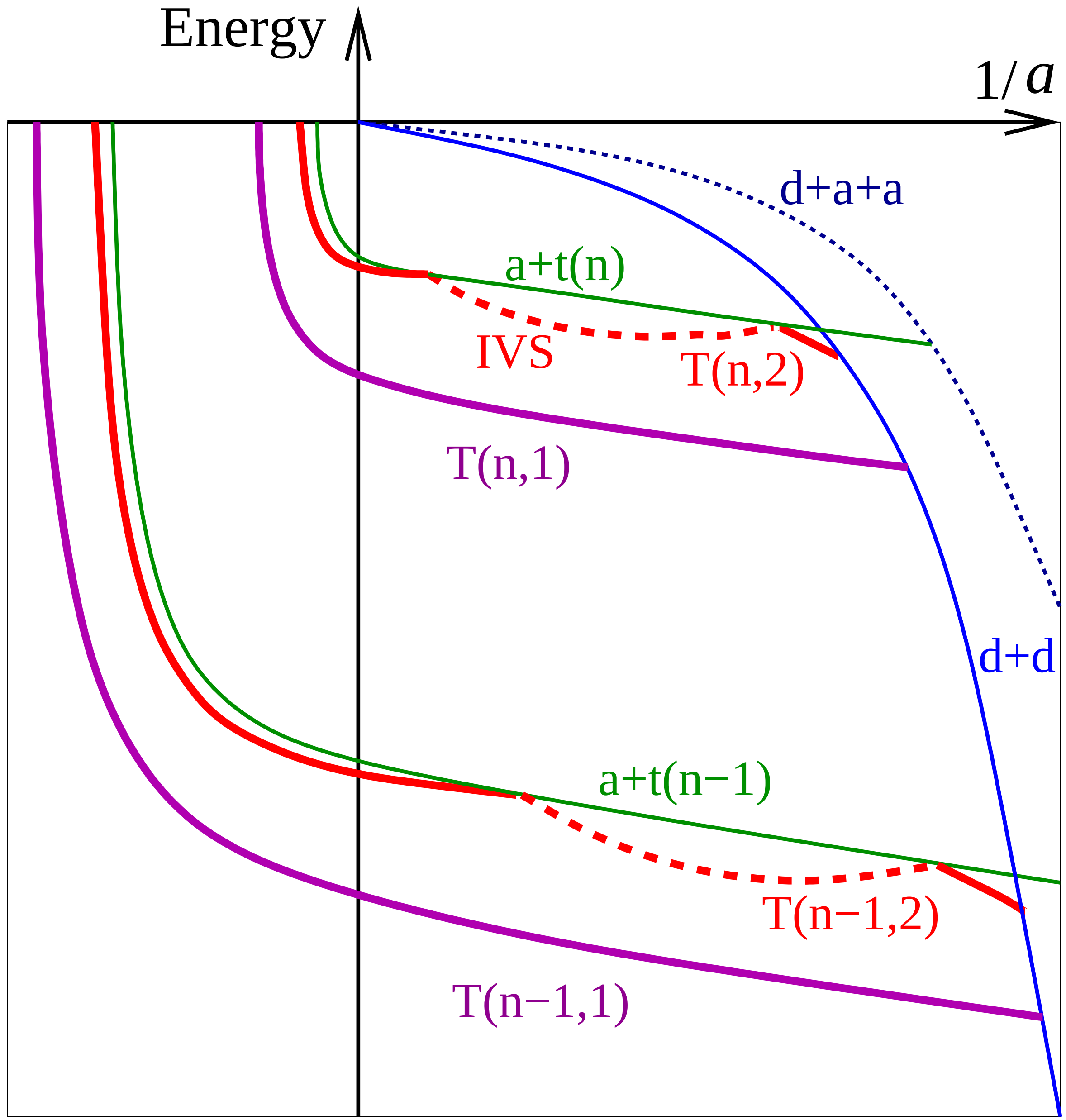
$$-B_{n,k} - i\Gamma_{n,k}/2 = -b_n + K_n^2/2\mu_1$$

Atom-trimer scattering length: UBS $\rightleftharpoons$ IVS

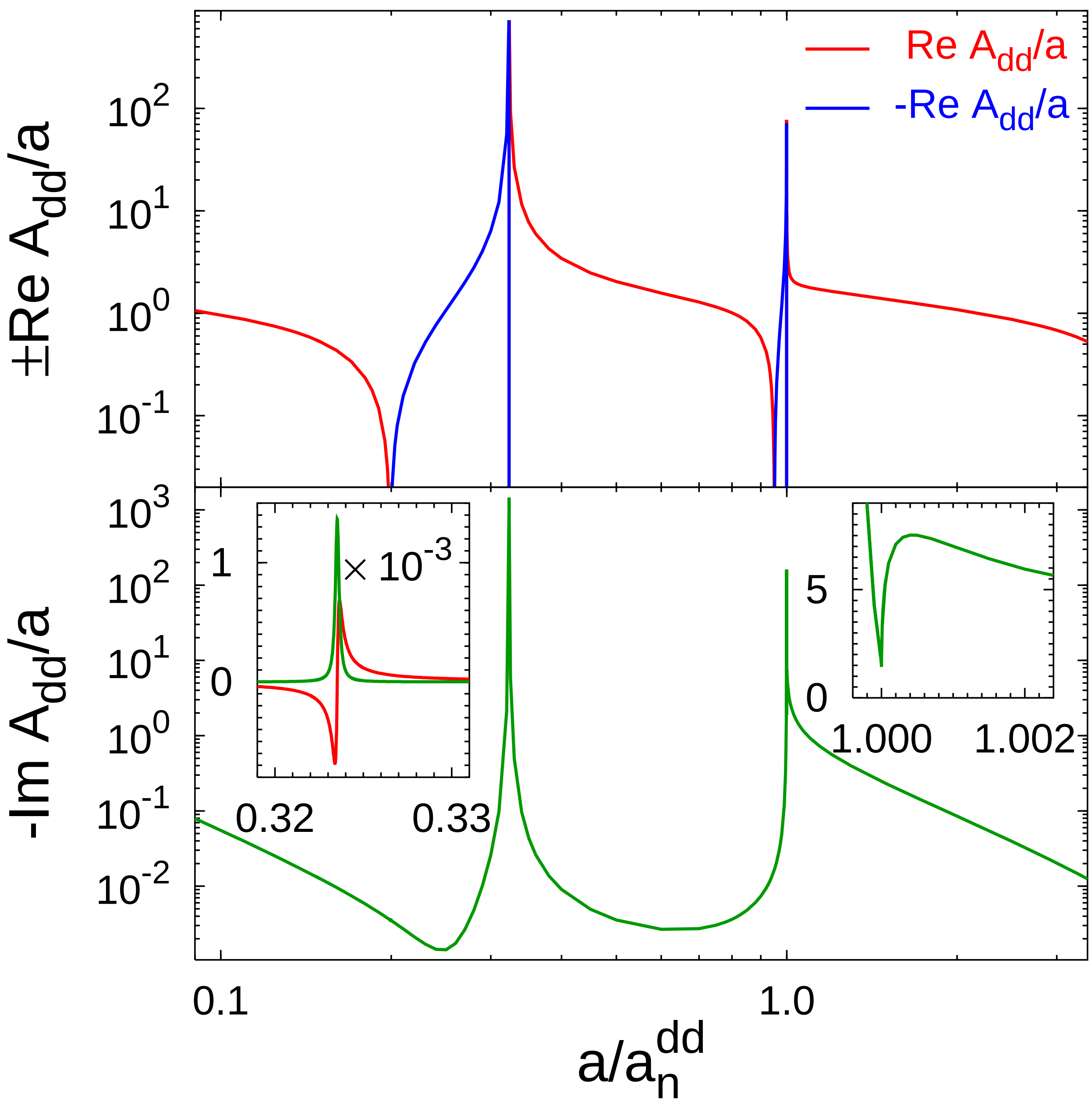


$T = 0$ atom-trimer relaxation: $\beta_n^0 = -(4\pi\hbar/\mu_1)\,\mathrm{Im}\,A_n$

Unstable tetramers



Dimer-dimer scattering length



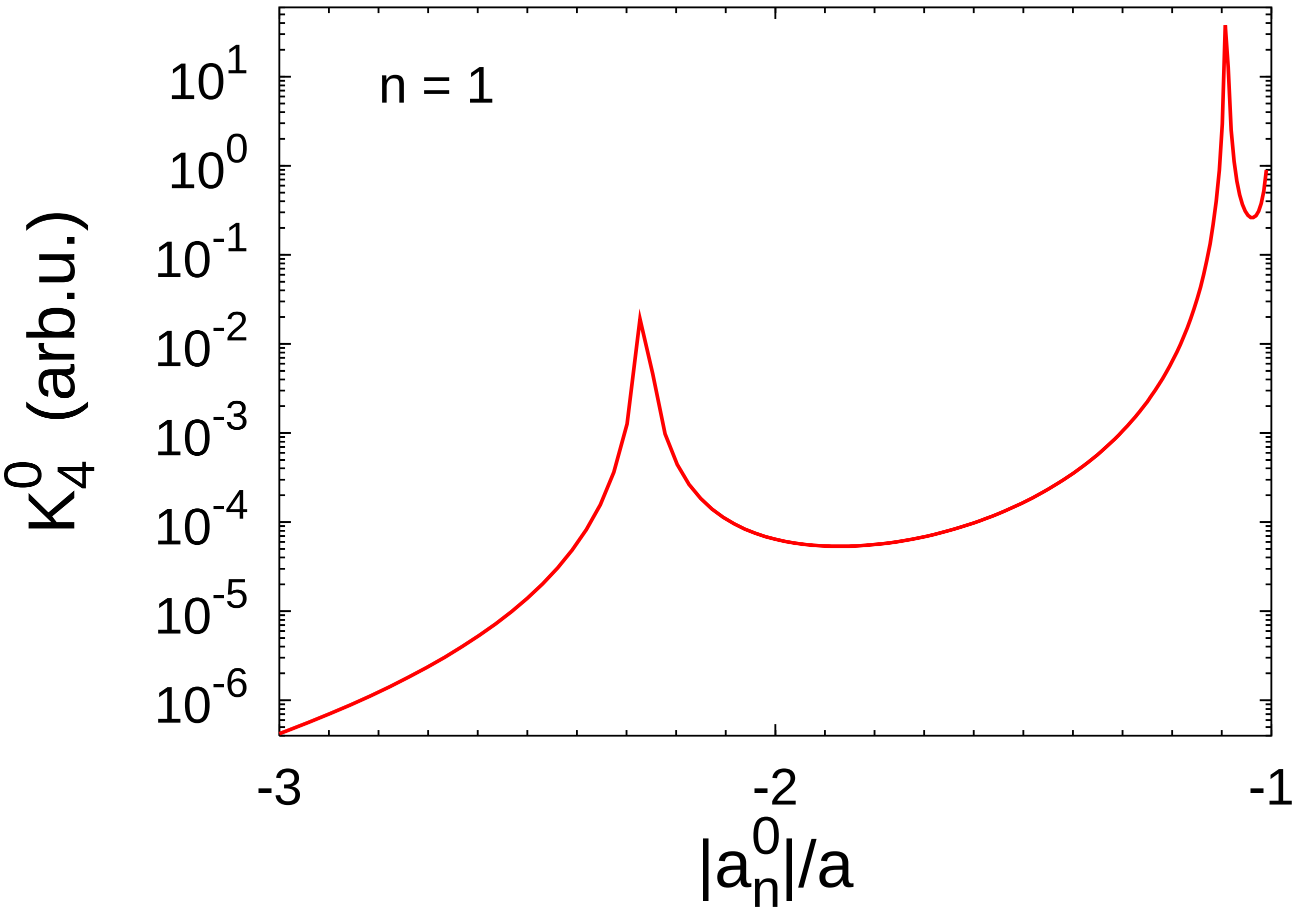
$$a_n^{dd} : b_n = 2b_d$$

$$a_{n,k}^{dd} : B_{n,k} \approx 2b_d$$
$$\Gamma_{n,k} = 0$$

$$a_{n,1}^{dd}/a_n^{dd} = 0.3235$$
$$a_{n,2}^{dd}/a_n^{dd} = 0.9995$$

(PRA 84, 022703)

Four-atom recombination at threshold



$$a_n^0 : b_n = 0$$

$$a_{n,k}^0 : B_{n,k} = 0$$

$$a_{n,1}^0 / a_n^0 = 0.426$$
$$a_{n,2}^0 / a_n^0 = 0.912$$

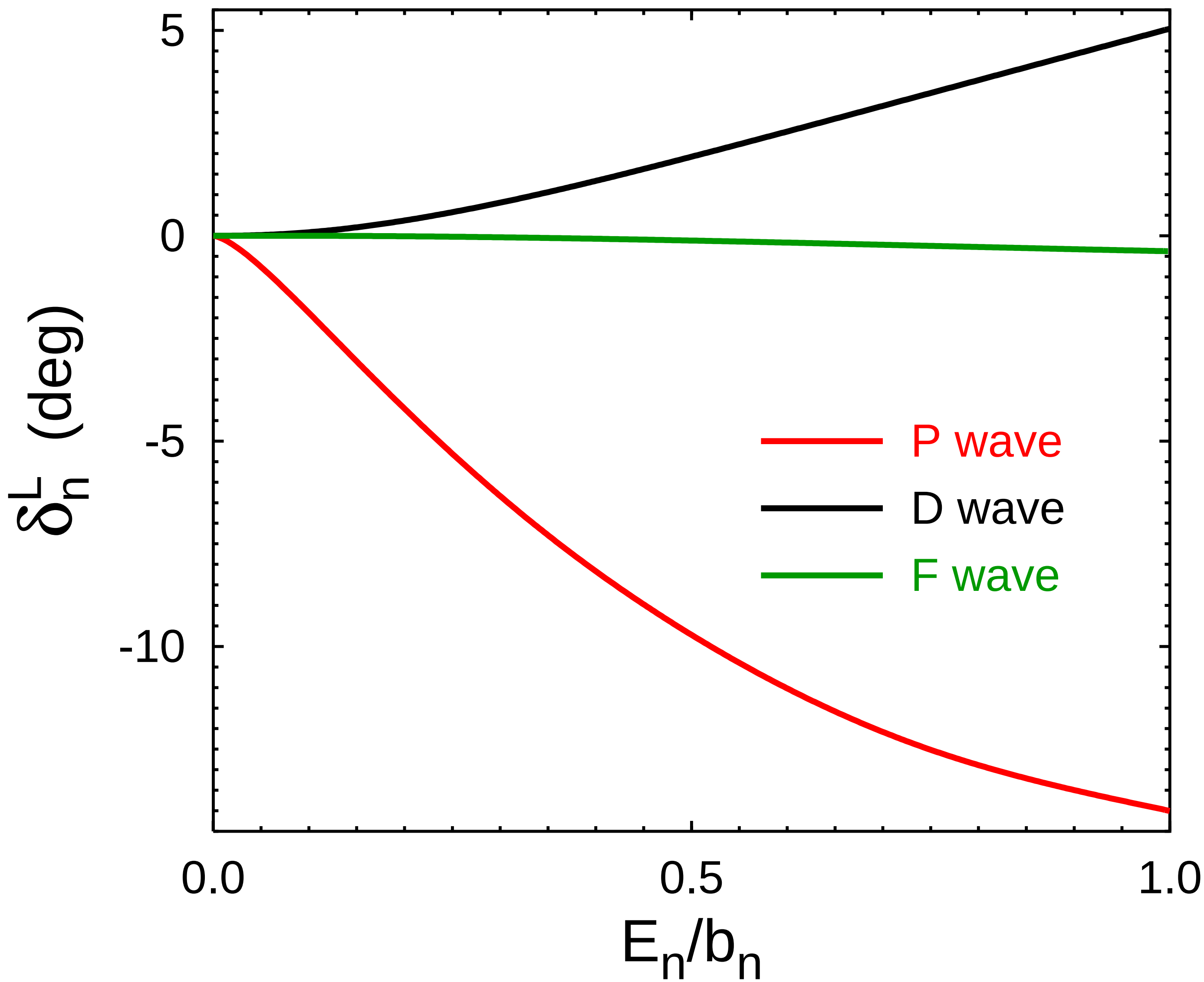
Comparison with other works

	$a^{dd}_{n,1}/a^{dd}_n$	$a^{dd}_{n,2}/a^{dd}_n$	$a^0_{n,1}/a^0_n$	$a^0_{n,2}/a^0_n$	n_{max}
A.D.	0.3235	0.9995	0.426	0.912	4
[UC]	0.352	0.981	0.43	0.90	2

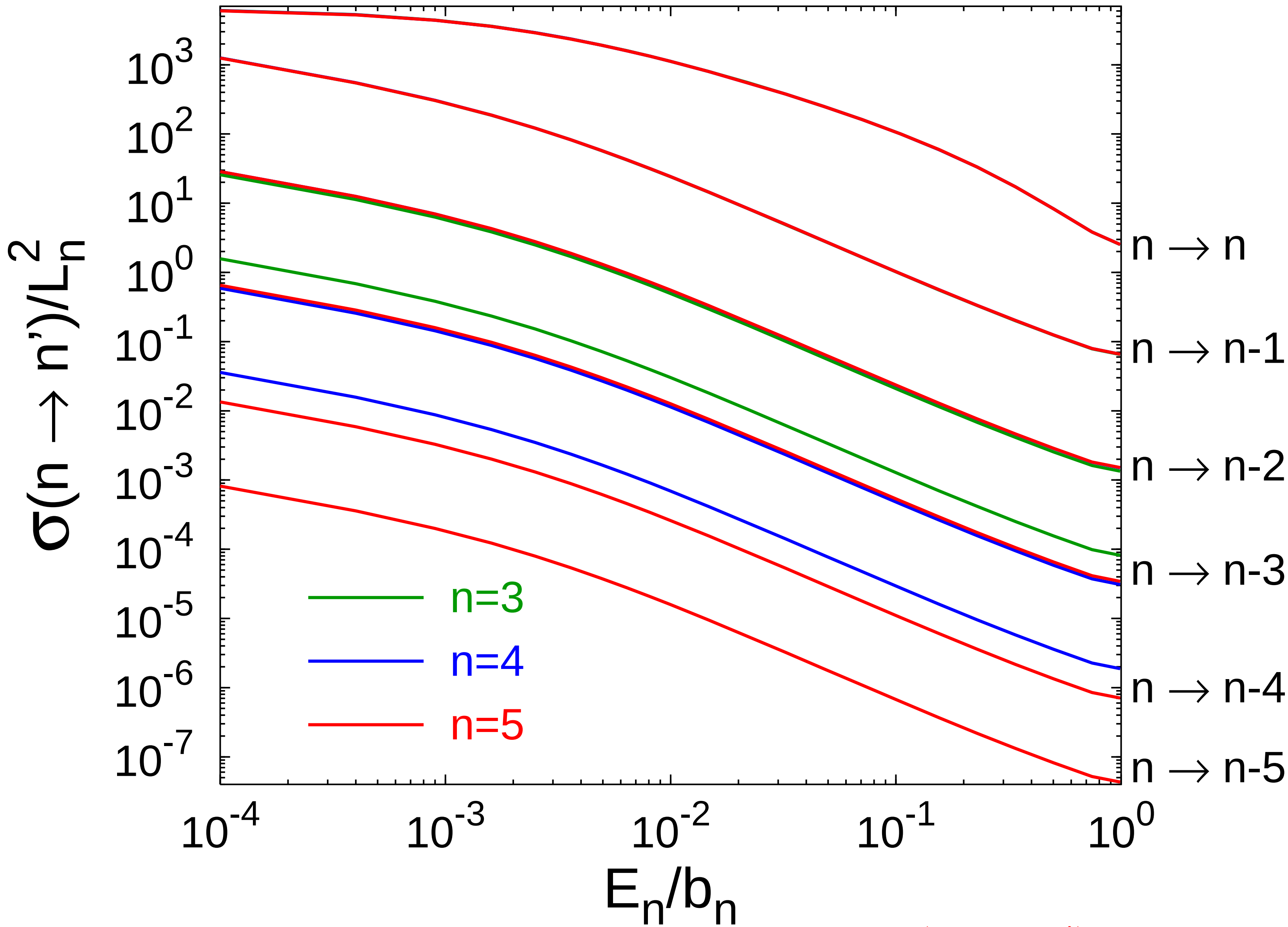
$(1 - a^{dd}_{n,2}/a^{dd}_n)$: 0.0005 vs 0.019

[UC: J. P. D'Incao et al., PRL 103, 033004 (2009),
J. von Stecher et al., Nature Phys. 5, 417 (2009)]

Atom-trimer scattering: $L > 0$ phase shifts $(a \to \infty)$



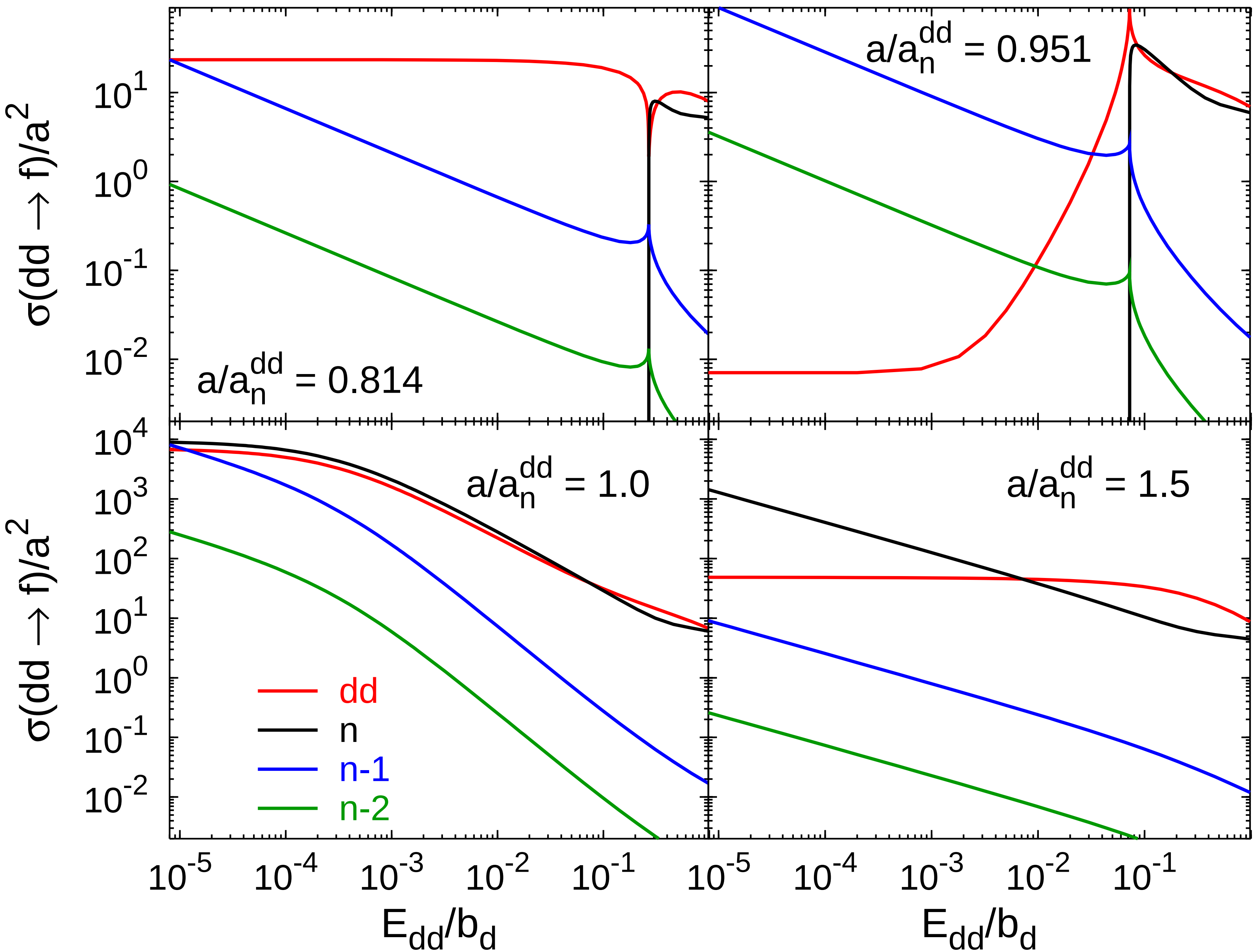
Atom-trimer scattering: cross sections $(a \to \infty)$



$$\frac{\sigma(n \to n')}{\sigma(n \to n'\text{-}1)} \approx 43.7$$

[PRA 82, 040701 (2010)]

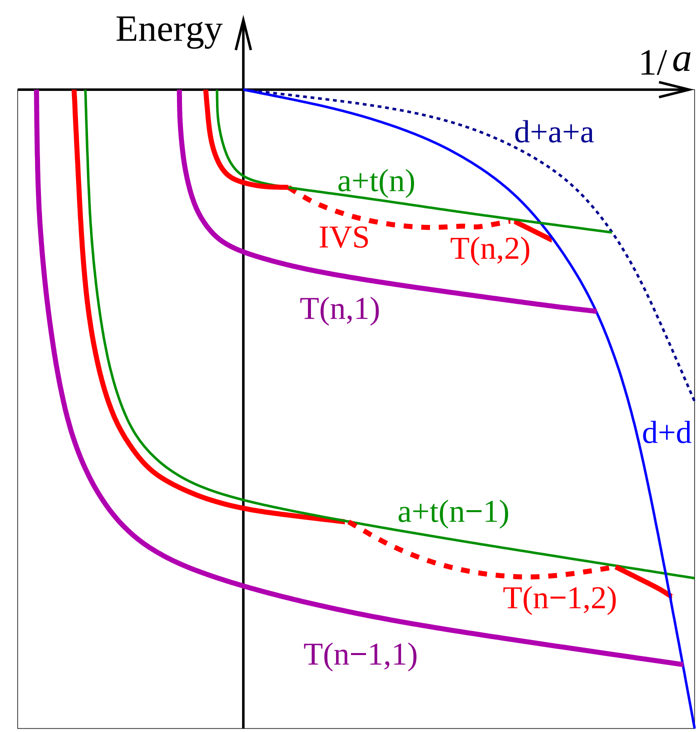
Dimer-dimer scattering: cross sections



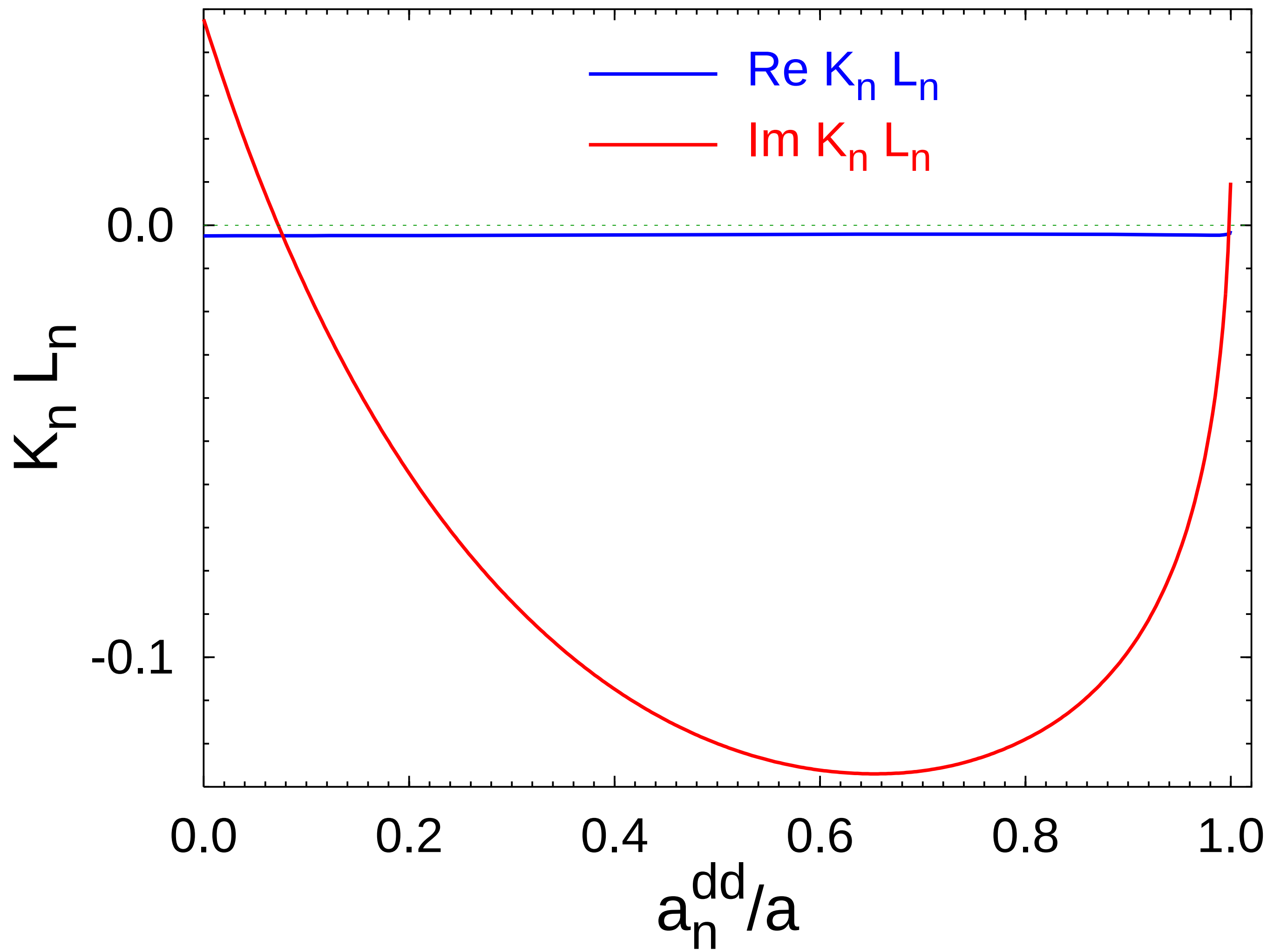
[PRA 84, 022703]

Summary

- four-boson AGS scattering equations in momentum space
- atom-trimer and dimer-dimer scattering
- properties of unstable tetramers: universal limit with high accuracy
- four-atom recombination
- future work: inclusion of deep dimers



Tetramer: UBS $\rightleftharpoons$ IVS



$$-1/A_n + \frac{1}{2} r_n K_n^2 - iK_n = 0$$

$$-B_{n,k} - i\Gamma_{n,k}/2 = -b_n + K_n^2/2\mu_1$$