

Fermionic Efimov States & A New Emerging Universality

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Collaborators
Theory

Pascal Naidon, Emiko Hiyama (RIKEN)

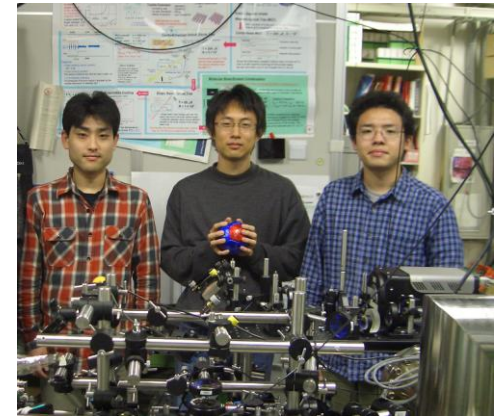


Experiment

Shuta Nakajima (Kyoto U.)

Munekazu Horikoshi (U. Tokyo)

Takashi Mukaiyama (U. Electro-Comm.)



P. Naidon, MU, PRL **103**, 073203 (2009); *Compt. Rend. Phys.* **12**, 13 (2011)
S. Nakajima, et al., PRL **105**, 023201 (2010); *ibid.* **106**, 143201 (2011)
P. Naidon, E. Hiyama, and M. U., arXiv: 1109.5807

What's the Efimov Effect?

V. Efimov, Phys. Lett. **33B**, 563 (1970)

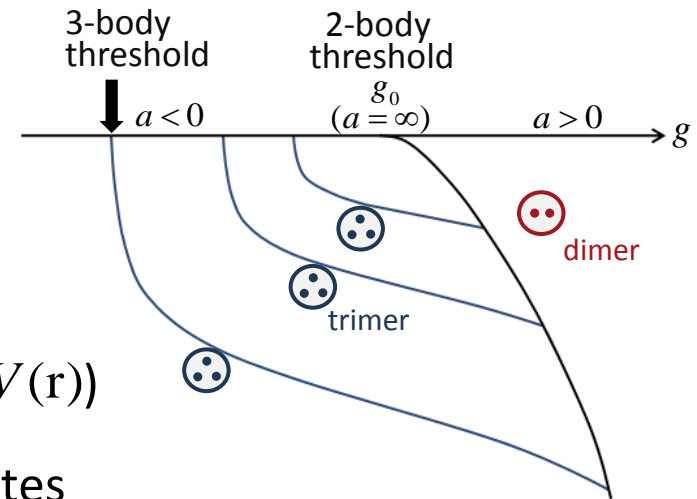
Sov. J. Nucl. Phys. **12**, 589 (1971)

3 spinless neutral particles, interacting through a potential

$$gV(\mathbf{r}), \quad V(\mathbf{r}) < 0$$

Near a 2-body threshold ($g = g_0$), an effective 3-body force emerges:

- resonant character: $a \gg r_0 = \text{range of forces}$
- universal long-range attraction $-R^{-2}$
(independent of the detailed structure of $V(\mathbf{r})$)
- supports an infinitely many 3-body bound states with discrete scaling symmetry and accumulation point at $g = g_0$



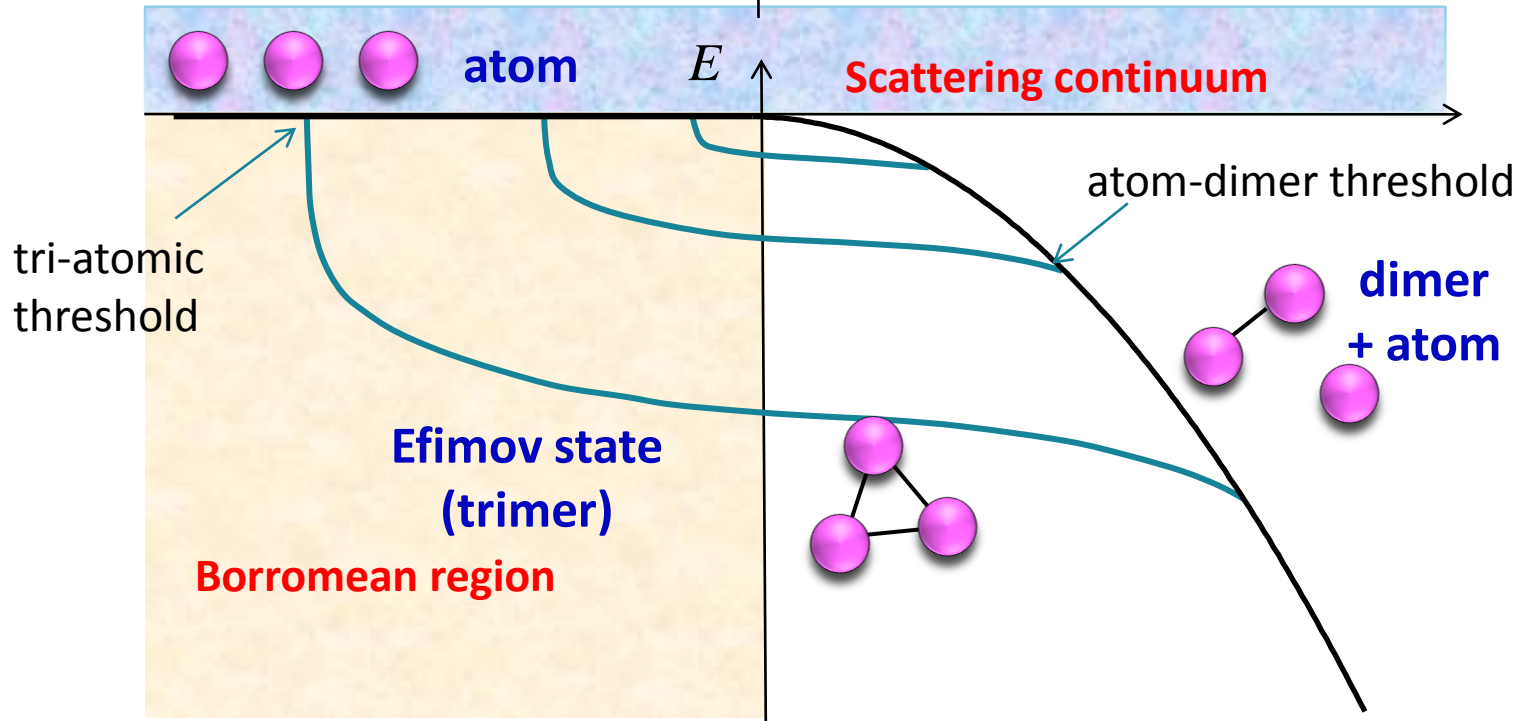
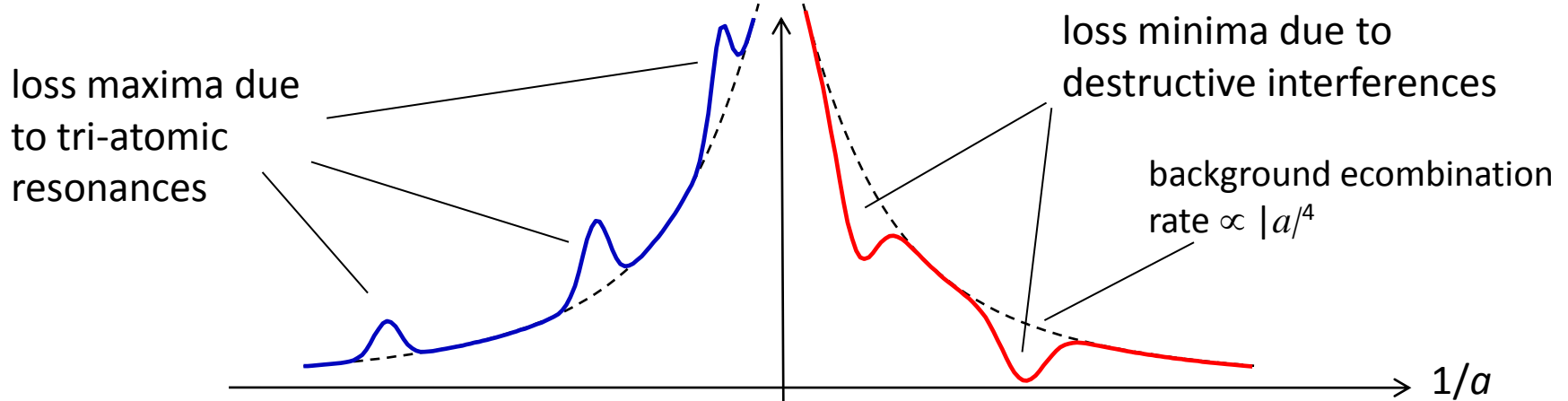
$$\frac{a_{n+1}}{a_n} \approx e^{\pi/s_0} \approx 22.7, \quad s_0 = 1.00624\dots \text{ (universal scaling parameter)}$$

“Matryoshka doll” scaling



Experimental Signatures: Loss Maxima and Minima

3-body loss coefficient



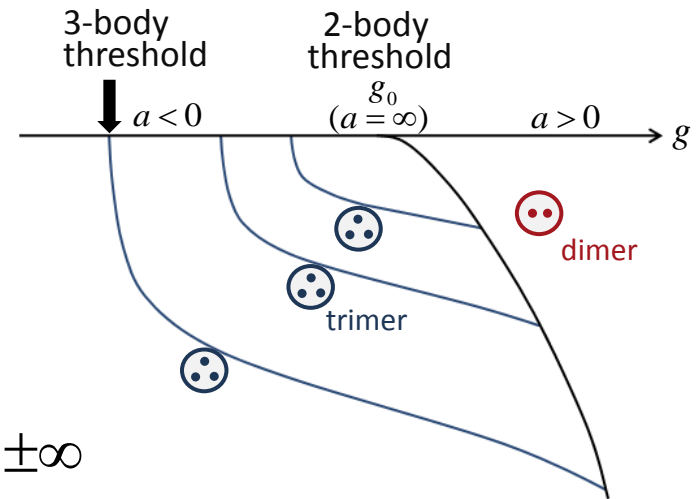
Some Remarks

- The universal attractive potential ($-R^{-2}$) permits **continuous** scale invariance:

$$H = -\nabla_R^2 - R^{-2} \xrightarrow{R \rightarrow \lambda R} \lambda^{-2} H$$

- However, because of the 3-body parameter κ_* only **discrete** scale invariance is permitted:

$$E_n \rightarrow e^{-\frac{2\pi}{s_0}(n-n_*)} \frac{\hbar^2 \kappa_*^2}{M} \text{ as } n \rightarrow \infty \text{ with } a = \pm\infty$$



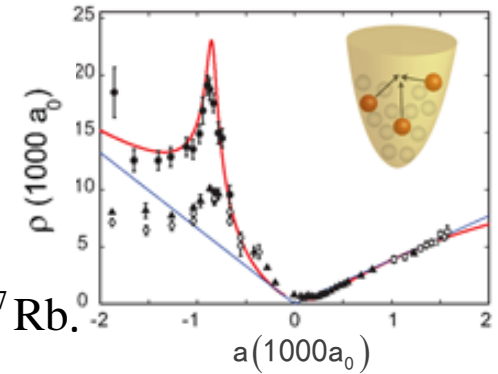
- The 3-body parameter is a free parameter in the Efimov theory.
- In principle, it can take any random value from one FBR to another, let alone from one atomic species to another.
- However, Nature seems to prefer otherwise! ← **main message of this talk**

Some Experimental Achievements in Ultracold Atomic Gases

2006 Efimov trimers identified in ^{133}Cs .

T. Kraemer, et al., *Nature* **440**, 315 (2006)

B. D. Esry, et al., *PRL* **83**, 1751 (1999) [theory]

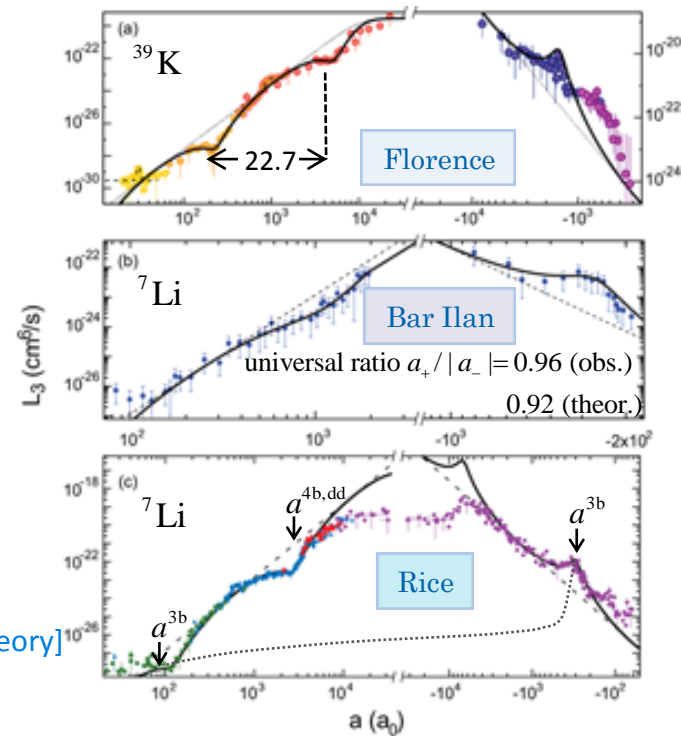


2008 Efimov trimers observed in a mass-imbalance system of ^{41}K & ^{87}Rb .

G. Barontini, et al., *PRL* **103**, 043201 (2009)

2009 Discrete scaling law between loss minima observed.

^{39}K M. Zaccanti, et al., *Nature Phys.* **5**, 586 (2009)



2009-2010 Universality across the Feshbach resonance

^7Li N. Gross, et al., *PRL* **103**, 163202 (2009)

PRL **105**, 103203 (2010)

cf. S. E. Pollack, et al., *Science* **326**, 1683 (2009)

M. Zaccanti, et al., *ibid.*

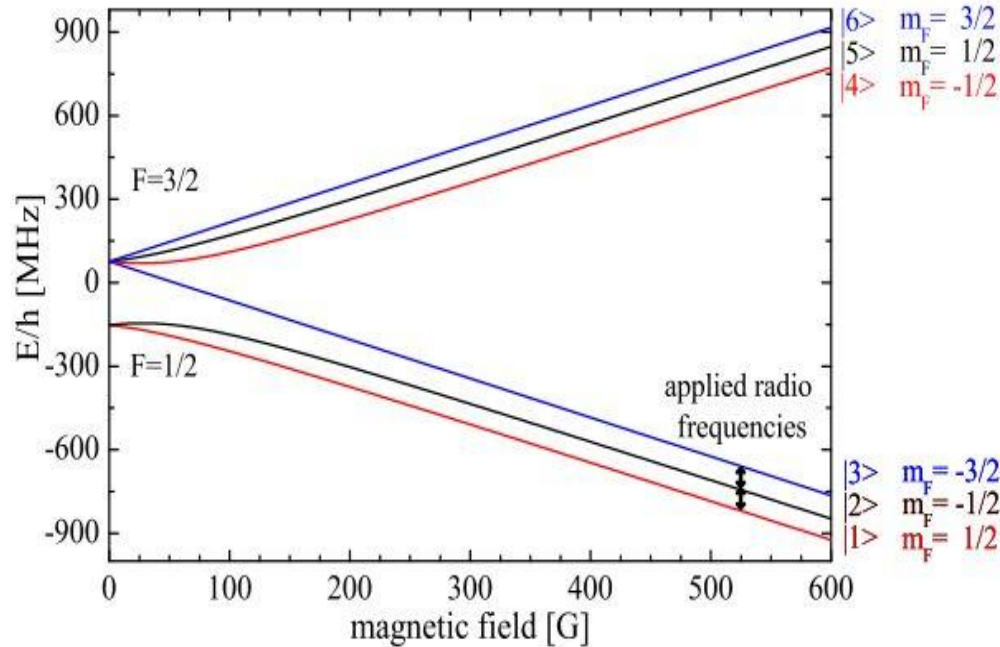
2009 A pair of universal tetramers for each Efimov trimer.

H. Hammer and L. Platter, *Eur. Phys. J. A* **32**, 113 (2007) [theory]

J. Von Stecher, et al., *Nature Physics* **5**, 417 (2009) [theory]

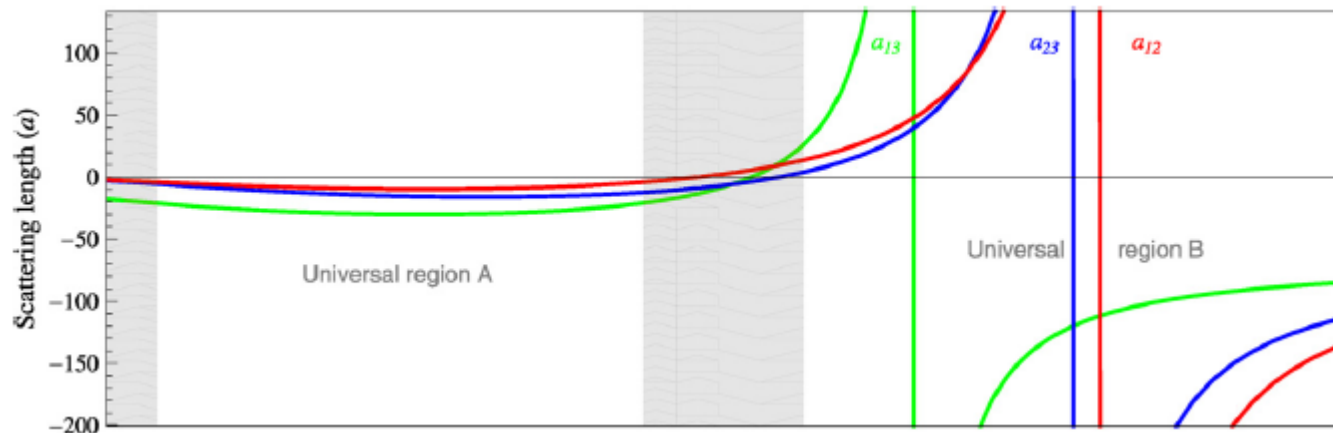
F. Ferlaino, et al., *PRL* **102**, 140401 (2009) [experiment]

Our System: 3-Component Fermi System of ${}^6\text{Li}$



$$\frac{1}{2} \text{ (electronic)} \otimes 1 \text{ (nuclear)} = \frac{1}{2} \oplus \frac{3}{2} \text{ (hyperfine)}$$

- Broad FBR for each pair of states
- Two universal regions (A, B)
- Can test different aspects of Efimov physics:
 - tri-atomic resonance
 - atom-dimer resonance



Enhanced Loss in a 3-Component Fermi System of ${}^6\text{Li}$

T. B. Ottenstein, et al., PRL **101**, 203202 (2008)

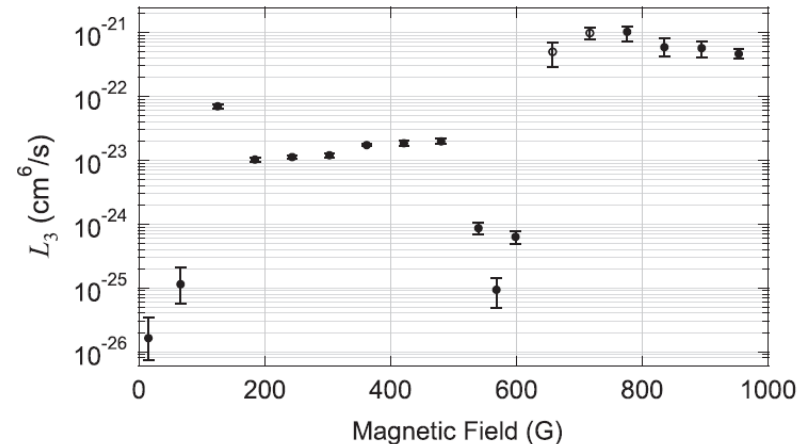
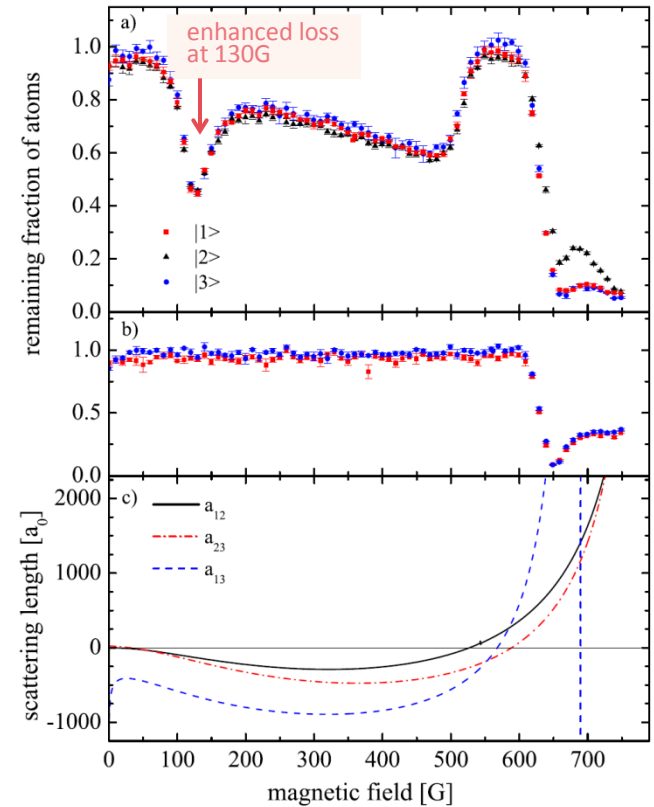
J. H. Huckans, et al., PRL **102**, 165302 (2009)

J.R. Williams, et al., PRL**103**, 130404 (2009)

➤ Stable if each pair of states alone is populated.

➤ Unstable at 130G only if all three states are populated.

→ smoking gun of Efimov states



Efimov Scenario confirmed by theories

effective field theory

E. Braaten, et. al., PRL **103**, 073202 (2009)

hyperspherical analysis

P. Naidon and MU, PRL **103**, 073203 (2009)

functional renormalization group theory

S. Floerchinger, et al., PRA **79**, 053633 (2009)

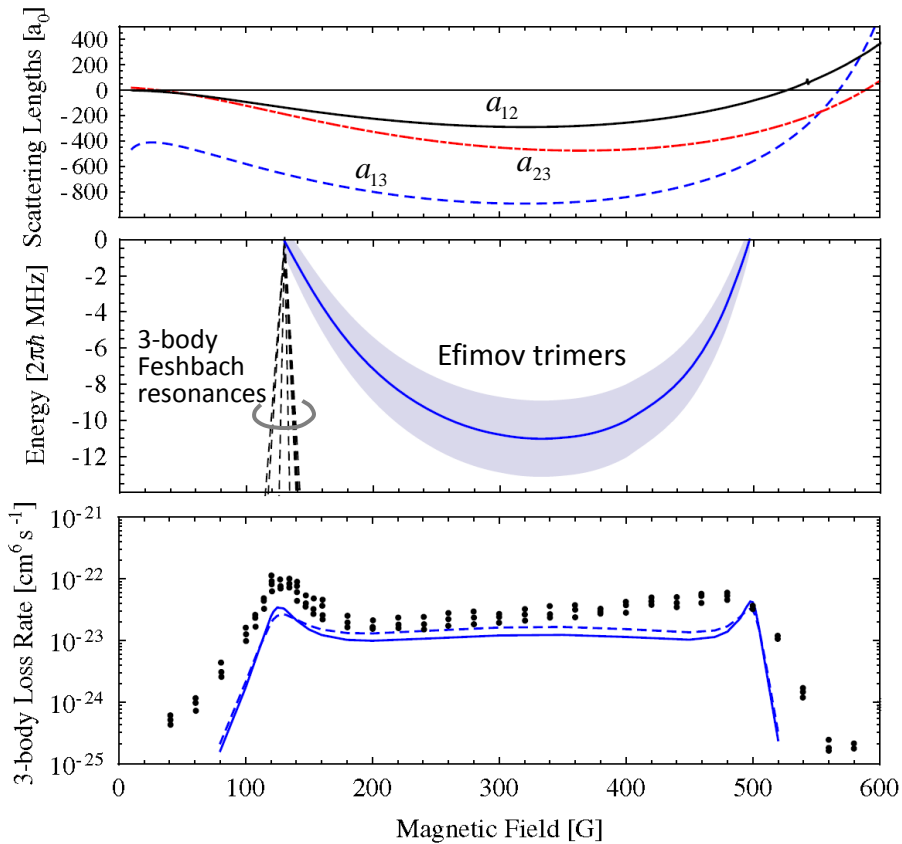
semiclassical approach

Seth T. Rittenhouse, PRA **81**, 040701 (R) (2010)

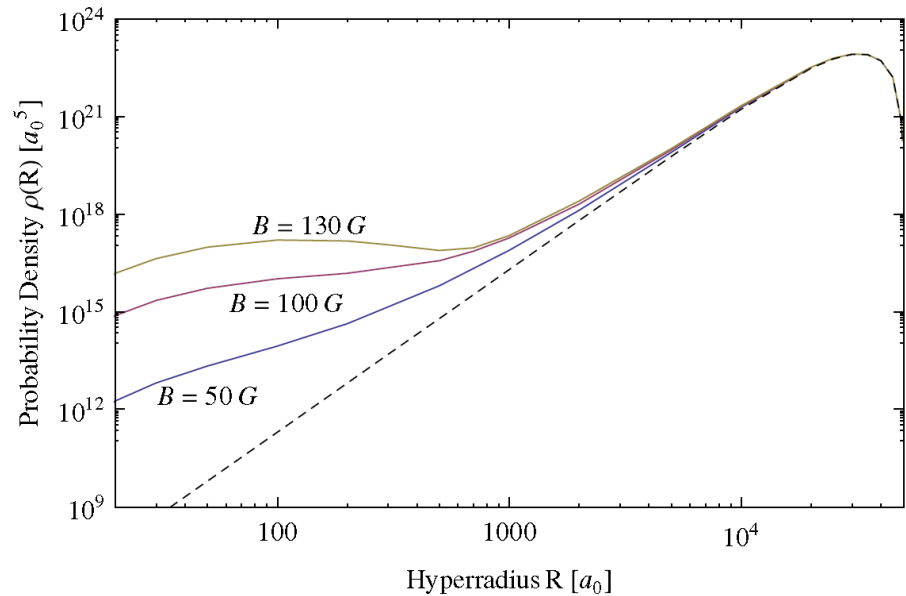
Efimov Trimer vs. Feshbach Scenarios

P. Naidon, MU, PRL **103**, 073203 (2009)

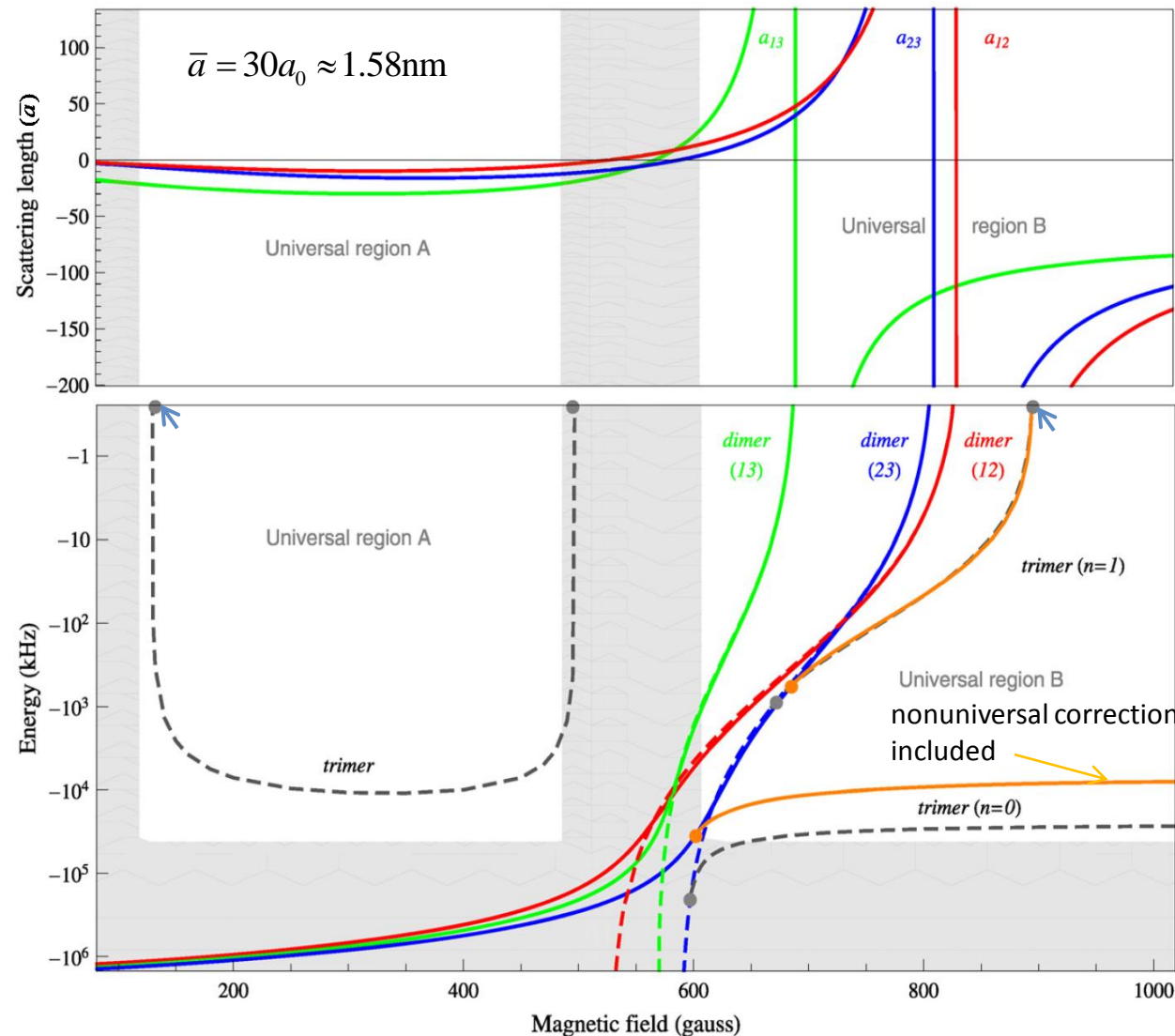
Theory confirms that the loss peak corresponds to a tri-atomic association threshold of Efimov trimers.



Physics: presence of an Efimov state enhances the probability density for 3 atoms to be found at short distance, which leads to an enhanced 3-body recombination.



A Possible Unification?



O'Hara group observed another 3-atom threshold at 895G and determined

$$\kappa_* = 6.9(2) \times 10^{-3} a_0^{-1}$$

This value is very close to

$$\kappa_* = 6.9(2) \times 10^{-3} a_0^{-1}$$

which was obtained in a different universal region A at 100G. [Braaten et al. PRL 103, 073202 (2010)]

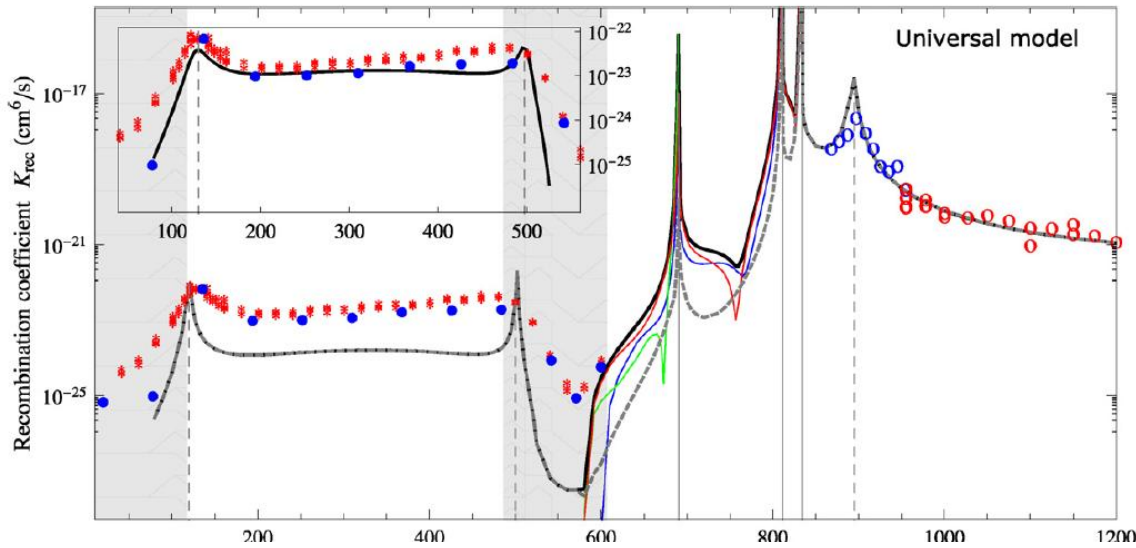


Suggests a possible unification of the two universal regions A and B.

cf. Innsbruck exp. on Cs
M. Berninger, et al., PRL **107**, 120401 (2011)

Single-Channel Model

P. Naidon, MU, C. R. Physique **12**, 13 (2011)



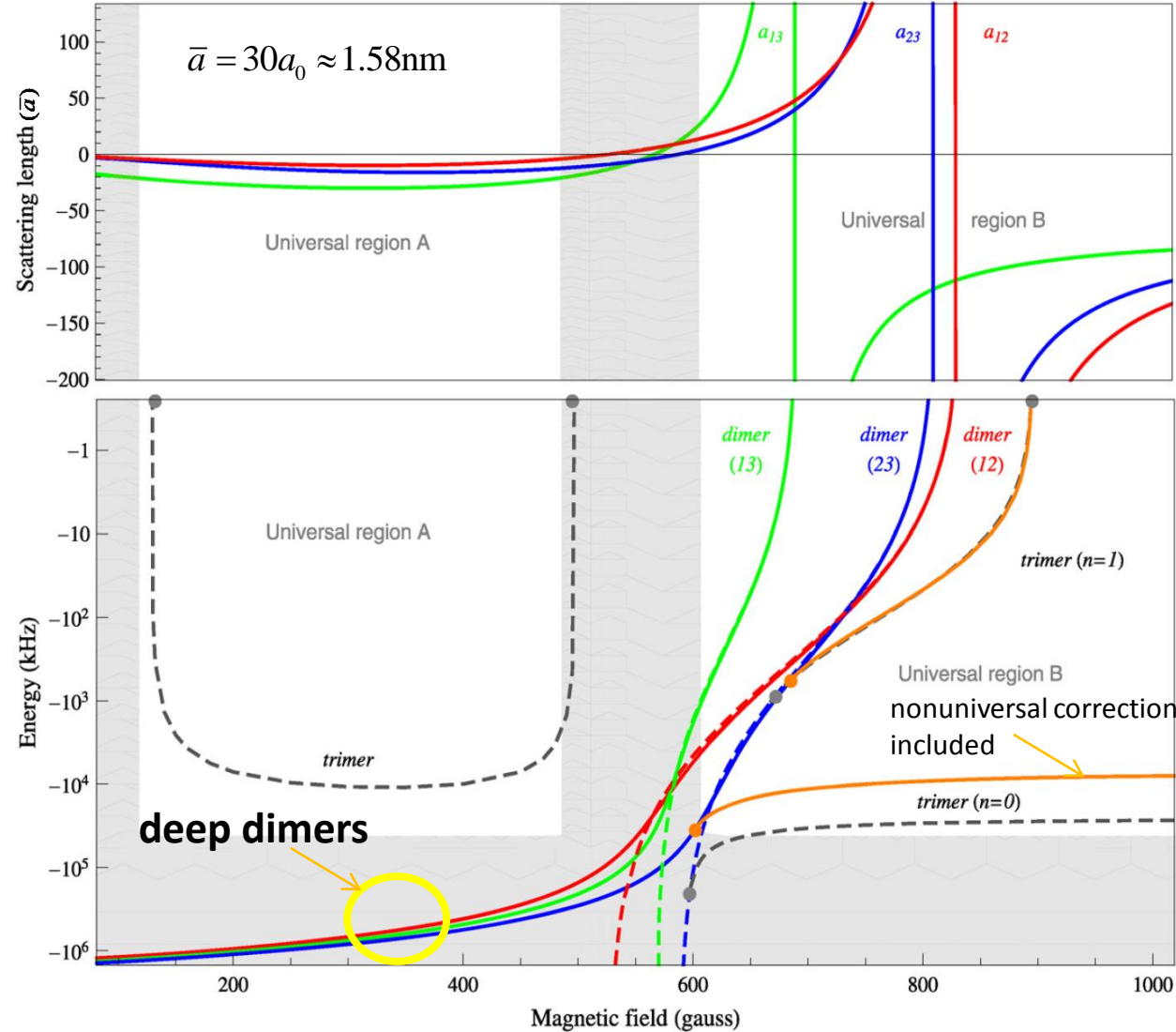
Data taken from

- * J. H. Huckans, et al.,
PRL **102**, 165302 (2009)
- T. B. Ottenstein, et al.,
PRL **101**, 203202 (2008)
- J.R. Williams, et al.,
PRL **103**, 130404 (2009)

- The single-channel model with energy-dependent scattering lengths and a fixed three-body parameter qualitatively reproduces an overall feature of the entire region.
- But not quite good for universal region A.

Why?

Qualitative Deviation from Universal Theory



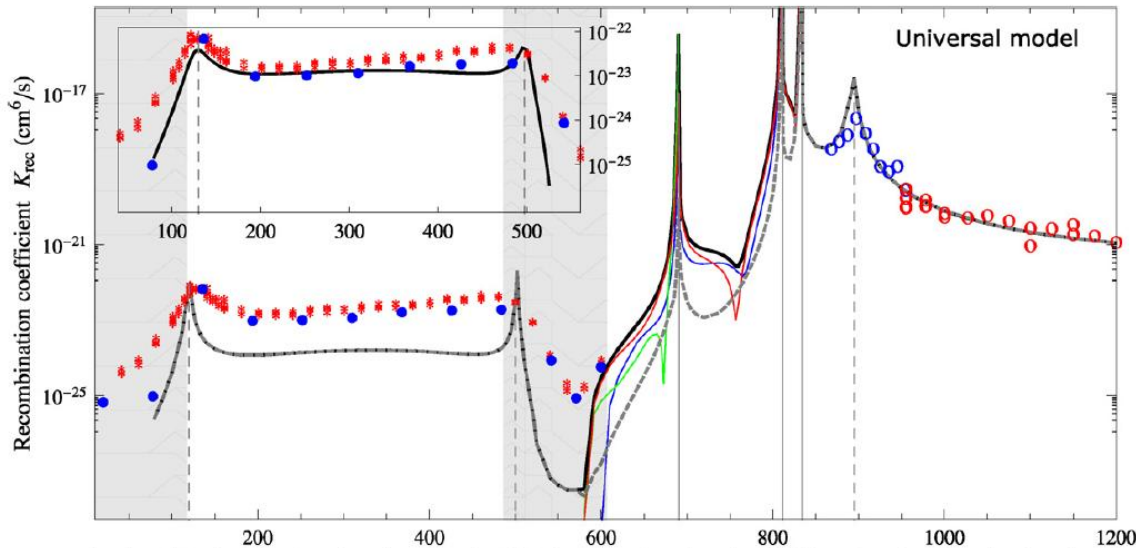
Deep dimers, which are not taken into account in the universal model, introduce effective-range corrections.



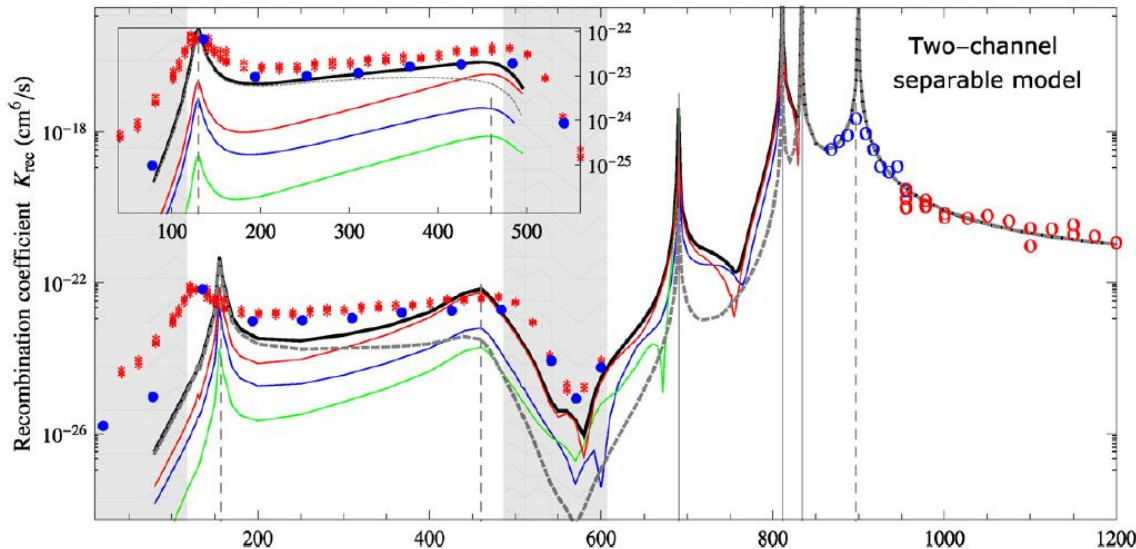
The coupled-channel nature matters!

Two-Channel Model

P. Naidon, MU, C. R. Physique **12**, 13 (2011)

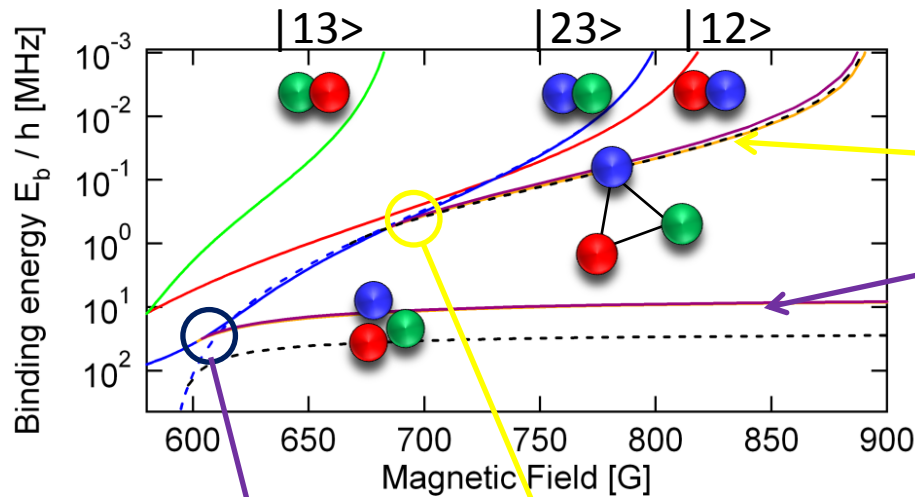


- * J. H. Huckans, et al.,
PRL **102**, 165302 (2009)
- T. B. Ottenstein, et al.,
PRL **101**, 203202 (2008)
- J.R. Williams, et al.,
PRL **103**, 130404 (2009)



- Two-channel model captures a global feature in the entire region.
- Zero-energy Efimov physics well described by a two-channel model with energy-dependent scattering length and finite-range corrections.

Atom-Dimer-Efimov Resonance



S. Nakajima *et al.*, PRL **105**, 023201 (2010)
 T. Lompe, *et al.*, PRL **105**, 103201 (2010)

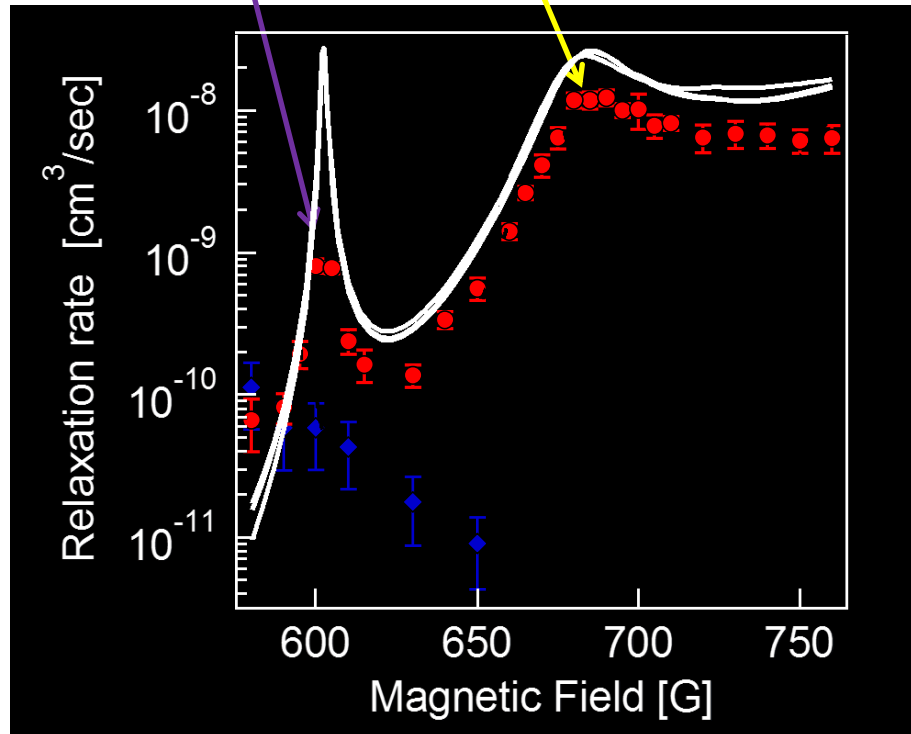
1st excited Efimov state

Ground-state Efimov state

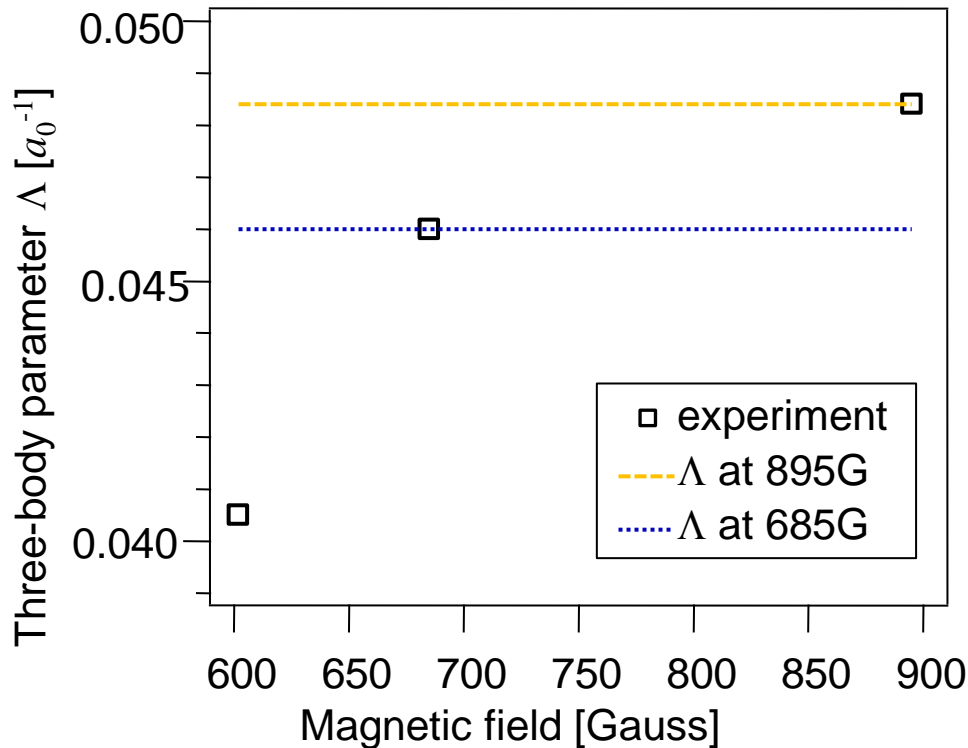
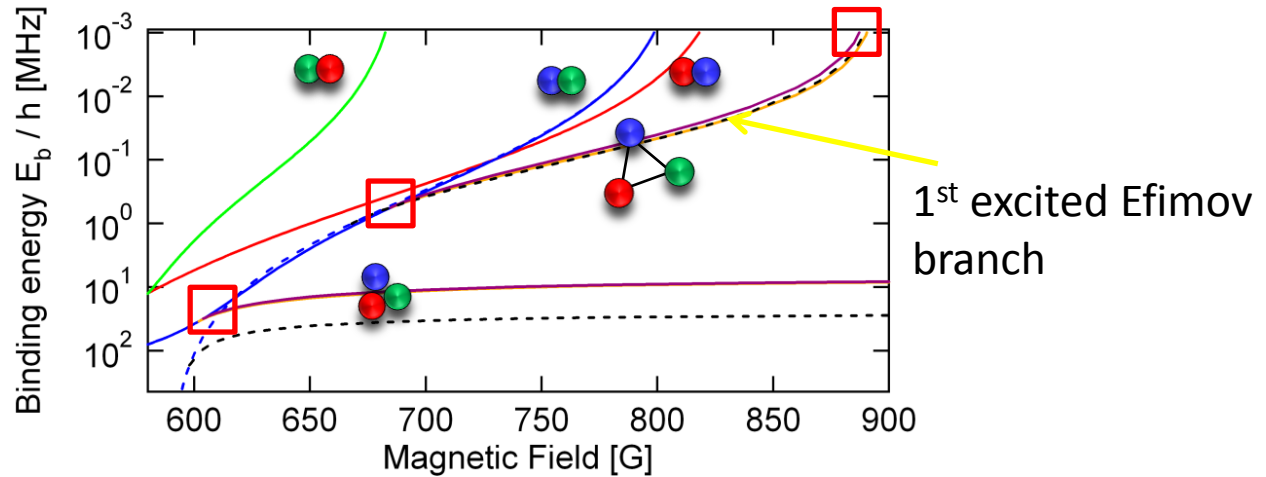
Two loss peaks associated with
 atom(1)+dimer (23) \leftrightarrow trimer
 For the ground and first excited
 state trimers observed.



New pieces of information on the
 three-body parameter for **nonzero**
energy Efimov trimers



Three-Body Parameter



The 3-body parameter measured at different points do not agree.

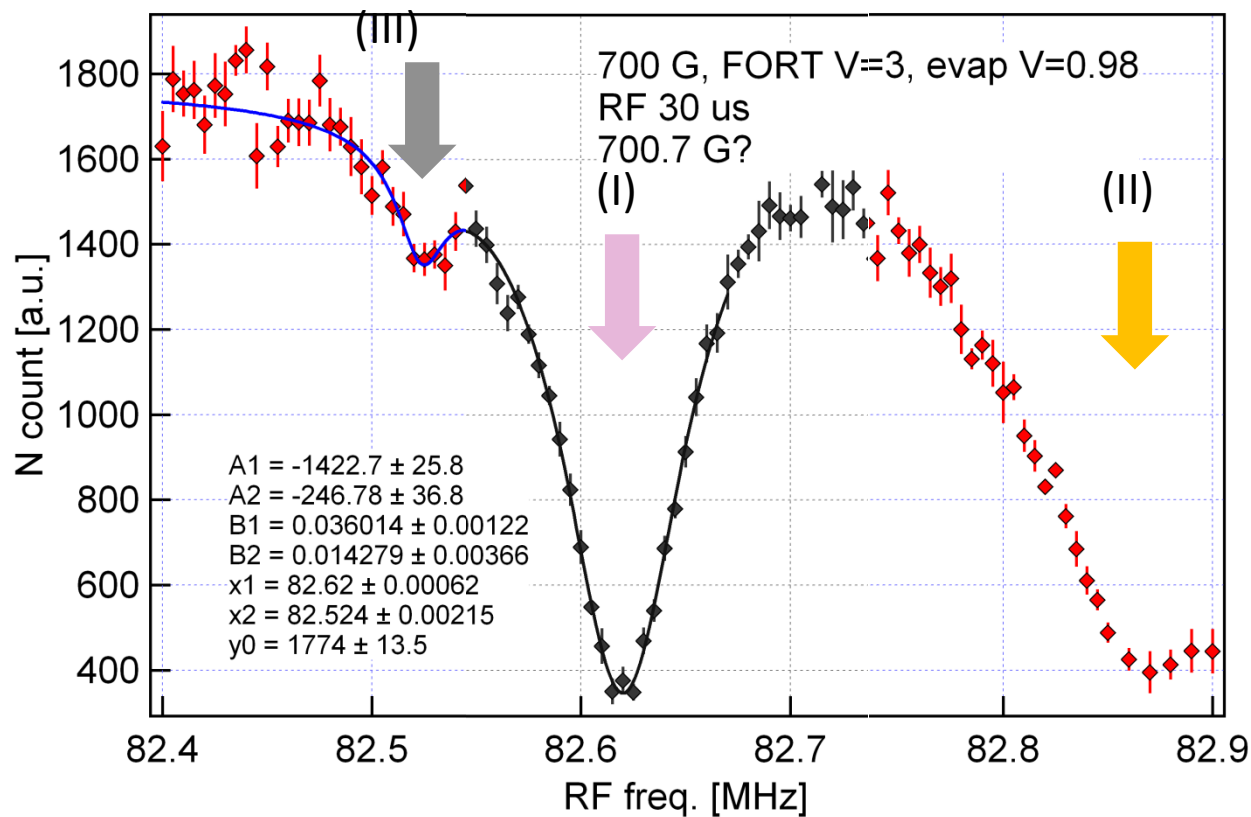
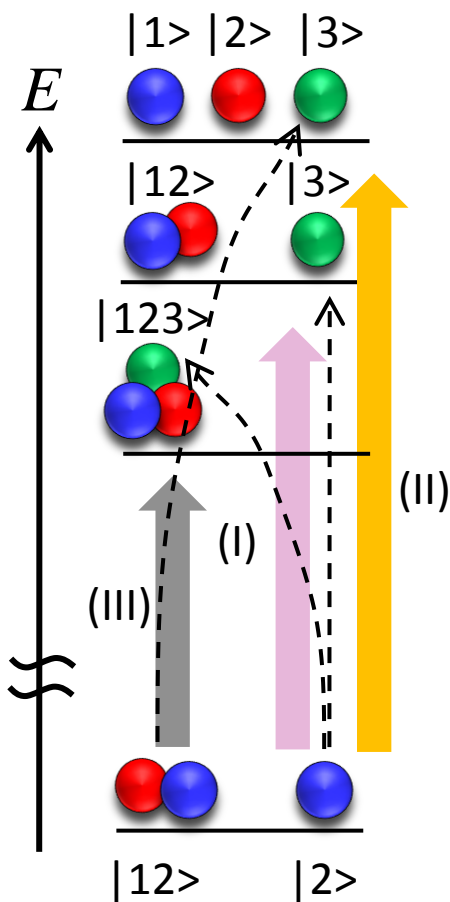
How does the 3-body parameter change along the 1st excited Efimov branch?

RF Association of Efimov Trimers

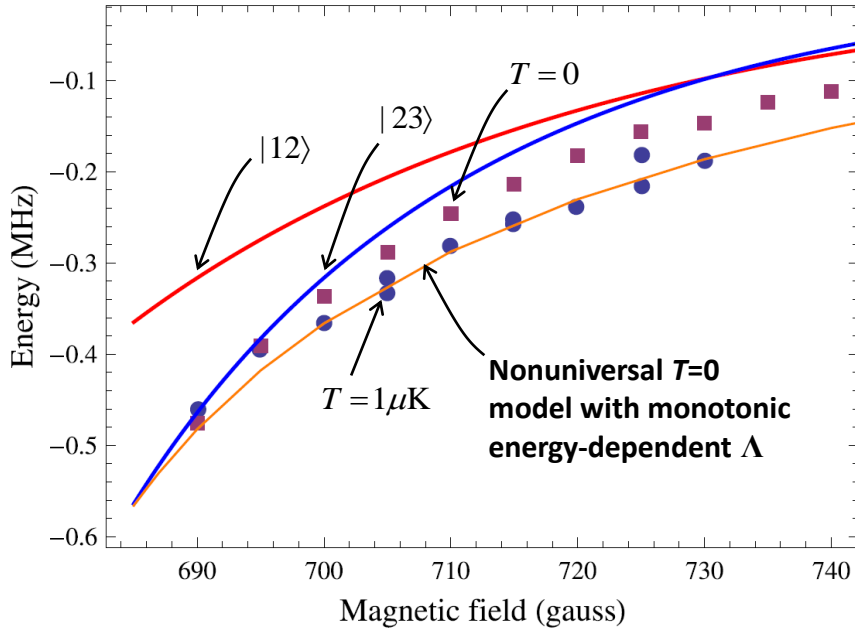
S. Nakajima, *et al.*, Phys. Rev. Lett. 106, 143201 (2011)

T. Lompe *et al.*, Science **330**, 940 (2010)

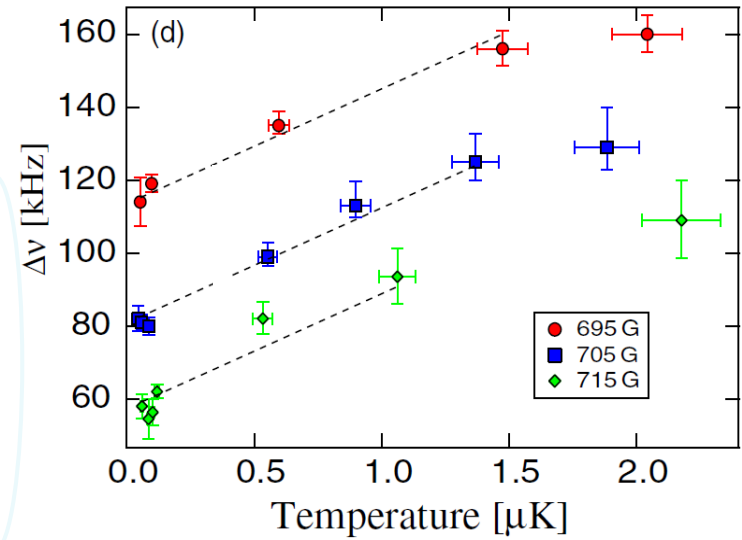
Number of atoms in state $|2\rangle$ vs RF frequency



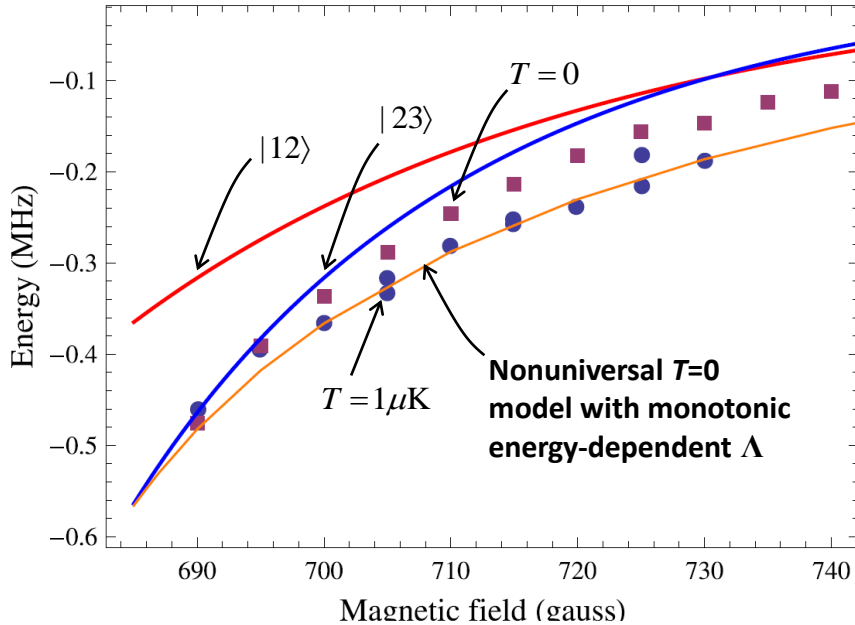
Temperature Dependence of Efimov Binding Energy



- S. Nakajima, et al., PRL **106**, 143201 (2011)
- T. Lompe, et al., Science **330**, 940 (2010)

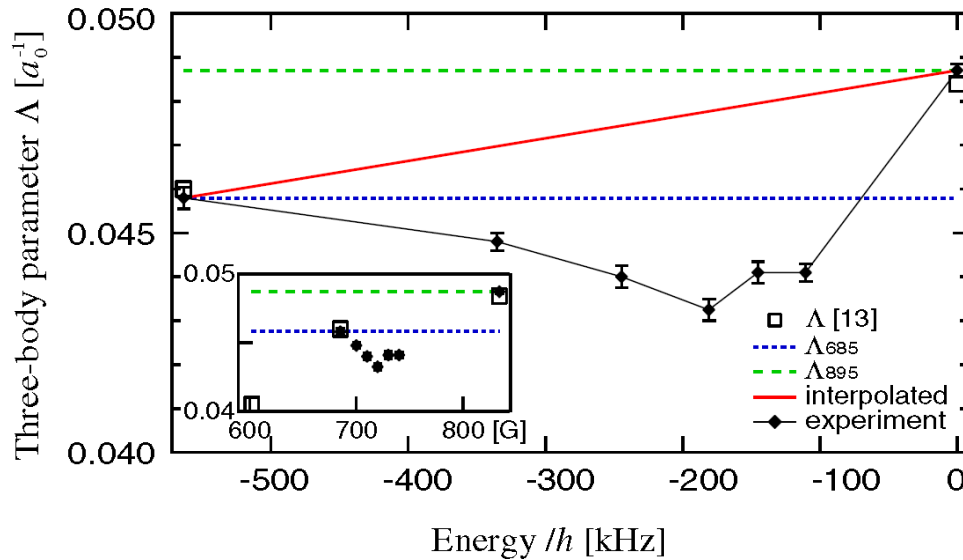


Energy Dependence of 3-Body Parameter



- S. Nakajima, et al., PRL **106**, 143201 (2011)
- T. Lompe, et al., Science **330**, 940 (2010)

- non-monotonic energy dependence of Λ as large as 15 %
 multichannel effect?
 finite-range effect?
 3-body physics?



- A small uncertainty ($\leq 0.7\%$) in a can cause a large (5%) variation of Λ .



Even more precise measurement of a is needed.

Back to Zero-Energy Efimov: Hidden Universality ?

- Evidence of near-constant (in log scale) three-body parameter in Cs (Innsbruck).
- Several alkali's (^{133}Cs , ^7Li , ^{85}Rb) give similar values in a_{vdW} .
- Let's check this theoretically using a realistic potential.

Results of He-4 Trimers

- Realistic LM2M2 potential [R.A. Aziz, M.J. Slaman, J. Chem. Phys. 94, 8047 (1991)]
- Solve Schrödinger equation with the Gaussian expansion method

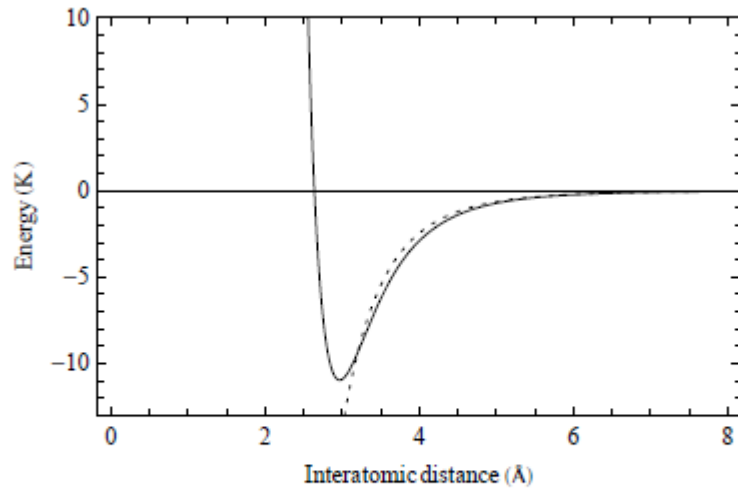
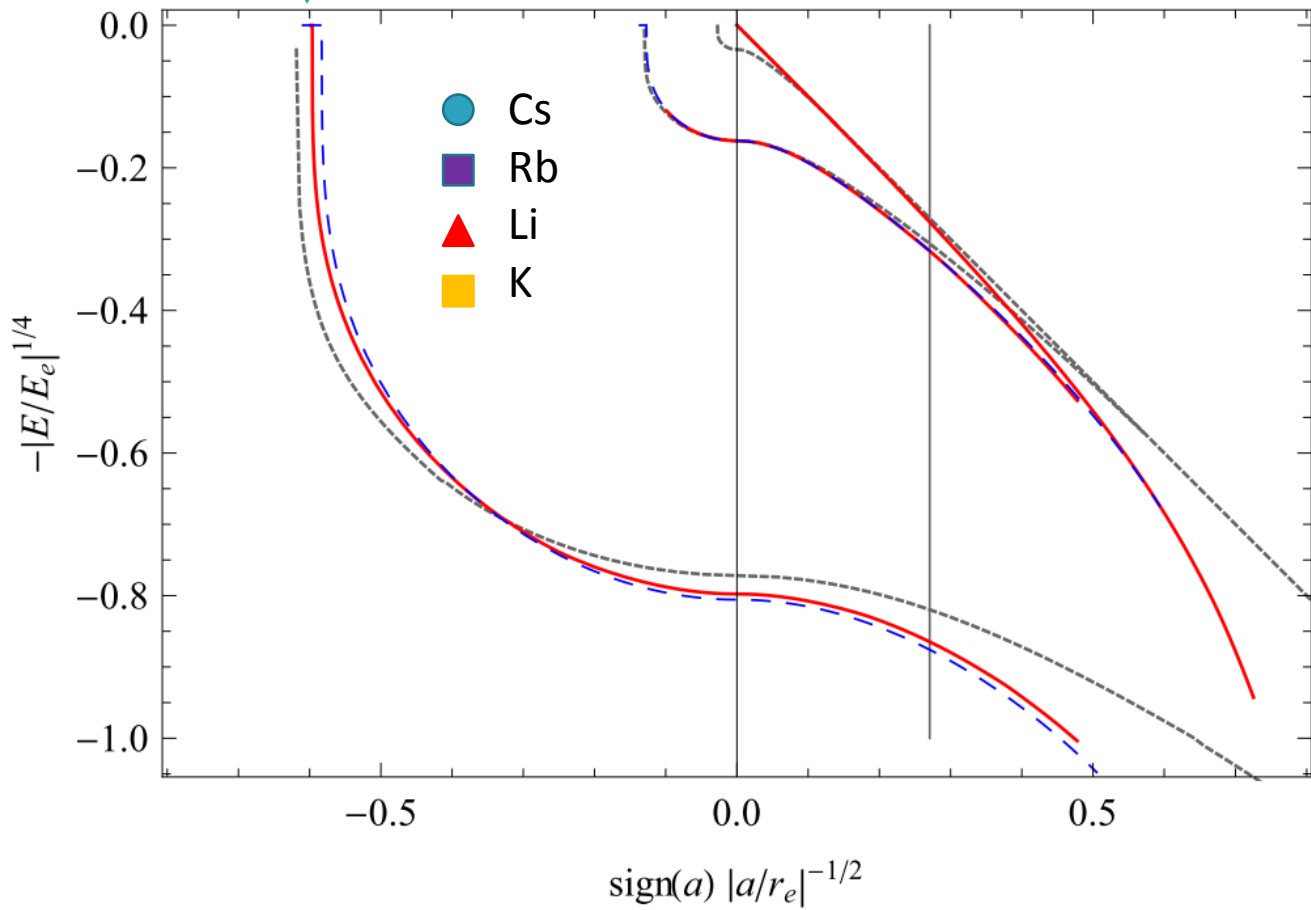


Figure 1: LM2M2 potential [25] used for the realistic calculations. The dotted curve indicates the van der Waals asymptote $-C_6/r^6$.

Let's Compare with Cs, Rb, K, Li

Cs133	Innsbruck
Rb85	JILA
Li6	Tokyo
Li7	Bar Ilan
K39	Florence

$a = 2.7r_e \longleftrightarrow a = 9.7\bar{a}$



- Realistic He
- - - Fitted Universal
- - - Separable Model

Calculations:



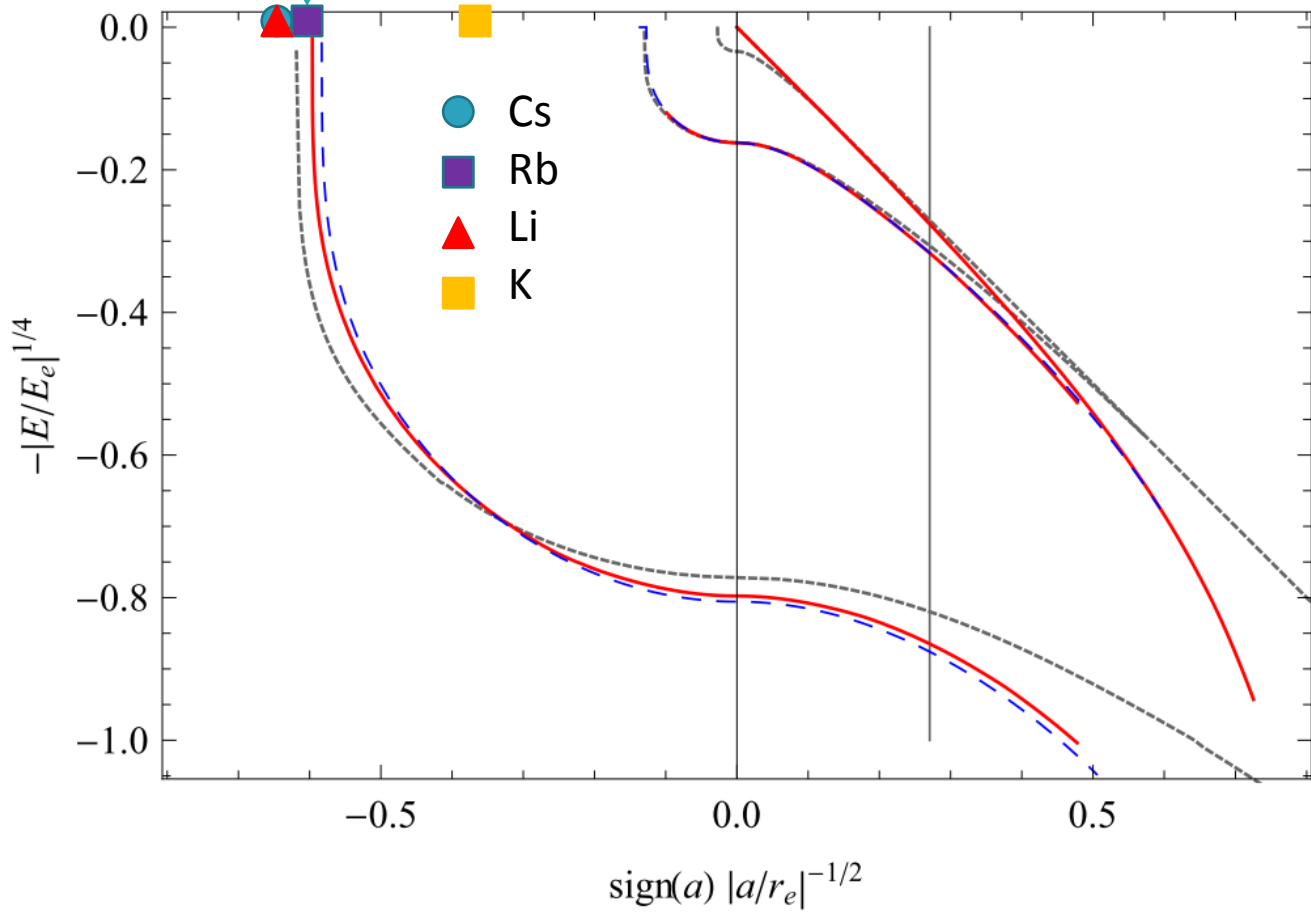
P. Naidon



E. Hiyama

He-4 Trimers: Comparison with Cs, Rb, K, Li

$a = 2.7r_e$ \longleftrightarrow $a = 9.7\bar{a}$



- Realistic He
- - - Fitted Universal
- - - Separable Model

Calculations:



P. Naidon



E. Hiyama

Conclusions

- Fermionic Li system provides a wealth of information about Efimov physics.
- $a(E)$ and effective range capture an overall feature of two universal regions (A and B) at zero energy.
- 3-body parameter shows irregular behavior (15%) at negative energy.
- Mounting evidence of near-constant 3-body parameter at zero energy not only across different universal regimes (A, B) but also across different species.