Sixth Workshop on the Critical Stability of Quantum Few-Body Systems

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Finite Range Effects in Three-body Recombination of Cold Atomic Gasses

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- By utilising a two-channel model, effectively describing the physics of Feshbach resonances

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The recombination rate is calculated and compared to experiment.

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- Comparison to experiments
- How we will deal with $N > 3$ particles

The Zero Range Model

The basic zero range model consists of free solutions to the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dr^2} = E\psi(r) \qquad \psi(r) = A\sin(kr + \delta(k))$$

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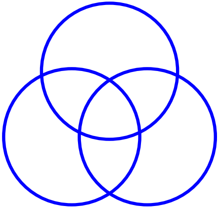

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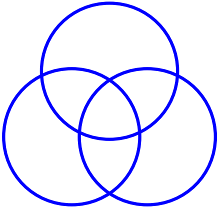

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Efimov effect: $E_n = E_0 \cdot (22.7)^{-2n}$, when $|a| \to \infty$.
Thomas effects: No lower bound on bound state energy.

The Zero Range Model With Finite Range

From scattering theory we also have the effective range expansion

$$\lim_{k \to 0} k \cot \delta = -\frac{1}{a} + \frac{1}{2} R k^2$$

$R =$ the effective range.

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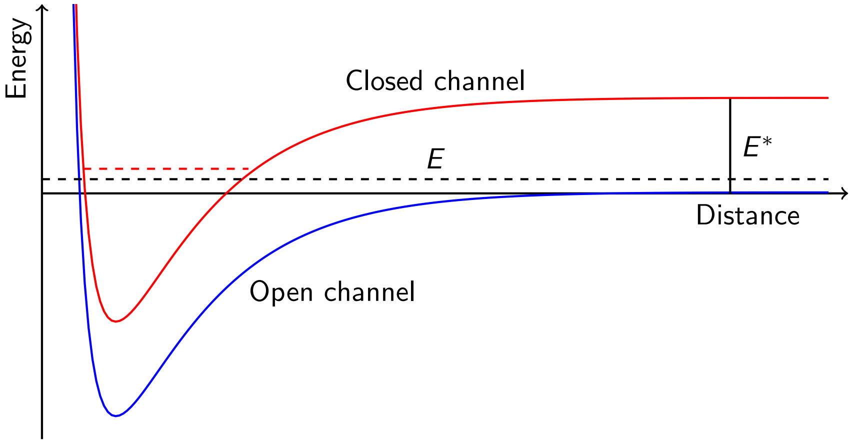
Thomas effect removed
Efimov effect persists

Feshbach Resonances



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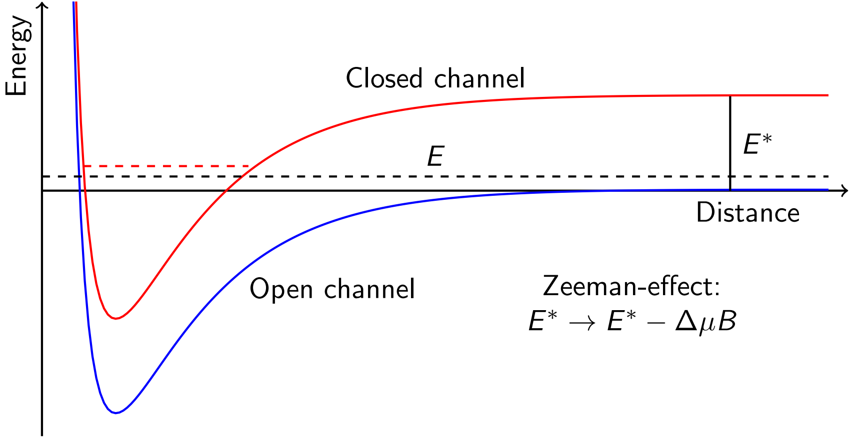

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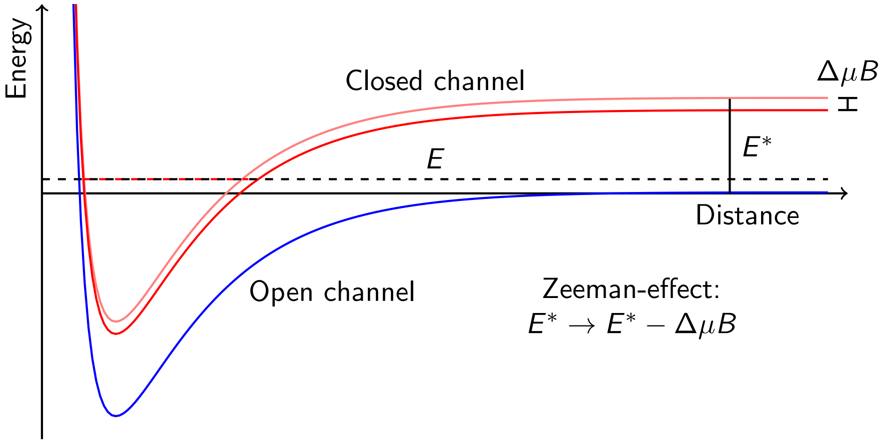

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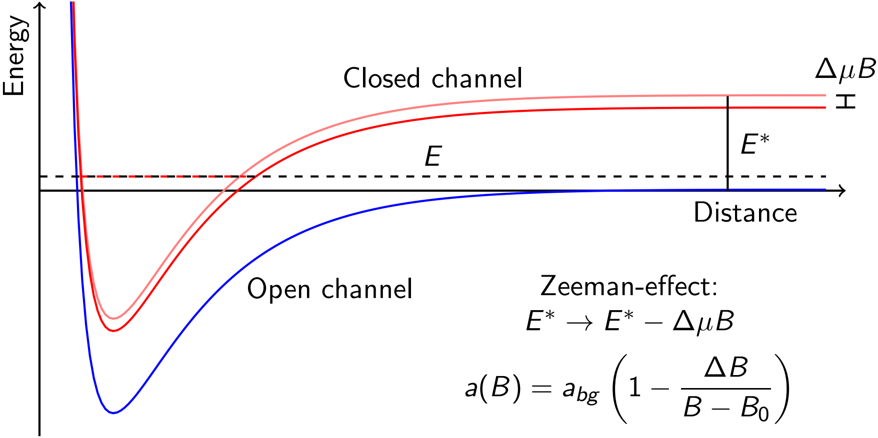


Zeeman-effect:

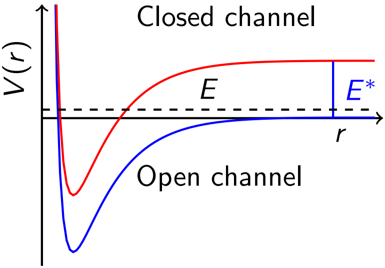
$$E^* \to E^* - \Delta\mu B$$

$$a(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

The Two-channel model

Taking an additional interaction channel into account

$$\psi(r) = \begin{bmatrix} u_c(r) \\ u_o(r) \end{bmatrix} \qquad \begin{aligned} u_c(r) &= \text{closed channel} \\ u_o(r) &= \text{open channel} \end{aligned}$$



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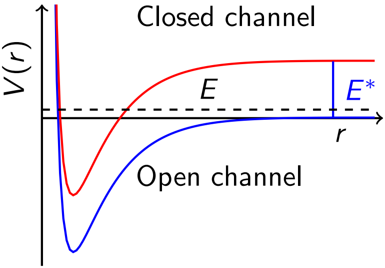

The Schrödinger equation is

$$-\frac{\hbar^2}{2m^*}u_c'' = (E - E^*)u_c$$
$$-\frac{\hbar^2}{2m^*}u_o'' = Eu_o$$

E^* = energy difference between channels.

m^* = the reduced mass.

The Two-channel model

The boundary condition becomes

$$\left.\frac{\psi'}{\psi}\right|_{r=0} = -\frac{1}{a} \quad \rightarrow \quad \begin{bmatrix} u_c' \\ u_o' \end{bmatrix}_{r=0} = \begin{bmatrix} -\frac{1}{a_c} & \beta \\ \beta & -\frac{1}{a_o} \end{bmatrix} \begin{bmatrix} u_c \\ u_o \end{bmatrix}_{r=0}$$

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$$u_c(r) = Be^{-\kappa_c r} \qquad\qquad u_o(r) = A\sin(k_o r + \delta)$$

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Inserted into the boundary condition gives

$$\lim_{k\to 0} k\cot\delta = -\frac{1}{a} + \frac{1}{2}Rk^2$$

where the scattering length and effective range are given by

$$\frac{1}{a} = \frac{1}{a_o} + \frac{\beta^2}{\kappa - \frac{1}{a_c}} \qquad\qquad R = \frac{-\beta^2}{\kappa\left(\kappa - \frac{1}{a_c}\right)^2}$$

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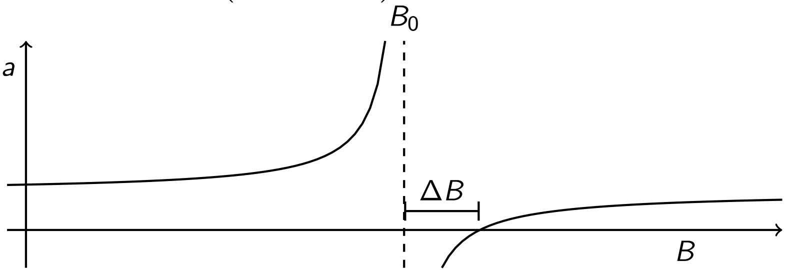

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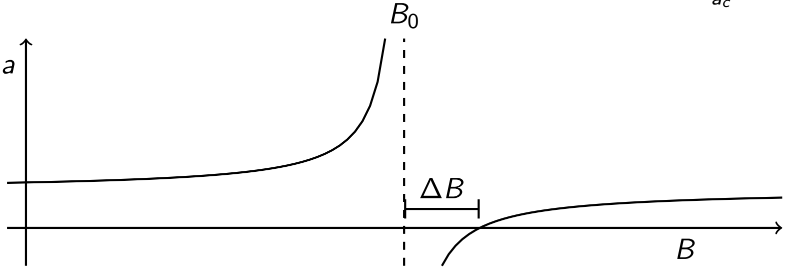

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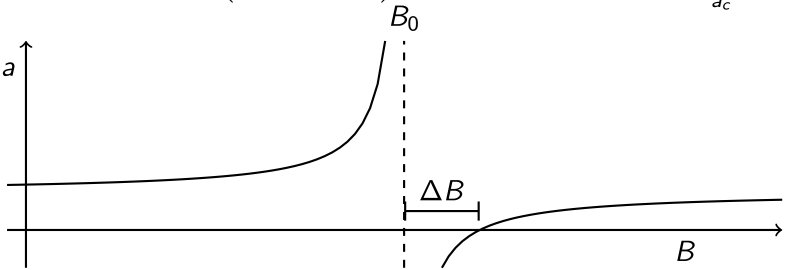
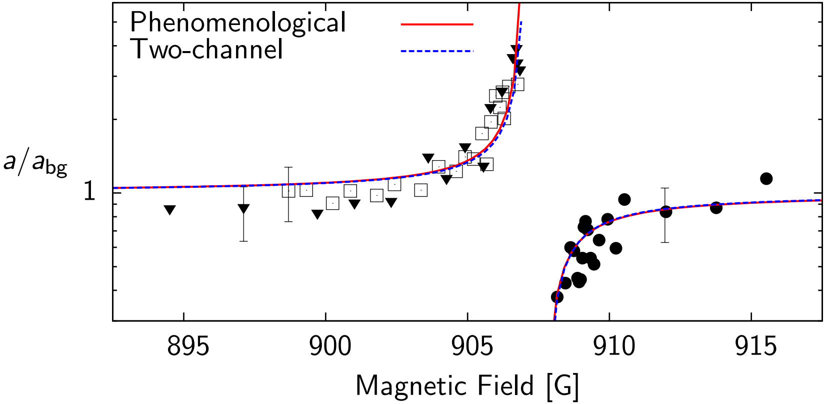


$$B_0 = \frac{1}{\Delta\mu}\left(E^* - E_0\right), \qquad \Delta B = \frac{1}{\Delta\mu}\frac{\hbar^2\kappa_0\beta^2 a_o}{m^*}$$

$$E_0 = \frac{\hbar^2\kappa_0^2}{2m^*}, \qquad R = -\frac{1}{a_o}\frac{\hbar^2}{m^*\Delta\mu\Delta B}$$

Feshbach resonance for ^{23}Na

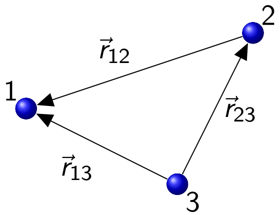


Data from Stenger et al, Phys. Rev. Lett., 82, (1999)

$$B_0 = 907\,\mathrm{G}, \qquad \Delta B = 0.7\,\mathrm{G}, \qquad R = -21 a_{bg}$$

Hyperspherical coordinates

From the Cartesian coordinates describing three particles . . .



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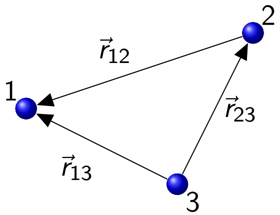

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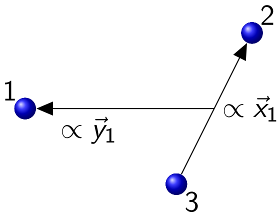

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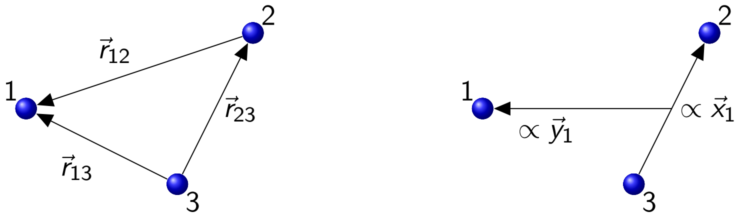

With the hyperradius ρ and hyperangle α_i given by

$$\rho^2 = x_i^2 + y_i^2 \qquad \rho \sin \alpha_i = x_i \qquad \rho \cos \alpha_i = y_i$$

Adiabatic Expansion

We expand the wavefunction on adiabatic basis states $\Phi_n(\rho, \Omega)$

$$\Psi(\rho, \Omega) = \rho^{-5/2} \sum_n f_n(\rho) \Phi_n(\rho, \Omega)$$

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$\Lambda =$ Grand angular momemtum operator.
Zero range potentials means $V = 0$.
At low energy only s-wave states are used.

Zero Range Angular Eigenvalue Equation

For zero-range potentials the solutions are

$$\Phi(\rho, \Omega) = \sum_{i=1}^{3} \frac{\varphi_i(\rho, \alpha_i)}{\sin(2\alpha_i)}, \qquad \varphi_i(\rho, \alpha_i) = N_i(\rho) \sin\left[\nu(\rho)\left(\alpha_i - \frac{\pi}{2}\right)\right]$$

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$$\frac{\nu\cos\left(\nu\frac{\pi}{2}\right) - \frac{8}{\sqrt{3}}\sin\left(\nu\frac{\pi}{6}\right)}{\sin\left(\nu\frac{\pi}{2}\right)} = \frac{\rho}{\sqrt{\mu}}\frac{1}{a} \qquad \lambda(\rho) = \nu^2 - 4$$

Effective Range Expansion Angular Eigenvalue Equation

The boundary condition for the effective range expansion

$$\left.\frac{\partial(\alpha_i\Phi)}{\partial\alpha_i}\right|_{\alpha_i=0} = \frac{\rho}{\sqrt{\mu_i}}\left[-\frac{1}{a_i}+\frac{1}{2}R_i k^2\right]\alpha_i\Phi\Big|_{\alpha_i=0}$$

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Two-channel Generalisation

The hyperspherical two-channel boundary condition is

$$\frac{\partial}{\partial \alpha_i}\begin{bmatrix}\alpha_i \Phi_c(\rho,\Omega_i)\\ \alpha_i \Phi_o(\rho,\Omega_i)\end{bmatrix}\Bigg|_{\alpha_i=0} = \frac{\rho}{\sqrt{\mu}}\begin{bmatrix}-\frac{1}{a_{i,c}} & \beta_i\\ \beta_i & -\frac{1}{a_{i,o}}\end{bmatrix}\begin{bmatrix}\alpha_i \Phi_c(\rho,\Omega_i)\\ \alpha_i \Phi_o(\rho,\Omega_i)\end{bmatrix}\Bigg|_{\alpha_i=0}$$

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yields

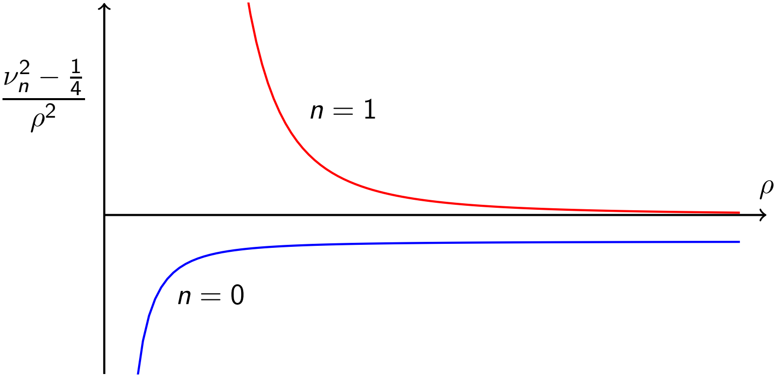
$$\frac{\rho^2\beta^2}{\mu}\sin\left(\nu\frac{\pi}{2}\right)\sin\left(\tilde{\nu}\frac{\pi}{2}\right) - f_o(\nu)f_c(\tilde{\nu}) = 0$$

$$f_l(\nu) = \nu\cos\left(\nu\frac{\pi}{2}\right) - \frac{8}{\sqrt{3}}\sin\left(\nu\frac{\pi}{6}\right) - \frac{\rho}{\sqrt{\mu}}\frac{1}{a_l}\sin\left(\nu\frac{\pi}{2}\right), \quad l = o, c.$$

$$\frac{1}{a} \approx \frac{1}{a_o} + \frac{\beta^2}{\sqrt{\mu}\kappa - \frac{1}{a_c}} \qquad R = \frac{-\beta^2}{\sqrt{\mu}\kappa\left(\sqrt{\mu}\kappa - \frac{1}{a_c}\right)^2}$$

Interpretation of Adiabatic Channels

$$\left(-\frac{d^2}{d\rho^2} + \frac{\nu_n^2 - 1/4}{\rho^2} - Q_{nn}(\rho) - \frac{2mE}{\hbar^2}\right) f_n(\rho) = 0$$



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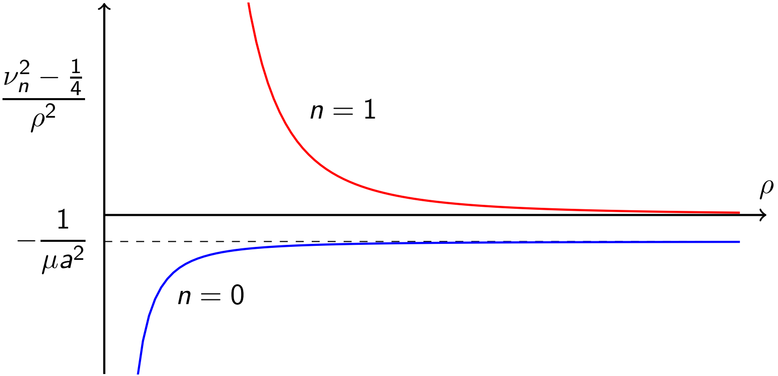

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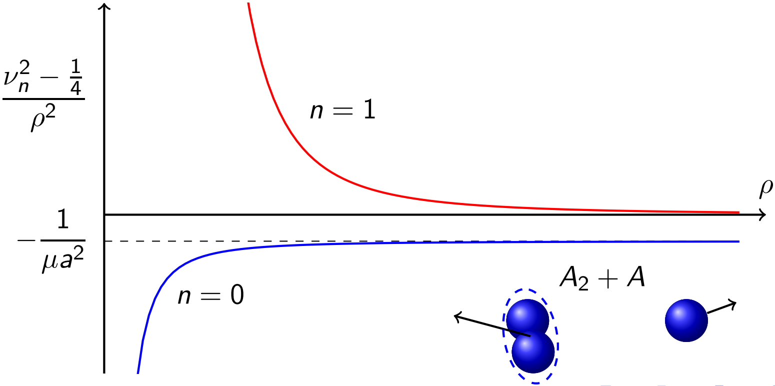

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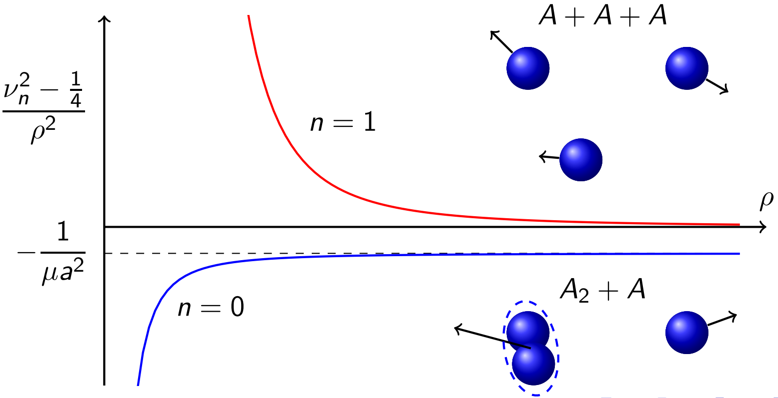

Analytical properties

One- and two-channel model

$$\nu_0(\rho) \xrightarrow{\rho\to\infty} \frac{i\rho}{\sqrt{\mu}a}\,, \qquad \nu_0(0) = 1.00624i$$

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Effective range model

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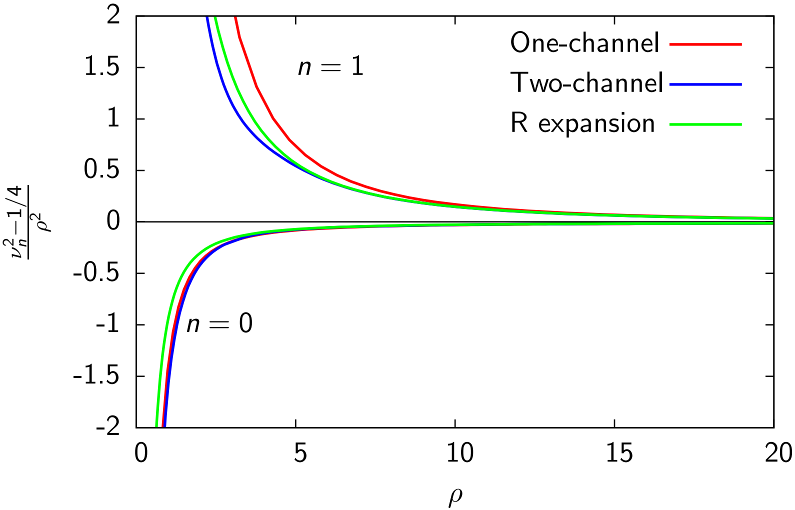
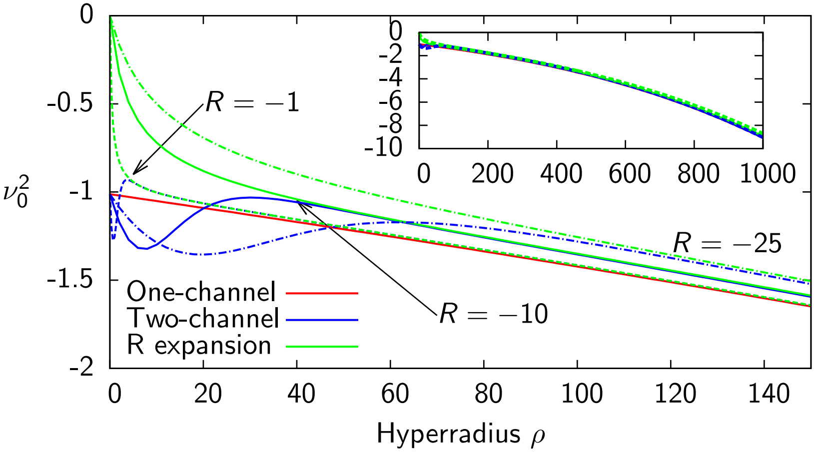
Efimov effect: $\exp(\pi/1.00624) \approx 22.7$.

Radial potentials



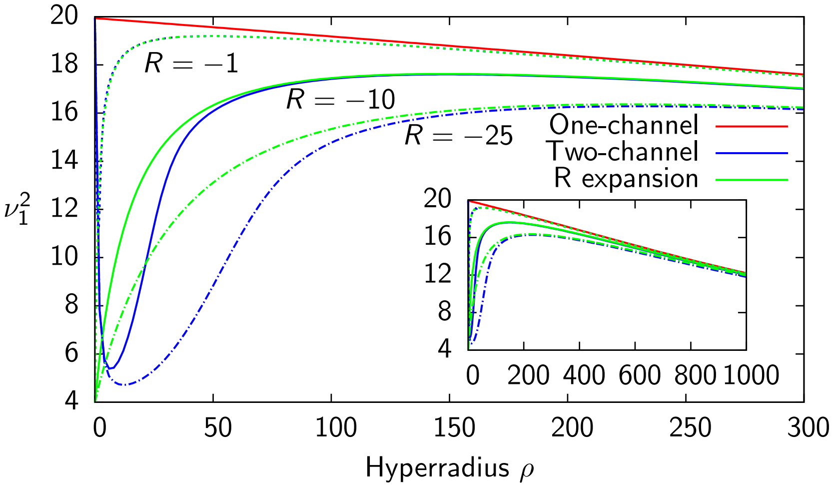
Eigenvalue solution

The $n = 0$ adiabatic eigenvalue solutions.



Eigenvalue solution

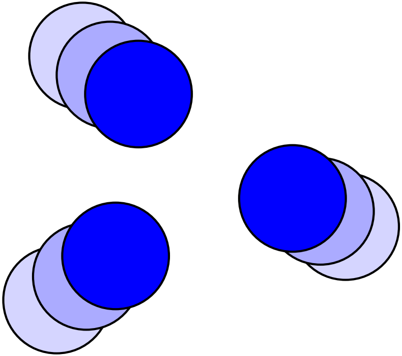
The $n = 1$ adiabatic eigenvalue solutions.



Recombination

Recombination is a three-body process in which

$$A + A + A \to A_2 + A$$



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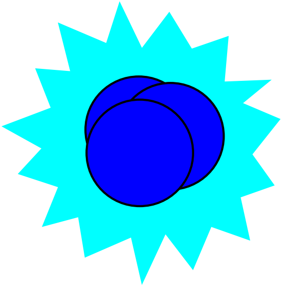

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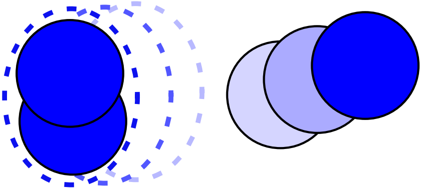

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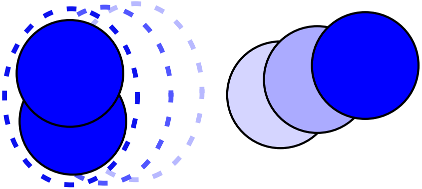


The loss rate due to recombination is given by

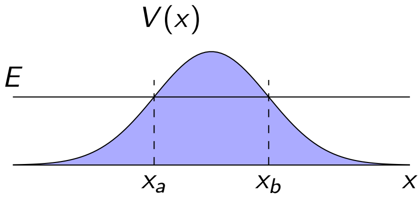
$$\dot{n} = -\alpha n^3$$

where n is the particle density and α is denoted the recombination coefficient.

The WKB-approximation

The WKB tunneling probability is

$$T \approx e^{-2S} \qquad iS = \frac{1}{\hbar}\int_{x_a}^{x_b} \sqrt{2m(E - V(x))}dx$$



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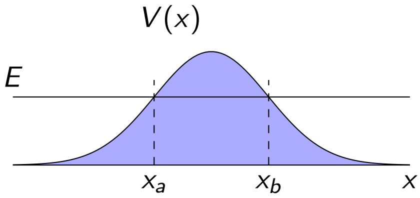

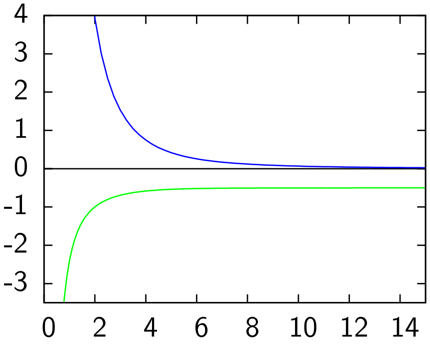

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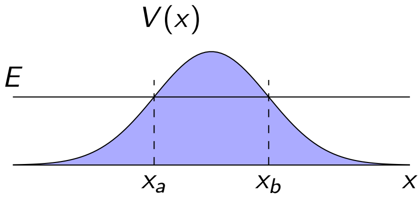

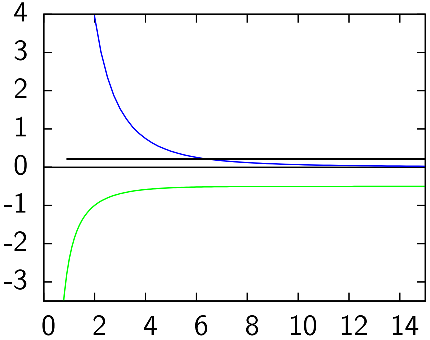

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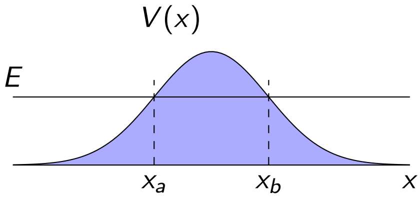

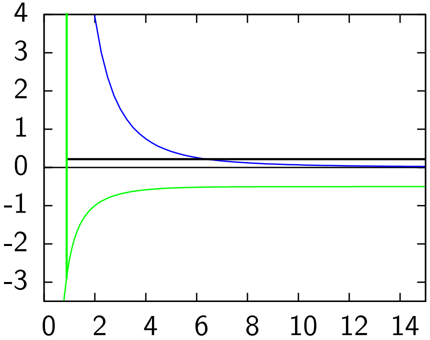

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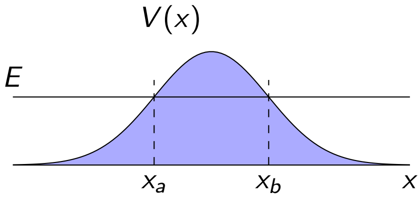

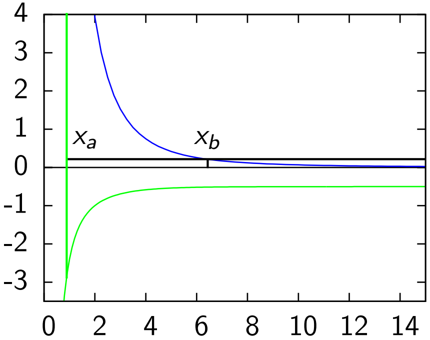

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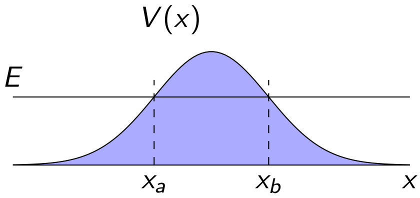
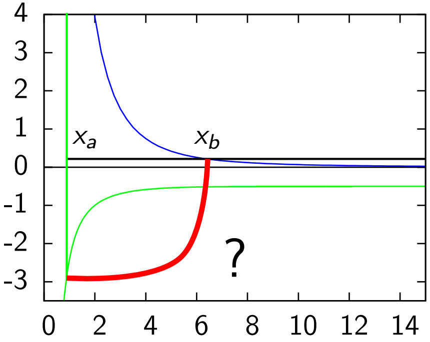
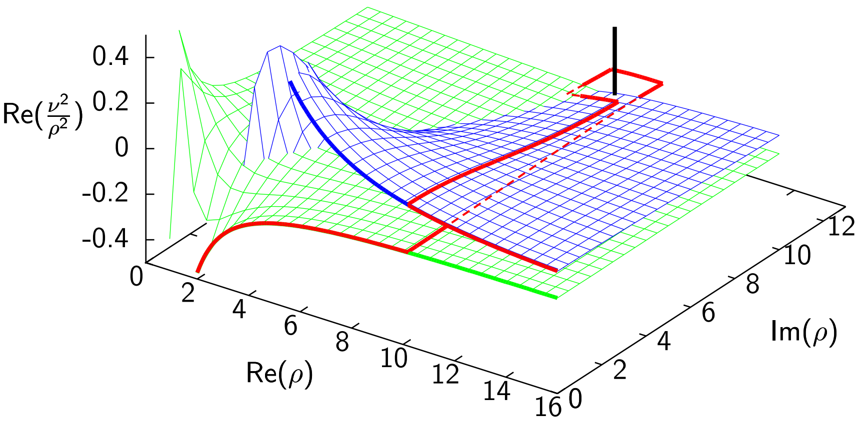




Hidden Crossing



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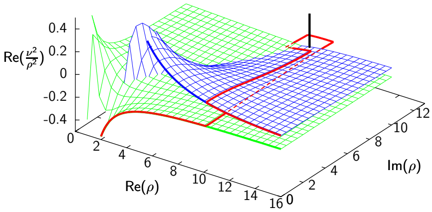


Transition probability

$$P(k) = e^{-2S}\sin^2\Delta$$

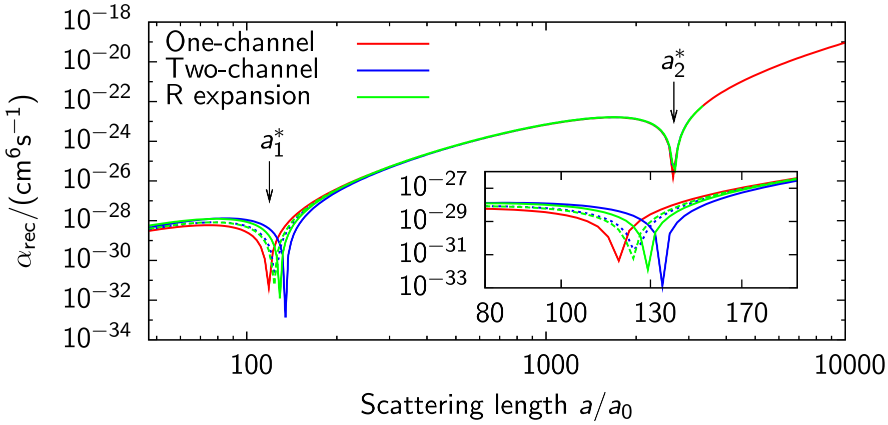
$$\Delta + iS = \int_c d\rho\sqrt{k^2 - \frac{\nu(\rho)^2}{\rho^2}}$$

$$\alpha_{\text{rec}} = 8(2\pi)^2 3\sqrt{3}\frac{\hbar}{\mu m}\lim_{k\to 0}\frac{P(k)}{k^4}$$

The phase Δ gives rise to interference effects.

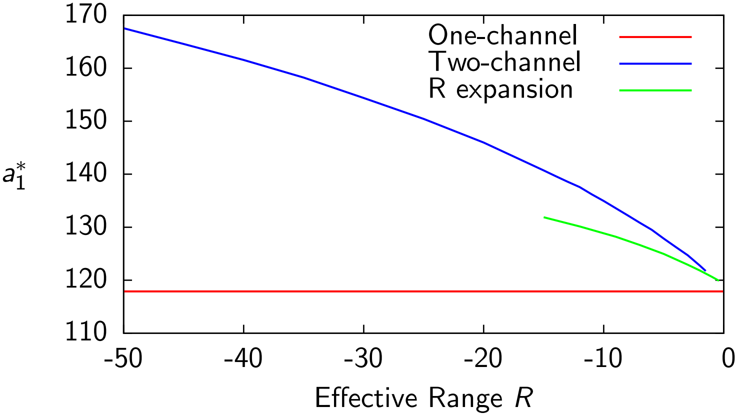
Recombination coefficient

The regularisation cut-off is chosen such that the recombination minimum at a_2^* is the same for all models.

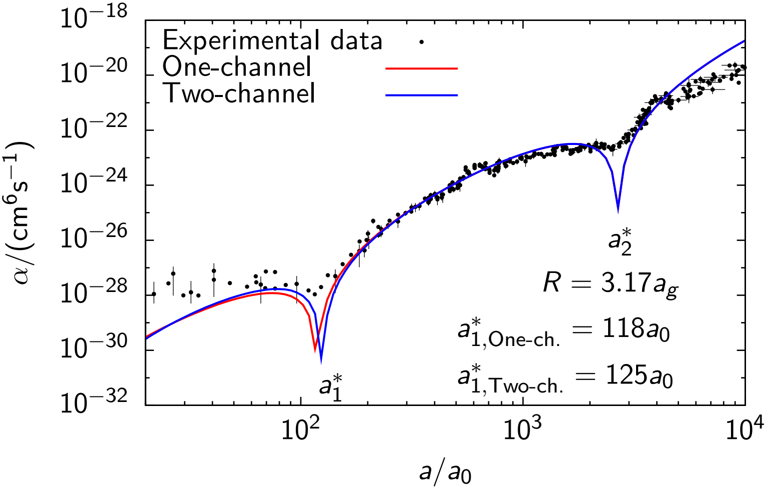


Recombination coefficient

Increasing $|R|$ whilst fixing a_2^* moves the minimum at a_1^*.



Recombination coefficient for ^7Li



Data from Hullet et al, Science, 326, (2009)

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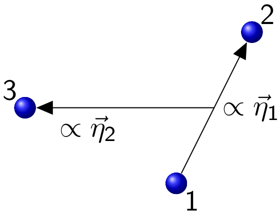
Next Steps

How to deal with $N > 3$



$$\frac{\rho}{\sqrt{\mu}}\frac{1}{a} = \frac{\nu\cos\left(\nu\frac{\pi}{2}\right) - \frac{8}{\sqrt{3}}\sin\left(\nu\frac{\pi}{6}\right)}{\sin\left(\nu\frac{\pi}{2}\right)}$$

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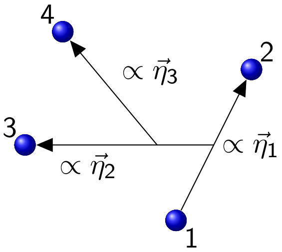

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Eigenvalue equation becomes:

$$\frac{\rho}{\sqrt{\mu}a} = T_{12} + T_{13} + T_{34}$$

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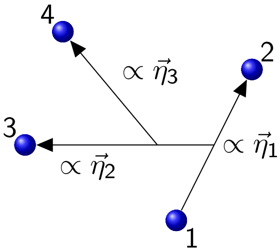

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Again solve for $\nu(rho)$ and apply hidden crossing method.