Exploring the Pion phenomenology within a fully covariant constituent quark model

Giovanni Salmè

in collaboration with

Tobias Frederico (ITA-CT, São Jose' dos Campos, Brazil)

Emanuele Pace and Giovanni Romanelli(Universita' di Roma "Tor Vergata" & INFN, Italy)

Barbara Pasquini (Universita' di Pavia & INFN, Italy)

Relativistic quark models for pion form factor

Phys. Lett. **B 581** (2004) 75; Phys. Rev. **D 73**, 074013 (2006) - J.P.B.C. de Melo, T. Frederico, E. Pace, G. S.

Pion Generalized Parton Distributions

Phys. Rev. **D 80** (2009) 05444021 - T. Frederico, E. Pace, B Pasquini, G. S.

Pion Tensor Generalized Parton Distribution in a CCQM

Few-Body Systems to be published - E. Pace, G. Romanelli, G. S.

Outline

1. Motivations
2. A primer on the Bethe-Salpeter Amplitudes (BSA)
3. The Mandelstam formula for the electromagnetic current of an interacting system
4. Living on the Light-front hyperplane
5. The Covariant Constituent Quark Model and the electromagnetic Pion observables
6. The future: the Nakanishi representation of BSA
7. Conclusions & Perspectives

Motivations

The fully understanding of the non perturbative regime of Quantum ChromoDynamics, the present theory of the strong interactions, still represents a paramount challenge. The perturbative regime, with its fundamental feature, the asymptotic freedom, has been experimentally investigated in great detail, and this has allowed us to access the short-distances behavior of the hadronic wave functions.

Recently, it has been recognized that a wealth of information on the partonic structure of hadrons is encoded in the Generalized Parton Distributions and the Transverse-momentum Distributions, as well. Those observables, that yield probability distributions of finding the constituents with given momenta in the father-hadron involved in both elastic and inelastic reactions, can be the pivotal quantities for investigating both from the theoretical and experimental sides the hadronic states. Presently, extensive theoretical and experimental (DVCS & SIDIS) research programs are being pursued to gain information on both GPD's and TMD's .

Goals: 1) a unified description of the observables through the Bethe-Salpeter Amplitudes of hadrons, or equivalently through the Light-Front wave functions (≡Fock expansion of the hadronic state); ii) paving the way from the purely phenomenological microscopic description to the one with a more consistent dynamical content.

Our present research program is based on

- first modeling the 4D quark-hadron vertex, namely the Bethe-Salpeter amplitude that, in Quantum Field Theory, plays, loosely speaking, the same role as the wave function in the non relativistic QM;
- then giving a phenomenological description of the observables (as many as possible) either i) experimentally investigated with electromagnetic probes or ii) evaluated within the Lattice QCD framework. The 4D Mandelstam formula of the current operator for a composed system is our primary tool.

Our study starts with the "simplest" hadronic system: the charged Pion ($u\bar{d}$ or $\bar{u}d$ in the valence component).

Fock decomposition of the pion state

$$|\pi\rangle = \underbrace{|q\bar{q}\rangle}_{\text{valence}} + \underbrace{|q\bar{q}\,q\bar{q}\rangle + |q\bar{q}\,g\rangle}_{\text{nonvalence}} \ldots\ldots$$

The next step will be an increasing of the dynamical content in the adopted Bethe-Salpeter amplitude. This goal will achieved by introducing an approach for approximating the solutions of the Bethe-Salpeter equation in Minkowski space (that recently is attracting great interest): the Nakanishi Perturbation Theory Integral Representation (in collaboration with T. Frederico and M. Viviani)

A primer on the Bethe-Salpeter Amplitude for a two-body systems

The BS amplitude is defined by the following matrix element of the time ordered product of two Heisenberg operators

$$\Phi(x_1^\mu, x_2^\mu, p^\mu) = \langle 0| T\left\{\varphi_H(x_1^\mu)\varphi_H(x_2^\mu)\right\}|p\rangle.$$

where $\langle 0|$ is the vacuum and $|p\rangle$ a state of the interacting system, with mass $p^2 = M^2$

The conjugate BS amplitude is

$$\bar{\Phi}(x_1^\mu, x_2^\mu, p^\mu) = \langle p| T\left\{\varphi_H^\dagger(x_1^\mu)\varphi_H^\dagger(x_2^\mu)\right\}|0\rangle$$

Translational invariance imposes to Φ the following form

$$\Phi(x_1, x_2, p) = \tilde{\Phi}(x, p)\, e^{-ip\cdot X},$$

where $X^\mu = (x_1^\mu + x_2^\mu)/2$, $x^\mu = x_1^\mu - x_2^\mu$ and $\widehat{\Phi}(x, p)$ the reduced amplitude.

Its Fourier transform, $\Phi(k, p)$, as follows

$$\Phi(k, p) = \int \frac{d^4x}{(2\pi)^4}\, e^{-ik\cdot x}\, \widehat{\Phi}(x, p)$$

where

$$p^\mu = p_1^\mu + p_2^\mu \qquad k^\mu = \frac{p_1^\mu - p_2^\mu}{2}$$

with $p_i^2 \neq m^2$

The vertex function, introduced for eliminating a trivial free-propagation, is defined in terms of the BS amplitude as

$$\Gamma(k,p) = \left[G_0^{12}(k,p)\right]^{-1} \; \Phi(k,p)$$

where

$$G_0^{(12)}(k,p) = G_0^{(1)} G_0^{(2)} =$$
$$= \frac{i}{(\frac{p}{2}+k)^2 - m^2 + i\epsilon} \; \frac{i}{(\frac{p}{2}-k)^2 - m^2 + i\epsilon} \,.$$

with $G_0^{(i)}$ the free propagators for the two constituents (no self-energy insertions).

The amplitude Φ satisfies a homogeneous BS equation for a bound state, or a inhomogeneous BS equation for the scattering case.

The BSE for bound states can be obtained from the analysis of the four-point interacting Green function near the poles, e.g.

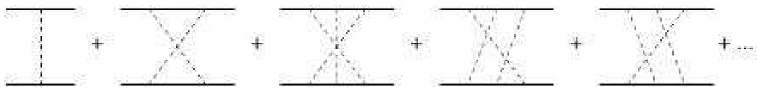
$$G^{(12)}(x_1, x_2, y_1, y_2) =$$
$$= \langle 0 | T \left\{ \varphi_H(x_1^\mu) \varphi_H(x_2^\mu) \varphi_H^\dagger(y_1^\mu) \varphi_H^\dagger(y_2^\mu) \right\} |0\rangle =$$
$$= i \frac{\Phi(x_1^\mu, x_2^\mu, p^\mu) \bar{\Phi}(y_1^\mu, y_2^\mu, p^\mu)}{p^2 - M^2 = i\epsilon} + \mathcal{R}$$

and the equation that yields G_{12}, while the BSE for scattering states comes from the half-off-shell Green function with its cuts

For a bound state one has the following *homogeneous* integral equation

$$\Phi_b(k,p) = G_0^{(12)}(k,p) \int \frac{d^4k'}{(2\pi)^4} i\,\mathcal{K}(k,k',p)\Phi_b(k',p),$$

where $i\,\mathcal{K}$ is the interaction kernel, containing irreducible diagrams only, but with self-energy insertions and vertex correction. A simplified picture is

$i\,\mathcal{K} \equiv$ 

N.B.: all the internal propagators and the interaction vertexes must be dressed.

The normalization reads

$$\int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4} \bar{\Phi}_b(k',p) \frac{\partial}{\partial p_\mu}$$
$$\left[G_0(k,p)^{-1}(2\pi)^2\delta^4(k'-k) - i\mathcal{K}(k',k,p)\right]\Phi(k,p) = 2ip^\mu$$

The *inhomogeneous* BSE, i.e. the field theoretic counter-part of the Lippman-Schwinger equation, is

$$\Phi^{(+)}(k,p,k_i) = (2\pi)^4 \delta^{(4)}(k-k_i) +$$
$$+G_0^{(12)}(k,p) \int \frac{d^4k'}{(2\pi)^4} i\,\mathcal{K}(k,k',p)\Phi^{(+)}(k',p,k_i)$$

where k_i the relative incoming momentum.

Attempts to solve BSE: only for bound states, primarily

Shortly

- Wick rotation: from Minkowski space to Euclidean space, namely $k_0 \to ik_0$
- Quasipotential reduction: 4D $\to$ 3D, with suitable prescriptions for dealing with the analytic dependence of Φ upon k_0, the relative energy, conjugated to the *relative time*
- Nakanishi representation of the vertex function (from the parametric form of the Feynman diagrams): 4D solutions, (Kusaka et al, PRD56 (1997); Carbonell and Karmanov, Few-body Syst. **49** (2011), for a review of their recent work.

Probing the hadron dynamics through electromagnetic observables

Elastic reaction

$$\gamma^*(q) + A(p_i) \to A(p_f)$$

$$\langle p_f | J^\mu(x) | p_i \rangle \quad \to \quad \text{elastic form factors}$$

Elastic form factors depend upon $(p_f - p_i)^2 = q^2$

Compton scattering in the Deep Inelastic regime

$$\gamma^*(q) + A(p_i) \to A(p_f) + \gamma^*(q')$$

$\langle p_f | T\{J^\mu(x)\; J^\nu(0)\} | p_i \rangle \to$ Generalized Parton Distributions

Generalized Parton Distributions depend $(p_f - p_i)^2 \neq q^2$, $(p_f - p_i) \cdot \hat{n}/(p_f + p_i) \cdot \hat{n}$ and $k_{qrk} \cdot \hat{n}/(p_f + p_i) \cdot \hat{n}$

Proper integration of GPD's (first moment of GPD's) yield elastic FF's

The Mandelstam Formula for the EM current for an interacting system

Our guidance $\Rightarrow$ the Mandelstam formula, that yields a covariant expression of the matrix elements of the electromagnetic current for hadrons.

A first application $\Rightarrow$ **Pion**

In the spacelike region one has

$$\langle p_f|J^\mu(0)|p_i\rangle \; = \; -\imath 2e\frac{m^2}{f_\pi^2} N_c \int \frac{d^4k}{(2\pi)^4} \overline{\Lambda}_\pi(k+q, P'_\pi) \; \times$$
$$\Lambda_\pi(k, P_\pi)\, Tr[S(k-P_\pi)\, \gamma^5 S(k+q)\, \Gamma^\mu(k,q)\, S(k)\, \gamma^5]$$

- $S(p) = \dfrac{1}{\not p - m + \imath\epsilon}$ is the constituent quark propagator
- $\gamma_5\, \Lambda_\pi(k, P_\pi) = [S(k-P_\pi)]^{-1} \Phi_\pi(k, P_\pi) [S(k)]^{-1}$ is the pion vertex function (known caveats...);
- $\Gamma^\mu(k,q)$ is the quark-photon vertex (q^μ the virtual photon momentum) $\rightarrow \; \gamma^\mu$

Challenge: How to perform the 4D integration, in presence of poles (see, e.g. the Dirac propagators) or much more complicated analytic structures (see, e.g. Λ) ?

Living on the Light-front hyperplane

A typical denominator in a Feynman diagram

$$k_0^2 - k_z^2 - k_\perp^2 - m^2 + i\epsilon =$$
$$= (k_0 - \sqrt{m^2 + k_z^2 + k_\perp^2} + i\epsilon)\,(k_0 + \sqrt{k_z^2 + k_\perp^2} - i\epsilon)$$

This leads to deal with two poles in the plane $\{\Re(k_0), \Im(k_0)\}$

A minimalist view of the issue

A simple change of variable leads to a different treatment of the analytic integration

$$k^\pm = k_0 \pm k_z$$

Then

$$k_0^2 - k_z^2 - k_\perp^2 - m^2 + i\epsilon =$$
$$= k^+ \left(k^- \frac{m^2 + k_\perp^2}{k^+} \right) + i\epsilon$$

Namely, one ends up with poles relative to different variables, k^+ and k^-

Indeed, the previous change of variables is very far-reaching.

Dirac (1949) explored the consequences within a Hamiltonian framework

He introduced the so-called Light-front Hamiltonian Dynamics, opening a new avenue in the description of relativistically (Poincaré covariant) interacting system, with a fixed number of particles.

In particular, it was turned out that the LF-boosts (combinations of standard boosts and transverse rotations) are not affected by the interaction. This substantially simplifies the description of reactions, where the final state has to be boosted.

The application to Quantum Field Theory (infinite degrees of freedom) has been equally fruitful. The operator P^+ is bound from below. Its spectrum is positive defined $P^+ \geq 0$. and this leads to an almost trivial vacuum. Indeed, in a theory with only massive particles, the vacuum is trivial: it is an empty vacuum.

In view of this, within a Light-front approach with massive particles, the Fock expansion of a state of an interacting system becomes meaningful, since the Fock states are constructed acting on the true vacuum.

Pion Vector Generalized Parton Distributions

Isoscalar and isovector pion GPD's in the light-cone gauge are

$$H^0_\pi(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \; \langle p'|\, \bar\psi_q(-\frac{z}{2})\,\gamma^+ \psi_q(\frac{z}{2})|p\rangle\Big|_{\tilde z=0}$$

$$H^1_\pi(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \; \langle p'|\bar\psi_q(-\frac{z}{2})\,\gamma^+ \tau_3\, \psi_q(\frac{z}{2})|p\rangle\Big|_{\tilde z=0}$$

$$\tilde z \equiv \{z^+ = z^0 + z^3, \boldsymbol{z}_\perp\}, \qquad \psi_q(z) = quark\ field\ isodoublet$$

$$\int_{-1}^{1} dx\, H^{I=1}(x,\xi,t) = \; F_\pi(t)$$

Pion Tensor Generalized Parton Distributions

$$\frac{P^+\Delta^j - P^j\Delta^+}{P^+ m_\pi} E^{I=0}_{\pi,T}(x,\xi,t) =$$

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\bar\psi_q(-\frac{1}{2}z) i\sigma^{+j} \psi_q(\frac{1}{2}z)|p\rangle\Big|_{\tilde z=0}$$

$$\frac{P^+\Delta^j - P^j\Delta^+}{P^+ m_\pi} E^{I=1}_{\pi,T}(x,\xi,t) =$$

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\bar\psi_q(-\frac{1}{2}z) i\sigma^{+j}\tau^3 \psi_q(\frac{1}{2}z)|p\rangle\Big|_{\tilde z=0}$$

They allow us to investigate the correlation between the quark polarization and its transverse momentum: tomography of the pion state !

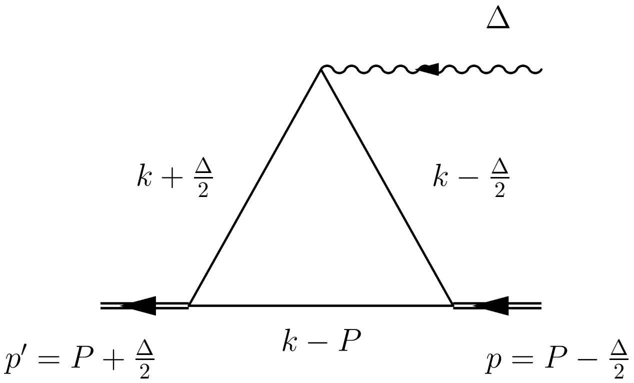
Diagrammatic picture of Deep Virtual Compton Scattering

For large value of the virtual photon four-momentum, the two electromagnetic vertexes, formally, shrink to one.....

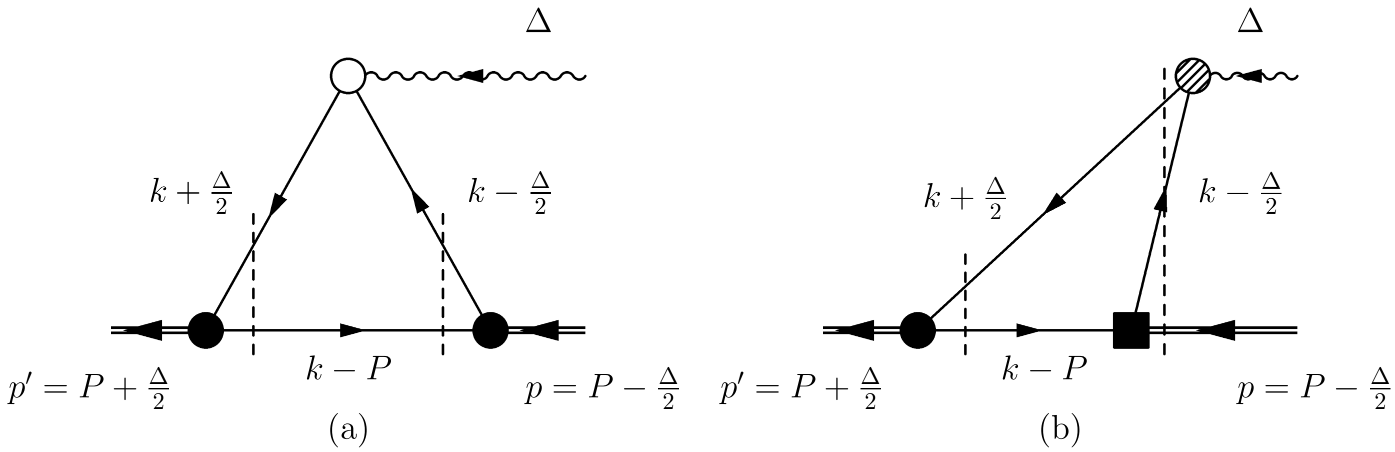
$t = \Delta^2 \qquad \Delta = p' - p$

$\xi = -\frac{\Delta^+}{2\ P^+} \qquad 2P = p + p'$

$x = \frac{k^+}{P^+} \qquad (1 \geq x \geq -1)$



LF time-ordered analysis of the pion GPD



active-quark valence region [DGLAP] — nonvalence region [ERBL]

$1 \geq x \geq |\xi|$ — $|\xi| > x > -|\xi|$

diagonal in the Fock space — non diagonal in the Fock space

Mellin Moments

for $H^u = H^0 + H^1$ and $E_T^u = E_T^0 + E_T^1$

$$\int_{-1}^{1} dx\, x^n H^u(x,\xi,t) = \sum_{i=0}^{n} (2\xi)^i A_{n+1,i}(t)$$

$$\int_{-1}^{1} dx\, x^n E_T^u(x,\xi,t) = \sum_{i=0}^{n} (2\xi)^i B_{n+1,i}(t)$$

$A_{n+1,i}(t)$ and $B_{n+1,i}(t)$ are isoscalar Generalized Form Factors if n is odd, and isovector GFF if n is even.

Those quantities are relevant for a comparison with Lattice QCD calculations.

Analytic covariant pion model with symmetric regulators

We use a pion Bethe-Salpeter amplitude (BSA) suggested by an effective Lagrangian [Frederico, Miller, PRD 45 (1992) 4207]

$$\Psi(k-P,p) = -\frac{m}{f_\pi}\, S\,(k-\Delta/2)\;\gamma^5\;\Lambda(k-P,p)\; S\,(k-P)$$

$m = 220$ MeV quark mass $\qquad$ $f_\pi = 92.4$ MeV decay constant

Two covariant symmetric forms for $\Lambda(k-P,p)$ are used :

i) a sum form

$$\Lambda_1 = \frac{C_1}{\left[(k-\Delta/2)^2 - m_R^2 + \imath\epsilon\right]} + \frac{C_1}{\left[(P-k)^2 - m_R^2 + \imath\epsilon\right]}$$

ii) a product form

$$\Lambda_2 = \frac{C_2}{\left[(k-\Delta/2)^2 - m_R^2 + \imath\epsilon\right]\left[(P-k)^2 - m_R^2 + \imath\epsilon\right]}$$

The parameter m_R (the only parameter we used) is fixed by reproducing f_π :

$m_R = 600 MeV$ $\qquad$ sum form

$m_R = 1200 MeV$ $\qquad$ product form

The constants C_1, C_2 are fixed through the FF normalization : $F_\pi(t=0) = 1$.

The formal expression for the u-quark vector GPD is given in Impulse Approximation (Mandelstam formula) by

$$2\, H^u(x,\xi,t) = -\imath\, N_c\, \mathcal{R}\ \times$$
$$\int \frac{d^4k}{(2\pi)^4}\, \delta[P^+x - k^+]\ \ V^+\ \Lambda(k-P,p')\ \ \Lambda(k-P,p)$$

$$V^+ = Tr\left\{S\,(k-P)\,\gamma^5\ S\,(k+\Delta/2)\,\gamma^+\ S\,(k-\Delta/2)\,\gamma^5\right\}$$

and the u-quark tensor GPD by

$$\frac{P^+\Delta^j - P^j\Delta^+}{iN_c\,\mathcal{R}\;P^+m_\pi} E_T^u(x,\xi,t) = \int \frac{d^4k}{(2\pi)^4}\delta[P^+x - k^+]\,\Lambda(k-P,p')$$

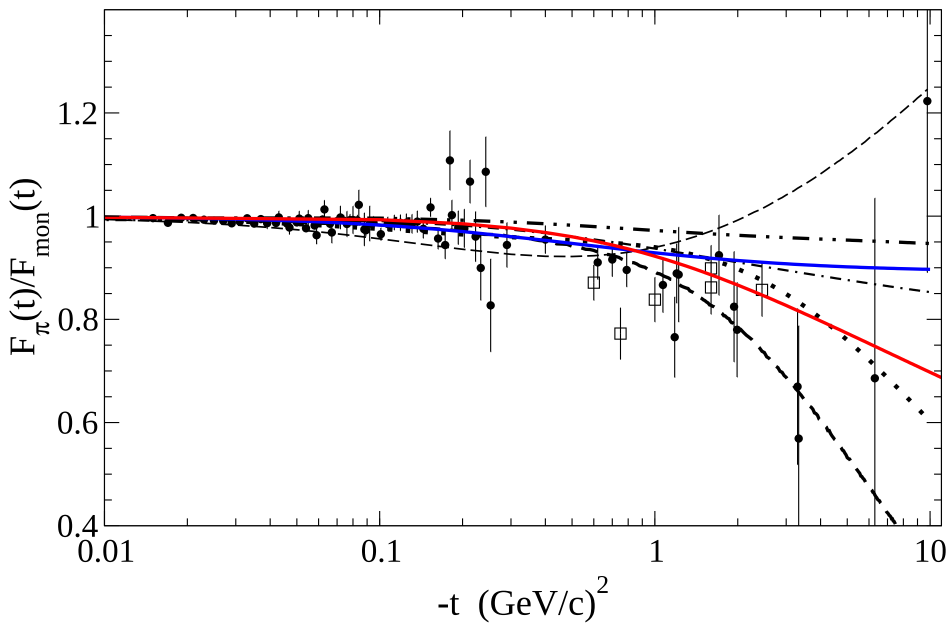
$$Tr[S(k-P)\gamma^5 S(k+\Delta/2)\gamma^+\gamma^j S(k-\Delta/2)\gamma^5]\,\Lambda(k-P,p)$$

$N_c = 3$ is the number of colors $\qquad \mathcal{R} = 2m^2/f_\pi^2$

The δ function imposes the active quark support $-|\xi| \leq x \leq 1$

Pion form factor

$$\int_{-1}^{1} dx\, H^{I=1}(x,\xi,t) = F_\pi(t)$$



Thin dashed line: covariant model, with the sum-form for BSA

Dotted line: covariant model, with the product-form BSA

Blue line and Red line: monopole and faster than monopole fit to Lattice data [Brommel et al., Eur. Phys. J. C51 (2007) 335]

Thick dashed line: Light-Front Hamiltonian dynamics (fixed number of d.o.f.) model with a Gaussian pion wave function.

$F_{mon}(t) = 1/(1 + |t|/m_\rho^2)$ $\qquad m_\rho = 770$ MeV

Models with an asymptotic decay slower than $F_{mon}(t)$, as the covariant sum-form model, yield a divergent charge density at short range.

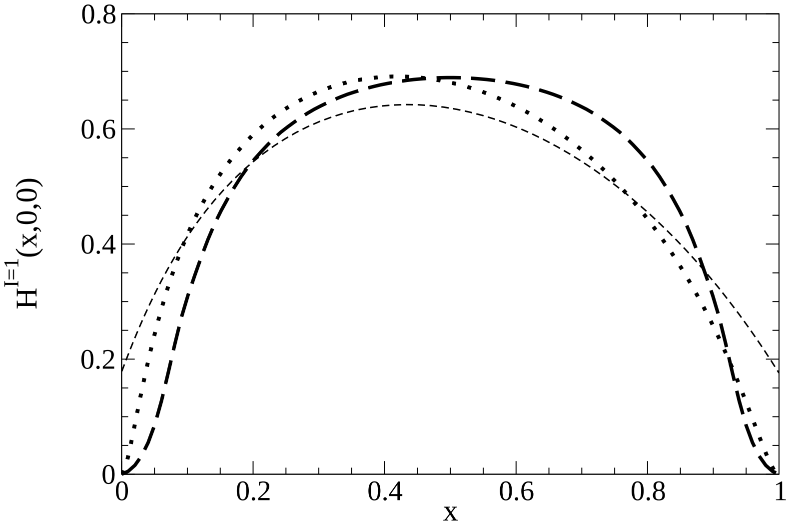
Pion longitudinal momentum distribution

From Deep Inelastic Scattering

$$u(x) = H^u(x,0,0) = 2\, H^{I=1}(x,0,0)$$

$$u(x) = \int d\mathbf{k}_\perp \; f_1(x, |\mathbf{k}_\perp|^2), \qquad (x \geq 0) \;\; .$$

At $\xi = 0$ the variable x coincides with the longitudinal fraction x_q



Thin dashed line: covariant model with the sum-form BSA

Dotted line: covariant model with the product-form BSA

Thick dashed line: LFHD model with a Gaussian wave function

Sum-form BSA is unable to yield vanishing values at the end points.

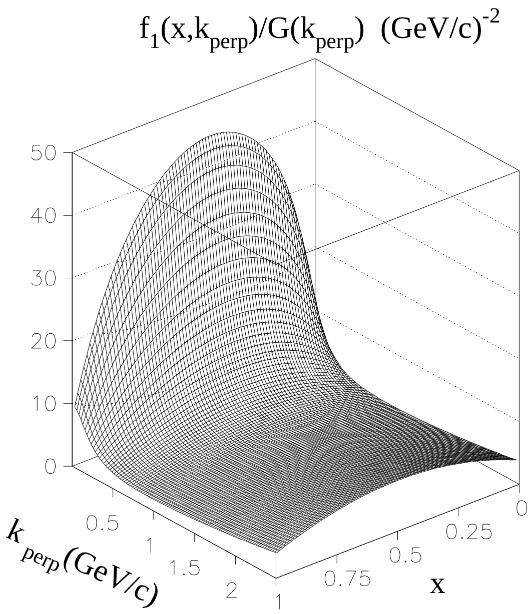
The covariant product-form model with a $|k_\perp|^4$ decay of the BSA, compatible with a BSE kernel dominated by the one-gluon-exchange (OGE), gives a consistent description of the tail of the form factor and of the end-point fall-off of the parton distribution.

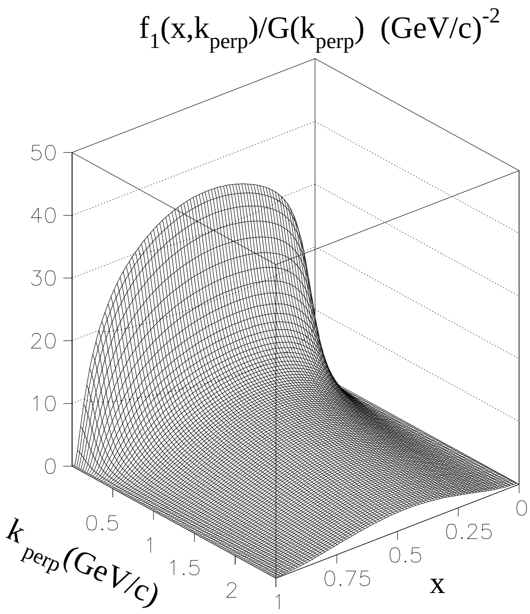
Both at $\;-t \to \infty\;$ and at $\;x \to 0\;$ or at $\;x \to 1\;$ the high momentum part of the pion state is probed.

Transverse-momentum dependent function, $f_1(x, |\boldsymbol{k}_\perp|^2)$

covariant symmetric model

sum-form BSA | product-form BSA





$G(|\boldsymbol{k}_\perp|) = 1/(1 + |\boldsymbol{k}_\perp|^2/m_\rho^2)^4$ $\quad\quad k_{perp} = |\mathbf{k}_\perp|$

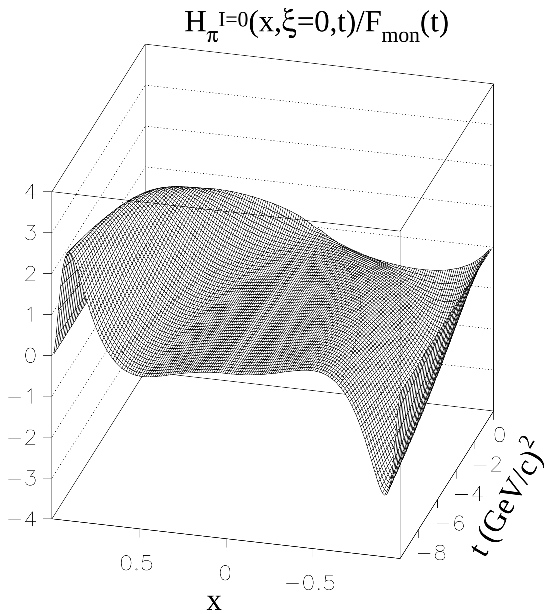
The normalization is given by $\int_0^1 dx \int d\mathbf{k}_\perp \; f_1(x, |\mathbf{k}_\perp|^2) = 1$

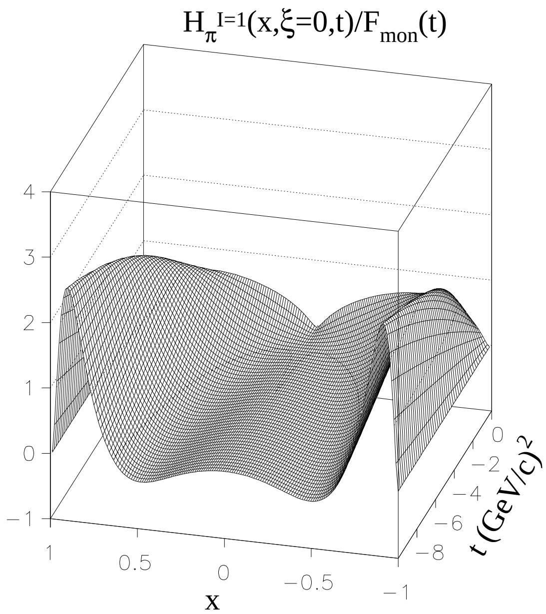
The product-form model has a faster $|k_\perp|$ falloff than the sum-form model.

Vector Generalized Parton Distributions $\xi = 0$

Isoscalar Isovector

covariant symmetric model (product-form BSA)





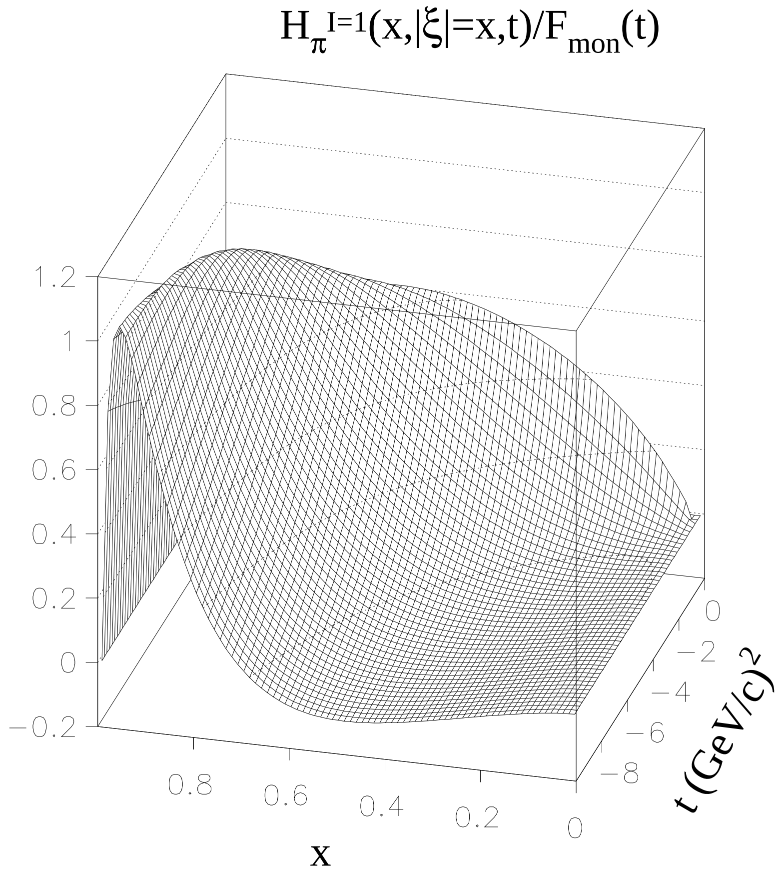
$\xi = 0 \quad \Rightarrow \quad$ valence region

As already noticed, as $-t \rightarrow \infty$, the maximum of GPD's moves from $x = 0.5$ towards $x = 1$.

Isovector Generalized Parton Distributions $|\xi| = x, \quad m_\pi = 0$

At $|\xi| = x$ one explores the transition from valence to non valence region. This kinematical regime should be relevant to study single spin asymmetry [Diehl, Phys.Rep. 388 (2003) 41].

covariant symmetric model (product form BSA)

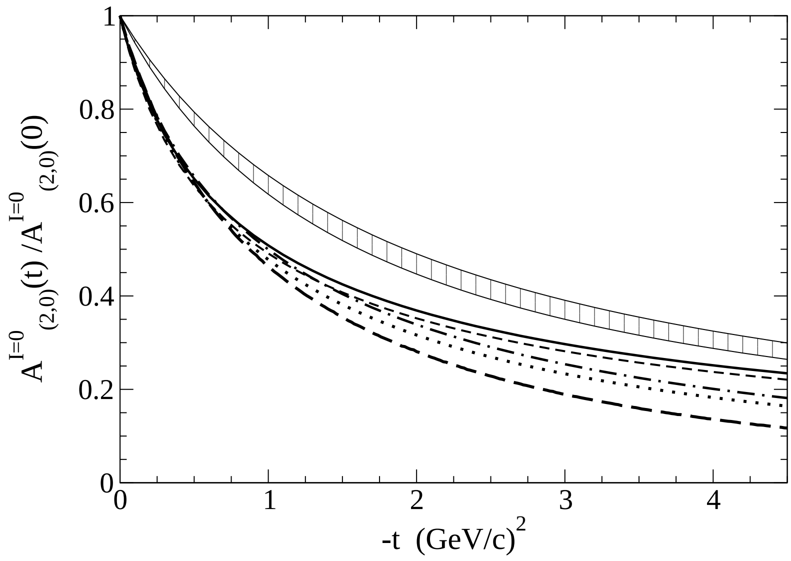


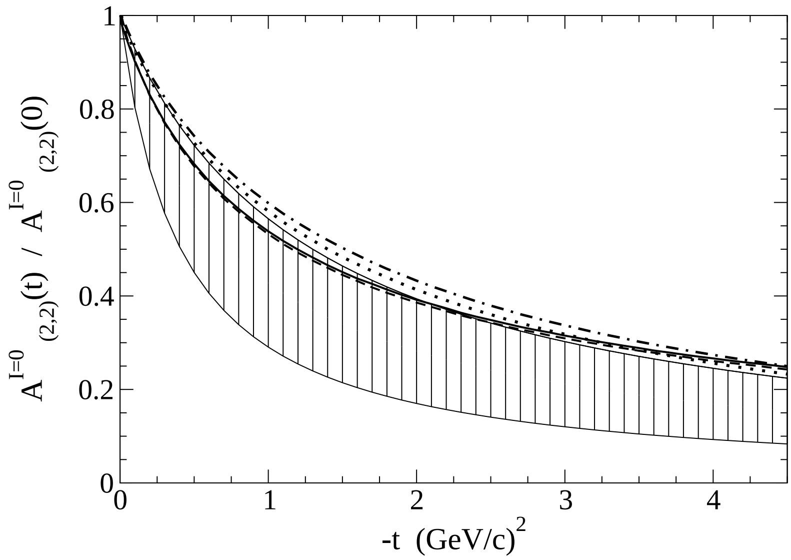
N.B. $|t| \rightarrow \infty$, the maximum of GPD moves from $x = 0.5 \rightarrow x = 1$.

. A similar analysis can be performed for the Tensor GPD

Generalized Form Factors

Vector isoscalar GFF $A_{2,0}^{I=0}$ and $A_{2,2}^{I=0}$





Solid, dashed and dotted lines : our model results with no evolution

Shaded area : Lattice results extrapolated at the physical pion mass and evaluated at an energy scale $\mu = 2$ GeV [Brommel et al. Phys. Rev. Lett. 101 (2008) 122001].

Ratio $A(t)/A(0)$ has been reported to get rid of the multiplicative effect of evolution [Broniowsky, Phys. Rev. D 82 (2010) 094001]

From Tensor GPD's

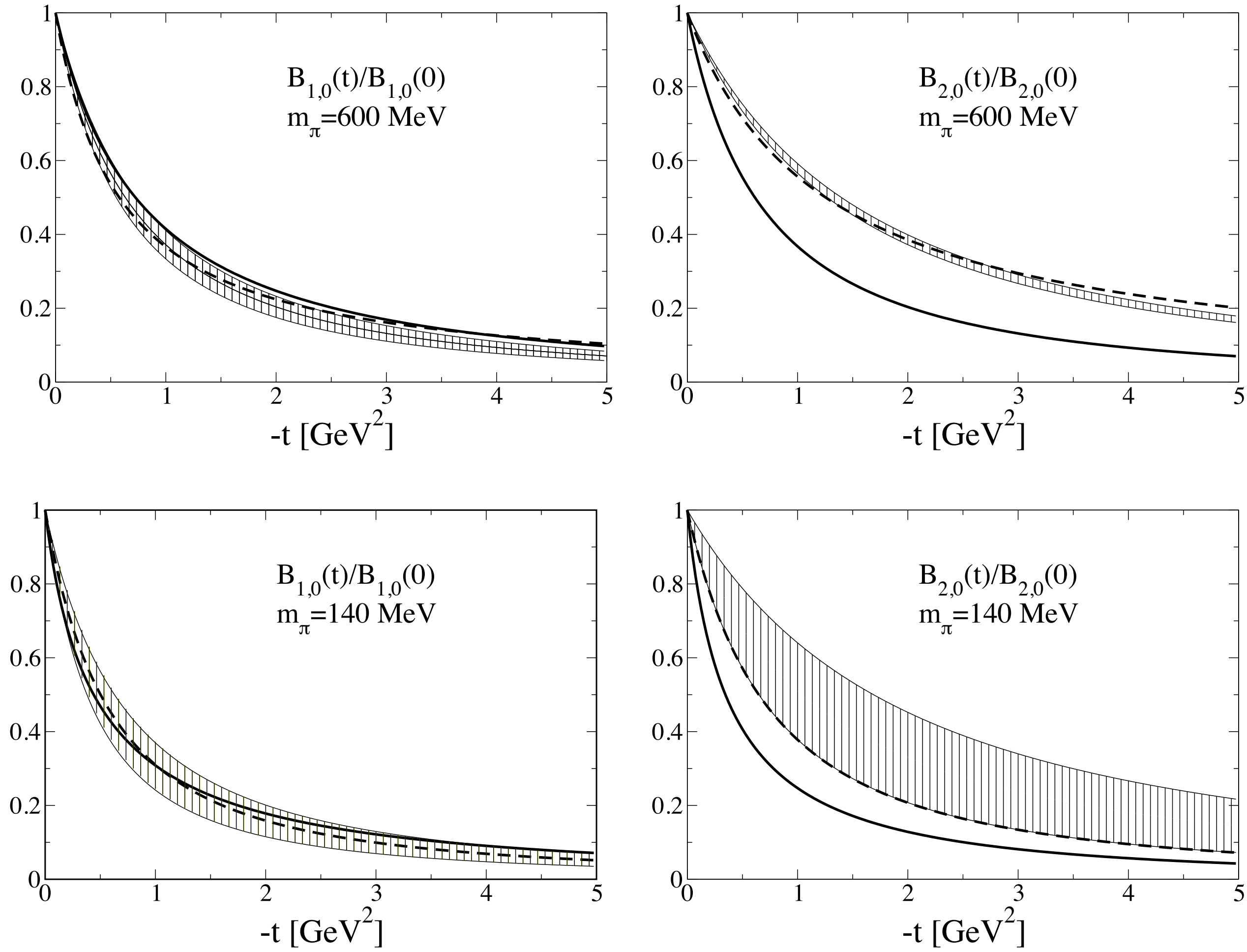


Fig. 4 Tensor GFFs, $B_{p,0}(t)/B_{p,0}(0)$. Left panels: $p = 1$; right panels: $p = 2$. Upper panels: $m_\pi = 600$ MeV; lower panels $m_\pi = 140$ MeV. Solid lines are the results of our CCQM. Shaded area: lattice data (upper and lower curves indicate the error band) [6]; dashed lines in the upper panels: χQM [7]; dashed lines in the lower panels: instanton vacuum model [8].

Impact Parameter Space

To get a density distribution of polarized quarks in the IPS, the Fourier Transform of the GFF is needed:

$$A_n(b_\perp) = \int d\vec{\Delta}_\perp e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} A_{n,0}(\Delta^2)$$

$$B_n(b_\perp) = \int d\vec{\Delta}_\perp e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} B_{n,0}(\Delta^2)$$

$A_n(b_\perp)$ yields the GFF in the IPS, and represents the probability density of finding an unpolarized parton in the pion at a certain distance $b_\perp$ from the transverse center of mass.

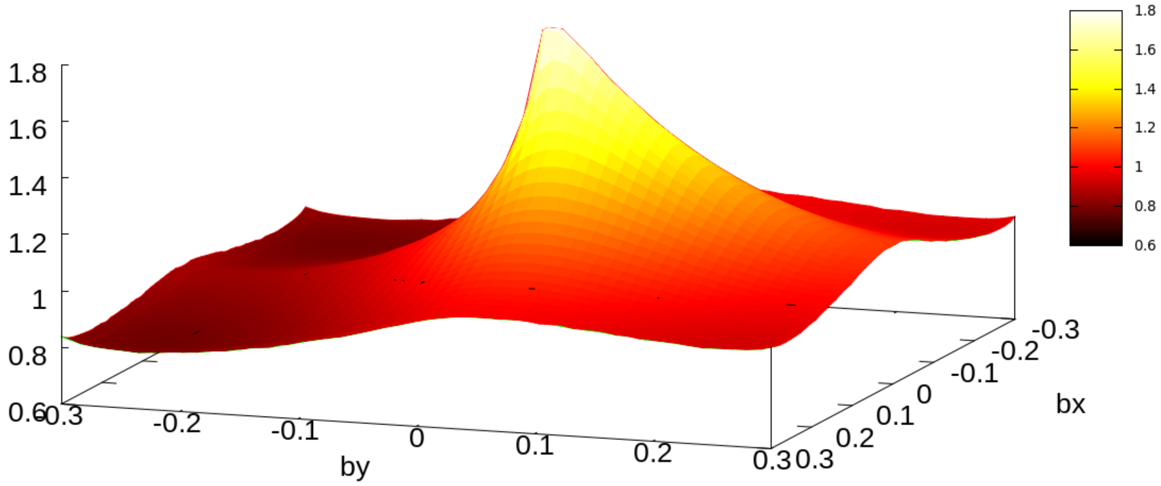
The probability density of finding a parton of a fixed transverse polarization, $\vec{s}$, at a certain distance $\vec{b}_\perp$ from the transverse center of mass is given by

$$\rho_n(\vec{b}_\perp, \vec{s}) = \frac{1}{2}\left[A_n(b_\perp) + \frac{s^i \epsilon^{ij} b^j}{b_\perp}\,\Gamma_n(b_\perp)\right]$$

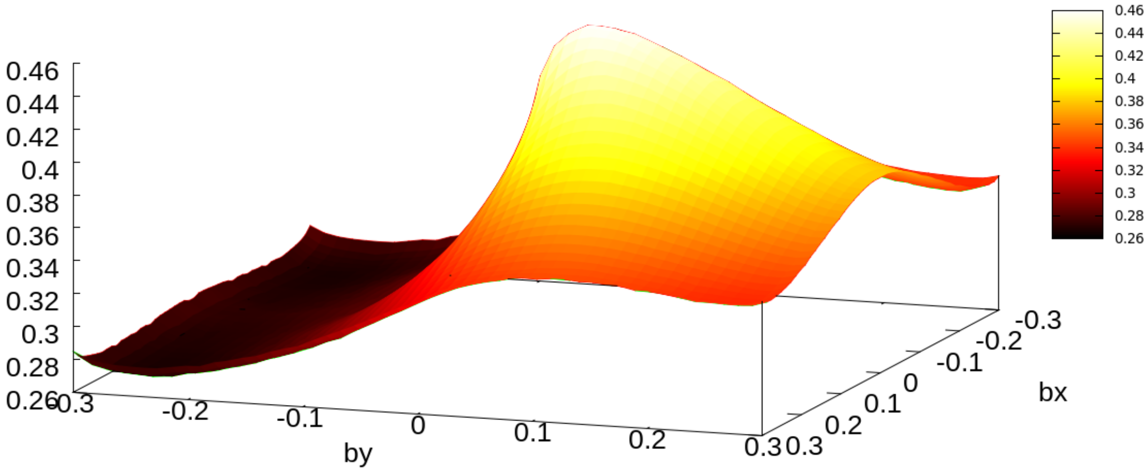
where

$$\Gamma_n(b_\perp) = -\frac{1}{2m_\pi}\frac{\partial\, B_n(b_\perp)}{\partial\, b_\perp}$$

$\rho_1(\vec{b}, s\hat{x})$



$\rho_2(\vec{b}, s\hat{x})$



The next step:
the Nakanishi representation of BSA

The successful description of the em observables of the pion, though not too much compelling with respect to other approaches, seems to suggests, within our approach, that the product-form of the Bethe-Salpeter amplitude is phenomenologically more effective.

The product form of the momentum part of the vertex function can be rewritten as

$$\Lambda = \frac{C}{\left[\left(\frac{p}{2}+k\right)^2 - m_R^2 + \imath\epsilon\right]\left[\left(\frac{p}{2}-k\right)^2 - m_R^2 + \imath\epsilon\right]} =$$

$$= \int_{-1}^{1} dz \int_{-\infty}^{\infty} d\gamma' \frac{\delta(\gamma' - m_R^2 + m^2)}{\left[\gamma' + k^2 + zk\cdot p - \kappa^2 + i\epsilon\right]^2}$$

with

$$\kappa^2 = m^2 - \frac{m_\pi^2}{4} \;\; \geq 0$$

This strongly **suggests** to resort to the Nakanishi integral representation of the Bethe-Salpeter Amplitude, based on the parametric form of the Feynman diagrams.

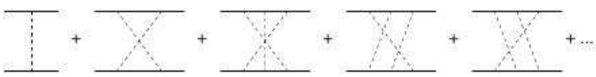
Nakanishi pointed out that the analytical behavior of each Feynman diagram (namely the denominator) can be put in the same, general form, through a smart change of variables (*once more a change of variables!*). This allowed him to give the general form of any multi-leg amplitude (two-leg vertex function, four-leg T-matrix, etc.)

For a simple, two-boson system interacting through the exchange of a boson (all massive), the vertex function (bound state) can be written within the Nakanishi integral representation as

$$\Gamma_b(k,p) = \int_{-1}^{1} dz \int_{-\infty}^{\infty} d\gamma' \frac{g(\gamma',z)}{[\gamma' + k^2 + zk\cdot p - \kappa^2 + i\epsilon]^n}$$

where the real function $g(\gamma', z)$ is the Nakanishi amplitude and $n \geq 1$. This expression can be seen as the result of an infinite sum of Feynman diagrams, all with the same denominator, after introducing the Nakanishi change of variables.

Once we have singled out the explicit analytic behavior, as already said, the Light-front formalism allows one to perform analytical integration in a very effective way. Then, one can find the integral equation to be fulfilled by the Nakanishi amplitude directly from the BS equation, where the kernel is given at some approximation order: ladder, cross-ladder etc. (J. Carbonell, V. Karmanov, EPJA 2006; EPJA 2011)

$i\,\mathcal{K} \equiv$ 

For the two-boson case, one has

$$\int_0^\infty d\gamma' \, \frac{g_b(\gamma', z)}{[\gamma' + \gamma + z^2 m^2 + (1-z^2)\kappa^2 - i\epsilon]^2} =$$

$$= \int_0^\infty d\gamma' \int_{-1}^{1} dz' \; V_b(\gamma, z; \gamma', z') g_b(\gamma', z').$$

where the kernel V_b, is related to the kernel $i\mathcal{K}$ in the BS equation, as follows

$$V_b(\gamma, z; \gamma', z') = \; ip^+ \; \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} \; G_0^{(12)}(k,p) \;\times$$

$$\int \frac{d^4k'}{(2\pi)^4} \frac{i\mathcal{K}(k,k',p)}{\left[k'^2 + p \cdot k' z' - \gamma' - \kappa^2 + i\epsilon\right]^3}$$

The integral equation for the Nakanishi amplitude can be put in a simpler form by using the uniqueness of the solution and furthermore it can be extended to the scattering states (T. Frederico, M. Viviani and G.S. in preparation)

Conclusions & Perspectives

- A simple covariant constituent quark model for the pion Bethe-Salpeter amplitude has been adopted for investigating the pion observables, that can be studied both by means of elastic electron scattering and DVCS. Moreover, beside the Generalized Parton distributions, the generalized form factors, more easily accessed by Lattice calculations, have been evaluated.

- It has been shown the relevance of a q-π vertex compatible with the OGE dominance, i.e. the product form, for describing the tail of the form factor and for obtaining a vanishing parton distribution at the end points.

- This phenomenological findings point to the application of a more fundamental approach, based on the Nakanishi representation of the BSA, and the consequent solutions of simple integral equation with a clear dynamical content.