Models of interaction and few-body problems in ultra-cold physics

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Collaborations:

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Orders of magnitude in ultracold atoms

- Typical size of traps: few μ m
- Number of atoms: 10^5
- Atomic density $10^{13} \lesssim n \lesssim 10^{15}$ atoms/cm^3
- Temperature $1\ \mathrm{nK} \lesssim T \lesssim \mu$ K
- Range of the interactions: $b \sim 10$ nm

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Scale separation:

$$\frac{\hbar^2 n^{2/3}}{m}, k_B T, \mu \cdots \ll \frac{\hbar^2}{m b^2} \sim m\,\mathrm{K}$$

Low energy: collective modes, bound states near scattering resonances . . .

“High” energy: usual molecules, clusters . . .

Tunability

- **Number of atoms and Temperature**

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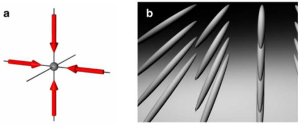
- **Species (bosons or/and fermions)**:
  - Alkali: ^{6}Li,^{7}Li,^{23}Na,^{39}K,^{40}K,^{85}Rb,^{87}Rb,^{133}Cs
  - more exotic: ^{52}Cr, ^{4}He*,^{84}Sr, ^{171}Yb, ^{173}Yb
  - ...and also polar molecules: (^{40}K-^{87}Rb)

. . .

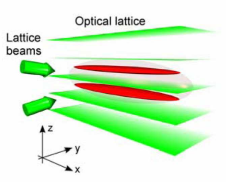
Tunability

- **High control of the trapping frequencies** isotropic or highly anisotropic traps

$\rightarrow$ quasi-1D or quasi-2D & optical lattice



I. Bloch *et al.* (MPQ-Garching)



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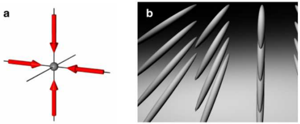

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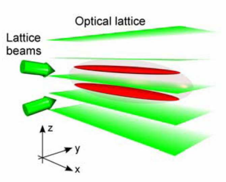


J. Dalibard *et al.* (LKB-Paris)

- **effective interaction of arbitrary strength**

Tragic fate of the gas

. . . But one essential fact:

the N-body ground state at T=0K is a solid 

$\Leftrightarrow$ many deep bound states for $2, 3, \ldots N$-atoms

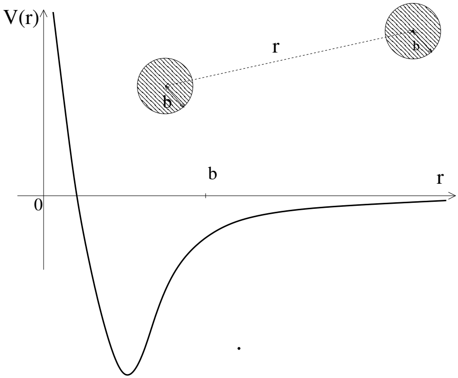
typical lifetime: few seconds to few minutes

Interaction between two atoms

- Large distance:

$$V(r) \simeq \frac{C_6}{r^6} \Rightarrow b = \left(\frac{mC_6}{\hbar^2}\right)^{1/4}$$

- Short distance: hard-core



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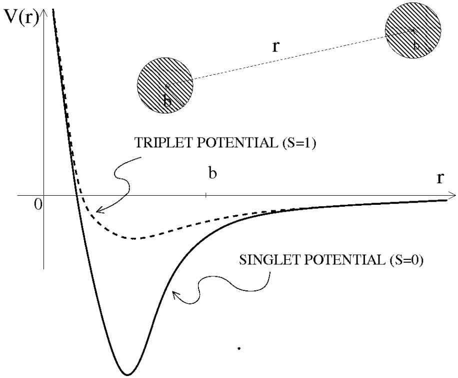
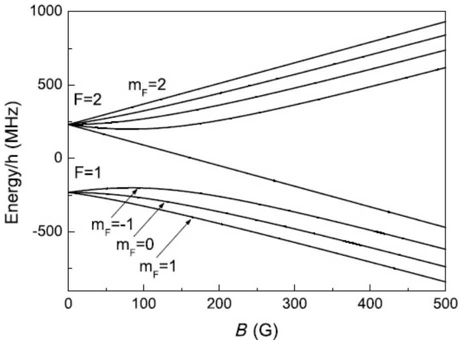


Atomic states in a magnetic field

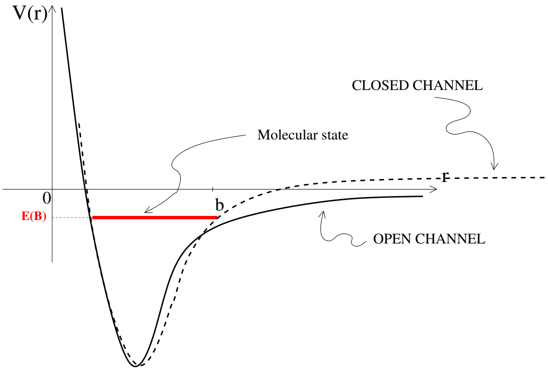


Example of the hyperfine states of Rb
[Chiara D'Errico et al., New J. Phys. **9** 223 (2007)].

$$\mathbf{F} = \mathbf{L} + \mathbf{S} + \mathbf{I}$$

Good quantum number: m_F

Magnetic Feshbach resonance

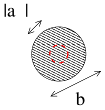


Resonant coupling

(Open channel) $\longleftrightarrow$ (molecular state)

Molecular state energy adjusted with an external magnetic field $\mathcal{B}$

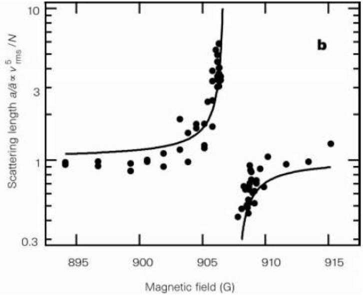
Tuning the *s* wave scattering length



$$-\infty < a < +\infty$$

s wave Feshbach resonance

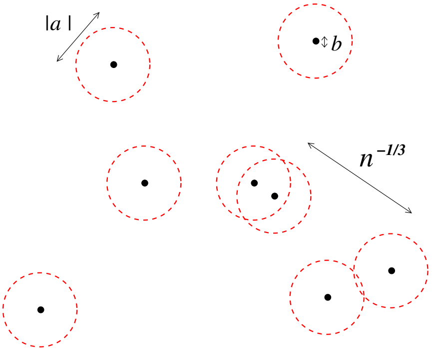
First achievement: group of Ketterle - 1998



s wave resonance $|a| \gg b$

$$a = a_{\rm bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

Scales for a broad *s*-wave resonance



$$b \ll |a|$$

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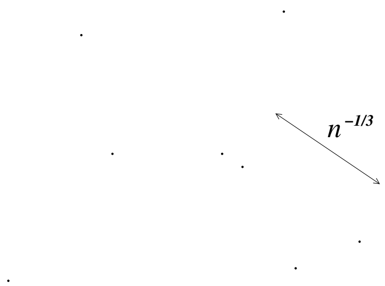
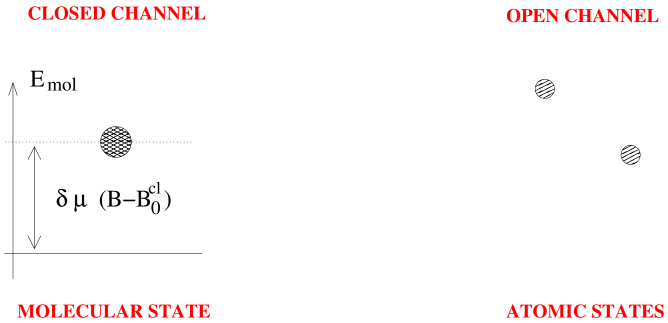


Unitary limit $b \to 0$ and $|a| = \infty$

Separable two-channel model

Y. Castin, M. Jona-Lasinio, C. Mora and L. P.



- $E_{\rm mol}$: molecular state energy

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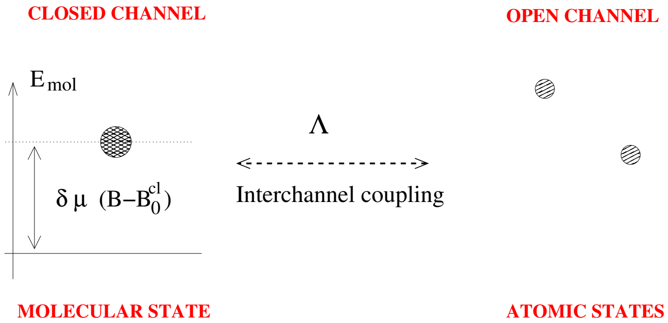

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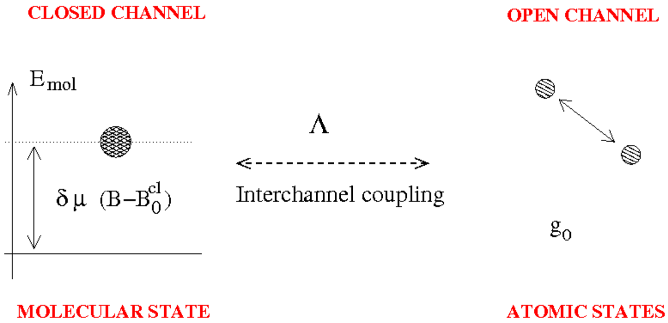

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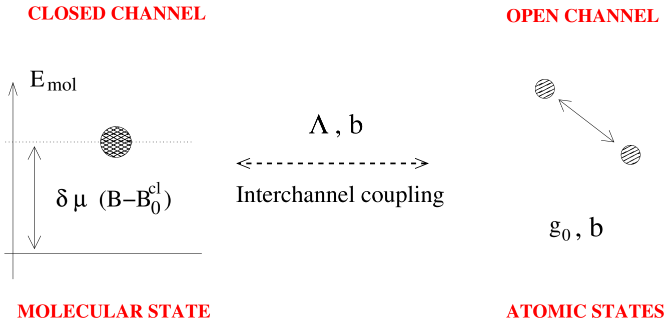


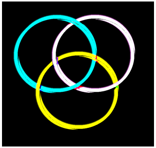
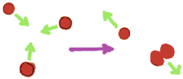
- E_{mol}: molecular state energy
- Λ: coupling strength (molecule-atomic pair)
- g_0: strength of the direct (atom-atom) interaction
- $b \sim$ range of the microscopic forces

Three identical bosons

$$|\Psi\rangle = (\text{atom} \otimes \text{atom} \otimes \text{atom}) \oplus \underbrace{(\text{atom} \otimes \text{molecule})}_{|\beta\rangle}$$

Equation for $|\beta\rangle \equiv$ generalized Skorniakov Ter Martirosian equation

Two issues:

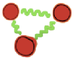
- Efimov states 
- Three-body recombination 

$$\frac{dN_{\mathrm{dim}}}{dt} = -\alpha_{\mathrm{rec}} \frac{N_{\mathrm{at}}(N_{\mathrm{at}}-1)(N_{\mathrm{at}}-2)}{L^6}.$$

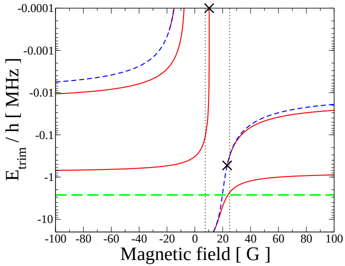
Theory *vs* experiments

M. Jona-Lasinio and L. P., Phys. Rev. Lett. **104**, 023201 (2010)

Trimers (^{133}Cs)



Group of R. Grimm (Innsbruck)



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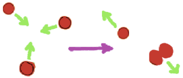

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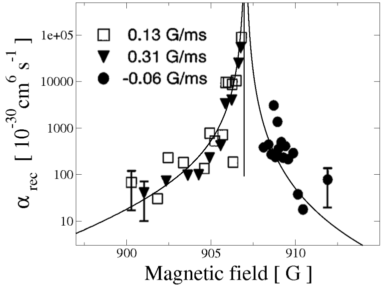

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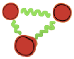

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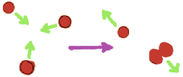

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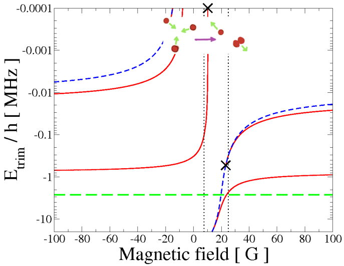

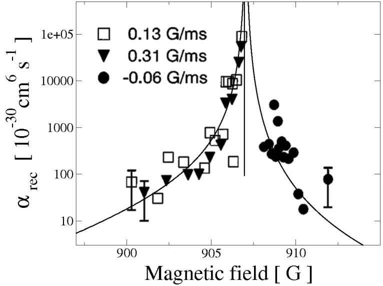

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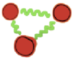

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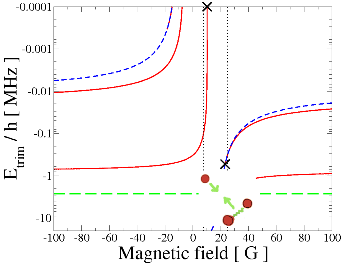

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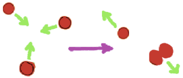

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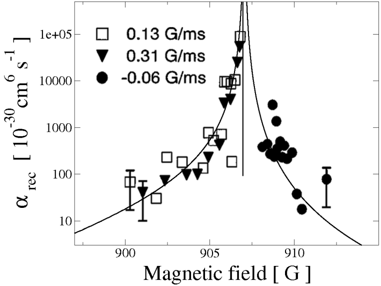

Three-body recombination near a narrow resonance

Resonance width $\Delta\mathcal{B} \Longrightarrow$ radius $R^\star = \dfrac{\hbar^2}{m a_{\rm bg} \delta\mu \Delta\mathcal{B}}$

Narrow resonance: $R^\star \gg |a_{\rm bg}|, b$

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- Efimov regime $|a| \gg |a_{\mathrm{bg}}|, R^\star$

  3-body parameter $= f(R^\star)$ (D.M. Petrov -2004)

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- Efimov regime $|a| \gg |a_{\rm bg}|, R^\star$

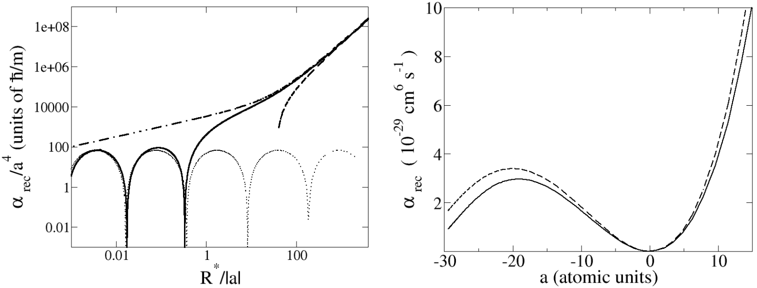
  3-body parameter $= f(R^\star)$ (D.M. Petrov -2004)
- Intermediary regime $R^\star(a - a_{\rm bg}) \gg a^2$

  $a_{\rm bg} < b\sqrt{\pi} \longrightarrow \exists$ only 1 shallow dimer: $E \simeq -\frac{\hbar^2}{mR^\star(a-a_{\rm bg})}$

$$\alpha_{\rm rec} \underset{R^\star \gg |a|}{=} \frac{192\pi^2\hbar}{m} \times a^2 \times \sqrt{3R^\star\left(a - a_{\rm bg}\right)^3} + \ldots$$

Prediction: Example of the Potassium

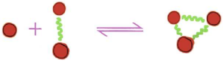
Narrow resonance near 752 G ($a_{\rm bg} < b\sqrt{\pi}$)



L.P. & M. Jona-Lasinio (arXiv:1109.3002).

Fano-Efimov resonances

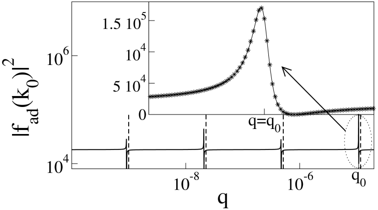
$$a_{\rm bg} > b\sqrt{\pi} \longrightarrow \exists \text{ dimer for } |a| = \infty$$

Resonances in the atom-dimer scattering 

Low energy model $\longrightarrow$ computation in the regime $R^\star \gg a_{\rm bg} \gg b$

Scattering amplitude $f_{\rm ad}$:
(units of the three-body parameter)

$$E = -\frac{\hbar^2 q^2}{m} = E_{\rm dimer} + \frac{3\hbar^2}{4m} k_0^2$$



L.P., Phys Rev A **82** 043633 (2010).

Searching for an exotic Efimov effect . . .

Three identical fermions

M. Jona-Lasinio, L. P. and Y. Castin, Phys. Rev. A **77**, 043611 (2008)

2 relevant parameters near the resonance: $f_1 = \frac{-1}{\frac{1}{k^2 \mathcal{V}_s} + \alpha_{\mathrm{res}} + ik}$

p wave resonance: $|\mathcal{V}_{\mathrm{s}}| \gg b^3$:

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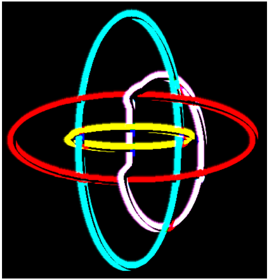
- $\alpha_{\mathrm{res}} \sim 1/b$: $\exists$ 2 Borromean states ... no Efimov effect
- 3-body recombination: universality without Efimov effect

$$\frac{dN_{\mathrm{dim}}}{dt} = 3(\frac{48\pi}{5})^2 \times \frac{\hbar k_F^4 N n^2}{m} \times \left(\frac{\mathcal{V}_{\mathrm{s}}^5}{3\alpha_{\mathrm{res}}} \right)^{\frac{1}{2}}$$

A 4-body Efimov effect at $|a| = \infty$?

Y. Castin, C. Mora, L. Pricoupenko, Phys. Rev. Lett. **105** 223201 (2010).



If $\exists$ 3-body Efimov effect $\Rightarrow$ no 4-body Efimov states

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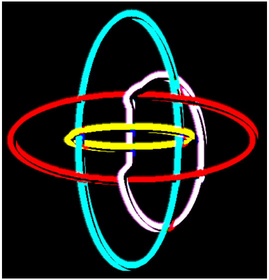
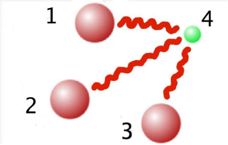


If $\exists$ 3-body Efimov effect $\Rightarrow$ no 4-body Efimov states

$\Rightarrow$
- < 2 interacting bosons
- 2 fermions $(M) \oplus 1$ impurity $(m) \longrightarrow m/M < 13.607$

3 identical fermions (M) $\oplus$ 1 impurity (m)



Efimov states for $\left(\frac{M}{m}\right)_{\text{crit}} < \frac{M}{m} < 13.607$?

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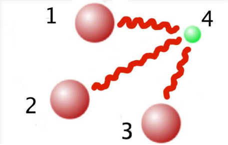

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Zero-range potential model for $\Psi(\mathbf{r}_1, \ldots \mathbf{r}_4)$:

diverges at $r_{i4} = 0$ $(i = 1 \ldots 3)$ as $D^{i \rightleftharpoons 4} \times \left(\frac{1}{a} - \frac{1}{r_{i4}}\right)$

[for fixed values of $M\mathbf{r}_4 + m\mathbf{r}_i$ and r_j $(j \neq i, 4)$]

solves the free-Schrodinger equation for $r_{i4} \neq 0$.

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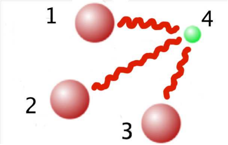

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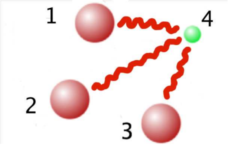

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- Translation invariance $\Rightarrow$ momentum representation

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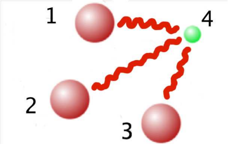

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[for fixed values of $M\mathbf{r}_4 + m\mathbf{r}_i$ and r_j ($j \neq i, 4$)]

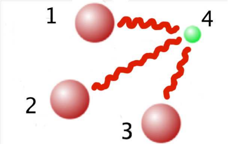
solves the free-Schrodinger equation for $r_{i4} \neq 0$.

Problem solved $\equiv$ determine $D^{i \leftrightharpoons 4}$

- Translation invariance $\Rightarrow$ momentum representation
- Contact condition + center of mass frame

$$D^{1 \leftrightharpoons 4} = D(\mathbf{k}_2, \mathbf{k}_3)$$

3 identical fermions (M) $\oplus$ 1 impurity (m)



Efimov states for $\left(\frac{M}{m}\right)_{\rm crit} < \frac{M}{m} < 13.607$?

Zero-range potential model for $\Psi(\mathbf{r}_1, \ldots \mathbf{r}_4)$:

diverges at $r_{i4} = 0$ ($i = 1 \ldots 3$) as $D^{i\leftrightharpoons 4} \times \left(\frac{1}{a} - \frac{1}{r_{i4}}\right)$

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- Translation invariance $\Rightarrow$ momentum representation
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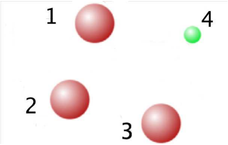
$$D^{1\leftrightharpoons 4} = D(\mathbf{k}_2, \mathbf{k}_3)$$

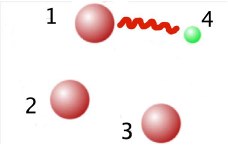
- Statistics $\Rightarrow$ $D^{2\leftrightharpoons 4} = -D(\mathbf{k}_1, \mathbf{k}_3)$ and $D^{3\leftrightharpoons 4} = -D(\mathbf{k}_2, \mathbf{k}_1)$.

Equation for the source amplitude D



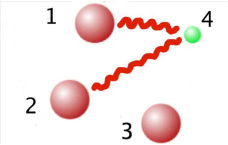
$$\left[\sum_{i=1}^{3} \frac{\hbar^2 k_i^2}{2M} + \frac{\hbar^2 k_4^2}{2m} - E\right] \langle \{\mathbf{k}_i\} | \Psi \rangle = 0$$

Equation for the source amplitude D



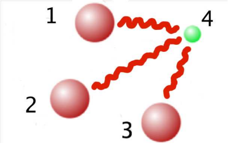
$$\left[\sum_{i=1}^{3}\frac{\hbar^2 k_i^2}{2M}+\frac{\hbar^2 k_4^2}{2m}-E\right]\langle\{\mathbf{k}_i\}|\Psi\rangle = -\frac{2\pi\hbar^2}{\mu}D(\mathbf{k}_2,\mathbf{k}_3)$$

Equation for the source amplitude D



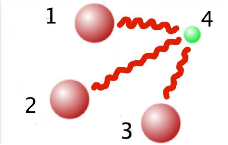
$$\left[\sum_{i=1}^{3}\frac{\hbar^2 k_i^2}{2M}+\frac{\hbar^2 k_4^2}{2m}-E\right]\langle\{\mathbf{k}_i\}|\Psi\rangle = \frac{2\pi\hbar^2}{\mu}[-D(\mathbf{k}_2,\mathbf{k}_3)+D(\mathbf{k}_1,\mathbf{k}_3)]$$

Equation for the source amplitude D



$$\left[\sum_{i=1}^{3} \frac{\hbar^2 k_i^2}{2M} + \frac{\hbar^2 k_4^2}{2m} - E\right] \langle \{\mathbf{k}_i\} | \Psi \rangle = \frac{2\pi\hbar^2}{\mu} [-D(\mathbf{k}_2, \mathbf{k}_3) + D(\mathbf{k}_1, \mathbf{k}_3) + D(\mathbf{k}_2, \mathbf{k}_1)]$$

Equation for the source amplitude D



$$\left[\sum_{i=1}^{3}\frac{\hbar^2 k_i^2}{2M}+\frac{\hbar^2 k_4^2}{2m}-E\right]\langle\{\mathbf{k}_i\}|\Psi\rangle = \frac{2\pi\hbar^2}{\mu}[-D(\mathbf{k}_2,\mathbf{k}_3)+D(\mathbf{k}_1,\mathbf{k}_3)+D(\mathbf{k}_2,\mathbf{k}_1)]$$

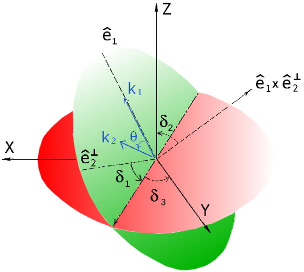
Contact condition $1 \leftrightharpoons 4$ for $E \leq 0$

$$\int \frac{d^3k_1}{2\pi^2}\frac{\mathrm{D}(\mathbf{k}_1,\mathbf{k}_3)+\mathrm{D}(\mathbf{k}_2,\mathbf{k}_1)}{\sum_{i=1}^{3}k_i^2+\frac{2M}{m+M}\sum\limits_{1\leq i<j\leq 3}\mathbf{k}_i\cdot\mathbf{k}_j-\frac{2ME}{\hbar^2}} = -\frac{\mathrm{D}(\mathbf{k}_2,\mathbf{k}_3)}{f(k_{\mathrm{rel}}^{1\leftrightharpoons 4})}$$

[Rq: $f(k) = -\dfrac{a}{1+ika}$ and $k_{\mathrm{rel}}^{1\leftrightharpoons 4}$, relative wave number (imaginary) of the pair (14)]

Symmetries considerations on $D(\mathbf{k}_1, \mathbf{k}_2)$

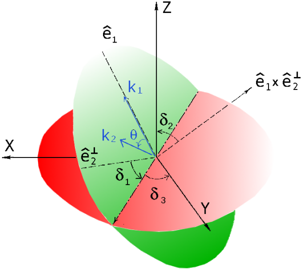
- Isotropic integral kernel:



$$D(\mathbf{k}_1, \mathbf{k}_2) = \sum_{l,m} Y^*_{lm}(\delta_2, \delta_1) f_{lm}(k_1, k_2, \theta)$$

Symmetries considerations on $D(\mathbf{k}_1, \mathbf{k}_2)$

- Isotropic integral kernel:



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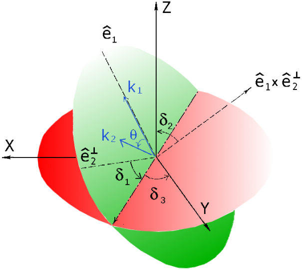

$$D(\mathbf{k}_1, \mathbf{k}_2) = \sum_{l,m} Y^*_{lm}(\delta_2, \delta_1) f_{lm}(k_1, k_2, \theta)$$

- Statistics: $D(\mathbf{k}_1, \mathbf{k}_2) = -D(\mathbf{k}_2, \mathbf{k}_1)$
- Unitarity $\rightarrow$ scale invariance: $D(\lambda\mathbf{k}_1, \lambda\mathbf{k}_2) = \lambda^{-\frac{7}{2}\pm s} D(\mathbf{k}_1, \mathbf{k}_2)$

4-body Efimov effect

- Integral equations on $f_{lm} \Longrightarrow$ set of exponents $\{s\}$

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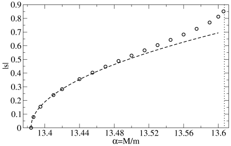
- Efimov effect $\leftrightarrow$ s imaginary



$\longrightarrow$ fix the phase for the asymptotic behavior of D as

$$\sqrt{{k_2}^2 + {k_1}^2} \to \infty$$

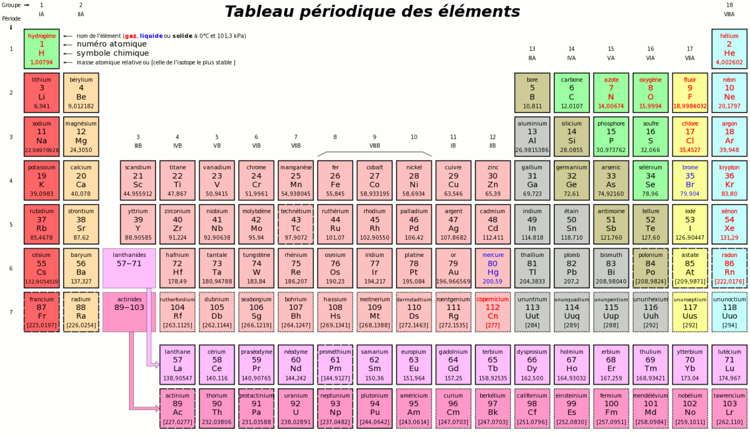
$\longrightarrow$ Spectrum with an accumulation point at $E = 0$:

$$E_n = E_{\mathrm{ref}} \times \exp\left(\frac{2n\pi}{|s|}\right)$$

Three gifts of nature

$$13.384 < \frac{M}{m} < 13.607$$

$^3He^*$-^{40}K: 13.25



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^{7}Li-^{95}Mo: 13.53

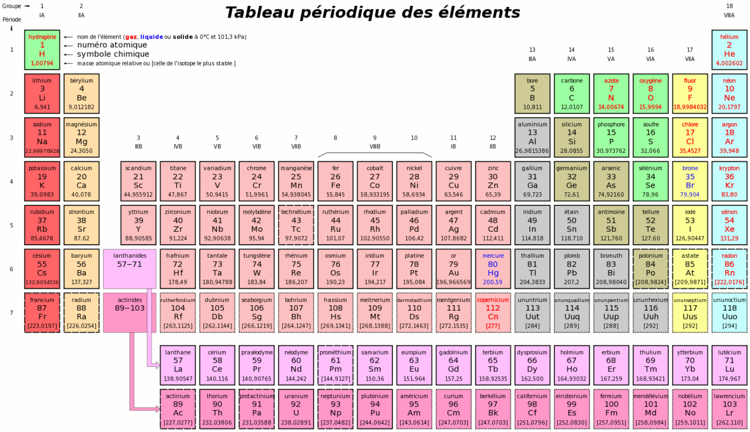

Tableau périodique des éléments

Groupe → / Période ↓

← nom de l'élément (gaz, liquide ou solide à 0°C et 101,3 kPa)
← numéro atomique
← symbole chimique
← masse atomique relative ou [celle de l'isotope le plus stable]

Période	1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8 VIIIB	9 VIIIB	10 VIIIB	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
1	hydrogène 1 H 1,00794																	hélium 2 He 4,002602
2	lithium 3 Li 6,941	bérylium 4 Be 9,012182											bore 5 B 10,811	carbone 6 C 12,0107	azote 7 N 14,00674	oxygène 8 O 15,9994	fluor 9 F 18,9984032	néon 10 Ne 20,1797
3	sodium 11 Na 22,98976928	magnésium 12 Mg 24,3050											aluminium 13 Al 26,9815386	silicium 14 Si 28,0855	phosphore 15 P 30,973762	soufre 16 S 32,066	chlore 17 Cl 35,4527	argon 18 Ar 39,948
4	potassium 19 K 39,0983	calcium 20 Ca 40,078	scandium 21 Sc 44,955912	titane 22 Ti 47,867	vanadium 23 V 50,9415	chrome 24 Cr 51,9961	manganèse 25 Mn 54,938045	fer 26 Fe 55,845	cobalt 27 Co 58,933195	nickel 28 Ni 58,6934	cuivre 29 Cu 63,546	zinc 30 Zn 65,39	gallium 31 Ga 69,723	germanium 32 Ge 72,61	arsenic 33 As 74,92160	sélénium 34 Se 78,96	brome 35 Br 79,904	krypton 36 Kr 83,80
5	rubidium 37 Rb 85,4678	strontium 38 Sr 87,62	yttrium 39 Y 88,90585	zirconium 40 Zr 91,224	niobium 41 Nb 92,90638	molybdène 42 Mo 95,94	technétium 43 Tc 97,9072	ruthérium 44 Ru 101,07	rhodium 45 Rh 102,90550	palladium 46 Pd 106,42	argent 47 Ag 107,8682	cadmium 48 Cd 112,411	indium 49 In 114,818	étain 50 Sn 118,710	antimoine 51 Sb 121,760	tellure 52 Te 127,60	iode 53 I 126,90447	xénon 54 Xe 131,29
6	césium 55 Cs 132,9054519	baryum 56 Ba 137,327	lanthanides 57–71	hafnium 72 Hf 178,49	tantale 73 Ta 180,94788	tungstène 74 W 183,84	rhénium 75 Re 186,207	osmium 76 Os 190,23	iridium 77 Ir 194,217	platine 78 Pt 195,084	or 79 Au 196,966569	mercure 80 Hg 200,59	thallium 81 Tl 204,3833	plomb 82 Pb 207,2	bismuth 83 Bi 208,98040	polonium 84 Po [208,9824]	astate 85 At [209,9871]	radon 86 Rn [222,0176]
7	francium 87 Fr [223,0197]	radium 88 Ra [226,0254]	actinides 89–103	rutherfordium 104 Rf [263,1125]	dubnium 105 Db [262,1144]	seaborgium 106 Sg [266,1219]	bohrium 107 Bh [264,1247]	hassium 108 Hs [269,1341]	meitnerium 109 Mt [268,1388]	darmstadtium 110 Ds [272,1463]	roentgenium 111 Rg [272,1535]	copernicium 112 Cn [277]	ununtrium 113 Uut [284]	ununquadium 114 Uuq [289]	ununpentium 115 Uup [288]	ununhexium 116 Uuh [292]	ununseptium 117 Uus [292]	ununoctium 118 Uuo [294]
				lanthane 57 La 138,90547	cérium 58 Ce 140,116	praséodyme 59 Pr 140,90765	néodyme 60 Nd 144,242	prométhium 61 Pm [144,9127]	samarium 62 Sm 150,36	europium 63 Eu 151,964	gadolinium 64 Gd 157,25	terbium 65 Tb 158,92535	dysprosium 66 Dy 162,500	holmium 67 Ho 164,93032	erbium 68 Er 167,259	thulium 69 Tm 168,93421	ytterbium 70 Yb 173,04	lutécium 71 Lu 174,967
				actinium 89 Ac [227,0277]	thorium 90 Th 232,03806	protactinium 91 Pa 231,03588	uranium 92 U 238,02891	neptunium 93 Np [237,0482]	plutonium 94 Pu [244,0642]	américium 95 Am [243,0614]	curium 96 Cm [247,0703]	berkélium 97 Bk [247,0703]	californium 98 Cf [251,0796]	einsteinium 99 Es [252,0830]	fermium 100 Fm [257,0951]	mendélévium 101 Md [258,0984]	nobélium 102 No [259,1011]	lawrencium 103 Lr [262,110]

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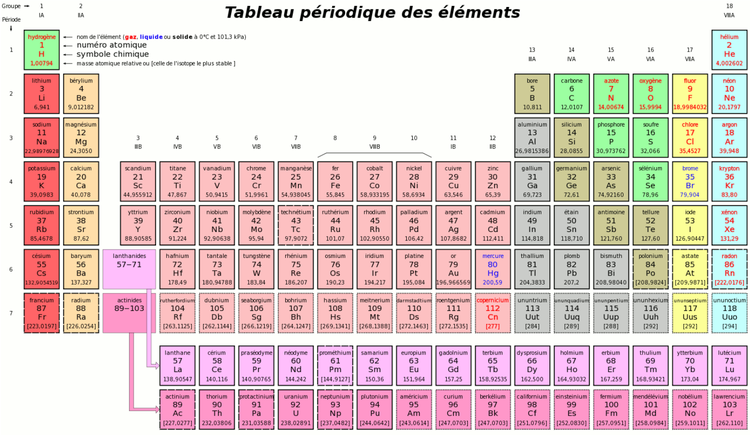
$$^{11}\text{B-}^{149}\text{Sm}: \quad 13.53$$



Three gifts of nature

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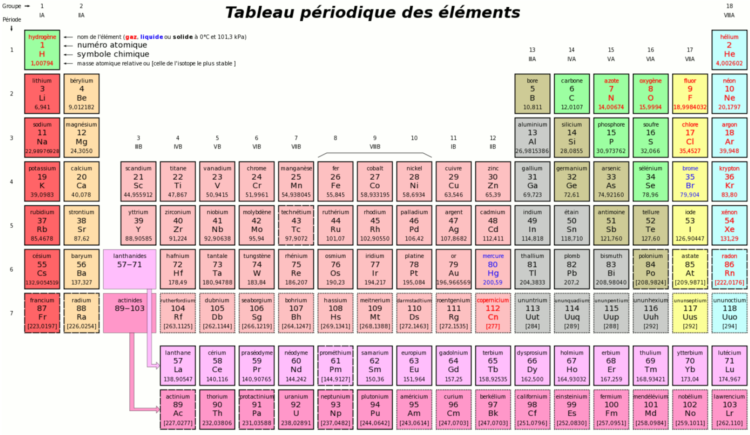
^{3}He*-^{41}Ca: 13.58



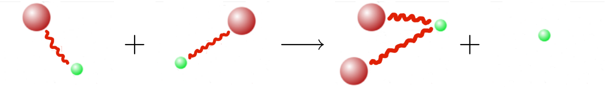
Conclusions

Zero-range model

- Easy to implement in the **k**-representation & in low-D

  L. P., Phys. Rev. A **83**, 062711 (2011).

- Critical stability in 2D !



Bosons: $\forall M/m$; Fermions: $M/m > 18.3$

L. P., P. Pedri, Phys. Rev. A **82**, 033625 (2010).

- Prediction of a 4-body Efimov effect

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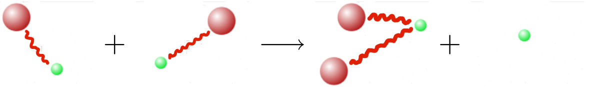

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Separable two-channel model

- Non universal corrections to Efimov physics
- Possibility to describe non-Efimovian but 'universal' physics
- Few-body equations of complexity $\equiv$ zero-range approximation

C. Mora, Y. Castin, L. P., Comptes Rendus Physique **12**, 71 (2011).