Beyond the Efimov effect

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Critical Stability
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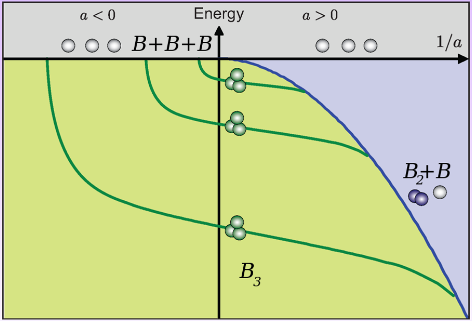
Efimov Effect

Three bodies with short-range interactions can have an *infinity* of three-body bound states even when no two of them are bound, *if* $\frac{|a|}{r_0} \to \infty$



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Why?

Adiabatic hyperspherical potential is

$$U = -\frac{s_0^2 + \frac{1}{4}}{2\mu R^2}, \qquad r_0 \ll R \ll |a|$$

Solutions are known analytically...

$s_0^2 \sim 1 > 0$ is supercritical, giving an infinity of states with

$$E_n = E_0 e^{-2\pi n / s_0}$$

Geometric spacing!

Strange effect

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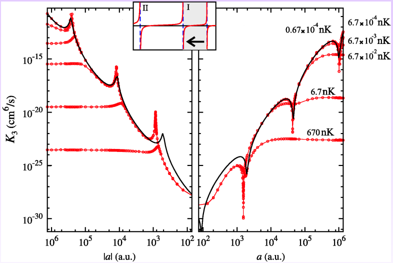
Why do we care?

- Efimov physics underlies all ultracold scattering, leaving imprint of Efimov states on ultracold observables

$$\text{Recombination} \quad B + B + B \longrightarrow B_2 + B$$

- Universality allows us to derive analytic expressions for observables:



D'Incao, Suno, Esry, PRL (2004)

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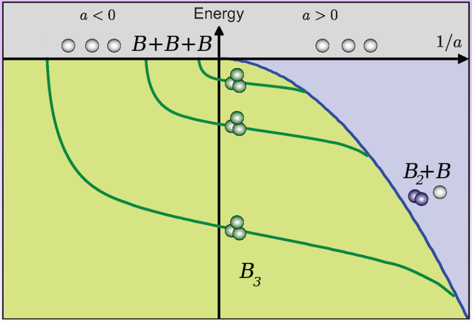

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What about other interactions?

Efimov's effect addresses short-range two-body interactions.

Q: What about long-range two-body interactions?

A: We know long-range potentials (like Coulomb) have infinity of three-body states — but also infinity of two-body states

But, what about attractive r^{-2} potential...

Recall that for

$$v(r) = -\frac{\alpha^2 + \frac{1}{4}}{2\mu r^2}$$

$\alpha^2 > 0$	supercritical	∞ of bound states
$\alpha^2 \leq 0$	subcritical	no bound states

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New class of weakly bound systems

The adiabatic hyperspherical equation

$$\left[\frac{\Lambda^2}{2\mu R^2} - \sum_{i<j}\frac{\alpha^2+\frac{1}{4}}{2\mu_{ij}r_{ij}^2}\right]\Phi_\nu = U_\nu(R)\Phi_\nu$$

is separable, guaranteeing

$$U_\nu(R) = -\frac{\alpha_\nu^2+\frac{1}{4}}{2\mu R^2}$$

Q: Is U subcritical or supercritical when α^2 is *subcritical*?!

A: Supercritical, sort of... $\alpha_0^2 \to -\infty$!
Three-body fall-to-the-center!

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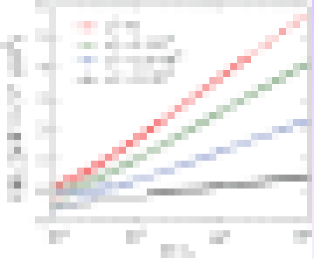
If it occurs in nature, then it will probably look more like

$$v(r) = -\frac{\alpha^2 + \frac{1}{4}}{2\mu r^2}, \qquad r \geq r_0$$

This “regularizes” the singularity, but also removes separability.

Empirically, for $J^\pi = 0^+$ bosons with subcritical α^2

$$U_\nu(R) = -\frac{\sqrt{\beta \ln(R/r_0) + \delta}}{2\mu R^2}$$



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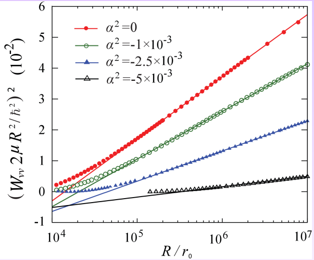

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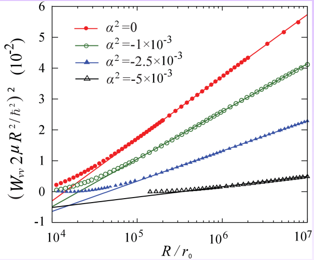

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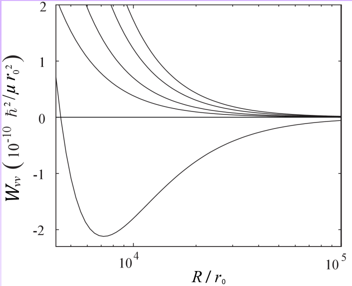
Three-body spectrum

Adiabatic hyperspherical potentials



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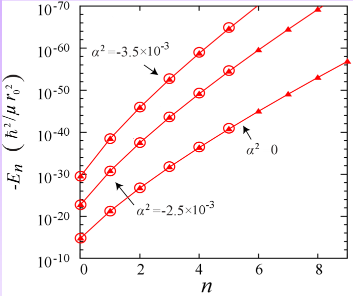

$$E_{n+1}/E_n = \exp\left(-\frac{2\pi}{[(\beta \ln\frac{\langle R\rangle_0}{r_0} - \frac{\beta}{2}\ln\frac{E_n}{E_0})^{1/2} - \frac{1}{4}]^{1/2}}\right)$$

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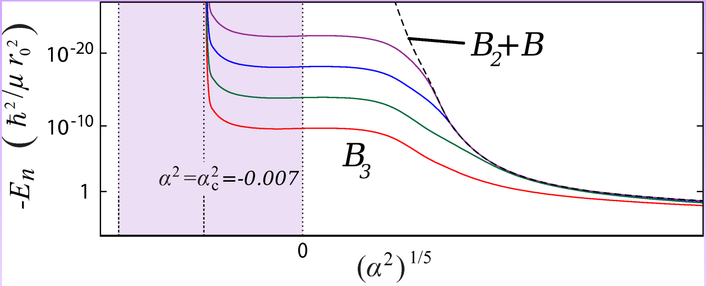


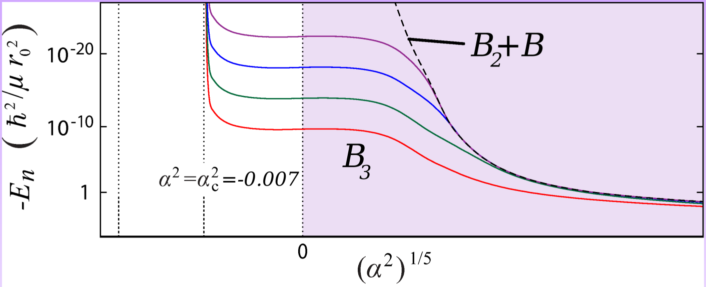
$$U_\nu \rightarrow E_{\nu l} - \frac{\alpha_{\text{eff}}^2 + 1/4}{2\mu R^2} \qquad \alpha_{\text{eff}}^2 = \frac{8}{3}\alpha^2 + \frac{5}{12} - \ell(\ell+1)$$

α_{eff}^2 *always* supercritical!

Three-body spectrum

Efimov effect
- Definition
- Inelastic processes

Beyond Efimov
- Separable
- Non-separable
- ?????? Effect
- ?????? vs Efimov
- Deeply-bound two-body states
- Other symmetries
- Four-body Efimov?



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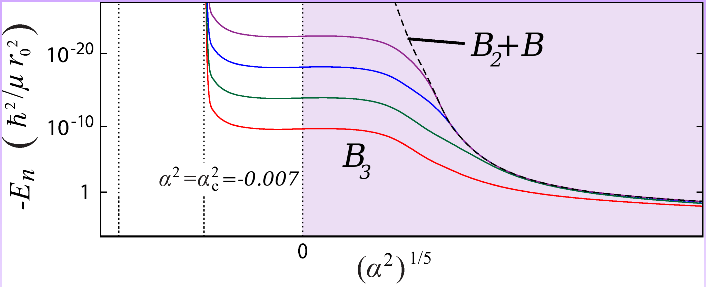

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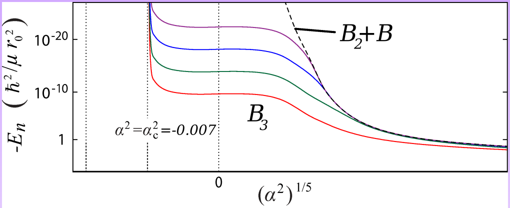
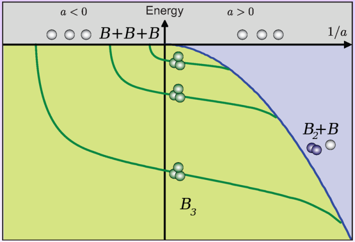
Compare again...





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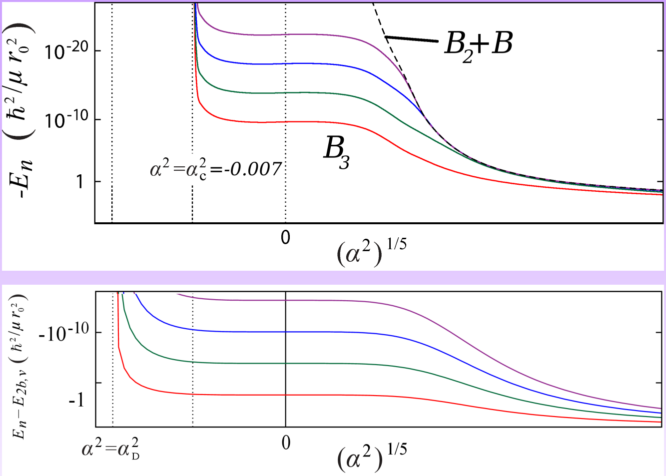

$$\alpha_D^2 = \frac{3}{8}\ell(\ell+1) - \frac{5}{32}$$

Other symmetries?

Effect exists for 0^+ bosons, what else?

We checked 1^+ identical, spin-polarized fermions...
No Efimov effect...

Consider effective two-body potential for $r \geq r_0$

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Solve for adiabatic hyperspherical potentials with $\alpha^2 \leq 2$, find empirically

$$U_0(R) \to -\frac{\alpha_{\rm eff}^2 + 1/4}{2\mu R^2} - \frac{\gamma}{2\mu \ln(R/r_0) R^2}$$

with

$$\alpha_{\rm eff}^2 = 5.24 \qquad \gamma = 4.19$$

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$\alpha_{\rm eff}^2$ supercritical! An infinity of three-body 1^+ fermion bound states with no two-body bound states

Effect persists down to α_c^2=1.6, where

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Four-body ?????? Effect

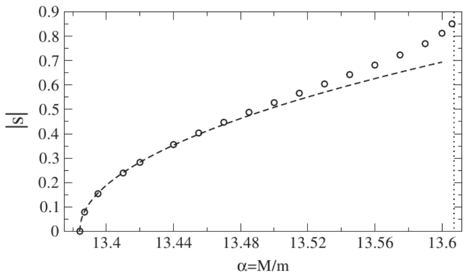
PRL 105, 223201 (2010) PHYSICAL REVIEW LETTERS week ending 26 NOVEMBER 2010

Four-Body Efimov Effect for Three Fermions and a Lighter Particle

Yvan Castin,[1] Christophe Mora,[2] and Ludovic Pricoupenko[3]

Found that for 1^+ $FFFX$ and $a_{FX} = \infty$, there is an Efimov effect for $13.384 \leq m_F/m_X \leq 13.607$:

$$U_0(R) = -\frac{s^2 + 1/4}{2\mu R^2}$$



Four-body ?????? Effect

How does this relate to our *three*-body effect?!

Consider $FFFX$ with $m_F \gg m_X$. Can approximately solve using Born-Oppenheimer:

- Integrate out light particle (X) motion
- Produces effective potential for heavy particle (F) motion
- Reduces problem to three-body: FFF!

For simplicity, approximate FFF Born-Oppenheimer surface with pairwise sum of FFX potentials... which are, for $a_{FX} = \infty$, Efimov potentials:

$$v_{F+F}(r) = -\frac{p_0^2 + 1/4}{2\mu r^2} = -\frac{\alpha^2 + 1/4}{2\mu r^2} + \frac{\ell(\ell+1)}{2\mu r^2}$$

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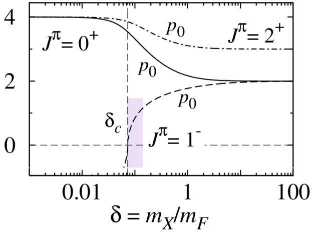

This is exactly our three-body fermion effect!
We thus know

$$U_0(R) \to -\frac{\alpha_{\text{eff}}^2 + 1/4}{2\mu R^2} - \frac{\gamma}{2\mu \ln(R/r_0) R^2}$$

We found an infinity of three-body states for

$$1.6 \leq \alpha^2 \leq 2$$

corresponding to

$$11.58 \leq m_F/m_X \leq 13.607$$
$$(13.384 \leq m_F/m_X \leq 13.607)$$

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Summary

- We have identified an effect that gives an infinity of three-body bound states in the absence of any two-body bound states — that is *not* the Efimov effect
- There are an infinity of such states even in the presence of two-body bound states
- Curious new "fall-to-the-center" problem
- Many other interesting questions to explore with these systems!
- "A new class of three-body states,"
  N. Guevara, Y. Wang, and B.D. Esry,
  arXiv:1110.0476