Critical Stability 2011
Erice, October 2011

Collapse of Bose-Einstein Condensate near Feshbach Resonance in Two-Channel Gross-Pitaevskii Model

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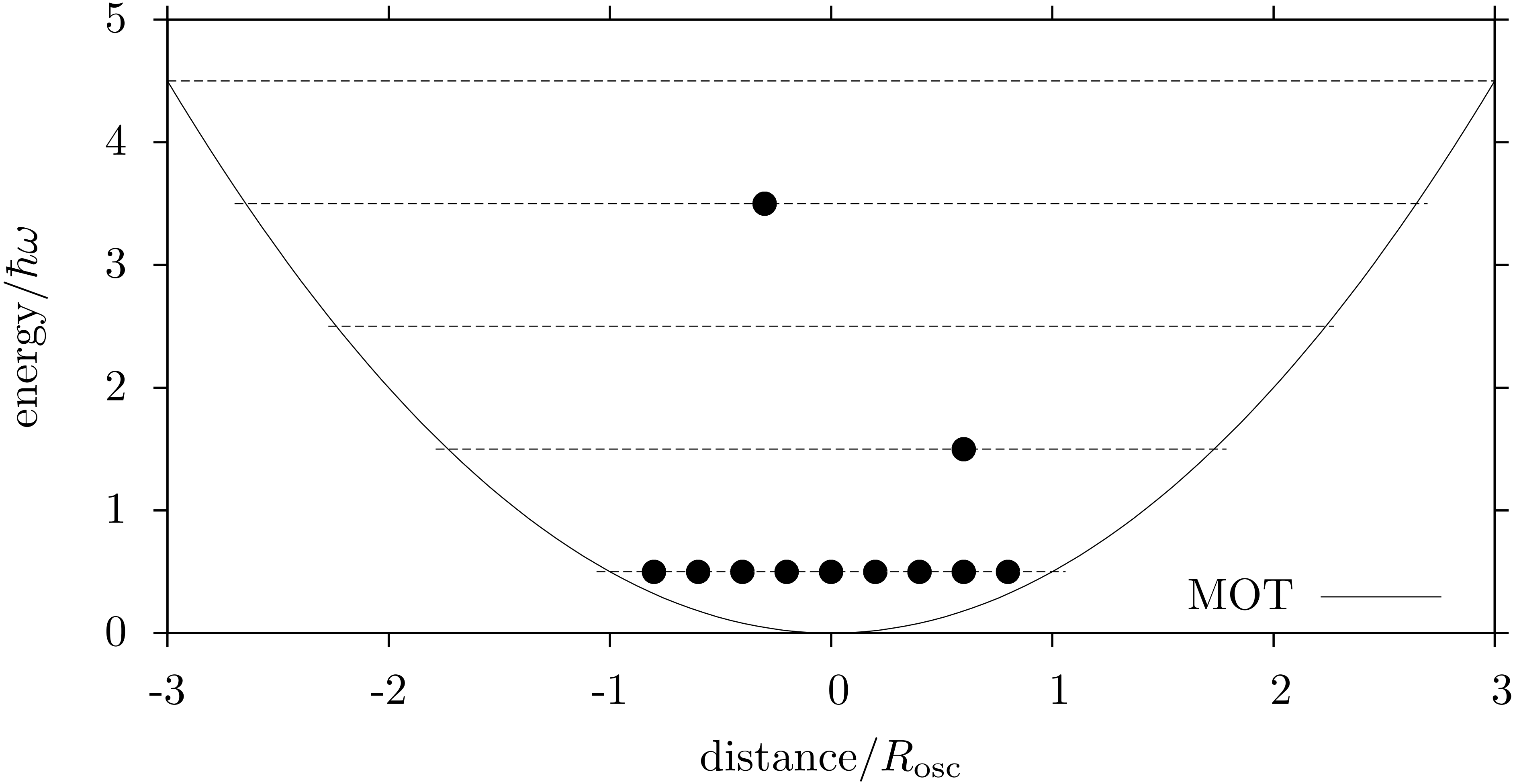
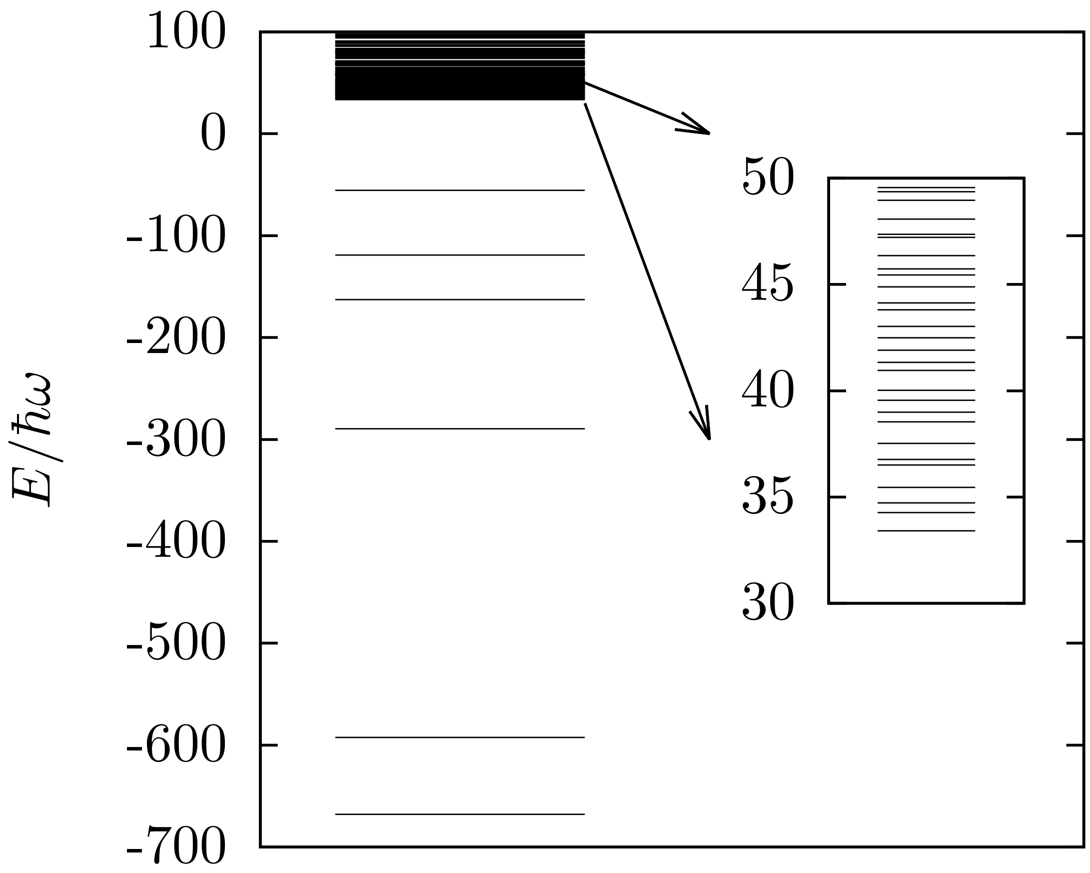
- Bose-Einstein condensates and Feshbach resonances;
- Gross-Pitaevskii equation;
  - Collapse of condensate near Feshbach resonance;
- Two-channel model of Feshbach resonances;
- Two-channel Gross-Pitaevskii equation;
  - Collapse of condensate in two-channel model;
- Outlook.



An artist's view of a BEC in a MOT.



(Part of) the spectrum of a system of $N = 20$ bosons in an oscillator trap.

N-body Hamiltonian with Skyrme force ($g = \frac{4\pi\hbar^2 a}{m}$),

$$H = \sum_{i=1}^{N} \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial \vec{r}_i^2} + V_{\text{ext}}(\vec{r}_i) \right) + \sum_{i<j} g\delta(\vec{r}_i - \vec{r}_j).$$

Hartree(-Fock) product wave-function ($na^3 \ll 1$),

$$\Phi(\vec{r}_1, \ldots, \vec{r}_N) = \prod_{i=1}^{N} \varphi(\vec{r}_i).$$

Variational principle,

$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0,$$

$\Rightarrow$ Gross-Pitaevskii equation ($N - 1 \approx N$),

$$\left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial \vec{r}^2} + V_{\text{ext}}(r) + Ng|\varphi|^2 \right) \varphi = E\varphi,$$

Take the trial single-particle wave-function in the Gaussian form

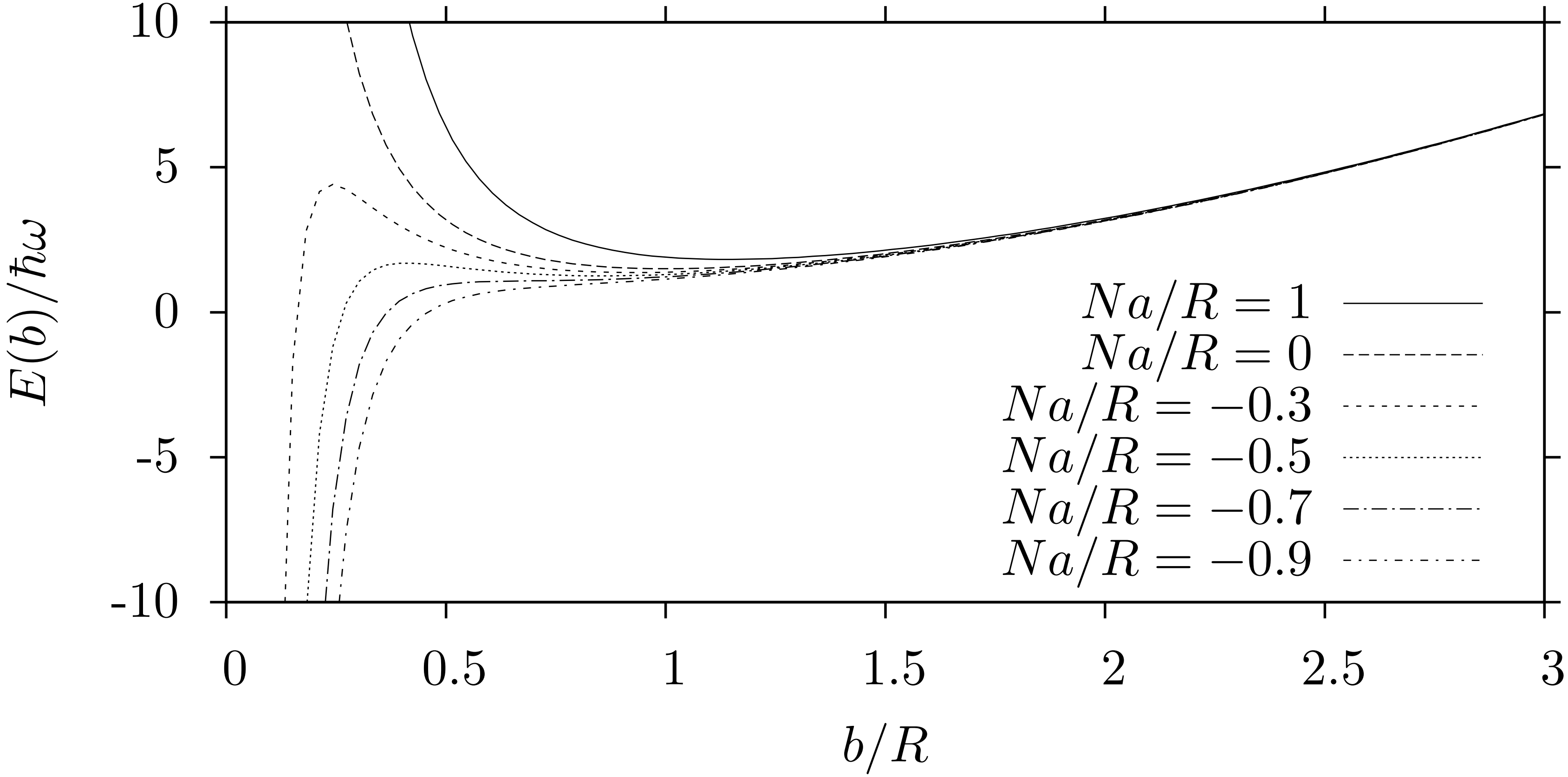
$$\varphi(r) = e^{-r^2/2b^2} \; .$$

The expectation value of the energy is then given as

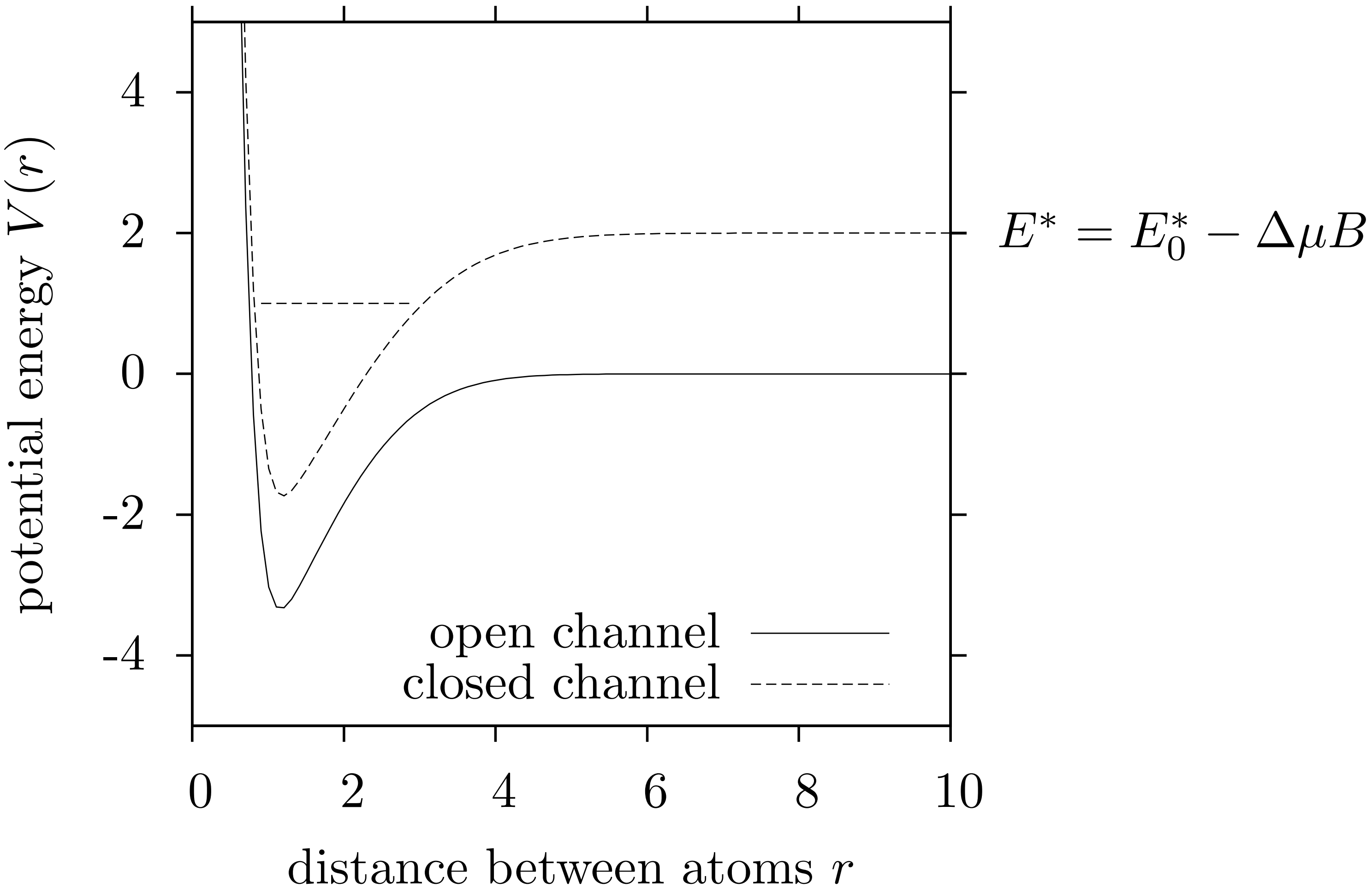
$$\frac{1}{\hbar\omega}\frac{1}{N}\frac{\langle\Phi|H|\Phi\rangle}{\langle\Phi|\Phi\rangle} = \frac{3}{4}\left(\frac{b}{R}\right)^{-2} + \frac{3}{4}\left(\frac{b}{R}\right)^{2} + \frac{1}{\sqrt{2\pi}}\frac{Na}{R}\left(\frac{b}{R}\right)^{-3} \; ,$$

where R is the oscillator length,

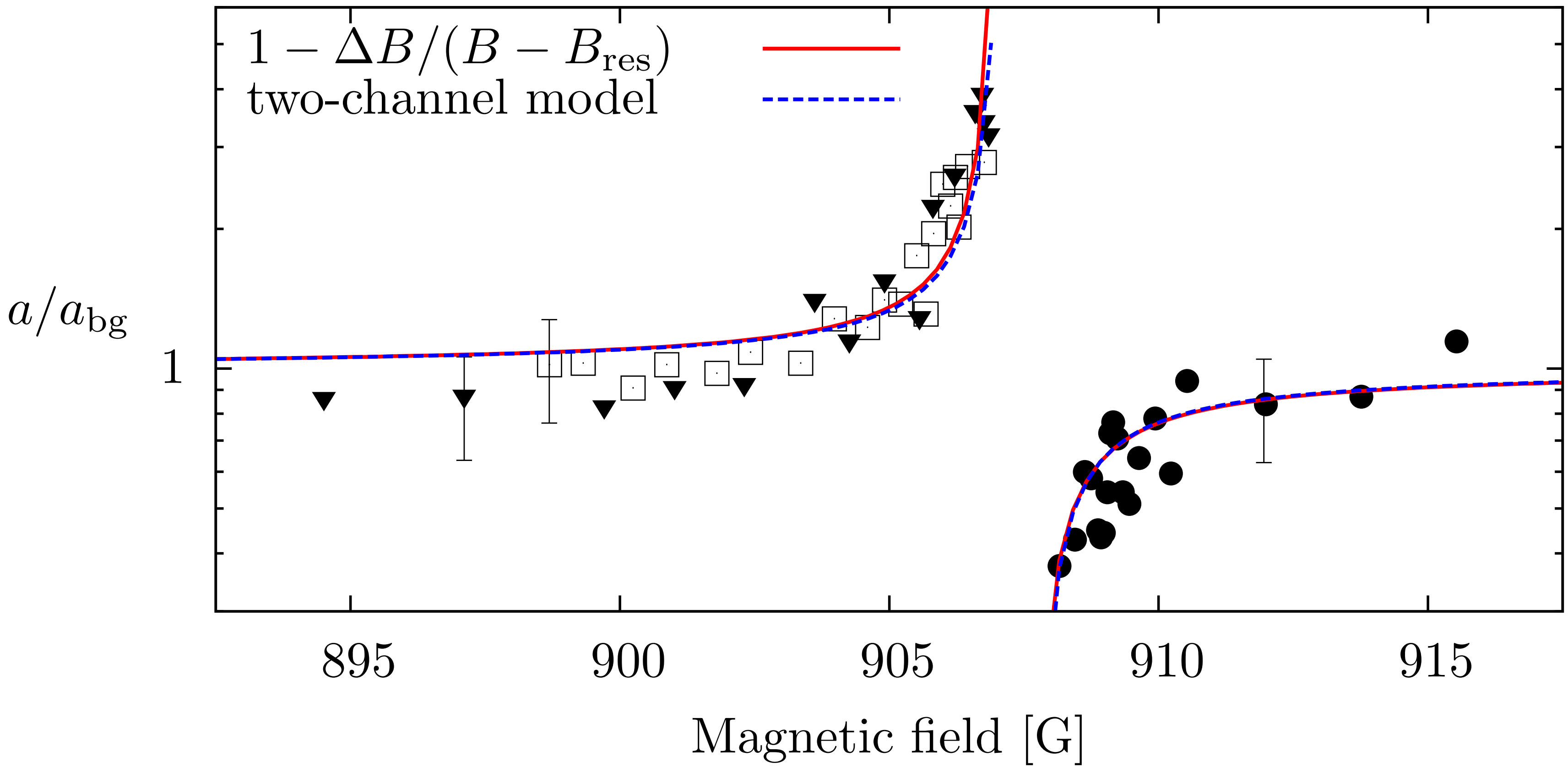
$$\hbar\omega = \frac{\hbar^2}{mR^2} \; .$$



Energy per particle of a system of N bosons as function of the range b of the trial Gaussian.



potential energy $V(r)$
4
2
0
-2
-4
0
2
4
6
8
10
distance between atoms r
open channel
closed channel
$E^* = E_0^* - \Delta\mu B$


$1 - \Delta B/(B - B_{\rm res})$
two-channel model
$a/a_{\rm bg}$
1
895
900
905
910
915
Magnetic field [G]

$$\begin{cases} \hat{h}|\alpha\rangle = E_\alpha|\alpha\rangle \\ \hat{h}|\beta\rangle = (E_\alpha + E^*)|\beta\rangle \end{cases}$$

$$H = \sum_{i=1}^{N} \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \vec{r}_i^2} + V(\vec{r}_i) + \hat{h}_i \right) + \sum_{i<j} \hat{g}^{(ij)}\delta(\vec{r}_i - \vec{r}_j)$$

$$\Psi(\vec{r}_1, \ldots, \vec{r}_N) = \prod_{i=1,N} (\varphi(\vec{r}_i)\alpha_i + \chi(\vec{r}_i)\beta_i)$$

$$\begin{cases} \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\vec{r}^2} + V + E^* + Ng_{\chi\chi\chi\chi}|\chi|^2 + 2Ng_{\varphi\chi\varphi\chi}|\varphi|^2\right)\chi & \\ \qquad +N\big(g_{\varphi\varphi\chi\chi}\chi^*\varphi + g_{\varphi\varphi\varphi\chi}|\varphi|^2 + 2g_{\chi\chi\chi\varphi}|\chi|^2\big)\varphi = E\chi, & \\ \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\vec{r}^2} + V + Ng_{\varphi\varphi\varphi\varphi}|\varphi|^2 + 2Ng_{\varphi\chi\varphi\chi}|\chi|^2\right)\varphi & \\ \qquad +N\big(g_{\varphi\varphi\chi\chi}\varphi^*\chi + 2g_{\varphi\varphi\varphi\chi}|\varphi|^2 + g_{\chi\chi\chi\varphi}|\chi|^2\big)\chi = E\varphi. & \end{cases}$$

Assume the single-particle wave-function in the form

$$\psi(\vec{r}) = e^{-r^2/2b^2}\left(|\alpha\rangle + B|\beta\rangle\right) .$$

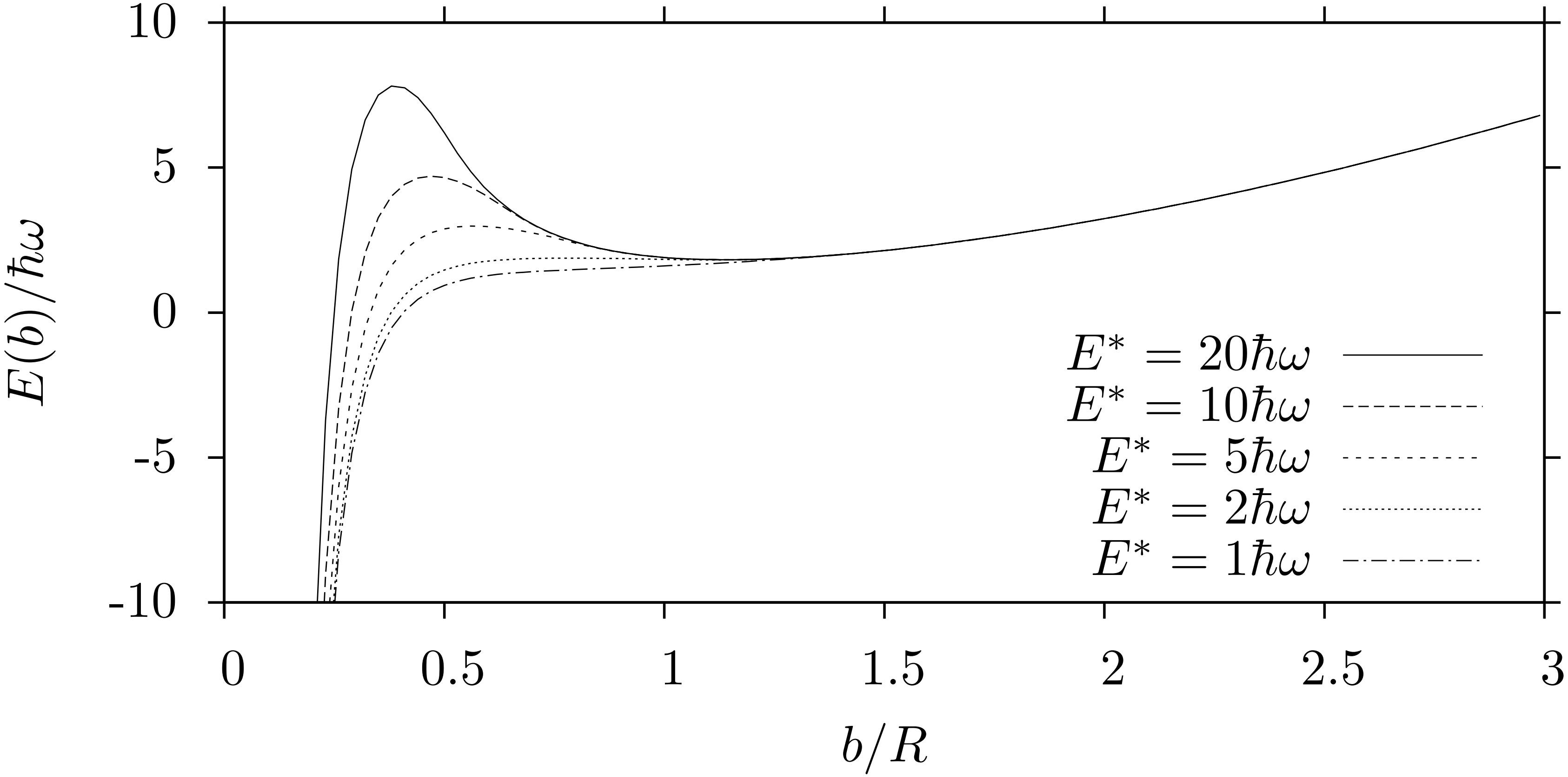
Leave for simplicity only the terms

$$a_{\varphi\varphi\varphi\varphi} \equiv a_{00},\ a_{\varphi\chi\varphi\chi} \equiv a_{11}, a_{\varphi\varphi\varphi\chi} \equiv a_{01}.$$

The energy is then given as

$$\frac{1}{\hbar\omega}\frac{1}{N}\frac{\langle\Phi|H|\Phi\rangle}{\langle\Phi|\Phi\rangle} = \frac{3}{4}\left(\frac{R^2}{b^2}+\frac{b^2}{R^2}\right)+\frac{E^*}{\hbar\omega}\frac{B^2}{1+B^2}+\frac{R^3}{b^3}\frac{Na_{00}}{\sqrt{2\pi}R}\frac{1+4\tilde{a}_{01}B+4\tilde{a}_{11}B^2}{(1+B^2)^2} ,$$

where $\tilde{a} \equiv a/a_{00}$ and $N-1 \approx N$.



Two-channel Gaussian variational calculation: expectation energy per particle for a system of N bosons in a trap as function of the range of the trial Gaussian.

- It is possible to formulate a two-channel Gross-Pitaevskii equation near a Feshbach resonance. The two-channel model naturally describes the physics of the Feshbach resonance.
- The two-channel Gross-Pitaevskii equation seems to be able to account for the collapse of the condensate when the scattering length is increased.