Gravitational waves from coalescing binaries

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Outline

1. Gravitational wave detectors
   - Natural detectors
   - Man-made detectors
     - Working principle
     - Status
     - Prospects

Università degli Studi di Urbino "Carlo Bo"

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Urbino Virgo Data Analysis group activities: search for coalescing binaries

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- Waveform modeling & data analysis implementation (RS)
- Experimental searches (Marica Branchesi, Gianluca Guidi, RS, Andrea Viceré)

- EM follow-up observations and development of image analysis procedures able to detect the EM counterparts (Marica Branchesi)

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GW interaction with point-particles

Physical distances are affected by GW's:

$$g_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}, \qquad ||h_{\mu\nu}|| \ll 1$$

$$L = \int_0^{\bar{L}} dx \sqrt{1 + h_{xx}} \simeq \bar{L} \left(1 + \frac{1}{2} h_{xx} \right)$$

or by geodesic equation deviation

$$\delta \ddot{L}^i = R^i{}_{tjt} L^j = -\frac{1}{2} \ddot{h}^{TT}_{ij} L^j$$

$$\begin{aligned} \text{Light path :} \quad & \delta\phi &= \; & 4\pi \delta L / \lambda \\ \text{EoM for test particle :} \quad & \ddot{x}^i + \omega^2 x^i &= \; & -\tfrac{1}{2} \ddot{h}^i_j x^j \end{aligned}$$

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Localized source:

$$h^{TT}_{ij}(t,x) \simeq \frac{4G_N}{|x|} \Lambda^{TT}_{ij;kl} \int d^3x' T_{ij}(t - |x - x'|) \sim \frac{G_N}{r} \ddot{Q}_{ij}$$

$$\frac{dE}{dAdt} = \frac{1}{16\pi G_N} \langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle \qquad Flux = \frac{G_N}{5} \langle \dddot{Q}_{ij} \dddot{Q}_{ij} \rangle$$

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The Hulse-Taylor binary pulsar

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GW's have been observed in the NS-NS binary system:

PSR B1913+16

Observation of orbital parameters ($a_p \sin \iota$, e, P, $\dot{\theta}$, γ, $\dot{P}$)

$\downarrow$

determination of m_p, m_c (1PN physics, GR)

Energy dissipation in GW's $\rightarrow \dot{P}^{(GR)}(m_p, m_c, P, e)$, compared with $\dot{P}^{(obs)}$

$$\frac{1}{2\pi}\phi = \int_0^T \frac{1}{P(t)}\,dt \simeq \frac{T}{P_0} - \frac{\dot{P}_0}{P_0^2}\frac{T^2}{2}$$

Weisberg and Taylor (2004)

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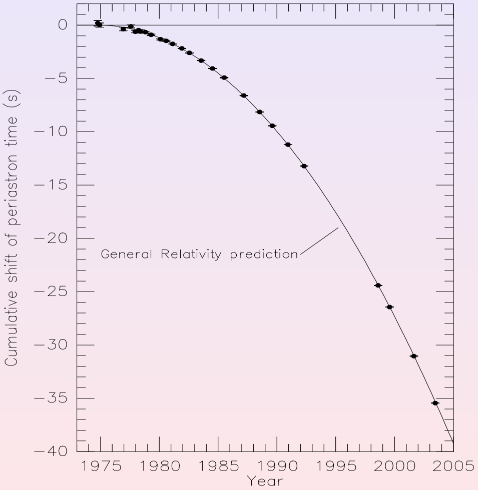
$$\frac{\dot{P}_{GR} - \dot{P}_{exp}}{\dot{P}} \sim 10^{-3}$$



Weisberg and Taylor (2004)

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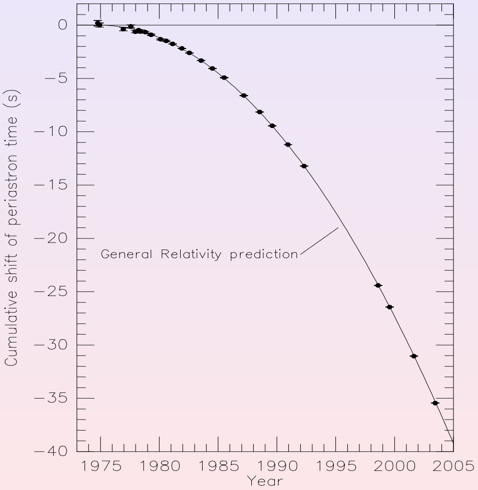

10 pulsars in NS-NS, still $\sim$ 100Myr for coalescence

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Large interferometers

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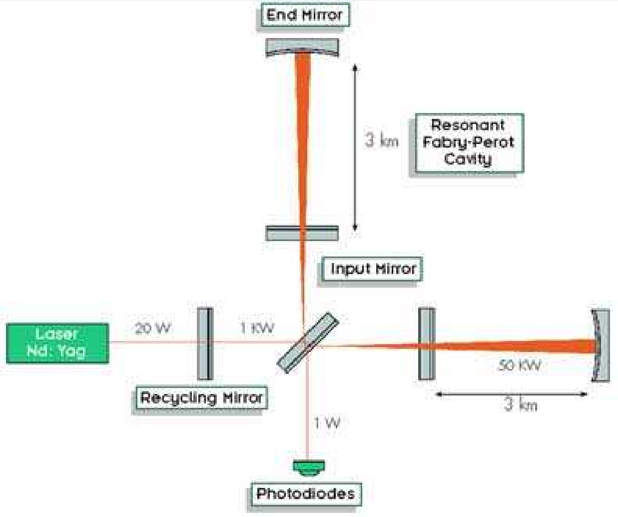

Detector Network

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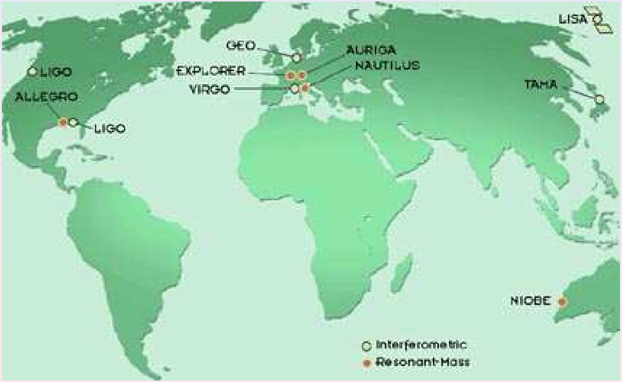

Sensitivity

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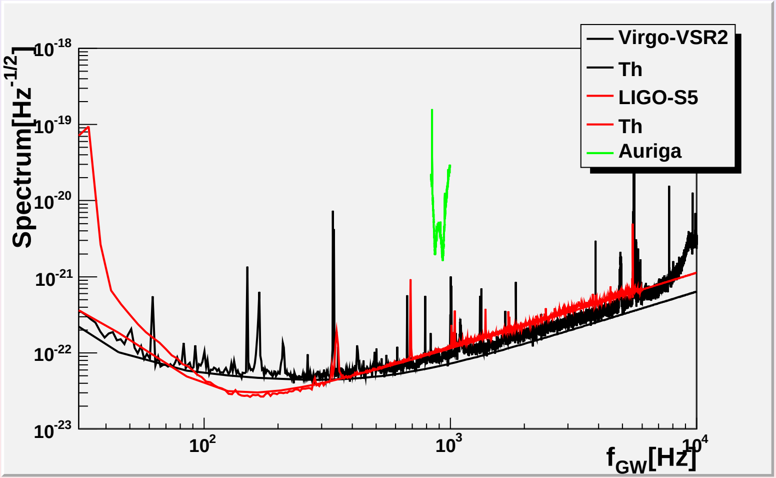

Prospects

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enLIGO/Virgo+ last run (S6/VSR3) ended in October 2010
• LIGO is now off for major hardware upgrade
• Virgo now in commissioning phase
Bar detectors in science run during 2011-2014
(possibly also GEO and Virgo)

- LIGO/Virgo Advanced: from 2014-2015
- LISA (>2020, pathfinder due in 2012)
- LCGT in $\sim$ 10 years, first 3 years funded last June
- AIGO project for a large interferometer in Australia $\sim$ 10 yrs
- ET project: new generation of large interferometers ($\sim$ 30-km-long arms)

Sensitivity of future detectors

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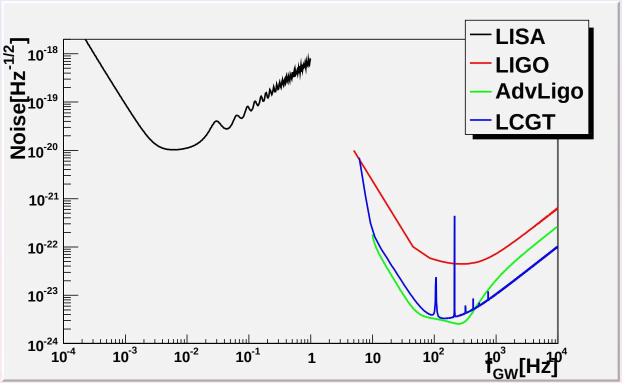

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Data analysis techniques in GW detection

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An experimental apparatus output: time series

$$s(t) = h(t) + n(t) \qquad h(t) = D^{ij} h_{ij}(t)$$

Noise is conveniently characterized by its spectral function

$$\langle \tilde{n}(f)\tilde{n}^*(f')\rangle = \delta(f - f') S_n(f) \qquad [Hz^{-1}]$$

Filtering enhances the sensitivity:

$$\text{filtered signal} \sim \frac{\langle hF \rangle}{\langle NN \rangle^{1/2} \langle FF \rangle^{1/2}}$$

maximized for $F \propto h/S_n$

$$SNR = \left[\int \frac{f|\tilde{h}(f)|^2}{S_n} d \ln f \right]^{1/2}$$

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How many coalescences can LIGO/Virgo see?

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LIGO S5 (ended in Sep 2007) could have seen a pair of $1.4 M_\odot$ NS ($50 M_\odot$ BH's)@ $r \sim 30$ (200) Mpc

	NS-NS	$50\ M_\odot$ BH-BH
Astrophysical rates $(\mathrm{L}_{10}^{-1}\mathrm{Myear}^{-1})$	$10 \div 10^3$	$10^{-1} \div 100$

Number of equivalent galaxies $N_{L_{10}}$ with blue luminosity L_{10} = 10^{10} blue solar lum.

$$N_{L_{10}}(D_H) = 0.02 \times \left(\frac{D_H}{Mpc}\right)^3$$

Present bound: $R_{BH-BH} \lesssim 10^4 \div 100\ \mathrm{Myr}^{-1}\ L_{10}^{-1}$

AdvLIGO/Virgo, reasonably favourable case:

$$R^{(\mathrm{obs})}_{NS-NS} \sim 100\mathrm{yr}^{-1} \qquad R^{(\mathrm{obs})}_{BH-BH} \sim 10^3\mathrm{yr}^{-1}$$

de Freitas Pacheco et al. PRD 2006

I. Mandell et al. PRD 2010

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Signal templates

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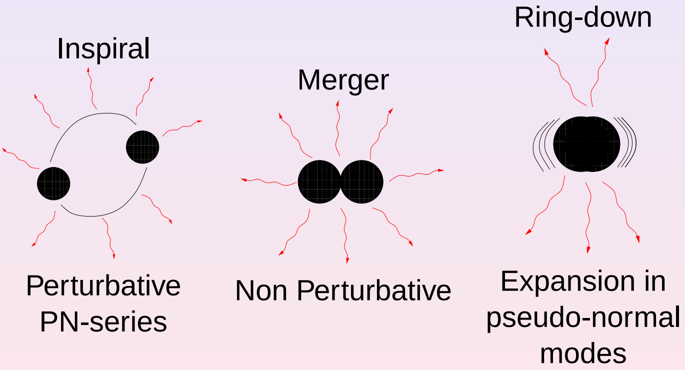
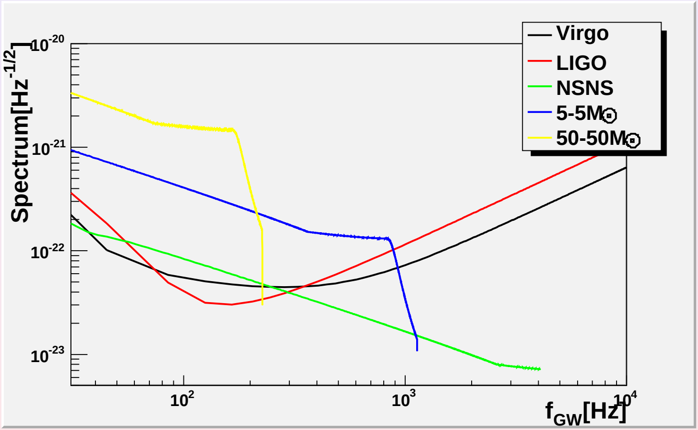


Sensitivity to binary inspiral



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The importance of the merger

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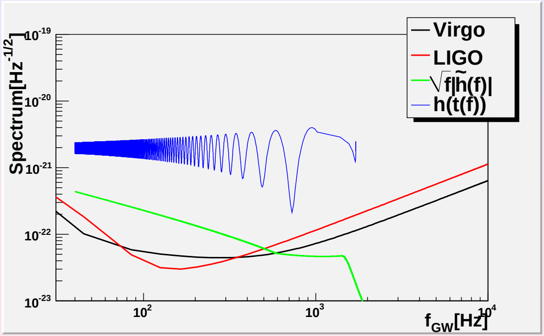

$$h(t(f)) vs. |\sqrt{f}\tilde{h}(f)|$$

Description of the three phases

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- Inspiral

  $$N_{cycles} \simeq 1.6 \cdot 10^4 \left(\frac{10\mathrm{Hz}}{f_{min}}\right)^{5/3} \left(\frac{1.2 M_\odot}{M_c}\right)^{5/3}$$

  $$\text{Sensitivity} \propto M_c^{5/3} \sqrt{N_{cycles}} \propto M^{1/3} \mu^{1/2}, \; f_{Max} \propto M^{-1}$$

- Merger

  Comparison with Numerical Relativity:
  NINJA to test search pipelines against Numerical Relativity Injections
  NRAR to test search waveforms (analytical and phenomenological)

- Ring-down

$$h(t) = \sum_{lmn} e^{-\tau_{lmn}(M,S)} \times [A \cos(\omega_{lmn}(M,S)t) + B \sin(\omega_{lmn}(M,S)t)]$$

Binary system & PN corrections: spinless

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Inspiral

Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$E(v) = -\tfrac{1}{2}\nu M v^2 \left(1 + \#(\nu) v^2 + \#(\nu) v^4 + \ldots\right)$$

$$P(v) \equiv -\frac{dE}{dt} = \frac{32}{5G_N} v^{10} \left(1 + \#(\nu) v^2 + \#(\nu) v^3 + \ldots\right)$$

E(v)(P(v)) known up to 3(3.5)PN, see Damour, Blanchet . . .

$$\frac{1}{2\pi}\phi(T) = \frac{1}{2\pi}\int^T \omega(t)dt = -\int^{v(T)} \frac{\omega(v)dE/dv}{P(v)} dv$$

$$\sim \int \left(1 + \#(\nu) v^2 + \ldots + \#(\nu) v^6 + \ldots\right) \frac{dv}{v^6}$$

Spinning binary systems & PN corrections

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Inspiral

$$\frac{d\phi}{dv} \propto \left[1 + \#(\nu)v^2 + \#(\nu, \mathbf{L}\cdot\mathbf{S_{1,2}})v^3 + \#(\nu, \mathbf{L}, \mathbf{S_{1,2}})v^4 + \ldots\right]$$

$$\frac{d\mathbf{L}}{dt} \propto \Omega(\nu, v, \mathbf{S_{1,2}}) \times \mathbf{L}$$

$$\frac{d\mathbf{S_{1,2}}}{dt} \propto \Omega(\nu, v, \mathbf{L}, \mathbf{S_{2,1}}) \times \mathbf{S_{1,2}}$$

+ finite size effects $\propto v^{10}$, but with possble large pre-factors for NS

Merger non-perturbative modeling

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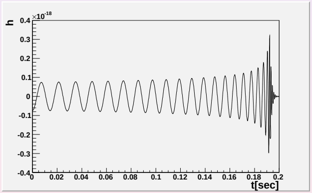
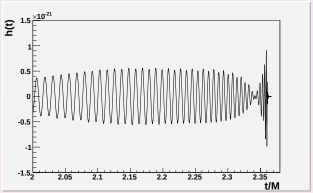
First complete analytical waveform from spinning binaries

$\Downarrow$



Spin-less
Buonanno et al. PRD 2006



Spinning
RS et al. CQG 2010

Phenomenological model

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- Inspiral

System is evolved (via a Taylor T4) approximant until a matching frequency f_m is reached:

$$\frac{d\phi}{dt} = \frac{v^3}{m} \qquad \frac{dv}{dt} = -\frac{F(v)}{dE/dv}$$

f_m is determined empirically.

- Phenomenological part

$$\frac{d\phi}{dt} = \frac{f_1}{1 - \frac{t}{T_A}} + f_0$$

f_0, f_1, T_P are determined by imposing continuity of $\dot{\phi}, \ddot{\phi}, \dddot{\phi}$.

- Ring down

When $d\phi/dt$ reaches $0.8 \times f_{RD}$, the ring down is attached

$$f_{RD} = f_{RD}(S_1, S_2, L, \eta)$$

Matched filtering and templates

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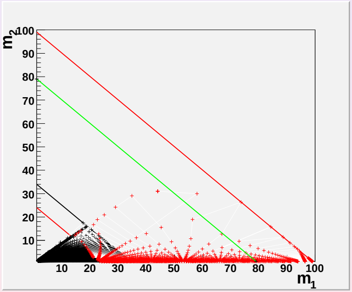
- Inspiral only
  $2.8 < M/M_\odot < 35$
- Inspiral+Merger+RD
  $25 < M/M_\odot < 100$,
  EOBNR non-perturbative template banks, calibrated on PN inspiral and numerically generated wf's
- Ring-down only
  $80 < M/M_\odot < 500$



The pipeline

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Matched-filter with $\sim$ few 10^3 templates, overlap > 0.95
$\rightarrow$ coincidence among detectors
$\rightarrow$ signal-base veto
$\rightarrow$ comparison to time-shifted data for loud triggers

Upper limits for R per space-time volume, given efficiency $\epsilon(\bar{x})$ at loudest signal with SNR $\bar{x}$

$$R \sim \frac{1}{TV\epsilon(\bar{x})}$$

ϵ estimated on software injections

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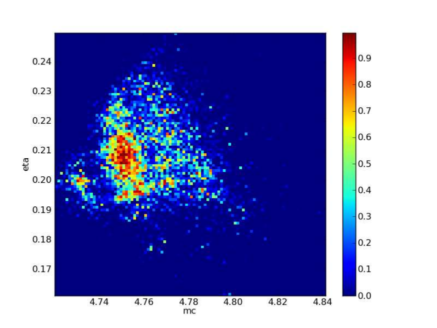
How to estimate binary's parameters?
template bank with spins: impractical $\rightarrow$ Bayesian inference:
15-dimensional parameter space θ random sampling $\rightarrow$
posterior probabilities and posterior density functions

$$p(\mathrm{data}|\theta, \mathcal{M}) \propto \mathcal{L}(\theta|\mathrm{data}, \mathcal{M})\pi(\theta, \mathcal{M})$$



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Effective field theory of Gravity

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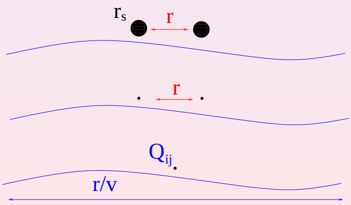
PN expansion of the fundamental GR Lagrangean can be computed via EFT methods

Goldberger and Rothstein 2004

Integrate out short-distance d.o.f. $\rightarrow$ coefficients of operators consistent with long-wavelength physics

- Very short scale r_s, internal structure: negligible until 5PN
- Short distance $\rightarrow$ potential gravitons $k_\mu \sim (v/r, 1/r)$
- Long wavelength $\rightarrow$ gravity waves $k_\mu \sim (v/r, v/r)$, background field



Integrating out potential gravitons

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$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + H_{\mu\nu} \\ S_{EH} &= -\frac{1}{32\pi G_N}\int d^4x\sqrt{g}\,R(H) \\ S_{pp} &\simeq -m\int dt\left(1+\frac{H_{00}}{2}+H_{0i}v_i+\frac{(H_{ij})v^iv^j}{2}\right) \end{aligned}$$

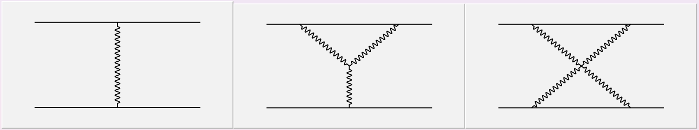
- Effective

$$S_{pp} = \int dt\left(\frac{1}{2}\sum_a m_a v_a^2 + \frac{G_N m_1 m_2}{r} + \dots\right)$$

Re-derivation of 2-body Lagrangean at 3PN order:
Computation of 80 Feynman diagrams via automatized algorithm, paving the way for higher order computations
arXiv:1104.1122, collaboration with S. Foffa, University of Geneva

Conservative dynamics

Classical massive particles (neutron stars, black holes . . .)
Scaling arguments associate Feynman diagrams with specific PN orders:



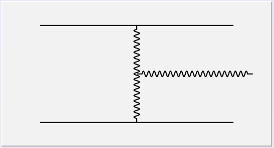
L $\qquad$ Lv^2 $\qquad$ Lv^4

$$V = -\frac{Gm_1m_2}{r}\left[1 - \frac{r_s}{2r} + \frac{1}{4}\left(\frac{r_s}{r}\right)^2\left(1 - 2\nu + 5\nu^2\right)\right]$$

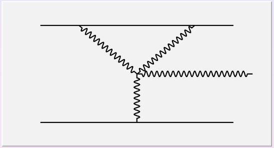
+v-dependent terms

Radiation diagrams

Long wavelength emitted gravitons



$0PN$

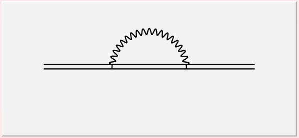


$1PN$

give rise to radiation coupling:

$$h_{ij}\left[\ddot{Q}_{ij}+\ldots\right]$$

Emitted power via optical theorem: Im



Precision test of gravity

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- GW have been observed from binary pulsars: $(\dot{P}_{exp} - \dot{P}_{th})/\dot{P}_{exp} \sim 10^{-3}$ (Hulse & Taylor)
  test of 1PN conservative physics and leading order dissipative effects
  Bound on triple interaction vertex: $\beta_3 < 2 \cdot 10^{-4}$
- Bayesian inference test: model comparison of different fundamental theories of gravity
  Disentangle theory from source parameters

(work in progress)

Coalescing binaries as standard sirens

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LISA and/or ground network can localize the sources (triangolation)

Complementarity with astrophysics: distance vs. red-shift

$$h_c \simeq \frac{1}{D}\left(G_N M_c\right)^{5/3}(f_e)^{2/3}\cos(\phi(t_e/M_c,\nu))\overset{t_r=t_e(1+z)}{\longrightarrow}$$

$$\frac{1}{D_L}\left(G_N M_c(z)\right)^{5/3}(f_r)^{2/3}\cos(\phi(t_r/M_c(z),\nu))$$

$$D_L \equiv D(1+z),\, M_c(z) \equiv M_c(1+z)$$

can measure the luminosity distance, complementarity with astrophysics: distance vs.red-shift

Standard sirens

Schutz '86, Holz & Hughes '08

Conclusions & Work in Progress

Gravitational wave detectors
Natural detectors
Man-made detectors
Working principle
Status
Prospects
Data Analysis
Coalescing binaries
Rates
Source modeling
Fundamental physics

GW's are out there and their detection will open a new window on the Universe:

- New way to detect compact objects in the Universe ($M_{bh} < 100 M_\odot$ for LIGO/Virgo, $M_{bh} < 10^7 M_\odot$ for LISA) and measure their properties
- Test of General Relativity/extensions, within and beyond post-Newtonian perturbation theory
  - derivation of 2-body Lagrangean at 4th PN order (collaboration with S. Foffa from Geneva University)
  - search within Bayesian inference methods for test of/deviations from General Relativity (problem with source parameter degeneracy, collaboration with Birmingham LIGO group)
- search for finite size effects from neutron stars in GW
- collaboration within NINJA and NRAR to improve complete waveforms of spinnning coalescing binaries

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Spare Slides

Quantum corrections are irrelevant

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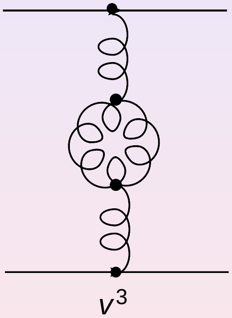

"Usual" rule for quantum weight $\hbar^{I-V} = \hbar^{L-1}$

Internal structure

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Fundamental coupling: $m\int d\tau$
Very short distance physics : eff. operators 2PN-correction to the potential:

$$c_R r_s^2 \int d\tau R + c_V r_s^2 \int d\tau R_{\mu\nu}\dot{x}^\nu \dot{x}^\nu$$

unphysical source bare-parameter redefinition (wiped away by coordinate re-definition)
First correction, 5PN (effacement principle, Damour '82):

$$c_e\, m r_s^4 \int d\tau R_{\mu\alpha\nu\beta} R^{\mu\ \nu}_{\ \gamma\ \delta}\ \dot{x}^\alpha \dot{x}^\beta \dot{x}^\gamma \dot{x}^\delta$$

$$c_m\, m r_s^4 \int d\tau \epsilon^{\mu\nu\rho\sigma} R_{\mu\alpha\nu\beta} R_{\rho\gamma\sigma\delta} \dot{x}^\alpha \dot{x}^\beta \dot{x}^\gamma \dot{x}^\delta$$

$c_e, c_m \propto (r_s/G_N m)^4$ can be large for neutron stars

Example of tagging of fundamental physics effects

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$\beta_{3,4}$ is a tag, not a viable modification of General Relativity
Effect on the phase:

$$\phi \propto \left(\frac{|t-t_c|}{M_c}\right)^{5/8} \times \left[1 - \frac{5}{2}\beta_3 + (a_1(\nu) + b_1(\beta_3, \beta_4, \nu))\, v^2 \right.$$
$$\left. + (a_2(\nu) + b_2(\beta_3, \beta_4, \beta_{LS}))\, v^3 \ldots \right]$$

$\beta_{3,4}$ effect $\rightarrow$ can be reabsorbed by shifting M_c, ν (m_1, m_2)
at PN order $\geq$ 1.5 degeneracy with spin-dependent terms
Need for use of other harmonics than the fundamental one to constrain $\beta_{3,4}$

Graviton self-interaction vertices

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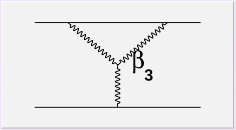
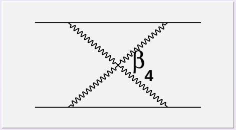
- Conservative dynamics



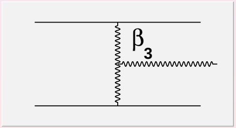
$1PN$



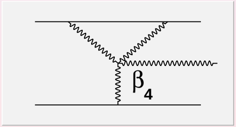
$2PN$

$$V \supset \beta_3 \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{r^2} + \beta_4 \frac{G_N^3 m_1^2 m_2^2}{r^3}$$

- Emission



$0PN$



$1PN$

$$L_{pp} \supset h_{ij} \left[\beta_3 (\nu M x_i \ddot{x}_j) + \frac{r_s}{r} \beta_4 (\nu M x_i \ddot{x}_j) \right]$$

Constraints

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At present: Hulse-Taylor Pulsar gives best constraint on non-conservative effect from β_3

$$\dot{P}_{\beta_3} = \dot{P}_{GR}(1 + c\beta_3) \qquad c \simeq 3.21$$

Given that $\frac{\dot{P}_{obs}}{\dot{P}_{GR}} - 1 \simeq 0.1\% \implies \beta_3 = (4.0 \pm 6.4) \cdot 10^{-4}$

β_3 already constrained by Lunar Laser Ranging, as @ 1PN

$$\beta_3 = \beta_{PPN} < 2 \cdot 10^{-4}$$

No interesting existing bound on β_4

Cannella et al. '09