Modeling the clustering of dark-matter haloes in resummed perturbation theories

arxiv:astro-ph/1012.4833

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Outline

- Introduction:

  - Linearized theory of large scale structure formation

  - Standard treatment

  - Validity
- Need for going to non-linearities: current approaches
- Renormalized Perturbation theory
- Going beyond matter power spectrum - issue of bias
- Results for bias predictions
- Summary

Introduction - from small to large scale

einen kurzen Kino Zeit

Large Scale Structure (LSS)

- Inhomogeneous Universe on small scales

  - Galaxy clusters and groups

  - Positions are correlated

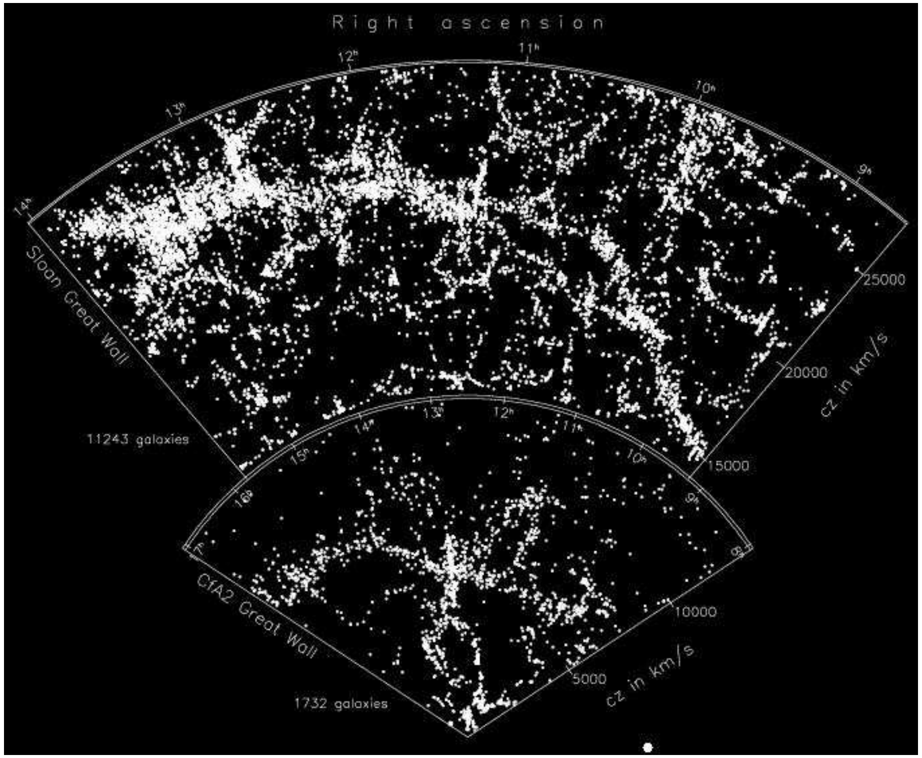
- Great wall

  - scale of $100h^{-1}Mpc$

  - Universe on length scales

    $> 200h^{-1}Mpc$ smooth

- Qn: Can we predict/ understand the structures and their evolution?



[SDSS + CfA1]

LSS - understanding the formation

$$
\begin{aligned}
\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) &= 0 && \text{Continuity equation} \\
\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{\nabla\mathbf{P}}{\rho} &= -\nabla\Phi && \text{Euler equation} \\
\nabla^2\Phi &= 4\pi G\rho && \text{Poisson equation}
\end{aligned}
$$

System of non-linear coupled differential equations, no analytical solution in general

- Continuity equation :- matter conservation
- Euler equation :- momentum conservation
- Poisson equation:- gravitational potential

Our interest: chasing the evolution of inhomogeneities

Linear Theory of LSS

Velocity field = homogeneous expansion + peculiar velocity $\mathbf{v}$

Density field = Average density field + density contrast $\delta(\mathbf{x}, t)$

$$\begin{aligned}
\frac{\partial \mathbf{v}}{\partial t} + \frac{\dot{a}}{a}\mathbf{v} + \frac{1}{a}(\mathbf{v}\cdot\nabla)\mathbf{v} &= -\frac{1}{a}\nabla\Phi \\
\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla\cdot[(1+\delta)\mathbf{v}] &= 0 \\
\nabla^2\Phi &= \frac{3H^2\Omega_m}{2a}\delta
\end{aligned}$$

Linearize and combine:

$$\frac{\partial^2\delta}{\partial t^2} + \frac{2\dot{a}}{a}\frac{\partial\delta}{\partial t} - \frac{3H_0^2\Omega_m}{2a^3}\delta = 0$$

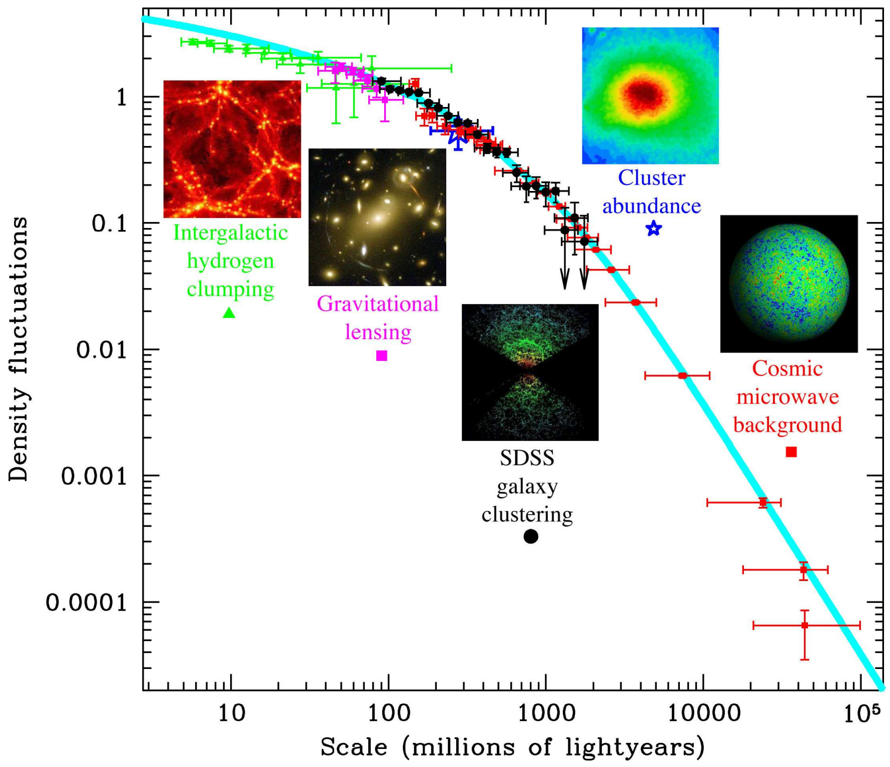
Has two solutions:

$\delta \propto a(t)$ Growing mode

$\delta \propto a^{-3/2}(t)$ Decaying mode

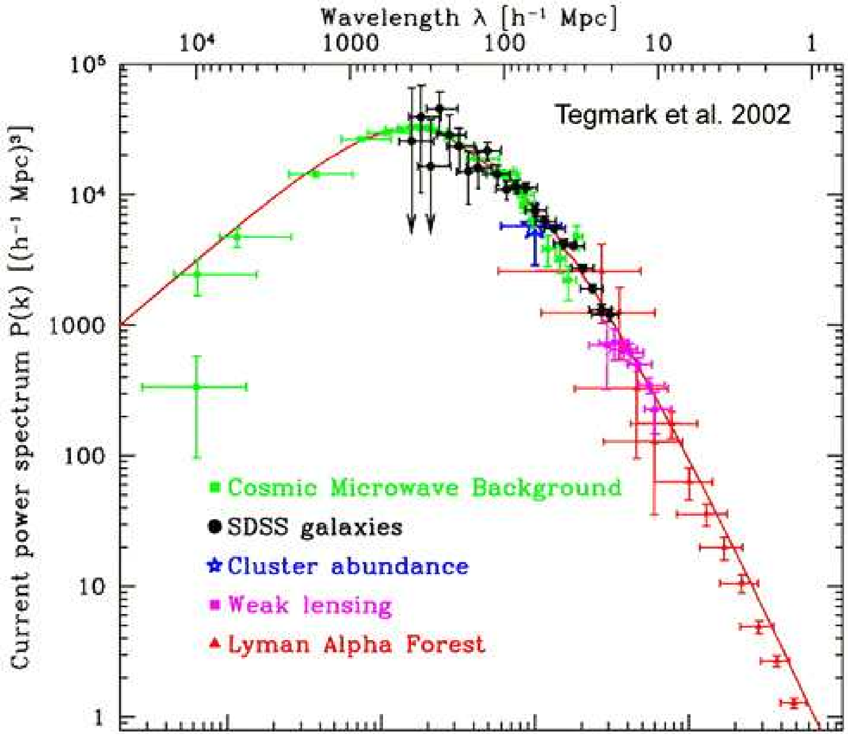
LSS - power spectrum

- Impossible to exactly simulate our universe - exact initial conditions unknown
- Statistically predict the properties
- Correlation function - measure of the statistical properties
- Two point correlation function - the matter power spectrum
- Three point correlation function - bi-spectrum . . .

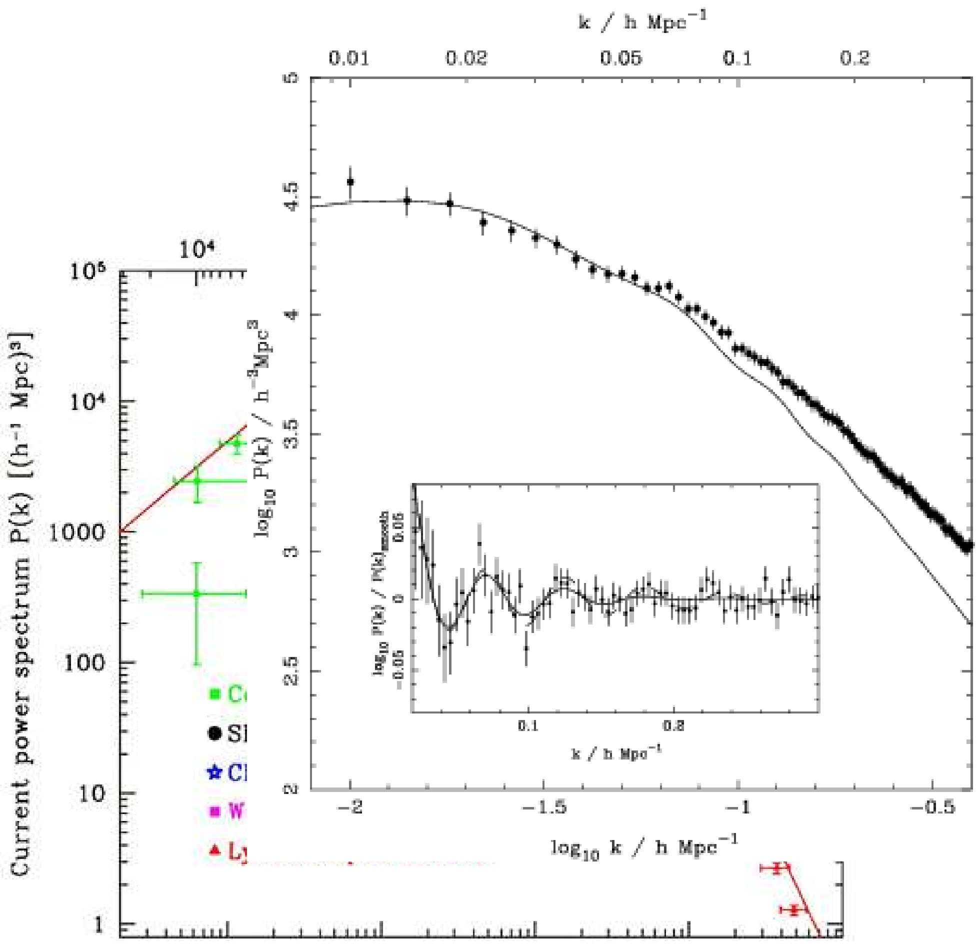


- $P(k)\delta_D(\mathbf{k}+\mathbf{k}') \equiv \langle \delta(\mathbf{k})\delta(\mathbf{k}') \rangle$

Non-linearities - where and why?

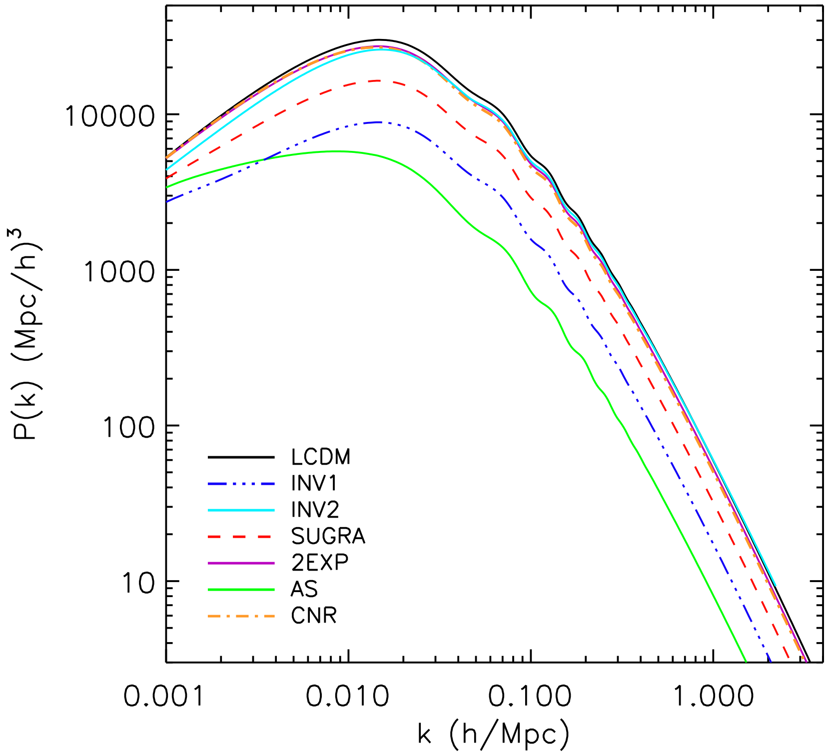


Non-linearities - where and why?



- BAO at $k \sim 0.1h/Mpc$
- Potential to constrain expansion history
- Can differentiate between different DE models

Non-linearities - where and why?



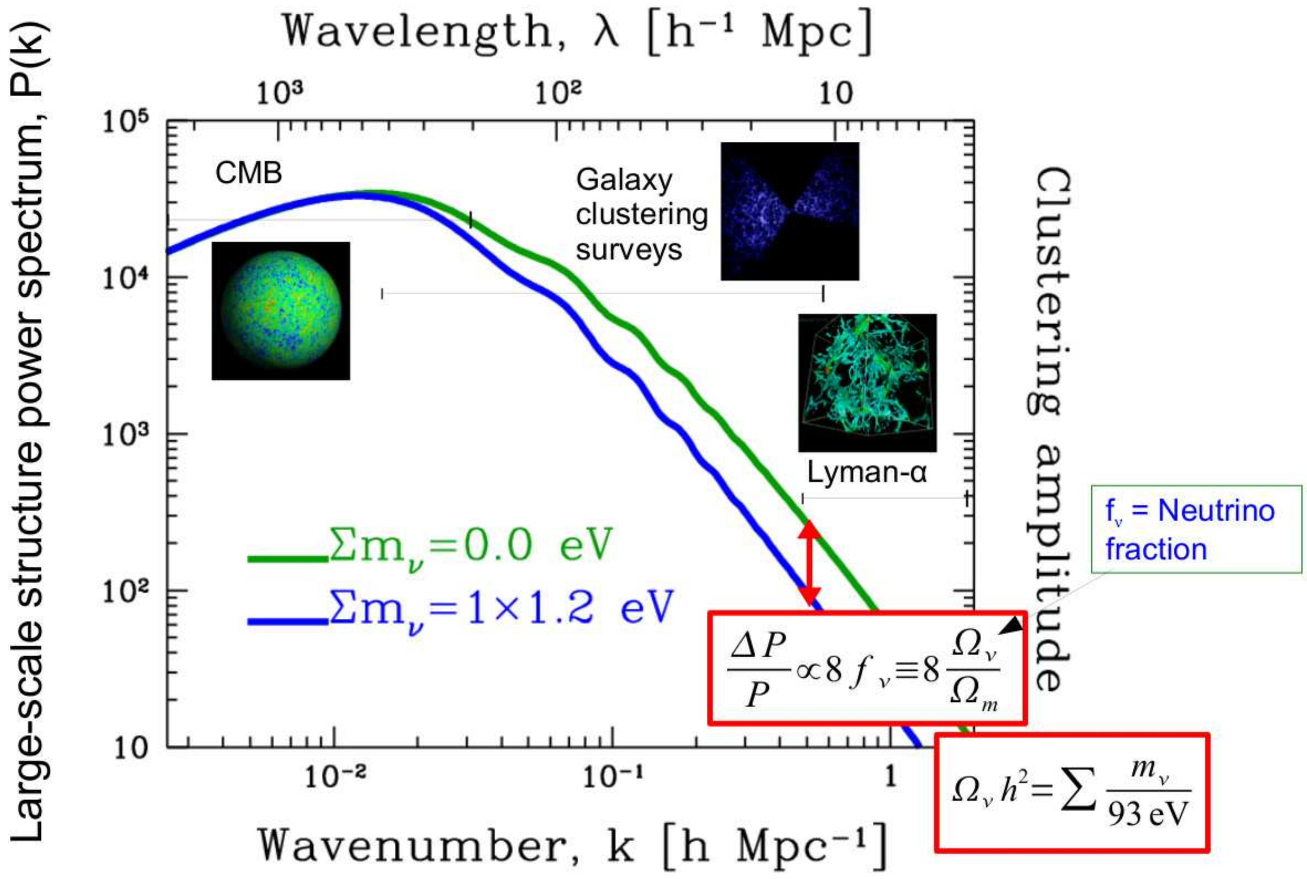
Jennings et.al. 1998

arXiv:0908.1394

- BAO at $k \sim 0.1h/Mpc$
- Potential to constrain expansion history
- Can differentiate between different DE models

Model	w_0	$w_{\rm m}$	$a_{\rm m}$	$\Delta_{\rm m}$
INV1	-0.4	-0.27	0.18	0.5
INV2	-0.79	-0.67	0.29	0.4
SUGRA	-0.82	-0.18	0.1	0.7
2EXP	-1.0	0.01	0.19	0.043
AS	-0.96	-0.01	0.53	0.13
CNR	-1.0	0.1	0.15	0.016

Non-linearities - where and why?



(Shamelessly) stolen from Y. Wong’s talk given at neutrino 2010

Non-linearities - current approaches

Zel’dovich approximation :-

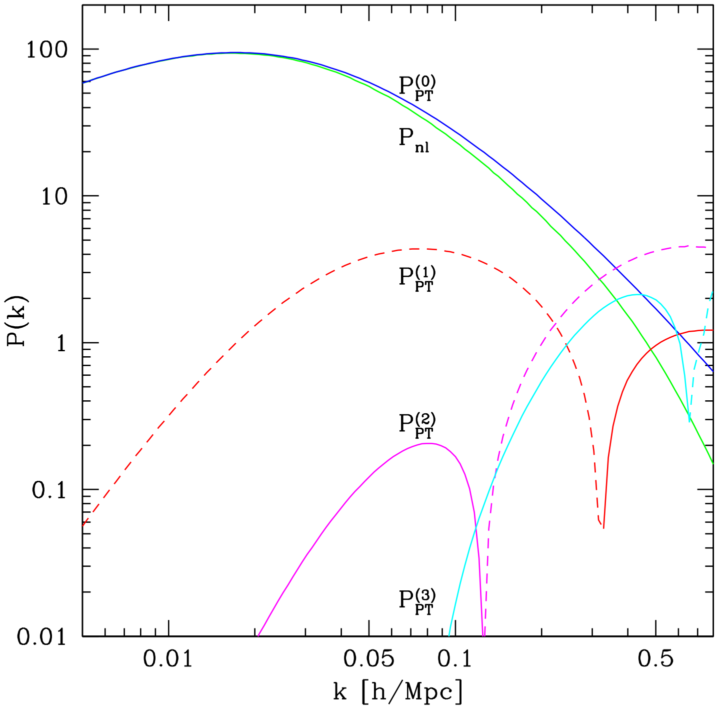
$$P(k) = \int \frac{d^3r}{(2\pi)^3} e^{\iota \mathbf{k}\cdot\mathbf{r}} \left[e^{-[k^2\sigma_v^2 - I(\mathbf{k},\mathbf{r})]} - 1 \right]$$

$$I(\mathbf{k},\mathbf{r}) \equiv \int d^3q (\mathbf{k}\cdot\mathbf{q})^2 \cos(\mathbf{k}\cdot\mathbf{q})\, P_L(q)/q^4$$

$$\sigma_v = I(k,0)/k^2$$

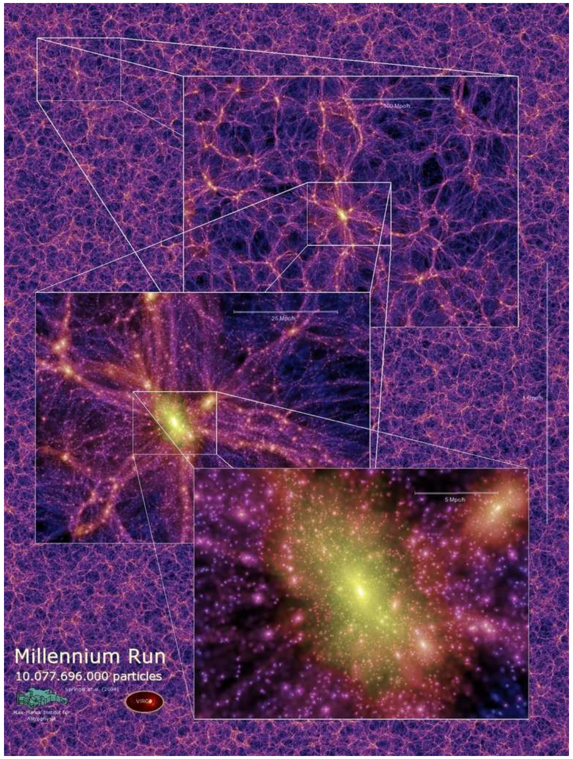
- Effective expansion in the amplitude of PS
- Delicate cancellations between different orders
- Improvement over linear theory

PS in Zel’dovich approximation



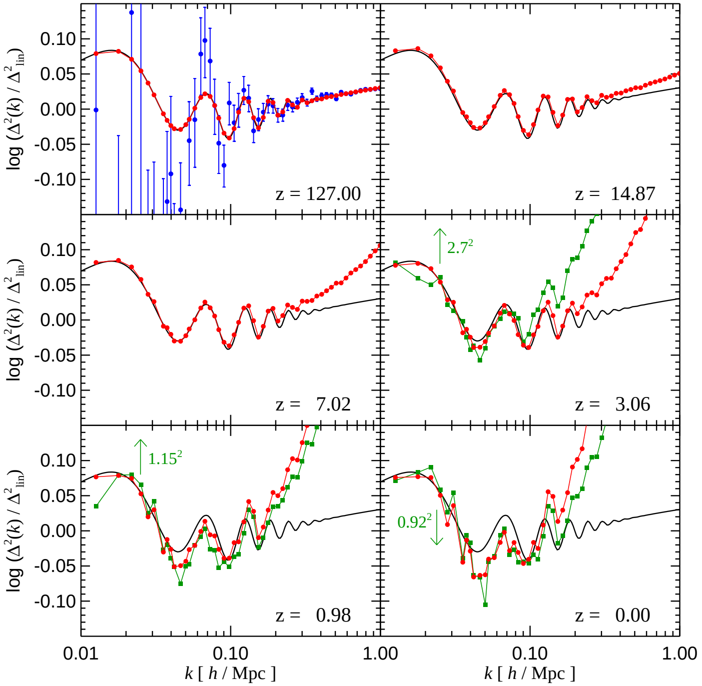
Non-linearities - current approaches

Numerical simulations



Millennium simulation

Power spectrum (normalized to smooth)



Actual realization of initial fluctuations

dark matter, galaxies

Scatter in initial realization due to finite number of modes

Springel et. al. 2005

arxiv:astro-ph/0504097

Perturbation Theory for cosmology

$$\frac{\partial\delta}{\partial\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0; \qquad \frac{\partial\mathbf{v}}{\partial\tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi; \qquad \nabla^2\phi = \frac{3}{2}\Omega_m\mathcal{H}\delta$$

In Fourier space with $\theta(x,\tau) \equiv \nabla\cdot\mathbf{v}(\mathbf{x},\tau)$

$$\frac{\partial\,\delta(\mathbf{k},\tau)}{\partial\,\tau} + \theta(\mathbf{k},\tau) + \int d^3\mathbf{q}\,d^3\mathbf{p}\,\delta_D(\mathbf{k}-\mathbf{q}-\mathbf{p})\alpha(\mathbf{q},\mathbf{p})\theta(\mathbf{q},\tau)\delta(\mathbf{p},\tau) = 0$$

$$\frac{\partial\,\theta(\mathbf{k},\tau)}{\partial\,\tau} + \mathcal{H}\,\theta(\mathbf{k},\tau) + \frac{3}{2}\mathcal{H}^2\delta(\mathbf{k},\tau) + \int d^3\mathbf{q}\,d^3\mathbf{p}\,\delta_D(\mathbf{k}-\mathbf{q}-\mathbf{p})\beta(\mathbf{q},\mathbf{p})\theta(\mathbf{q},\tau)\theta(\mathbf{p},\tau) = 0$$

Mode - mode coupling controlled by:-

$$\alpha(\mathbf{p},\mathbf{q}) = \frac{(\mathbf{p}+\mathbf{q})\cdot\mathbf{p}}{p^2}\,, \qquad \beta(\mathbf{p},\mathbf{q}) = \frac{(\mathbf{p}+\mathbf{q})^2\,\mathbf{p}\cdot\mathbf{q}}{2\,p^2q^2}$$

Linear approximation

$$\alpha(\mathbf{p},\mathbf{q}) = \beta(\mathbf{p},\mathbf{q}) = 0$$

No mode-mode coupling

$$\frac{\partial\,\delta(\mathbf{k},\tau)}{\partial\,\tau} + \theta(\mathbf{k},\tau) = 0$$
$$\frac{\partial\,\theta(\mathbf{k},\tau)}{\partial\,\tau} + \mathcal{H}\,\theta(\mathbf{k},\tau) + \frac{3}{2}\mathcal{H}^2\delta(\mathbf{k},\tau) = 0$$

$$\Omega_m = 1 \rightarrow \mathcal{H} \sim a^{1/2}$$
$$\downarrow$$

$$\begin{aligned} \delta(\mathbf{k},\tau) &= \delta(\mathbf{k},\tau_i)\left(\frac{a(\tau)}{a(\tau_i)}\right)^m \\ -\frac{\theta(\mathbf{k},\tau)}{\mathcal{H}} &= m\delta(\mathbf{k},\tau) \end{aligned} \qquad m = \begin{cases} 1 & \text{Growing mode} \\ \frac{-3}{2} & \text{Decaying mode} \end{cases}$$

Compactifying PT for cosmology

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{v}] = 0; \qquad \frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi; \qquad \nabla^2\phi = \frac{3}{2}\Omega_m \mathcal{H}\delta$$

Define

$$\left(\begin{array}{c} \varphi_1(\mathbf{k},\eta) \\ \varphi_2(\mathbf{k},\eta) \end{array} \right) \equiv e^{-\eta} \left(\begin{array}{c} \delta(\mathbf{k},\eta) \\ -\theta(\mathbf{k},\eta)/\mathcal{H} \end{array} \right) \qquad \eta = \log\frac{a}{a_{in}} \qquad \mathbf{\Omega} = \left(\begin{array}{cc} 1 & -1 \\ -3/2 & 3/2 \end{array} \right)$$

Then (assuming EdS cosmology) we can write:-

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b(\mathbf{k},\eta) = e^{\eta}\gamma_{abc}(\mathbf{k},\,-\mathbf{p},\,-\mathbf{q})\varphi_b(\mathbf{p},\eta)\,\varphi_c(\mathbf{q},\eta)\,,$$

With mode-mode coupling $\gamma_{abc}(\mathbf{k},\mathbf{p},\mathbf{q})$ $(a,b,c,=1,2)$

$$\gamma_{121}(\mathbf{k},\,\mathbf{p},\,\mathbf{q}) = \gamma_{112}(\mathbf{k},\,\mathbf{q},\,\mathbf{p}) = \frac{1}{2}\,\delta_D(\mathbf{k}+\mathbf{p}+\mathbf{q})\,\alpha(\mathbf{p},\mathbf{q})\,,$$
$$\gamma_{222}(\mathbf{k},\,\mathbf{p},\,\mathbf{q}) = \delta_D(\mathbf{k}+\mathbf{p}+\mathbf{q})\,\beta(\mathbf{p},\mathbf{q})\,,$$

Perturbation Theory for cosmology

The action is given by

$$
\begin{aligned}
S &= \int d\eta \,[\chi_a(-\mathbf{k},\eta)\,(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b(\mathbf{k},\eta) \\
&\quad - e^\eta \gamma_{abc}(-\mathbf{k},-\mathbf{p},-\mathbf{q})\chi_a(\mathbf{k},\eta)\varphi_b(\mathbf{p},\eta)\varphi_c(\mathbf{q},\eta)]
\end{aligned}
$$

$$
\begin{aligned}
Z[J_a,\, K_b;\, \varphi_a(0)] &\equiv \int \mathcal{D}\varphi_a(\eta_f) \int \mathcal{D}''\varphi_a \mathcal{D}\,\chi_b \;\times \\
&\quad \exp\left\{ i \int_0^{\eta_f} d\eta\, \chi_a(\delta_{ab}\partial_\eta + \Omega_{ab})\varphi_b - e^\eta\, \gamma_{abc}\chi_a\varphi_b\varphi_c + J_a\varphi_a + K_a\chi_a \right\}
\end{aligned}
$$

Averaging the probabilities over the initial conditions with a statistical weight function for the physical fields $\varphi_a(0)$,

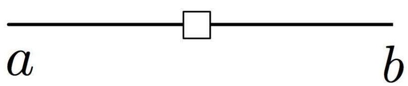
$$
Z[J_a,\, K_b;\, C's] = \int \mathcal{D}\varphi_a(0)\, W[\varphi_a(0),\, C's]\, Z[J_a,\, K_b;\, \varphi_a(0)]\,.
$$

Gaussian initial conditions, the weight function reduces to the form

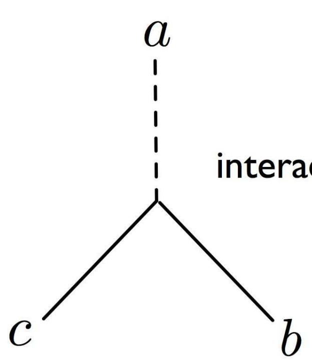
$$
W[\varphi_a(0),\, C_{ab}] = \exp\left\{ -\frac{1}{2}\varphi_a(\mathbf{k},\, 0) C_{ab}(k)\varphi_b(-\mathbf{k},\, 0) \right\},
$$

Feynman diagrams and all that . . .

propagator: $-i\, g_{ab}(\eta_a, \eta_b)$



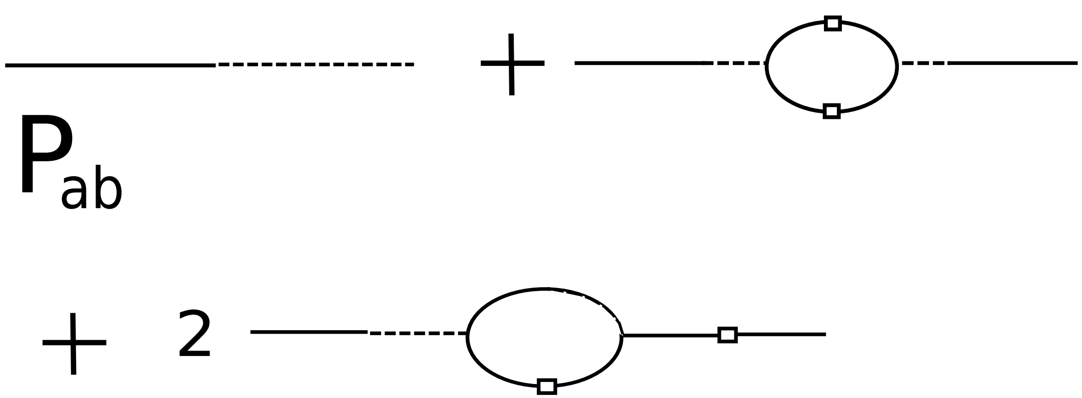
power spectrum: $P^L_{ab}(\eta_a, \eta_b; \mathbf{k})$



interaction vertex: $-i\, e^{\eta}\, \gamma_{abc}(\mathbf{k_a}, \mathbf{k_b}, \mathbf{k_c})$

Tree level diagrams

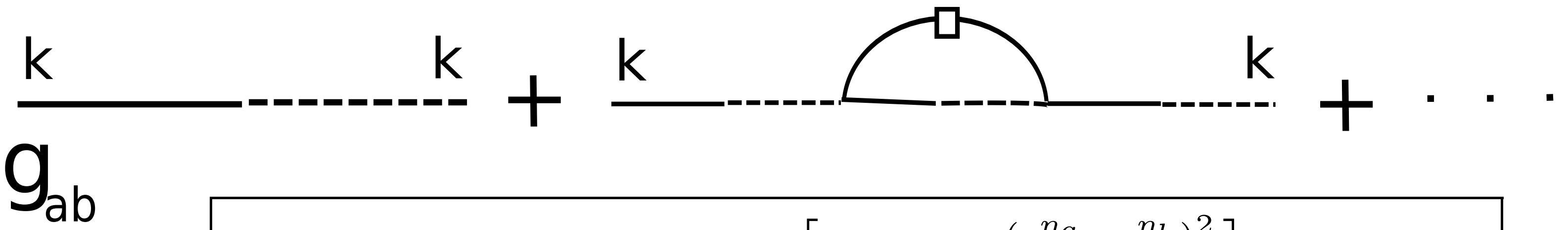
- Propagator - linear evolution
- Power spectrum - initial conditions
- Vertex - nonlinearities



Loop corrections to power spectrum

Does it work?

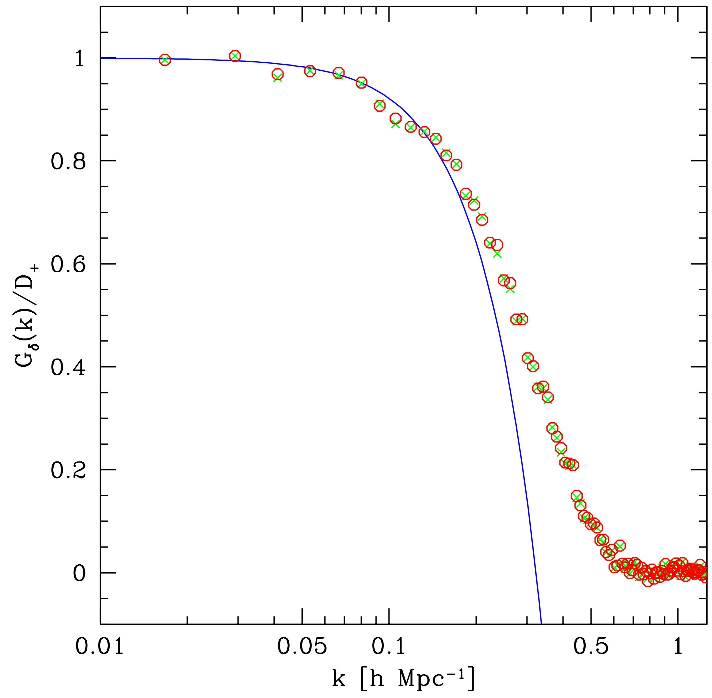
Propagator in large-k limit



$$\boxed{G_{ab}(k;\eta_a,\eta_b) = g_{ab}(\eta_a,\eta_b)\left[1 - k^2\sigma^2\frac{(e^{\eta_a}-e^{\eta_b})^2}{2}\right] + O(k^4\sigma^4)}$$

$$\left(\sigma \equiv \frac{1}{3}\int d^3q\,\frac{P^0(q)}{q^2}\right)$$

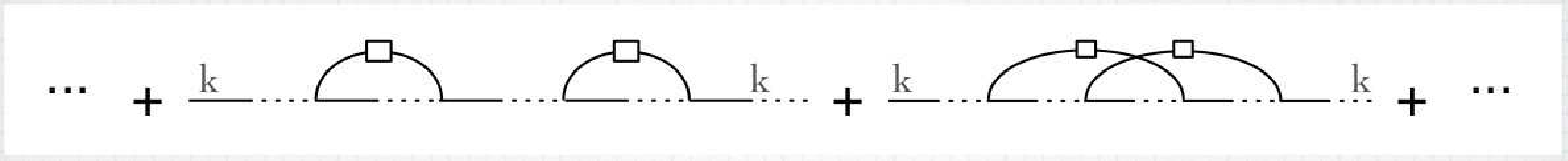
$(\sigma e^{\eta_a})^{-1} \simeq 0.15hMpc^{-1}$, in the BAO range



PT blows up in the BAO range

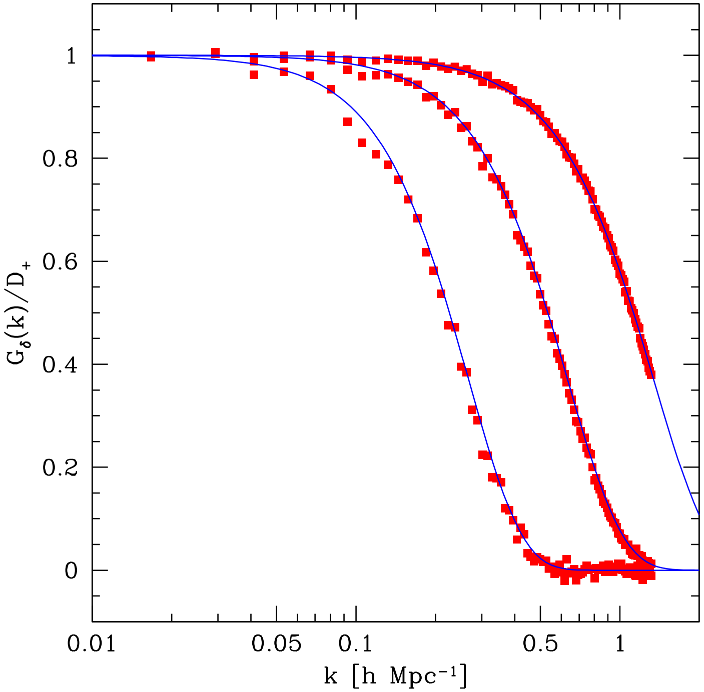
Renormalized perturbation theory

Different contributions can be resumed

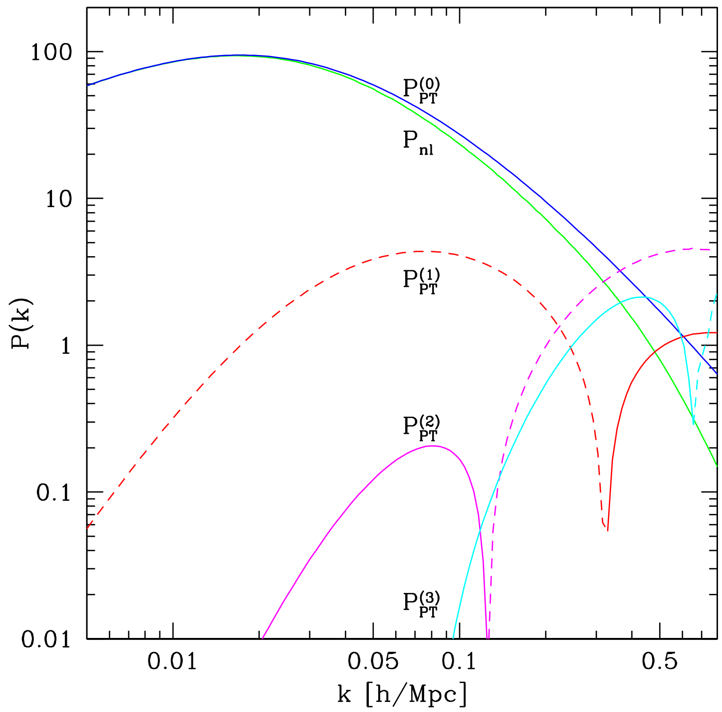


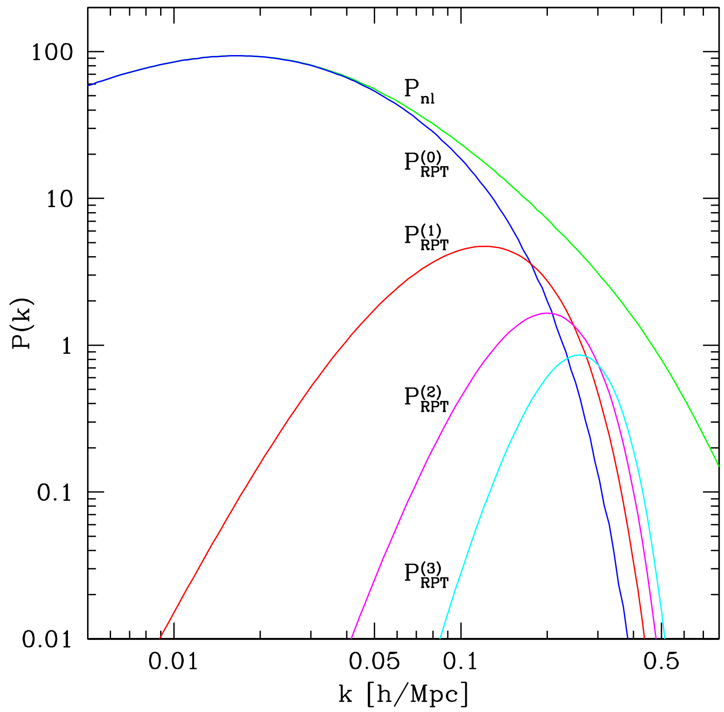
$$\boxed{G_{ab}(k,\eta) = g_{ab}(\eta)\exp\left(-k^2\sigma_v^2(\exp(\eta)-1)^2/2\right)}$$

- Exponential damping in the BAO range
- Represents the effect of multiple interactions
- Memory loss



Results

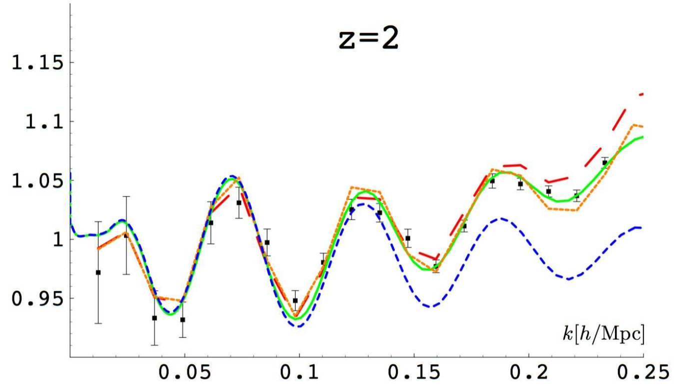


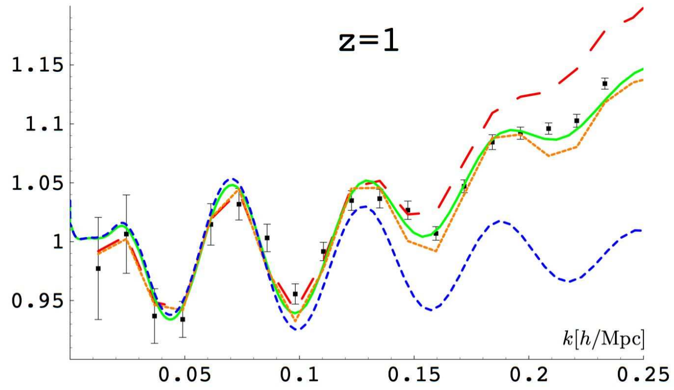


- Results from PT using Zel'dovich approximation
- Fine cancellation between different loop orders

- Results from PT using field theory methods
- Clearly improved results

Matter power spectra





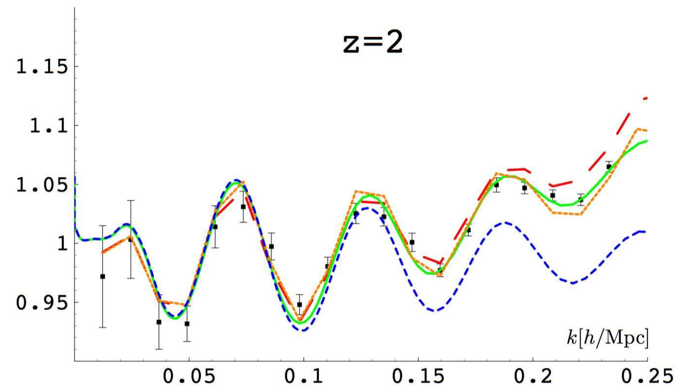
RPT, one loop, linear, simulations, halo approach

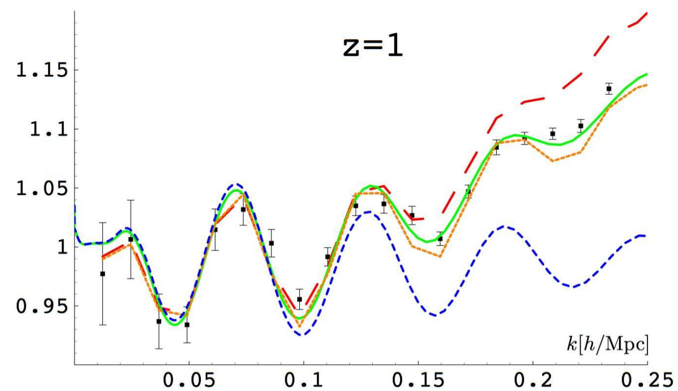
- RG equations for propagator and PS can be written down
- Result of using the RG for propagator and PS
- RG for propagator

$$\boxed{\partial_\lambda \frac{\partial^2 W_\lambda}{\partial J_a(\mathbf{k},\eta_a)\partial K_b(\mathbf{k},\eta_a)} = -\delta(\mathbf{k}+\mathbf{k}')\partial_\lambda G_{ab,\lambda}(k,\eta_a,\eta_b)}$$

- Suppression of PS from RG at $k \sim 0.25hMpc^{-1}$ due to failure of analytical approximations

Matter power spectra





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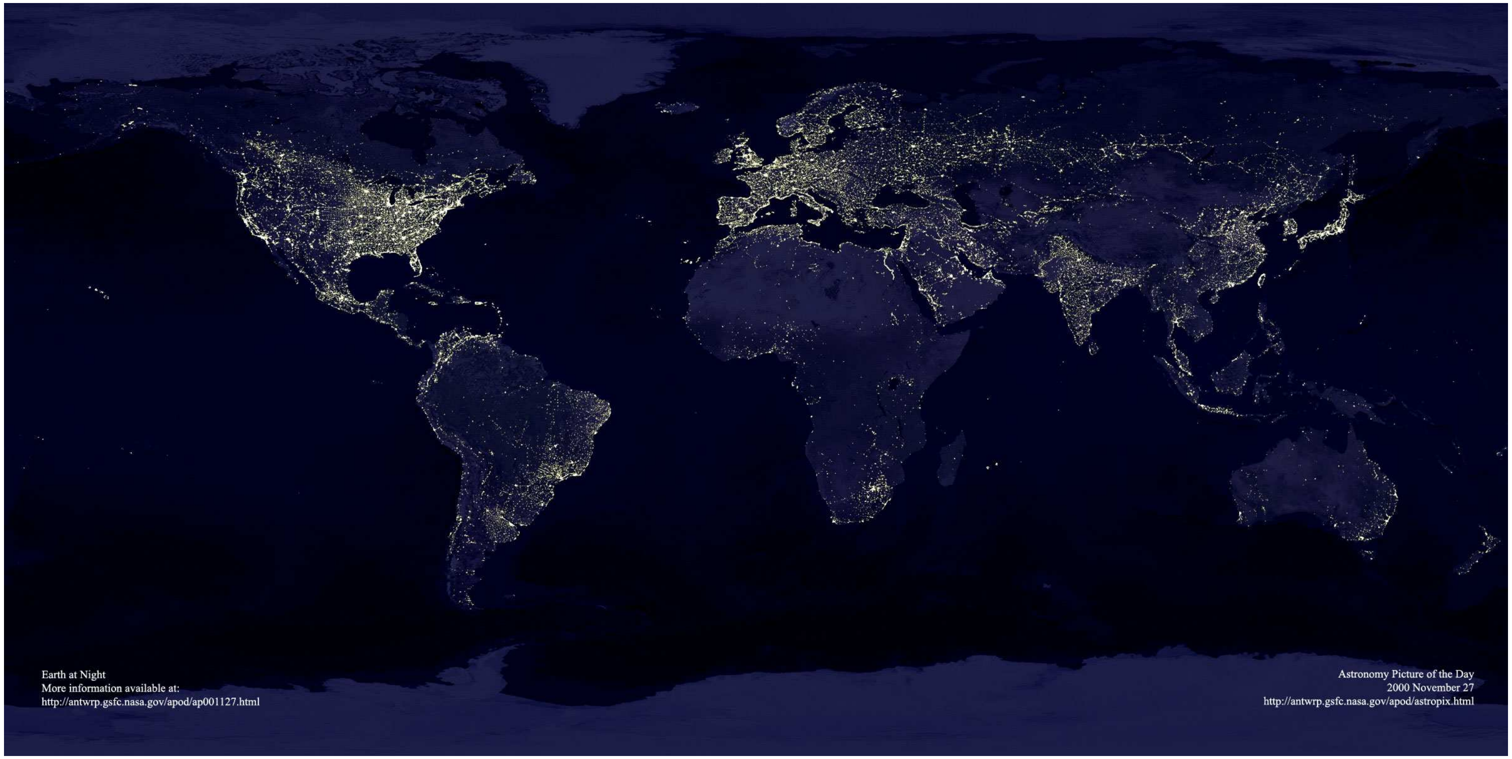
- Suppression of PS from RG at $k \sim 0.25 hMpc^{-1}$ due to failure of analytical approximations

What we want: Matter power spectrum.
What we observe: Galaxies
Is that the same?

Tracing the matter with galaxies - bias



Does absence of light mean absence of land?

Galaxies do not trace dark matter distribution in general

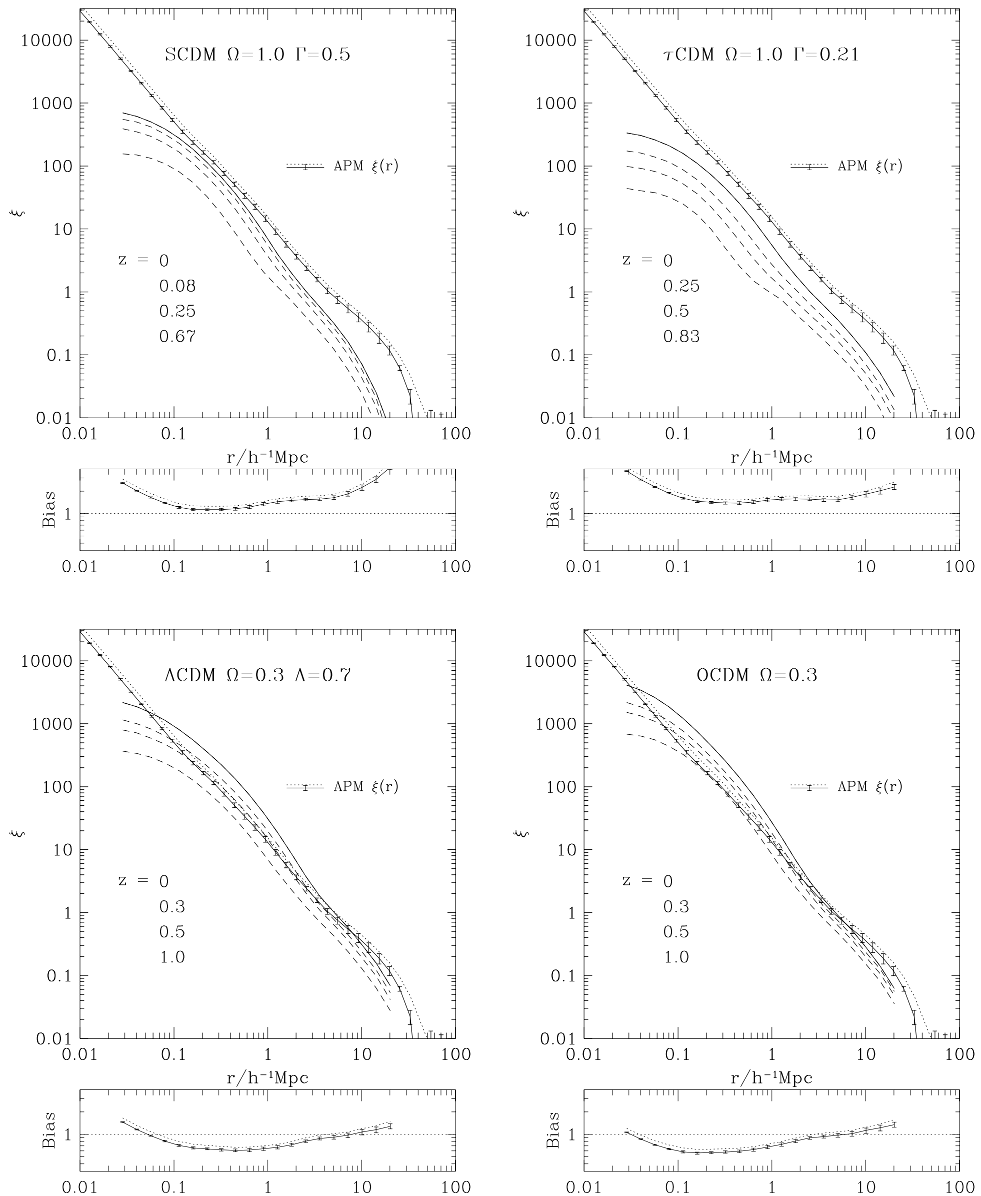
Tracing the matter with galaxies - bias

APM galaxy survey

Mass correlation function

Different cosmologies

Bias in general non-linear and local



Virgo, consortium, 98, arXiv:astro-ph/970901

Chasing the bias - can we do it?

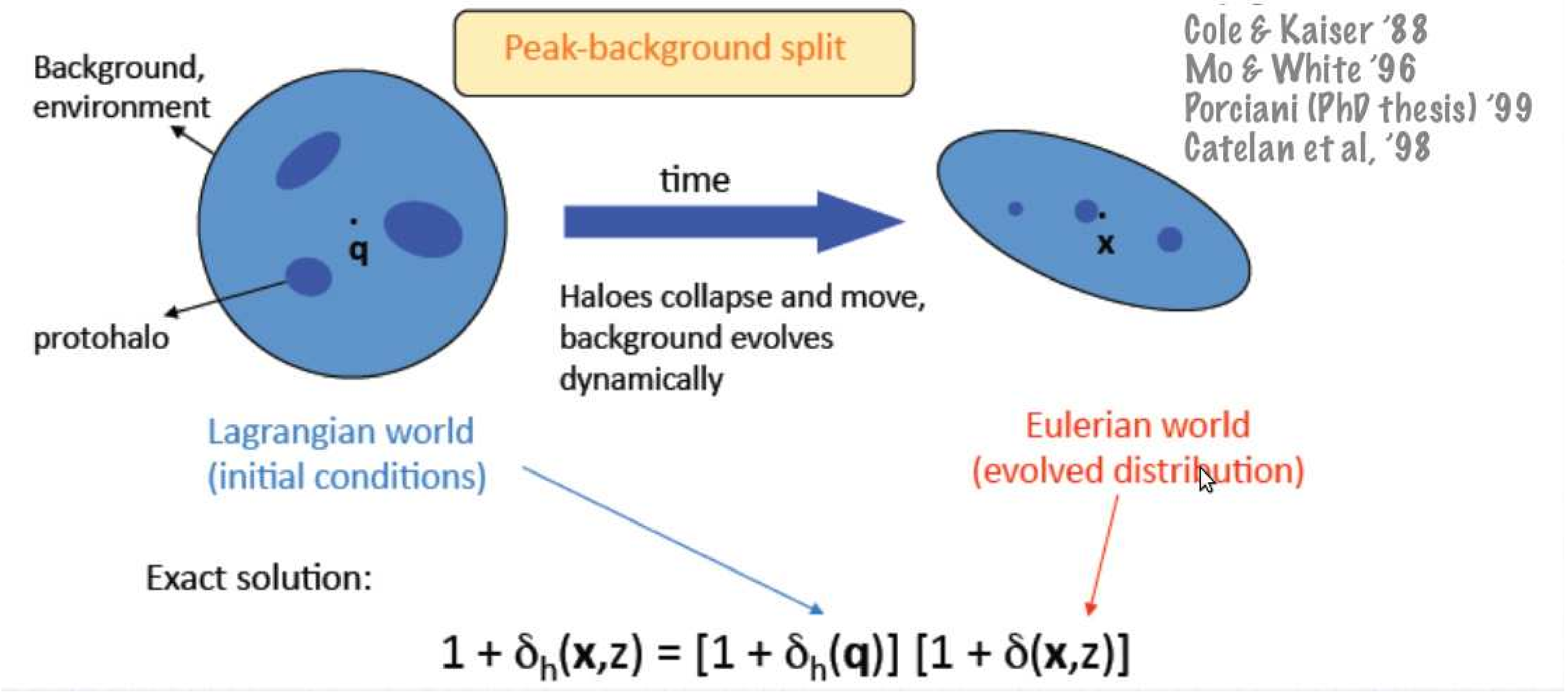
Chasing the bias - can we do it?

Chasing the bias - can we do it?

- Galaxies live in dark matter haloes
- Haloes themselves are biased against the background matter field
- Understand halo bias as a first step to understand galaxy bias
- Halo power spectrum suffers from the issue of shot noise.
- Shot noise, break down of the fluid assumption for discrete lumps of haloes
- Should predict/calculate the halo power spectrum, but we calculate the cross-power spectrum to avoid dealing with shot noise.

Question: Given initial model of halo bias, can we predict it's evolution?

Formalism



- Final haloes can be traced to their initial position (proto-haloes)
- proto-haloes are conserved
- Follow the evolution of center of mass of proto-haloes

Formalism

$$
\begin{aligned}
\frac{\partial \delta_h}{\partial \tau} + \nabla \cdot [(1+\delta_h)\mathbf{v}] &= 0 \qquad \text{Proto-haloes are conserved} \\
\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} &= -\nabla\phi \\
\nabla^2\phi &= \frac{3}{2}\Omega_m \mathcal{H}\delta
\end{aligned}
$$

Haloes identified at $z = 0$, traced back to initial position - called proto haloes

Extension of the previous formalism to three fluid system

$$
\mathbf{\Omega} = \begin{pmatrix} 1 & -1 & 0 \\ -3/2 & 3/2 & 0 \\ 0 & -1 & 1 \end{pmatrix}
$$

$$
\begin{aligned}
&\gamma_{121}(\mathbf{k},\, \mathbf{p},\, \mathbf{q}) = \frac{1}{2}\,\delta_D(\mathbf{k}+\mathbf{p}+\mathbf{q})\,\alpha(\mathbf{p},\mathbf{q})\,, \\
&\gamma_{121}(\mathbf{k},\, \mathbf{p},\, \mathbf{q}) = \gamma_{112}(\mathbf{k},\, \mathbf{q},\, \mathbf{p})\,, \\
&\gamma_{222}(\mathbf{k},\, \mathbf{p},\, \mathbf{q}) = \delta_D(\mathbf{k}+\mathbf{p}+\mathbf{q})\,\beta(\mathbf{p},\mathbf{q})\,, \\
&\gamma_{323}(\mathbf{k},\, \mathbf{p},\, \mathbf{q}) = \gamma_{332}(\mathbf{k},\, \mathbf{q},\, \mathbf{p}) = \gamma_{121}(\mathbf{k},\, \mathbf{p},\, \mathbf{q})
\end{aligned}
$$

Initial halo-halo and matter-halo (cross) power spectrum fitted via

$P_{33}(k) = (b_1 + b_2 \cdot k^2)^2 P_{11}(k)\exp(-k^2 R^2)$

$P_{13}(k) = (b_1 + b_2 \cdot k^2) P_{11}(k)\exp(-k^2 R^2/2)$

Initial conditions

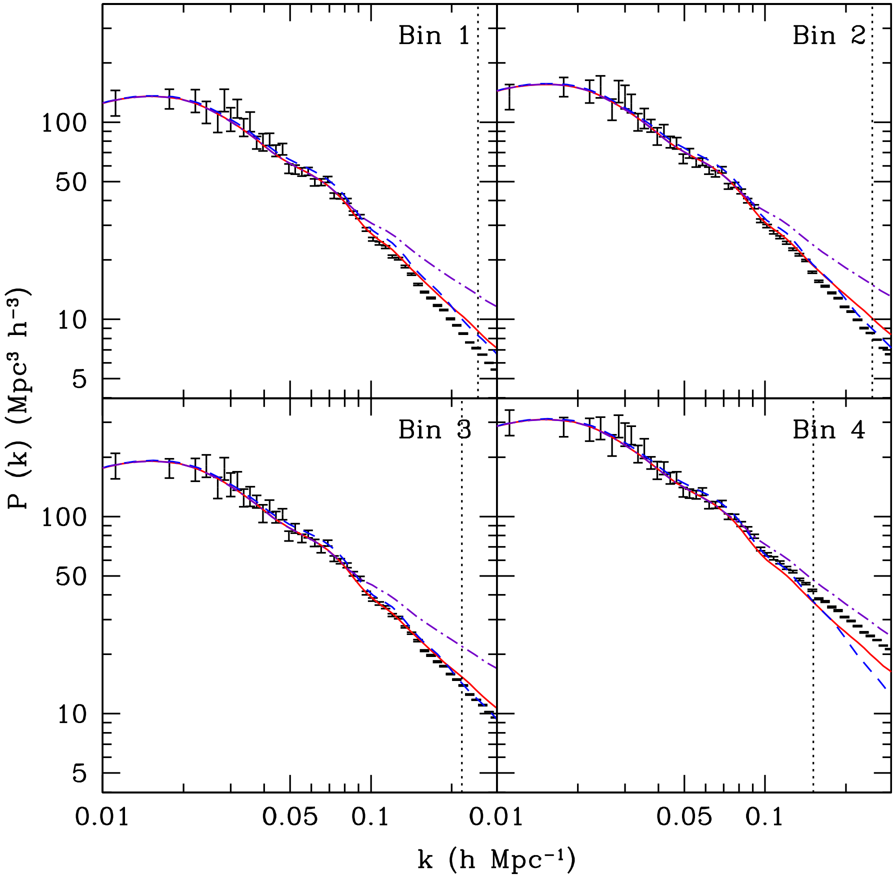
- 1024^3 dark-matter particles within a periodic cubic box $Lbox = 1200h^1$ Mpc
- ΛCDM model, Gaussian initial conditions and cosmological parameters: $h = 0.701, \sigma_8 = 0.817, ns = 0.96, \Omega_m = 0.279, \Omega_b = 0.0462, \Omega_\Lambda = 0.721$
- Assume initial relation between halo and matter fluctuations as $\delta_h(k) = (b_1 + b_2 \cdot k^2)\delta_m(k)$
- Fit initial power spectrum using above relations and follow the evolution

Analysis for four different mass bins:

Bin	Mass range ($10^{13} M_\odot/h$)
Bin 1	$1.24 - 1.8$
Bin 2	$1.8 - 3.4$
Bin 3	$3.4 - 10$
Bin 4	> 10

Note: Huge haloes in fourth bin

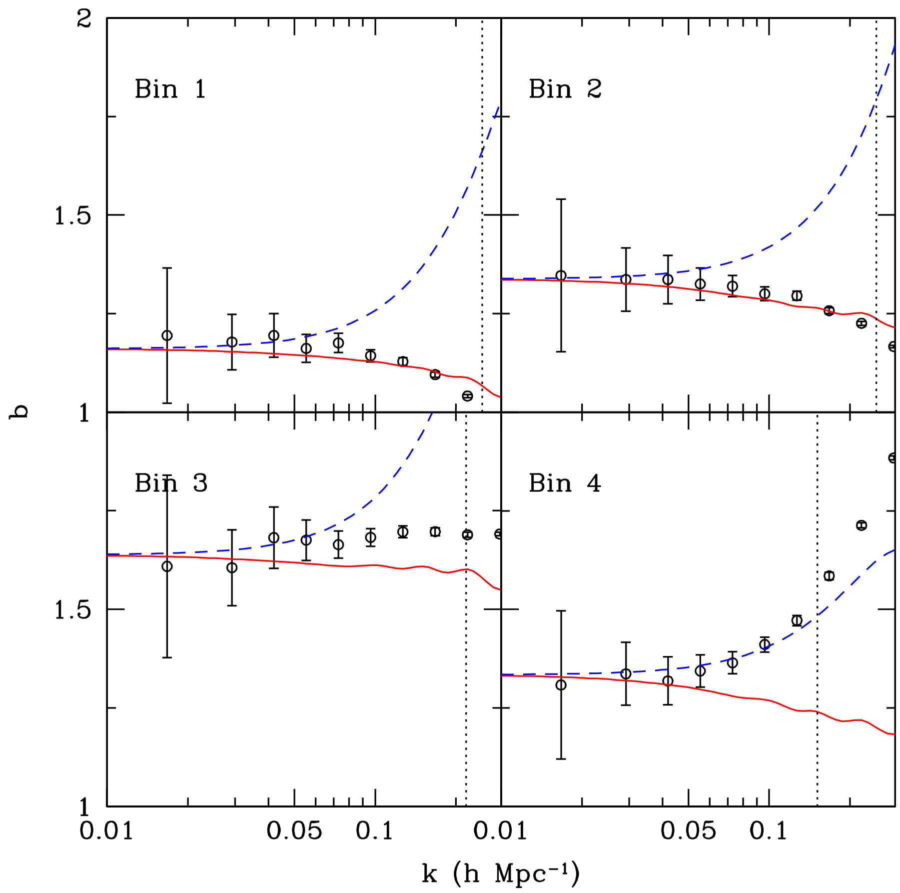
Results - cross PS at $z = 0$



Simulations, linear, 1-loop, RPT

- Zero the initial bispectra
- First three bins, linear theory overpredicts the power on mildly non-linear scales
- One-loop power spectrum corrects only on very large scale
- Renormalization corrects it up to a smaller scale, before starting to fail
- The fourth bin, everything fails
- Very massive haloes are large and rare in the initial conditions, therefore less suited for the fluid approximation.

Results - bias



$b \equiv P_{mh}/P_m$

Simulations, linear, 1-loop, RPT

- In linear-theory - bias always increases with scale
- Renormalized theory follows the scale dependence of $b(k)$
- Nearly constant bias for the third bin
- Linear model performs better in the last bin

Conclusions

- Understanding results of current generation galaxy surveys demmands understanding of perturbations on non-linear scales
- Predicting power spectrum with the help of field theory helps thoeoretical understanding of evolution
- Renormalized perturbation thoery improves the predictions in the regions interested for BAO
- Bias is one of the major issues in connecting observed parameters to thoeretical predictions
- The current RPT apporach can be extended to include haloes as fluids and predicts evolution of bias
- Renormalized perturbation theory can help connecting observables to theoretical predictions