The Higgs potential in type II seesaw

G. Moultaka

Laboratoire Charles Coulomb (L2C)
CNRS & University of Montepllier II

GDR-Terascale, Lyon, April 18-20, 2011

The Higgs Potential in the Type II Seesaw Model

A. Arhrib[1,2], R. Benbrik[2,3,4], M. Chabab[2],
G. Moultaka* [5,6], M. C. Peyranère[7,8], L. Rahili[2], J. Ramadan[2]

[1] *Département de Mathématiques, Faculté des Sciences et Techniques, Tanger, Morocco*
[2] *Laboratoire de Physique des Hautes Energies et Astrophysique*
Département de Physiques,Faculté des Sciences Semlalia,Marrakech, Morocco
[3] *Faculté Polydisciplinaire, Université Cadi Ayyad, Sidi Bouzid, Safi-Morocco*
[4] *Instituto de Fisica de Cantabria (CSIC-UC), Santander, Spain*
[5] *Université Montpellier 2, Laboratoire Charles Coulomb UMR 5221, F-34095 Montpellier, France*
[6] *CNRS, Laboratoire Charles Coulomb UMR 5221, F-34095 Montpellier, France*
[7] *Université Montpellier 2, Laboratoire Univers & Particules de Montpellier UMR 5299, F-34095 Montpellier, France*
[8] *CNRS/IN2P3, Laboratoire Univers & Particules de Montpellier UMR 5299, F-34095 Montpellier, France*

April 17, 2011

Abstract

The Standard Model Higgs sector, extended by one weak gauge triplet of scalar fields with a very small vacuum expectation value, is a very promising setting to account for neutrino masses through the so-called type II seesaw. In this paper we consider the general renormalizable doublet/triplet Higgs potential of this model. We perform a detailed study of its main dynamical features that depend on five dimensionless couplings and one mass parameter after spontaneous symmetry breaking, and highlight the implications for the Higgs phenomenology. In particular, we determine *i)* the complete set of tree-level unitarity constraints on the couplings of the potential and *ii)* the exact tree-level *all directions* boundedness from below constraints on these couplings. When combined, these constraints delineate precisely the theoretically allowed parameter space domain within our perturbative approximation. Among the seven physical Higgs states of this model, the mass of the lighter (heavier) $\mathcal{CP}_{even}$ state h^0 (H^0) will always satisfy a theoretical *upper (lower) bound* that is reached for a critical value μ_c of μ (the mass parameter controlling triple couplings among the doublet/triplet Higgses). Saturating the unitarity constraints we find an upper bound $m_{h^0} < \mathcal{O}(500-700\text{GeV})$, while the upper bound for the remaining Higgses lies in the several tens of TeV. However, the actual masses can be much lighter. We identify two regimes corresponding to $\mu \gtrsim \mu_c$ and $\mu \lesssim \mu_c$. In the first regime the Higgs sector is typically very heavy and only h^0 which becomes SM-like could be accessible to the LHC. In contrast, in the second regime, somewhat overlooked in the literature, most of the Higgs sector is light, and in particular the heaviest state H^0 becomes SM-like, the lighter states being (doubly) charged, $\mathcal{CP}_{odd}$ or a decoupled $\mathcal{CP}_{even}$, possibly leading to a distinctive phenomenology at the colliders.

**corresponding author*

Outline

Introductory motivations

The model

Electroweak symmetry breaking

Dynamical constraints
- Unitarity constraints
- Boundedness from below

Higgs mass bounds

phenomenological implications

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

  No: *simply add a* ν_R *and a standard Yukawa coupling* $\rightarrow$ *Dirac mass* $+$ perhaps a Majorana mass

$\rightarrow$ mysterious... SM singlet only gravitationally coupled !?
$\rightarrow$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\rightarrow$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\rightarrow$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

  No: *simply add a ν_R and a standard Yukawa coupling* $\rightarrow$ *Dirac mass* $+$ perhaps a Majorana mass

$\rightarrow$ mysterious... SM singlet only gravitationally coupled !?
$\rightarrow$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\rightarrow$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\rightarrow$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

No: *simply add a ν_R and a standard Yukawa coupling* $\rightarrow$ *Dirac mass* $+$ perhaps a Majorana mass

$\rightarrow$ mysterious... SM singlet only gravitationally coupled !?
$\rightarrow$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\rightarrow$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\rightarrow$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

No: *simply add a ν_R and a standard Yukawa coupling* $\rightarrow$ *Dirac mass* $+$ perhaps a Majorana mass

$\rightarrow$ mysterious... SM singlet only gravitationally coupled !?
$\rightarrow$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\rightarrow$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\rightarrow$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

No: *simply add a ν_R and a standard Yukawa coupling* $\to$ *Dirac mass* $+$ perhaps a Majorana mass

$\to$ mysterious... SM singlet only gravitationally coupled !?
$\to$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\to$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\to$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

  No: *simply add a ν_R and a standard Yukawa coupling* $\rightarrow$ *Dirac mass* $+$ perhaps a Majorana mass

$\rightarrow$ mysterious... SM singlet only gravitationally coupled !?
$\rightarrow$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\rightarrow$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\rightarrow$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

  No: *simply add a ν_R and a standard Yukawa coupling* $\rightarrow$ *Dirac mass* $+$ perhaps a Majorana mass

$\rightarrow$ mysterious... SM singlet only gravitationally coupled !?
$\rightarrow$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\rightarrow$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\rightarrow$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

  No: *simply add a ν_R and a standard Yukawa coupling* $\rightarrow$ *Dirac mass* $+$ perhaps a Majorana mass

$\rightarrow$ mysterious... SM singlet only gravitationally coupled !?
$\rightarrow$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\rightarrow$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\rightarrow$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

  No: *simply add a ν_R and a standard Yukawa coupling* $\rightarrow$ *Dirac mass* $+$ perhaps a Majorana mass

$\rightarrow$ mysterious... SM singlet only gravitationally coupled !?

$\rightarrow$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\rightarrow$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\rightarrow$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

  No: *simply add a ν_R and a standard Yukawa coupling* $\rightarrow$ *Dirac mass* $+$ perhaps a Majorana mass

$\rightarrow$ mysterious... SM singlet only gravitationally coupled !?
$\rightarrow$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\rightarrow$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\rightarrow$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

  No: *simply add a ν_R and a standard Yukawa coupling* $\rightarrow$ *Dirac mass* $+$ perhaps a Majorana mass

$\rightarrow$ mysterious... SM singlet only gravitationally coupled !?
$\rightarrow$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\rightarrow$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\rightarrow$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

  No: *simply add a* ν_R *and a standard Yukawa coupling* $\rightarrow$ *Dirac mass* $+$ perhaps a Majorana mass

$\rightarrow$ mysterious... SM singlet only gravitationally coupled !?
$\rightarrow$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\rightarrow$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\rightarrow$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

Introductory motivations

are there hints for physics beyond the Standard Model?

- Naturalness or hierarchy problems are often overstated
- Dark matter!
- Neutrino masses? No and Yes

  No: *simply add a ν_R and a standard Yukawa coupling* $\rightarrow$ *Dirac mass* $+$ perhaps a Majorana mass

$\rightarrow$ mysterious... SM singlet only gravitationally coupled !?
$\rightarrow$ more elegant (but not necessary!), ν_R charged under some GUT group... e.g. spinorial rep. of $SO(10)$

$\rightarrow$ seesaw mechanisms

In this talk we will have in mind the type II seesaw $\rightarrow$ neutrinos masses without an extra ν_R

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

The model

This sector consists of the standard Higgs weak doublet H and a colorless scalar field Δ transforming as a triplet under the $SU(2)_L$ gauge group with hypercharge $Y_\Delta = 2$:
$H \sim (1,2,1)$ and $\Delta \sim (1,3,2)$ under $SU(3)_c \times SU(2)_L \times U(1)_Y$.

$$Q = I_3 + \frac{Y}{2}$$

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \quad \text{and} \quad H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$\mathcal{L} = (D_\mu H)^\dagger (D^\mu H) + Tr(D_\mu \Delta)^\dagger (D^\mu \Delta) - V(H,\Delta) + \mathcal{L}_{Yukawa} + \ldots$$

$$\begin{aligned} V(H,\Delta) = & -m_H^2 H^\dagger H + M_\Delta^2 Tr(\Delta^\dagger \Delta) + [\mu(H^T i\sigma_2 \Delta^\dagger H) + \text{h.c.}] \\ & + \frac{\lambda}{4}(H^\dagger H)^2 + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) \\ & + \lambda_2 (Tr\Delta^\dagger \Delta)^2 + \lambda_3 Tr(\Delta^\dagger \Delta)^2 \\ & + \lambda_4 H^\dagger \Delta \Delta^\dagger H \end{aligned}$$

Electroweak symmetry breaking

$$\langle\Delta\rangle = \begin{pmatrix} 0 & 0 \\ v_t/\sqrt{2} & 0 \end{pmatrix} \quad \text{and} \quad \langle H\rangle = \begin{pmatrix} 0 \\ v_d/\sqrt{2} \end{pmatrix}$$

one finds after minimization of the potential the following necessary conditions:

$$\begin{aligned} M_\Delta^2 &= \frac{2\mu v_d^2 - \sqrt{2}(\lambda_1+\lambda_4)v_d^2 v_t - 2\sqrt{2}(\lambda_2+\lambda_3)v_t^3}{2\sqrt{2}v_t} \\ m_H^2 &= \frac{\lambda v_d^2}{4} - \sqrt{2}\mu v_t + \frac{(\lambda_1+\lambda_4)}{2}v_t^2 \end{aligned}$$

8 parameters $\longrightarrow$ 7 parameters with $v \equiv \sqrt{v_d^2 + 2v_t^2} = 246\text{GeV}$

Electroweak symmetry breaking

$\rightarrow$ 10 scalar states: 7 massive physical Higgses, $h^0, H^0, A^0, H^\pm, H^{\pm\pm}$ and 3 Goldstone bosons

$$m^2_{H^{\pm\pm}} = \frac{\sqrt{2}\mu v_d^2 - \lambda_4 v_d^2 v_t - 2\lambda_3 v_t^3}{2v_t}$$

$$m^2_{H^\pm} = \frac{(v_d^2 + 2v_t^2)[2\sqrt{2}\mu - \lambda_4 v_t]}{4v_t}$$

$$m^2_A = \frac{\mu(v_d^2 + 4v_t^2)}{\sqrt{2}v_t}$$

$$m^2_{h^0,H^0} = \frac{1}{2}[A + C \mp \sqrt{(A-C)^2 + 4B^2}]$$

$$A = \frac{\lambda}{2}v_d^2 \quad , \quad B = v_d[-\sqrt{2}\mu + (\lambda_1 + \lambda_4)v_t] \quad , \quad C = \frac{\sqrt{2}\mu v_d^2 + 4(\lambda_2 + \lambda_3)v_t^3}{2v_t}$$

$\rightarrow$ three mixing angles α, β, β'.

Electroweak symmetry breaking

→ 10 scalar states: 7 massive physical Higgses, $h^0, H^0, A^0, H^\pm, H^{\pm\pm}$ and 3 Goldstone bosons

$$m^2_{H^{\pm\pm}} = \frac{\sqrt{2}\mu v_d^2 - \lambda_4 v_d^2 v_t - 2\lambda_3 v_t^3}{2v_t}$$

$$m^2_{H^\pm} = \frac{(v_d^2 + 2v_t^2)[2\sqrt{2}\mu - \lambda_4 v_t]}{4v_t}$$

$$m^2_A = \frac{\mu(v_d^2 + 4v_t^2)}{\sqrt{2}v_t}$$

$$m^2_{h^0,H^0} = \frac{1}{2}[A + C \mp \sqrt{(A-C)^2 + 4B^2}]$$

$$A = \frac{\lambda}{2}v_d^2 \quad , \quad B = v_d[-\sqrt{2}\mu + (\lambda_1 + \lambda_4)v_t] \quad , \quad C = \frac{\sqrt{2}\mu v_d^2 + 4(\lambda_2 + \lambda_3)v_t^3}{2v_t}$$

→ three mixing angles α, β, β'.

Dynamical constraints

- tree-level unitarity constraints from scalar and gauge boson scattering
- conditions for a bounded from below potential
- absence of charge breaking minima?
- metastable gauge symmetric vacuum?
- tachyonless states
- spontaneous CP violation?

Higgs spectrum ?

Dynamical constraints

- tree-level unitarity constraints from scalar and gauge boson scattering
- conditions for a bounded from below potential
- absence of charge breaking minima?
- metastable gauge symmetric vacuum?
- tachyonless states
- spontaneous CP violation?

Higgs spectrum ?

Dynamical constraints

- tree-level unitarity constraints from scalar and gauge boson scattering
- conditions for a bounded from below potential
- absence of charge breaking minima?
- metastable gauge symmetric vacuum?
- tachyonless states
- spontaneous CP violation?

Higgs spectrum ?

Dynamical constraints

- tree-level unitarity constraints from scalar and gauge boson scattering
- conditions for a bounded from below potential
- absence of charge breaking minima?
- metastable gauge symmetric vacuum?
- tachyonless states
- spontaneous CP violation?

Higgs spectrum ?

Dynamical constraints

- tree-level unitarity constraints from scalar and gauge boson scattering
- conditions for a bounded from below potential
- absence of charge breaking minima?
- metastable gauge symmetric vacuum?
- tachyonless states
- spontaneous CP violation?

Higgs spectrum ?

Dynamical constraints

- tree-level unitarity constraints from scalar and gauge boson scattering
- conditions for a bounded from below potential
- absence of charge breaking minima?
- metastable gauge symmetric vacuum?
- tachyonless states
- spontaneous CP violation?

Higgs spectrum ?

Dynamical constraints

- tree-level unitarity constraints from scalar and gauge boson scattering
- conditions for a bounded from below potential
- absence of charge breaking minima?
- metastable gauge symmetric vacuum?
- tachyonless states
- spontaneous CP violation?

Higgs spectrum ?

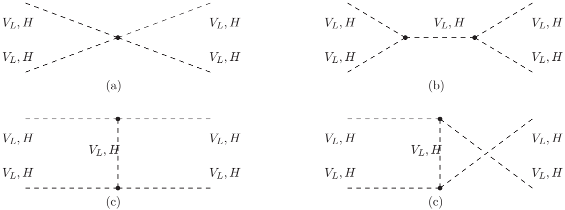
Dynamical constraints

- tree-level unitarity constraints from scalar and gauge boson scattering
- conditions for a bounded from below potential
- absence of charge breaking minima?
- metastable gauge symmetric vacuum?
- tachyonless states
- spontaneous CP violation?

Higgs spectrum ?

Dynamical constraints

Tree-level unitarity:

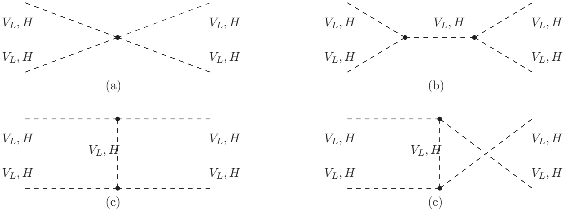


a 27×27 S matrix composed of 5 submatrices $\mathcal{M}_1(6 \times 6)$, $\mathcal{M}_2(7 \times 7)$, $\mathcal{M}_3(2 \times 2)$, $\mathcal{M}_4(8 \times 8)$, and $\mathcal{M}_5(4 \times 4)$

partial wave analyses $\rightarrow |a_0| \leq 1$

Dynamical constraints

Tree-level unitarity:

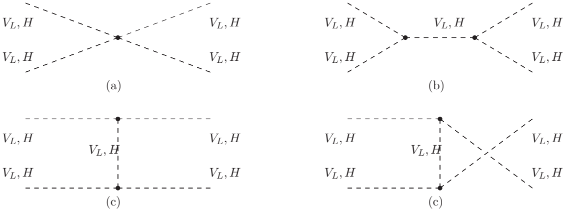


a 27×27 S matrix composed of 5 submatrices $\mathcal{M}_1(6 \times 6)$, $\mathcal{M}_2(7 \times 7)$, $\mathcal{M}_3(2 \times 2)$, $\mathcal{M}_4(8 \times 8)$, and $\mathcal{M}_5(4 \times 4)$

partial wave analyses $\rightarrow$ $|a_0| \leq 1$

Dynamical constraints

Tree-level unitarity:



a 27×27 S matrix composed of 5 submatrices $\mathcal{M}_1(6 \times 6)$, $\mathcal{M}_2(7 \times 7)$, $\mathcal{M}_3(2 \times 2)$, $\mathcal{M}_4(8 \times 8)$, and $\mathcal{M}_5(4 \times 4)$

partial wave analyses $\rightarrow$ $|a_0| \leq 1$

Dynamical constraints

Tree-level unitarity:

$$|\lambda_1 + \lambda_4| \leq \kappa\pi \tag{1}$$

$$|\lambda_1| \leq \kappa\pi \tag{2}$$

$$|2\lambda_1 + 3\lambda_4| \leq 2\kappa\pi \tag{3}$$

$$|\lambda| \leq 2\kappa\pi \tag{4}$$

$$|\lambda_2| \leq \frac{\kappa}{2}\pi \tag{5}$$

$$|\lambda_2 + \lambda_3| \leq \frac{\kappa}{2}\pi \tag{6}$$

$$|\ \lambda + 4\lambda_2 + 8\lambda_3 \pm \sqrt{(\lambda - 4\lambda_2 - 8\lambda_3)^2 + 16\lambda_4^2}\ | \leq 4\kappa\pi \tag{7}$$

$$|\ 3\lambda + 16\lambda_2 + 12\lambda_3 \pm \sqrt{(3\lambda - 16\lambda_2 - 12\lambda_3)^2 + 24(2\lambda_1 + \lambda_4)^2}\ | \leq 4\kappa\pi \tag{8}$$

$$|2\lambda_1 - \lambda_4| \leq 2\kappa\pi \tag{9}$$

$$|\ 3\lambda_2 + \lambda_3 \pm \sqrt{(\lambda_2 + \lambda_3)^2 + 4\lambda_3^2}\ | \leq \kappa\pi \tag{10}$$

Dynamical constraints

Tree-level unitarity:

$$|\lambda_1 + \lambda_4| \leq \kappa\pi \tag{1}$$

$$|\lambda_1| \leq \kappa\pi \tag{2}$$

$$|2\lambda_1 + 3\lambda_4| \leq 2\kappa\pi \tag{3}$$

$$|\lambda| \leq 2\kappa\pi \tag{4}$$

$$|\lambda_2| \leq \frac{\kappa}{2}\pi \tag{5}$$

$$|\lambda_2 + \lambda_3| \leq \frac{\kappa}{2}\pi \tag{6}$$

$$|\ \lambda + 4\lambda_2 + 8\lambda_3 \pm \sqrt{(\lambda - 4\lambda_2 - 8\lambda_3)^2 + 16\lambda_4^2}\ | \leq 4\kappa\pi \tag{7}$$

$$|\ 3\lambda + 16\lambda_2 + 12\lambda_3 \pm \sqrt{(3\lambda - 16\lambda_2 - 12\lambda_3)^2 + 24(2\lambda_1 + \lambda_4)^2}\ | \leq 4\kappa\pi \tag{8}$$

$$|2\lambda_1 - \lambda_4| \leq 2\kappa\pi \tag{9}$$

$$|\ 3\lambda_2 + \lambda_3 \pm \sqrt{(\lambda_2 + \lambda_3)^2 + 4\lambda_3^2}\ | \leq \kappa\pi \tag{10}$$

Dynamical constraints

Tree-level Boundedness From Below:

- Keep only the quartic operators

$$V^{(4)}(H,\Delta) = \frac{\lambda}{4}(H^\dagger H)^2 + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) + \lambda_2 (Tr \Delta^\dagger \Delta)^2$$
$$+\lambda_3 Tr(\Delta^\dagger \Delta)^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H$$

Dynamical constraints

Tree-level Boundedness From Below:

- Keep only the quartic operators

$$V^{(4)}(H,\Delta) = \frac{\lambda}{4}(H^\dagger H)^2 + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) + \lambda_2 (Tr\Delta^\dagger \Delta)^2 + \lambda_3 Tr(\Delta^\dagger \Delta)^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H$$

Dynamical constraints

Tree-level Boundedness From Below:

- Keep only the quartic operators

$$V^{(4)}(H,\Delta) = \frac{\lambda}{4}(H^\dagger H)^2 + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) + \lambda_2 (Tr \Delta^\dagger \Delta)^2 \\ + \lambda_3 Tr(\Delta^\dagger \Delta)^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H$$

Dynamical constraints

Tree-level Boundedness From Below:

In the literature one finds only very partial answers;

e.g. field space directions where only the electrically neutral components are non vanishing

$$V_0^{(4)} = \frac{\lambda}{4}|\phi^0|^2 + (\lambda_2 + \lambda_3)|\delta^0|^2 + (\lambda_1 + \lambda_4)|\phi^0|^2|\delta^0|^2$$

lead to the sufficient and necessary conditions

$$\begin{aligned} \lambda &> 0 \\ \lambda_2 + \lambda_3 &> 0 \\ \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} &> 0 \end{aligned}$$

→ it becomes more complicated in other directions!

Dynamical constraints

Tree-level Boundedness From Below:

In the literature one finds only very partial answers;
e.g. field space directions where only the electrically neutral components are non vanishing

$$V_0^{(4)} = \frac{\lambda}{4}|\phi^0|^2 + (\lambda_2 + \lambda_3)|\delta^0|^2 + (\lambda_1 + \lambda_4)|\phi^0|^2|\delta^0|^2$$

lead to the sufficient and necessary conditions

$$\begin{aligned} \lambda &> 0 \\ \lambda_2 + \lambda_3 &> 0 \\ \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} &> 0 \end{aligned}$$

→ it becomes more complicated in other directions!

Dynamical constraints

Tree-level Boundedness From Below:
In the literature one finds only very partial answers;
e.g. field space directions where only the electrically neutral components are non vanishing

$$V_0^{(4)} = \frac{\lambda}{4}|\phi^0|^2 + (\lambda_2 + \lambda_3)|\delta^0|^2 + (\lambda_1 + \lambda_4)|\phi^0|^2|\delta^0|^2$$

lead to the sufficient and necessary conditions

$$\begin{aligned} \lambda &> 0 \\ \lambda_2 + \lambda_3 &> 0 \\ \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} &> 0 \end{aligned}$$

→ it becomes more complicated in other directions!

Dynamical constraints

Tree-level Boundedness From Below:

In the literature one finds only very partial answers;
e.g. field space directions where only the electrically neutral components are non vanishing

$$V_0^{(4)} = \frac{\lambda}{4}|\phi^0|^2 + (\lambda_2 + \lambda_3)|\delta^0|^2 + (\lambda_1 + \lambda_4)|\phi^0|^2|\delta^0|^2$$

lead to the sufficient and necessary conditions

$$\begin{aligned} \lambda &> 0 \\ \lambda_2 + \lambda_3 &> 0 \\ \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} &> 0 \end{aligned}$$

$\rightarrow$ it becomes more complicated in other directions!

Dynamical constraints

e.g. of a 3-field direction $(\phi^+, \delta^0, \delta^{++})$:

$$
\begin{aligned}
V^{(4)} &= (\lambda_2+\lambda_3)\,|\delta^0|^4 + 2\,\lambda_2\,|\delta^0|^2\,|\delta^{++}|^2 + (\lambda_2+\lambda_3)\,|\delta^{++}|^4 \\
&\quad + \lambda_1\,|\delta^0|^2\,|\phi^+|^2 + (\lambda_1+\lambda_4)\,|\delta^{++}|^2\,|\phi^+|^2 + \frac{\lambda}{4}\,|\phi^+|^4
\end{aligned}
$$

$$
\begin{aligned}
&\lambda > 0 \wedge \lambda_2+\lambda_3 > 0 \wedge \sqrt{\lambda(\lambda_2+\lambda_3)} + \lambda_1 > 0 \wedge \\
&\Big(\Big(\frac{(\lambda_2+\lambda_3)\Big(\lambda\lambda_2^2 + \lambda_1^2(\lambda_3-\lambda_2) + 2\lambda_1\lambda_3\lambda_4 + \lambda_4^2(\lambda_2+\lambda_3)\Big)}{\lambda_2(\lambda_1+\lambda_4)} < 0 \wedge \\
&\Big((\lambda_3(2\lambda_2+\lambda_3) > 0 \wedge \lambda_1+\lambda_4 > 0 \wedge \lambda_2 < 0) \vee \Big(\lambda_2 > 0 \wedge \lambda(\lambda_2+\lambda_3) > (\lambda_1+\lambda_4)^2 \\
&\wedge \lambda_1+\lambda_4 < 0\Big)\Big)\Big) \vee (\lambda_2 > 0 \wedge \lambda_1+\lambda_4 > 0) \vee \Big(\lambda(\lambda_2+\lambda_3) > (\lambda_1+\lambda_4)^2 \wedge \lambda_3(2\lambda_2+\lambda_3) > 0 \\
&\wedge \sqrt{-\lambda_3(2\lambda_2+\lambda_3)\Big((\lambda_1+\lambda_4)^2 - \lambda(\lambda_2+\lambda_3)\Big)} + \lambda_1\lambda_3 > \lambda_2\lambda_4\Big)\Big)
\end{aligned}
$$

Dynamical constraints

e.g. of a 3-field direction $(\phi^+, \delta^0, \delta^{++})$:

$$
\begin{aligned}
V^{(4)} \;=\;& (\lambda_2+\lambda_3)\,|\delta^0|^4 + 2\,\lambda_2\,|\delta^0|^2\,|\delta^{++}|^2 + (\lambda_2+\lambda_3)\,|\delta^{++}|^4 \\
& + \lambda_1\,|\delta^0|^2\,|\phi^+|^2 + (\lambda_1+\lambda_4)\,|\delta^{++}|^2\,|\phi^+|^2 + \frac{\lambda}{4}\,|\phi^+|^4
\end{aligned}
$$

$$
\begin{aligned}
&\lambda > 0 \wedge \lambda_2+\lambda_3 > 0 \wedge \sqrt{\lambda(\lambda_2+\lambda_3)} + \lambda_1 > 0 \wedge \\
&\Big(\Big(\frac{(\lambda_2+\lambda_3)\Big(\lambda\lambda_2^2 + \lambda_1^2(\lambda_3-\lambda_2) + 2\lambda_1\lambda_3\lambda_4 + \lambda_4^2(\lambda_2+\lambda_3)\Big)}{\lambda_2(\lambda_1+\lambda_4)} < 0 \wedge \\
&\Big((\lambda_3(2\lambda_2+\lambda_3) > 0 \wedge \lambda_1+\lambda_4 > 0 \wedge \lambda_2 < 0) \vee \Big(\lambda_2 > 0 \wedge \lambda(\lambda_2+\lambda_3) > (\lambda_1+\lambda_4)^2 \\
&\wedge \lambda_1+\lambda_4 < 0\Big)\Big)\Big) \vee (\lambda_2 > 0 \wedge \lambda_1+\lambda_4 > 0) \vee \Big(\lambda(\lambda_2+\lambda_3) > (\lambda_1+\lambda_4)^2 \wedge \lambda_3(2\lambda_2+\lambda_3) > 0 \\
&\wedge \sqrt{-\lambda_3(2\lambda_2+\lambda_3)\Big((\lambda_1+\lambda_4)^2 - \lambda(\lambda_2+\lambda_3)\Big)} + \lambda_1\lambda_3 > \lambda_2\lambda_4\Big)\Big)
\end{aligned}
$$

Dynamical constraints

e.g. of a 3-field direction $(\phi^+, \delta^0, \delta^{++})$:

$$
\begin{aligned}
V^{(4)} \;=\;& (\lambda_2+\lambda_3)\,|\delta^0|^4 + 2\,\lambda_2\,|\delta^0|^2\,|\delta^{++}|^2 + (\lambda_2+\lambda_3)\,|\delta^{++}|^4 \\
& + \lambda_1\,|\delta^0|^2\,|\phi^+|^2 + (\lambda_1+\lambda_4)\,|\delta^{++}|^2\,|\phi^+|^2 + \frac{\lambda}{4}\,|\phi^+|^4
\end{aligned}
$$

$$
\begin{aligned}
&\lambda > 0 \wedge \lambda_2+\lambda_3 > 0 \wedge \sqrt{\lambda(\lambda_2+\lambda_3)} + \lambda_1 > 0 \wedge \\
&\Bigg(\Bigg(\frac{(\lambda_2+\lambda_3)\Big(\lambda\lambda_2^2 + \lambda_1^2(\lambda_3-\lambda_2) + 2\lambda_1\lambda_3\lambda_4 + \lambda_4^2(\lambda_2+\lambda_3)\Big)}{\lambda_2(\lambda_1+\lambda_4)} < 0 \wedge \\
&\Big((\lambda_3(2\lambda_2+\lambda_3) > 0 \wedge \lambda_1+\lambda_4 > 0 \wedge \lambda_2 < 0) \vee \Big(\lambda_2 > 0 \wedge \lambda(\lambda_2+\lambda_3) > (\lambda_1+\lambda_4)^2 \\
&\wedge \lambda_1+\lambda_4 < 0\Big)\Big)\Bigg) \vee (\lambda_2 > 0 \wedge \lambda_1+\lambda_4 > 0) \vee \Big(\lambda(\lambda_2+\lambda_3) > (\lambda_1+\lambda_4)^2 \wedge \lambda_3(2\lambda_2+\lambda_3) > 0 \\
&\wedge \sqrt{-\lambda_3(2\lambda_2+\lambda_3)\Big((\lambda_1+\lambda_4)^2 - \lambda(\lambda_2+\lambda_3)\Big)} + \lambda_1\lambda_3 > \lambda_2\lambda_4\Big)\Bigg)
\end{aligned}
$$

there are 10 such 3-field directions

(up to gauge transformations) with as many different conditions

!!!

...and this is not exhausting all possibilities, $4-$, $5-$,...field dir?

there are 10 such 3-field directions
(up to gauge transformations) with as many different conditions
!!!

...and this is not exhausting all possibilities, $4-$, $5-$,...field dir?

there are 10 such 3-field directions
(up to gauge transformations) with as many different conditions
!!!
...and this is not exhausting all possibilities, $4-$, $5-$,...field dir?

Dynamical constraints

Tree-level Boundedness From Below: The most general solution

$$
\begin{aligned}
r &\equiv \sqrt{H^\dagger H + Tr\Delta^\dagger \Delta} \\
H^\dagger H &\equiv r^2 \cos^2\gamma \\
Tr(\Delta^\dagger \Delta) &\equiv r^2 \sin^2\gamma \\
(H^\dagger \Delta\Delta^\dagger H)/(H^\dagger H Tr\Delta^\dagger \Delta) &\equiv \xi \\
Tr(\Delta^\dagger \Delta)^2/(Tr\Delta^\dagger \Delta)^2 &\equiv \zeta
\end{aligned}
$$

$$V^{(4)}(r, \tan\gamma, \xi, \zeta) = \frac{r^4}{4(1+\tan^2\gamma)^2}(\lambda + 4(\lambda_1 + \xi\lambda_4)\tan^2\gamma + 4(\lambda_2 + \zeta\lambda_3)\tan^4\gamma)$$

$$0 \leq \tan\gamma < +\infty$$

$$0 \leq \xi \leq 1 \quad \text{and} \quad \frac{1}{2} \leq \zeta \leq 1$$

Dynamical constraints

Tree-level Boundedness From Below: The most general solution

$$
\begin{aligned}
r &\equiv \sqrt{H^\dagger H + Tr\Delta^\dagger\Delta} \\
H^\dagger H &\equiv r^2\cos^2\gamma \\
Tr(\Delta^\dagger\Delta) &\equiv r^2\sin^2\gamma \\
(H^\dagger\Delta\Delta^\dagger H)/(H^\dagger H Tr\Delta^\dagger\Delta) &\equiv \xi \\
Tr(\Delta^\dagger\Delta)^2/(Tr\Delta^\dagger\Delta)^2 &\equiv \zeta
\end{aligned}
$$

$$V^{(4)}(r,\tan\gamma,\xi,\zeta) = \frac{r^4}{4(1+\tan^2\gamma)^2}(\lambda+4(\lambda_1+\xi\lambda_4)\tan^2\gamma+4(\lambda_2+\zeta\lambda_3)\tan^4\gamma)$$

$$0 \leq \tan\gamma < +\infty$$

$$0 \leq \xi \leq 1 \quad \text{and} \quad \frac{1}{2} \leq \zeta \leq 1$$

Tree-level Boundedness From Below: The most general solution

$$
\begin{aligned}
r &\equiv \sqrt{H^\dagger H + Tr\Delta^\dagger\Delta} \\
H^\dagger H &\equiv r^2 \cos^2\gamma \\
Tr(\Delta^\dagger\Delta) &\equiv r^2 \sin^2\gamma \\
(H^\dagger\Delta\Delta^\dagger H)/(H^\dagger H Tr\Delta^\dagger\Delta) &\equiv \xi \\
Tr(\Delta^\dagger\Delta)^2/(Tr\Delta^\dagger\Delta)^2 &\equiv \zeta
\end{aligned}
$$

$$V^{(4)}(r, \tan\gamma, \xi, \zeta) = \frac{r^4}{4(1+\tan^2\gamma)^2}(\lambda + 4(\lambda_1 + \xi\lambda_4)\tan^2\gamma + 4(\lambda_2 + \zeta\lambda_3)\tan^4\gamma)$$

$$0 \leq \tan\gamma < +\infty$$

$$0 \leq \xi \leq 1 \quad \text{and} \quad \frac{1}{2} \leq \zeta \leq 1$$

Dynamical constraints

Tree-level Boundedness From Below: The most general solution

$$
\begin{aligned}
r &\equiv \sqrt{H^\dagger H + Tr\Delta^\dagger\Delta} \\
H^\dagger H &\equiv r^2\cos^2\gamma \\
Tr(\Delta^\dagger\Delta) &\equiv r^2\sin^2\gamma \\
(H^\dagger\Delta\Delta^\dagger H)/(H^\dagger H Tr\Delta^\dagger\Delta) &\equiv \xi \\
Tr(\Delta^\dagger\Delta)^2/(Tr\Delta^\dagger\Delta)^2 &\equiv \zeta
\end{aligned}
$$

$$V^{(4)}(r,\tan\gamma,\xi,\zeta) = \frac{r^4}{4(1+\tan^2\gamma)^2}(\lambda+4(\lambda_1+\xi\lambda_4)\tan^2\gamma+4(\lambda_2+\zeta\lambda_3)\tan^4\gamma)$$

$$0 \leq \tan\gamma < +\infty$$

$$0 \leq \xi \leq 1 \quad \text{and} \quad \frac{1}{2} \leq \zeta \leq 1$$

Dynamical constraints

Tree-level Boundedness From Below: The most general solution

$$
\begin{aligned}
r &\equiv \sqrt{H^\dagger H + Tr\Delta^\dagger\Delta} \\
H^\dagger H &\equiv r^2 \cos^2\gamma \\
Tr(\Delta^\dagger\Delta) &\equiv r^2 \sin^2\gamma \\
(H^\dagger\Delta\Delta^\dagger H)/(H^\dagger H Tr\Delta^\dagger\Delta) &\equiv \xi \\
Tr(\Delta^\dagger\Delta)^2/(Tr\Delta^\dagger\Delta)^2 &\equiv \zeta
\end{aligned}
$$

$$V^{(4)}(r, \tan\gamma, \xi, \zeta) = \frac{r^4}{4(1+\tan^2\gamma)^2}(\lambda + 4(\lambda_1 + \xi\lambda_4)\tan^2\gamma + 4(\lambda_2 + \zeta\lambda_3)\tan^4\gamma)$$

$$0 \leq \tan\gamma < +\infty$$

$$0 \leq \xi \leq 1 \quad \text{and} \quad \frac{1}{2} \leq \zeta \leq 1$$

Dynamical constraints

$$\lambda > 0 \ \ \& \ \ \lambda_2 + \zeta\lambda_3 > 0 \ \ \& \ \ \lambda_1 + \xi\lambda_4 + \sqrt{\lambda(\lambda_2 + \zeta\lambda_3)} > 0,$$

$$\forall\zeta \in [\frac{1}{2}, 1], \forall\xi \in [0, 1]$$

$$\lambda \geq 0 \tag{11}$$

$$\lambda_2 + \lambda_3 \geq 0 \tag{12}$$

$$\lambda_2 + \frac{\lambda_3}{2} \geq 0 \tag{13}$$

$$\lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0 \tag{14}$$

$$\lambda_1 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} \geq 0 \tag{15}$$

$$\lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0 \tag{16}$$

$$\lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} \geq 0 \tag{17}$$

Dynamical constraints

$$\lambda > 0 \;\; \& \;\; \lambda_2 + \zeta\lambda_3 > 0 \;\; \& \;\; \lambda_1 + \xi\lambda_4 + \sqrt{\lambda(\lambda_2 + \zeta\lambda_3)} > 0,$$

$$\forall\zeta \in [\frac{1}{2}, 1], \forall\xi \in [0, 1]$$

$$\lambda \geq 0 \tag{11}$$

$$\lambda_2 + \lambda_3 \geq 0 \tag{12}$$

$$\lambda_2 + \frac{\lambda_3}{2} \geq 0 \tag{13}$$

$$\lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0 \tag{14}$$

$$\lambda_1 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} \geq 0 \tag{15}$$

$$\lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0 \tag{16}$$

$$\lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} \geq 0 \tag{17}$$

Dynamical constraints

combining all constraints $\rightarrow$

$$0 \leq \lambda \leq \frac{2}{3}\kappa\pi$$

$$\lambda_2 + \lambda_3 \geq 0 \;\;\&\;\; \lambda_2 + \frac{\lambda_3}{2} \geq 0$$

$$\lambda_2 + 2\lambda_3 \leq \frac{\kappa}{2}\pi$$

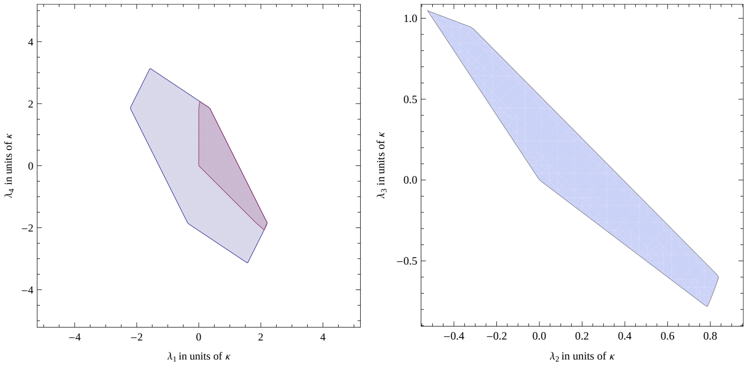
$$4\lambda_2 + 3\lambda_3 \leq \frac{\kappa}{2}\pi$$

$$\lambda_2 - 2\lambda_3 - \sqrt{(\lambda_2 - \frac{\kappa}{2}\pi)(9\lambda_2 - \frac{5}{2}\kappa\pi)} \leq \frac{\kappa}{2}\pi$$

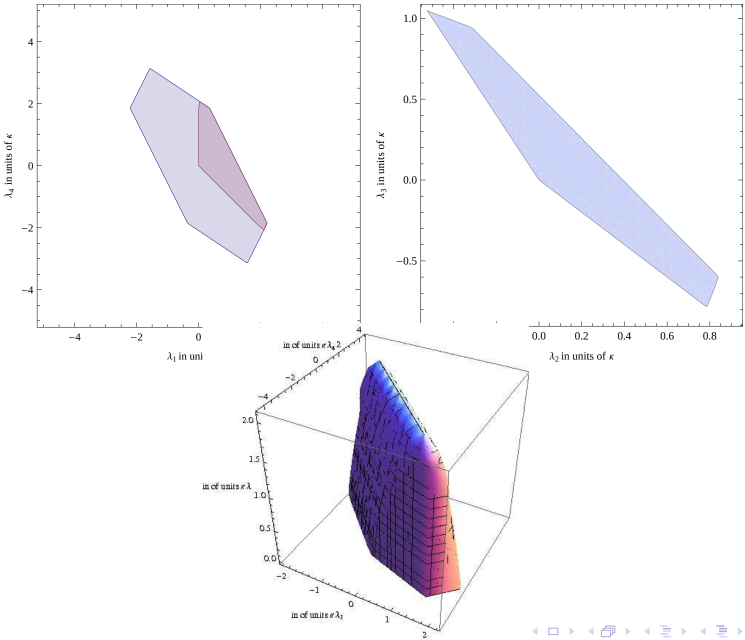
$$|\lambda_4| \leq \min \sqrt{(\lambda \pm 2\kappa\pi)(\lambda_2 + 2\lambda_3 \pm \frac{\kappa}{2}\pi)}$$

$$|2\lambda_1 + \lambda_4| \leq \sqrt{2(\lambda - \frac{2}{3}\kappa\pi)(4\lambda_2 + 3\lambda_3 - \frac{\kappa}{2}\pi)}$$

Dynamical constraints



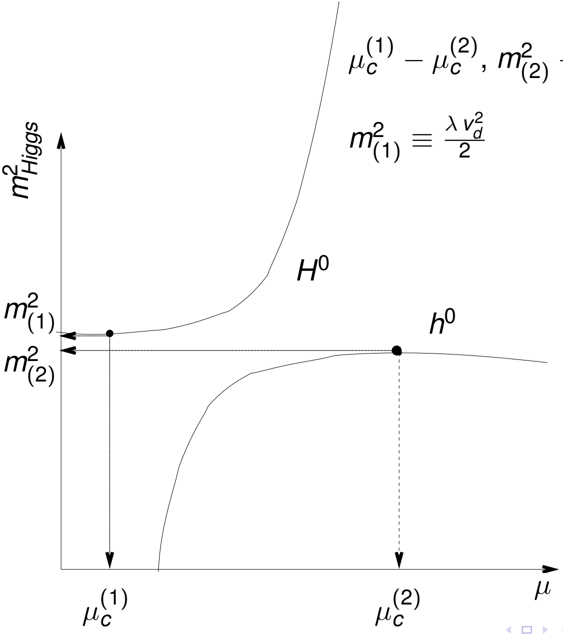
Dynamical constraints



Higgs mass bounds

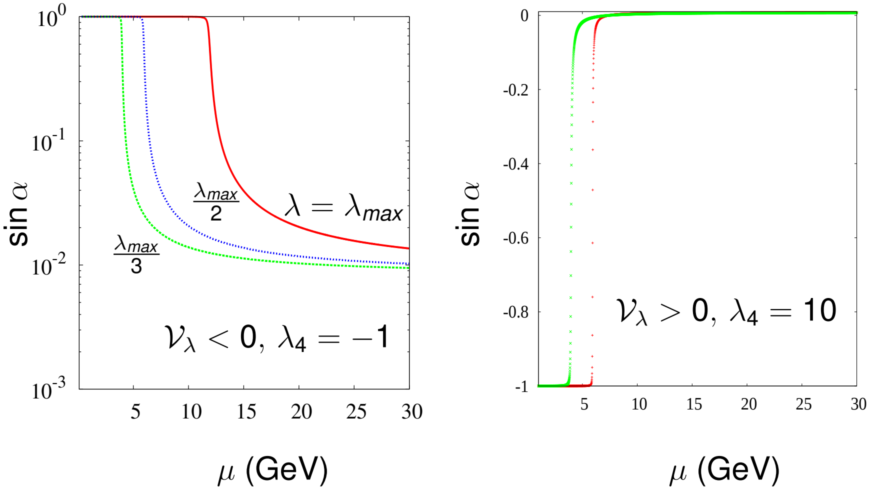
$$\mu_c^{(1)} - \mu_c^{(2)},\ m^2_{(2)} - m^2_{(1)} \sim V_\lambda$$

$$m^2_{(1)} \equiv \frac{\lambda\, v_d^2}{2}$$



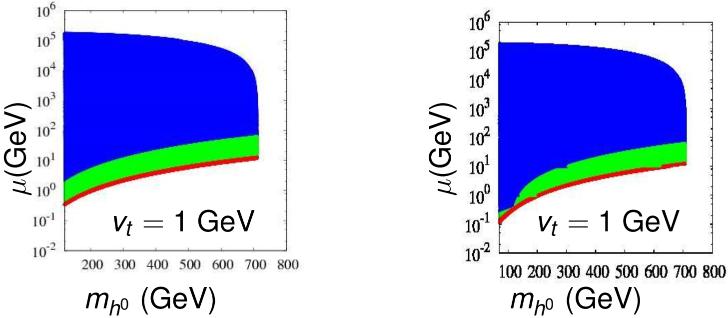
phenomenological implications

$$h^0 = \cos\alpha\, h + \sin\alpha\, \xi^0 \quad , \quad H^0 = -\sin\alpha\, h + \cos\alpha\, \xi^0$$



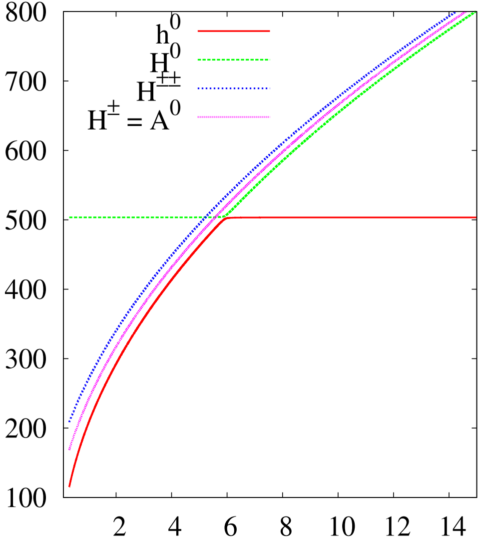
$v_t = 1$ GeV, $\lambda_{max} = 16\pi/3$, $\lambda_2 = \lambda_3 = 0.1$, $\lambda_1 = 0.5$

phenomenological implications



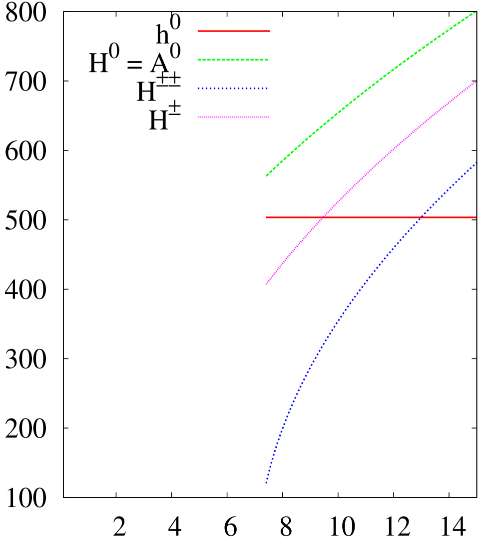
$10^{-1} \leq |\sin\alpha| \leq 1$ (red), $10^{-2} \leq |\sin\alpha| \leq 10^{-1}$ (green), $10^{-3} \leq |\sin\alpha| \leq 10^{-2}$ (blue)

phenomenological implications



$\mathcal{V}_\lambda < 0, \ \lambda_4 = -1$

phenomenological implications



$\mathcal{V}_\lambda > 0,\ \lambda_4 = 10$

Preliminary conclusions

- an $SU(2)$ triplet Higgs extension of the SM could be motivated by small neutrino masses
- the doublet-triplet Higgs sector has by itself a very rich structure and phenomenology
- a very good handel on theoretical constraints (in contrast with to two-Higgs doublet models for instance)
- theoretical lower (upper) bounds in the CP-even sector
- high μ regimes, all non-SM Higgses decouple quickly
- low μ regimes, the SM-like Higgs is the heaviest (H_0)!

  h^0 decouples quickly; not necessarily the lightest Higgs!
  distinctive $H^{\pm\pm}$ phenomenology ?
- exclusions from existing bounds? precision tests? model-dependence?