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Improving sub-Nyquist MRI reconstruction performance

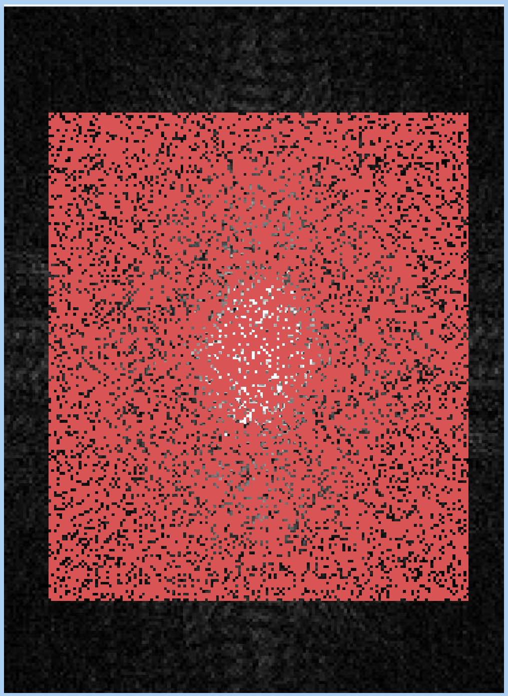
BASP Frontiers Workshop (September 4-9, 2011)

Authors: Jan Aelterman, Hiêp Luong, Bart Goossens, Aleksandra Pižurica, Wilfried Philips

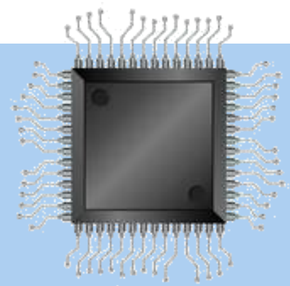
Fourier Acquisition in MRI

SIGNAL ACQUISITION

K-Space



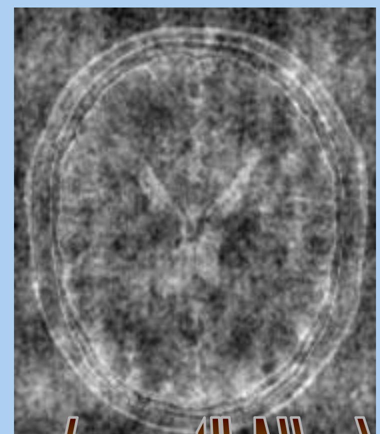
Randomly Subsampled



SIGNAL PROCESSING

FFT

Image Space



"incoherent" Aliasing

Problem Formulation

$$\hat{\vec{x}} = \arg\min_{\vec{x}} |\vec{S}\vec{x}| \quad \text{s.t.} \quad \left\| \vec{y} - \vec{F}\vec{x} \right\| = 0$$

FFT/CG/Gauss-Seidel

STEP 1.1.1

$$\vec{x}_{j+1} = \arg\min_{\vec{x}} \frac{\lambda_{df}}{2}\left\|\vec{y}_i - \vec{F}\vec{x}\right\|^2 + \frac{\lambda_b}{2}\left\|\vec{d}_j - \vec{S}\vec{x} - \vec{b}_l\right\|^2$$

Bregman

STEP 1.1.2

$$\vec{d}_{j+1} = \arg\min_{\vec{d}} |d| + \frac{\lambda_b}{2}\left\|\vec{d}_j - \vec{S}\vec{x}_{j+1} - \vec{b}_l\right\|^2$$

Soft Thresholding

Bregman

STEP 1.2

$$\vec{b}_{l+1} = \vec{b}_l + \vec{S}\vec{x}_{j+1} - \vec{d}_{j+1}$$

STEP 2

$$\vec{y}_{i+1} = \vec{y}_i + \vec{y} - \vec{F}\vec{x}_{j+1}$$

[Goldstein et al.; *SIAM J. Imaging Sci 2009*][Aelterman et al.; *Elsevier Sigpro 2011*]

- A number of improvement ideas:
  - Image representation
  - Parallel Imaging
  - K-space trajectory
  - Prior model

$$\hat{\vec{x}} = \arg\min_{\vec{x}} |\vec{S}\vec{x}| \quad \text{s.t.} \quad \left\| \vec{y} - \vec{F}\vec{x} \right\| = 0$$

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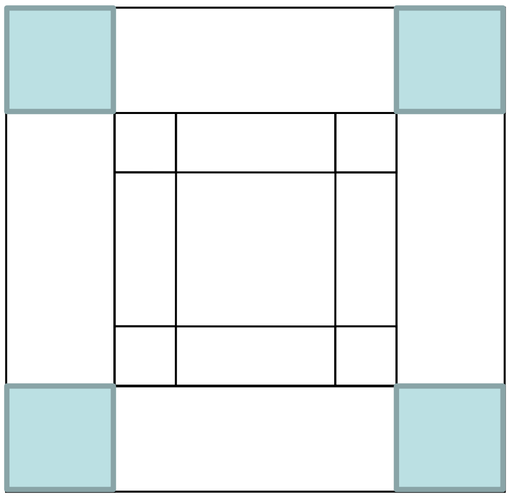
  - Prior model

Improvement 1: the discrete shearlet transform

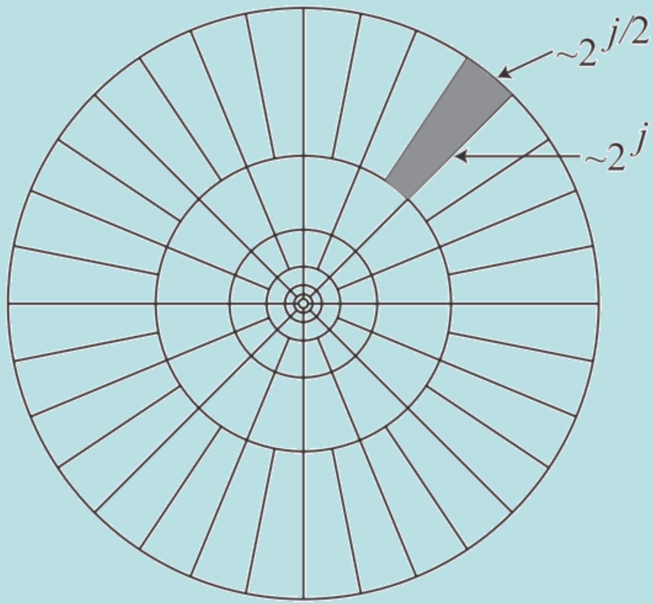
Real DWT



Approximation error:

$$\epsilon_M \leq C M^{-1}$$

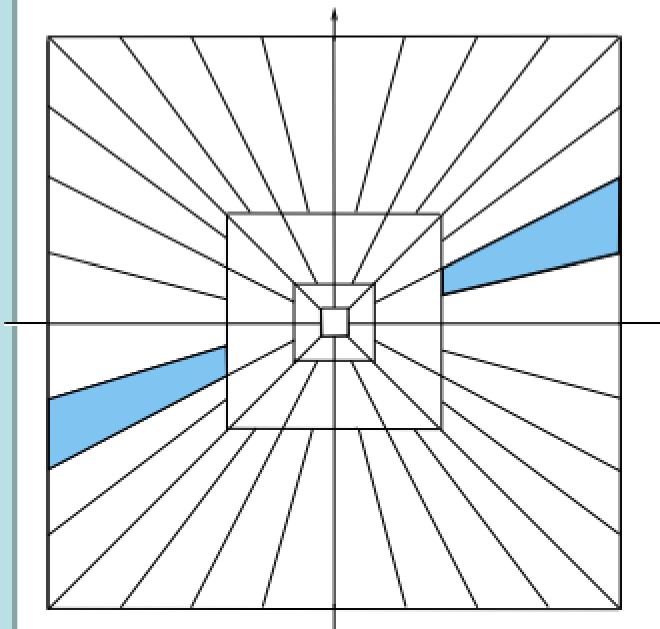
curvelet



Approximation error:

$$\epsilon_M \leq C(\log M)^3 M^{-2}$$

shearlet



Approximation error:

$$\epsilon_M \leq C(\log M)^3 M^{-2}$$

optimal for piecewise C^2 images

[Candės et al.; *Multiscale mod. 2006*]

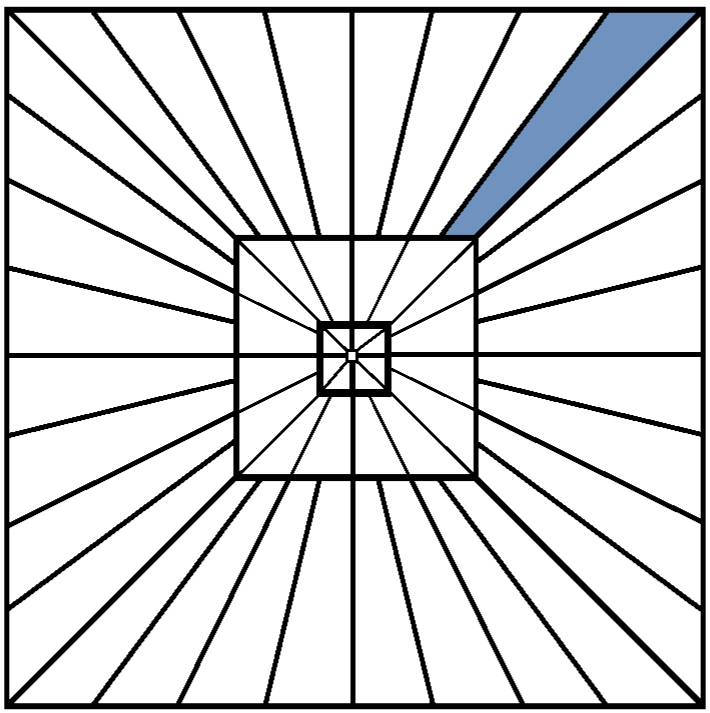
[Guo et al.; *SIAM Mat. An. 2007*] [Easley et al; *Appl. Comput.*

Improvement 1: the discrete shearlet transform

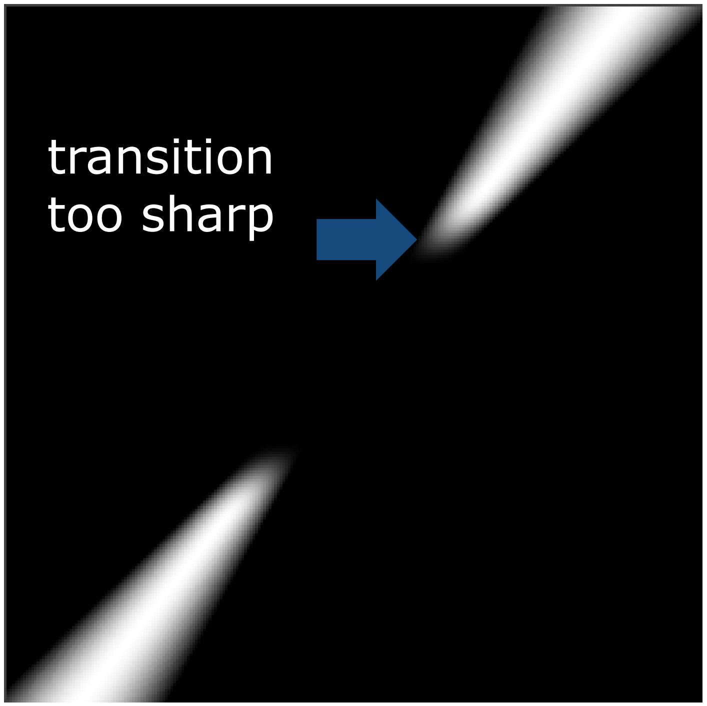
25% Fourier acquisition
+ reconstruction with resolution increase of 50%

Improvement 1: the discrete shearlet transform

Ideal spectral support
of 1 upperscale shearlet

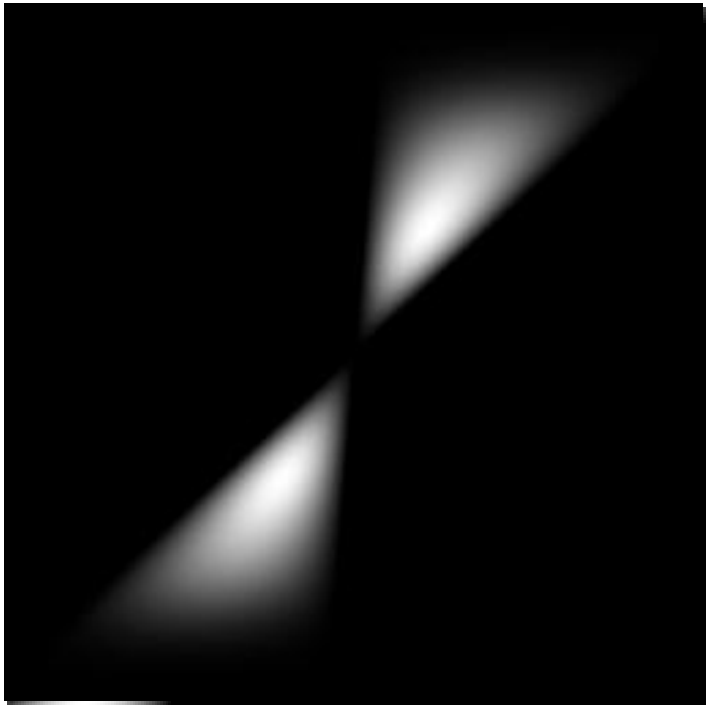


Power Spectral Density
of the corresponding shearlet filter

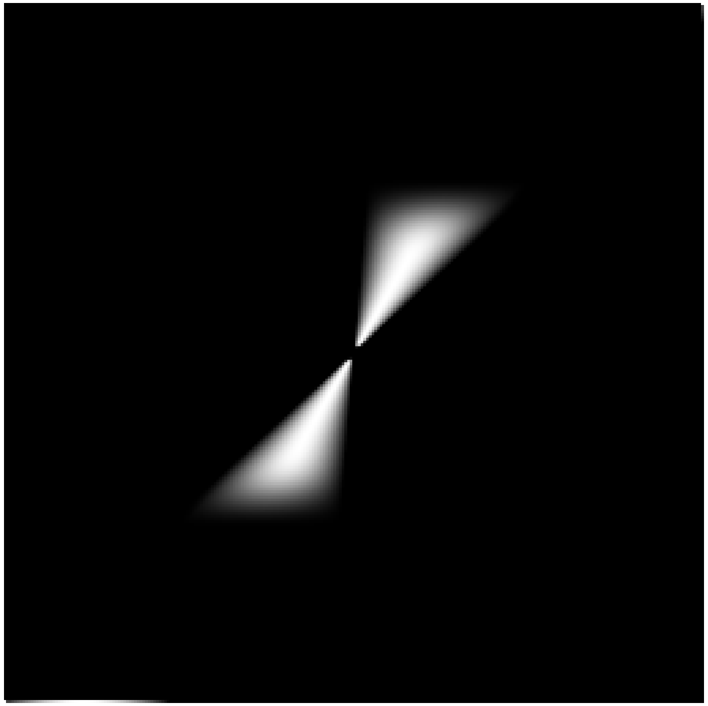


Improvement 1: the discrete shearlet transform

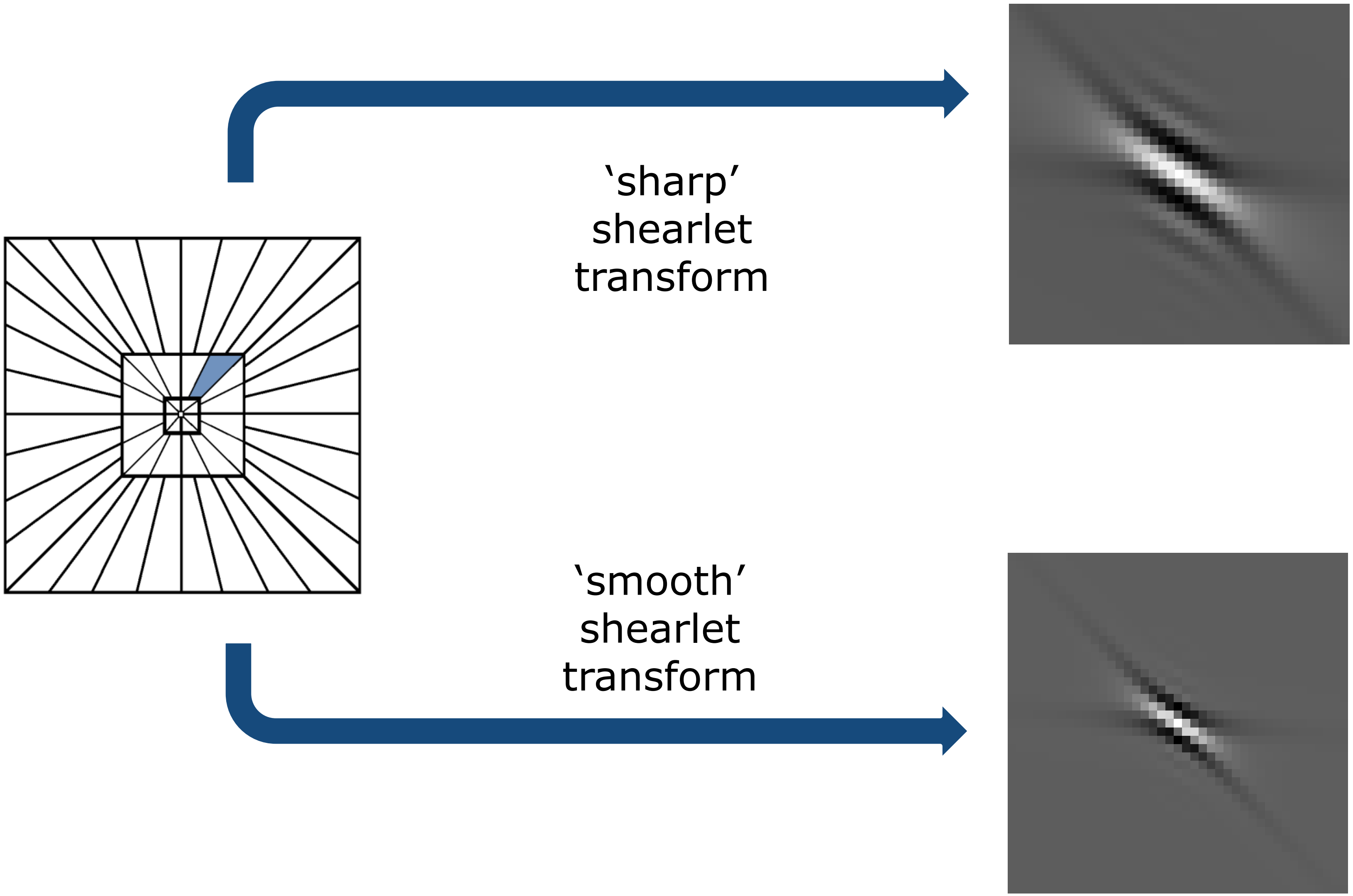
Power Spectral Density
of the smooth filter



Power Spectral Density
of the original shearlet filter

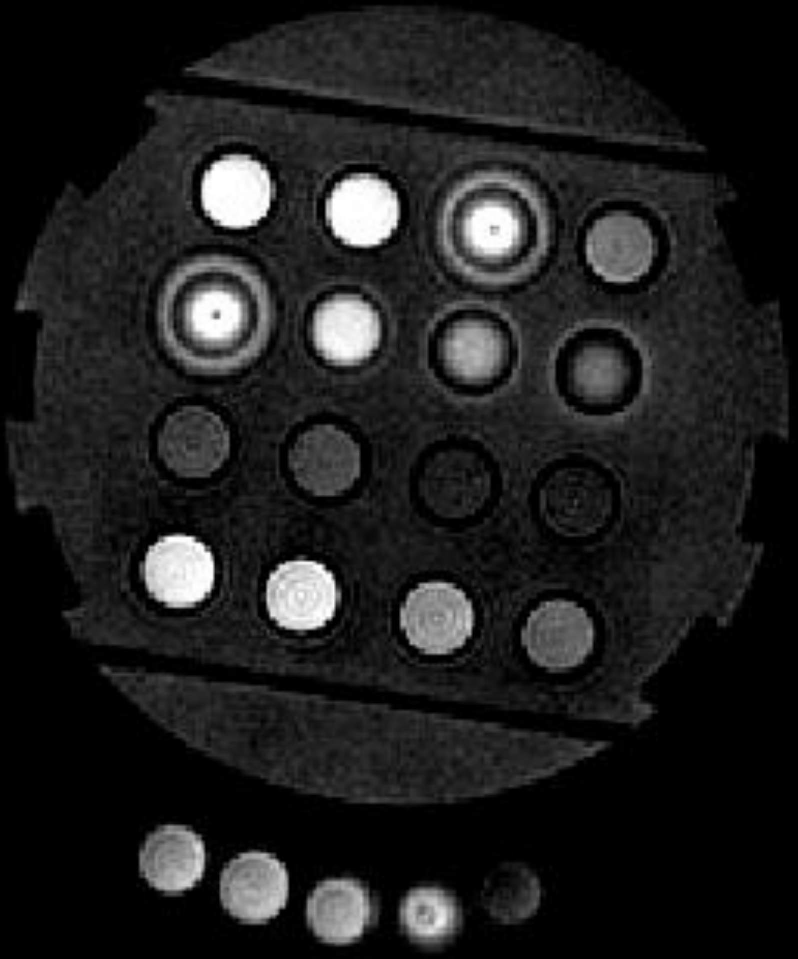


Improvement 1: the discrete shearlet transform

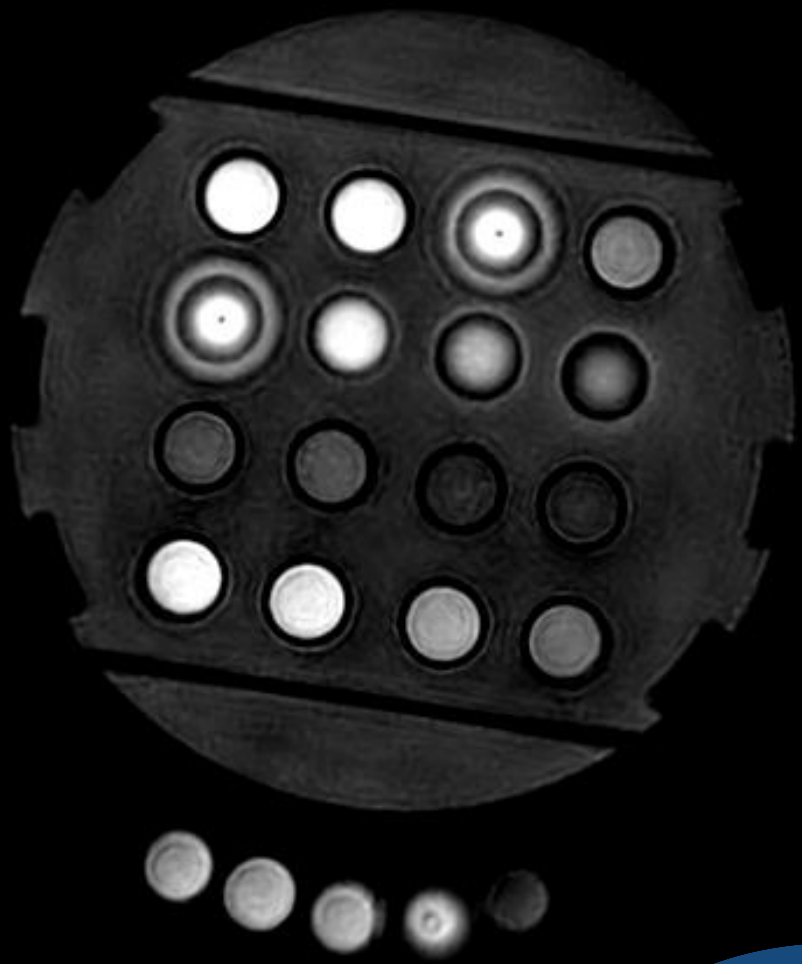


Spirally Acquired Data

Naive reconstruction



'sharp' shearlets



Anatomical Data

Only TV
35.6dB

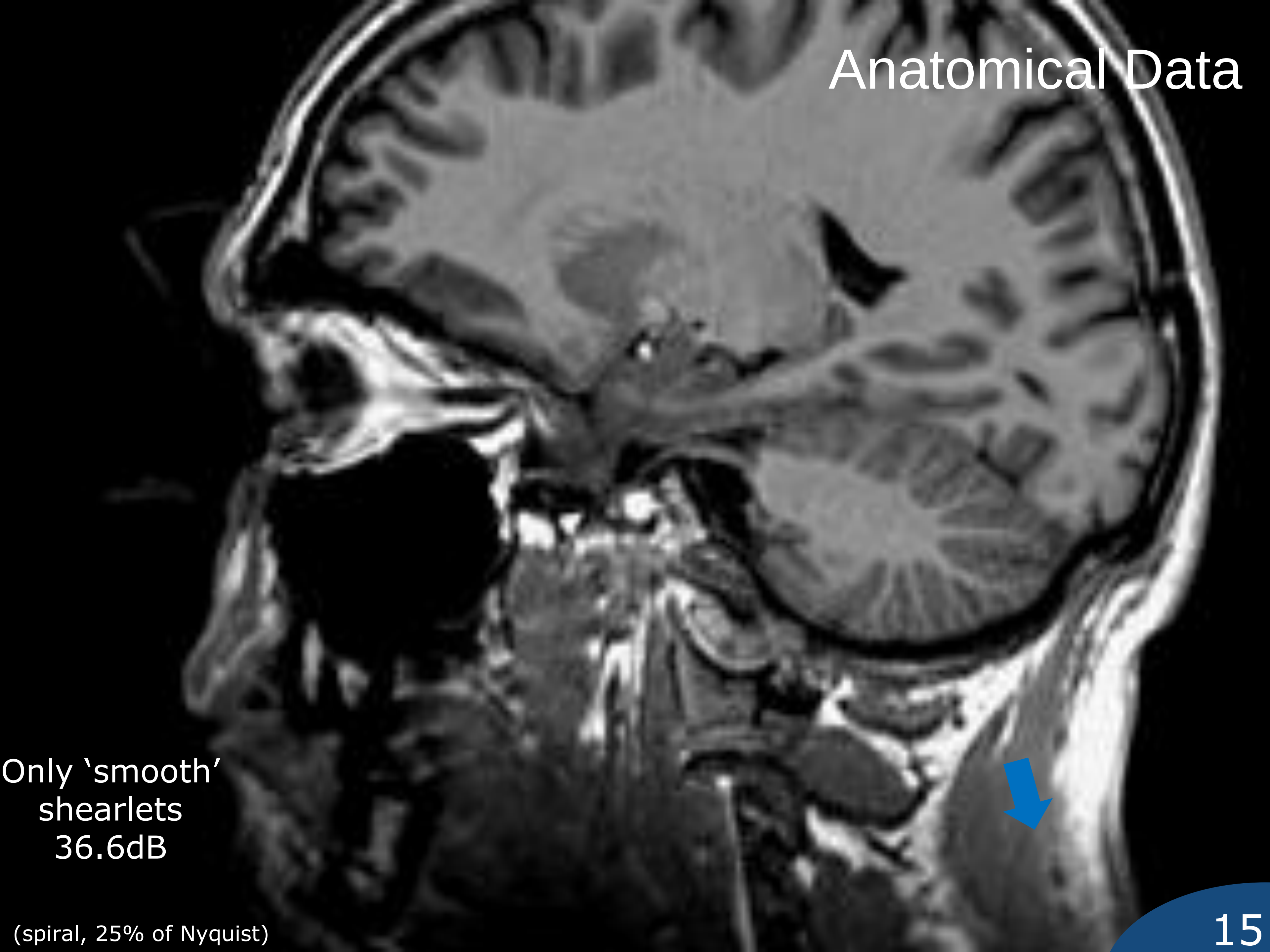
(spiral, 25% of Nyquist)

Anatomical Data

Only ‘sharp’ shearlets
36.5dB

(spiral, 25% of Nyquist)

Anatomical Data



Only ‘smooth’ shearlets
36.6dB

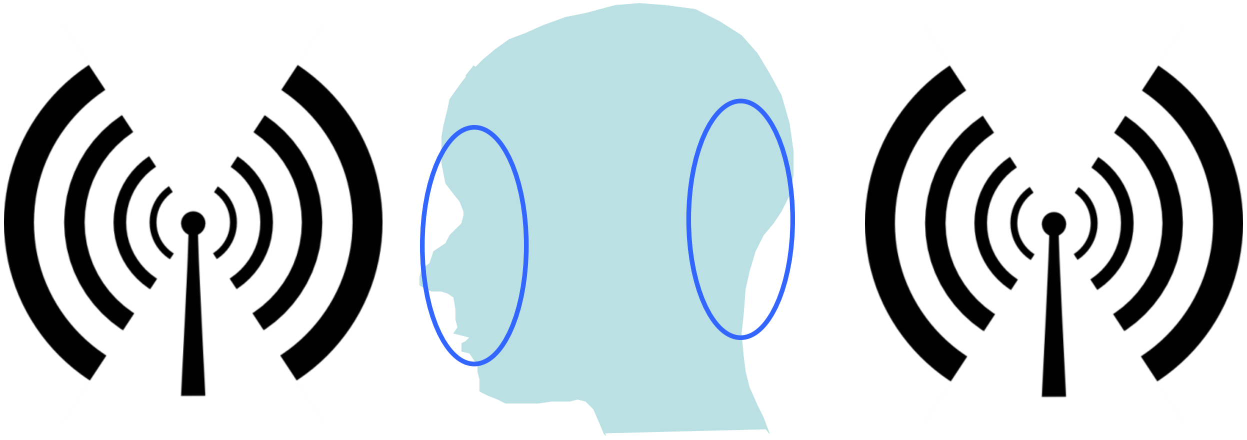
(spiral, 25% of Nyquist)

- A number of improvement ideas:
  - Image representation
  - Parallel Imaging
  - K-space trajectory
  - Prior model

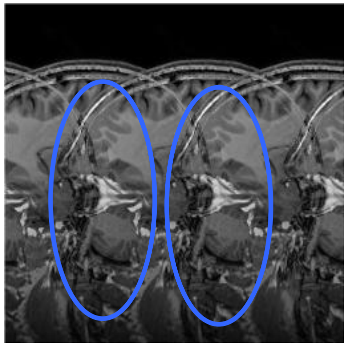
email: jan.aelterman@telin.ugent.be

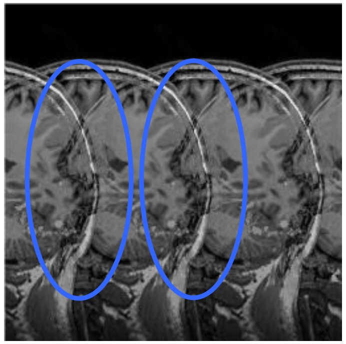
Improvement 2: Parallel Imaging

- Subsampled MRI with multiple receiver antennas (e.g. SPIR-iT [Lustig et al.; *MRM 2009*]):



Clear aliases of the front of the head





Clear aliases of the back of the head

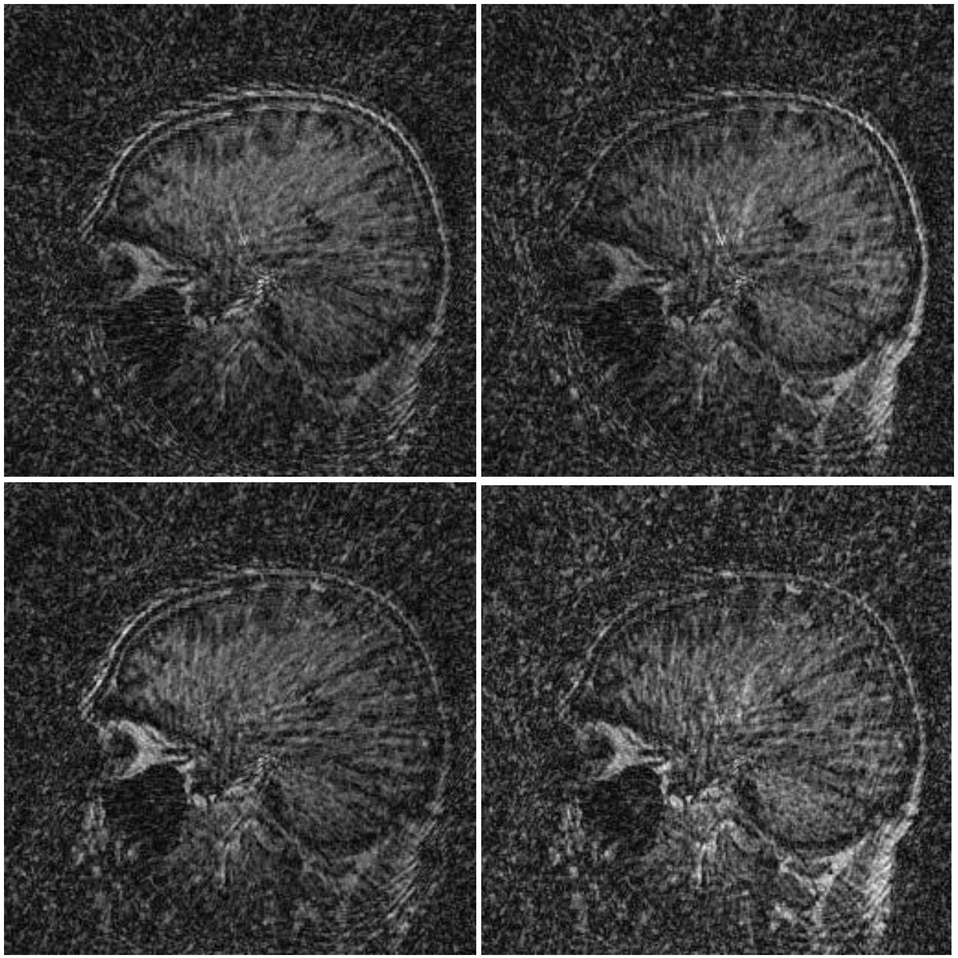
Improvement 2: Parallel Imaging

- Compressive Sensing:
  - Sub-Nyquist MRI data
  - Regularized reconstruction exploiting sparsity
  - Estimation of missing K-space data
- Parallel Imaging:
  - Sub-Nyquist MRI data
  - well-posed reconstruction through large number of different coil sensitivities
  - “Calculation” of missing K-space data

Complementary techniques

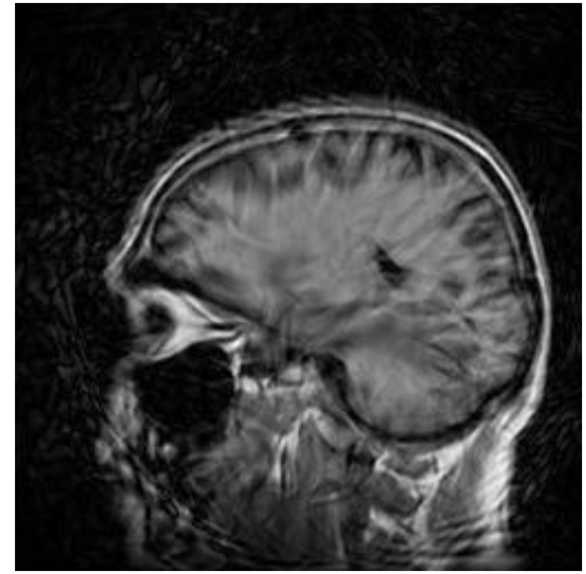
Experiment: joint vs. Separate CS-pMRI

- 4 coils, 3% sub-Nyquist sampled



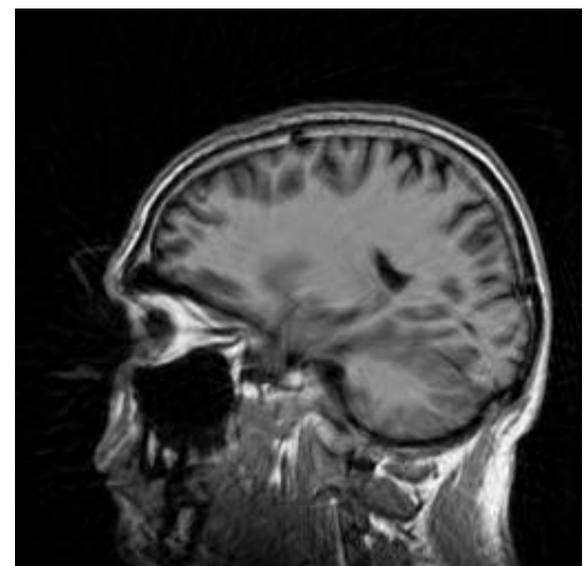
Single coil rec, then combining

PSNR = 23dB



Joint reconstruction

PSNR = 28dB



- A number of improvement ideas:
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  - K-space trajectory
  - Prior model

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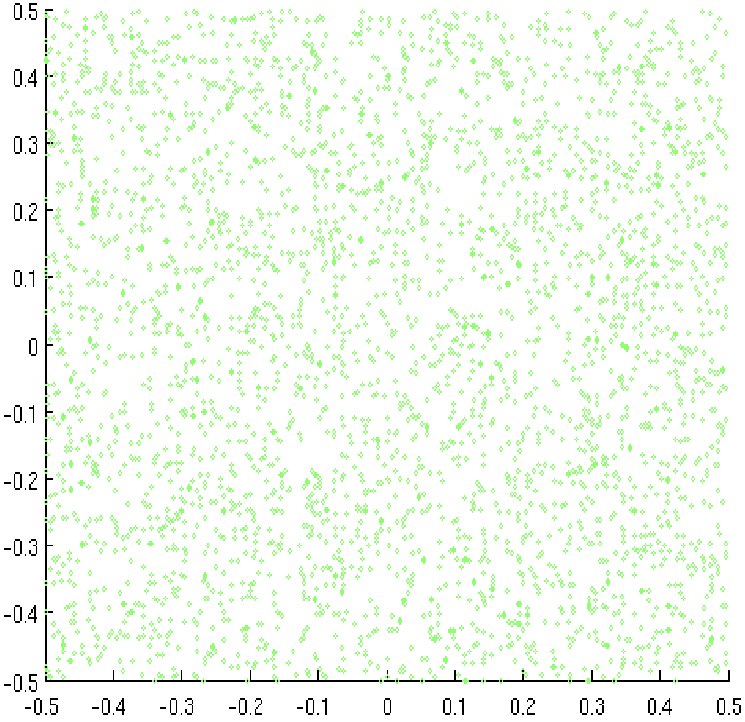
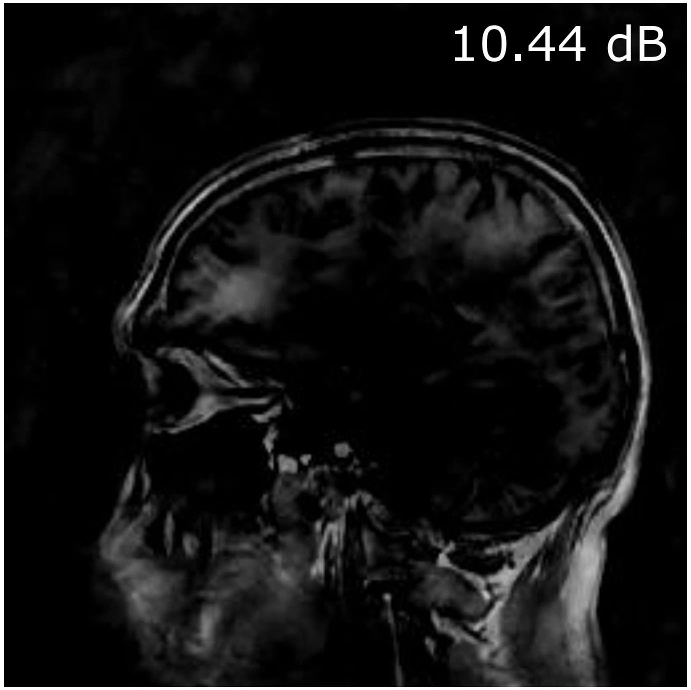
Improvement 3: Improved Trajectory

- At first:

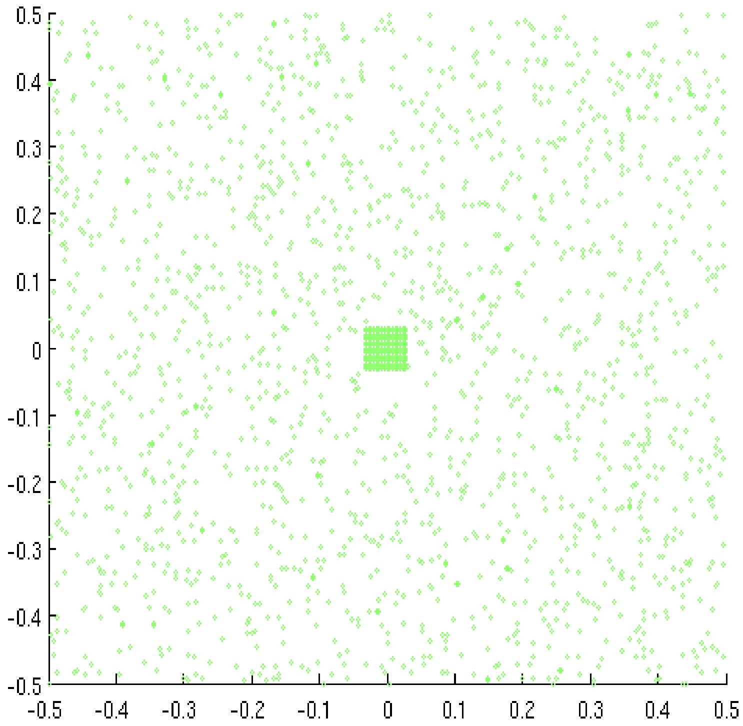


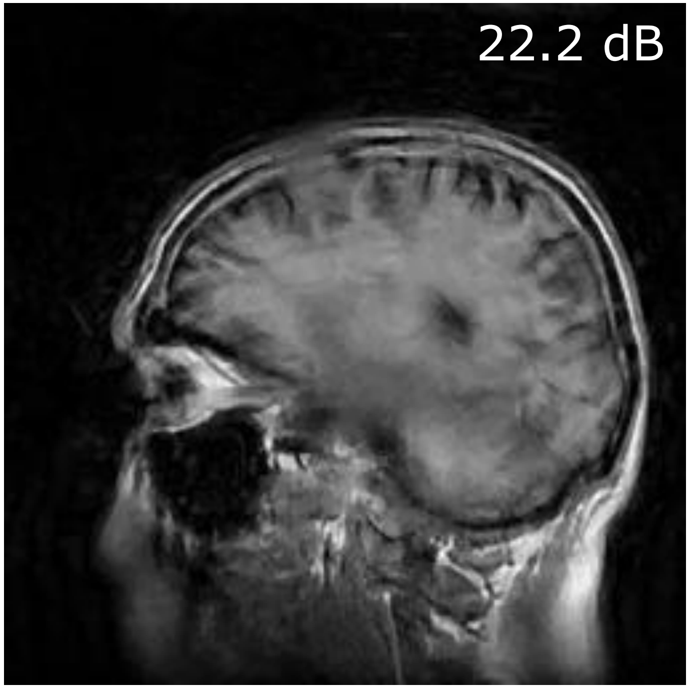


(10% of Nyquist)

Improvement 3: Improved Trajectory

- But better is:

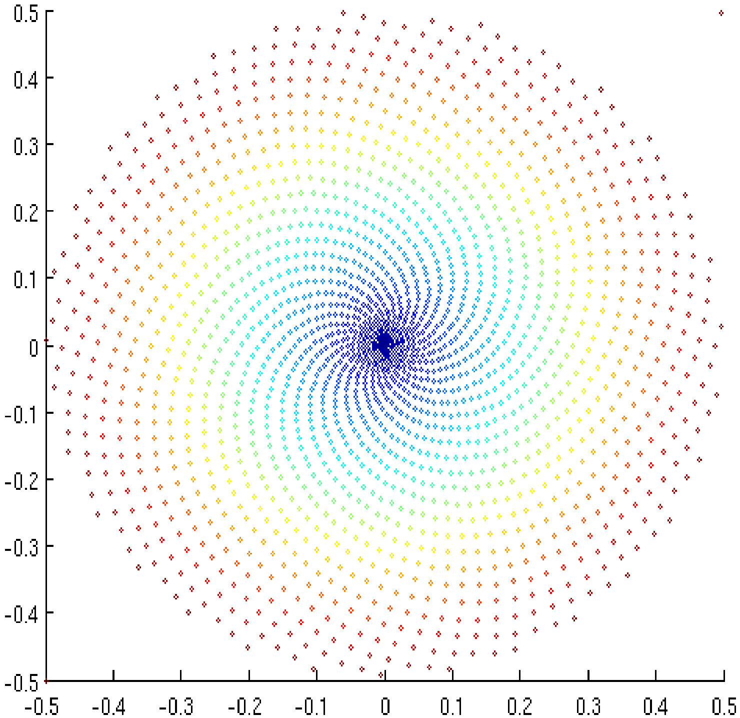


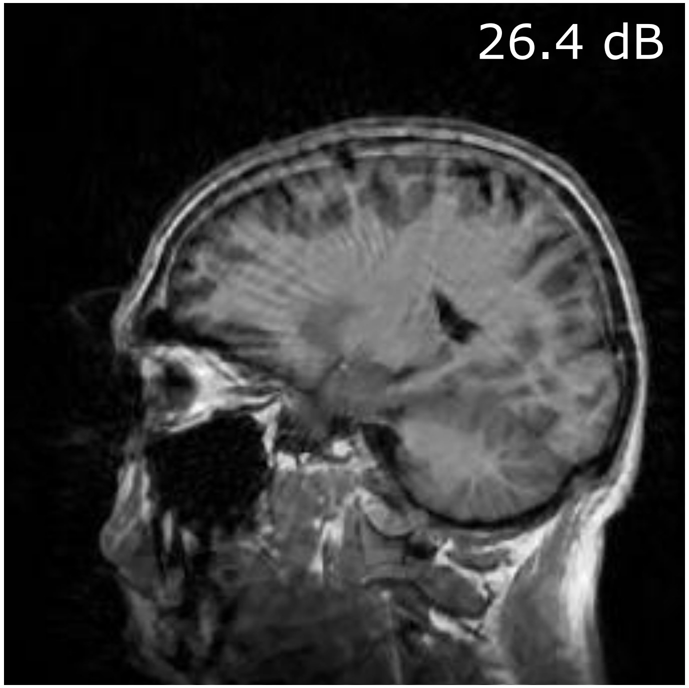


(10% of Nyquist)

Improvement 3: Improved Trajectory

- Or:



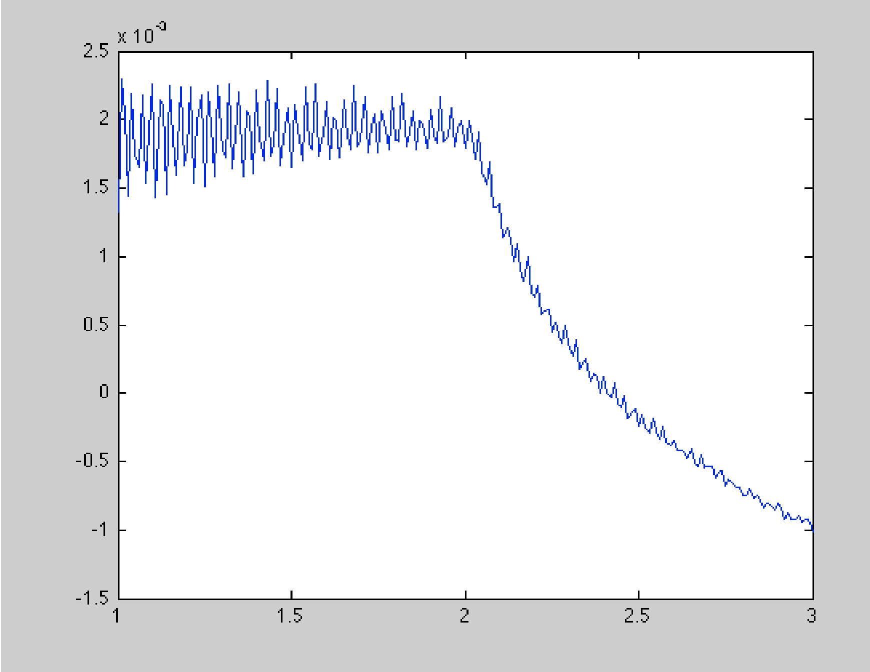


(10% of Nyquist)

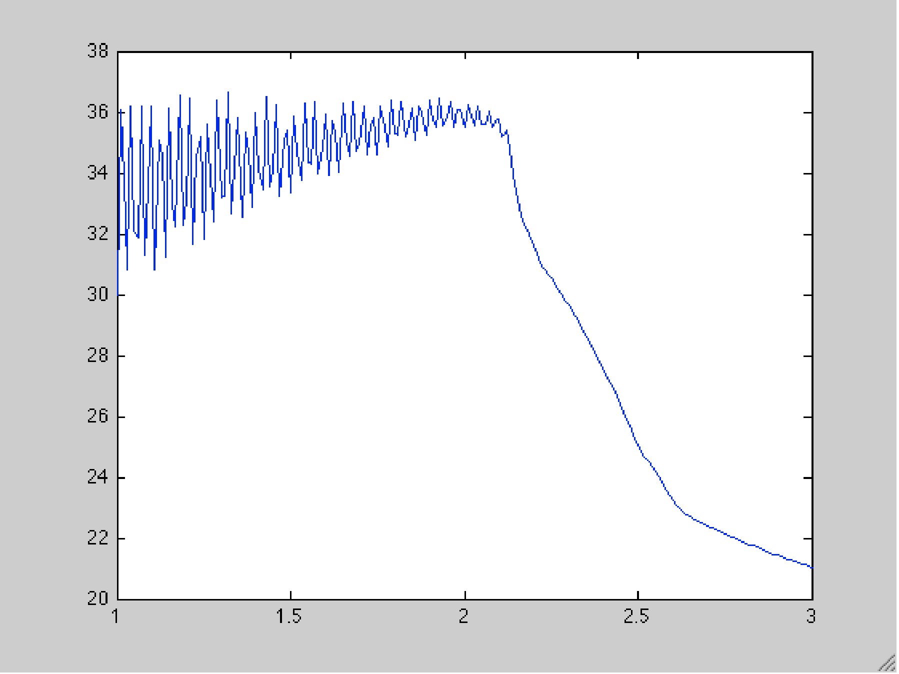
Need Regridding!

Improvement 3: Improved Trajectory

- Archimedean spirals with varying radii:



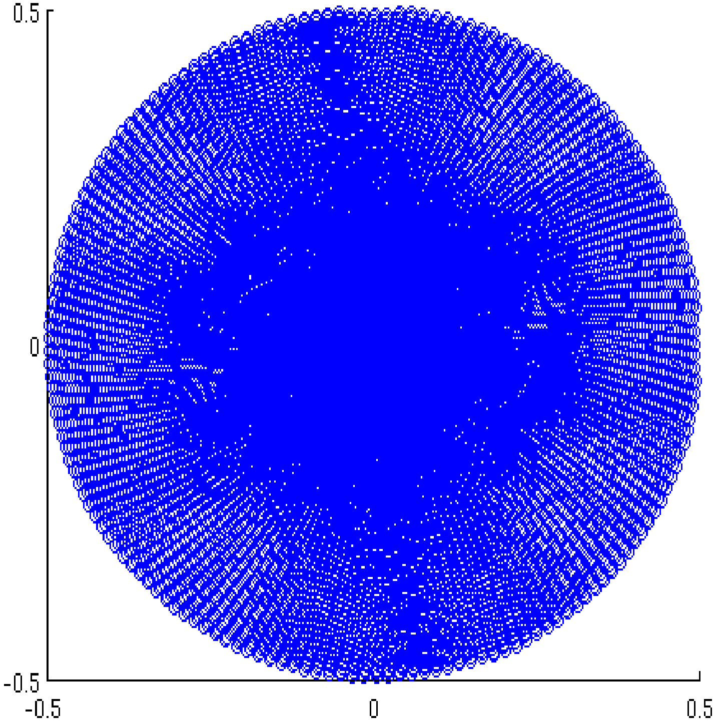
prediction

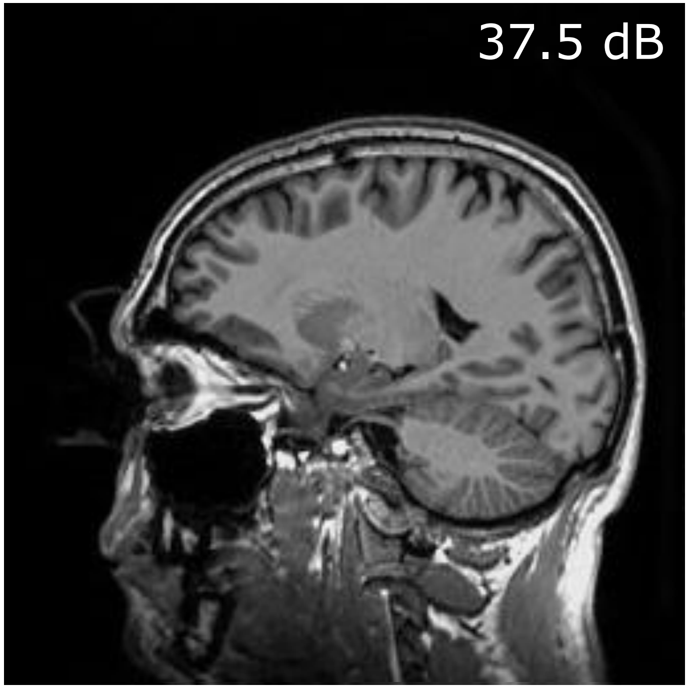


ground truth

Improvement 3: Improved Trajectory

- For a more moderate speedup:

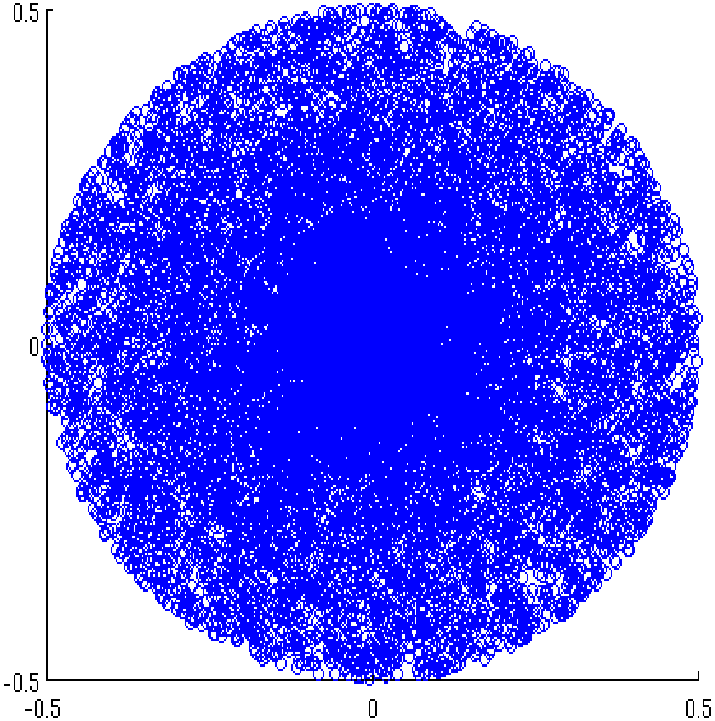


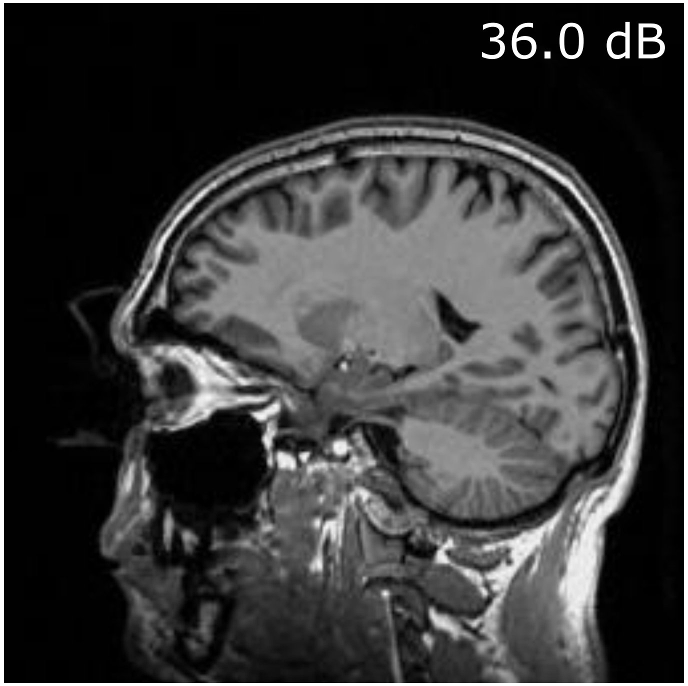


(25% of Nyquist)

Improvement 3: Improved Trajectory

- Curiously:





(25% of Nyquist)

Improvement 3: Improved Trajectory

The MRI Imaging equation:

$$\nu(\mathbf{k}) = \int_{\mathbb{R}^2} \rho(\mathbf{x}) e^{i\pi w |\mathbf{x}|^2} e^{-2i\pi \mathbf{k}\cdot\mathbf{x}} d^2\mathbf{x}$$

Phase Scrambling

- well-known in MRI (high Dynamic, reduce aliasing)
- obtained through dedicated coils or RF pulses

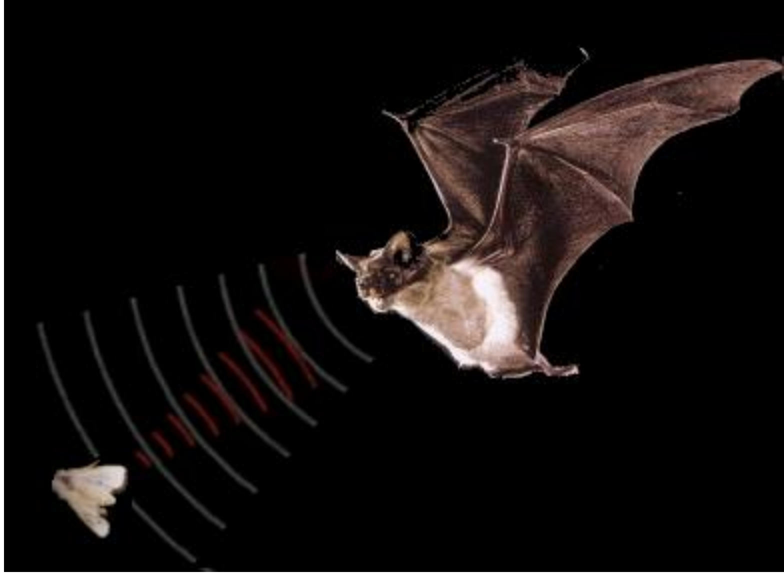
[Puy et al.; *ISMRM 2011*]

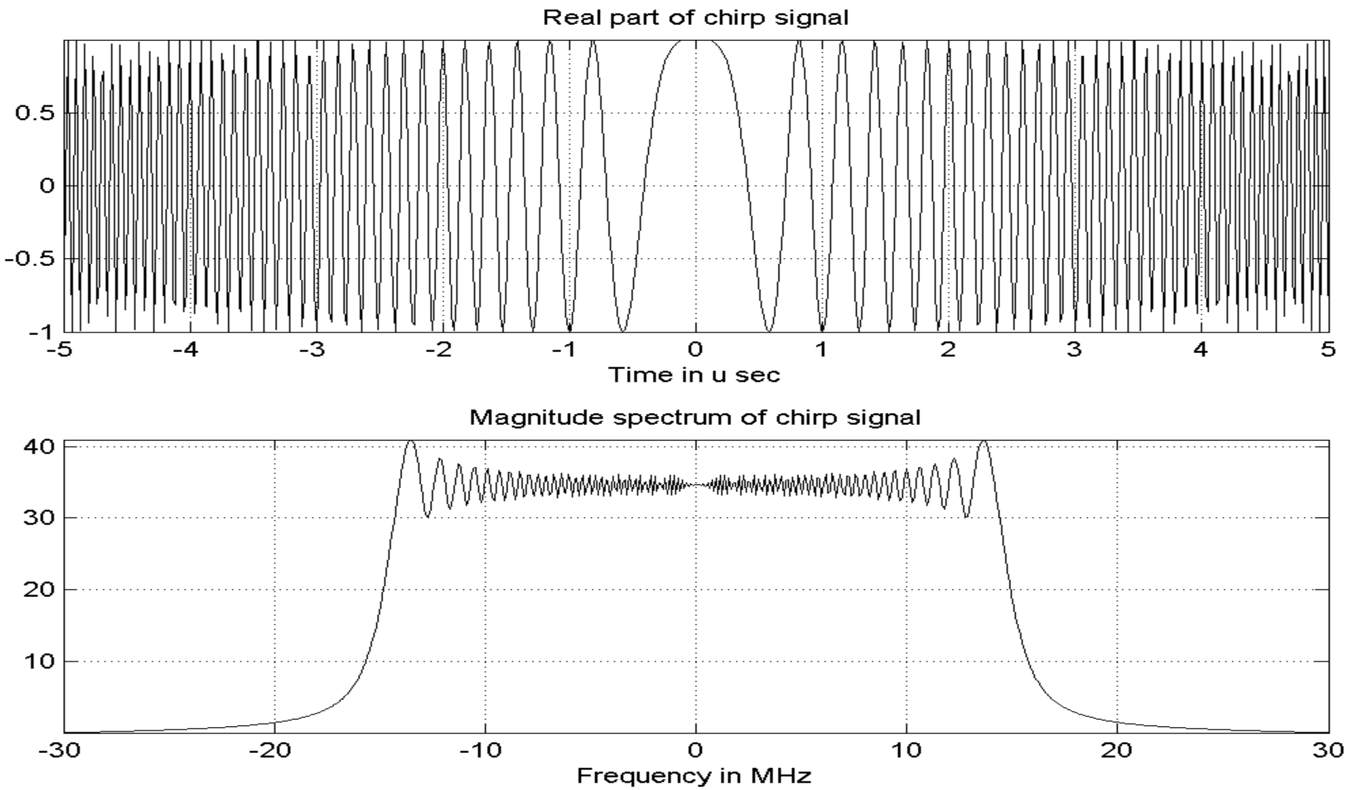
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Increases signal bandwidth

Some Biology

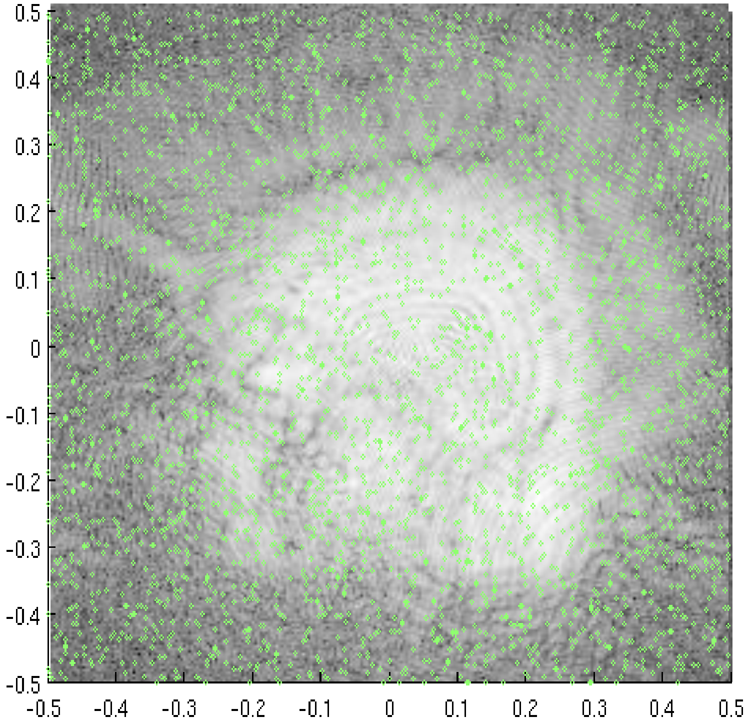
-Distance Estimation through correlating with received echos
-Spatially compact autocorrelation function: Broadband signal
-Sweep over all frequencies: A Chirp!

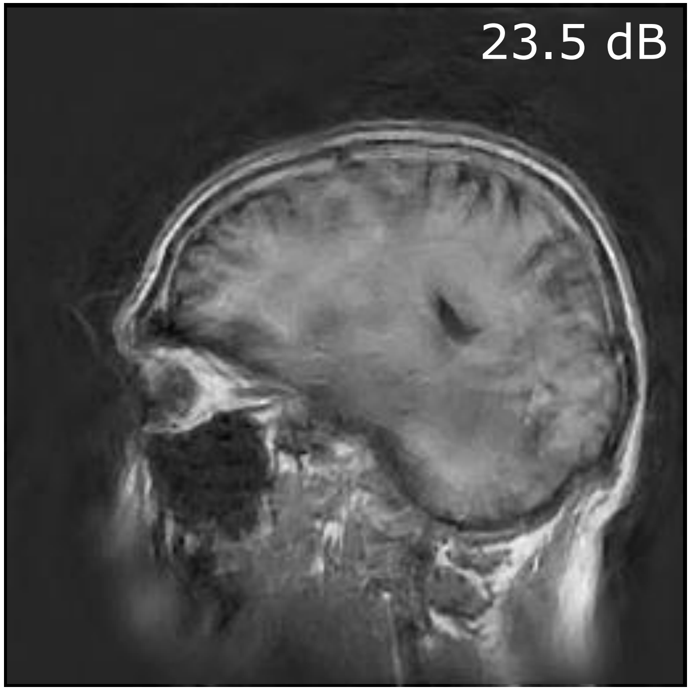




Improvement 3: Improved Trajectory

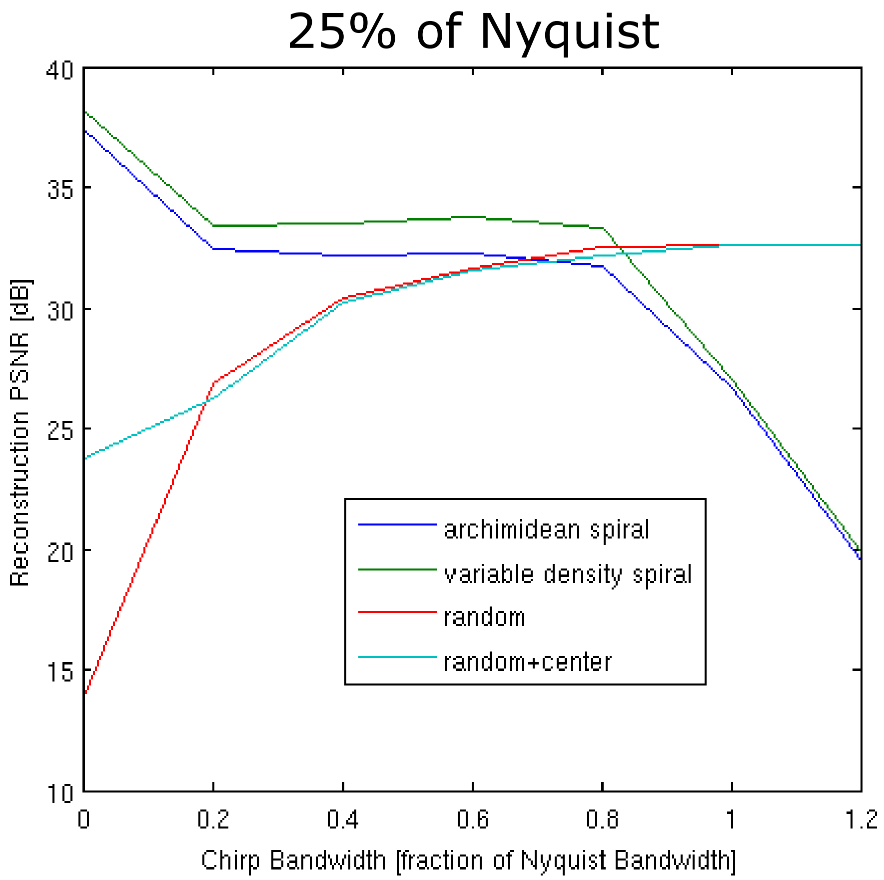
- Great effect!

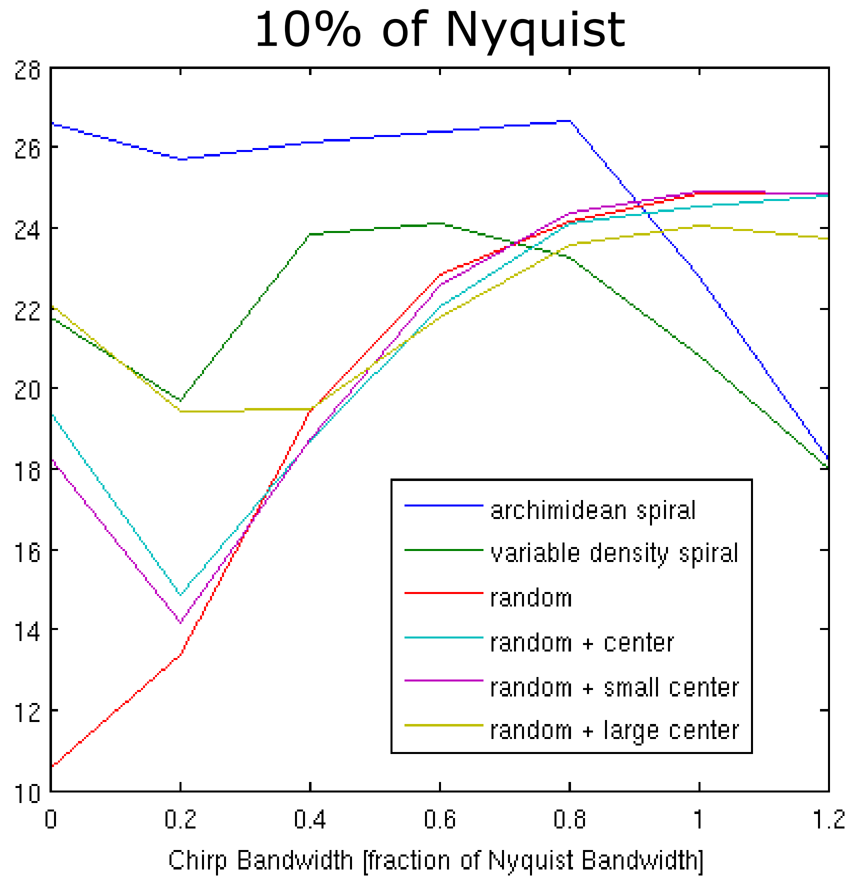




(10% of Nyquist)

Improvement 3: Improved Trajectory





- A number of improvement ideas:
  - Image representation
  - Parallel Imaging
  - K-space trajectory
  - Prior model

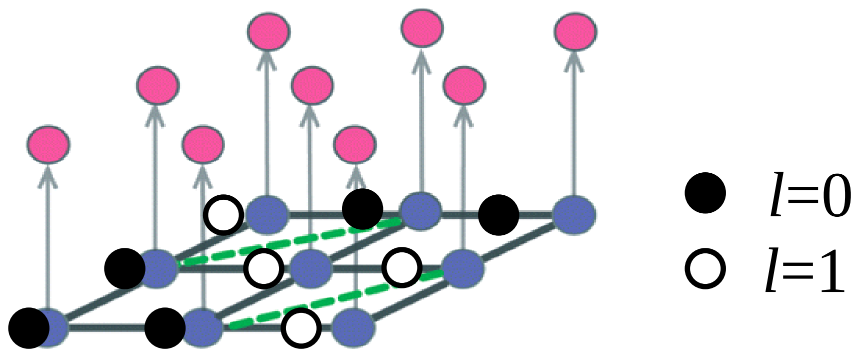
Improvement 4: Prior Model

- Using structure in addition to sparsity. Many recent developments include:
  - **Group sparsity, Group Lasso, Block sparsity, Tree sparsity, Graph sparsity** [Huang et al, *ICML*'09], **Model based sparsity** [Baraniuk et al, *IEEE Trans IT* 2010]

- We encode spatial structure using **Markov Random Field**. Closely related work (not reported on MRI):
  - **LaMP** – Lattice Matching Pursuit [Cevher et al; *Sig Proc Mag* 2010]

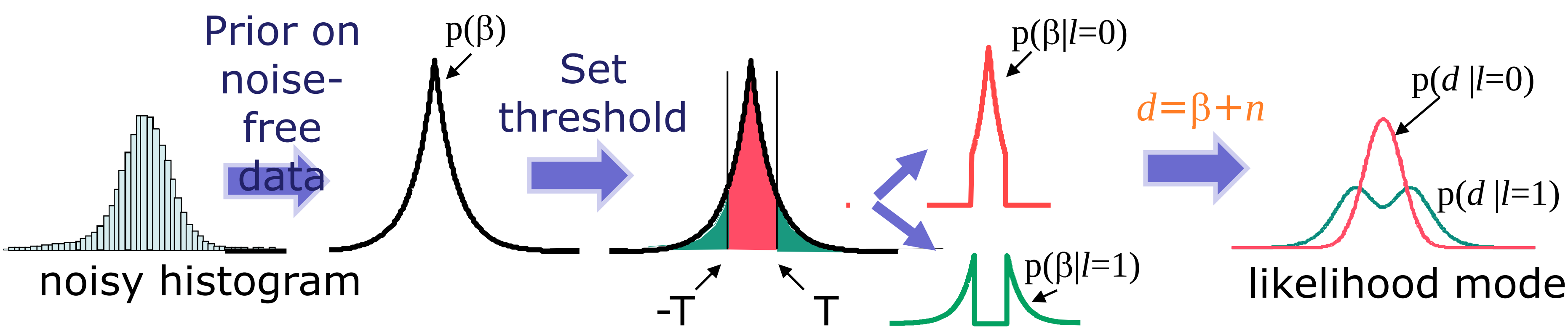
Improvement 4: Prior Model

Observable random field $\boldsymbol{d} = \{d_1, d_2 \ldots d_N\}$

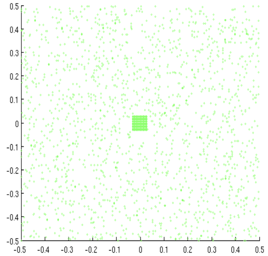


Hidden random field $\boldsymbol{l} = \{l_1, l_2 \ldots l_N\}$

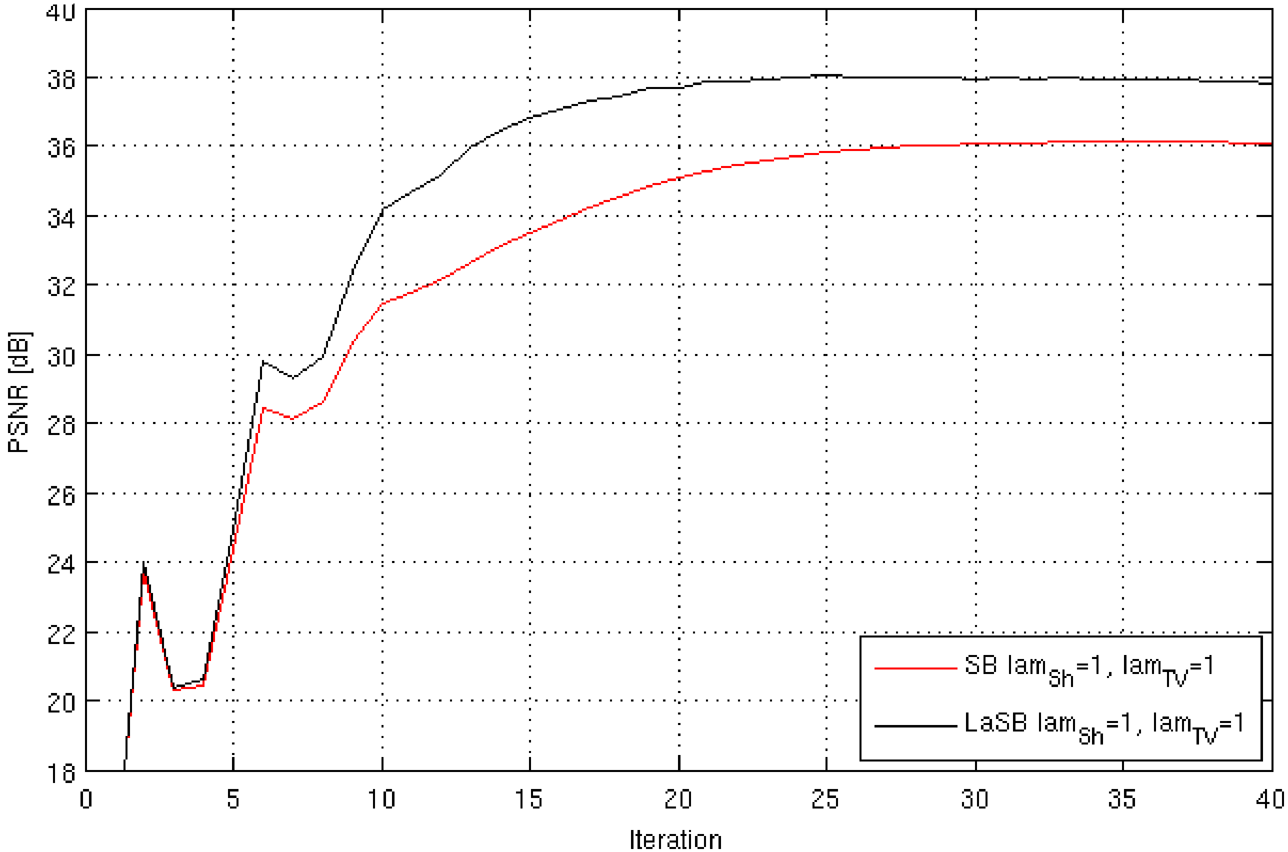
$$V_C(l_i, l_j) = \begin{cases} -\gamma, & l_i = l_j \\ \gamma, & l_i \neq l_j \end{cases}, \quad \gamma > 0$$



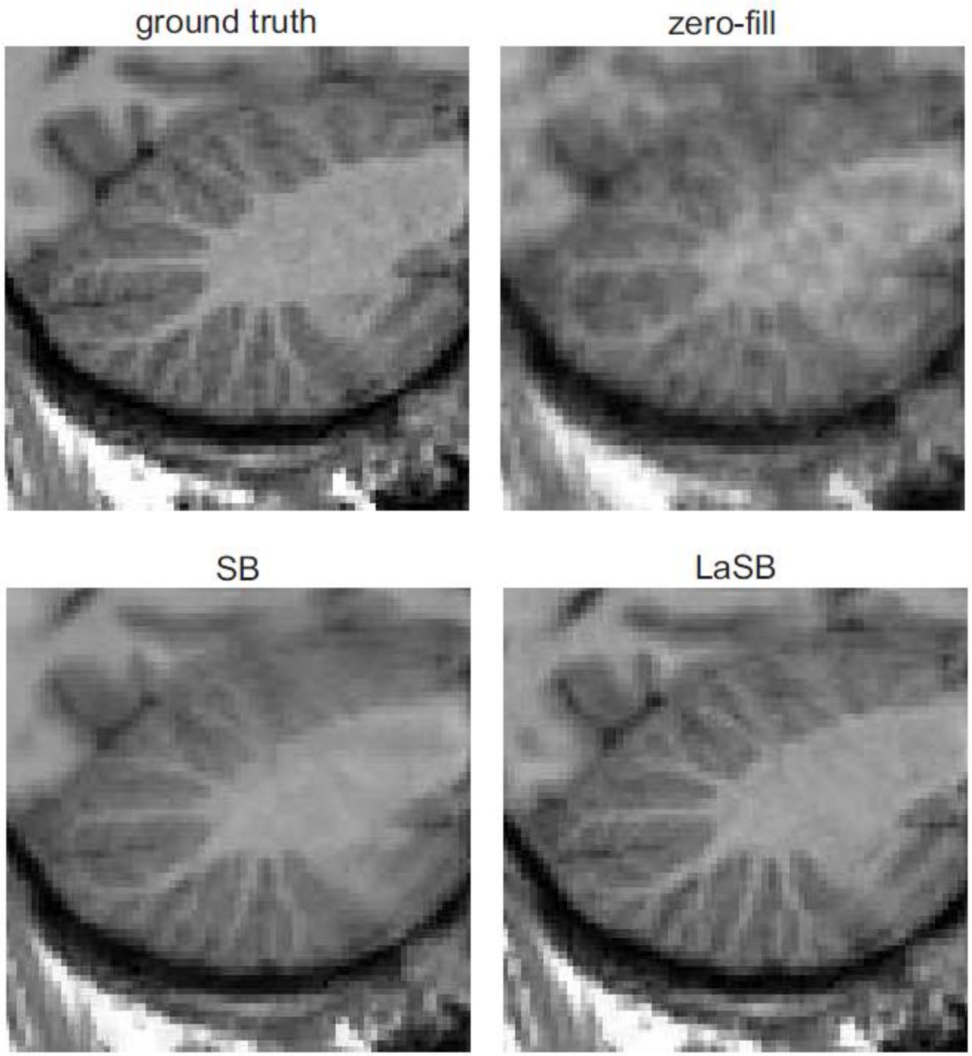
Improvement 4: Prior Model



Random subsampling with center low pass, 50%

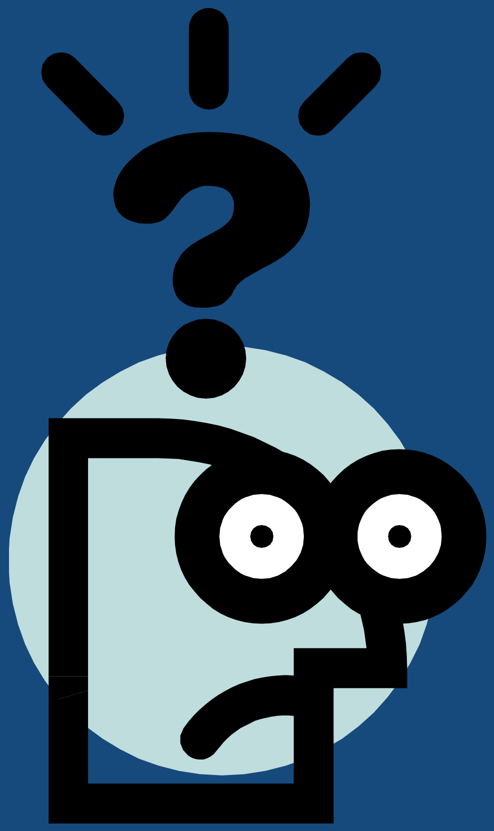


Improvement 4: Prior Model



- In conclusion:
  - In order to maximize reconstruction performance, a combination of optimal image representation, parallel imaging, a well designed trajectory (!) and properly handling spatial structure in the regularization are needed.

Any questions?



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Thank you for your attention.

email: jan.aelterman@telin.ugent.be