Calibrating And Correcting Direction-Dependent Effects In Radio Interferometry

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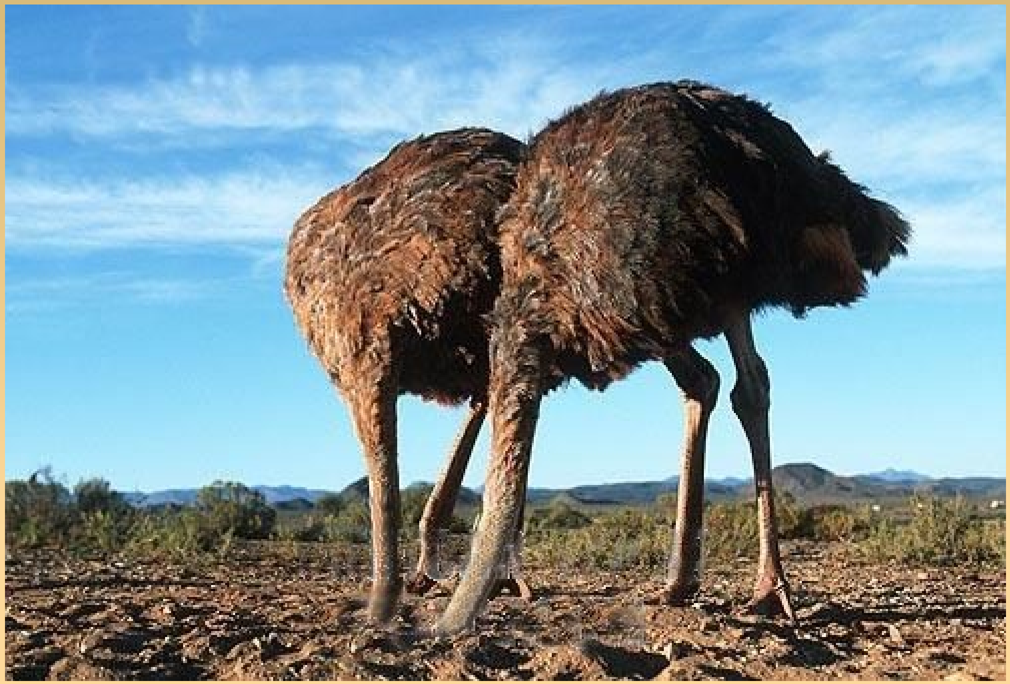
ASTRON

Introduction: DDEs

- Classical interferometry assumes that every antenna "sees" the **same** sky attenuated by the **same** antenna spatial response pattern, a.k.a. the "primary beam" (PB).
  - ...we calibrate for a single per-antenna gain (thus, direction-independent)
- Direction-dependent effects (DDEs): antenna-, time- and direction-dependent gain variations:
  - PB stability (sky rotation, pointing errors, mechanical dish deformation, etc.)
  - Ionospheric refraction (at lower frequencies)
- Generally not known *a priori*.

DDEs: The Traditional Approach

- Pick your battles:
  choose the field of view carefully, keeping all the bright stuff in the center
- Traditional calibrations absorbs all effects towards the dominant source



- DDEs cause increasing distortion towards the edges, but only the *faint* stuff is subject to them, so we ignore them
- **NOT** possible with the new brood of telescopes:
  - Wider fields of view, new regimes, higher sensitivity
  - "Low unit cost" (i.e. we build them out of cheap junk)

DDEs Westerbork-Style
(Luxury Problems?)

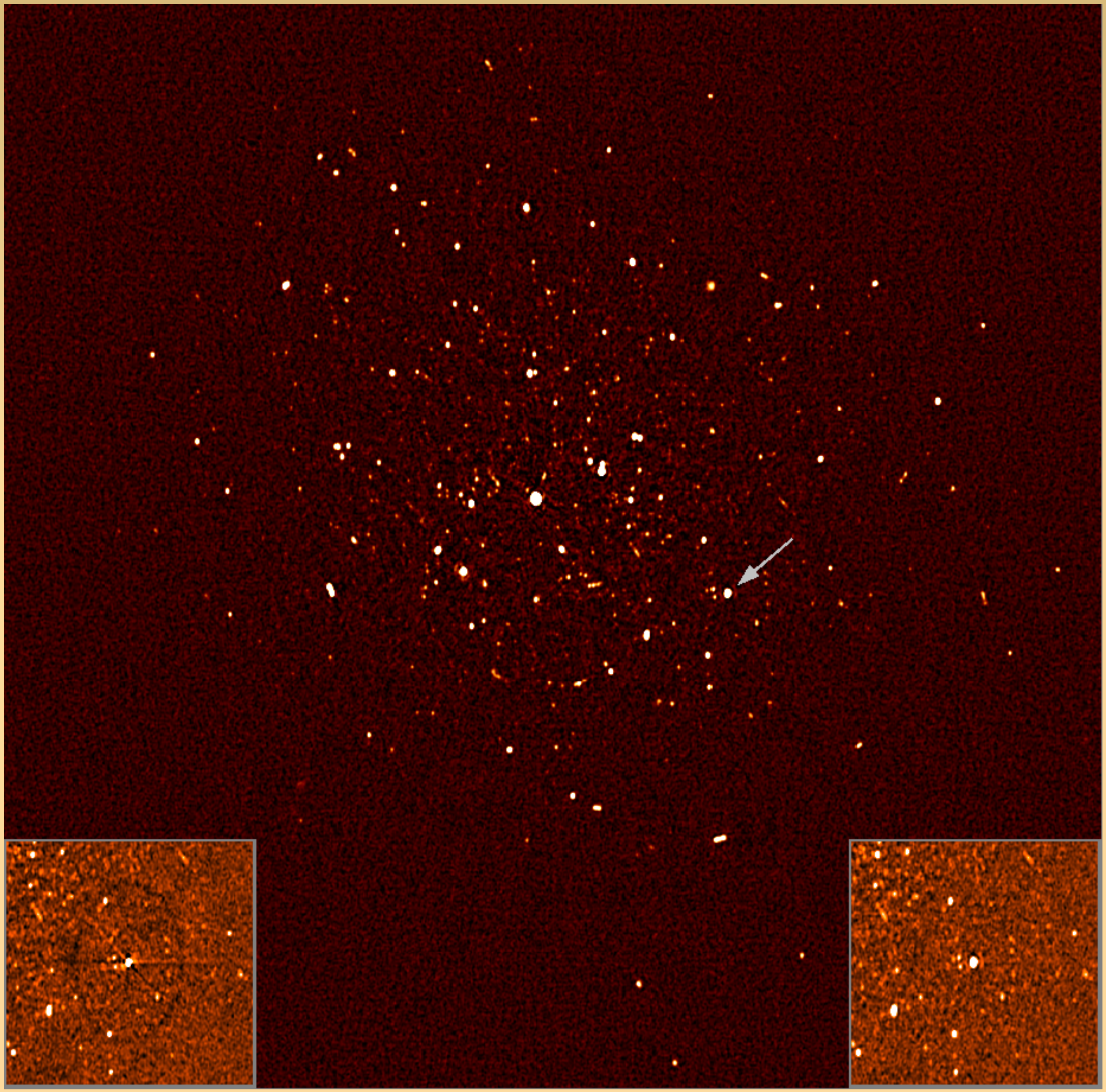
3C147 @21cm
Single 12h
WSRT synthesis

1,600,000:1 DR

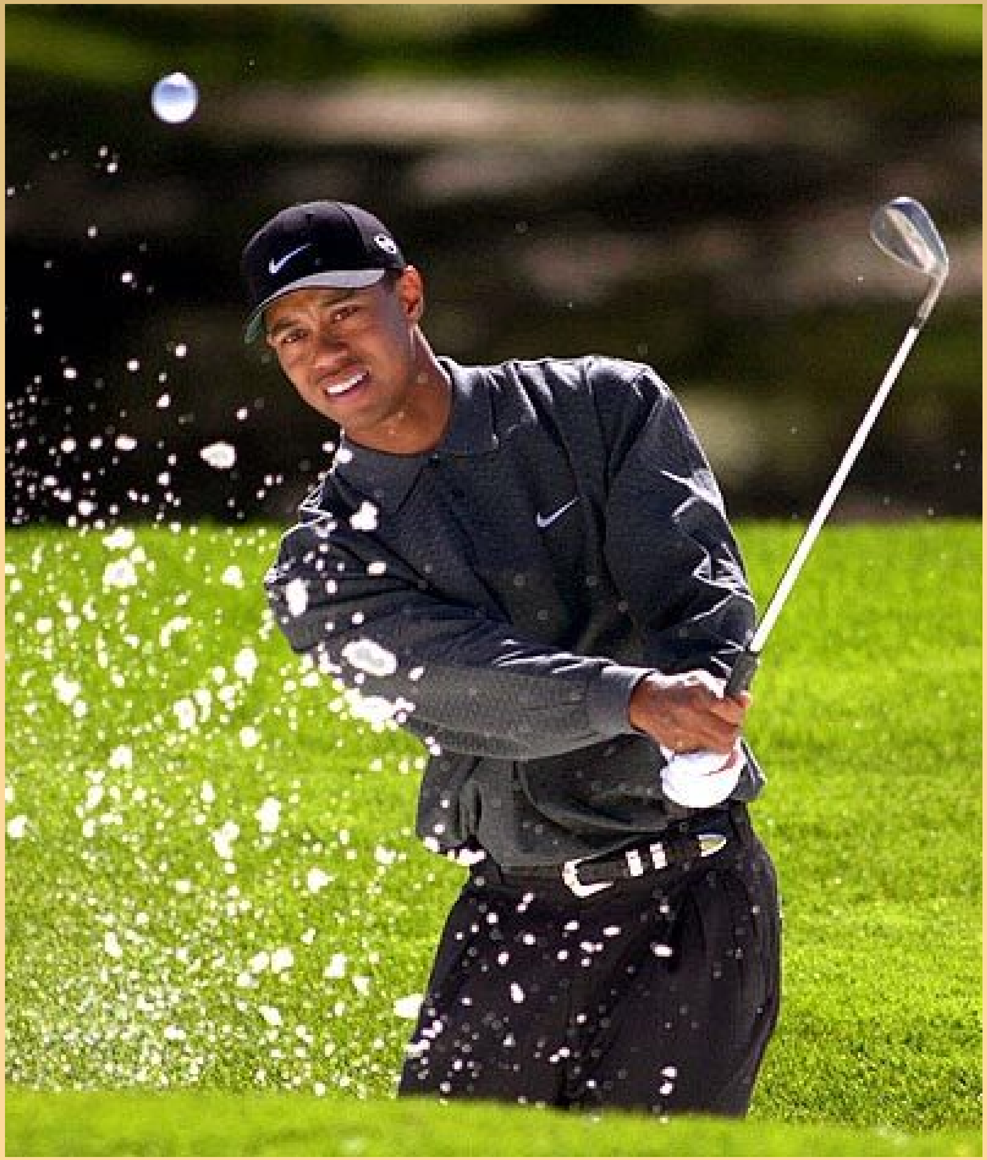
Such DR made possible by WSRT's extremely stable design (equatorial mounts ⇒stationary beams, etc.)

Nonetheless, this map is deep enough to show DDEs.

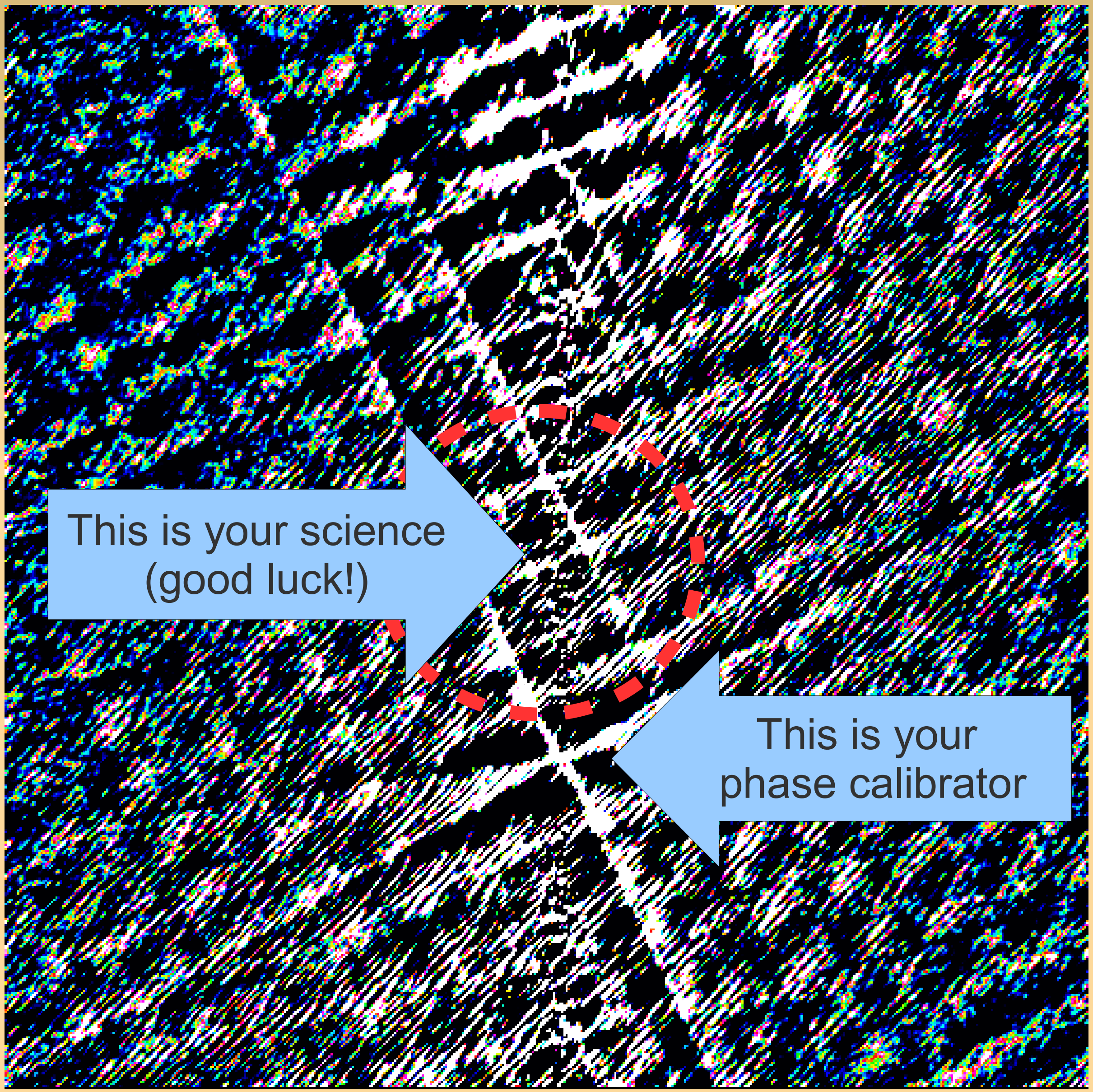
Cleaned up via application of *differential gains*.



Just a Luxury Problem?



DDEs: Not Always a Luxury Problem
(Courtesy of Ian Heywood)



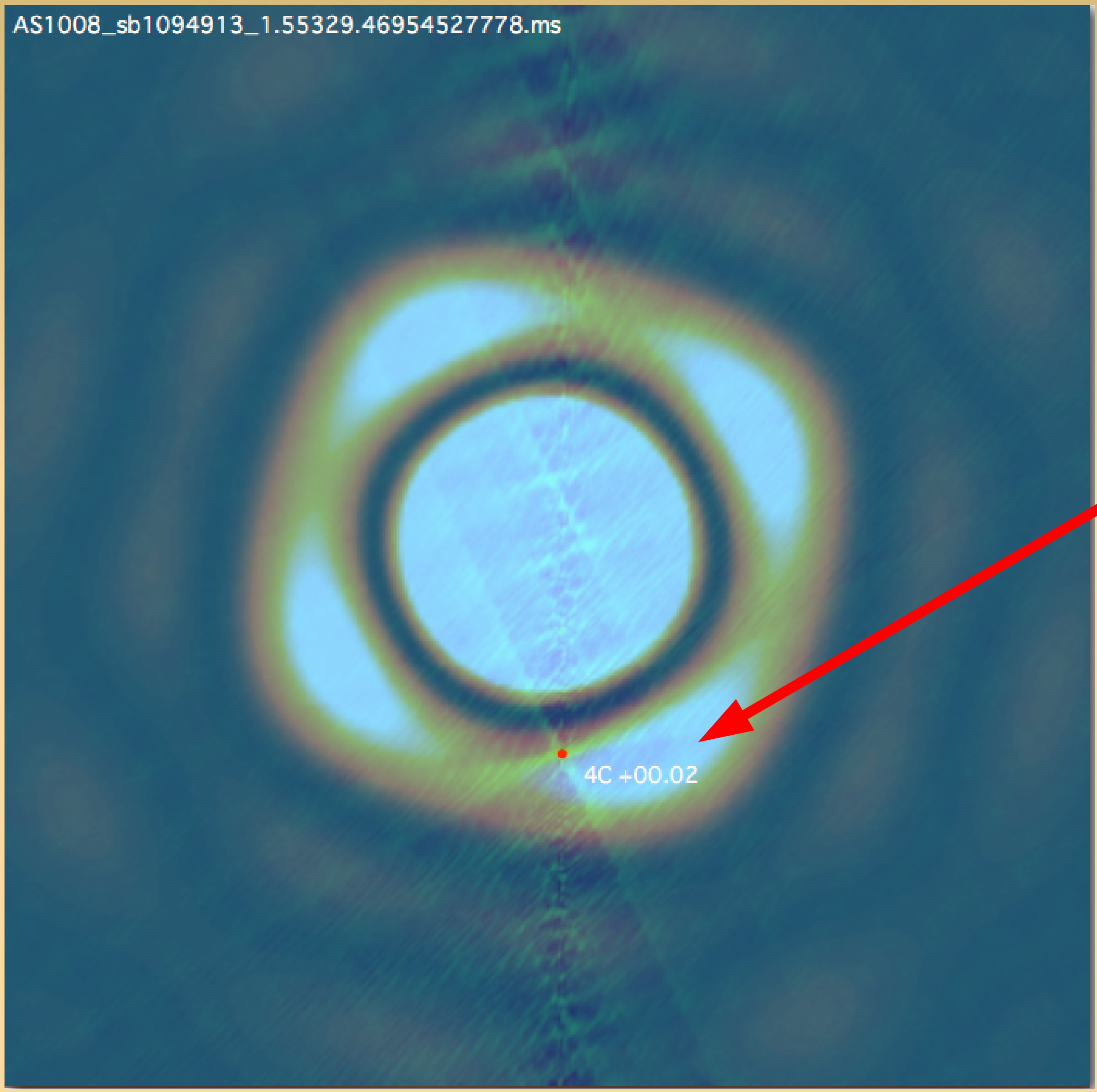
EVLA 8 GHz: Looking for sub-mm galaxies and QSOs in the William Herschel Deep Field.

Dominant effect: bright calibrator source rotating through first sidelobe of the primary beam.

(This also has a horrible PSF, being an equatorial field.)

Brightness scale 0~50μJy

Keep Your Friends Close,
and your calibrators as far away as you can...

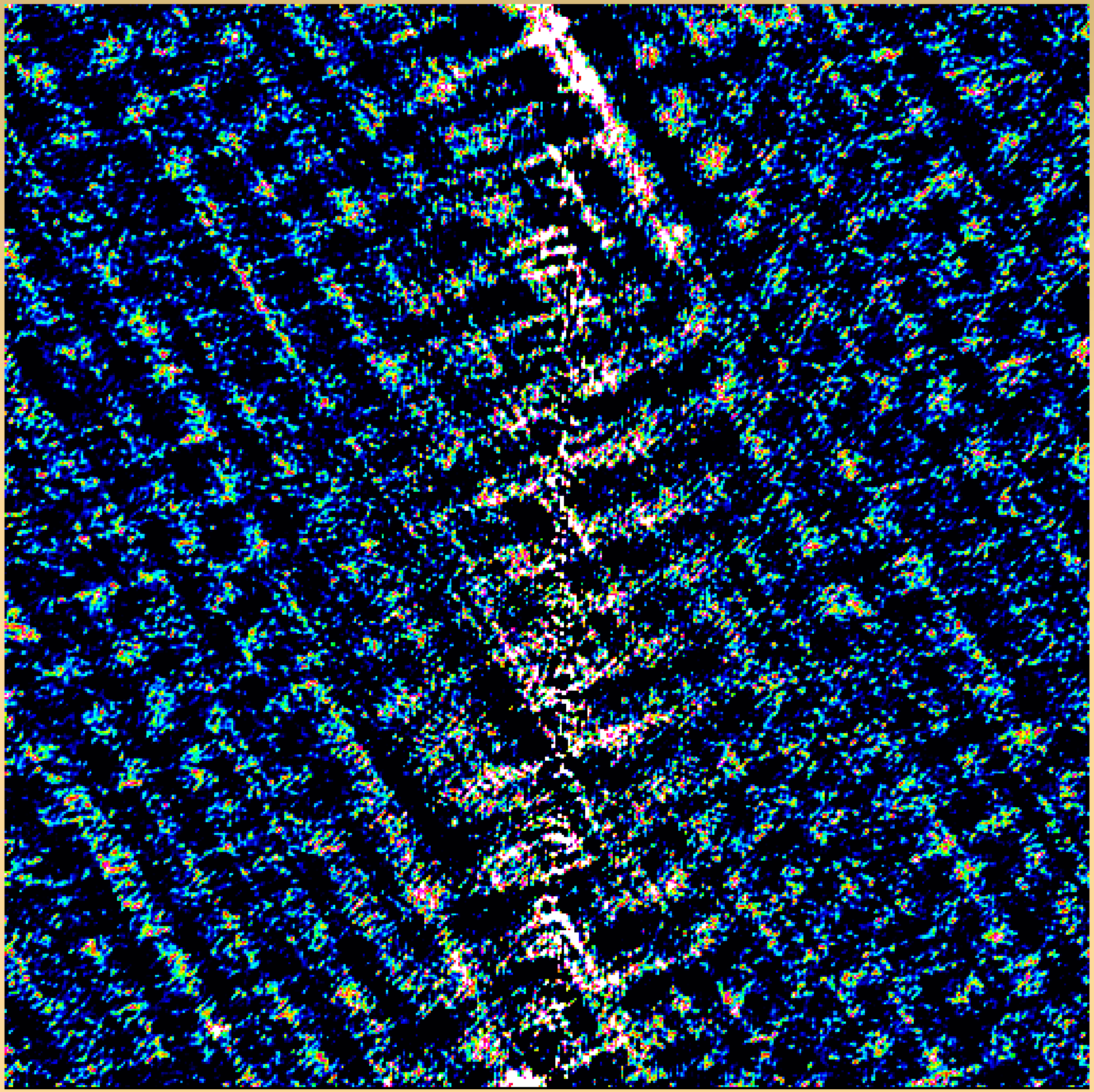


An approximation of the primary beam response, overlaid on top of the image.

As the sky rotates, the sidelobes of the PB sweep over the source, thus making it effectively *time-variable*.

Brightness scale 0~50μJy

Deconvolution Doesn't Help...

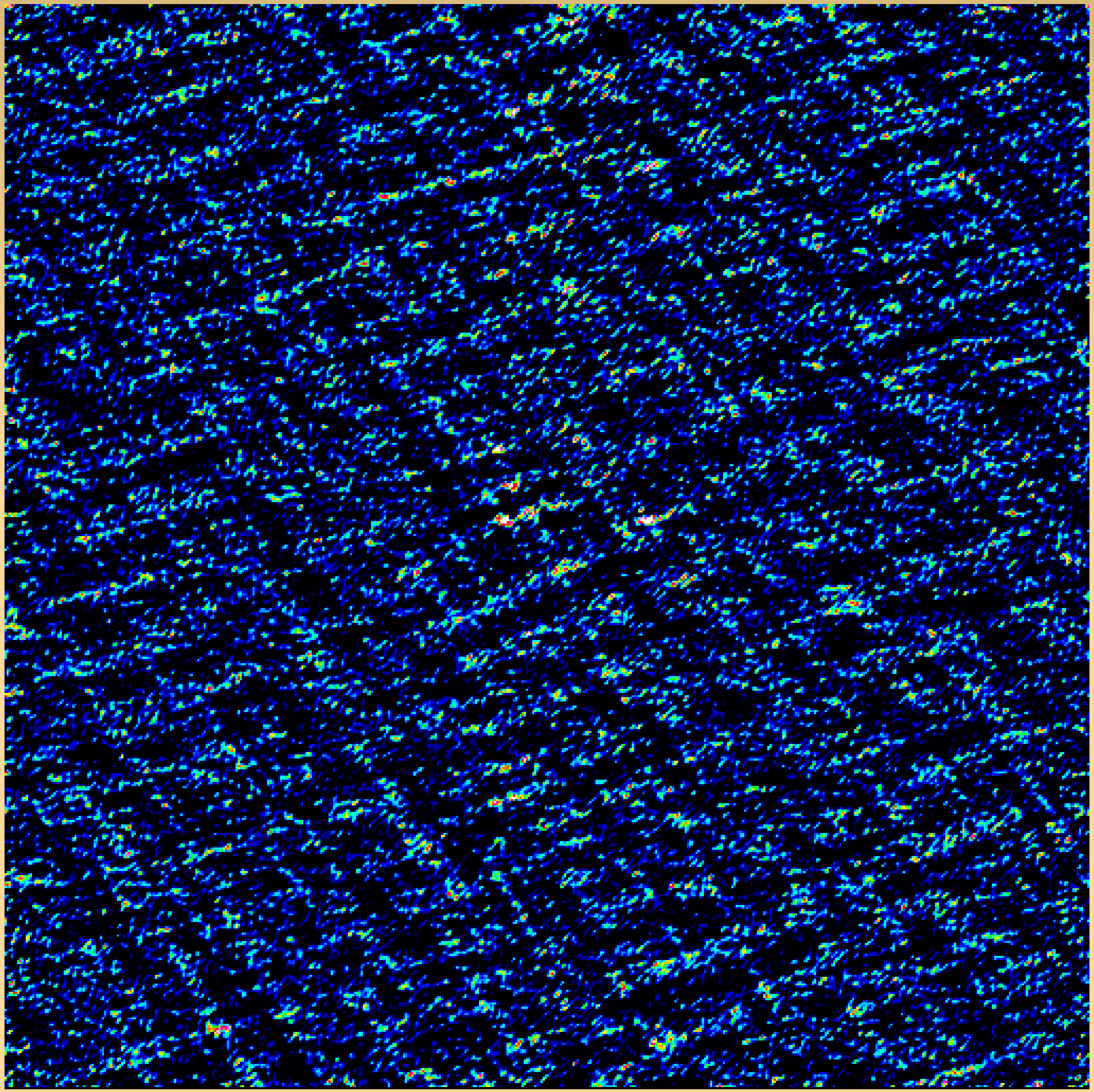


Residual image, after deconvolution.

The contaminating source cannot be deconvolved away properly, due to its *instrumental* time-variability.

Brightness scale 0~50μJy

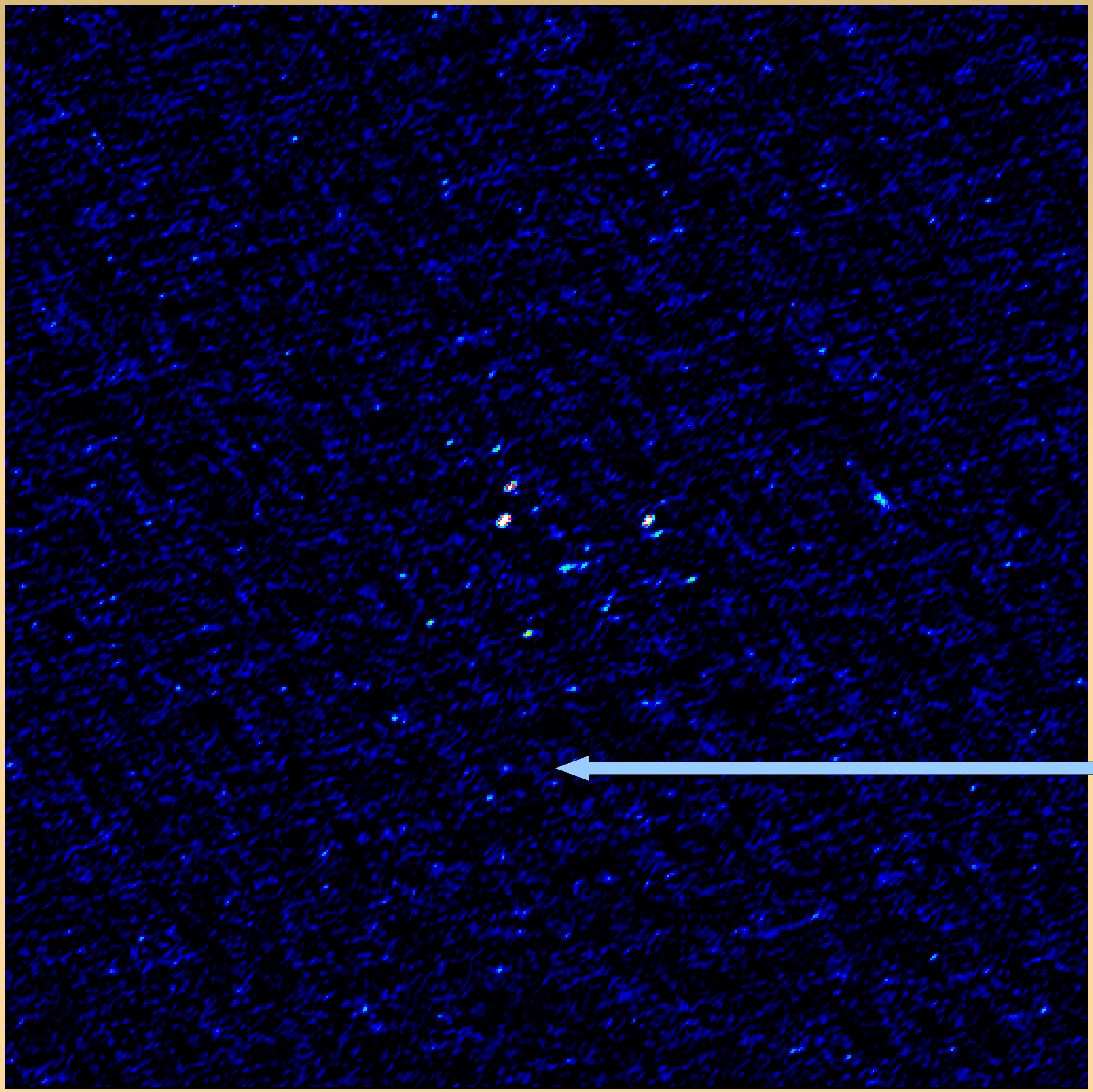
Differential Gains To The Rescue



Residual image after applying differential gain solutions to the contaminating source

Brightness scale 0~50μJy

Multi-Band Image



Multi-band residual image: noise-limited, no trace of contaminating source.

Phase calibrator used to be here

Brightness scale 0~50μJy

Two Broad Approaches To DDE Calibration

- Direction-dependent solutions (peeling, differential gains, etc.)
  - Treat the gain towards each direction as an independent solvable parameter
- Model-based approaches (pointing selfcal, ionospheric models, warped snapshot imaging, etc.)
  - Fit a “global” model to the DDE in question, solve for model parameters

Direction-Dependent Gains (and subtraction in the *uv*-plane)

- Given a model for the dominant source components, solve for direction-dependent gain terms:

$$\overbrace{\mathbf{V}_{pq}}^{\text{model visibilities}} = \underbrace{\sum_s \overbrace{\mathbf{J}_p^{(s)}}^{\text{gain, source } s \text{, station } p} \overbrace{\mathbf{X}_{pq}^{(s)}}^{\text{source model } s} \overbrace{\mathbf{J}_q^{(s)\dagger}}^{\text{gain, source } s \text{, station } q}}_{\text{sum over sources}} \rightarrow \overbrace{\mathbf{D}_{pq}}^{\text{observed visibilities}}$$

2×2 visibility matrix

2×2 Jones matrix

- Image the residual visibilities *{**R**=**V**-**D**}*
  - These are still subject to the same *relative* level of DDEs, but the *absolute* error level is lower.
- The subtracted source components can always be "restored" back into the resulting image

Differential Gains

- DoFs proliferate quickly, so it is better to use e.g.:

$$\boldsymbol{V}_{pq} = \underbrace{\boldsymbol{G}_p}_{\text{overall gain}} \underbrace{\left(\sum_s \underbrace{\boldsymbol{dE}_p^{(s)}}_{\text{differential gain}} \underbrace{\boldsymbol{E}_p^{(s)}}_{\text{nominal beam}} \underbrace{\boldsymbol{X}_{pq}^{(s)}}_{\text{source model}} \boldsymbol{E}_q^{(s)\dagger} \boldsymbol{dE}_q^{(s)\dagger} \right)}_{\text{sum over sources}} \boldsymbol{G}_q^\dagger$$

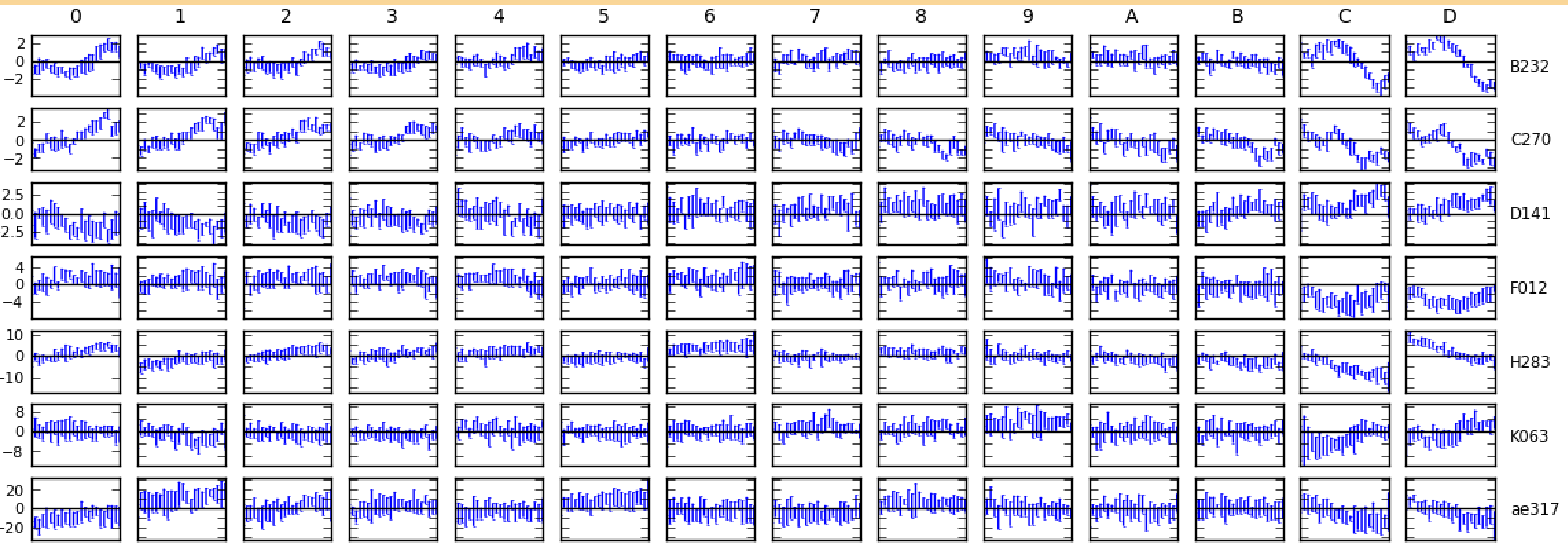
- *Direction-independent* gains $\boldsymbol{G}$ vary on short time-frequency scales
- Nominal beam model $\boldsymbol{E}$ accounts for the bulk of the DDE
- $\boldsymbol{dE}$ accounts for the small and slow direction-dependent variations (… hopefully)

A.k.a. “The Flyswatter”

- **The Good: it swats sources**
  - Point-and-shoot: dE's can completely eliminate contaminating sources, making for great maps.
- **The Bad: it swats sources**
  - Mashes together all information on both the source and all DDEs towards it
- **The Ugly: it proliferates degrees of freedom**
  - Fundamental and computational limits on how many dE's you can have
  - LOFAR EoR project: up to 60 per antenna

The Ugly, continued...

- ...and makes no use of spatial continuity.
- So we'd really like to learn to fit some “global” DDE models instead
- Example: 3C147 field, dE-phase solutions:



The QMC* Project:
a "controlled" experiment

- Pick a field containing a cluster of reasonably bright off-center sources
- Observe this at Westerbork
- Introduce deliberate (and secret!) pointing errors during observation
- Attempt to recover these during the reduction

**) Named in honour of the now-defunct WSRT Quality Monitoring Committee. Yes, the Dutch do love to establish committees. Fortunately, so do the Russians.*

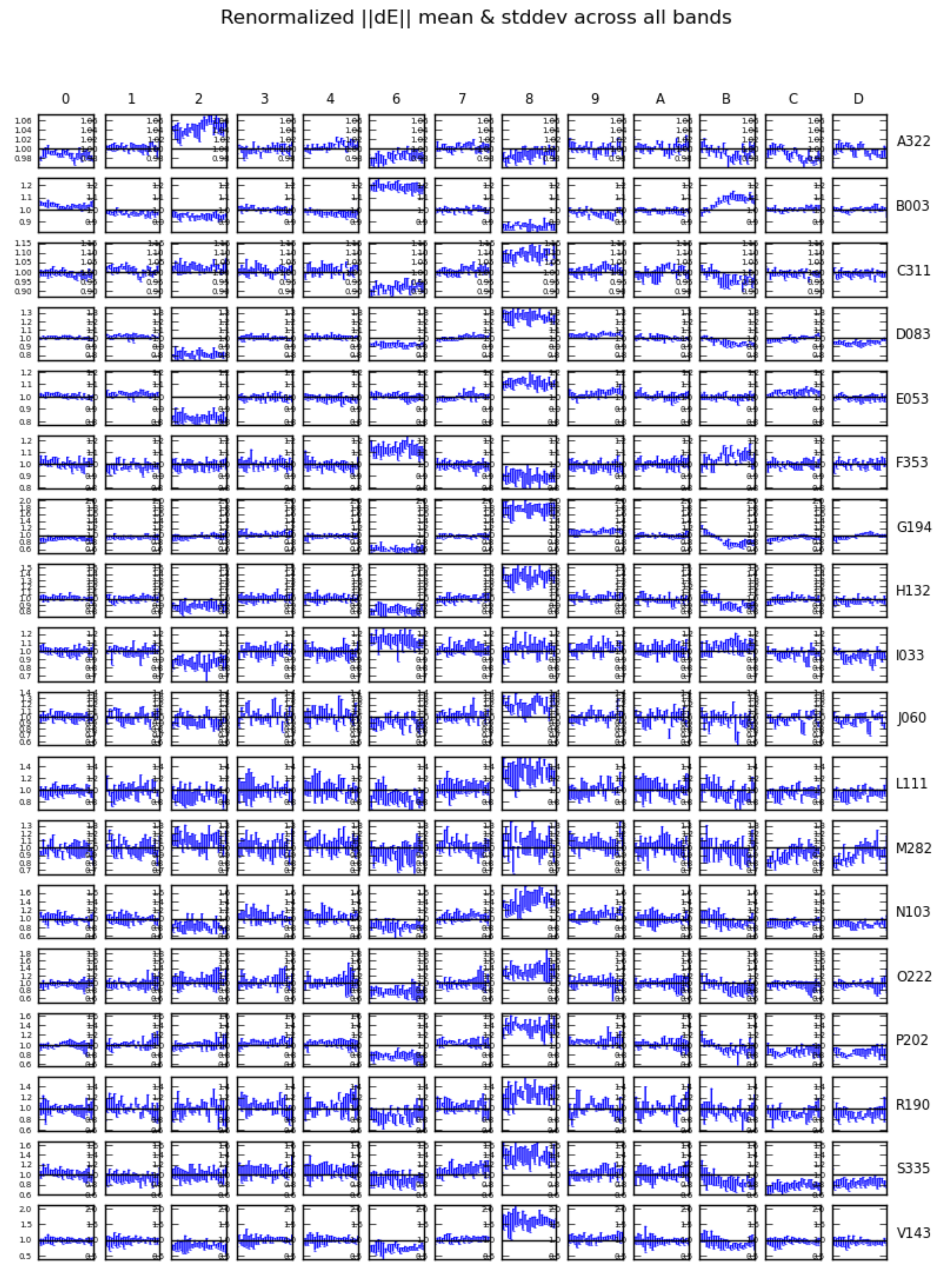
The QMC2 Field (01515+6736)
(a radio astronomer's worst nightmare)

- >10 moderately bright off-center sources
- The type of field that usually has radio astronomers running away screaming...

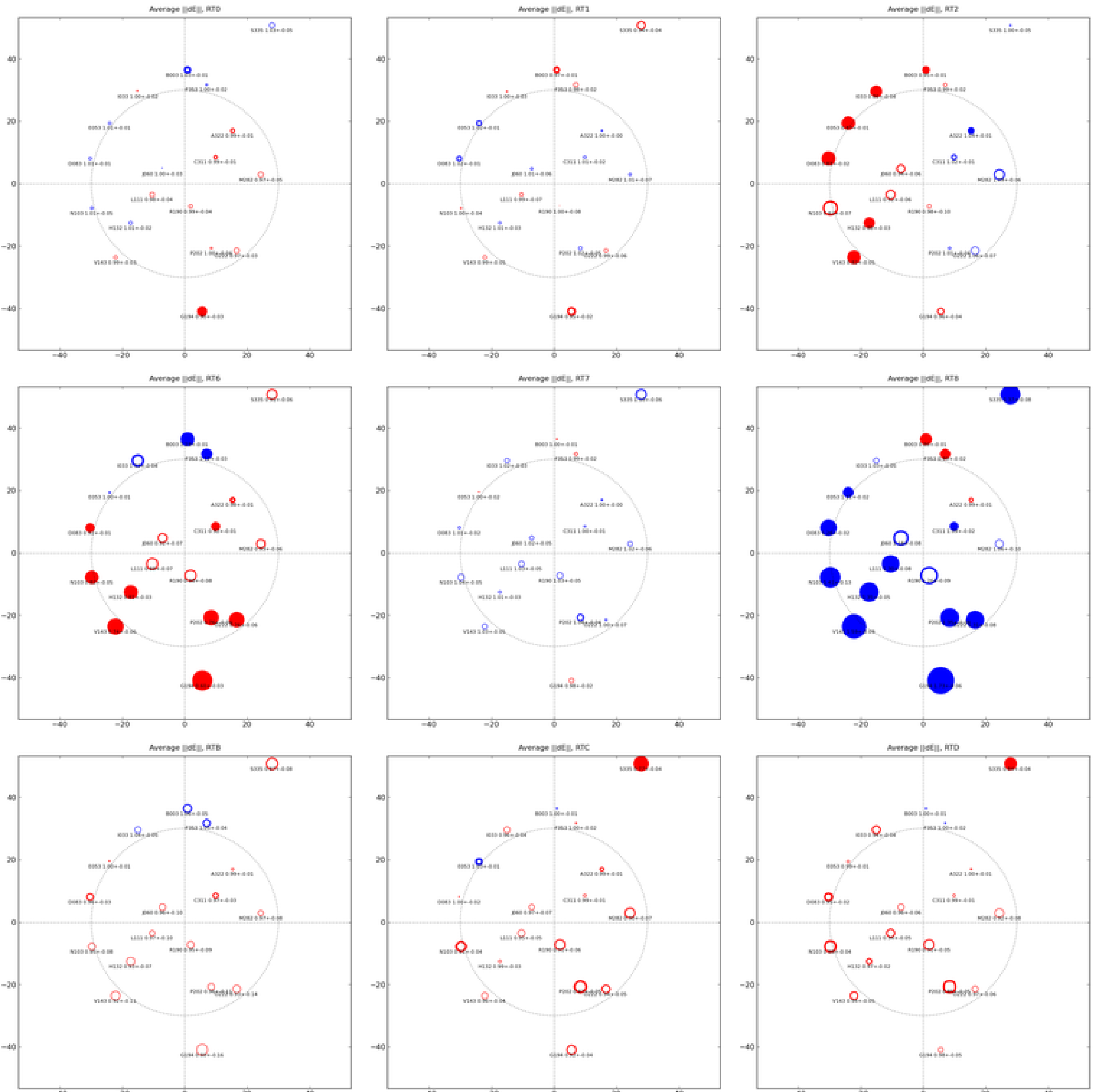
- But perfect for our purposes!

QMC2 2010Jul21

- ||dE|| solutions suggest a static mispointing of three antennas
- ...and a time-variable mispointing of RTB ("Hans's susprise")
- Observatory confirmed that this was consistent with the mispointings they had put in.



“Rogues' Gallery” Plot



Plots of mean ||dE|| per antenna, at proper positions within the field.
Colour/size indicates
||dE||>1, <1.

Solving For Pointing Errors

- ||dE|| plots are a reliable indicator of mispointing
  - But only a very rough one...
- Can we recover the actual pointing offsets?
- *Pointing selfcal* algorithm (S. Bhatnagar)
  - Solves for first-order approximation via FFT
- DFT-based pointing solutions
  - A brute-force modeling approach
  - Not as efficient, but more flexible

DFT Pointing Solutions

$$V_{pq} = \underbrace{G_p}^{\text{overall gain}} \underbrace{\left(\sum_s \overbrace{E_p^{(s)}}^{\text{beam}} \overbrace{X_{pq}}^{\text{source model}} E_q^{(s)\dagger} \right)}_{\text{sum over sources}} G_q^\dagger$$

$$\boldsymbol{E}_p(l,m,\nu) = E(l+\Delta l_p, m+\Delta m_p, \nu),$$

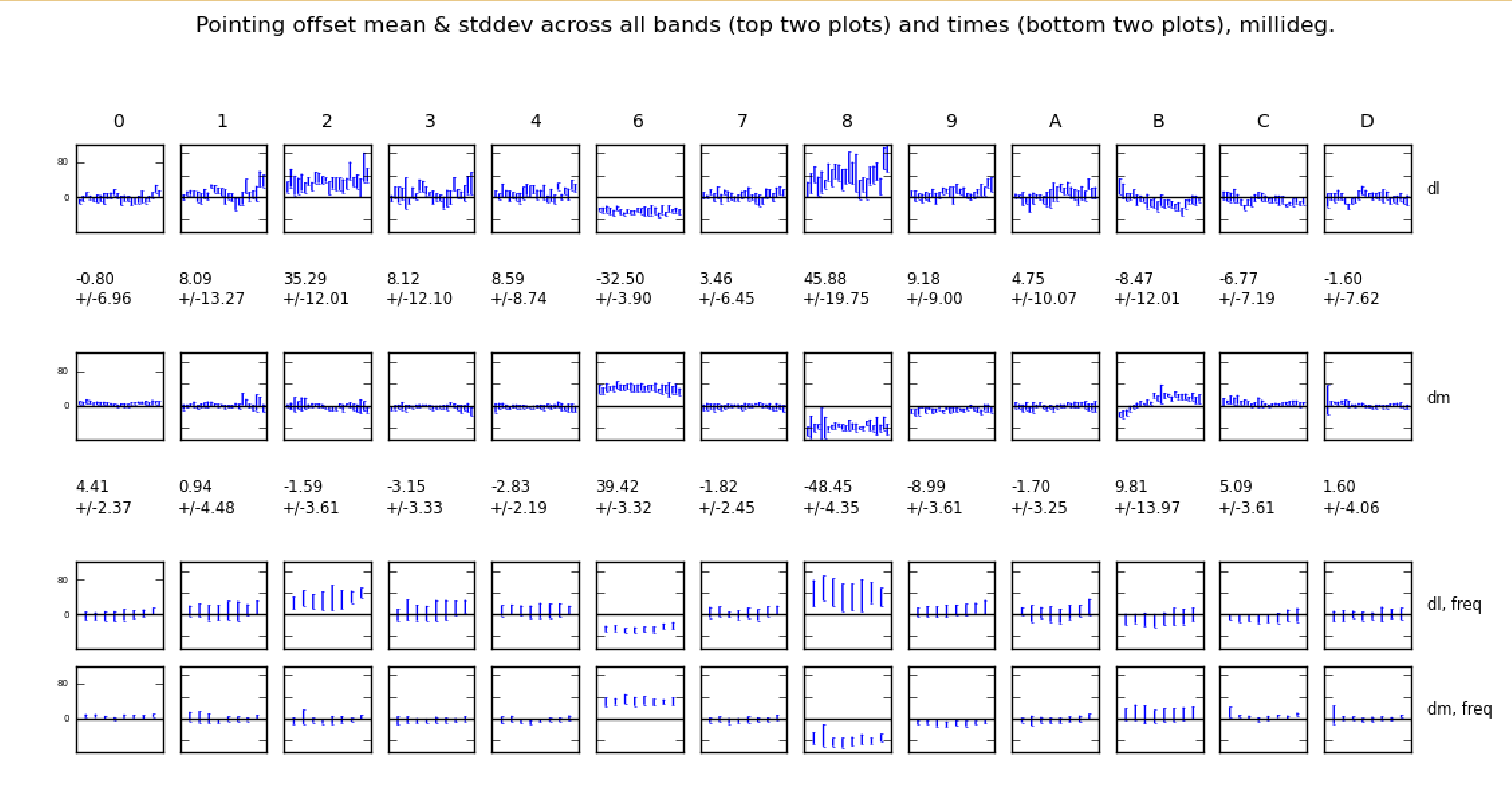
where $E(l,m,\nu)$ is a primary beam model.

...and solve for the offsets $\Delta l_p, \Delta m_p$.

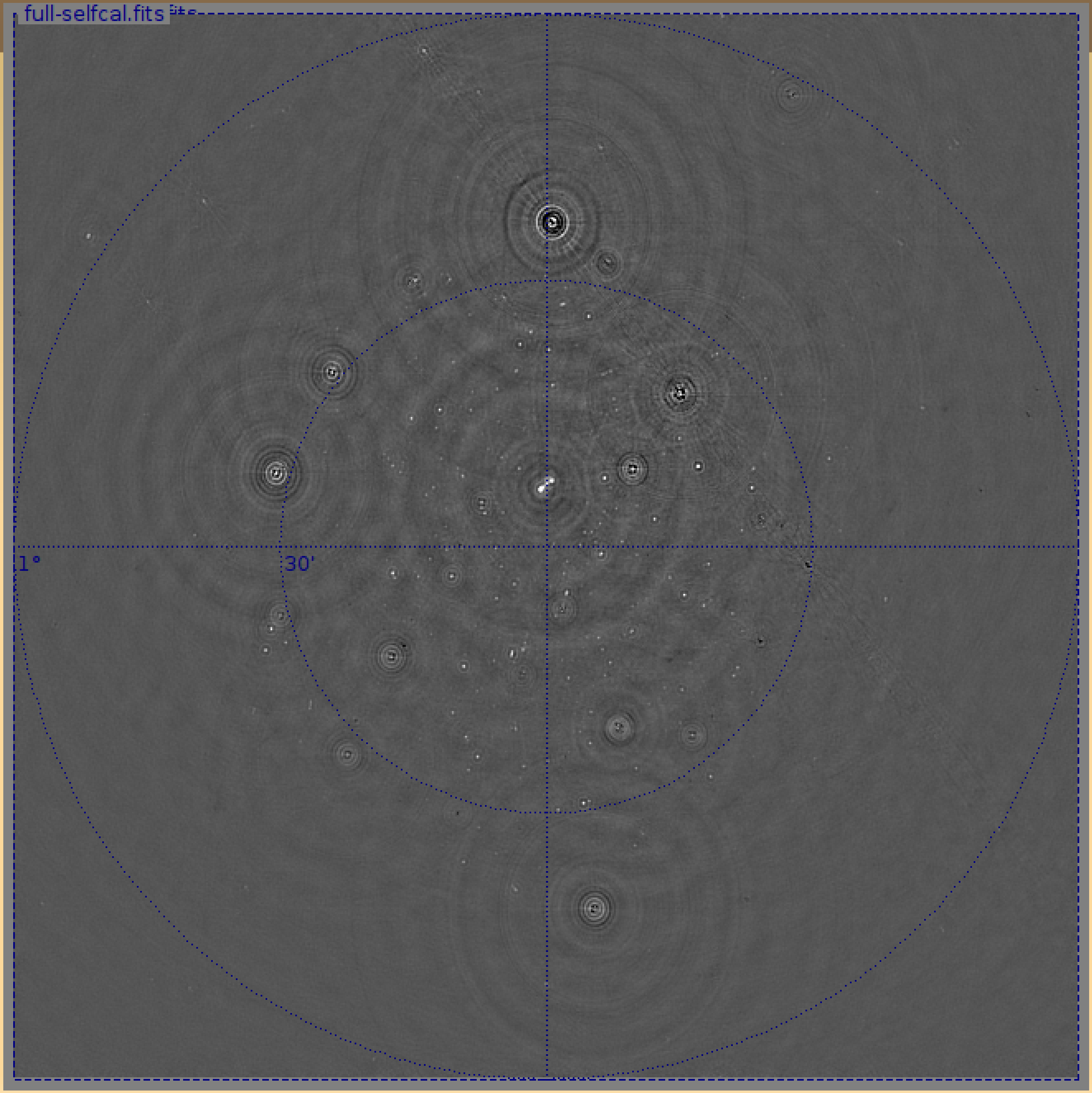
Standard WSRT beam model: $E(l,m,\nu) = \cos^3\left(C\nu\sqrt{l^2+m^2}\right)$

P.E. Solutions
as a function of time

- Recovered solutions consistent with deliberate mispointings, but <u>underestimate</u> them:

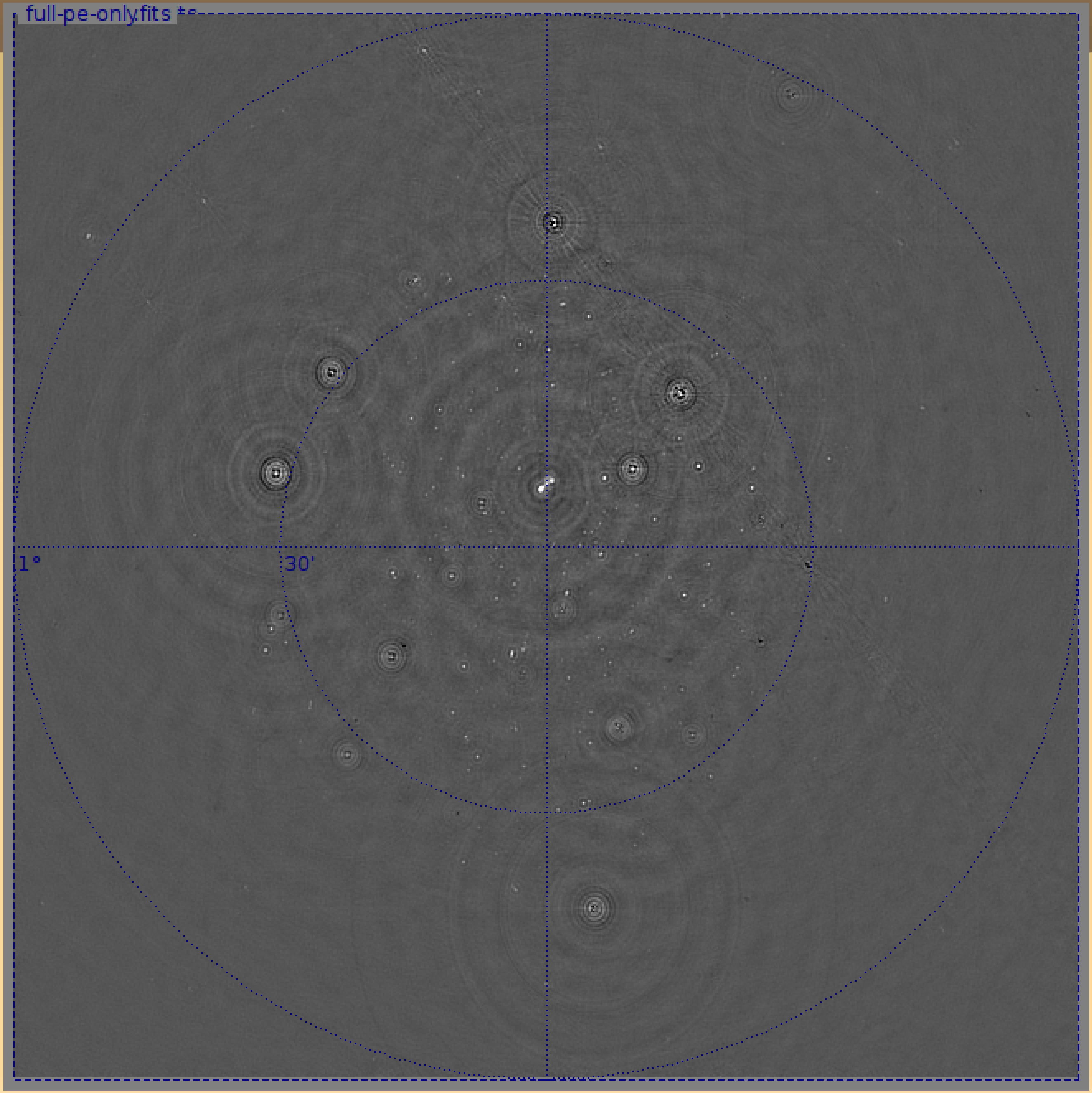


Not so impressive...



Residual image, post-selfcal

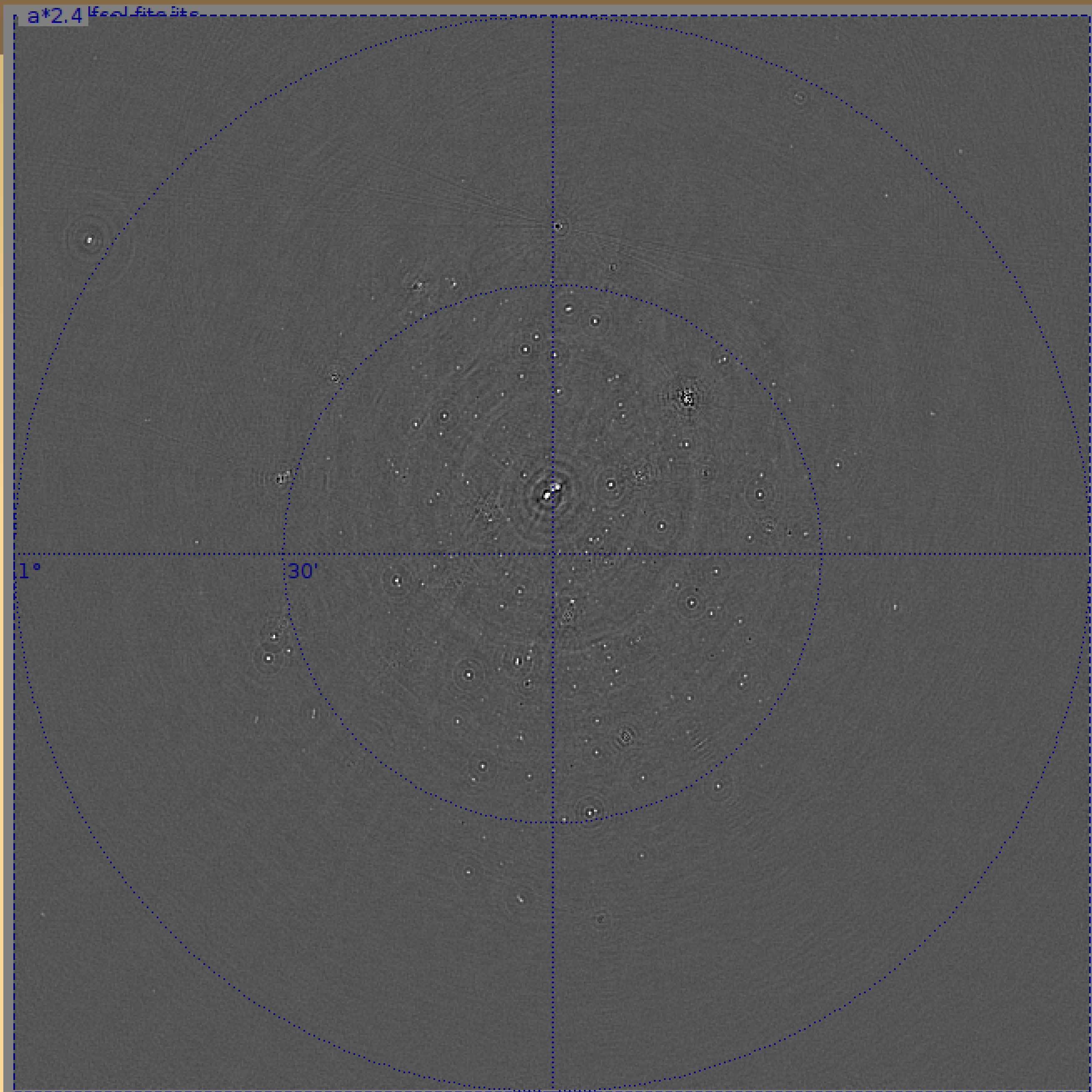
A Marginal Improvement



Residual image,
post-selfcal,
with pointing error
solutions

(Note how this relative lack of improvement is consistent with S.Bhatnagar's pointing selfcal results.)

Nowhere Near The Flyswatter...



Residual image, post-selfcal, with differential gains.

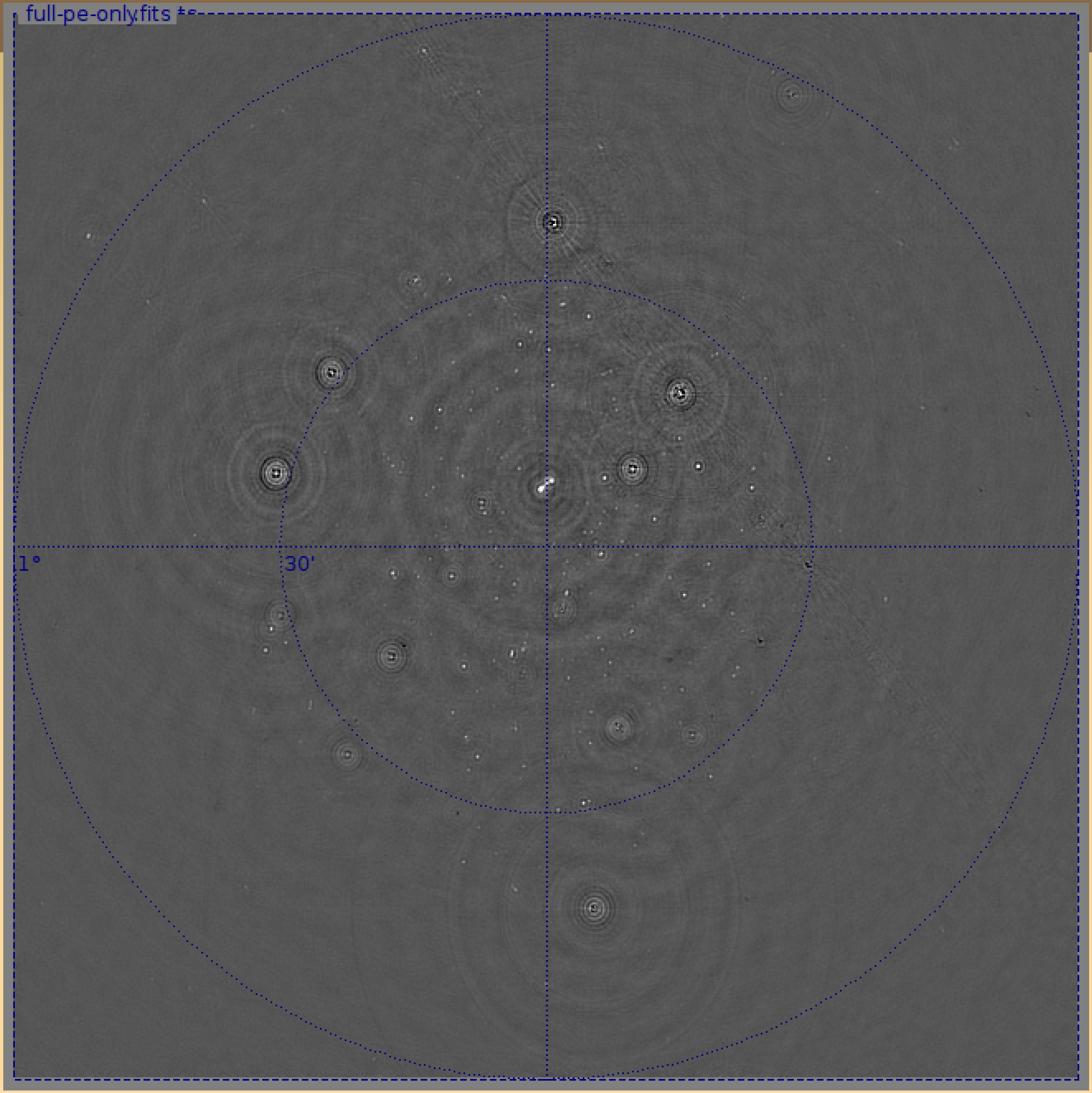
Parameterizing The Beam

- The advantage of the DFT approach is that we can introduce other parameters into the primary beam model.
- Just as a random example, we can introduce a per-antenna beam scale s_p:

$$\mathbf{E}_p(l,m,\nu)=E(l+\Delta l_p, m+\Delta m_p, s_p, \nu),$$
$$E(l,m,s,\nu)=\cos^3(C\nu s\sqrt{l^2+m^2})$$

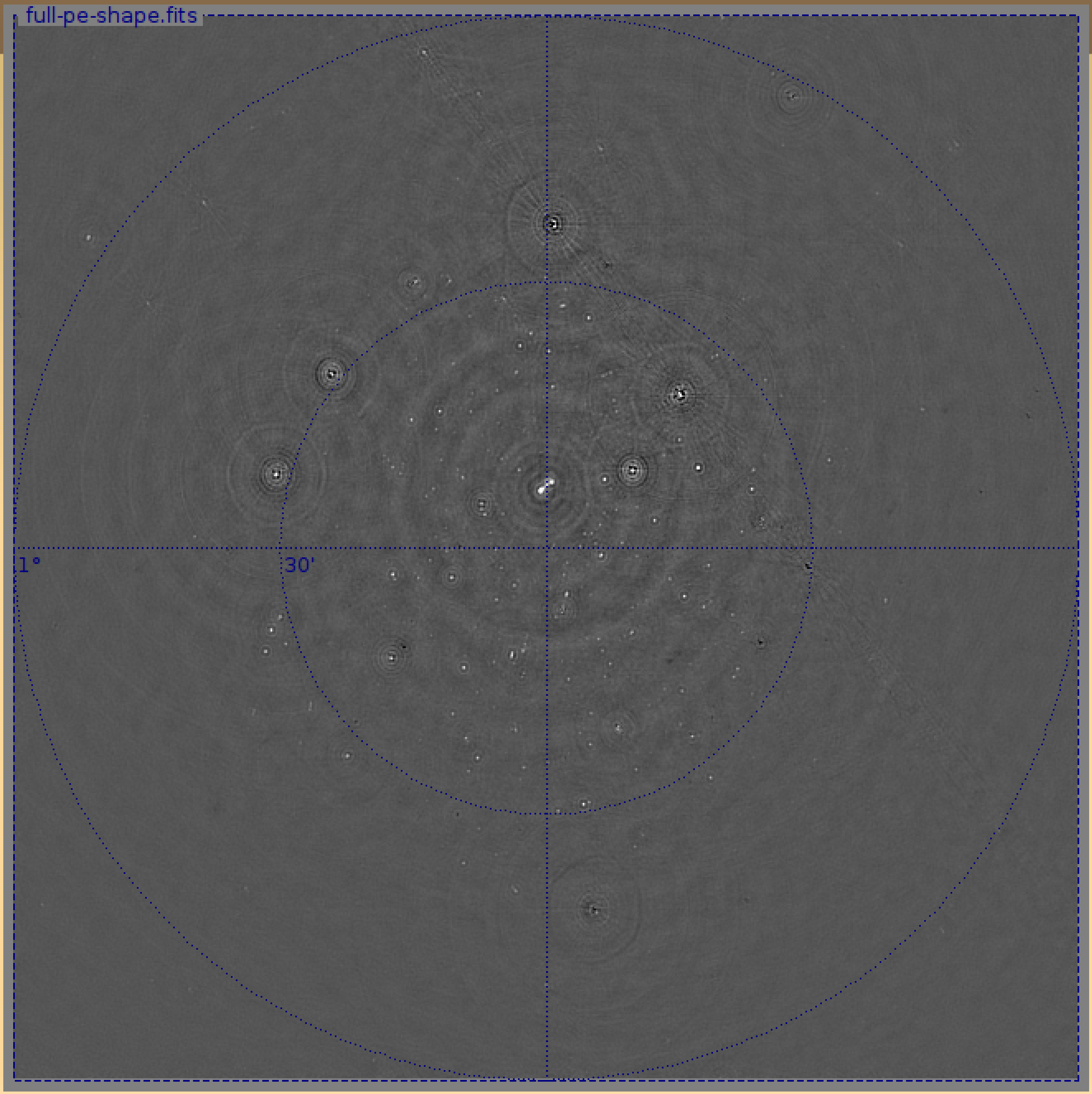
- And then treat s_p as a solvable.

P.E. Solution Only



Residual image,
post-selfcal,
with pointing error
solutions

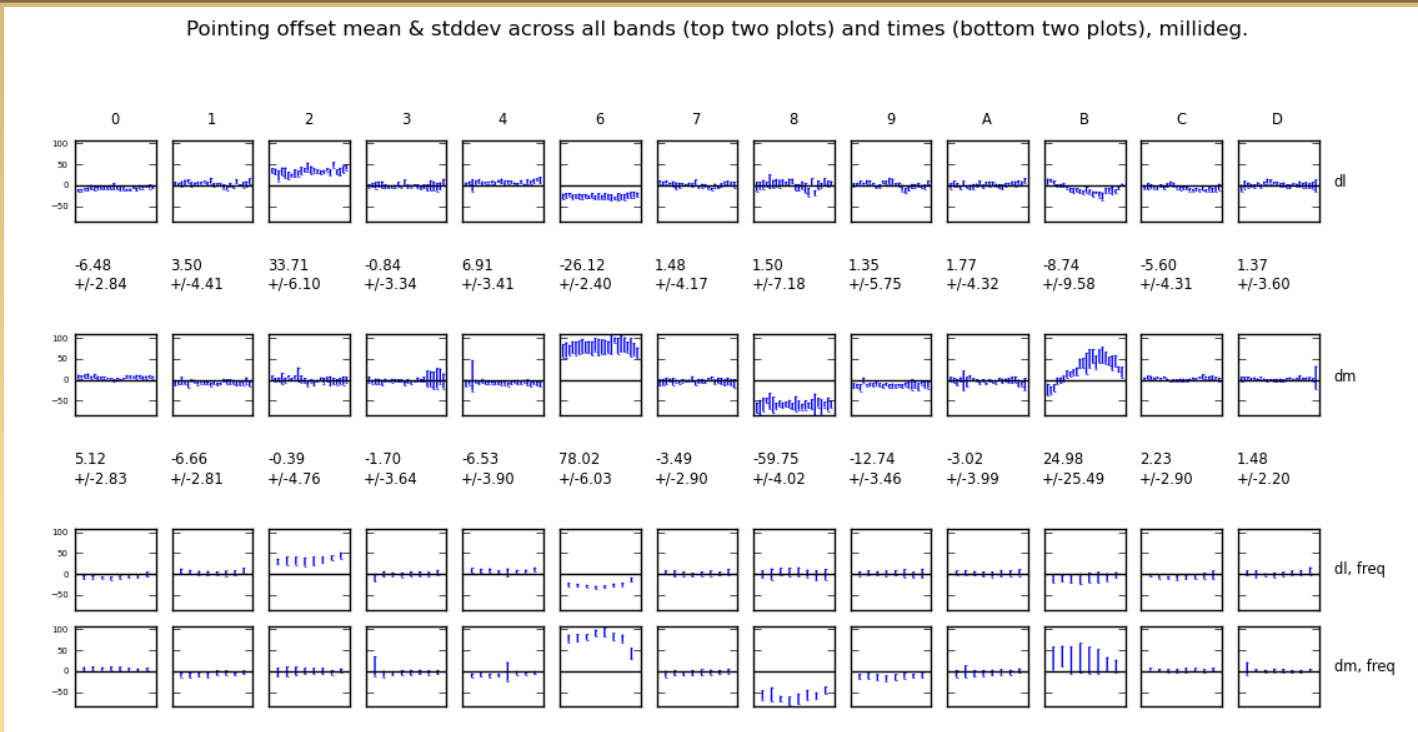
P.E. + Beam Extent

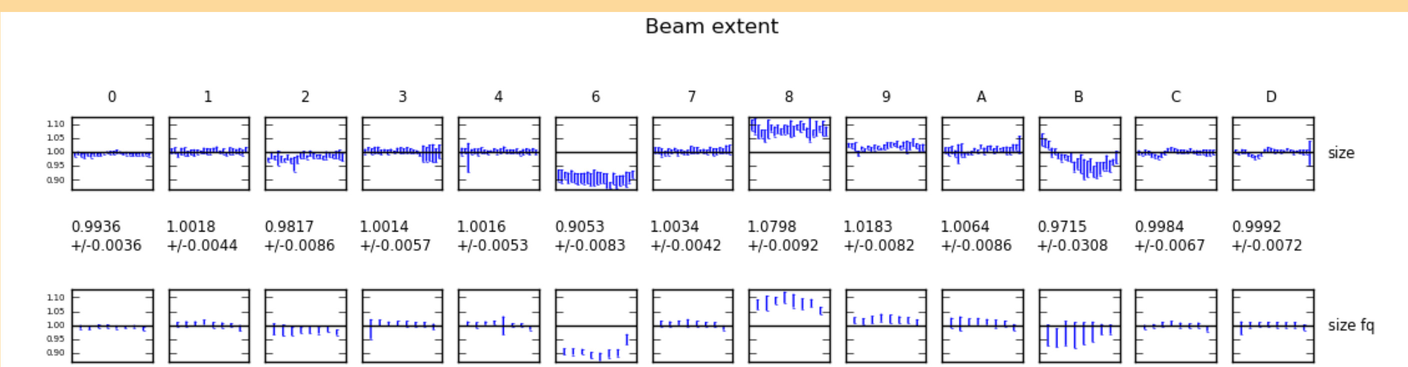


Residual image, post-selfcal, with pointing error and beam extent solutions

...not as good as differential gains, but an improvement!

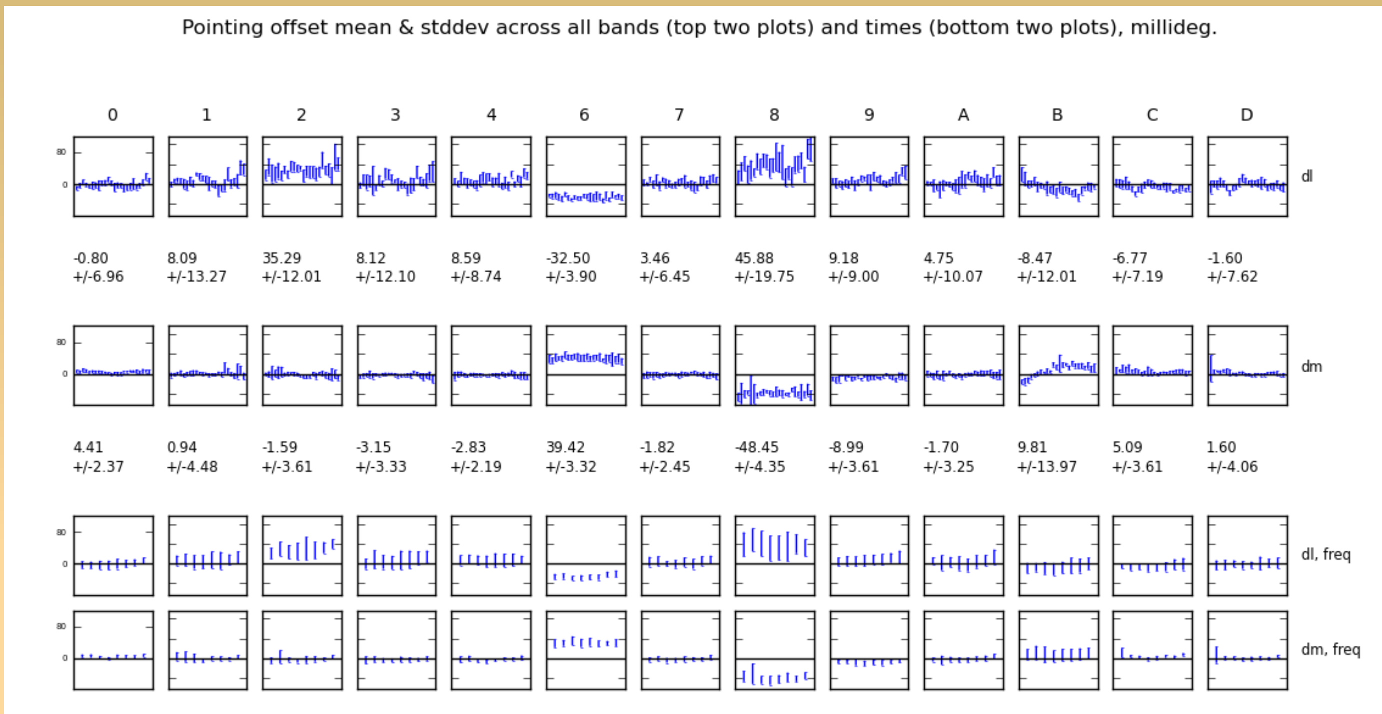
Now Back To The Pointing Plots





- Beam “extent” and pointing offset solutions are strongly coupled
- “Extent” solutions are non-physical (±10%!)
- Pointing offsets are recovered better

Compare To The PE-Only Case



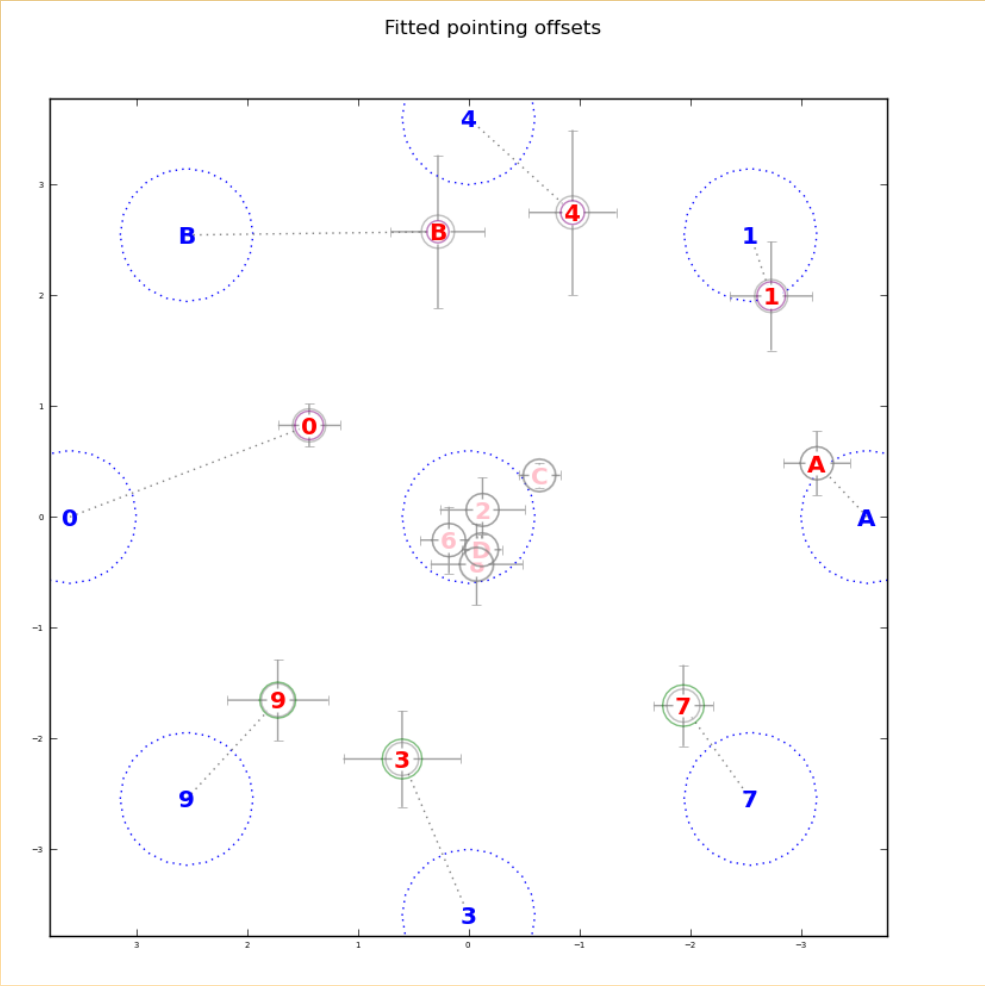
- P.E. solutions without a beam “extent” show more variance
- ...and underestimate the true offsets
- Obviously the extra degree of freedom is compensating for something else, but what exactly?

- Tentative conclusion: P.E. solutions are limited by the accuracy of the beam model.
- ...as are the final maps: **KNOW THY BEAMS!**

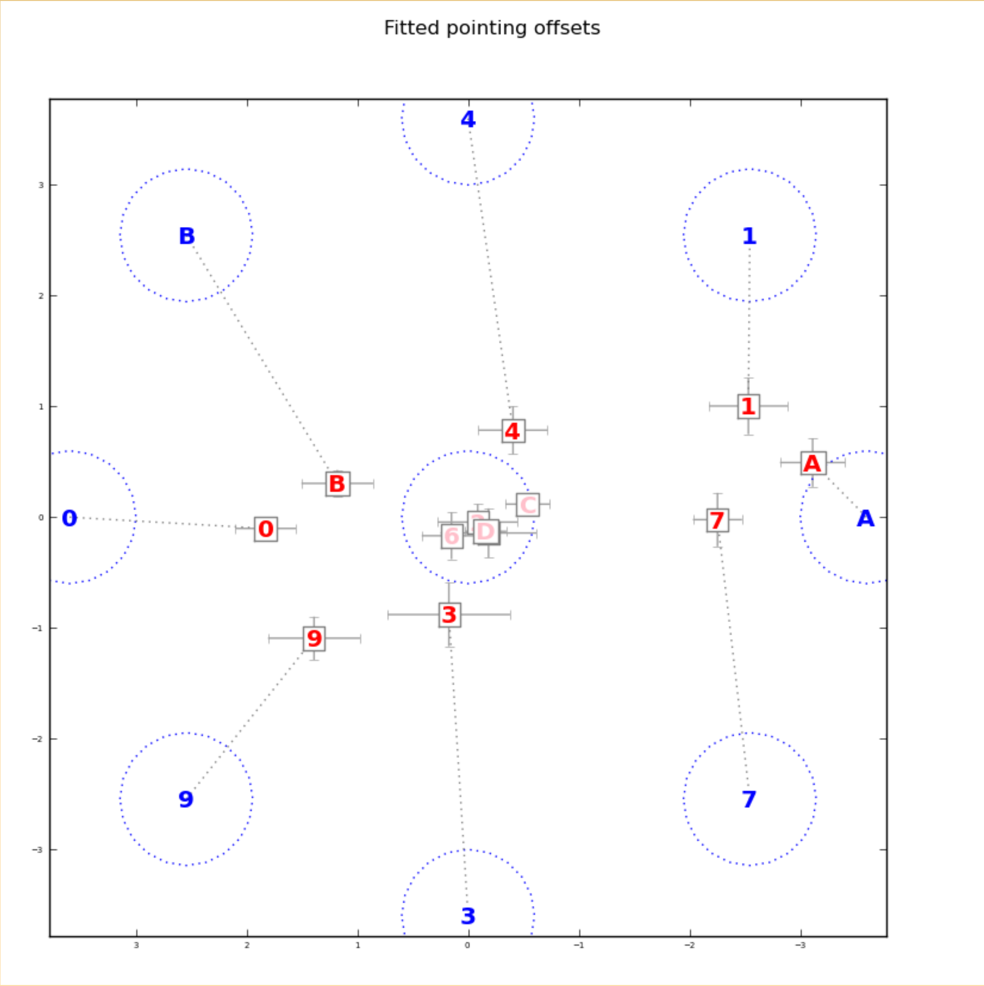
A Different Observation (8 antennas mispointed in 8 directions)

- Plot of actual vs. fitted pointing offsets

With a solvable beam extent

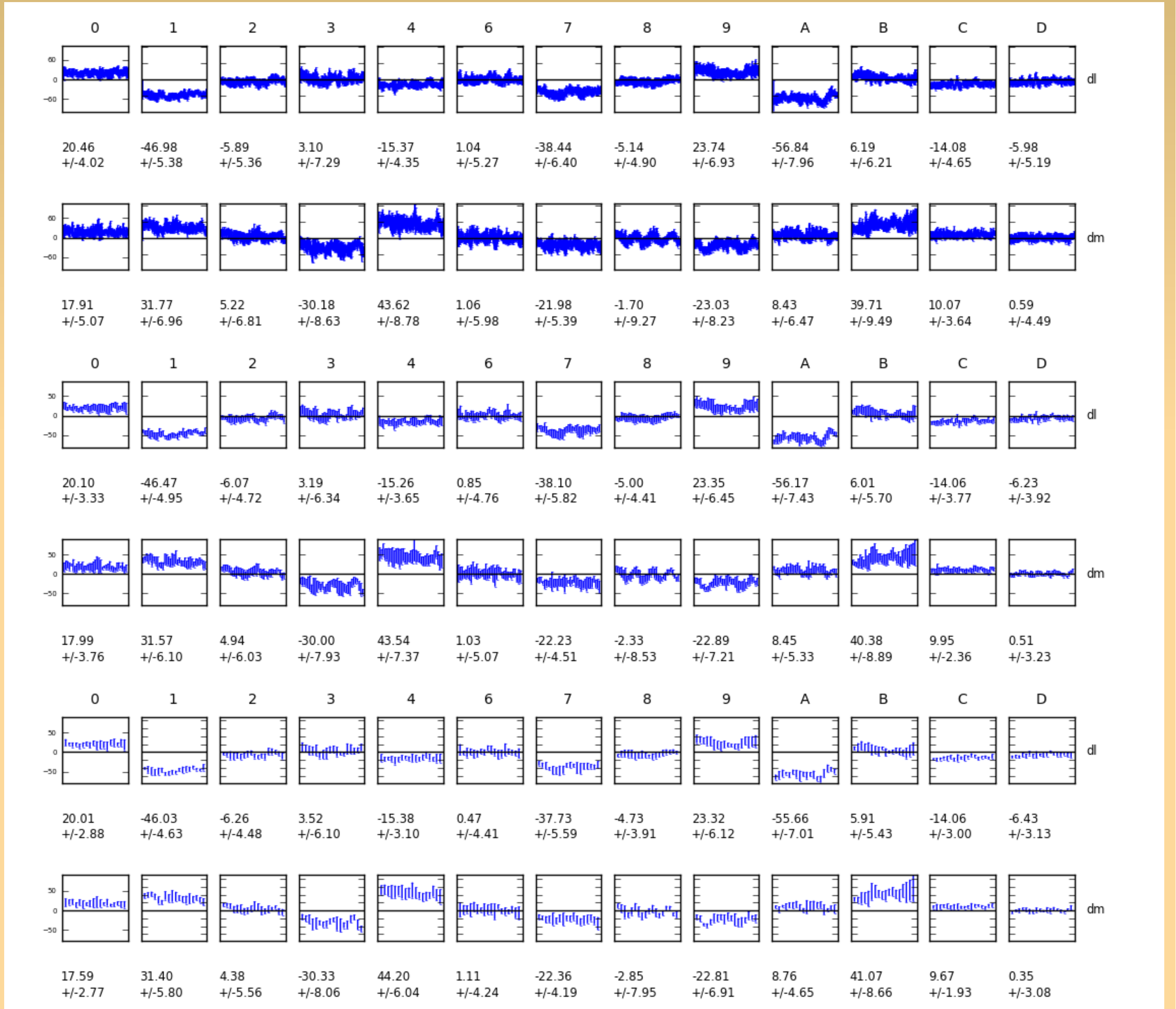


Without a solvable beam extent



Yet Another Twist: Solving For P.E. On Shorter Time Scales

- Solutions every 30 sec, 2.5 min and 5 min.
- Longer time scales: decreased variance (higher SNR)
- Diminishing returns above 5 min.
- Show a striking feature unnoticed on the previous (30 min) plots...



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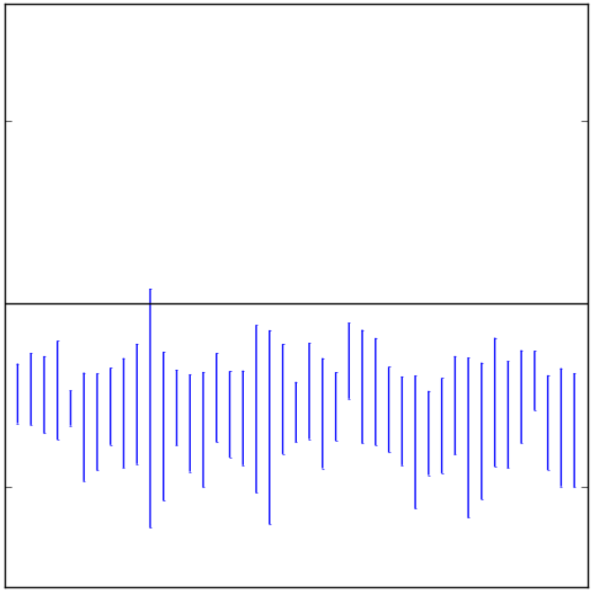
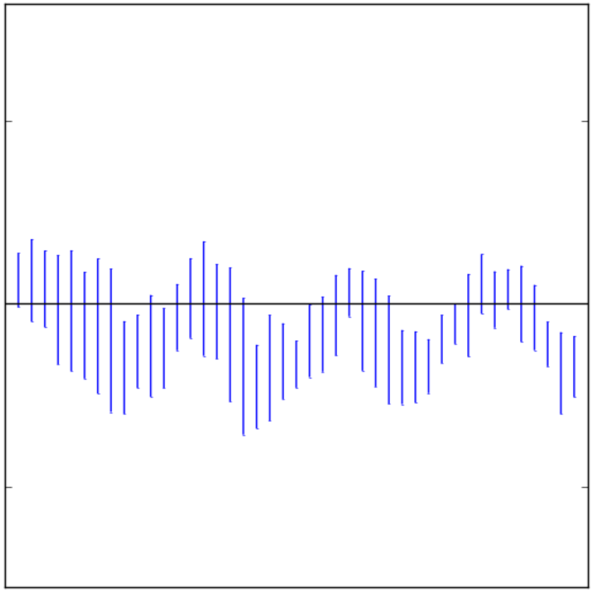
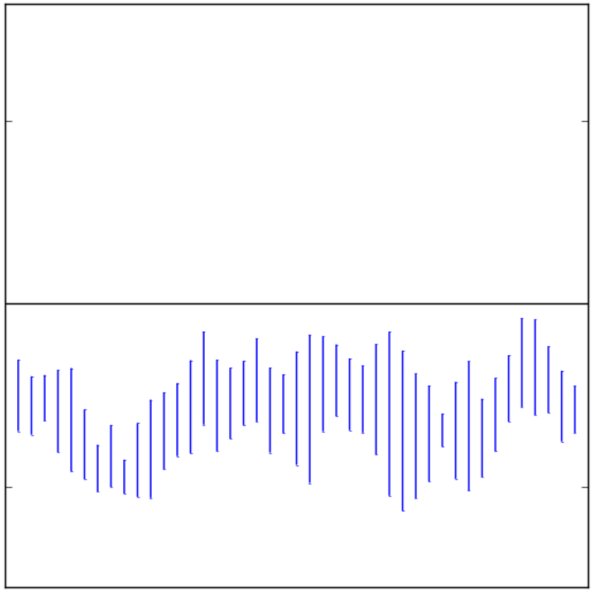
And Now, Applying Sophisticated Model Fitting Techniques...



mean -29.19
+/- 4.60



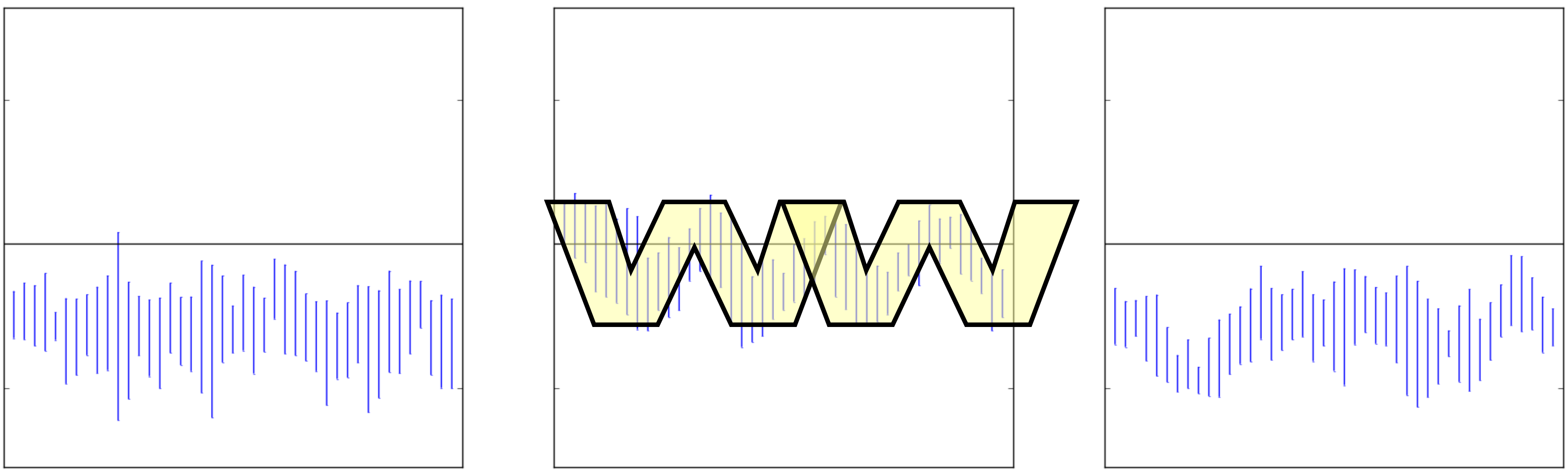
mean -7.36
+/- 7.95



mean -29.87
+/- 7.13

And Now, Applying Sophisticated Model Fitting Techniques...

Westerbork Wobble!



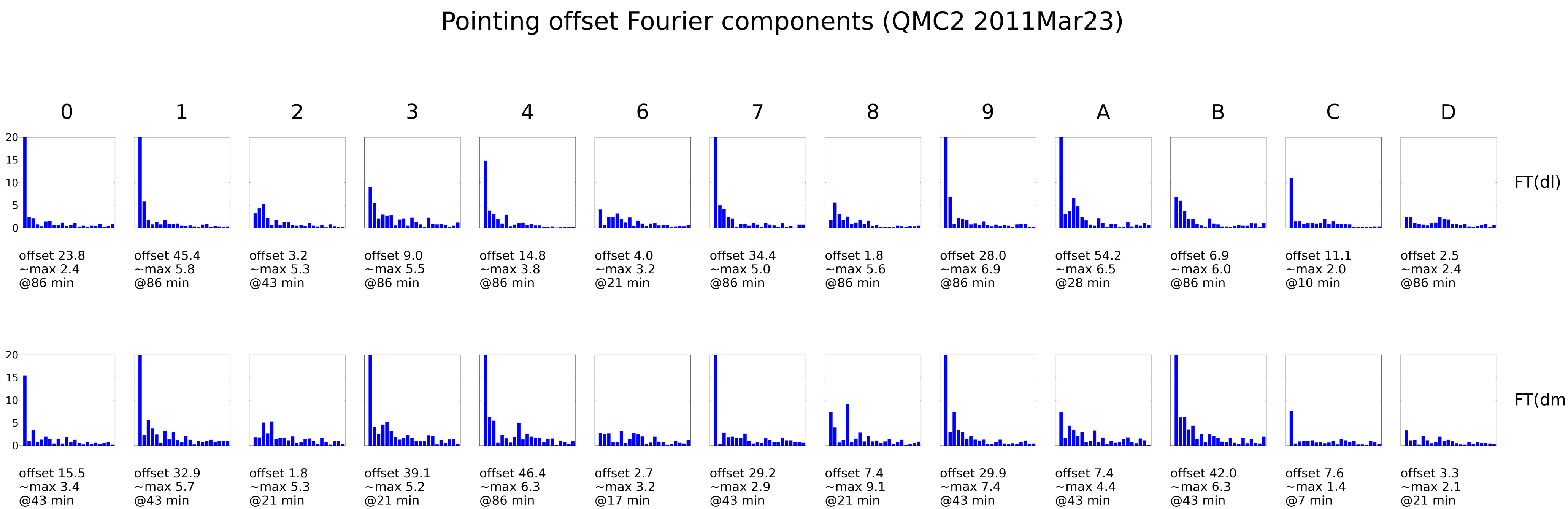
mean -29.19
+/- 4.60

mean -7.36
+/- 7.95

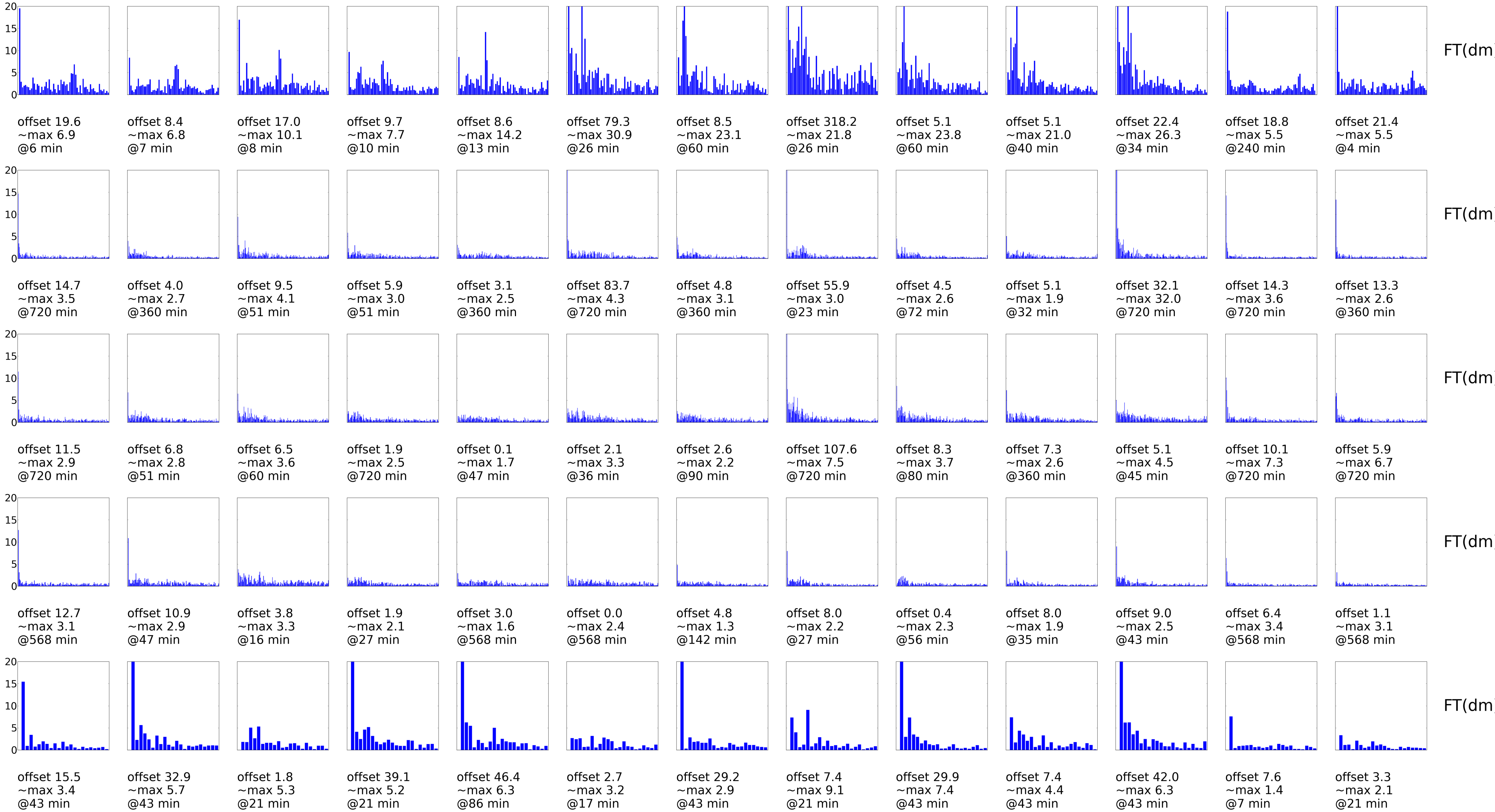
mean -29.87
+/- 7.13

The Wobble

- A periodic (~20 min) variation in the pointing of 10-20 mdeg.
- Shows up in other observations, on **<u>other</u>** antennas (to varying extent)
- We can identify dominant “wobbly modes” by taking a Fourier transform of the P.E. solutions, and examining the amplitudes:



Wobbling Across 5 Epochs



Two Approaches To Correction

- Subtraction in the *uv*-plane
  - Subtract a model of each “corrupted” source directly from the raw visibility data
- AW-projection or facet imaging
  - Applies a single (convolution-based) or per-facet correction during imaging
  - Requires “global” DDE model (or interpolation between solutions?)
  - Accuracy limited by DDE model

The Relative Trade-Offs

(A wholly inaccurate but fully management-compliant graph)

"accuracy"

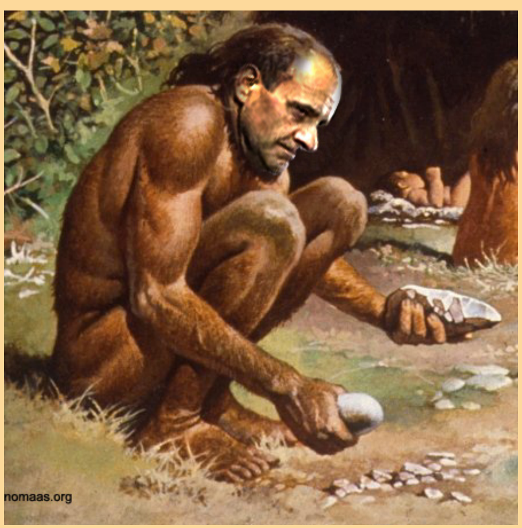
Differential gains + *uv*-plane subtraction

Differential gains + A-proj or facets

"Global" model fit + *uv*-plane subtraction

"Global" model fit + A-proj or facets

Traditional FFT-based imaging

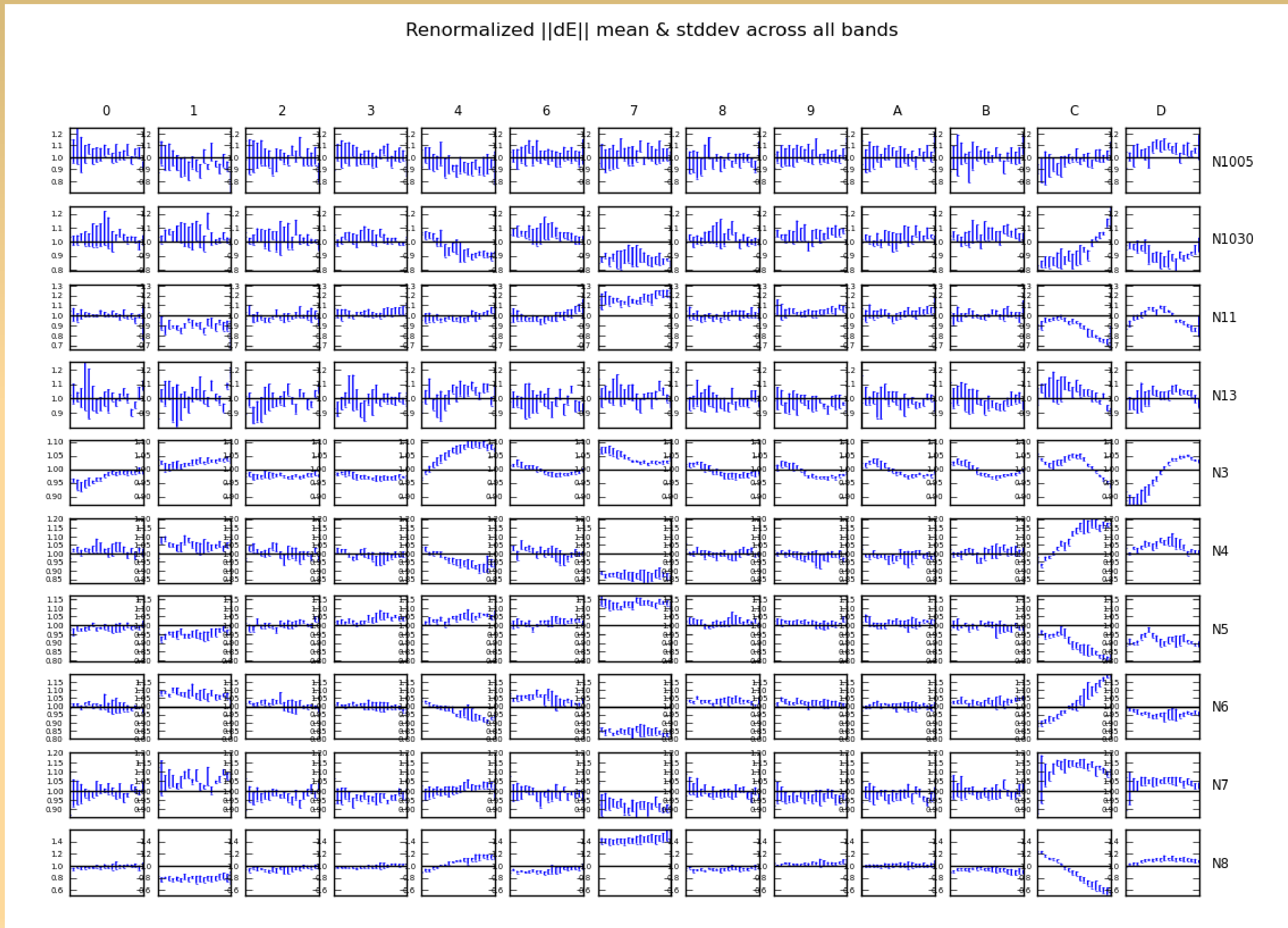


"efficiency"
(# of DoFs, scalability)

DDEs vs. Source Structure

- We've been taking sky models for granted
- In real life, these need to be bootstrapped from the observations themselves.
- ...where it can be very difficult to decouple DDEs from spatial source structure.
  - Our QMC2 field has point-like sources only
- Unmodeled source structure...
  - ...is either partially absorbed into differential gain solutions
  - ...or else contaminates the “global” model fits

DDEs vs. Source Structure II:
(an example from a different observation)



Conclusions

- Differential gains work
  - ...slowly
- “Global model” DDE solutions (pointing selfcal, DFT pointing, etc.) work faster
  - ...but less accurately
- Can be combined / traded off
- Accuracy limited by models, so
  - **KNOW THY BEAMS!**
  - **DEAL WITH THY SOURCE STRUCTURE!**