Stochastic Models for Sparse Signals with Application to Imaging

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ABSTRACT

Traditional MRI reconstruction techniques are linear: They can be motivated statistically by assuming that the underlying signal is a realization of a Gaussian stationary process and that the measurements are corrupted by additive Gaussian noise. In recent years, there has been a shift of paradigm towards nonlinear reconstruction techniques that impose sparsity constraints, either through a wavelet-domain ℓ_1 penalty, or through the minimization of a nonquadratic regularization functional such as total variation (TV).

In this talk, we shall present an extended class of signal models that are ruled by stochastic differential equations (SDEs) driven by white Lévy noise. When the excitation (or innovation) is Gaussian, the proposed model is equivalent to the traditional one. Of special interest is the property that the signals generated by non-Gaussian linear SDEs tend to be sparse by construction; they also admit a concise representation in some adapted wavelet basis. Moreover, these processes can be (approximately) decoupled by applying a discrete version of the whitening operator (e.g., a finite-difference operator). The corresponding log-likelihood functions, which are nonquadratic, can be characterized analytically. In particular, this allows us to uncover a Lévy processes that results in a maximum a posteriori (MAP) estimator that is equivalent to total variation. We make the connection with current methods for the recovery of sparse signals and present some examples of MAP reconstruction of MR images with sparse priors.

References

[1] M. Guerquin-Kern, M. Häberlin, K.P. Pruessmann, M. Unser, "A Fast Wavelet-Based Reconstruction Method for Magnetic Resonance Imaging," *IEEE Transactions on Medical Imaging*, in press.

[2] M. Unser, P.D. Tafti, "Stochastic Models for Sparse and Piecewise-Smooth Signals," *IEEE Transactions on Signal Processing*, vol. 59, no. 3, pp. 989-1006, March 2011.