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The applications of compressive sensing to radio astronomy

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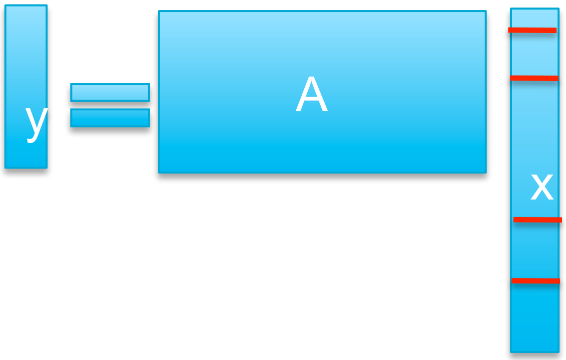
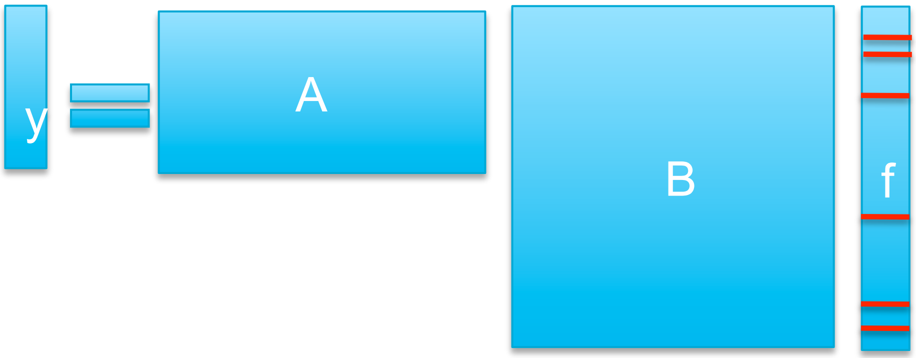
Introduction of CS

- CS says that we can reconstruct a signal using far fewer measurements than required by the Nyquist sampling theory, provided that the signal is sparse or there is a sparse representation of the signal with a respective given basis function dictionary



OR



CS vs Nyquist-Shannon sampling

Compressive sampling
2004

Nyquist-Shannon sampling
1950s

CS	NS
Indirect sampling	Direct sample
Nonuniform sampling Sparsity decides	Uniform sampling Maximum frequency decides
Reconstruction step required	No need reconstruction step

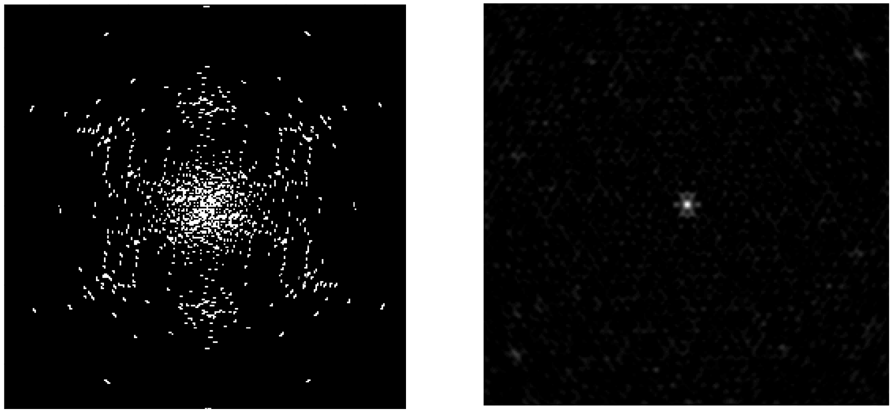
All photos are from Wikipedia

Outline

- Two investigated applications of compressive sensing/ sampling (CS) to radio astronomy:
  - Image Deconvolution
  - Faraday Rotation Measure Synthesis
- Conclusion

Australian Square Kilometer Array Pathfinder (ASKAP)

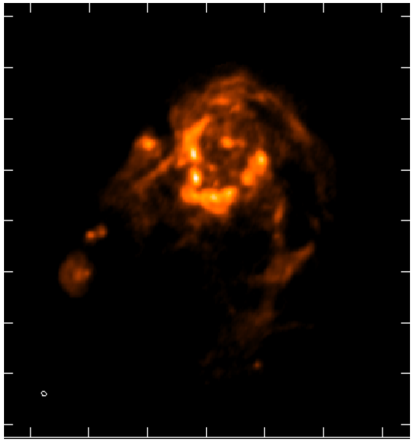
- 36 antennas (12m in diameter) in West Australia
- The smallest separation of 22m
- The longest baseline 6km
- 30 of them locating within a 2km radius
- 6 long baseline antennas to capture high frequency details



ASKAP UV coverage and its PSF

Isotropic Undecimated Wavelet Transform (IUWT)

- In the IUWT, the non-orthogonal Astro filter bank (the low pass filter: h1D= [1, 4, 6, 4, 1]/16, the high pass filter: g1D= δ − h1D= [−1, −4, −10, −4, −1]/16) is adopted
- Many of sources in the universe are isotropic
- Advantage of IUWT is that we can use the low pass filter only for implementation



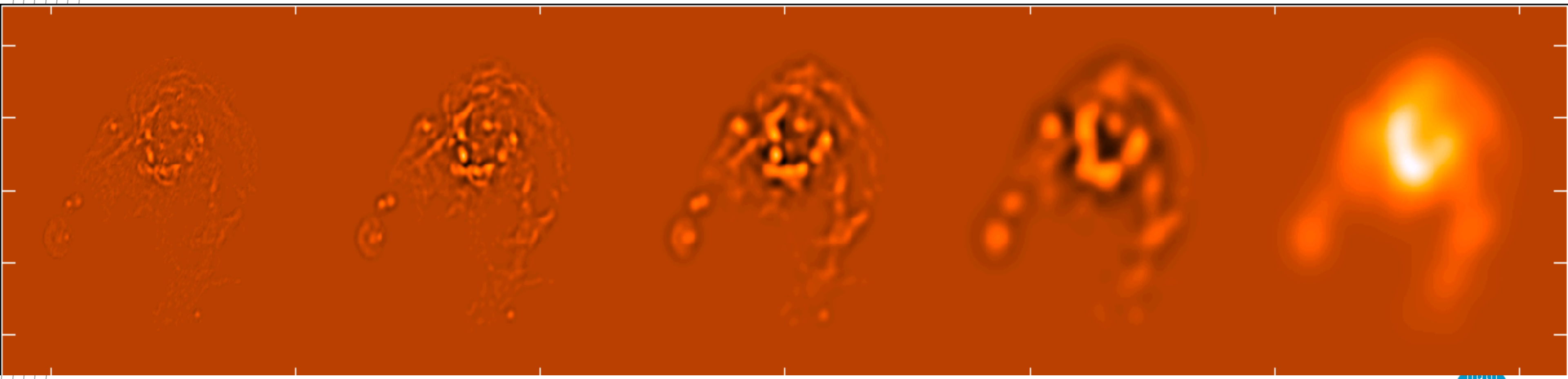


Image Deconvolution

- Multiplication (Fourier domain)= Convolution (spatial domain)
- Based on Van Citter-Zernike theorem

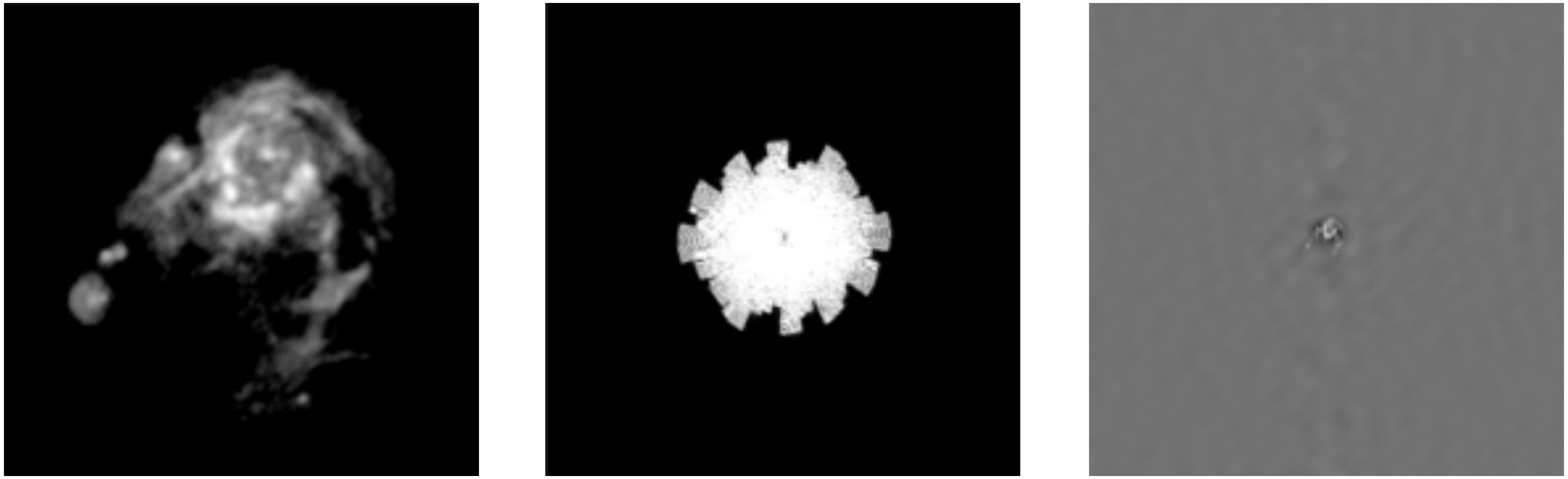
$$\mathrm{MFI} = \mathrm{V}$$

- Introduce a new image deconvolution method: Isotropic Undecimated Wavelet Transform (IUWT)-based CS method

$$\min \|\alpha\|_{l_1} \quad s.t.\ \|MFW^{-1}\alpha - V\|_{l_2} \leq \epsilon. \qquad \mathrm{WI} = \alpha$$

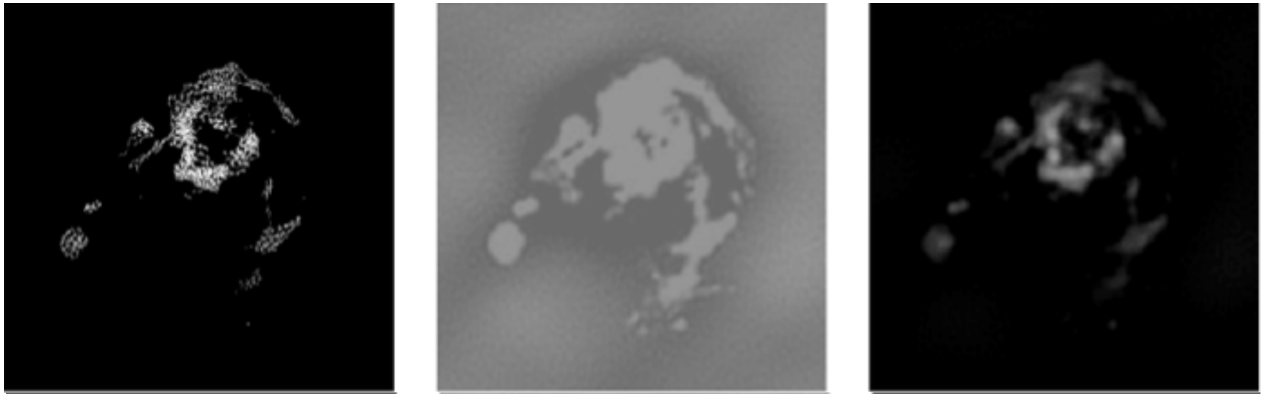
- Fast Iterative Shrinkage Thresholding Algorithm (FISTA) is used to solve the above problem
- Reweighted FISTA can improve the above results further!

Results

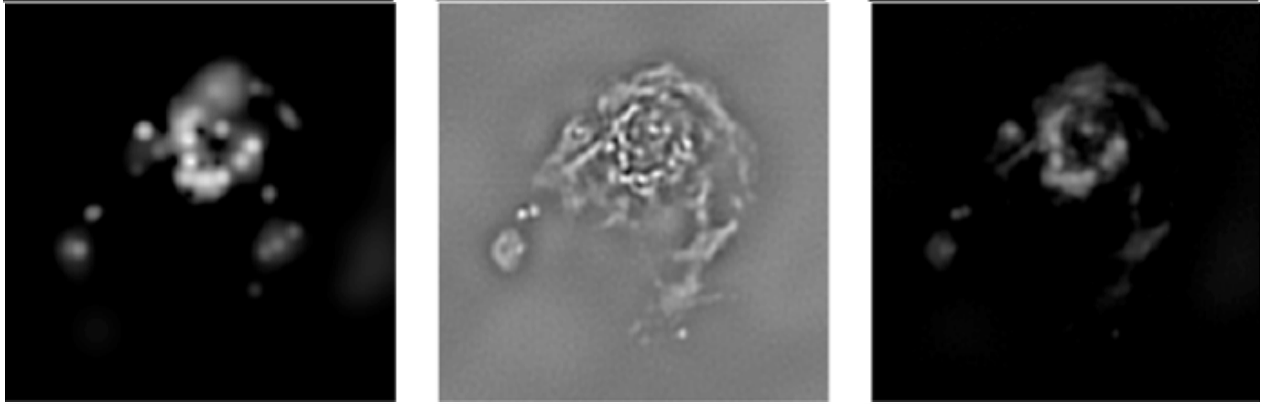


- Declination -45 degrees; Right ascension 12h30m00.00 (epoch J2000) image size 2048*2048;
- Frequency range: 700M-1GHz; 30 channels with 10MHz bandwidth; Integration time is 60 seconds; observing time is 1 hour. The system temperature is set to 50K in this test.

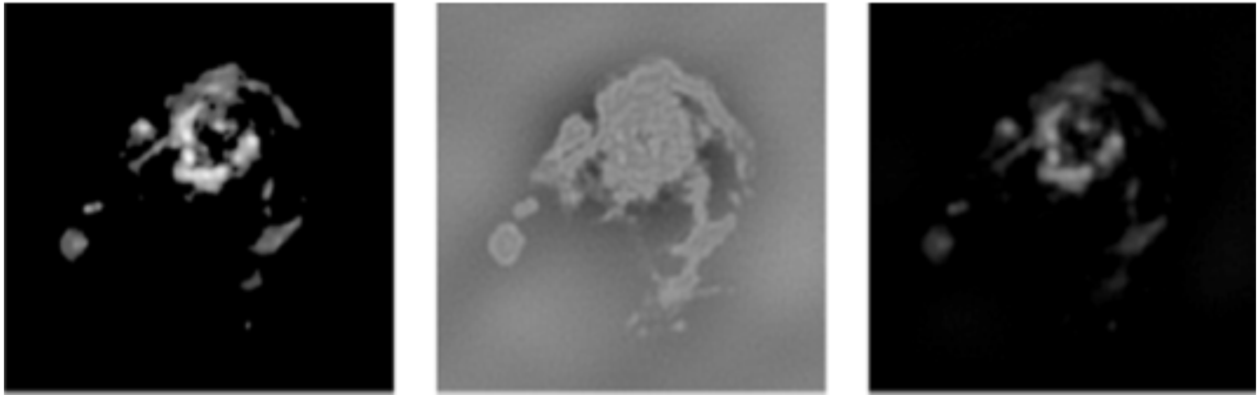
Results



Hogbom CLEAN

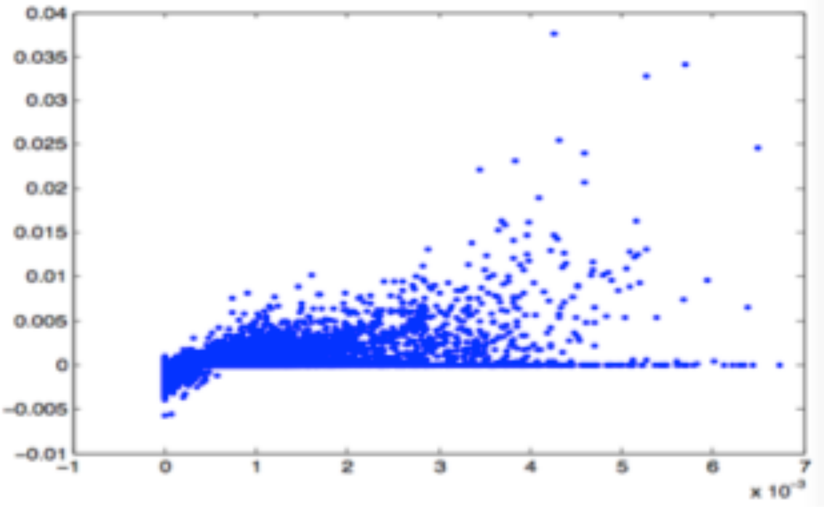


Multi-scale CLEAN

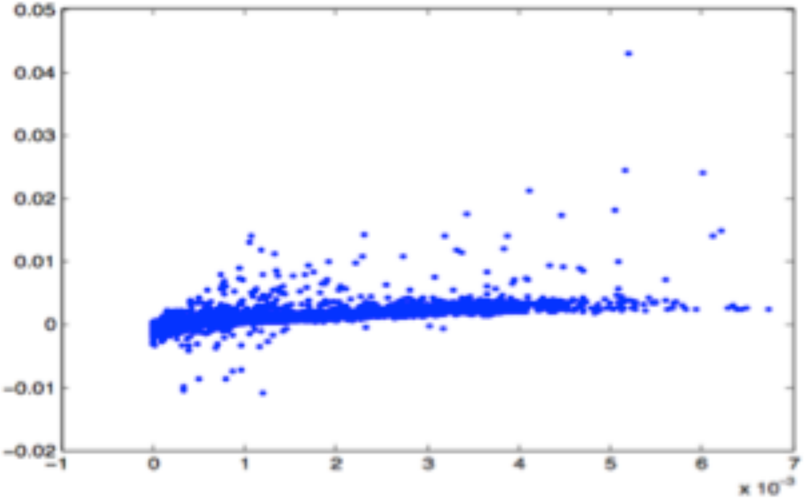


IUWT-based-CS

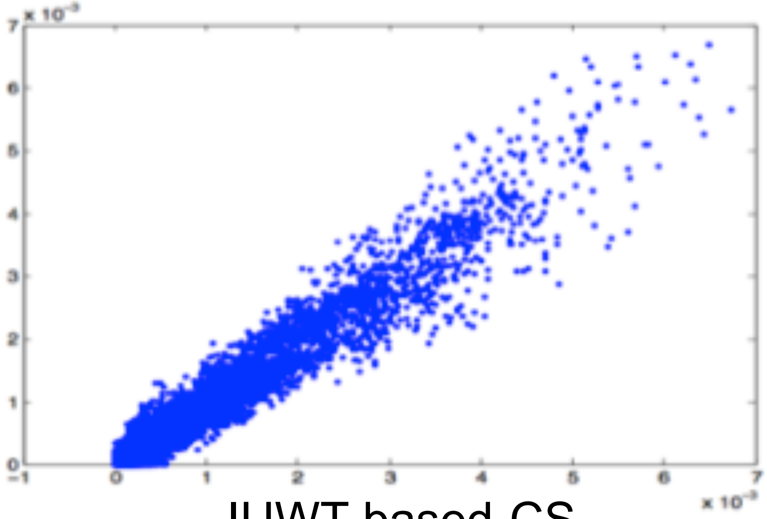
Results with the uniform weighting



Hogbom CLEAN

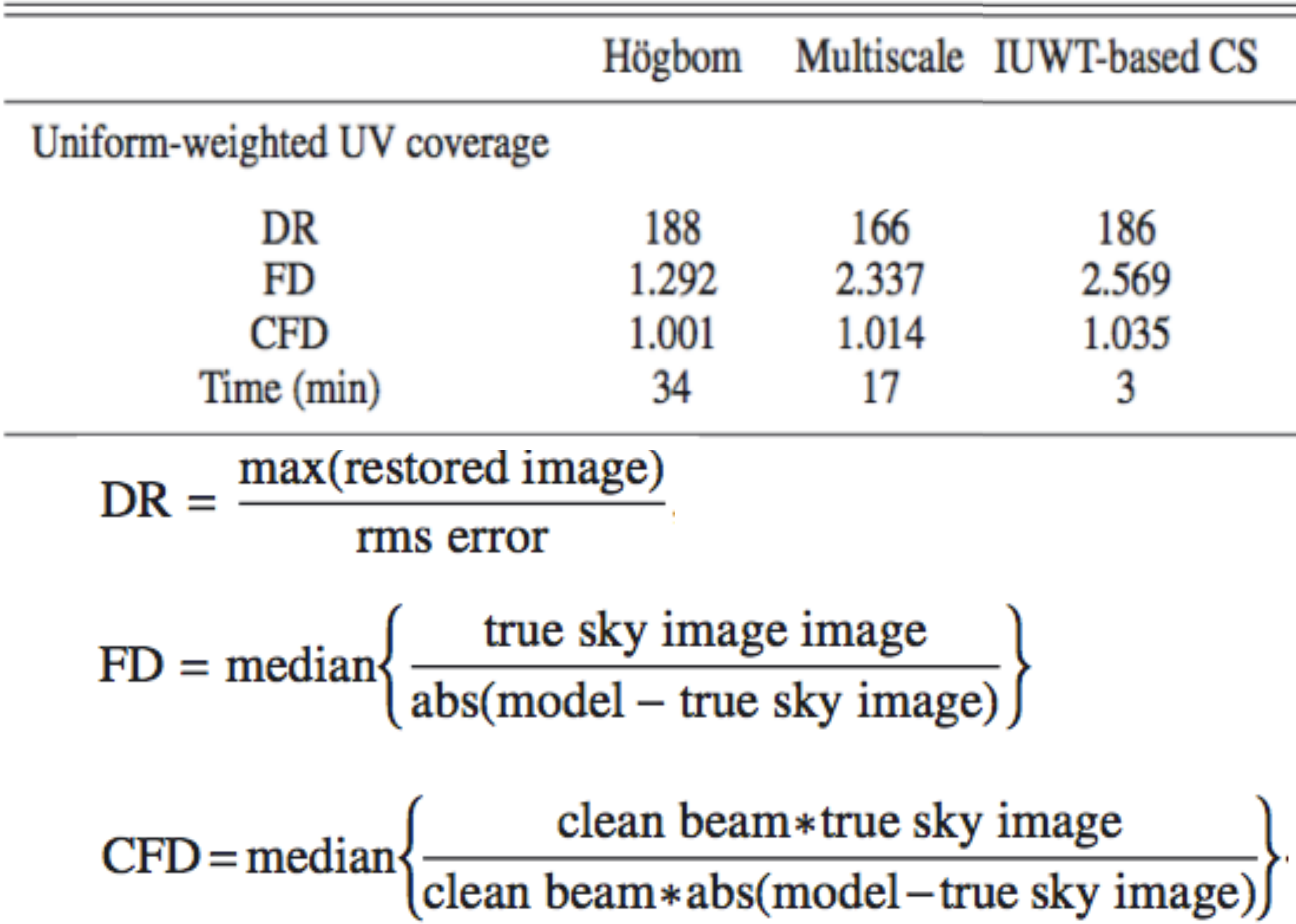


Multi-scale CLEAN



IUWT-based-CS

Numerical comparison results



	Högbom	Multiscale	IUWT-based CS
Uniform-weighted UV coverage			
DR	188	166	186
FD	1.292	2.337	2.569
CFD	1.001	1.014	1.035
Time (min)	34	17	3

$$\mathrm{DR} = \frac{\max(\text{restored image})}{\text{rms error}},$$

$$\mathrm{FD} = \mathrm{median}\left\{\frac{\text{true sky image image}}{\mathrm{abs}(\text{model} - \text{true sky image})}\right\}$$

$$\mathrm{CFD} = \mathrm{median}\left\{\frac{\text{clean beam} * \text{true sky image}}{\text{clean beam} * \mathrm{abs}(\text{model} - \text{true sky image})}\right\}.$$

Outline

- Two investigated applications of compressive sensing/ sampling (CS) to radio astronomy:
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  - Faraday Rotation Measure Synthesis
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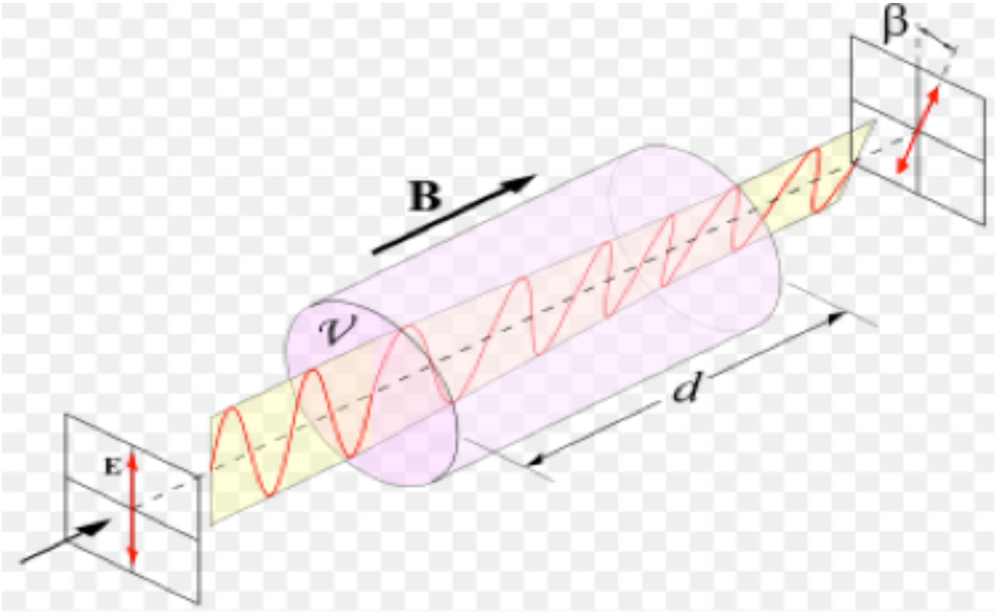
The applications of CS to radio astronomy
---Faraday rotation measure synthesis

- A physical phenomenon:

  The angle of linear polarization radiation which propagates through magnetic fields is rotated as a function of frequency



From Wikipedia

- Usage:

  To study magnetic fields of galaxies

Faraday rotation measure synthesis

- Faraday dispersion function F(φ) , which describes the intrinsic polarized flux per unit Faraday depth φ, and its relationship with the complex polarized emission P(λ^2) as follows:

$$P(\lambda^2) = \int_{-\infty}^{\infty} F(\phi)\mathrm{e}^{2i\phi\lambda^2}\,\mathrm{d}\phi$$

where λ is the wavelength. Note that, P can also be written as P = Q + iU, where Q and U represent the emission of Stokes Q and U, respectively

$$\phi(r) = 0.81 \int_{source}^{observor} n_e B dr$$

where B is the magnetic field strength in micro-Gauss; n_e is the electron density; r is the path length in parsecs

Faraday rotation measure synthesis

- Obviously, to reconstruct the Faraday dispersion function

$$F(\phi) = \frac{1}{\pi} \int_{-\infty}^{\infty} P(\lambda^2) \mathrm{e}^{-2\mathrm{i}\phi\lambda^2} \, \mathrm{d}\lambda^2$$

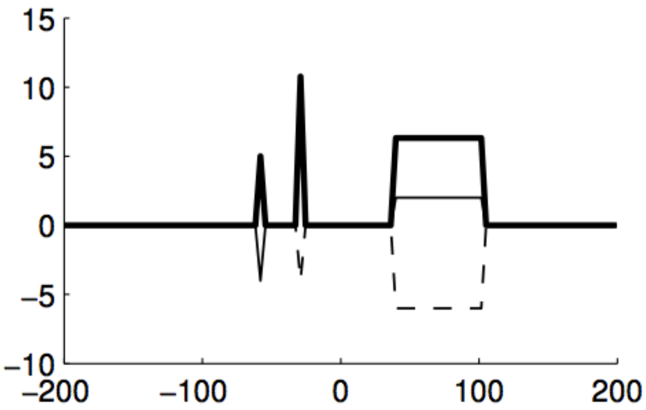
- Problems:
  - Can not observe at wavelengths when $\lambda^2 < 0$
  - Nor do we observe at all wavelength when $\lambda^2 > 0$

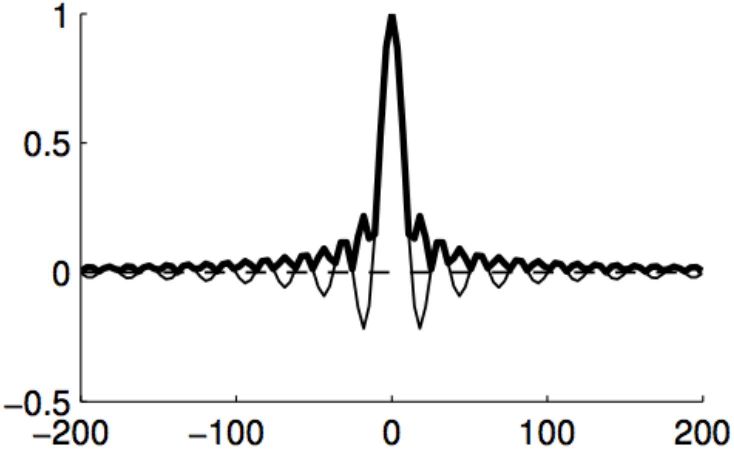
- Expression in equation:

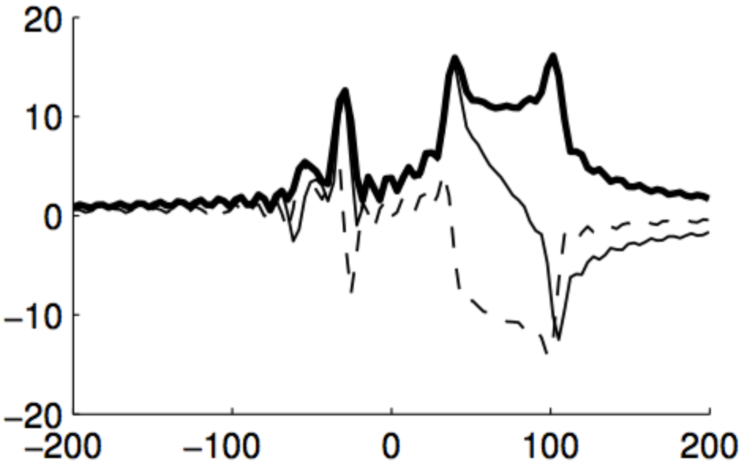
$$Y f = \tilde{p}$$

$$Y(j, N/2 + k) = \mathrm{e}^{2\mathrm{i}\phi_k \lambda_j^2}, j = 1, \cdots, m; k = 1 - N/2, \cdots, N/2.$$

Example


15
10
5
0
–5
–10
–200
–100
0
100
200


1
0.5
0
–0.5
–200
–100
0
100
200


20
10
0
–10
–20
–200
–100
0
100
200

Our solutions

- Faraday thin sources: CS-RM-Thin

$$\min \{\|\mathrm{Re}(\mathbf{f})\|_{l_1} + \|\mathrm{Im}(\mathbf{f})\|_{l_1}\} \quad s.t.\ \mathbf{Y}\mathbf{f} = \widetilde{\mathbf{p}},$$

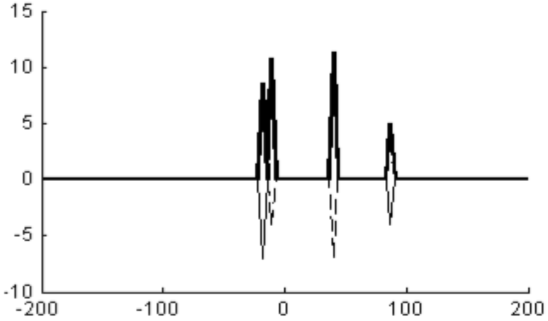
- Faraday thick sources: CS-RM-Thick

$$\min \{\|\mathbf{W} \cdot \mathrm{Re}(\mathbf{f})\|_{l_1} + \|\mathbf{W} \cdot \mathrm{Im}(\mathbf{f})\|_{l_1}\} \quad s.t.\ \mathbf{Y}\mathbf{f} = \widetilde{\mathbf{p}}$$

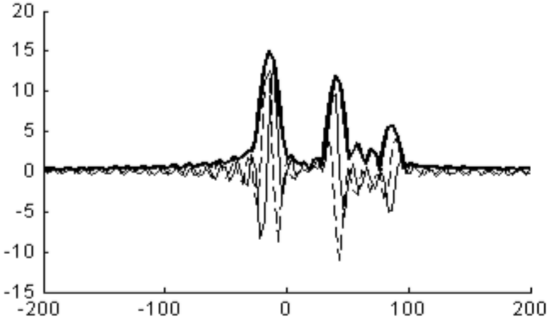
- Faraday mixed sources: CS-RM-Mix

$$\begin{aligned}\min \{&\|\mathrm{Re}(\mathbf{f}_{\mathrm{thin}})\|_{l_1} + \|\mathrm{Im}(\mathbf{f}_{\mathrm{thin}})\|_{l_1} + \|\mathbf{W} \cdot \mathrm{Re}(\mathbf{f}_{\mathrm{thick}})\|_{l_1} \\ &+\|\mathbf{W} \cdot \mathrm{Im}(\mathbf{f}_{\mathrm{thick}})\|_{l_1}\} \quad s.t.\ \mathbf{Y}\mathbf{f} = \widetilde{\mathbf{p}},\end{aligned}$$

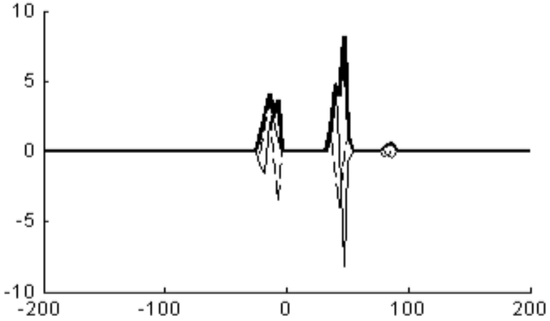
Results for Faraday Thin sources



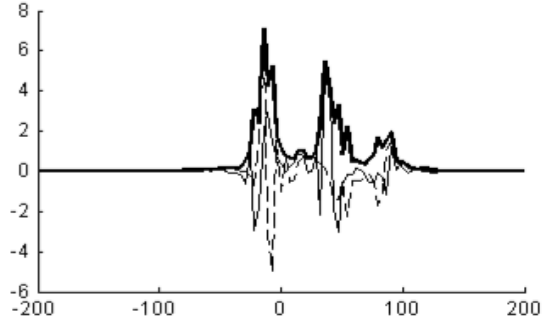
Original



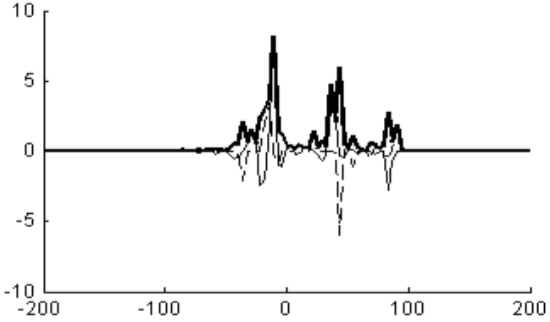
Dirty curve



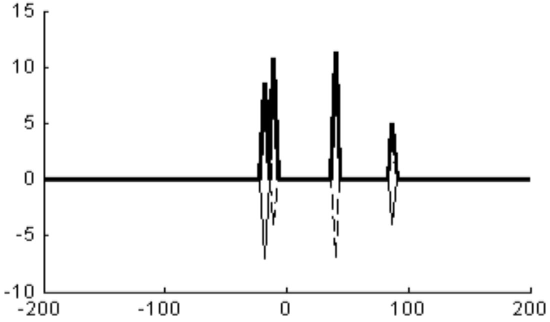
RM-CLEAN



CS-RM-Thick

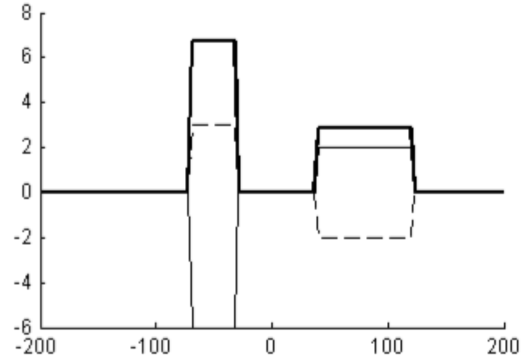


CS-RM-Mix

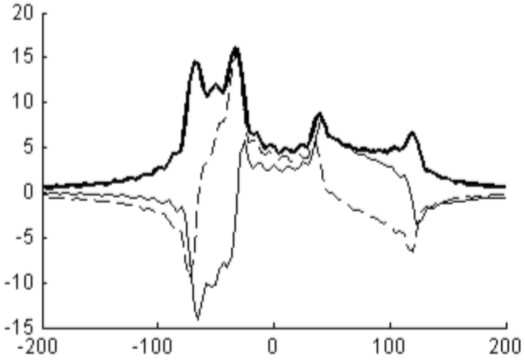


CS-RM-Thin

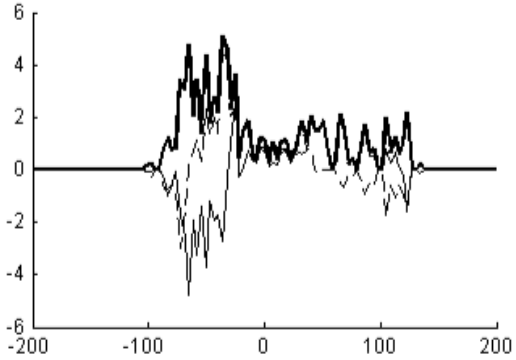
Results for Faraday thick sources



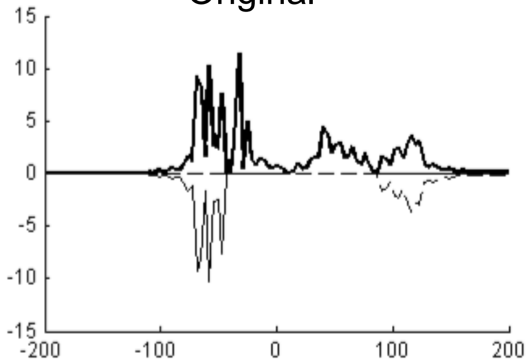
Original



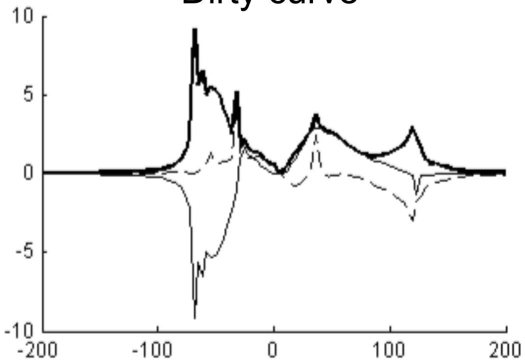
Dirty curve



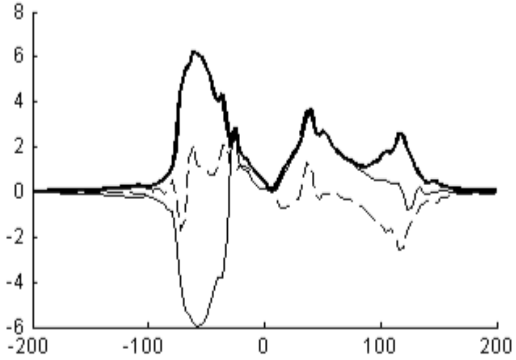
RM-CLEAN



CS-RM-Thin

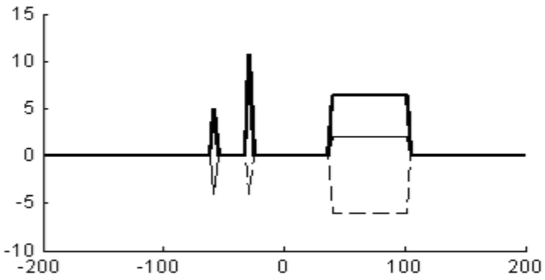


CS-RM-Mix



CS-RM-Thick

Results for Faraday mixed sources



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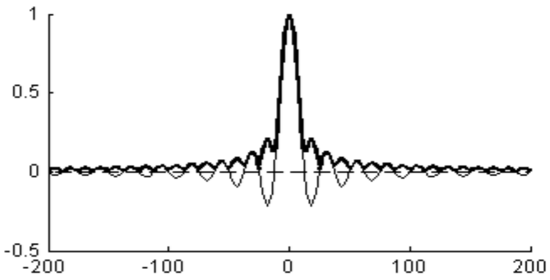
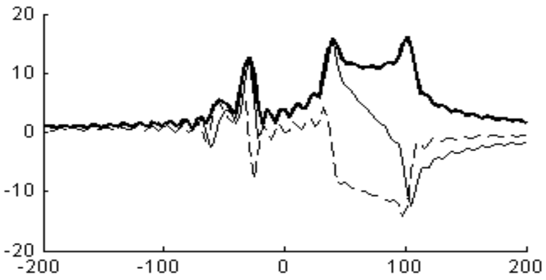
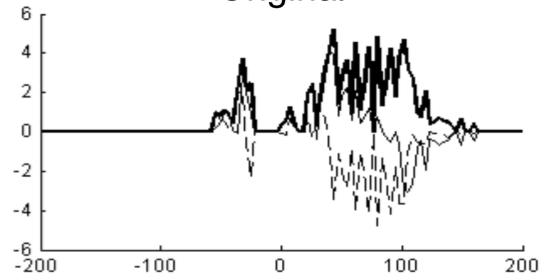


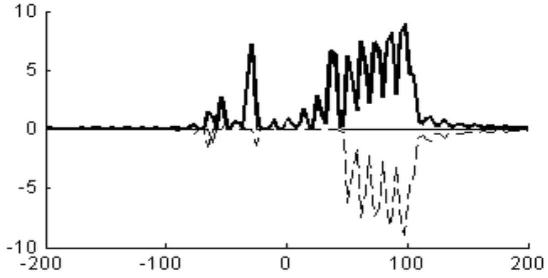
RMSF



Dirty curve



RM-CLEAN



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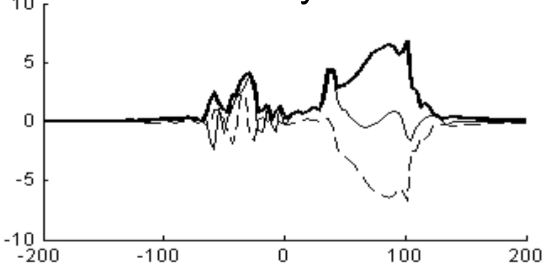
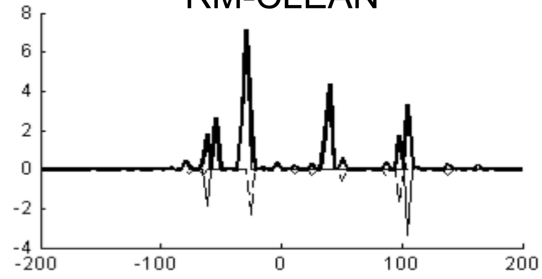
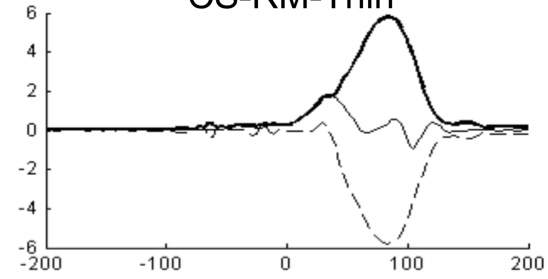


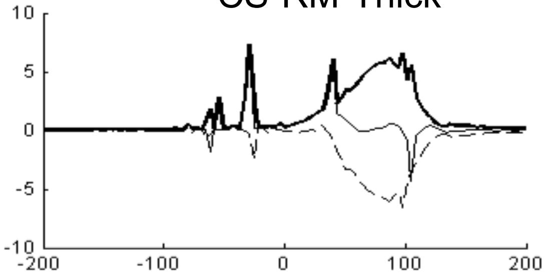
CS-RM-Thick



Thin structures, CS-RM-Mix



Thick structures, CS-RM-Mix



CS-RM-Mix

Miscellaneous

- Acknowledgement to Jean-Luc Starck, Tim Cornwell and Frank de Hoog
- Publications
  - Feng Li, Tim Cornwell and Frank de Hoog, “The Application of Compressive Sampling to Radio Astronomy

    I: Deconvolution”, ***Astronomy & Astrophysics,*** *Volume* 528, 2011

  - Feng Li, Shea Brown, Tim Cornwell and Frank de Hoog, “The Application of Compressive Sampling to Radio Astronomy

    II: Faraday Rotation Measure Synthesis”, Accepted to ***Astronomy & Astrophysics***
- Code can be found at http://code.google.com/p/csra

Conclusion

- Applying the CS reconstruction step to solve ill-conditioned problems in radio astronomy
- Two investigated applications to radio astronomy:
  - For image deconvolution given a radio telescope array
  - For Faraday rotation measure synthesis
- Detailed comparison can be found in our papers

Questions?

Thank you

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