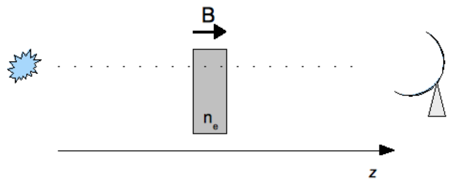
Compressed Sensing for Rotation Measure Synthesis

Anna Scaife

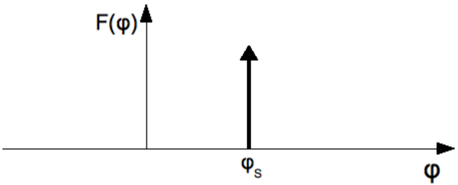
BASP 2011, Villars-sur-Ollon

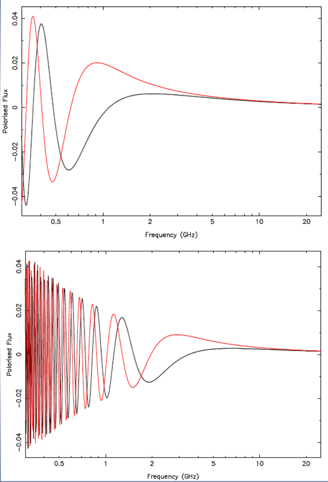
Faraday Rotation



$$\phi = 0.81 \int_{\mathrm{pc}} \frac{n_e}{\mathrm{cm}^{-3}} \frac{B_{||}}{\mu\mathrm{G}} \mathrm{d}z \,\mathrm{rad\,m}^{-2}$$

For an external screen: $\phi = RM$





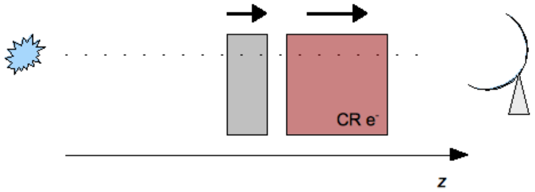
Faraday Depth

In general:

$$\phi \neq RM, \quad RM = \frac{\mathrm{d}\chi(\lambda^2)}{\mathrm{d}\lambda^2} \text{ where } \chi = \frac{1}{2}\tan^{-1}\frac{U}{Q}.$$

For the single source:

$$\chi(\lambda^2) = \chi_0 + \phi\lambda^2, \text{ therefore } \frac{\mathrm{d}\chi(\lambda^2)}{\mathrm{d}\lambda^2} = \phi = RM.$$



Multiple Faraday structures:

$$P(\lambda^2) = \int F(\phi) e^{2i\phi\lambda^2} \mathrm{d}\phi \text{ (Burn 1966)}$$

That single source again:

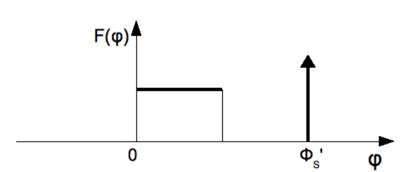
$$F(\phi) = \delta(\phi - \phi_0) \rightarrow P(\lambda^2) = e^{2i\phi_0\lambda^2} = \cos(2\phi_0\lambda^2) + i\sin(2\phi_0\lambda^2) = Q + iU$$

RM Synthesis

The Faraday dispersion function is a Fourier relationship:

$$P(\lambda^2) = \int F(\phi) e^{2i\phi\lambda^2} d\phi \quad \text{(Burn 1966)}$$

$$F(\phi) = \int P(\lambda^2) e^{-2i\phi\lambda^2} d\phi$$



Similarly to the relationship between the *uv* and image planes in aperture synthesis it is not fully sampled:

$$P(\tilde{\lambda}^2) = W(\lambda^2) P(\lambda^2)$$

We get a response function similar to that of a PSF:

$$RMSF(\phi) = \frac{\int_{-\infty}^{\infty} W(\lambda^2) e^{-2i\phi\lambda^2} d\lambda^2}{\int_{-\infty}^{\infty} W(\lambda^2) d\lambda^2}$$

Brentjens & de Bruyn 2005

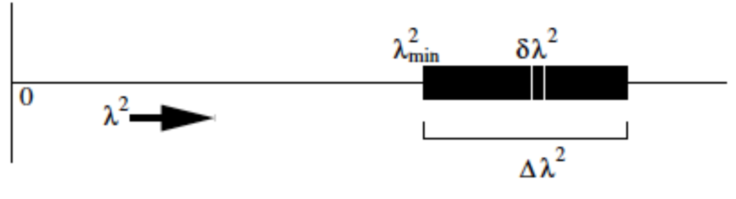
RM Synthesis

Resolution is a function of coverage in λ^2:

$$\delta\phi \approx \frac{2\sqrt{3}}{\Delta\lambda^2}$$

Sensitivity to maximum scale in ϕ is a function of resolution in λ^2:

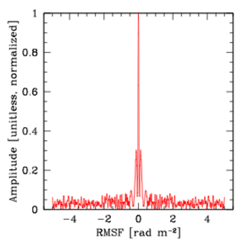
$$||\phi_{\max}|| \approx \frac{\sqrt{3}}{\delta\lambda^2}$$

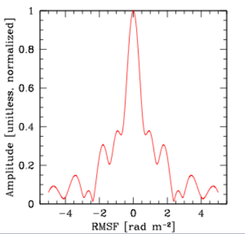


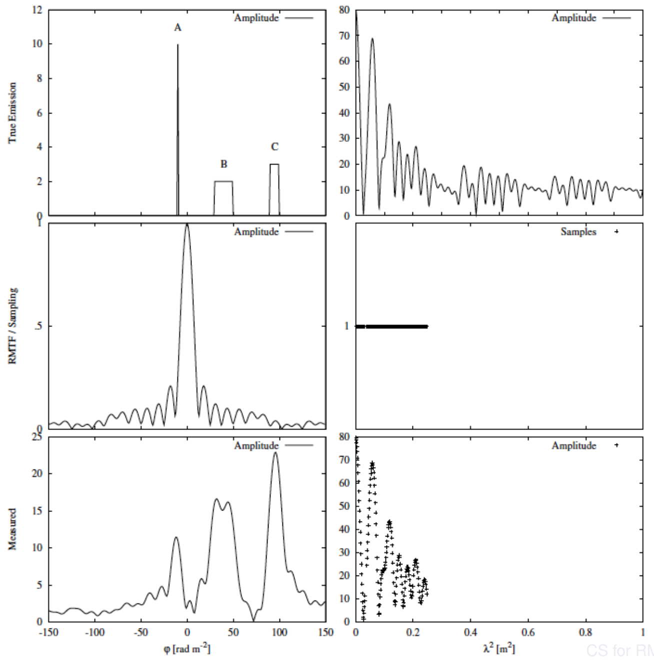
RM Synthesis

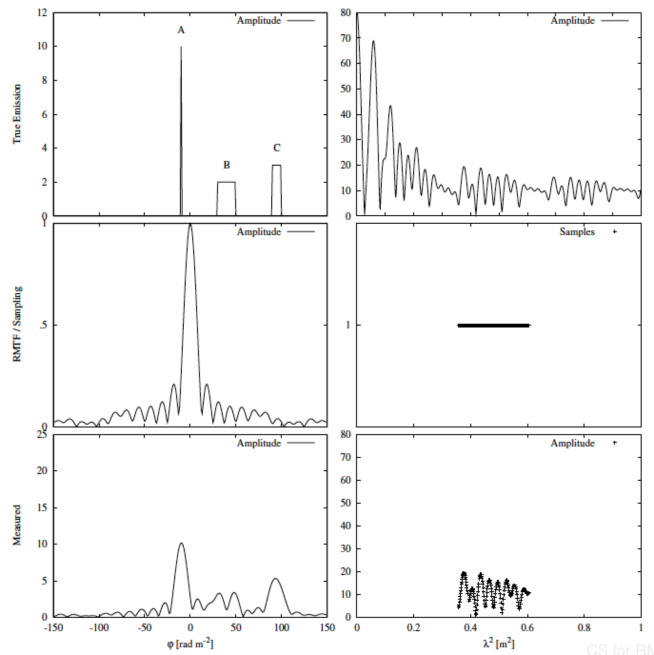
- RMSF from 30-50 MHz + 60-80 MHz: $\delta\phi = 0.05\,\mathrm{rad\,m^{-2}}$, $\phi_{\max} = 19\,\mathrm{rad\,m^{-2}}$
- RMSF from 120-150 MHz + 180-210 MHz: $\delta\phi = 1.0\,\mathrm{rad\,m^{-2}}$, $\phi_{\max} = 1200\,\mathrm{rad\,m^{-2}}$

Heald 2009

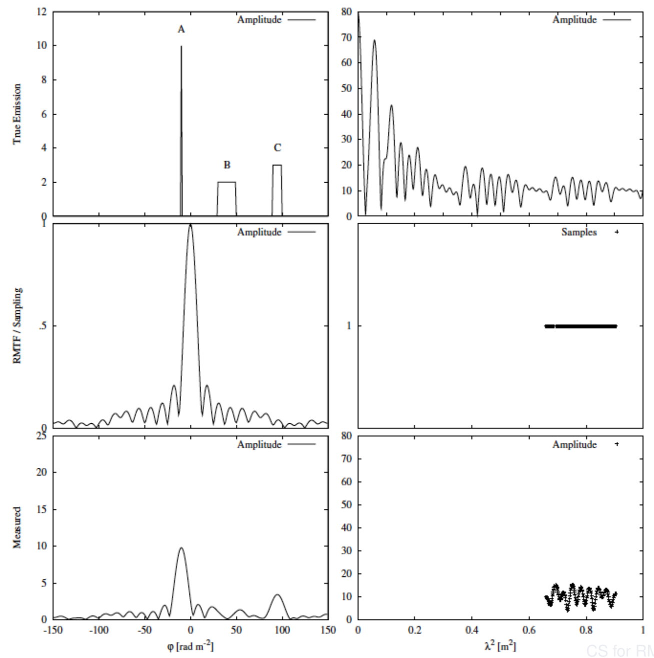





True Emission
RMTF / Sampling
Measured
Amplitude
Samples
A
B
C
φ [rad m^-2]
λ^2 [m^2]

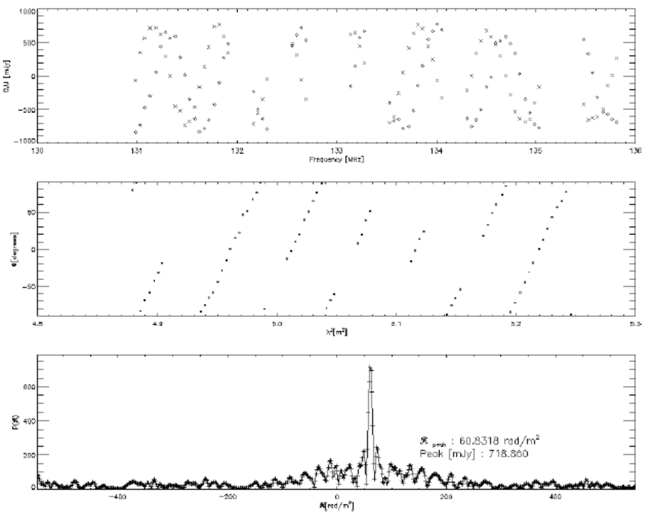


True Emission
RMTF / Sampling
Measured
Amplitude
Samples
A
B
C
φ [rad m^{-2}]
λ^2 [m^2]

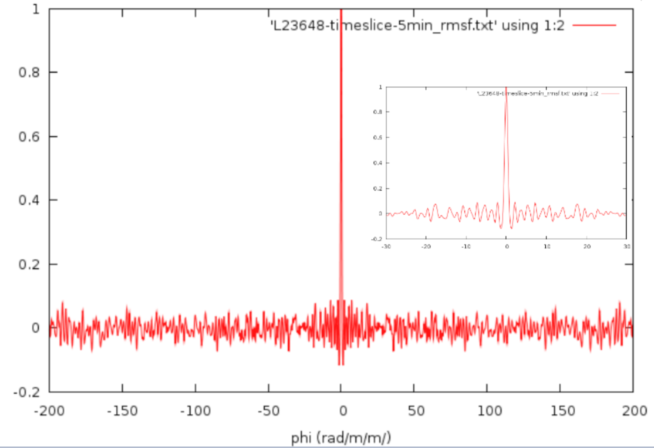


Amplitude
A
B
C
True Emission
RMTF / Sampling
Samples
Measured
φ [rad m^-2]
λ^2 [m^2]

LOFAR Early Results

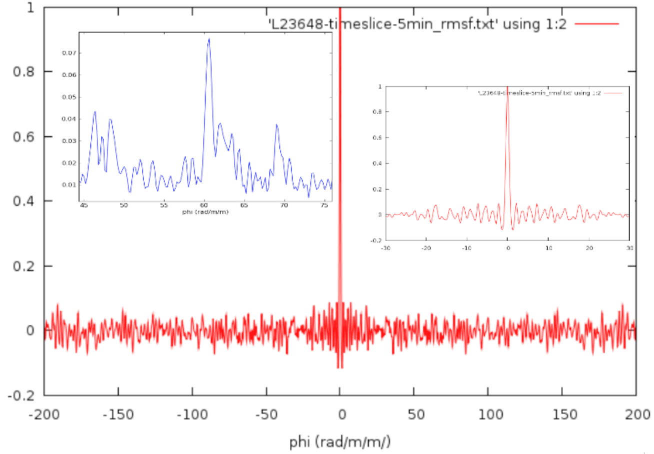


LOFAR Early Results



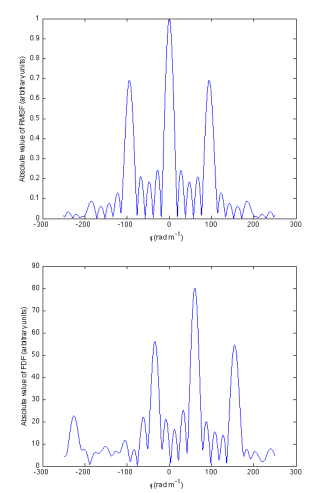
Andreas Horneffer

LOFAR Early Results



Andreas Horneffer

RM Clean

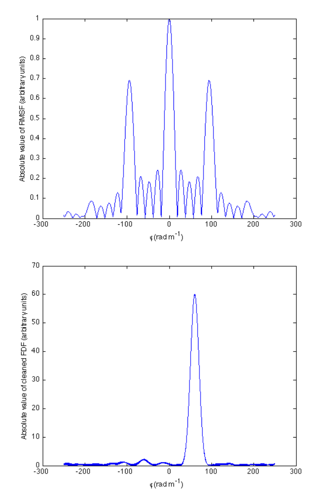


RM Clean (Heald 2009)

Works in the same way as standard CLEAN

Iterative subtraction of a δ-fnc scaled by a loop gain factor.

RM Clean

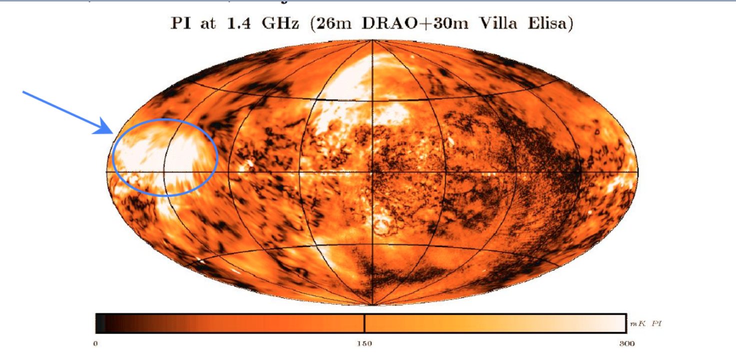


RM Clean (Heald 2009)

Works in the same way as standard CLEAN

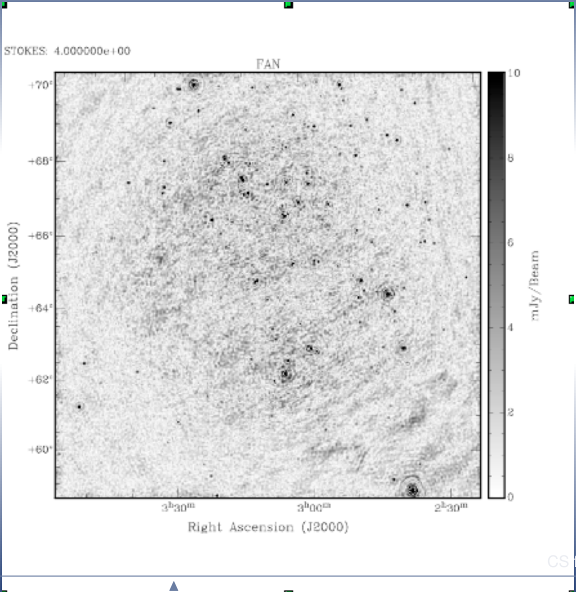
Iterative subtraction of a δ-fnc scaled by a loop gain factor.

Fan region

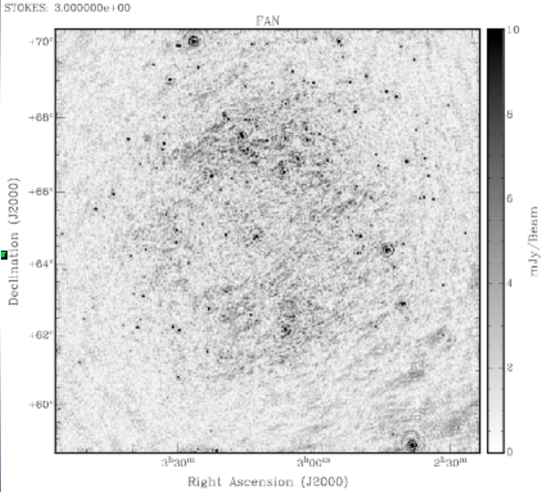


Marijke Haverkorn

Fan region


STOKES: 4.000000e+00
FAN
+70°
+68°
+66°
+64°
+62°
+60°
Declination (J2000)
3h30m
3h00m
2h30m
Right Ascension (J2000)
10
8
6
4
2
0
mJy/Beam

Fan region



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+64°
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Declination (J2000)
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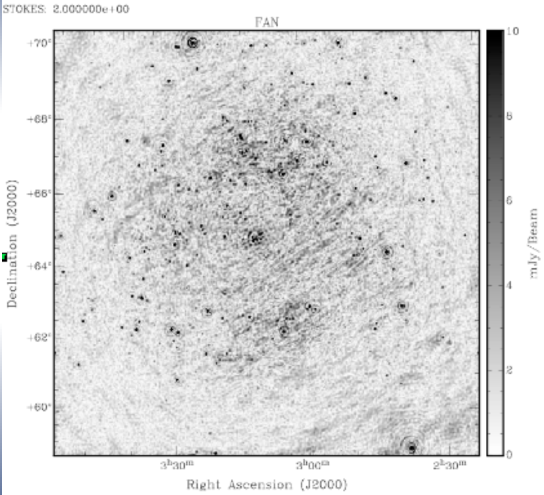
2h30m
Right Ascension (J2000)
10
8
6
4
2
0
mJy/Beam

Fan region



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+64°
+62°
+60°
Declination (J2000)
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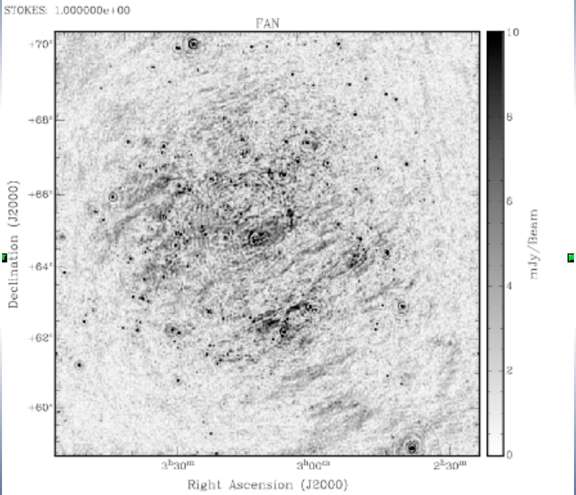
3h00m
2h30m
Right Ascension (J2000)
10
8
6
4
2
0
mJy/Beam

Fan region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Declination (J2000)
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2h30m
Right Ascension (J2000)
10
8
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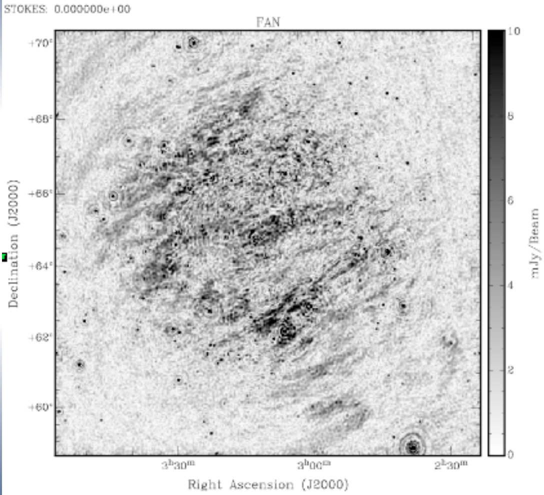
mJy/Beam

Fan region



STOKES: 0.000000e+00
FAN
+70°
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Declination (J2000)
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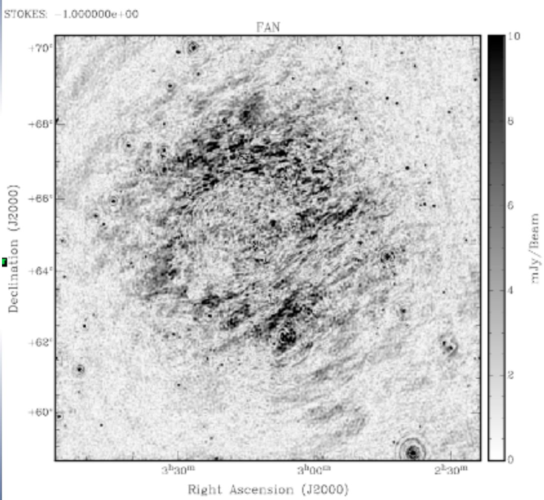

STOKES: 1.000000e+00
FAN
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+68°
+66°
+64°
+62°
+60°
Declination (J2000)
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3h00m
2h30m
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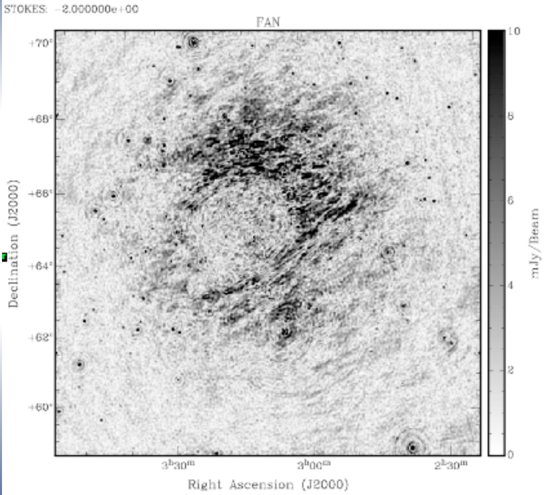
Fan region



STOKES: -2.000000e+00
FAN
+70°
+68°
+66°
+64°
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+60°
Declination (J2000)
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3^h00^m
2^h30^m
Right Ascension (J2000)
10
8
6
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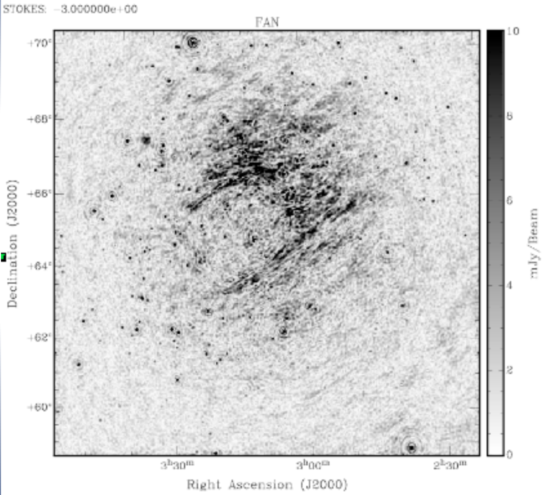
mJy/Beam

Fan region



STOKES: -3.000000e+00
FAN
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+62°
+60°
Declination (J2000)
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3^h00^m
2^h30^m
Right Ascension (J2000)
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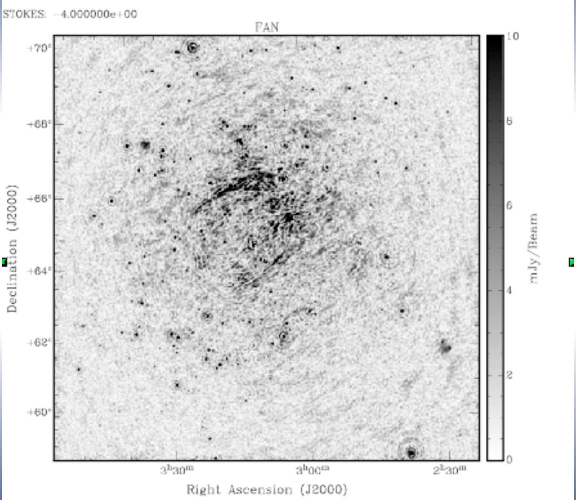
Fan region


STOKES: −4.000000e+00
FAN
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+64°
+62°
+60°
Declination (J2000)
3h30m
3h00m
2h30m
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10
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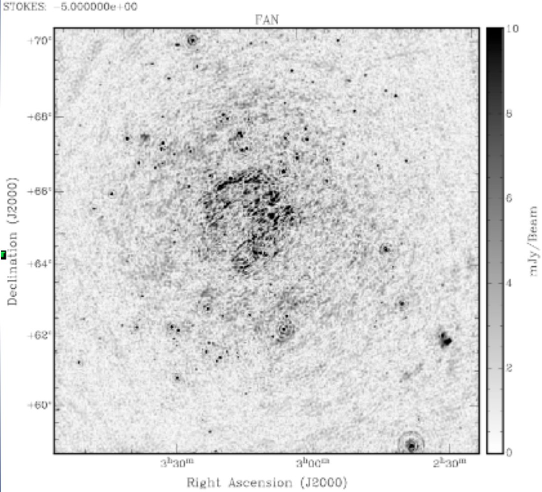
Fan region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FAN
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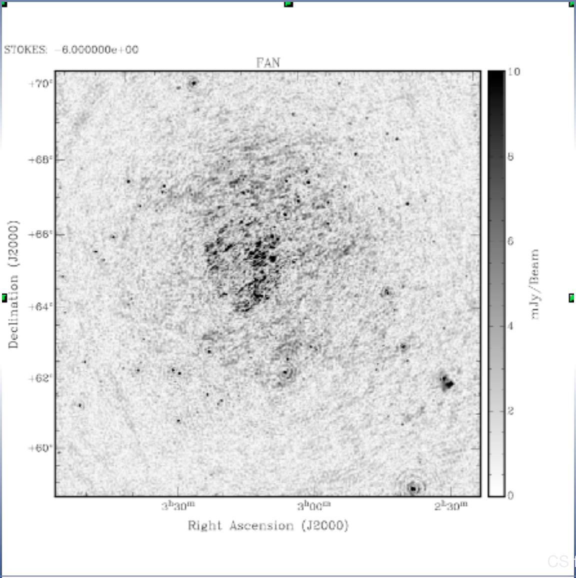
+68°
+66°
+64°
+62°
+60°
Declination (J2000)
3^h30^m
3^h00^m
2^h30^m
Right Ascension (J2000)
10
8
6
4
2
0
mJy/Beam

Fan region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+62°
+60°
Declination (J2000)
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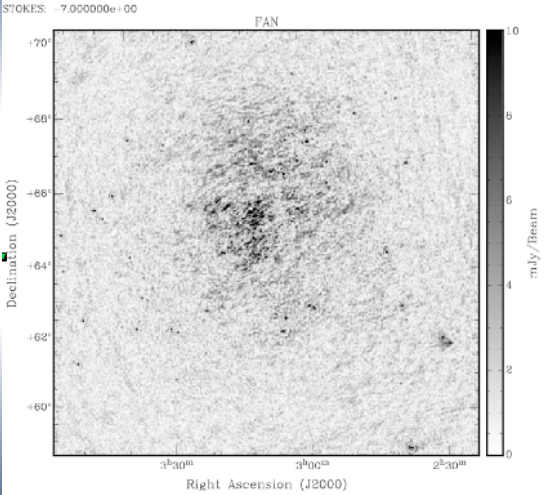
3h00m
2h30m
Right Ascension (J2000)
10
8
6
4
2
0
mJy/Beam

Fan region



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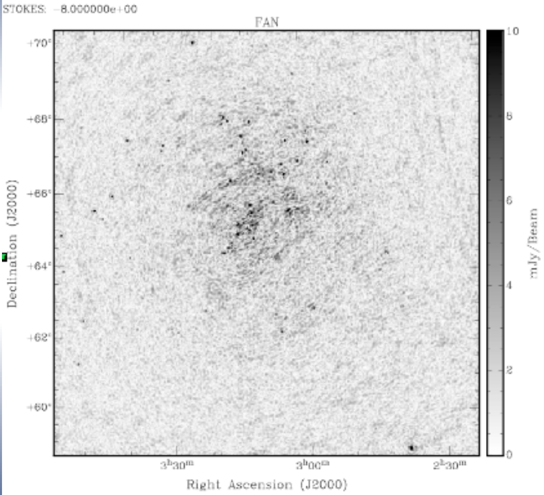
FAN
Declination (J2000)
+70°
+68°
+66°
+64°
+62°
+60°
Right Ascension (J2000)
3h30m
3h00m
2h30m
mJy/Beam
10
8
6
4
2
0

Fan region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+62°
+60°
Declination (J2000)
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3h00m
2h30m
Right Ascension (J2000)
10
8
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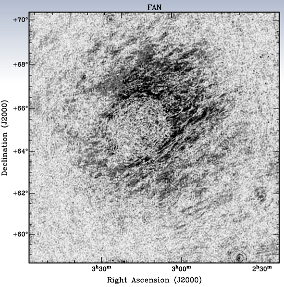
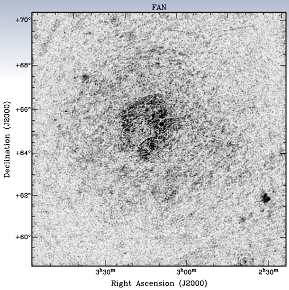
2
0
mJy/Beam

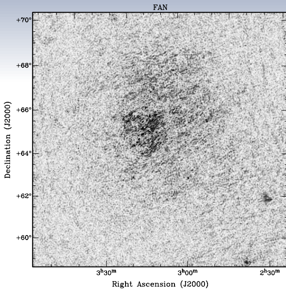
Extended emission (Fan region)



RM=2 rad m^{-2}



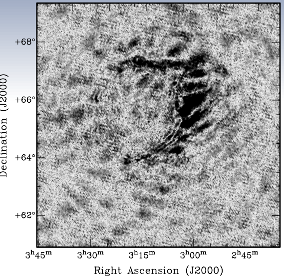
RM=5 rad m^{-2}



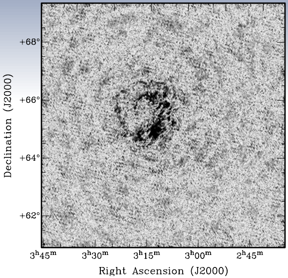
RM=7 rad m^{-2}

WSRT

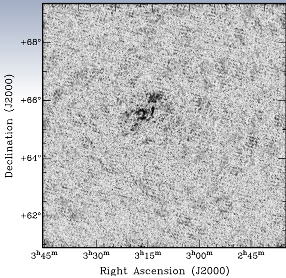
Extended emission (Fan region)



RM=-2 rad m^{-2}



RM=-5 rad m^{-2}



RM=-7 rad m^{-2}

LOFAR: Marco Iacobelli & Marijke Haverkorn

RM Synthesis

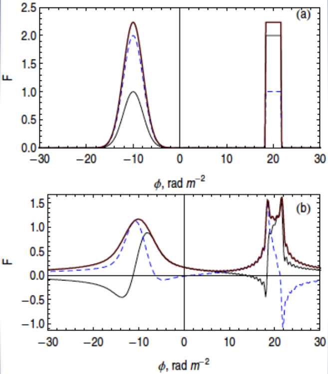
Faraday spectra are **complex**: the modulus defines the emission and the phase the PA

$$P(\lambda^2) = \int \epsilon(z) \mathrm{e}^{2i\chi(z)} \mathrm{e}^{2i\phi(z)\lambda^2} \mathrm{d}z$$

$$F(\phi) = \epsilon(\phi) \mathrm{e}^{2i\chi(\phi)} \left(\frac{\mathrm{d}\phi}{\mathrm{d}z}\right)^{-1}$$

Standard RM Synthesis does not recover the complex components as there is no information at $\lambda^2 < 0$

Requires a degree of inference about the underlying signal distribution

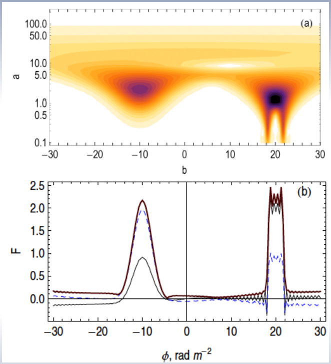


Frick et al. 2010

RM Synthesis

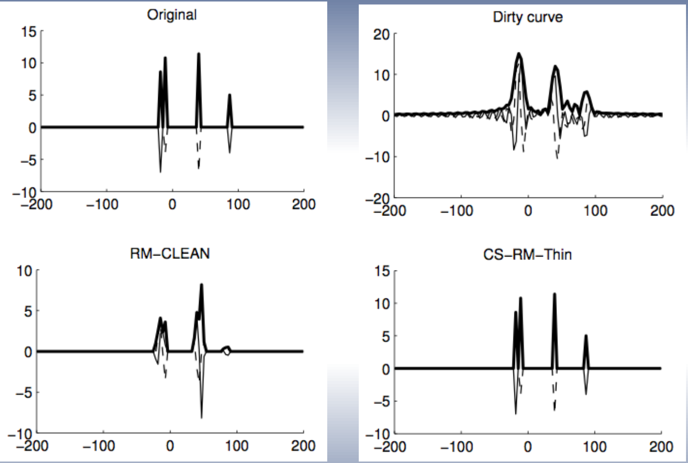
Wavelet based RM Synthesis can recover real and imaginary parts of $F(\phi)$ more accurately

Requires a degree of inference about the underlying signal distribution $\rightarrow$ symmetry of dispersion function



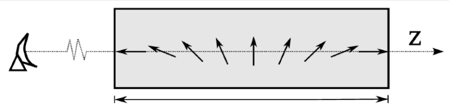
Frick et al. 2010

CS for Faraday Thin Sources



Li et al. 2011

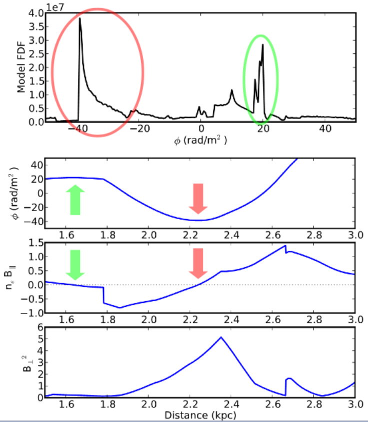
Faraday Caustics



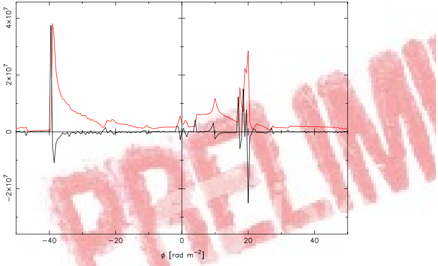
(Bell, Enßlin & Junklewitz 2011)

Caused by reversals of the B-field along the l.o.s.

Leads to Heaviside functions in the Faraday dispersion spectrum

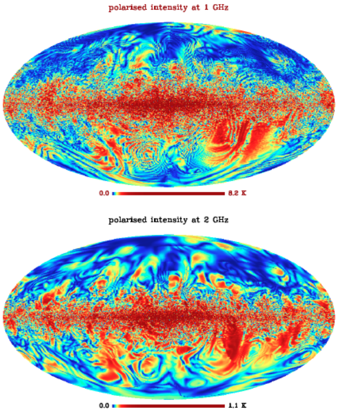


Sparsity of Faraday Caustics



TV norm (TV_ϵ):

$$\min ||x||_{\mathrm{TV}} \quad \text{subject to} \quad \chi^2 \leq \epsilon^2$$



(Waelkens+ 2009)

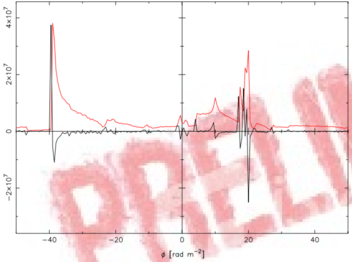
Sparsity of Faraday Caustics

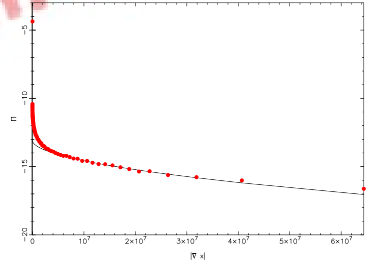
Statistical TV norm (STV$_\epsilon$):

$$\Pi(\nabla x) \propto \mathrm{e}^{-\left|\frac{x}{b}\right|^{\alpha}}$$

$$\min ||x||_{\mathrm{TV}}^{\alpha} \quad \text{subject to} \quad \chi^2 \leq \epsilon^2$$

Hammurabi simulations (Waelkens et al. 2009): $\alpha = 0.624 \pm 0.013$



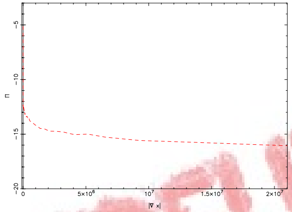


TV norm (TV$_\epsilon$):

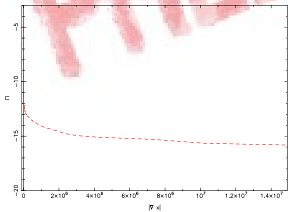
$$\min ||x||_{\mathrm{TV}} \quad \text{subject to} \quad \chi^2 \leq \epsilon^2$$

Variation with Galactic Co-ords

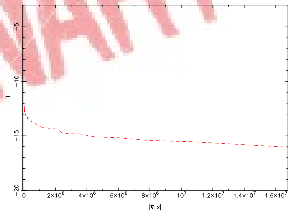
$b = 40^\circ$



$b = 130^\circ$



$\ell = 0^\circ$



The exact form of the GGD is a function of Galactic latitude & longitude

Conclusions

- The RM-Synthesis problem is similar in some respects to the Aperture Synthesis problem
- Differences in the underlying signal cause necessary variations of approach - prior knowledge is required for RM-Synthesis
- Sparsity can be used as a prior in a number of circumstances
    - Point-like objects in FD
    - *Faraday Caustics*
- Basis Pursuit approaches should be possible