University of Zurich[UZH]

ETH Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Accelerating Dynamic MRI

Uniform and random undersampling

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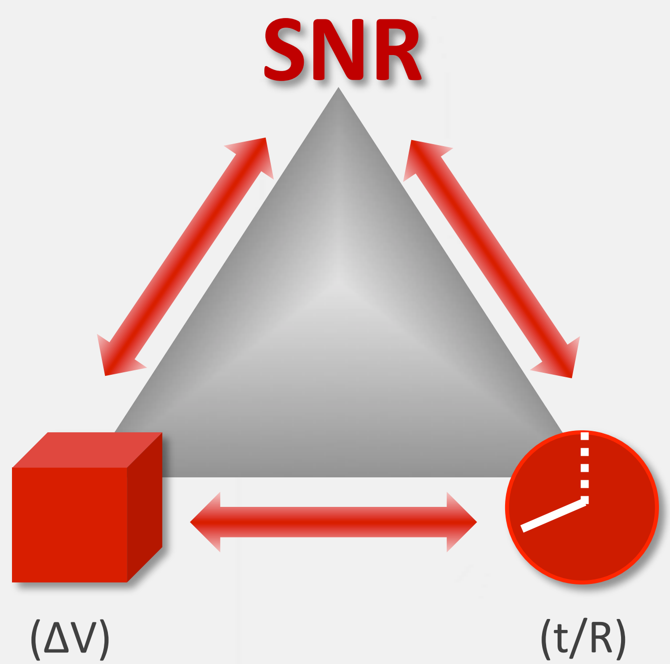
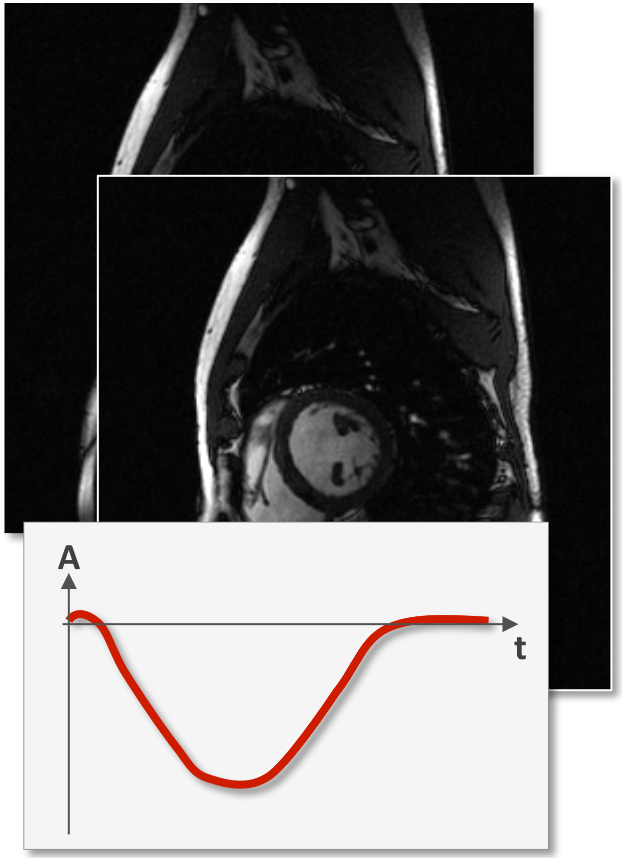
SNR – Speed



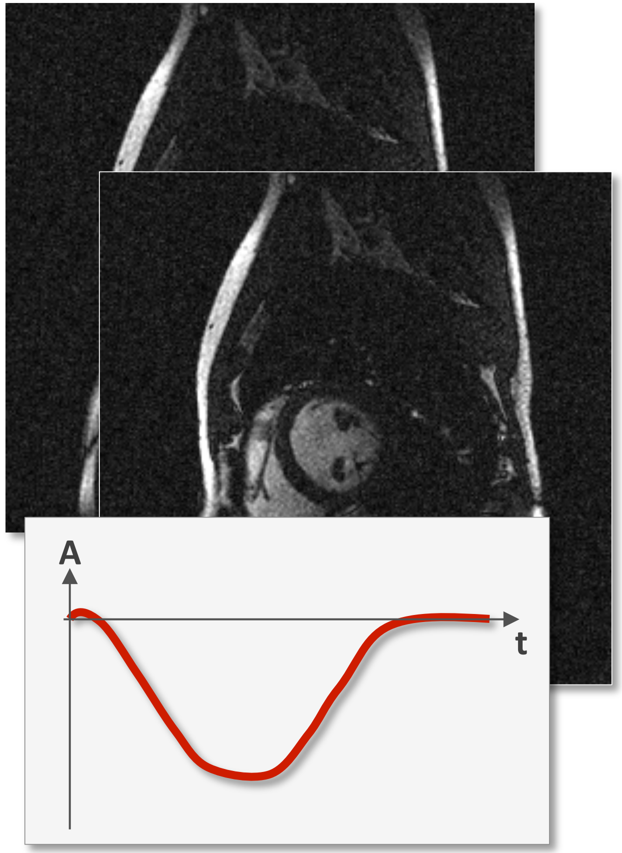
$$SNR \propto \Delta V \sqrt{t} / \sqrt{R}$$

SNR – Speed

SNR = 32



SNR = 8



Reconstruction

Encoding: $$\vec{d} = E\,\vec{m} + \vec{\eta}$$

Decoding: $$\vec{i} = F\,\vec{d}$$

	MR Signal		Noise
Data	Encoding matrix	E	Ψ
Image	Depiction	FE	$F\Psi F^H$
		↓ Identity	↓ Minimum

Reconstruction formulas

Constrain

$$FE = Id$$

Solutions

Pseudo-inverse: $F = (E^H E)^{-1} E^H$

Optimum SNR inverse: $F = (E^H \Psi^{-1} E)^{-1} E^H \Psi^{-1}$

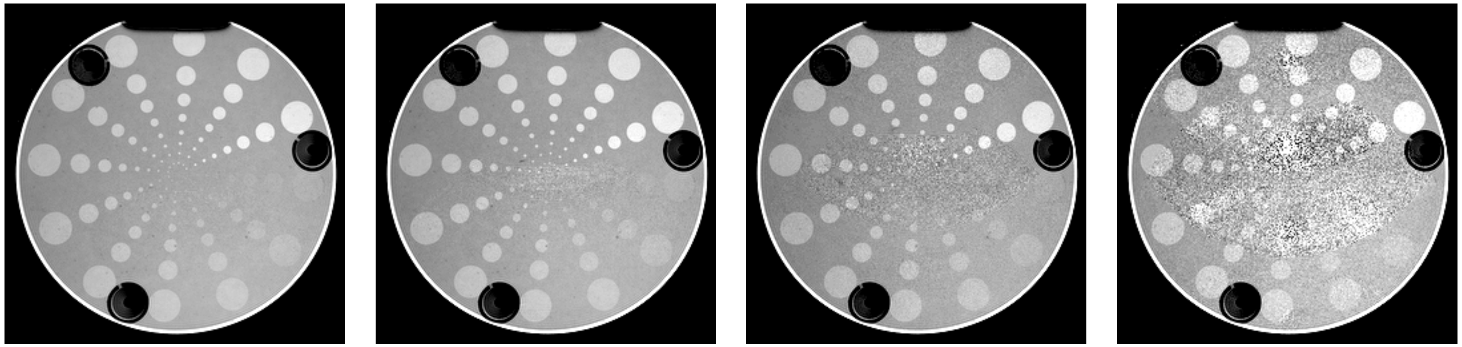
Regularized solution:

$R < N_c$ $\quad F = (E^H \Psi^{-1} E + \lambda \Theta^{-1})^{-1} E^H \Psi^{-1}$

$R > N_c$ $\quad F = \Theta E^H (E \Theta E^H + \lambda \Psi)^{-1}$

Geometry factor

R = 2.0 **R = 2.4** **R = 3.0** **R = 4.0**

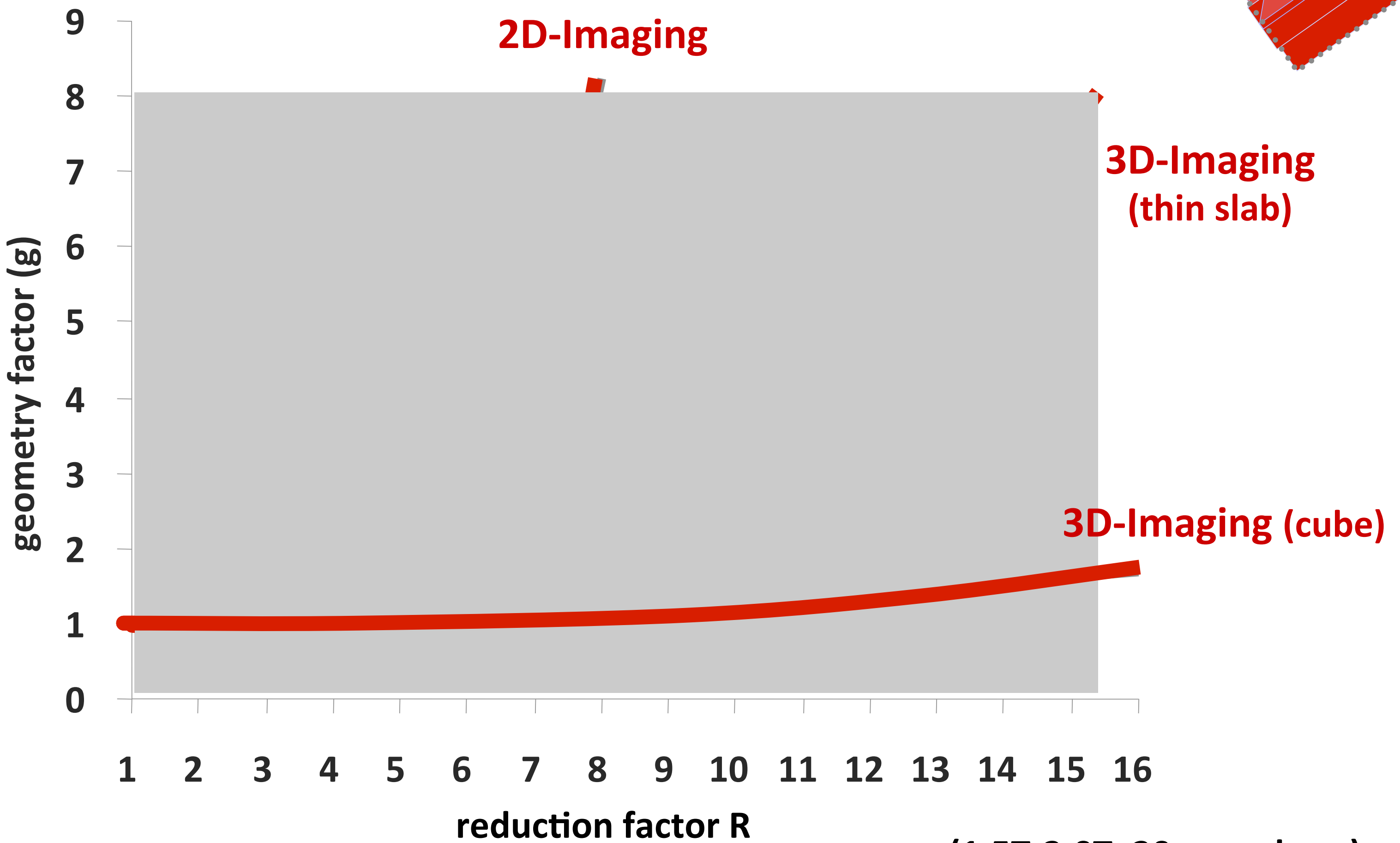


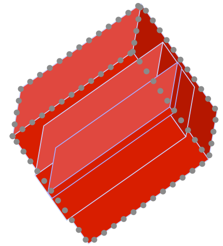
$$SNR^{SENSE} = \frac{SNR^{Full}}{\sqrt{R} \cdot g(\vec{x})}$$

$$g(\vec{x}) = \sqrt{\left(E^H E\right)_{i,i} \left(\left(E^H E\right)^{-1}\right)_{i,i}} \geq 1$$

Pruessmann KP et al MRM 1999

Parallel imaging – ultimate performance



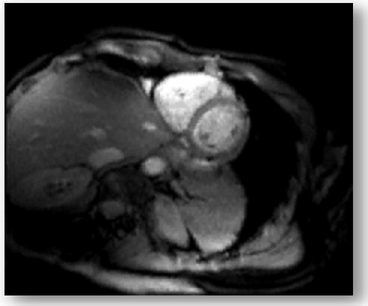


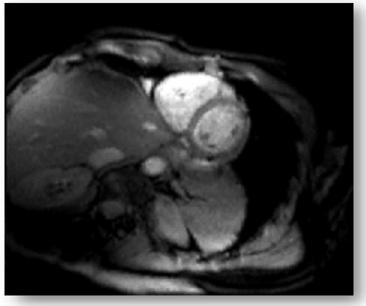
(1.5T-3.0T, 20 cm sphere)

Wiesinger F et al. MRM 2004

How much information is redundant?

- Transform data to a **sparser** coefficient space
- Find **model** that suitably **links** coefficients
- Retain **key** coefficients

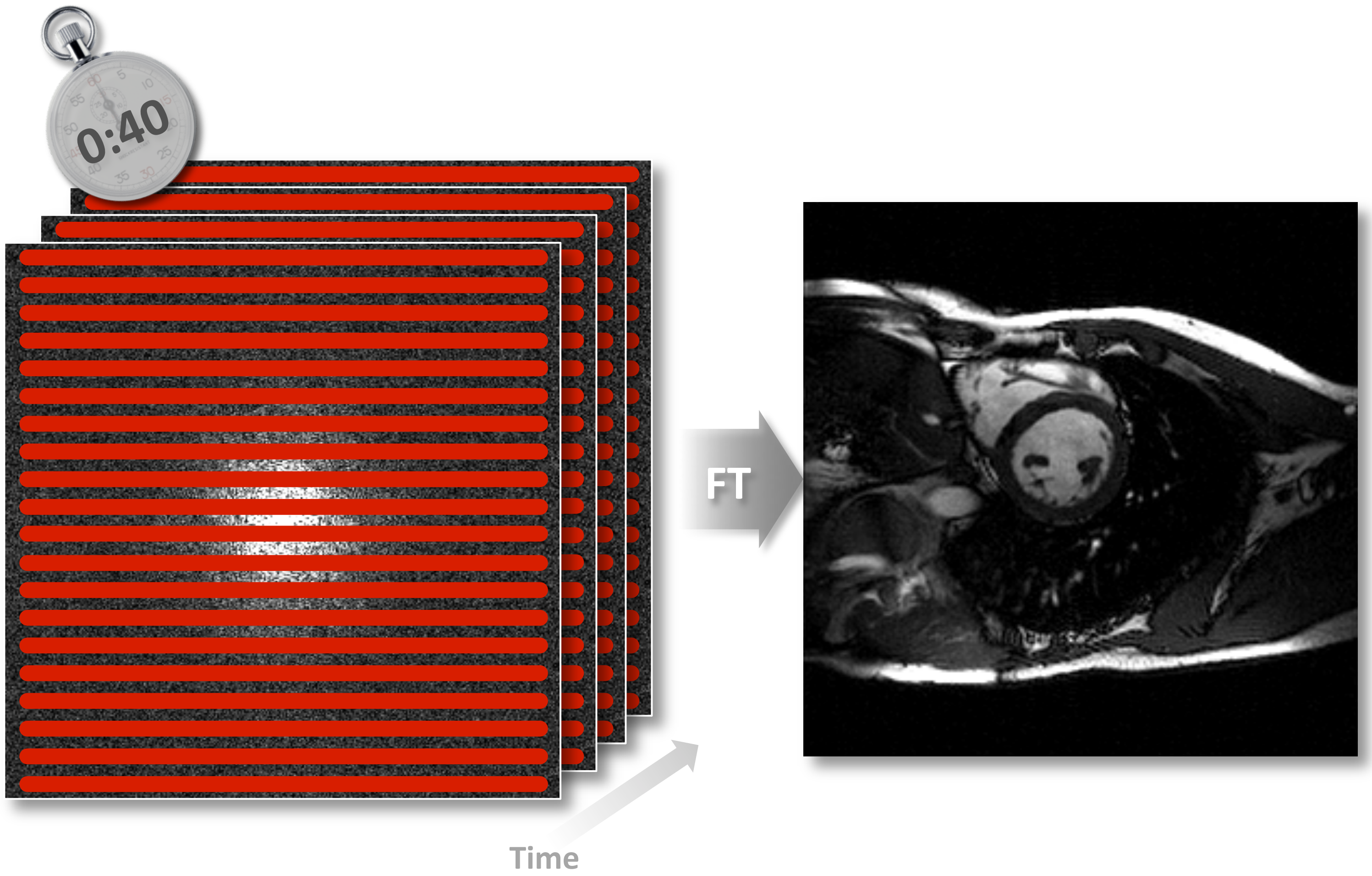




Original:	16 bits/pixel	16 bits/pixel
Entropy rate[1)]:	3.3 bits/pixel	1.0 bits/pixel
"theoretical" R:	4.8	16.0

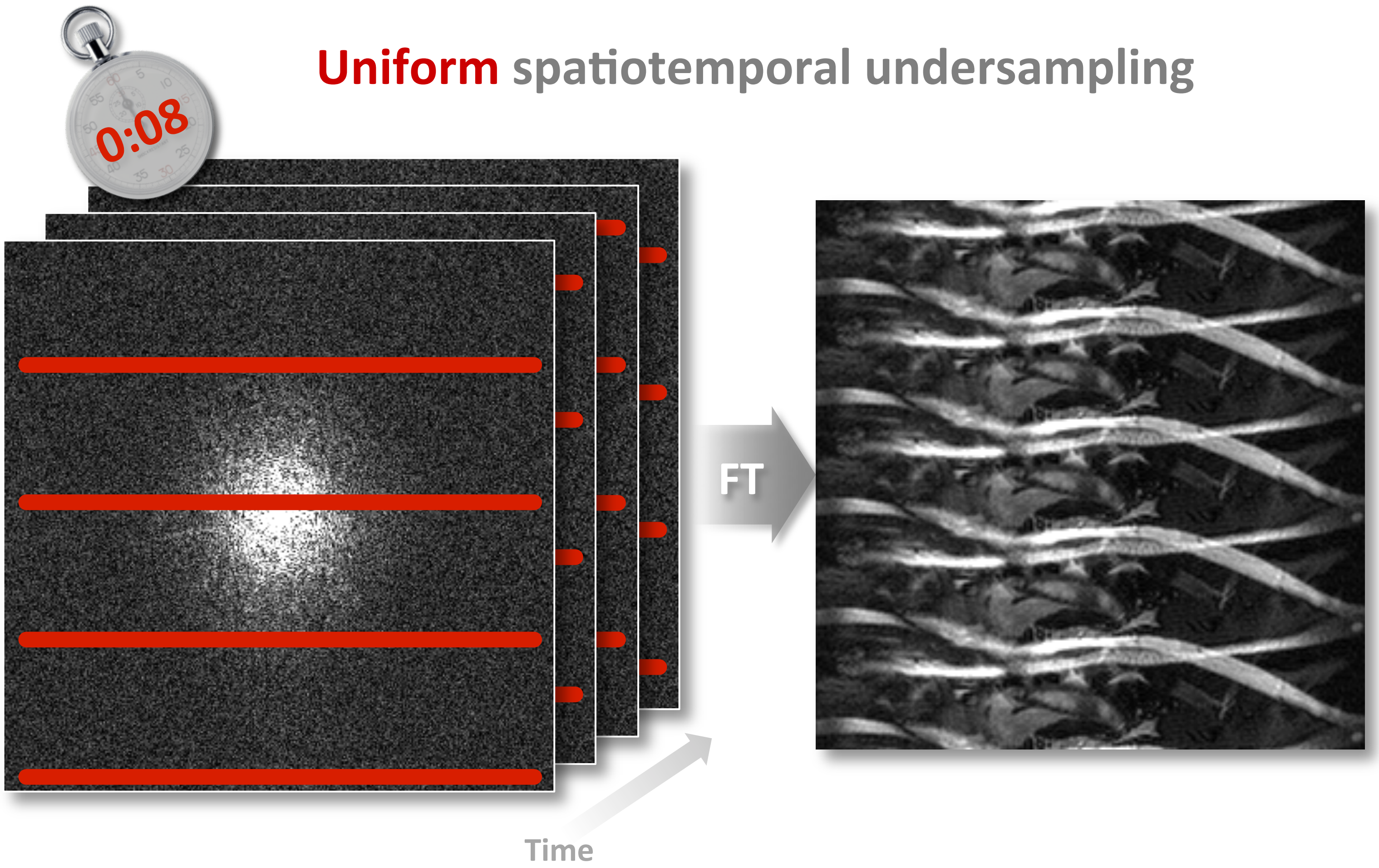
[1)] Cosine Transform, Huffman encoding

k-t undersampling

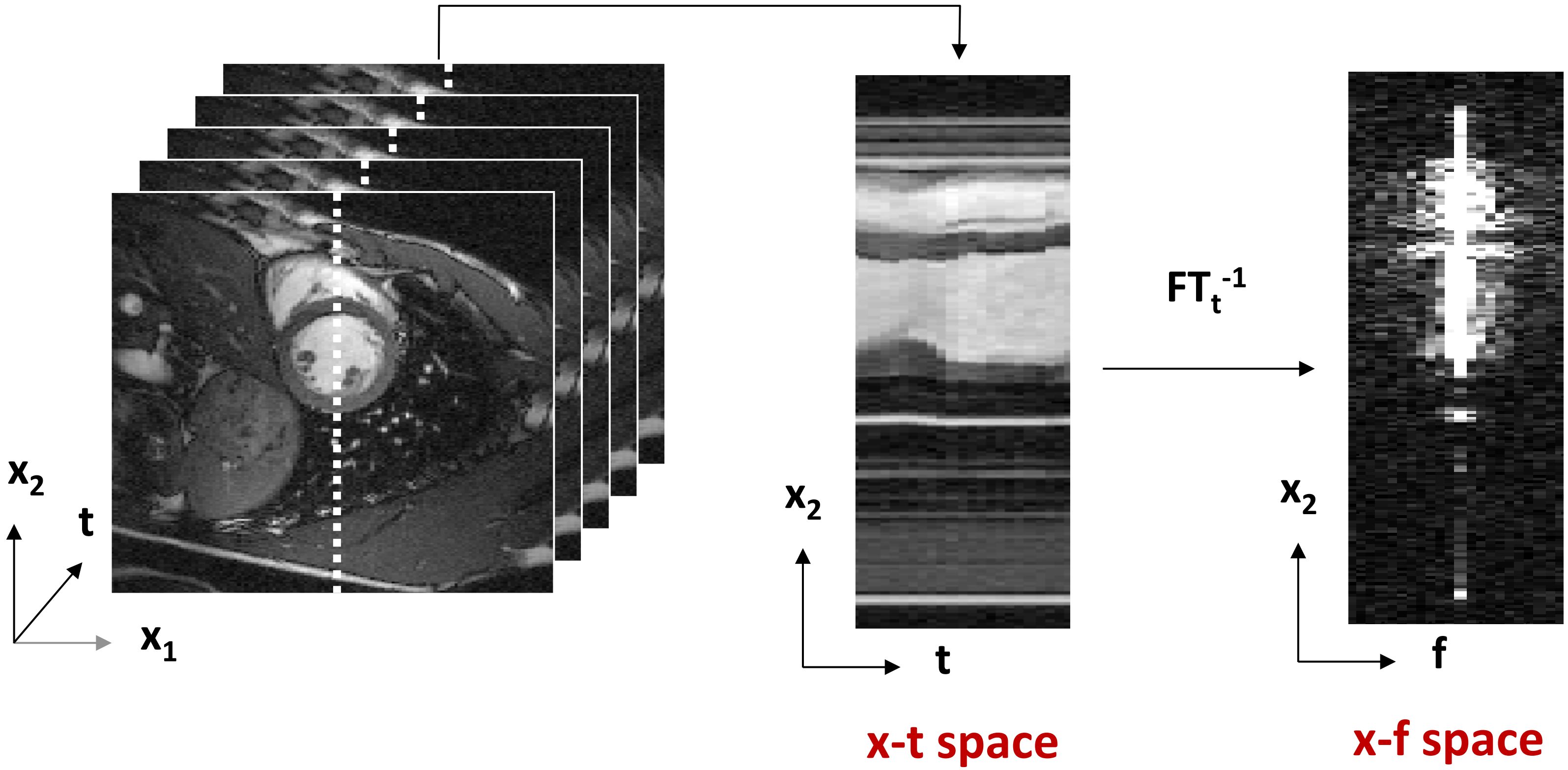


Uniform k-t undersampling

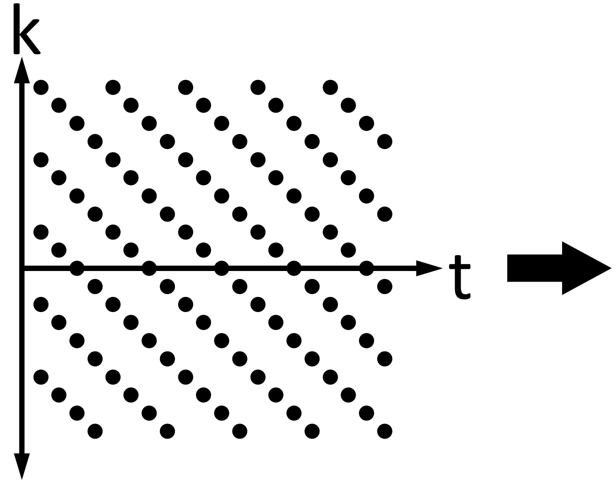
Uniform spatiotemporal undersampling



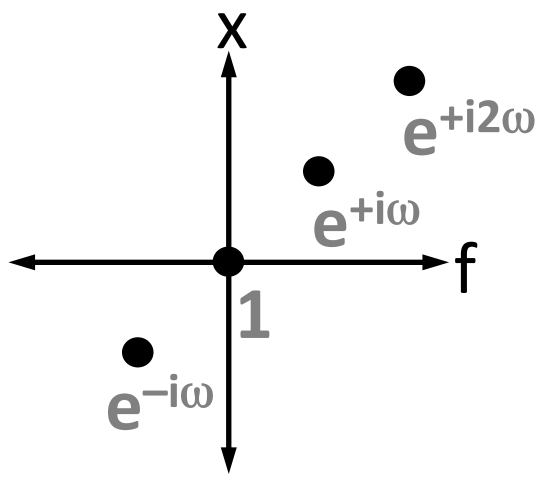
x-t → x-f space



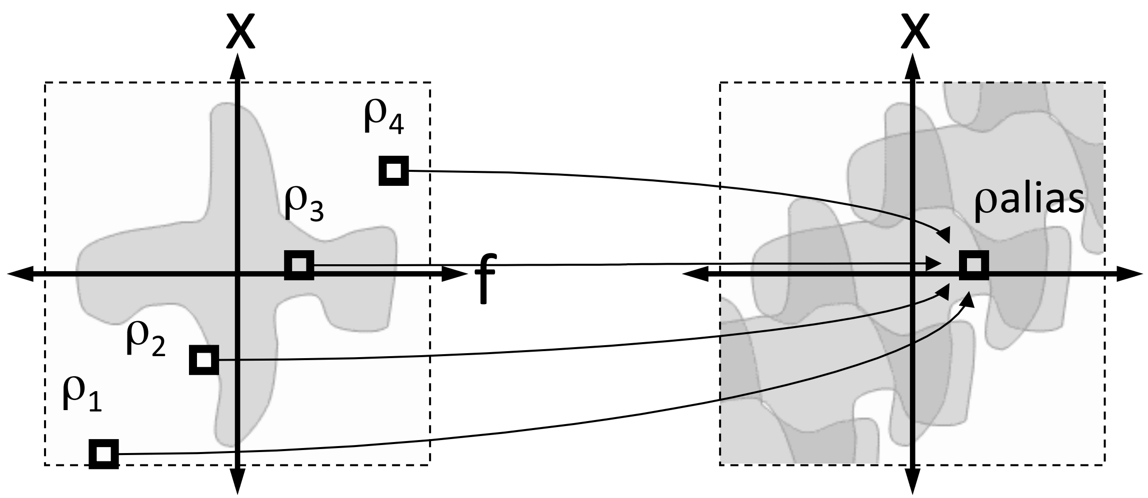
x-t → x-f space



k-t sampling pattern



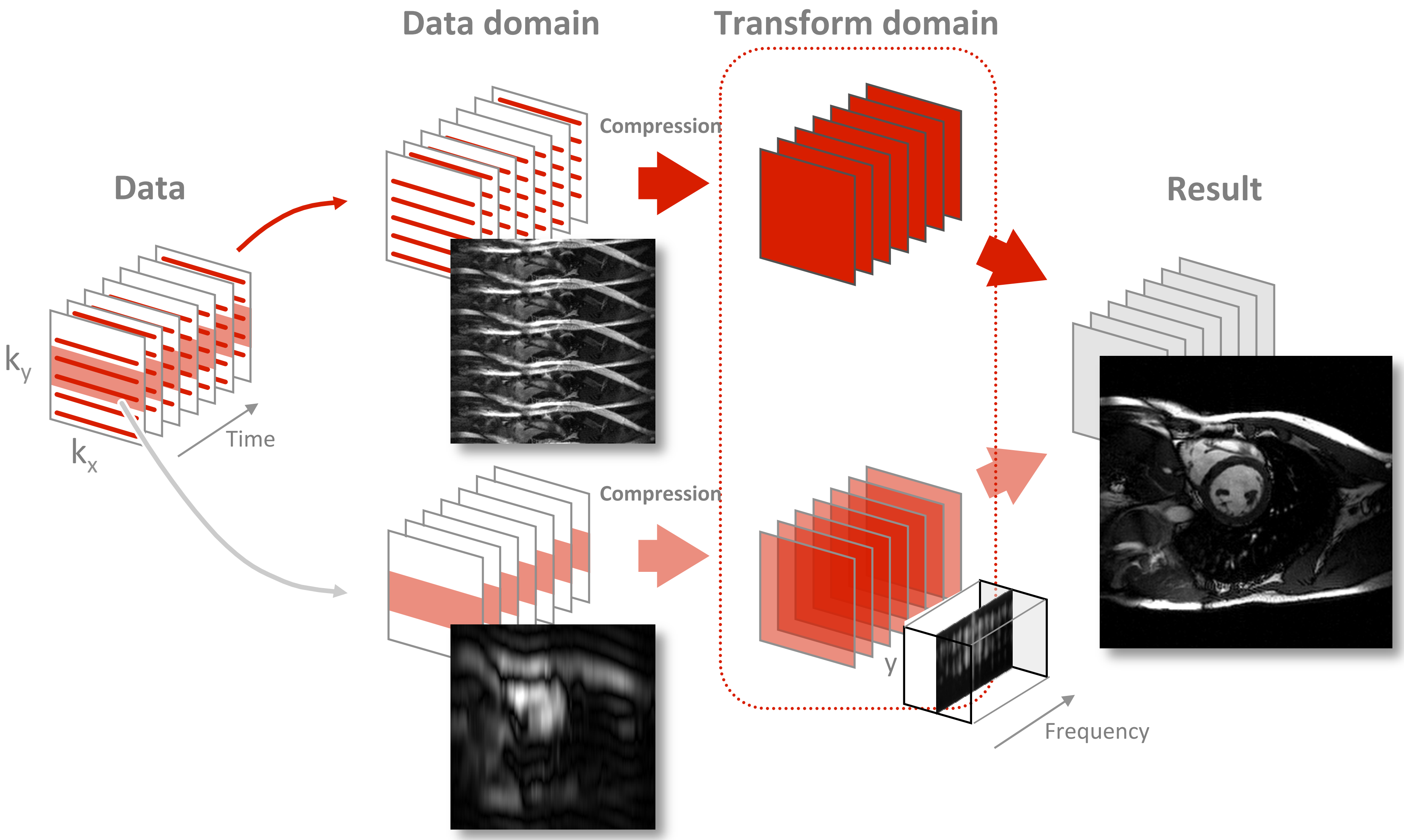
Point spread function



True object signal

After aliasing

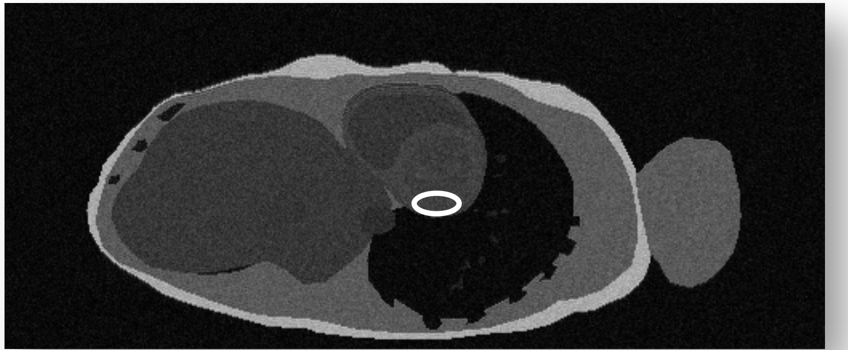
k-t BLAST / k-t SENSE



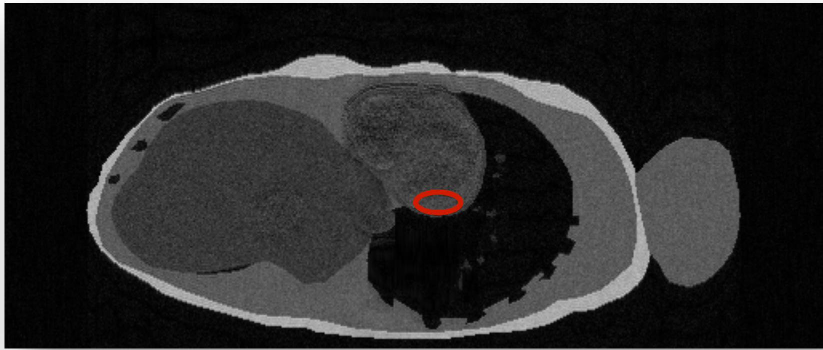
J Tsao et al. MRM 2003

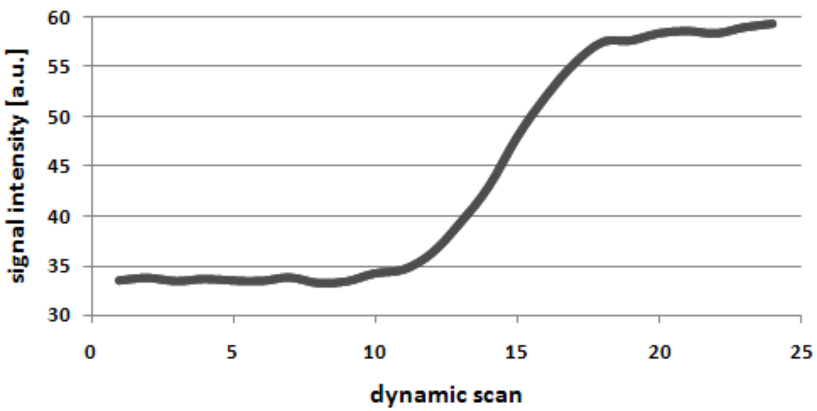
k-t SENSE – Temporal fidelity

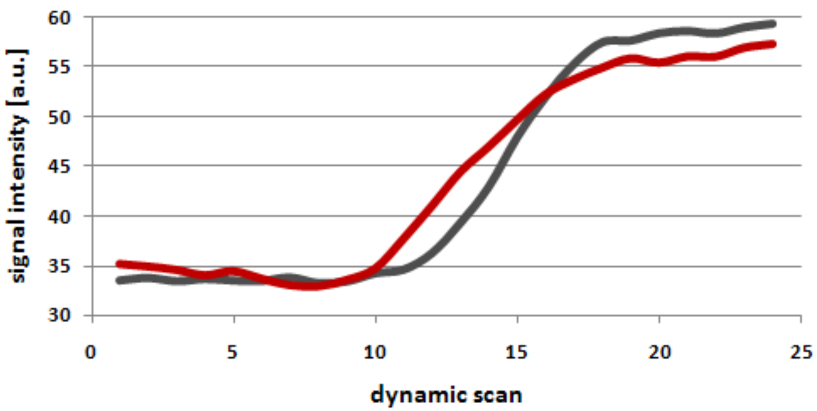
Reference



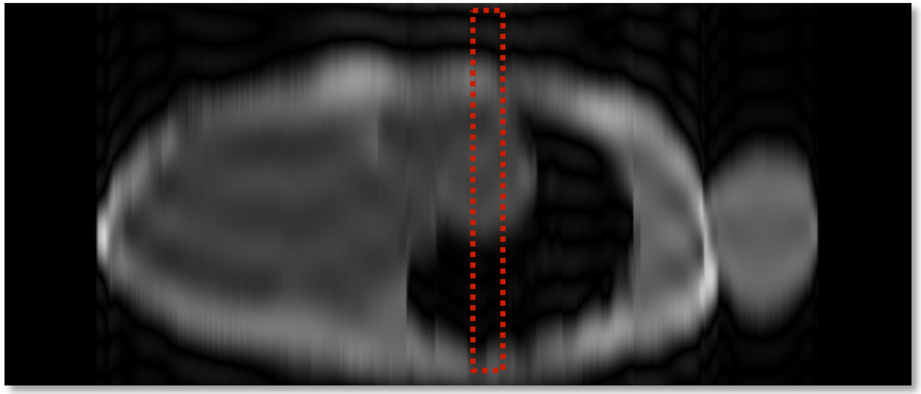
12x *k-t* SENSE



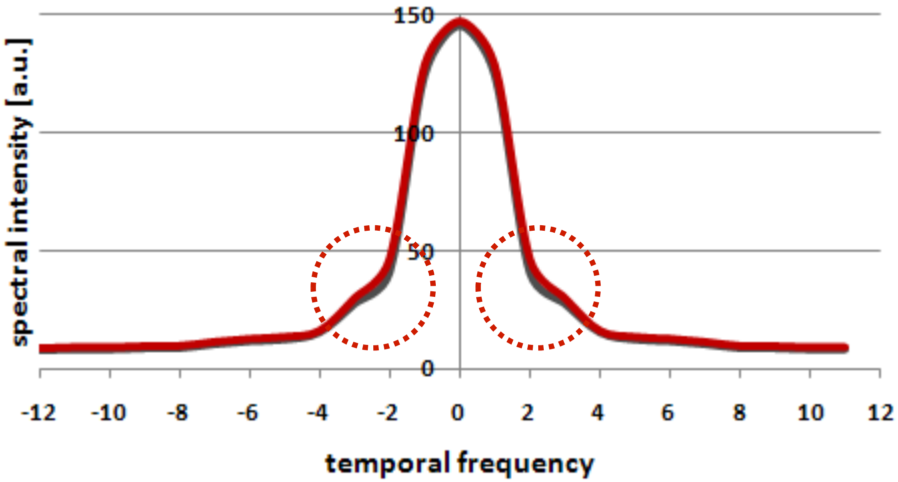




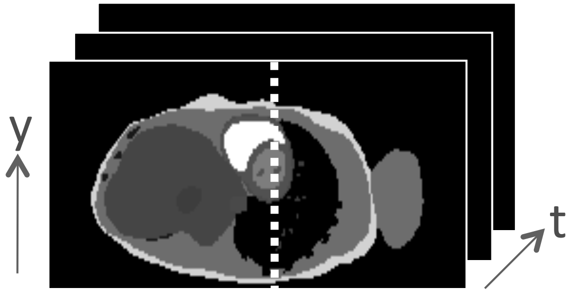
k-t SENSE – Training data



$$i = \mathbf{M^2}\, E^H(E\mathbf{M^2}E^H+\sigma^2)^+ m_{alias}$$

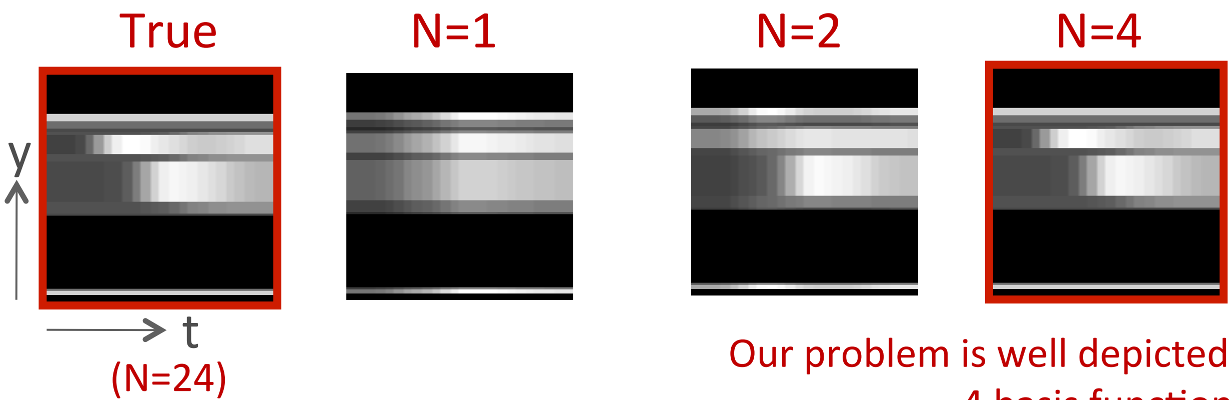


Reconsidering temporal signals



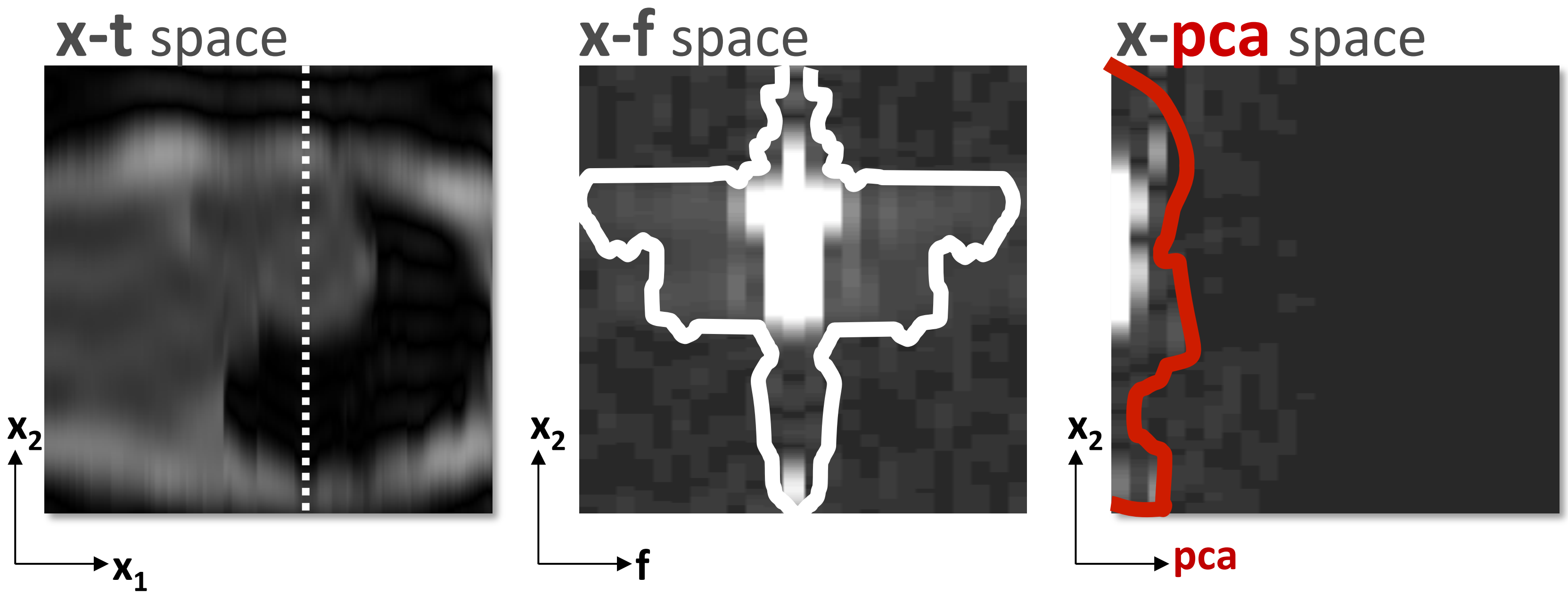
Spatial Weighting **Basis functions**

$$m(y,t) = UEV^{H} = WB = \sum_{n=1}^{N} W(y,n)B(n,t)$$



Our problem is well depicted with as few as 4 basis functions

PCA space



48 frames

4 weights determine 48 frames

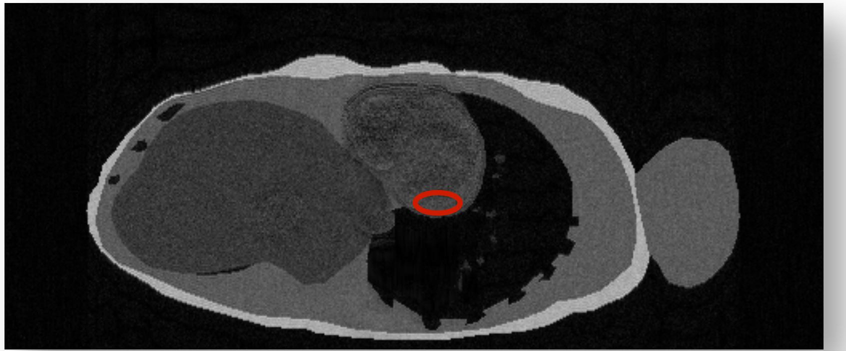
48 equations
in 48x12 unknowns

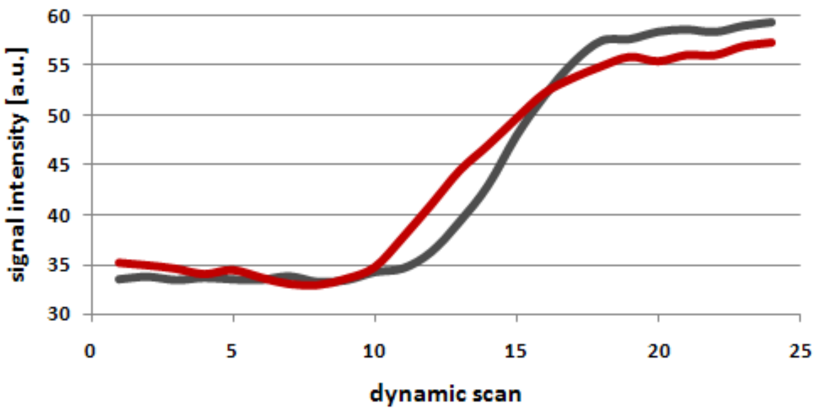
48 equations
in 4x12 unknowns

Pedersen H et al. MRM 2009

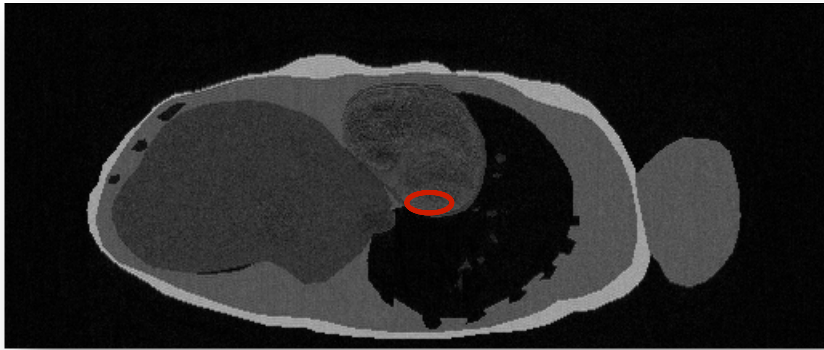
k-t SENSE vs k-t PCA

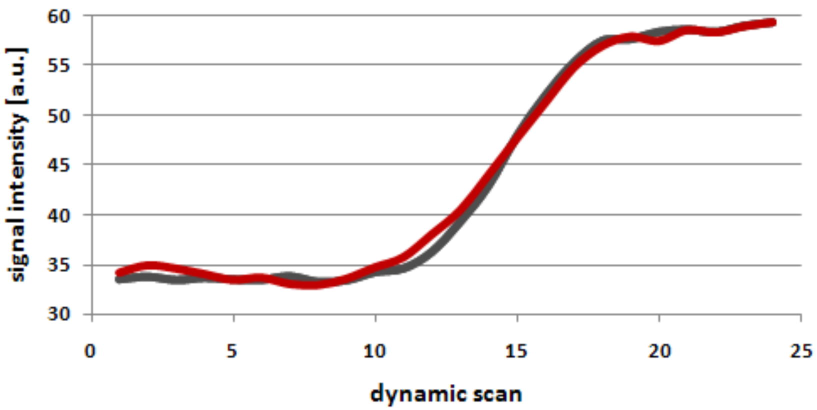
12x *k-t* SENSE



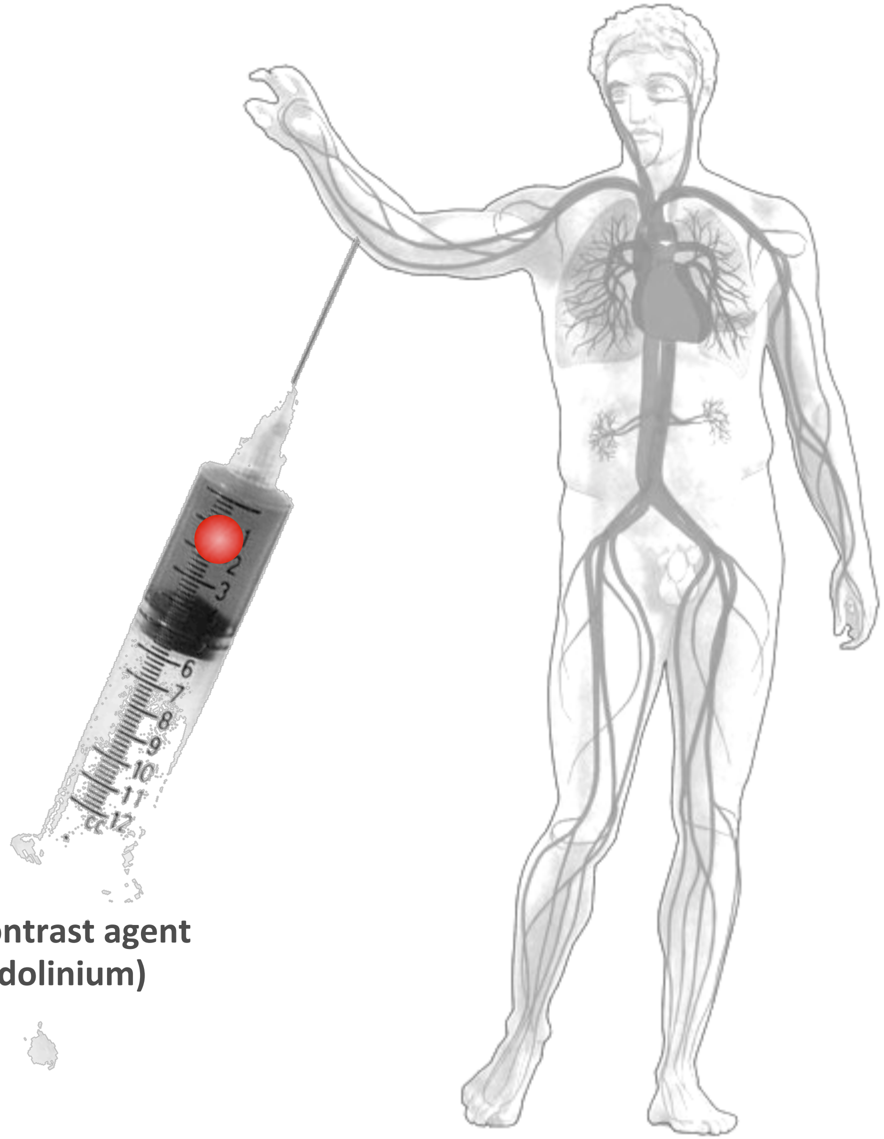


12x *k-t* PCA





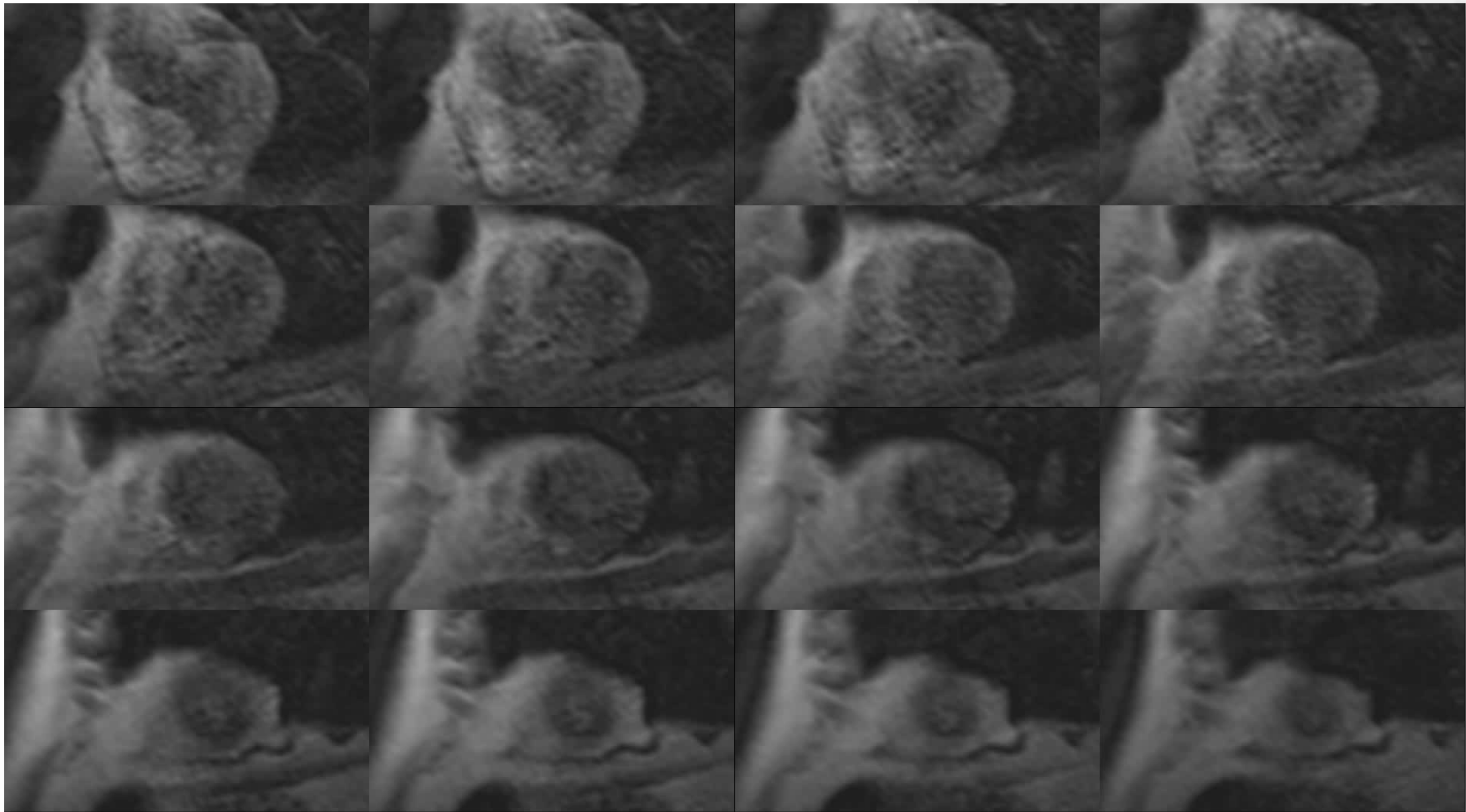
Perfusion imaging



MR Contrast agent
(Gadolinium)

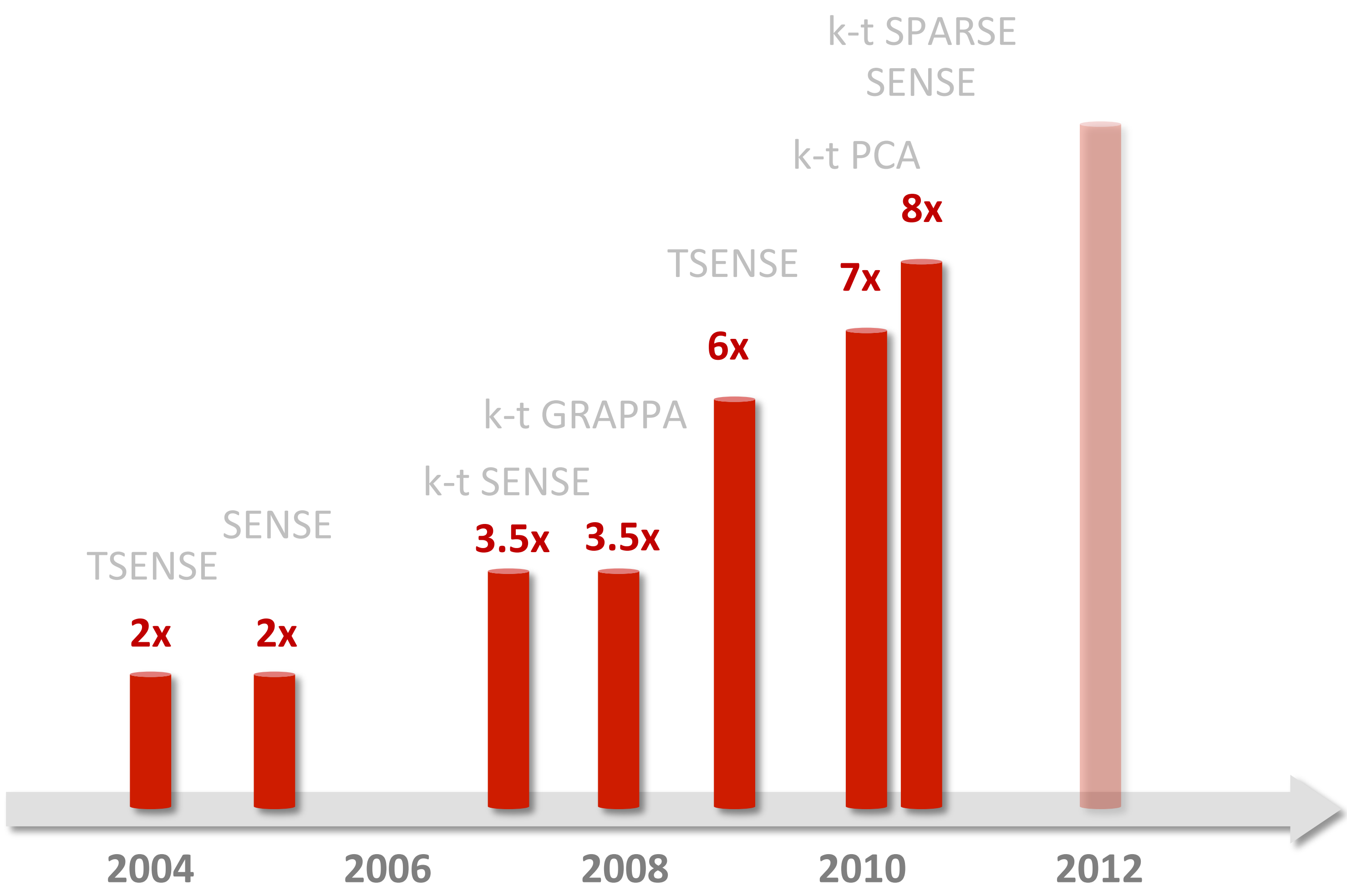
3D Perfusion imaging

10x 3D k-t PCA (2.2 x 2.2 mm²)



R. Manka et al. JACC 2011

Speed-up – Perfusion imaging



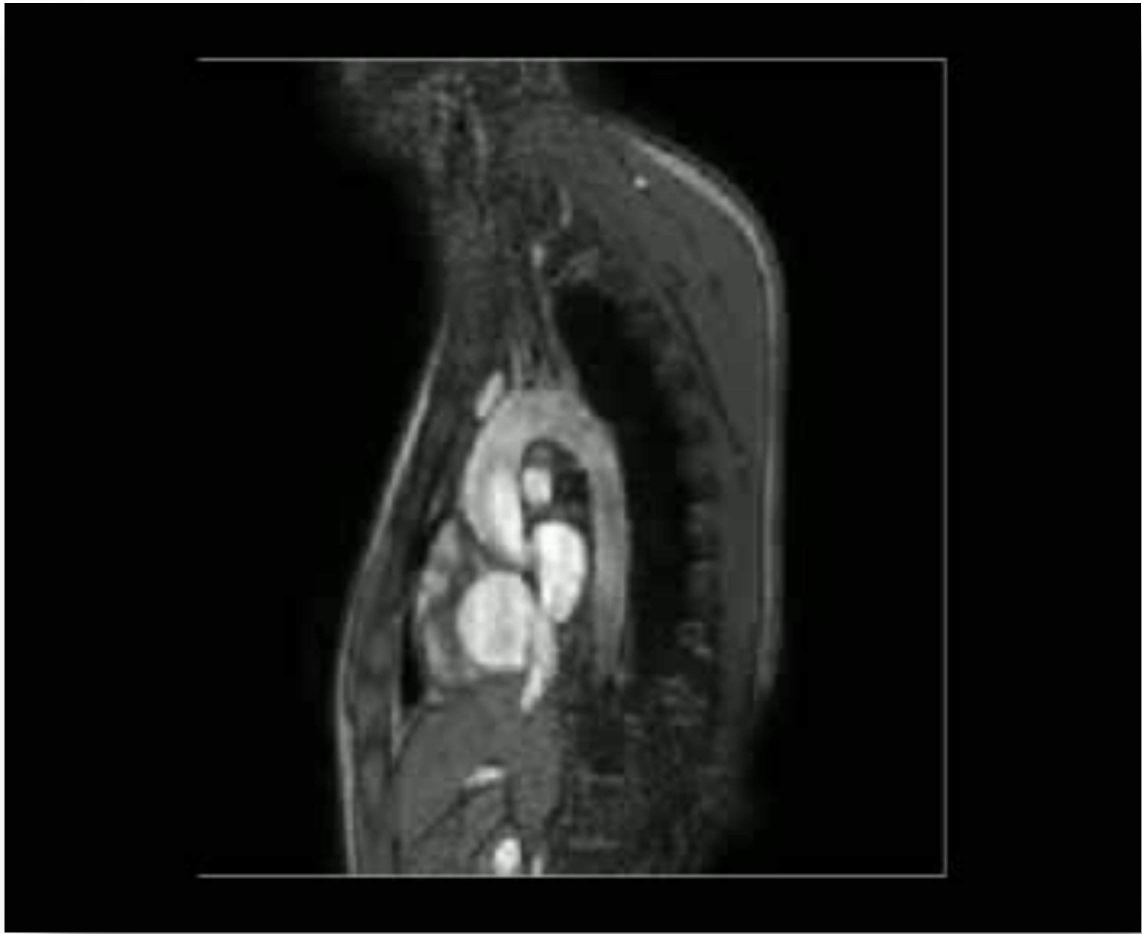
Kellman P et al. MRM 2004
Plein S et al. Radiology 2005

Plein S et al. MRM 2007
Jung B et al. JMRI 2008

Nayak KS et al. JCMR 2008
Vitanis V et al. MRM 2010

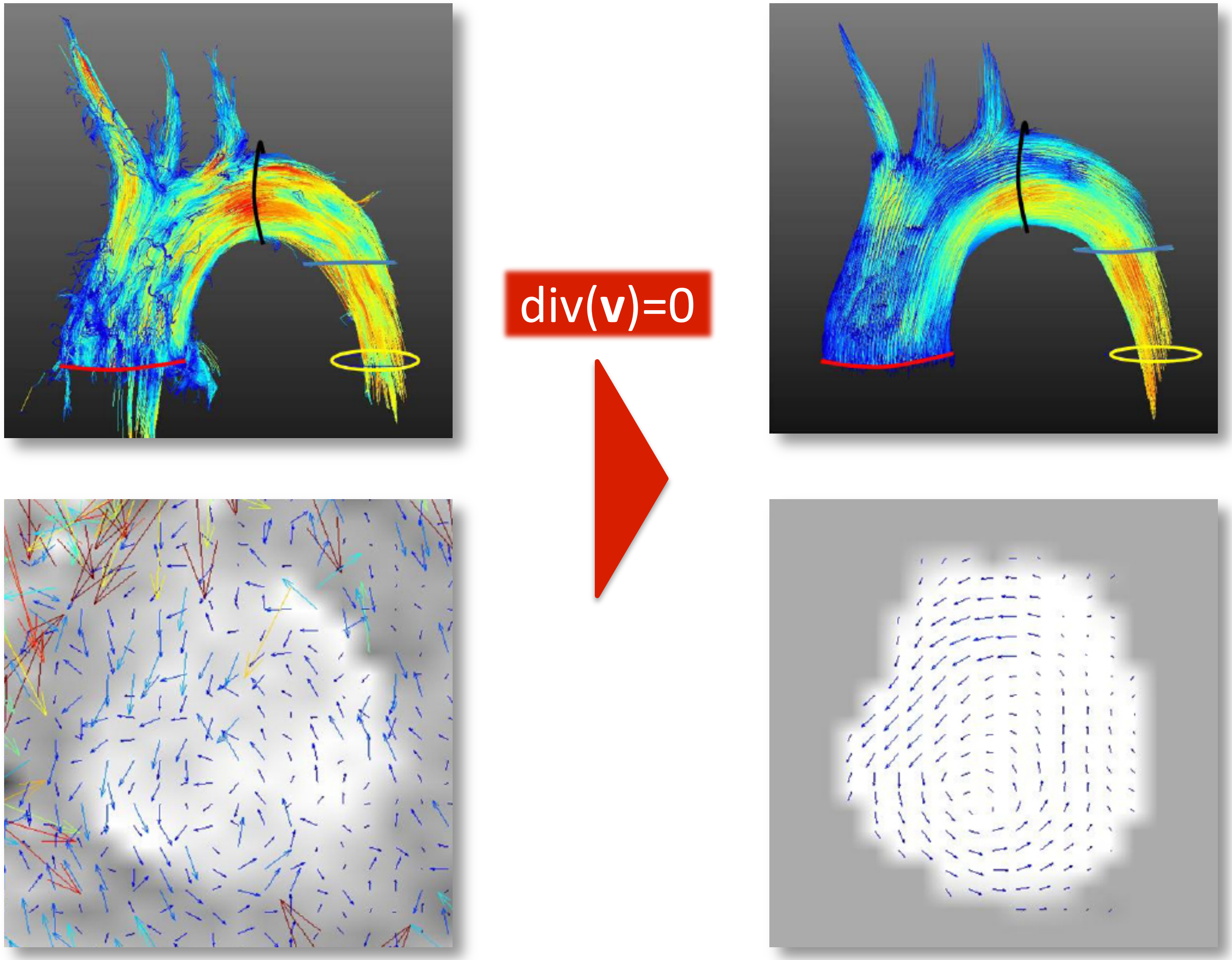
Otazo R et al. MRM 2010

8x 3D k-t PCA



G. Crelier, GyroTools

Divergence-free constraint

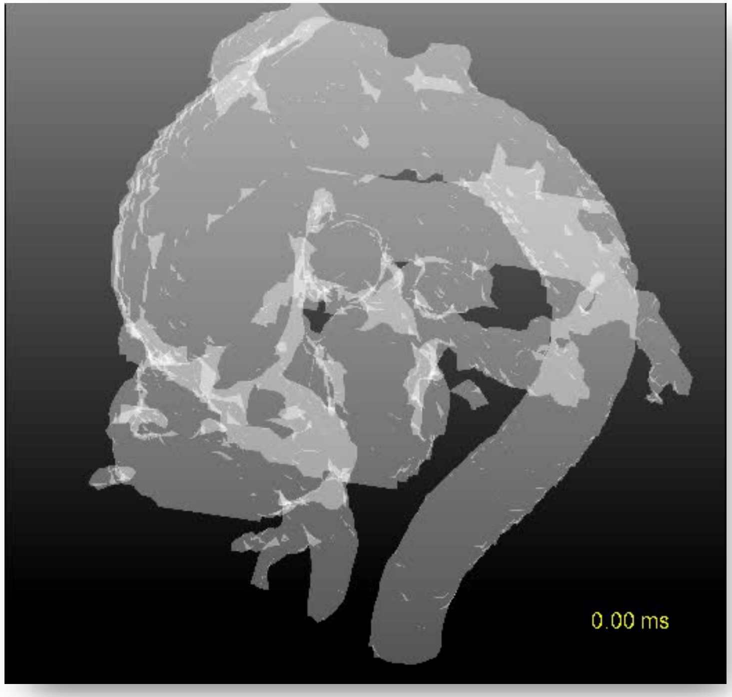


J. Busch et al. ISMRM 2011

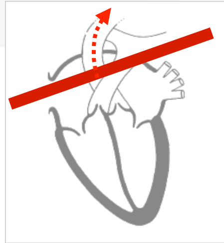
3D Blood flow quantification

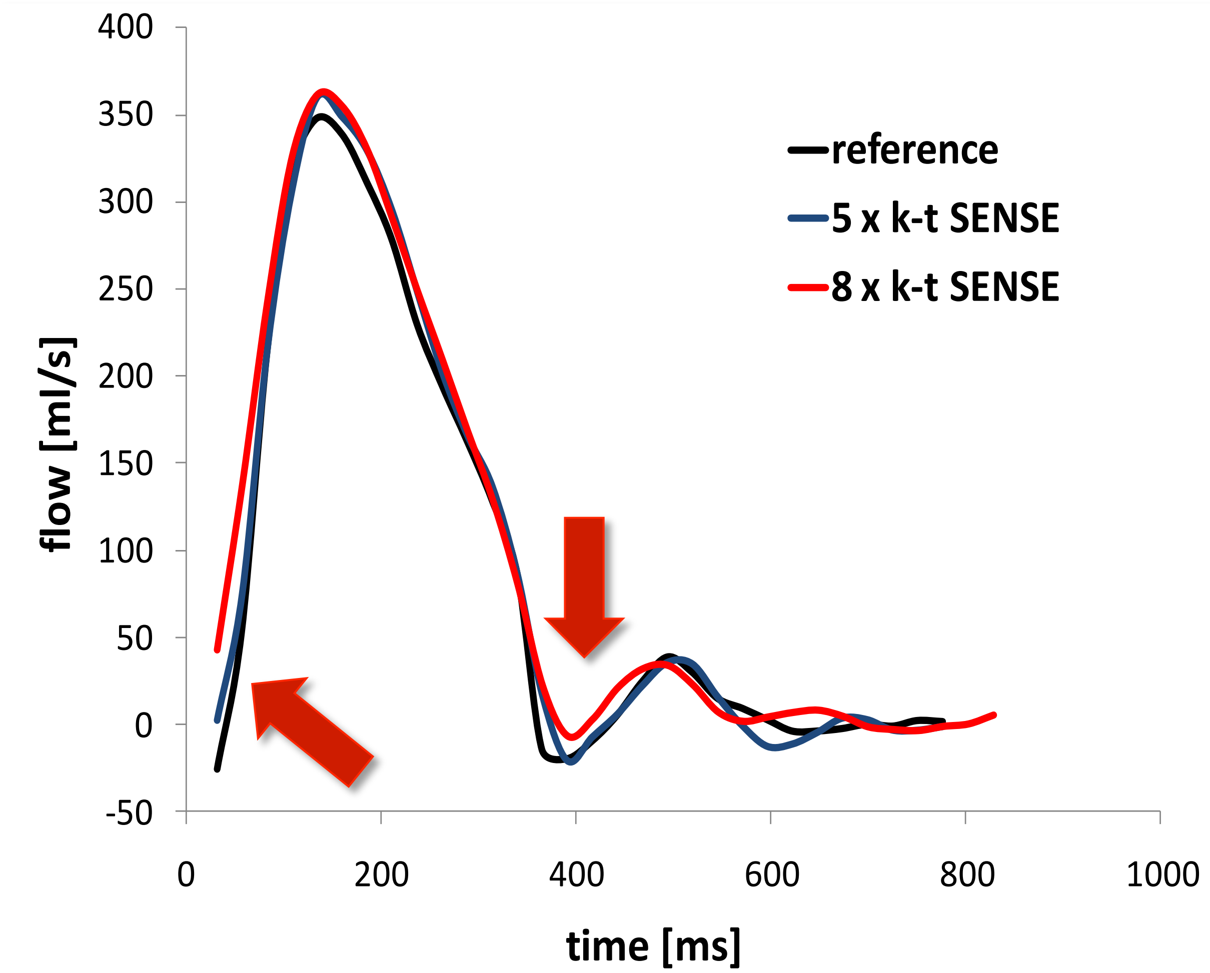
Healthy volunteer

Patient with dilated aorta



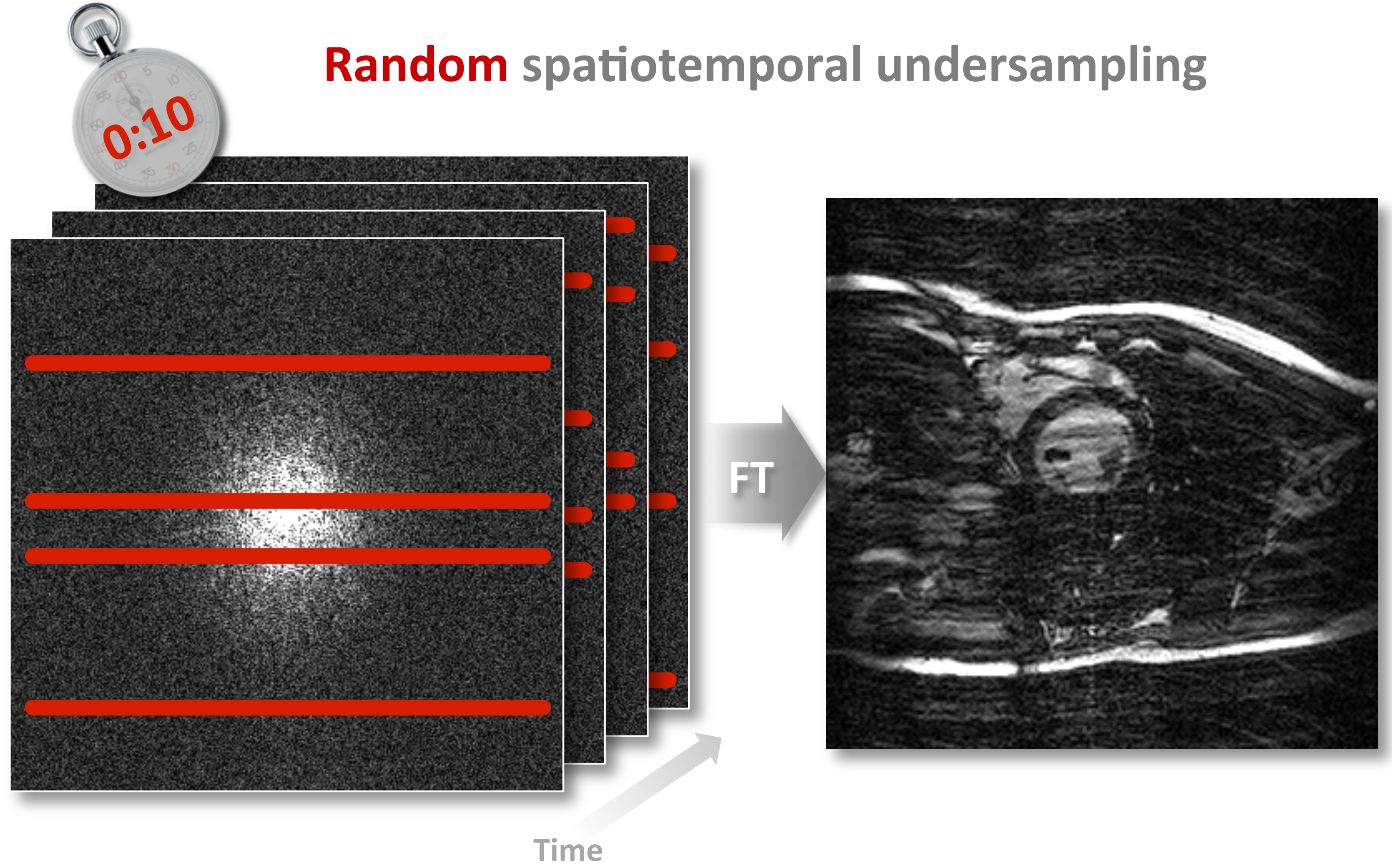
Limitations



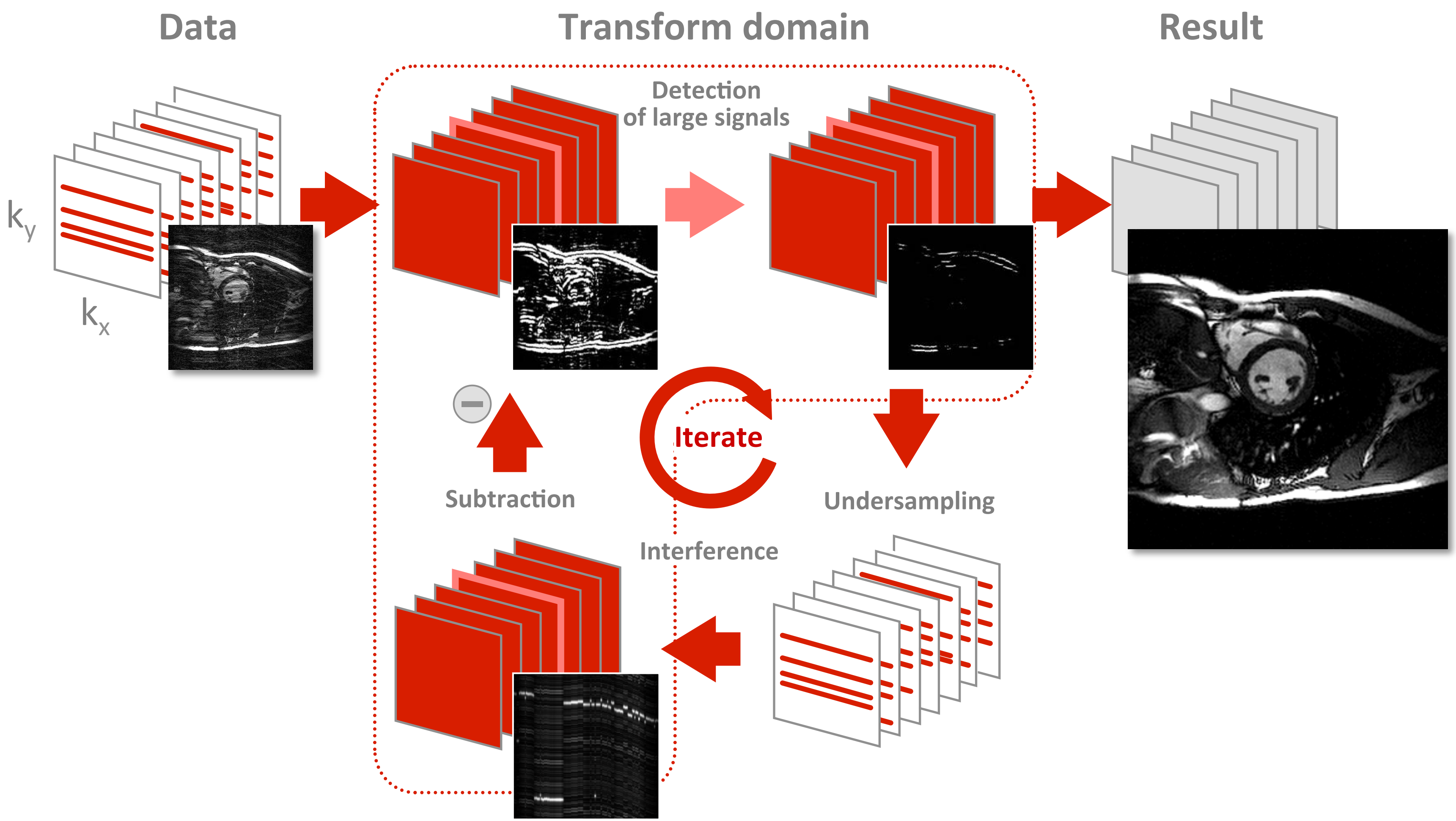


Random undersampling

Random spatiotemporal undersampling



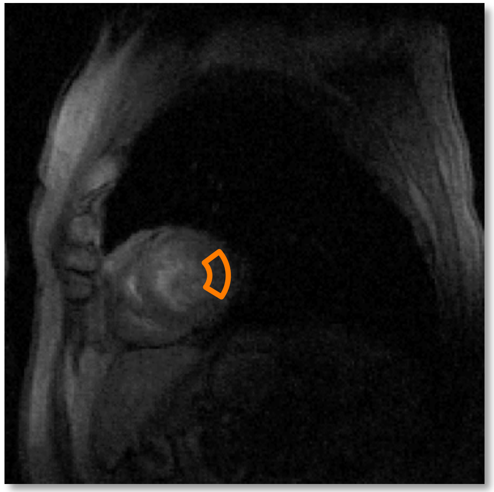
Compressed Sensing



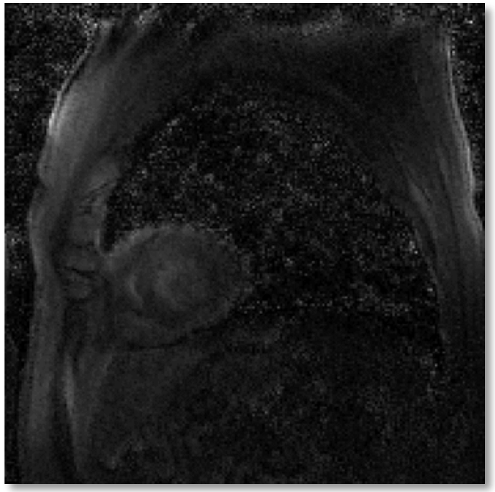
M. Lustig et al. MRM 2007

Perfusion imaging

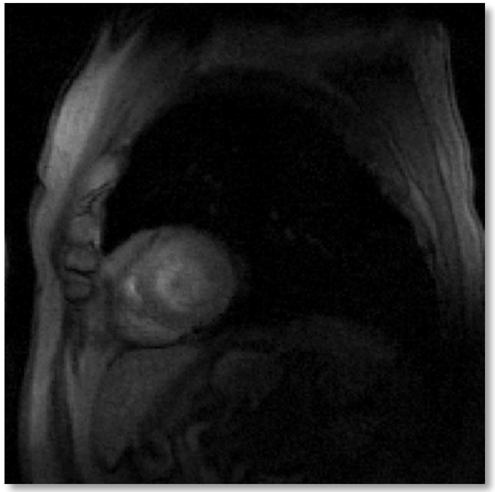
Reference

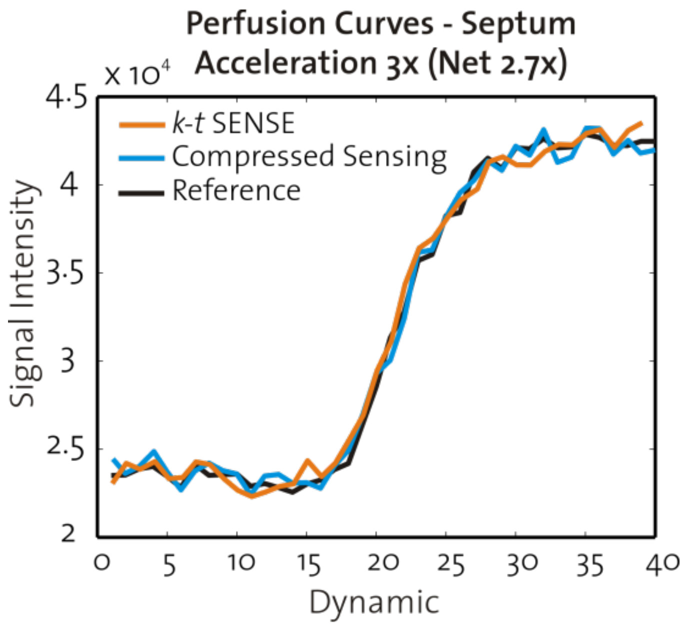


3x *k-t* CS



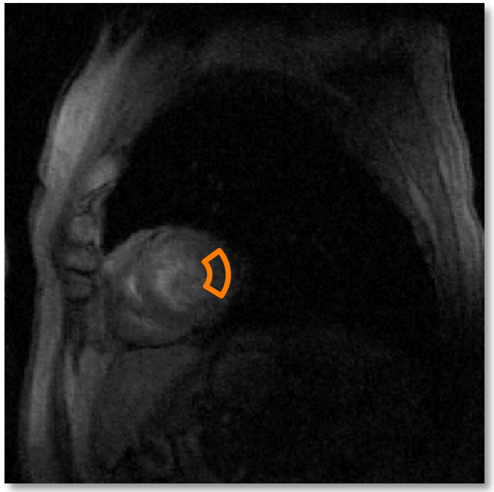
3x *k-t* SENSE



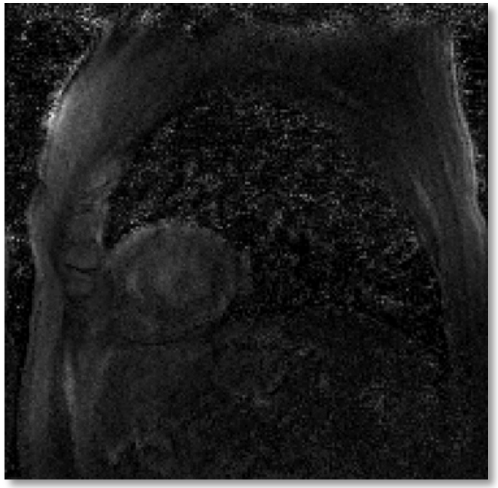


V. Vitanis et al. ISMRM 2008

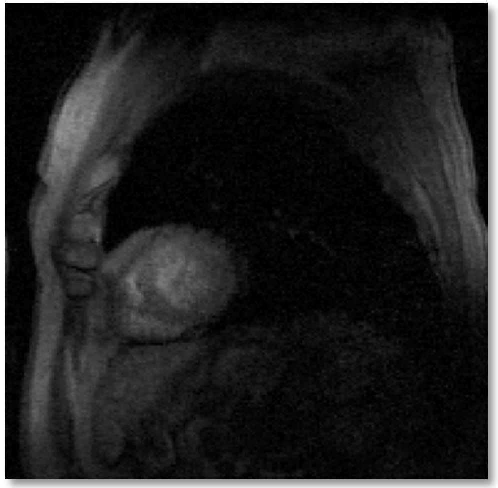
Reference



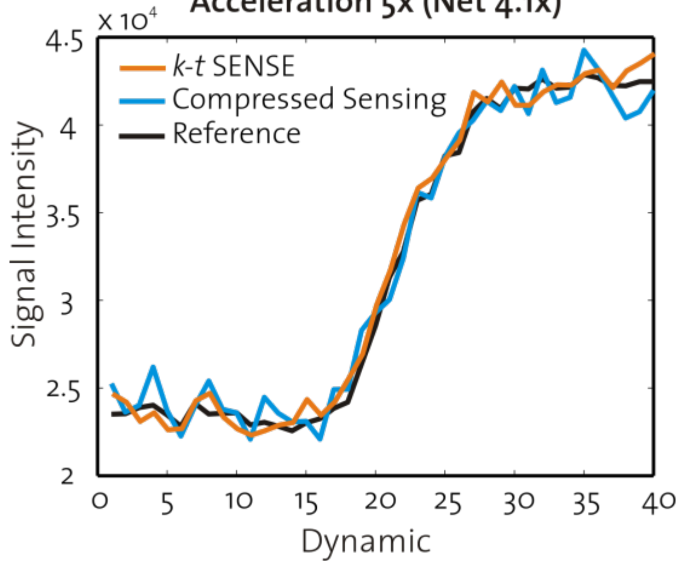
5x *k-t* CS



5x *k-t* SENSE

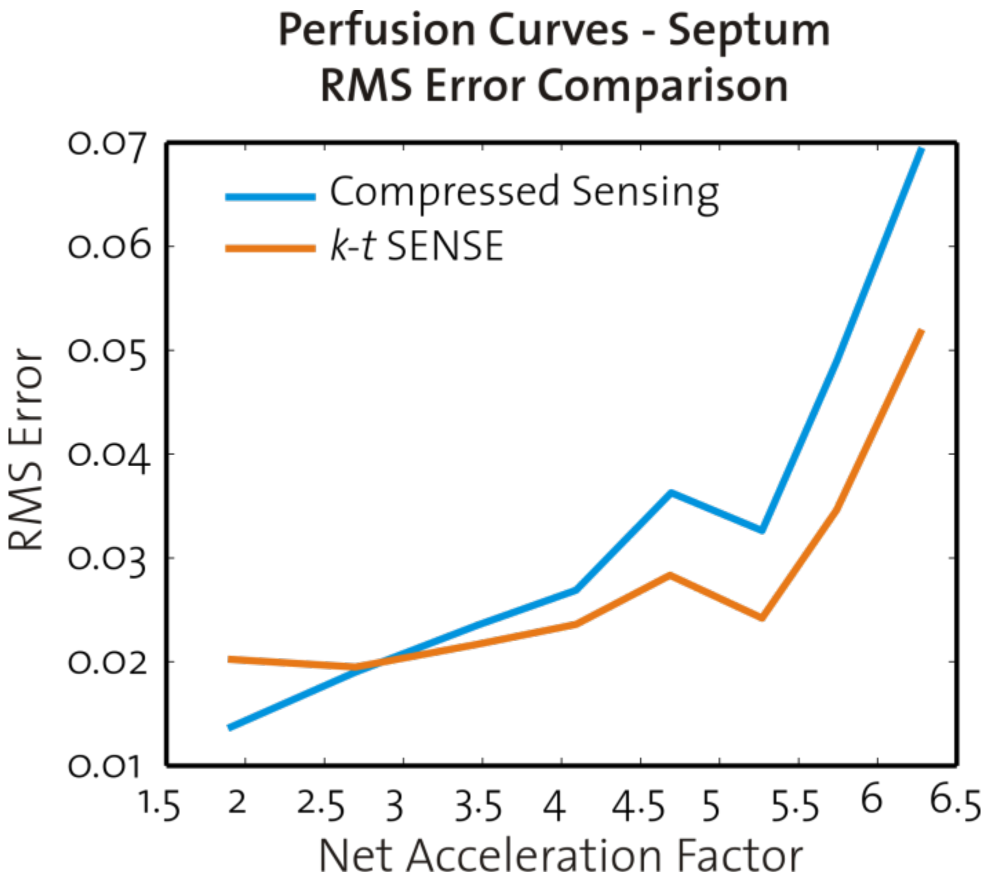


Perfusion Curves - Septum
Acceleration 5x (Net 4.1x)



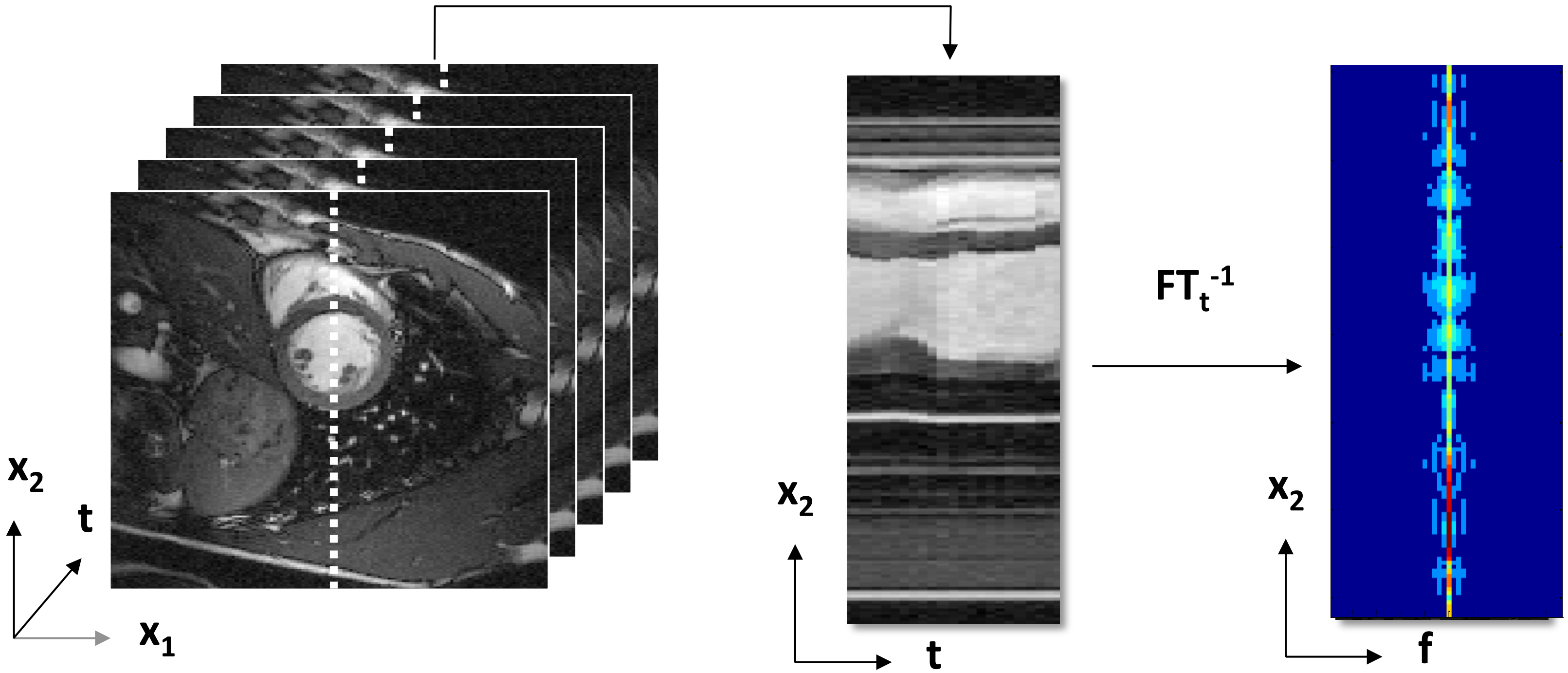
V. Vitanis et al. ISMRM 2008

Random versus uniform undersampling



V. Vitanis et al. ISMRM 2008

k-t group sparsity intensity (k-t GSI)

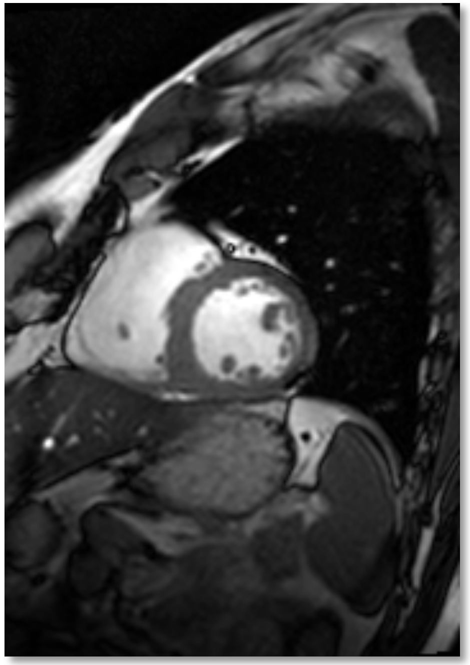


$$\min_{\mathbf{m}} \left\| i^{g} \right\|_{1,2} = \left\| i_1^{g} \right\|_{1,2} + \left\| i_2^{g} \right\|_{1,2} + \ldots \left\| i_k^{g} \right\|_{1,2} \text{ s.t. } \left\| E_u i - d \right\|_2 \leq \varepsilon$$

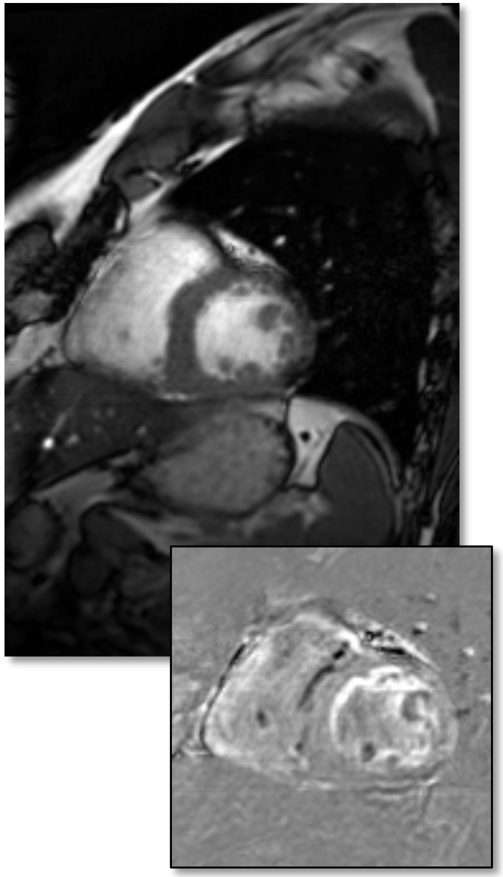
M. Usman et al. MRM 2011, C. Prieto et al. MRM 2011

k-t GSI and k-t SPARSE

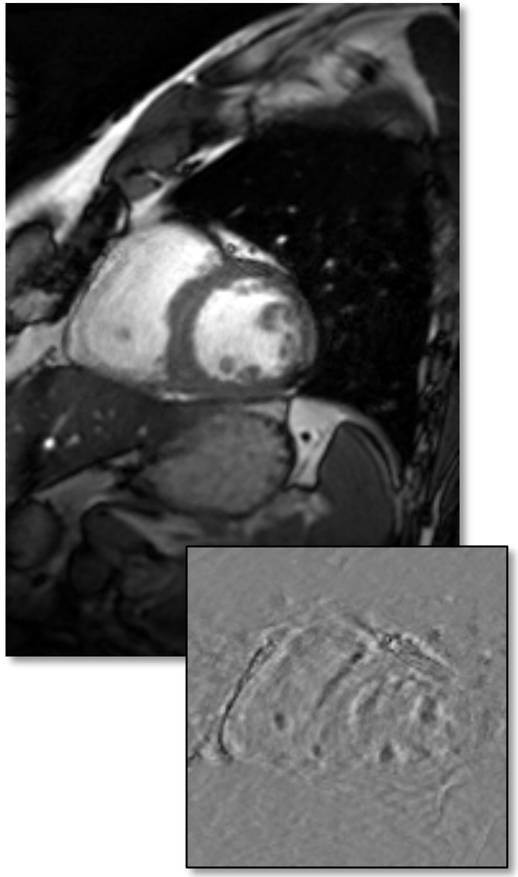
Reference



9x *k-t* SPARSE

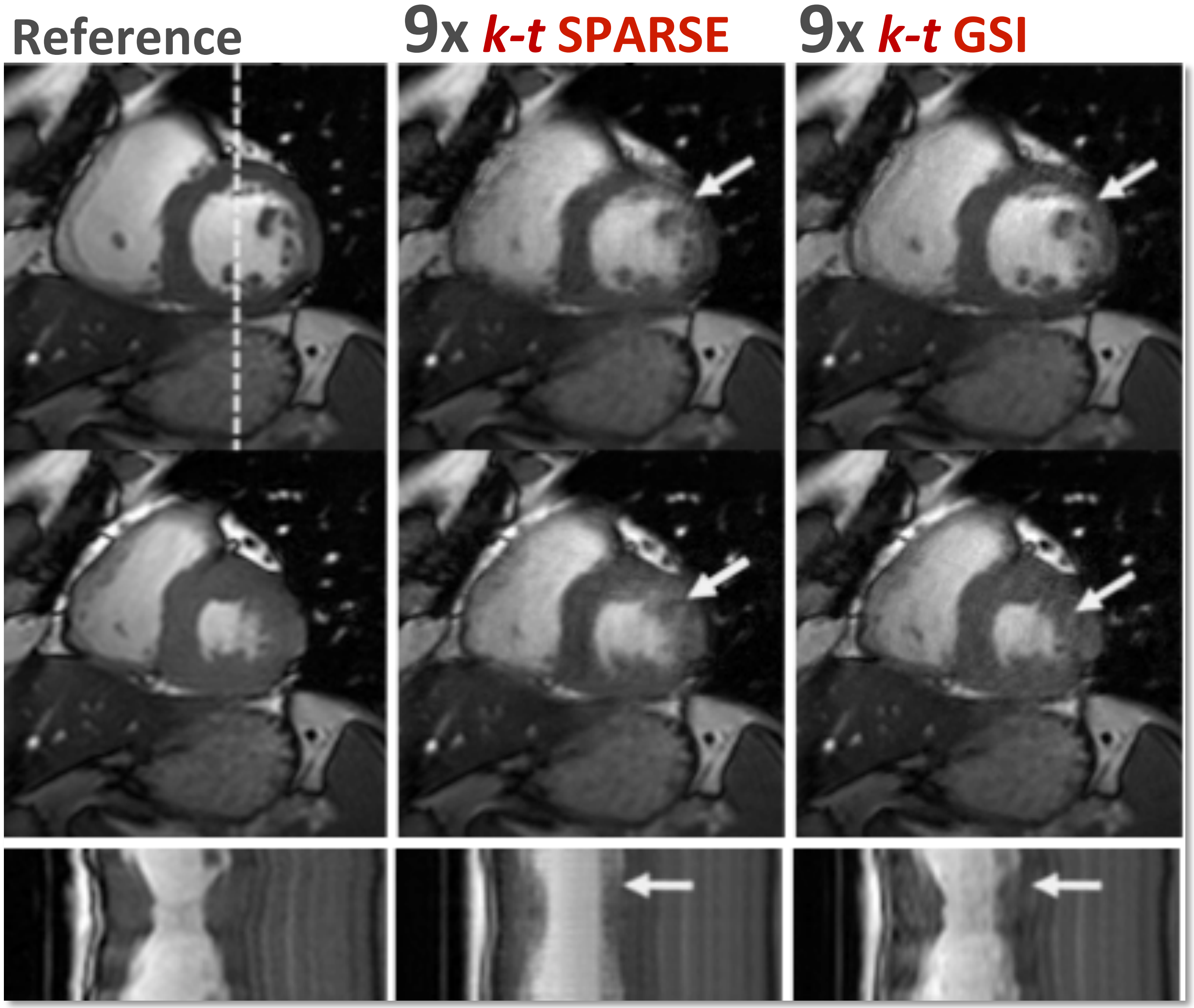


9x *k-t* GSI



M. Lustig et al. MRM 2007, M. Usman et al. MRM 2011, C. Prieto et al. MRM 2011

k-t GSI and k-t SPARSE



M. Lustig et al. MRM 2007, M. Usman et al. MRM 2011, C. Prieto et al. MRM 2011

Summary

- Significant spatiotemporal correlation in dynamic data
- Compact data representation in x-f and x-pc spaces
- Uniform undersampling of 2-8x with L2 reconstruction
- Random undersampling of 9x by exploiting group sparsity
- Temporal filtering results if undersampling exceeds limits
- Combination with parallel imaging to effort higher factors