Imaging Sciences and
Biomedical Engineering

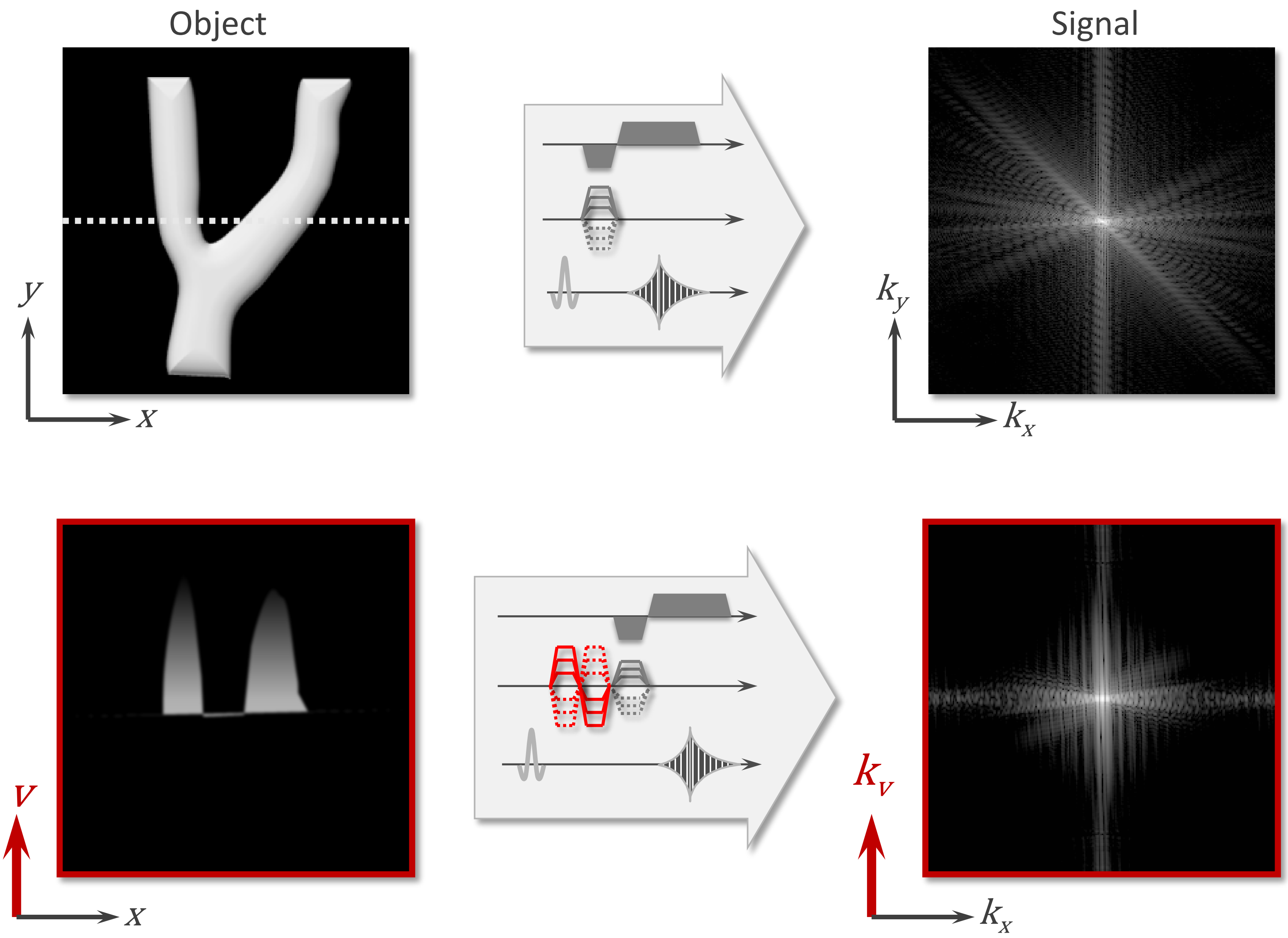
Performance of l1-norm minimizing regularizers on phase/magnitude reconstruction in flow encoded MRI

2011 BASP Frontiers Wokshop

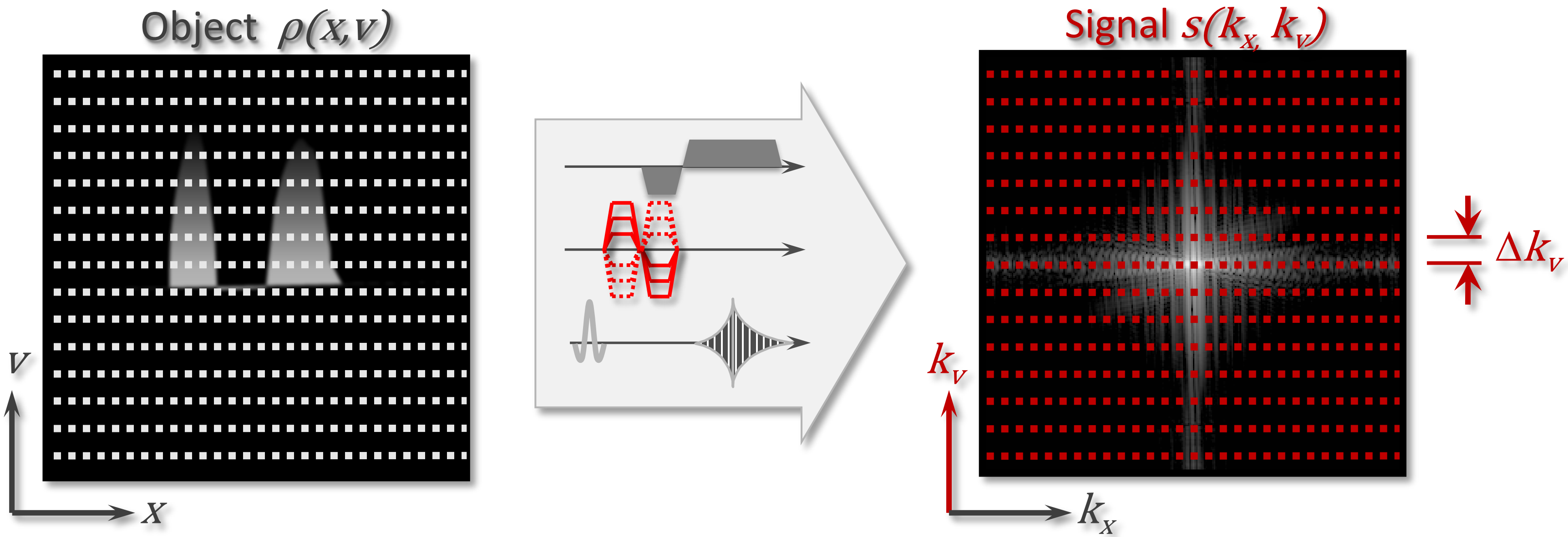
Claudio Santelli

Flow encoding

Fourier velocity encoding (FVE)



Fourier velocity encoding (FVE)



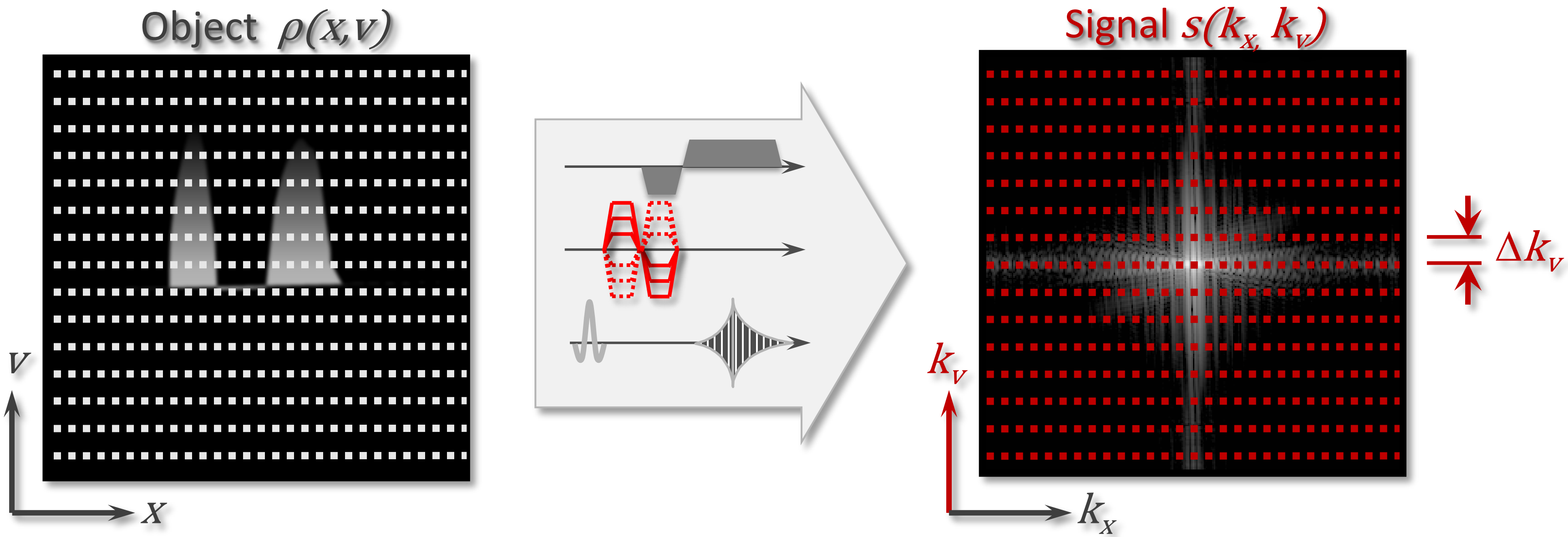
$$s(\mathbf{k_x}(t),\mathbf{k_v}(t)) = \iint \rho(\mathbf{x},\mathbf{v})\,e^{-j\,[\mathbf{k_x}(t)\cdot\mathbf{x} + \mathbf{k_v}(t)\cdot\mathbf{v}]}\,d\mathbf{x}\,d\mathbf{v}$$

$$\mathbf{k_x}(t) = \gamma \int_0^t \mathbf{G}(\tau)\,d\tau \quad , \quad \mathbf{k_v}(t) = \gamma \int_0^t \tau\mathbf{G}(\tau)\,d\tau$$

FOV … field-of-view

FOS … field-of-speed

Fourier velocity encoding (FVE)



$$s(n,m) = \sum_{x_i} \sum_{v_j} \rho(x_i, v_i) e^{-jn\,\Delta k_x x_i} e^{-jm\,\Delta k_v v_j}$$

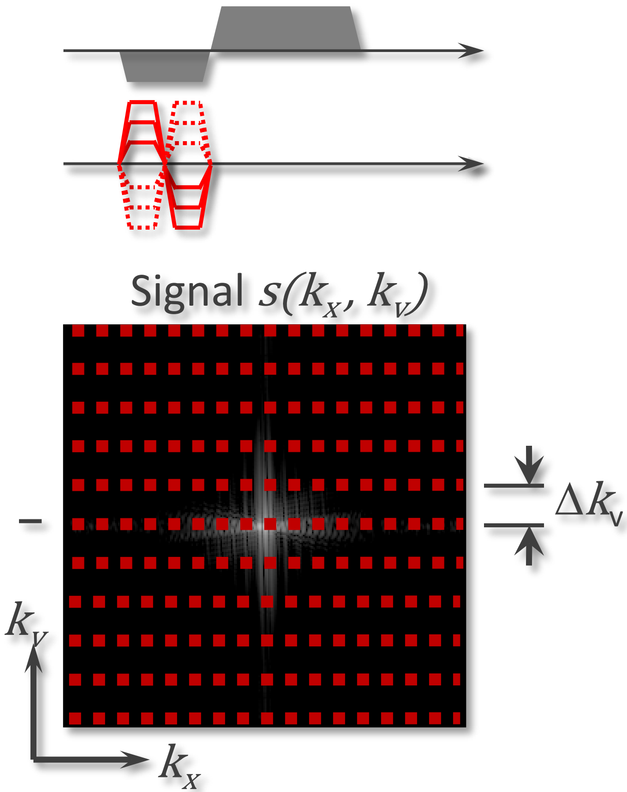
$$\Delta k_x = \frac{2\pi}{FOV} \quad , \quad \Delta k_v = \frac{2\pi}{FOS} = \frac{\pi}{V_{enc}}$$

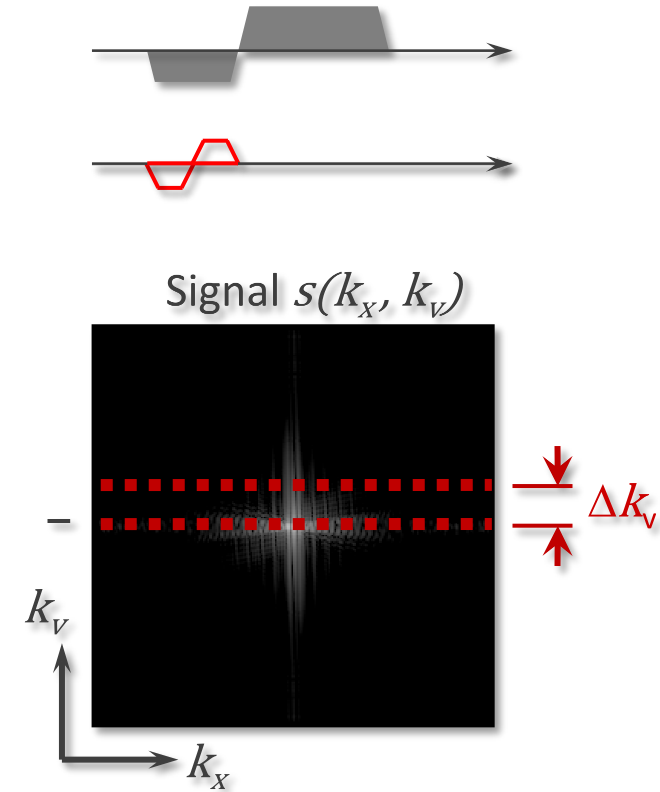
FOV … field-of-view

FOS … field-of-speed

$$\mathbf{s} = \mathbf{E}\boldsymbol{\rho}$$

FVE vs. phase contrast (PC)





$$\rho(x,v) = \sum_{k_x} \sum_{k_v} s(k_x, k_v) e^{j k_x x} e^{j k_v v}$$

$$\rho_A(x,y) = \sum_{k_x} s(k_x, k_v = 0) e^{j k_x x}$$

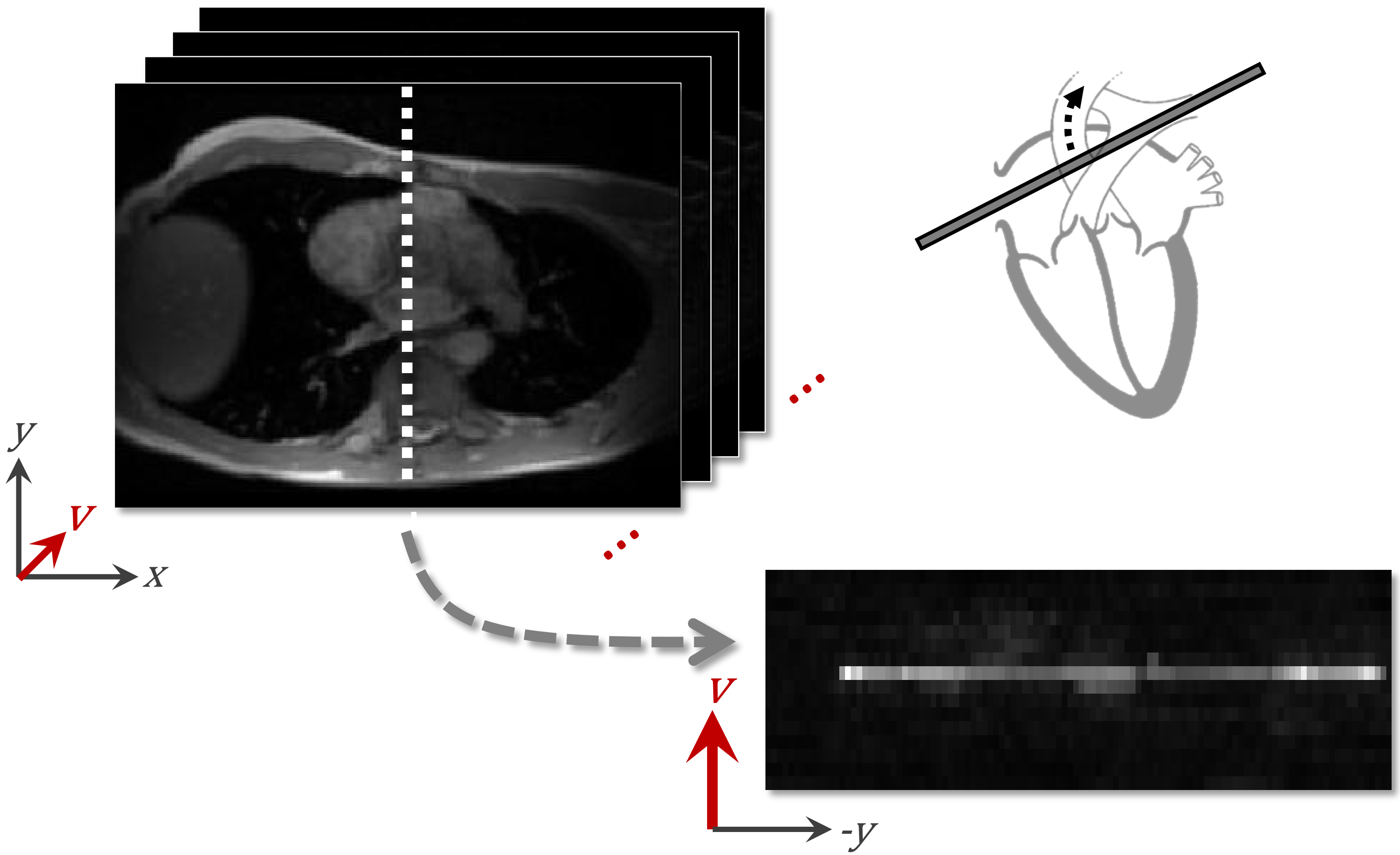
$$\rho_B(x,y) = \sum_{k_x} s(k_x, k_v = \Delta k_v) e^{j k_x x} e^{j \Delta k_v v}$$

Phase difference

$$\angle \rho(x,y) = \angle\left(\rho_A^* \cdot \rho_B\right) = \Delta k_v v$$

Fourier velocity encoding

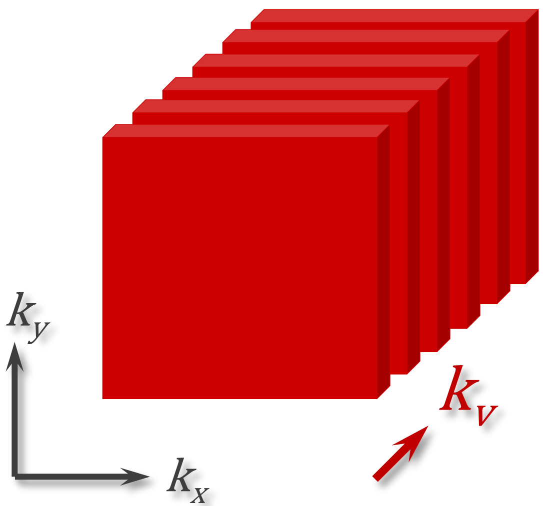
- **Velocity distributions in ascending aorta**

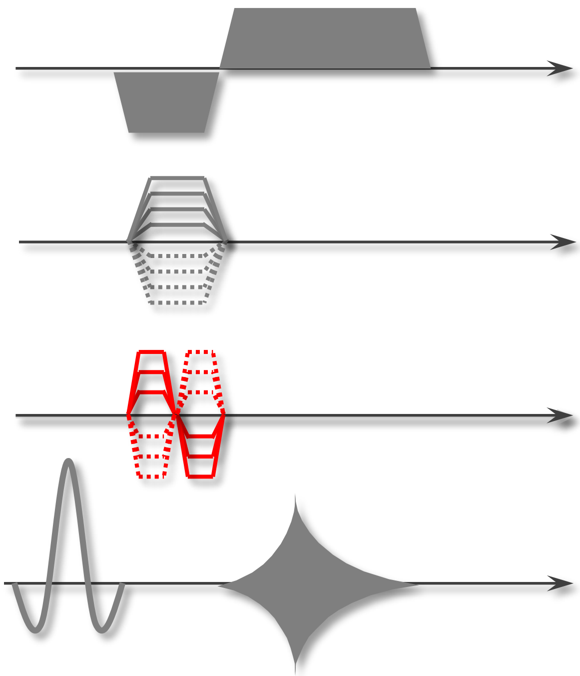


Baltes C, et al.

Fourier velocity encoding

- **Scan time**





$$t_{scan} \propto N_y \, \boldsymbol{N_v}$$

Speed-up

SPIRiT

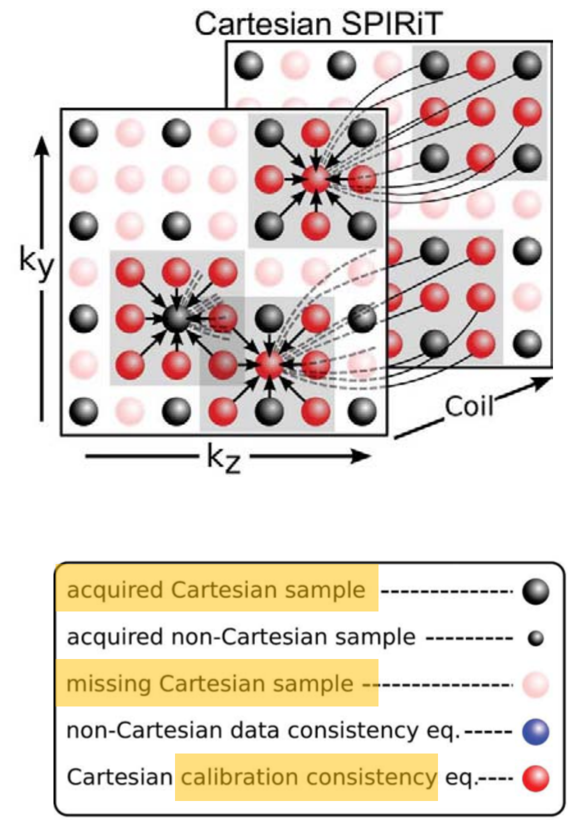
- "Iterative self-consistent parallel imaging reconstruction"

- Full and shift invariant interpolation kernel, i.e. convolution in k-space

- $\mathbf{s}_c = \mathbf{G}\mathbf{s}_c$ Consistency equation for *all* k-space points

  $\mathbf{s}_c$: Cartesian k-space over all coils

  $\mathbf{G}$: Interpolation matrix (Convolution operator)



Calibration consistency:

Minimize $\|(\mathbf{G}-\mathbf{I})\mathbf{s}_c\|_2^2$

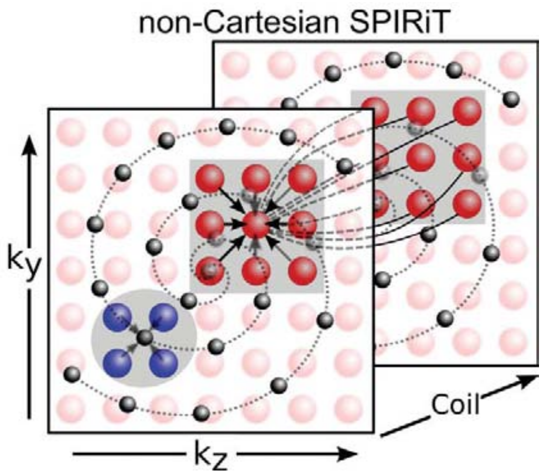
Lustig M et al. MRM 2010

- Data consistency:

  Minimize $\|(\mathbf{G}-\mathbf{I})\mathbf{s}_c\|_2^2$, s.t. $\|\mathbf{D}\mathbf{s}_c-\mathbf{s}\|_2^2 \le \varepsilon$

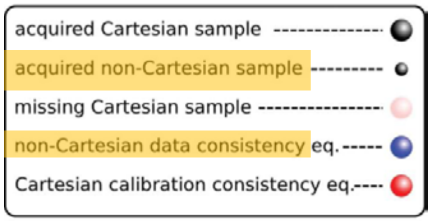
  $\mathbf{D}$: Linear Operator which maps reconstructed

  Cartesian k-space to the acquired data $\mathbf{s}$



- Image priors

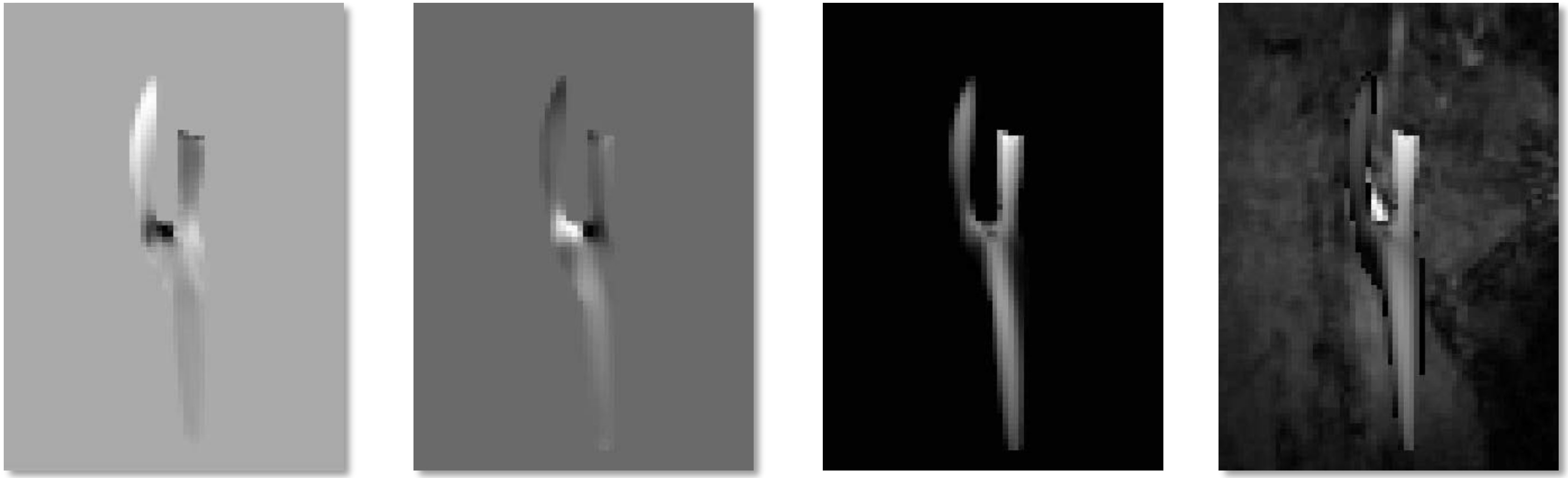
$$\underset{\mathbf{s}_c}{\operatorname{argmin}} \ \|\mathbf{D}\mathbf{s}_c-\mathbf{s}\|_2^2+\lambda_1\|(\mathbf{G}-\mathbf{I})\mathbf{s}_c\|_2^2+\sum_{j\geq 2}\lambda_j R_j(\mathbf{s}_c)$$



Lustig M et al. MRM 2010

SPIRiT – PC (Cartesian)

- CFD model (SNR = 30, 66 x 89 x 33 voxels)



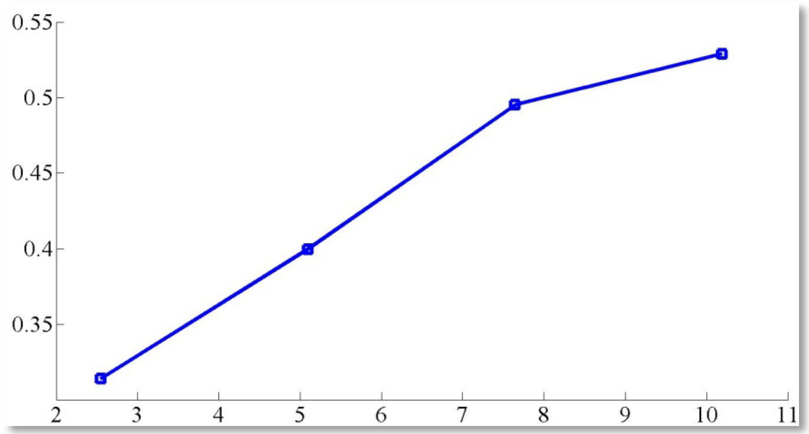
- Image priors (wavelet, total variation)

$$\underset{\mathbf{s}_c}{\operatorname{argmin}} \; \|\mathbf{D}\mathbf{s}_c - \mathbf{s}\|_2^2 + \lambda_1 \|(\mathbf{G}-\mathbf{I})\mathbf{s}_c\|_2^2 + \lambda_2 \|\boldsymbol{\Psi}\boldsymbol{\mathcal{F}}^{-1}\mathbf{s}_c\|_1 + \lambda_3 \|\boldsymbol{\nabla}\boldsymbol{\mathcal{F}}^{-1}\mathbf{s}_c\|_1$$

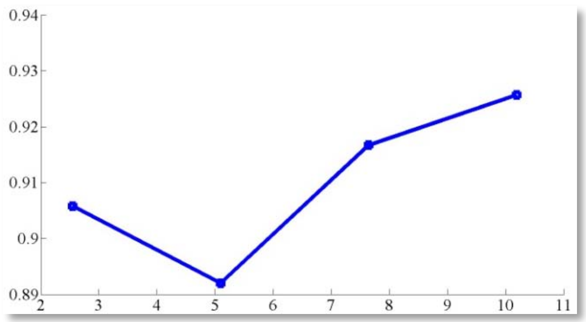
SPIRiT – PC (Cartesian)

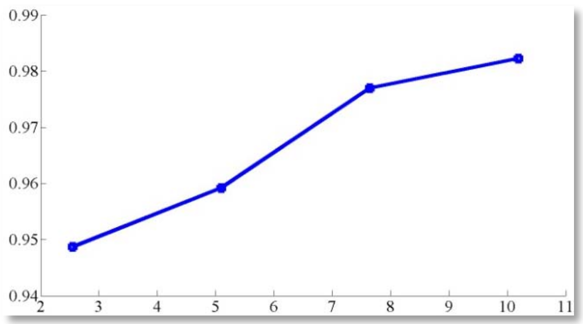
- RMS error vs. undersampling factor

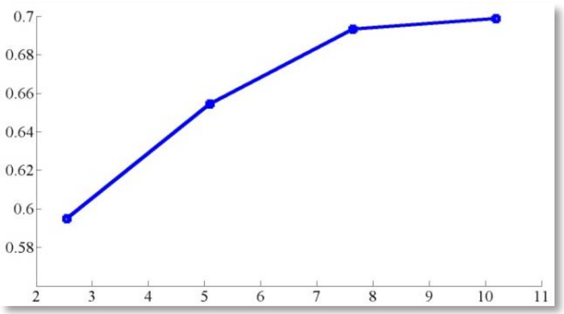
- Magnitude



- Phase







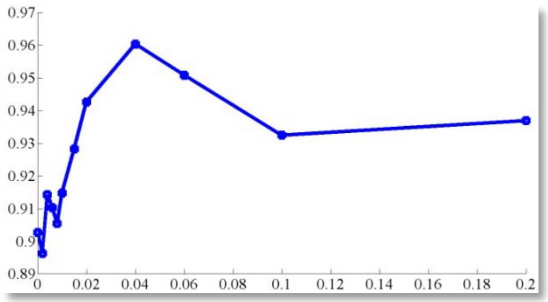
Segm 1 **Segm 2** **Segm 3**

Duerst E

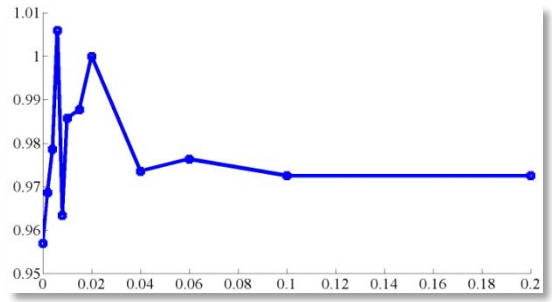
SPIRiT – PC (Cartesian)

- Phase error vs regularization weights (7.5x undersampling)

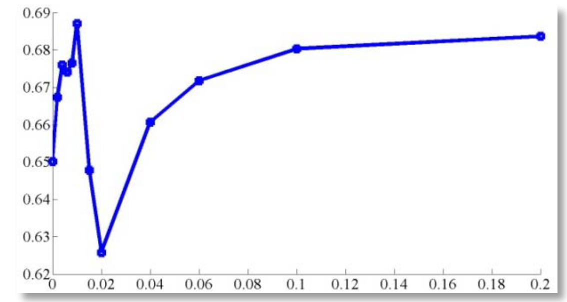
$$\underset{\mathbf{s}_c}{\operatorname{argmin}} \; \|\mathbf{D}\mathbf{s}_c - \mathbf{s}\|_2^2 + \lambda_1\|(\mathbf{G}-\mathbf{I})\mathbf{s}_c\|_2^2 + \lambda_2\|\nabla\mathcal{F}^{-1}\mathbf{s}_c\|_1$$



Segm 1

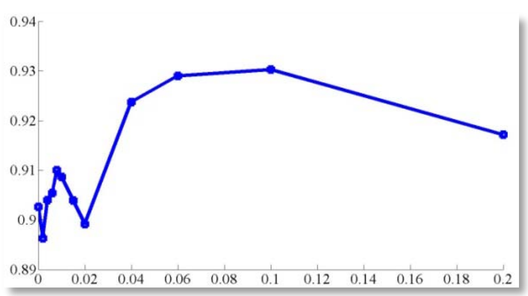


Segm 2

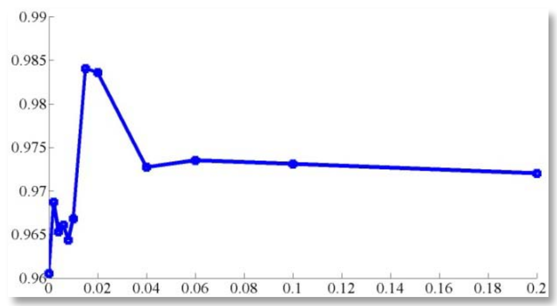


Segm 3

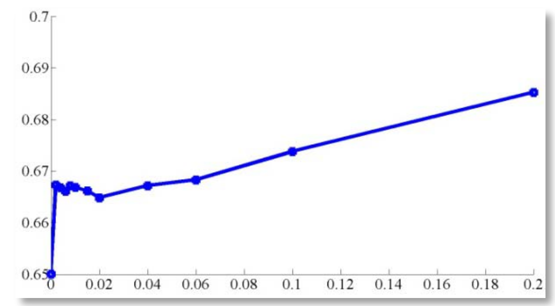
$$\underset{\mathbf{s}_c}{\operatorname{argmin}} \; \|\mathbf{D}\mathbf{s}_c - \mathbf{s}\|_2^2 + \lambda_1\|(\mathbf{G}-\mathbf{I})\mathbf{s}_c\|_2^2 + \lambda_2\|\Psi\mathcal{F}^{-1}\mathbf{s}_c\|_1$$



Segm 1



Segm 2

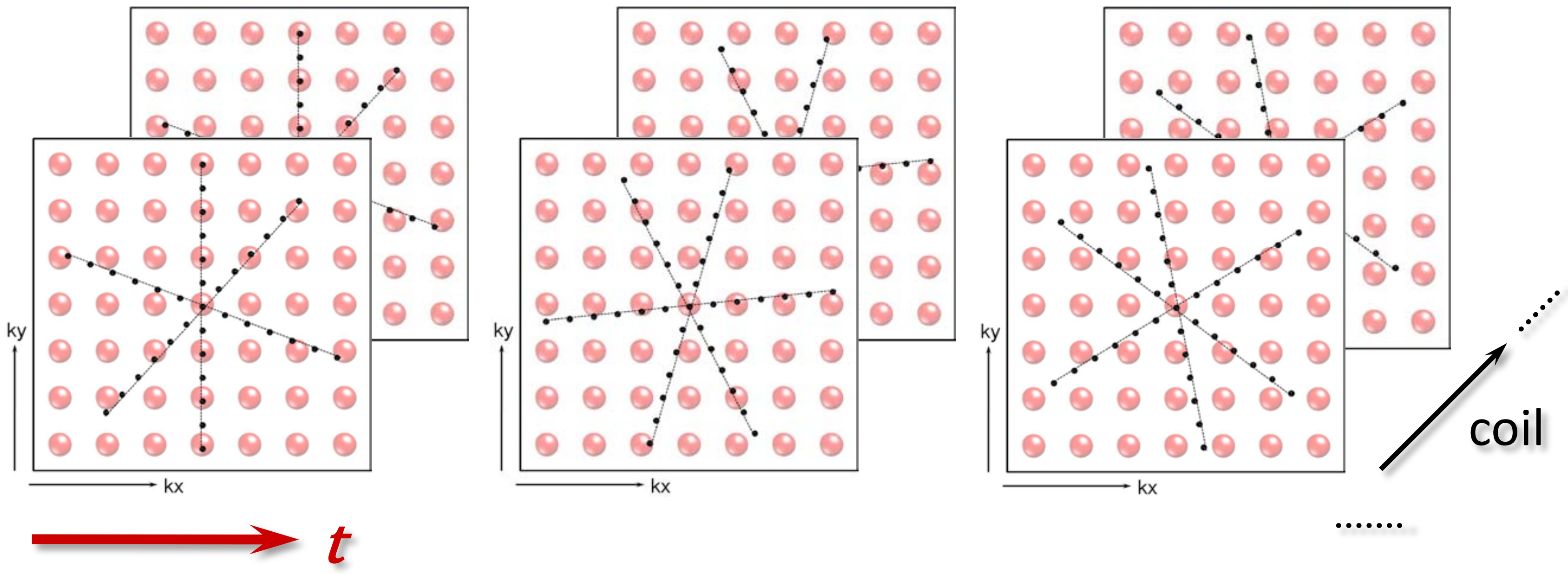


Segm 3

Duerst E

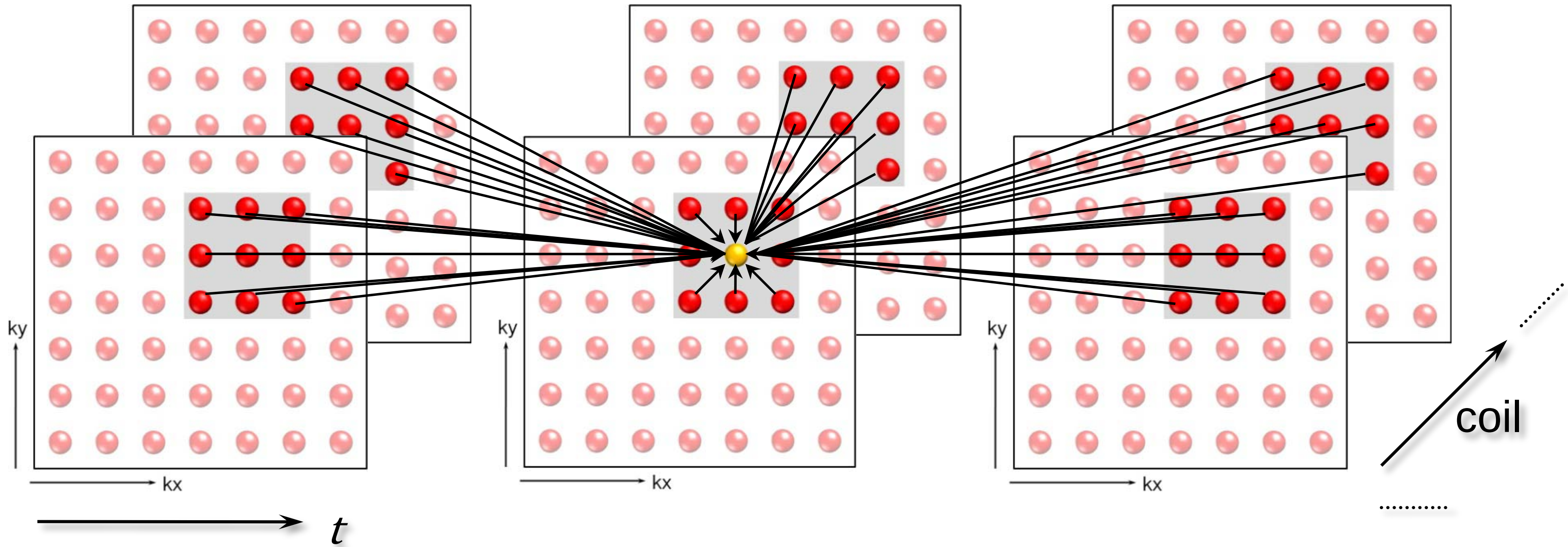
Dynamic imaging

- Dynamic imaging
- Arbitrary k-space trajectories (e.g. golden angle profile)



Dynamic imaging

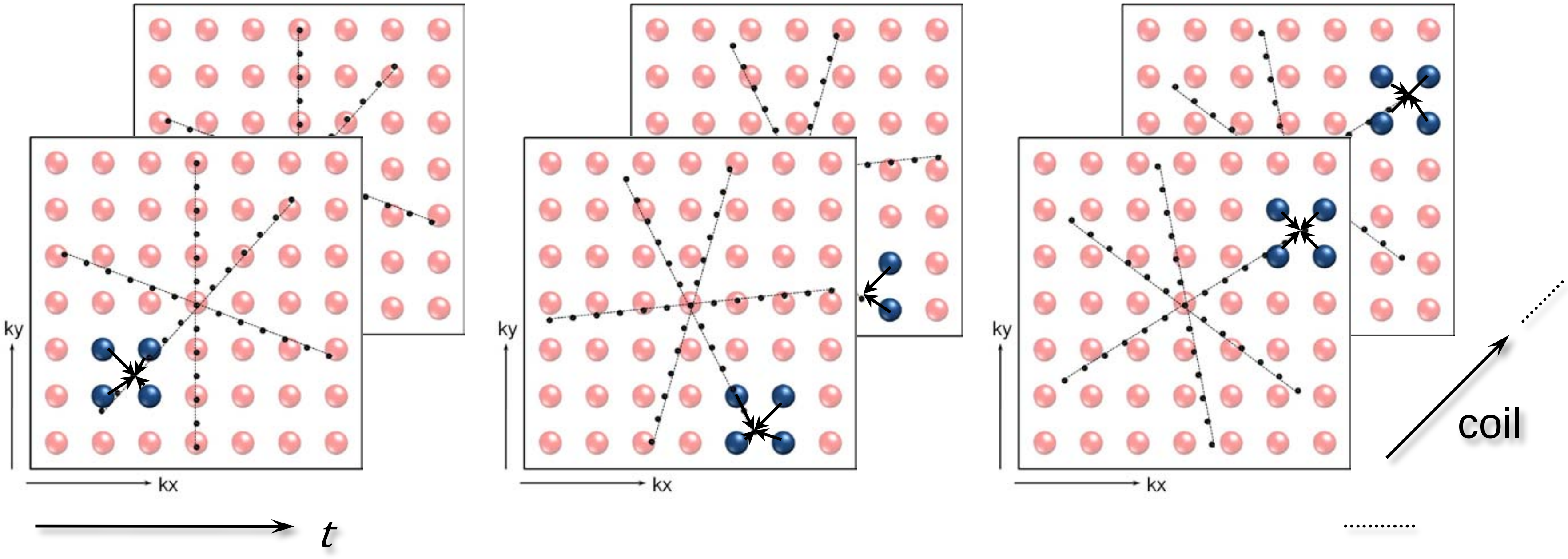
- Calibration consistency



Minimize $\|(\mathbf{G}-\mathbf{I})\mathbf{s}_c\|_2^2$

Dynamic imaging

- Data consistency



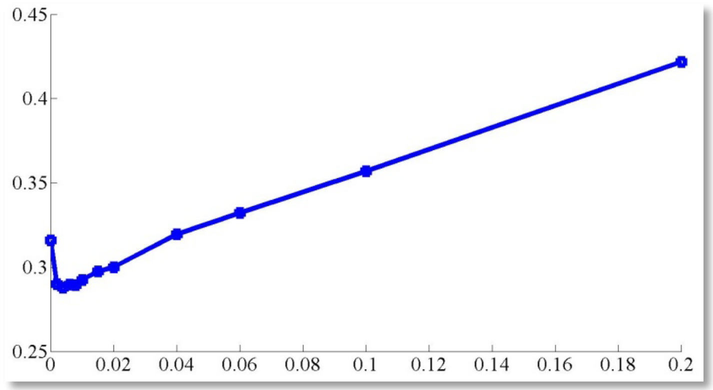
Minimize $\|(\mathbf{G}-\mathbf{I})\mathbf{s}_c\|_2^2$, s.t. $\|\mathbf{D}\mathbf{s}_c-\mathbf{s}\|_2^2 \leq \varepsilon$

Temporal Fourier transform – PC (Cartesian)

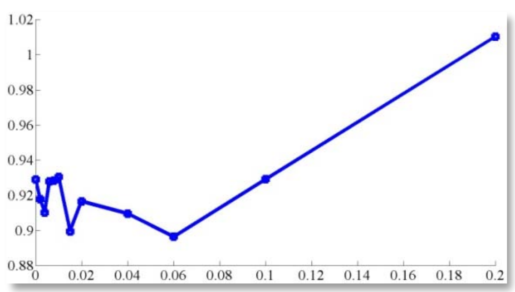
- RMS error vs. regularization weight

$$\underset{\mathbf{s}_c}{\operatorname{argmin}} \ \|\mathbf{D}\mathbf{s}_c - \mathbf{s}\|_2^2 + \lambda_1 \|(\mathbf{G}-\mathbf{I})\mathbf{s}_c\|_2^2 + \lambda_2 \|\mathcal{F}_t\mathcal{F}^{-1}\mathbf{s}_c\|_1$$

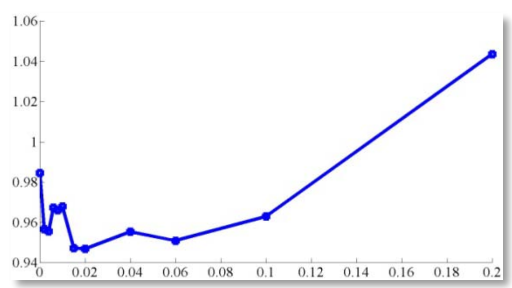
- Magnitude



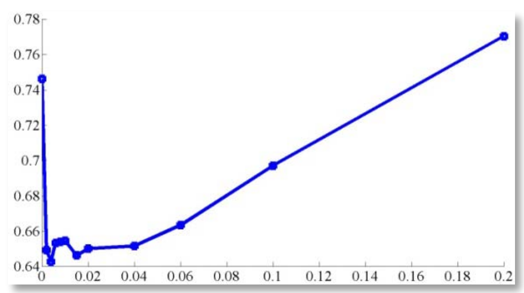
- Phase



Segm 1



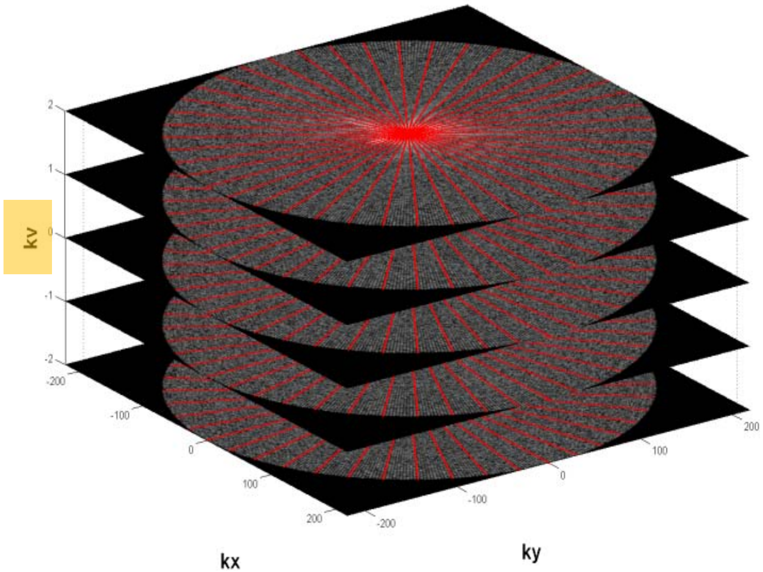
Segm 2

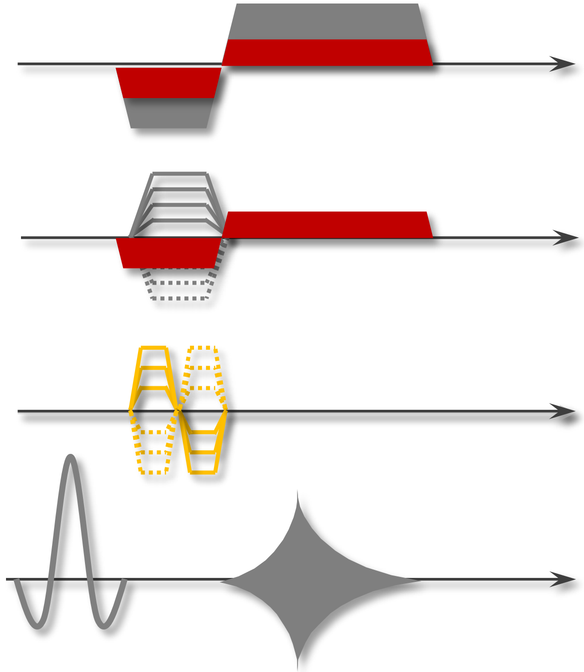


Segm 3

Radial FVE (rFVE)

- **Data acquisition**

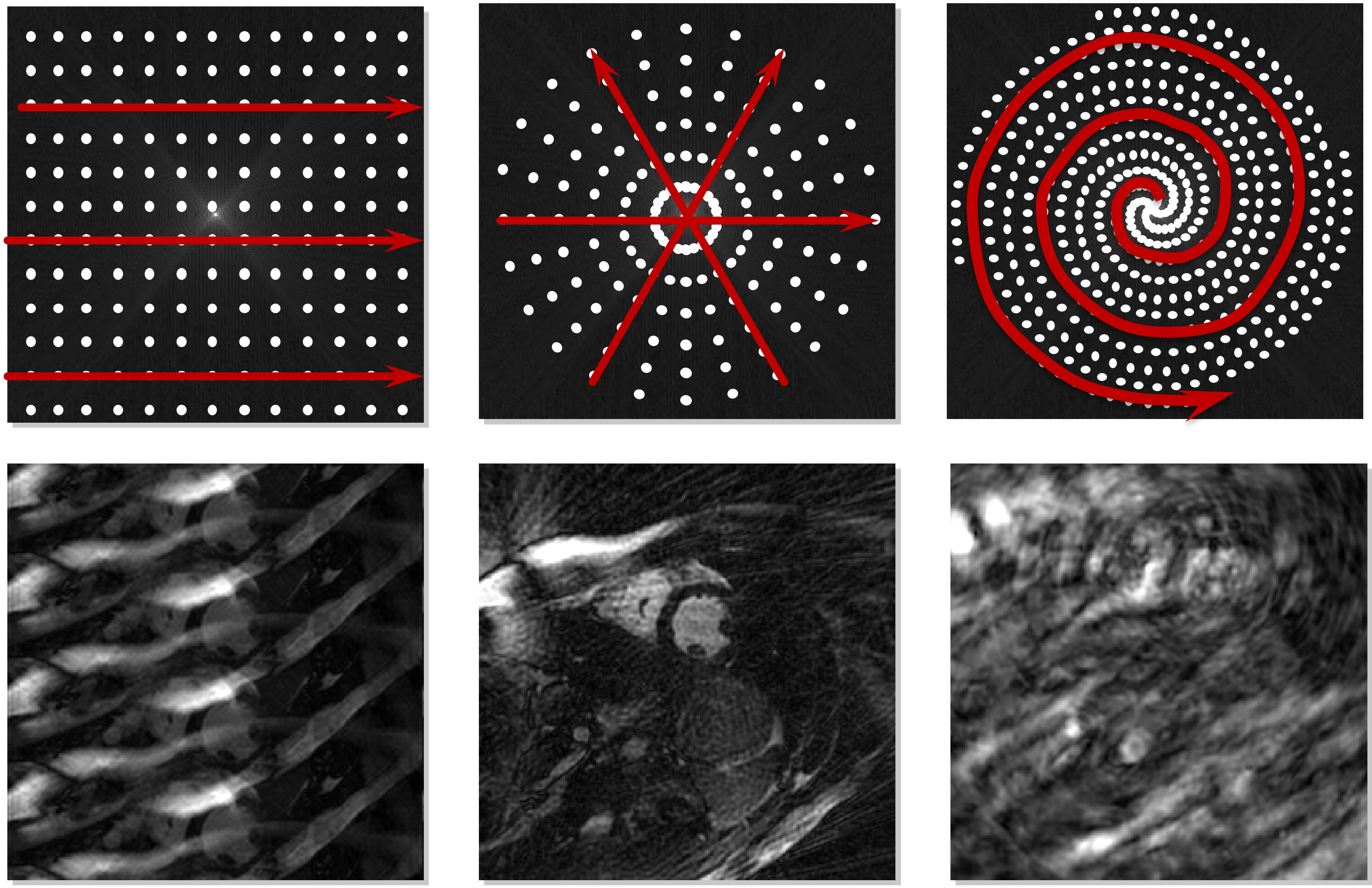




- **Why radial?**
  - Shorter minimum TE
  - Oversampled center of k-space (Low res., training/acquisition stage, contrast)
  - Motion (Low sensitivity, streak artifacts, motion tracking)

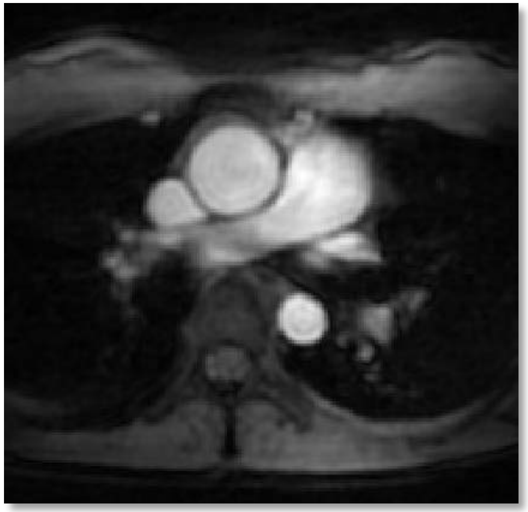
Radial FVE (rFVE)

– Undersampling artifacts (4x, Cartesian, radial, spiral)

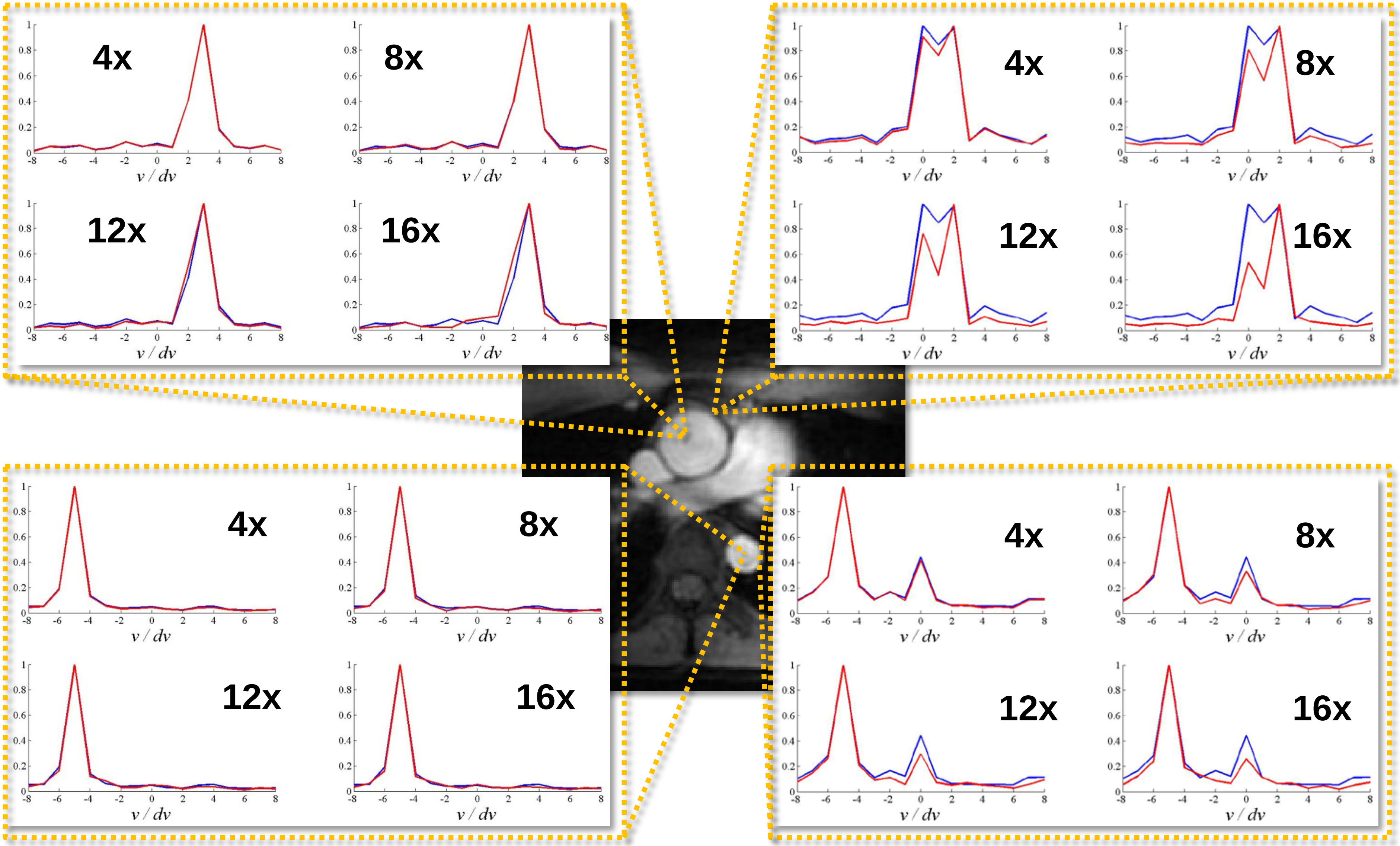


In-vivo - k-t rFVE

- 2D radial (FFE)
- FOV = 250 x 250 mm, voxel size = 2 x 2 x 10 mm
- 24 heart phases, 35.5 ms, 157.1 % (100% radial Nyquist)
- 16 velocity encodes + 1 ref
- Kernel size = 7 x 7 x 3 (k_x - k_y - t), 30 x 30 x 24 calibration area



In-vivo - k-t rFVE



Discussion

Discussion

- Wavelet- or TV-l1 minimization seem not to be suitable for frame-by-frame reconstruction in dynamic PC-MRI
- Exploiting temporal correlations with temporal FT as a sparsifier can improve phase reconstruction accuracy
- rFVE has the potential to significantly accelerate FVE, and therefore, to assign complex flow patterns
  - Inclusion of priors (e.g. sparsity)

$$\underset{\mathbf{s}_c}{\operatorname{argmin}} \ \|\mathbf{D}\mathbf{s}_c - \mathbf{s}\|_2^2 + \lambda_1 \|(\mathbf{G} - \mathbf{I})\mathbf{s}_c\|_2^2 + \sum_{j \geq 2} \lambda_j R_j(\mathbf{s}_c)$$

  - Self-gating and motion correction

$$\underset{\mathbf{s}_c}{\operatorname{argmin}} \ \|\mathbf{D}\mathbf{T}\mathbf{s}_c - \mathbf{y}\|_2^2 + \lambda \|(\mathbf{G} - \mathbf{I})\mathbf{T}\mathbf{s}_c\|_2^2$$