A general framework for static and dynamic tomography with regularity constraints

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in collaboration with

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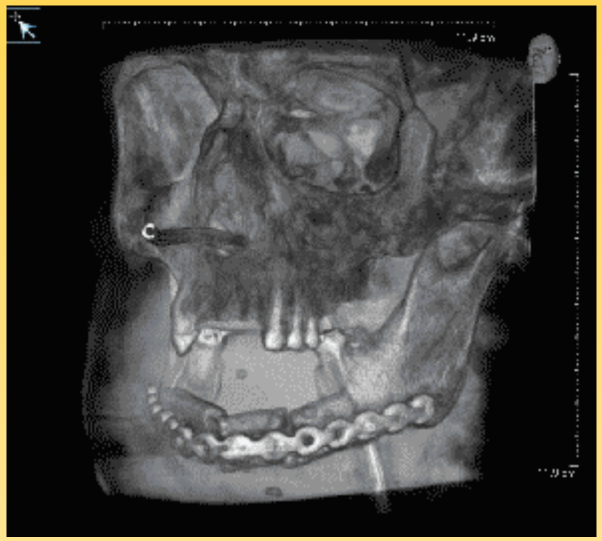
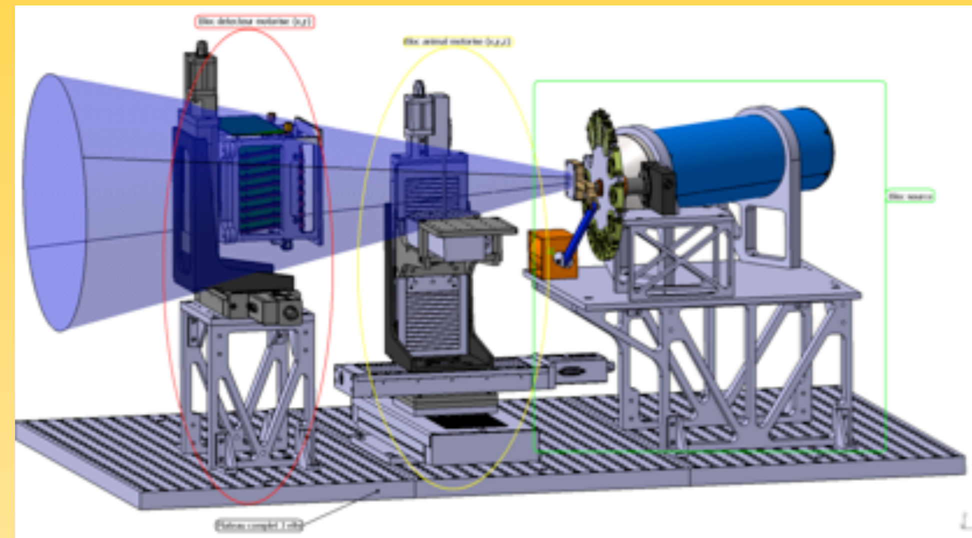
BASP Frontiers, 4th - 9th September 2011 - Villars, Switzerland

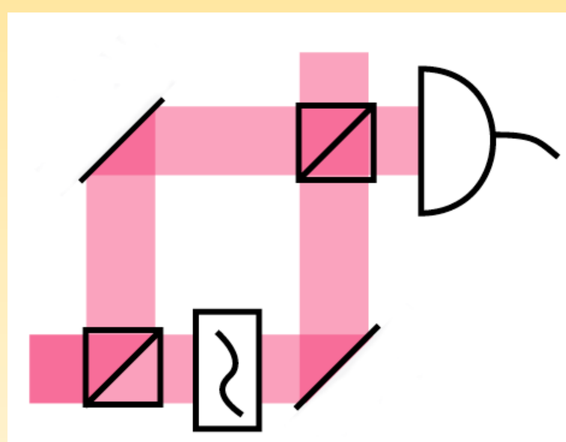
Tomography ...

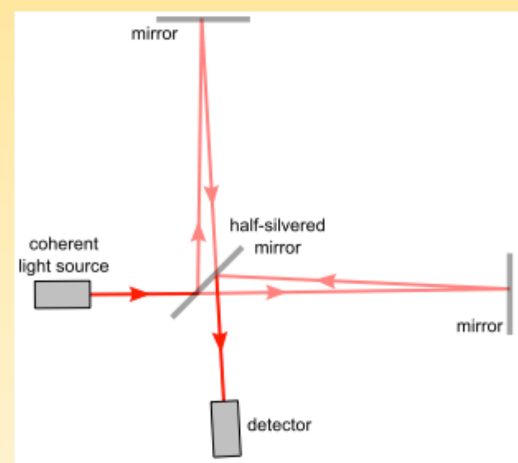
- Absorption tomography (X-ray tomography)





- Phase tomography (with interferometry)





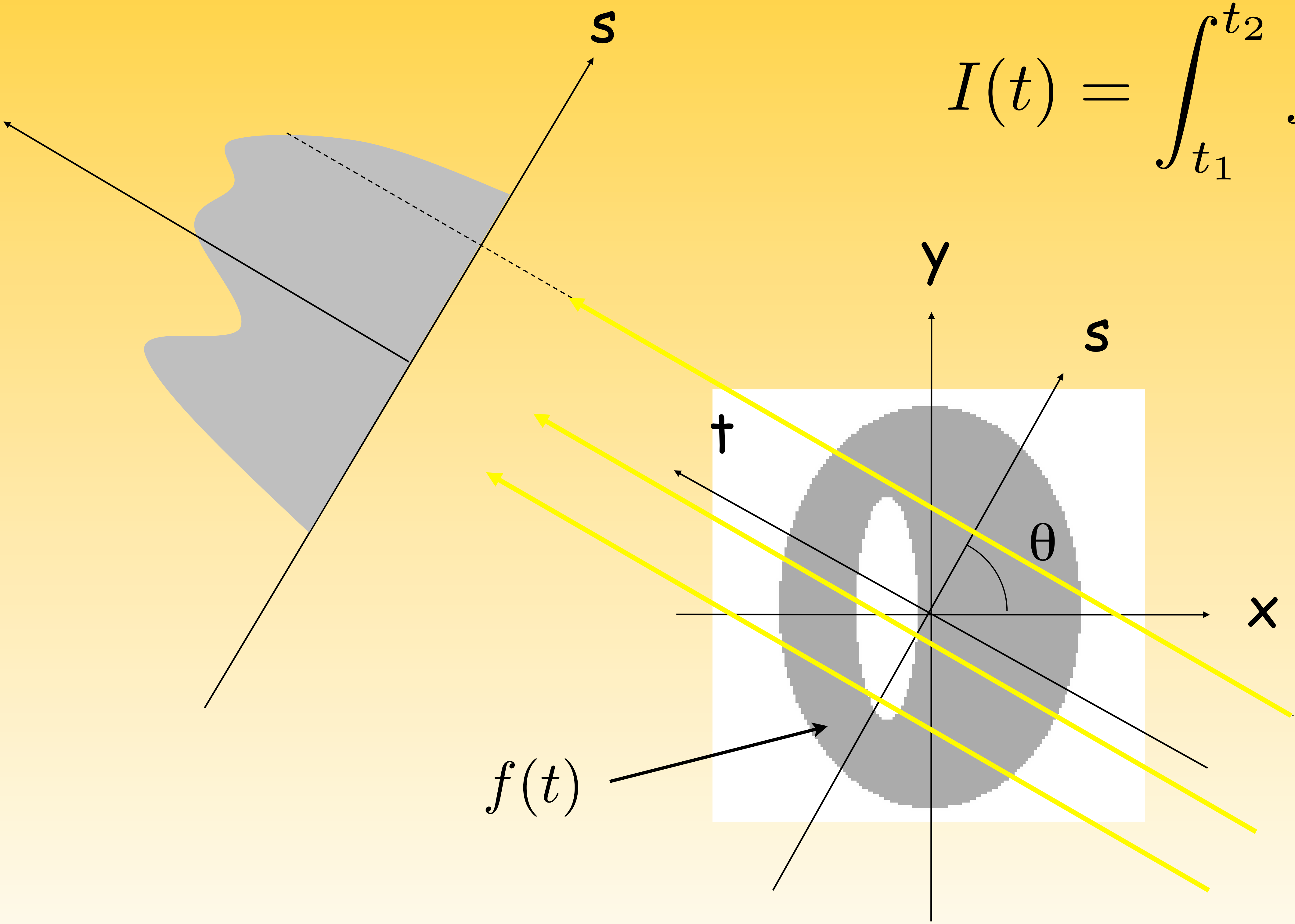
- ... and also Optical Computed, Ultrasound, Deflection, Positron Emission Tomographies ...

Outline

- I - Recalls on tomography
- II - Two biomedical applications:
  - a - Cone-Beam Computerized Tomography
  - b - Positron Emission Tomography
- III - An astronomical application : Solar Rotational Tomography
- Conclusion

I - Recalls on Tomography

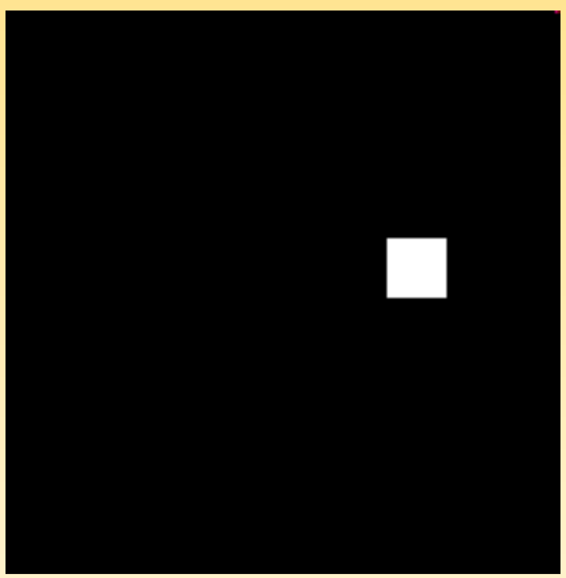
$$I(t) = \int_{t_1}^{t_2} f(t)dt$$



I - Recalls on Tomography

- Basis of tomography : data in 1D + angle, object in 2D.

Image to reconstruct



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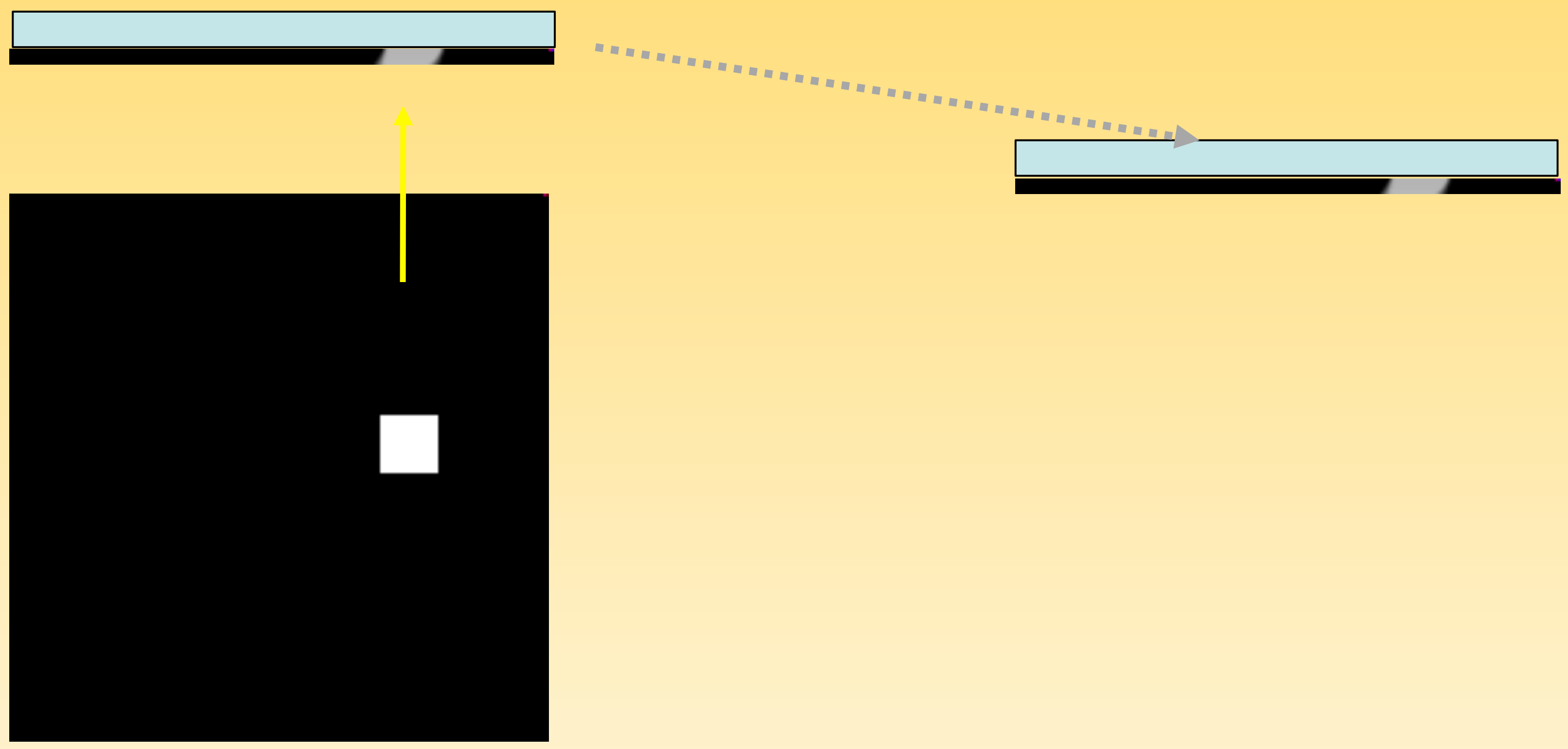

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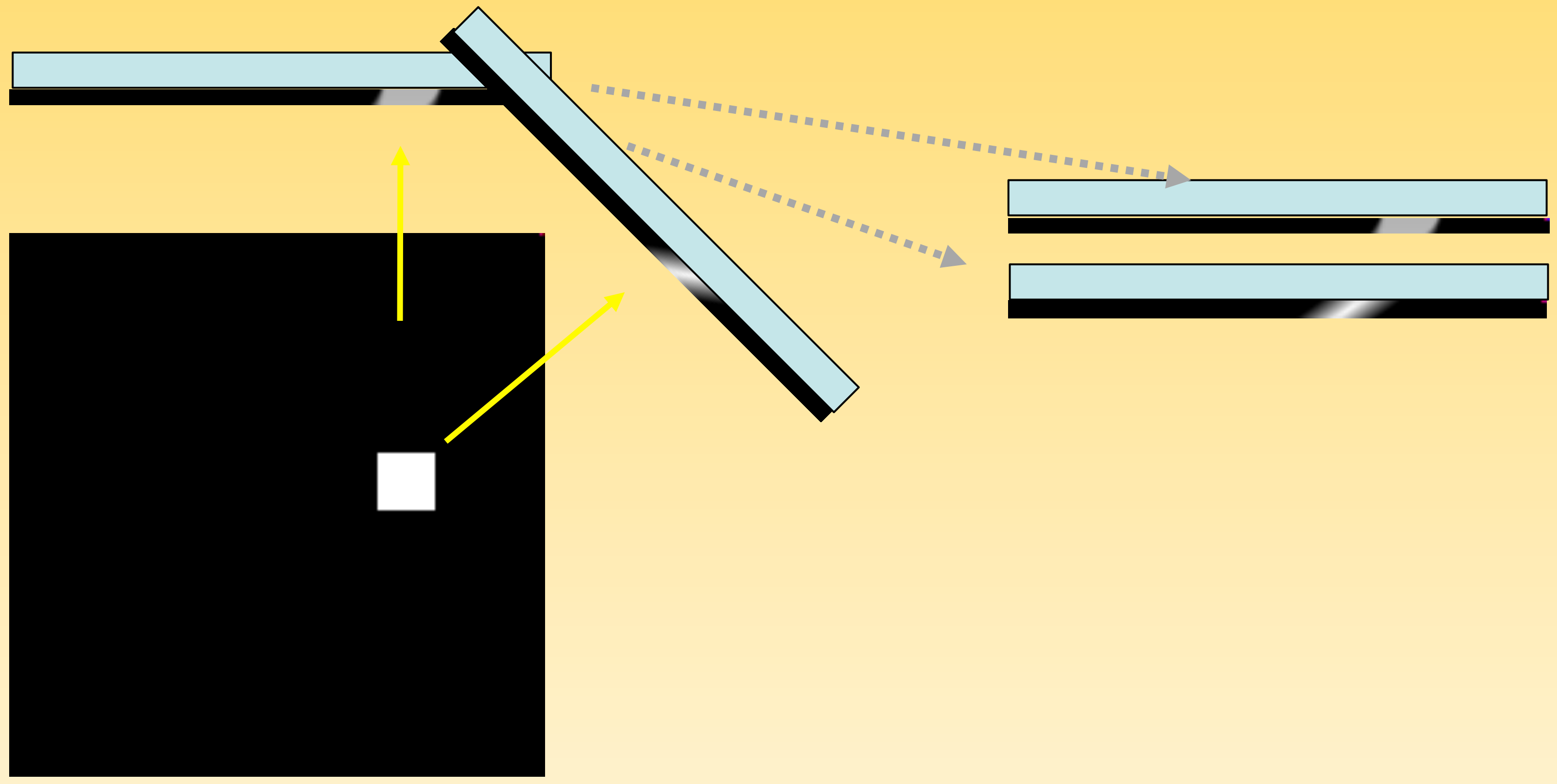

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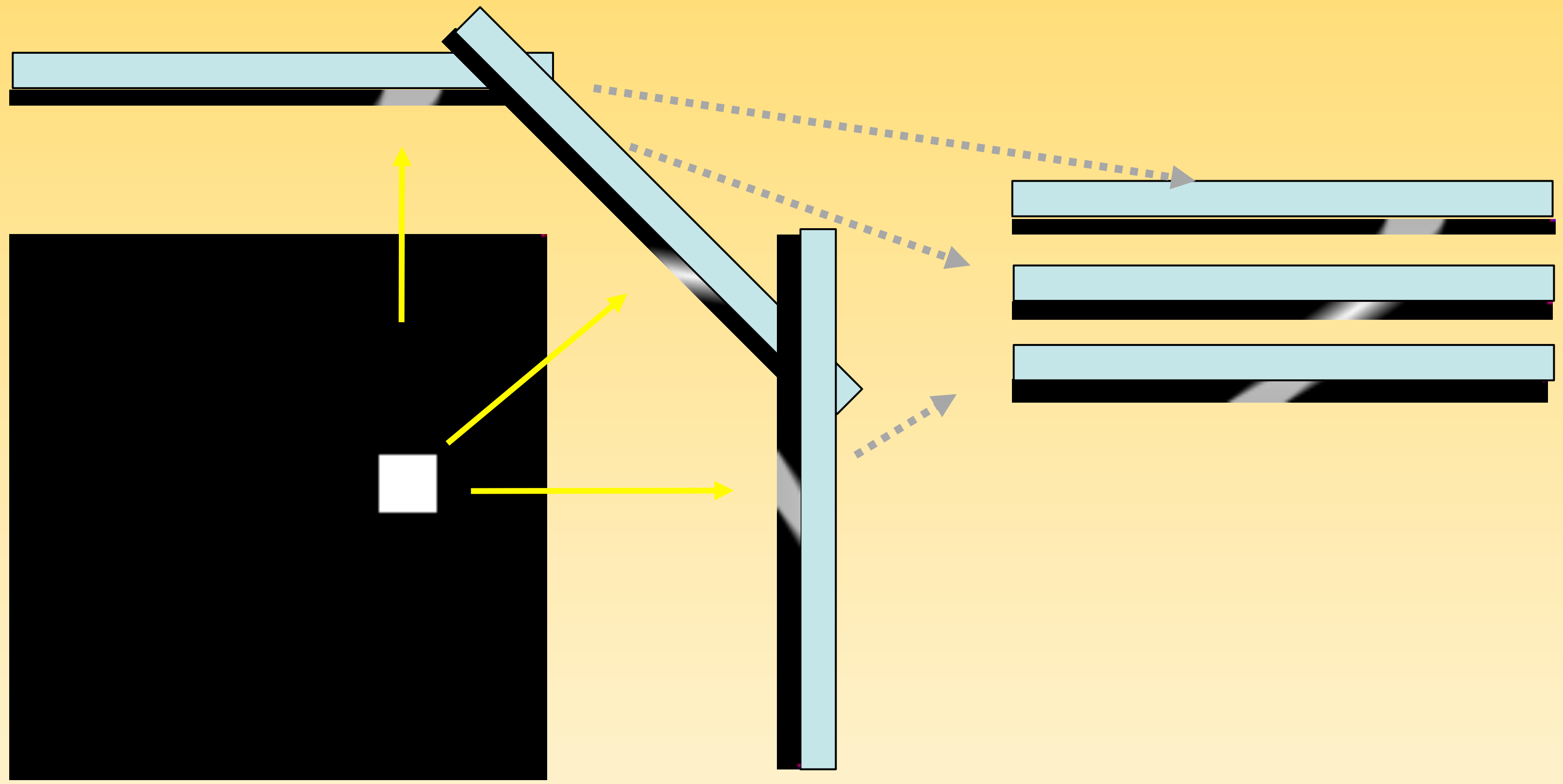

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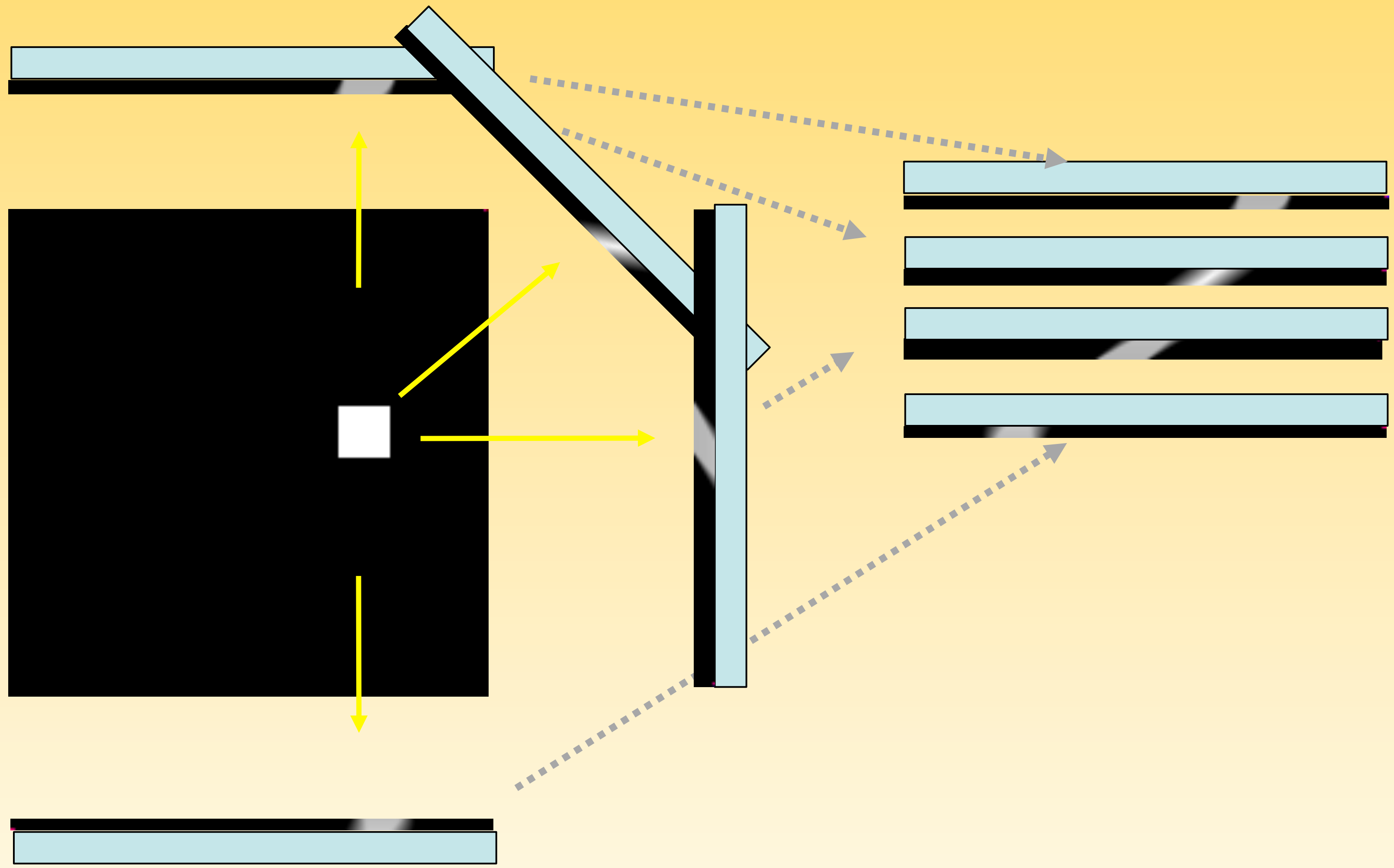

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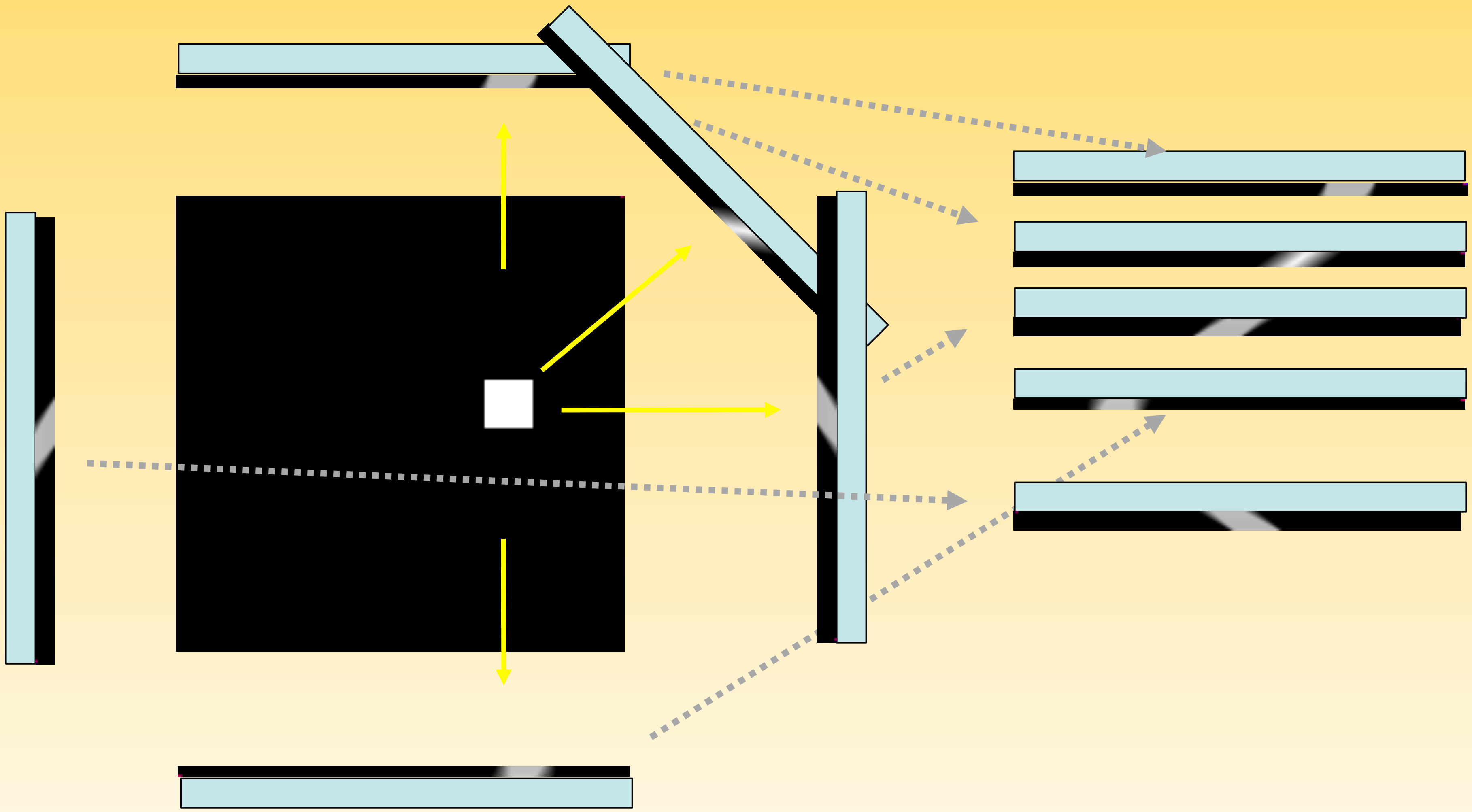

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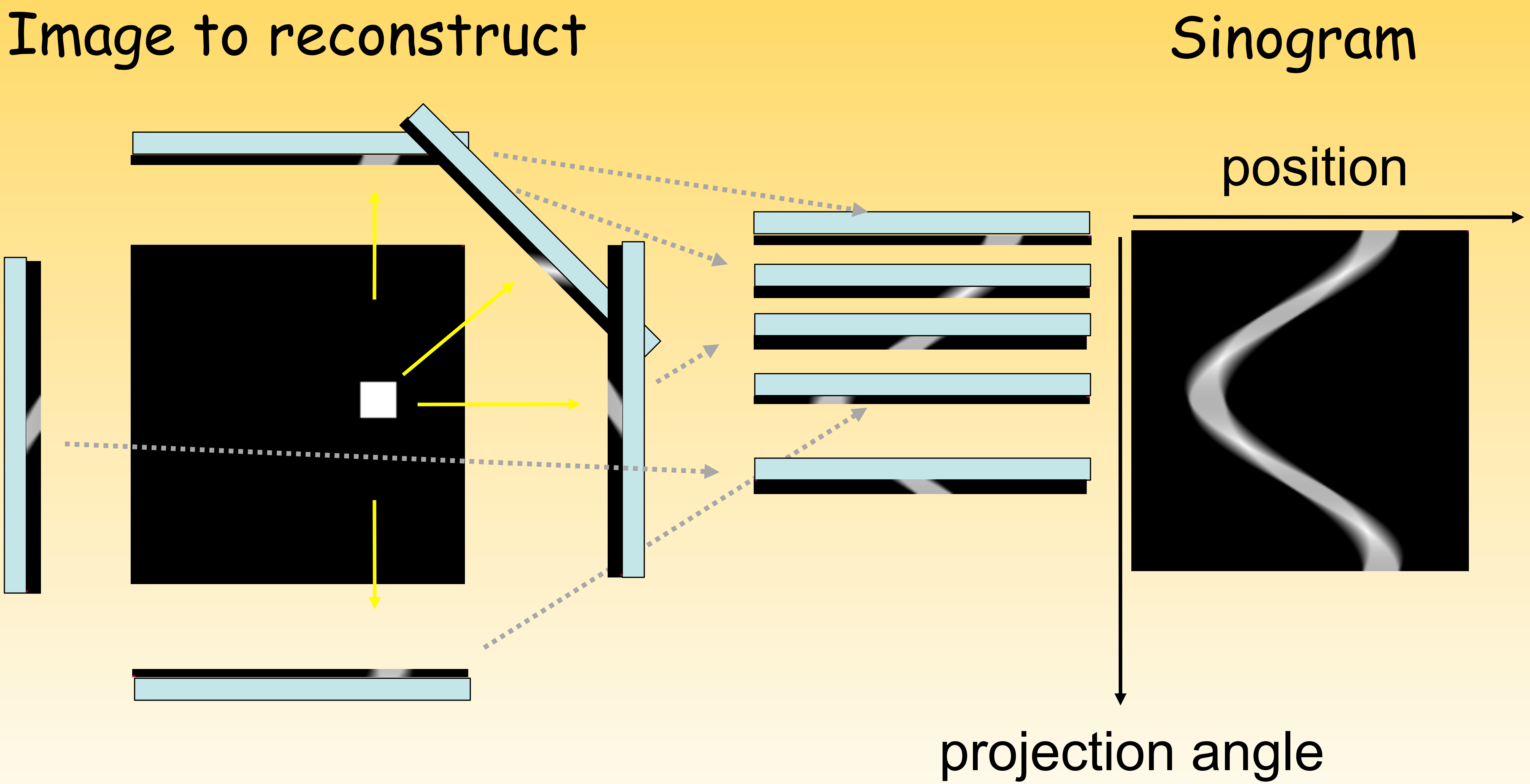
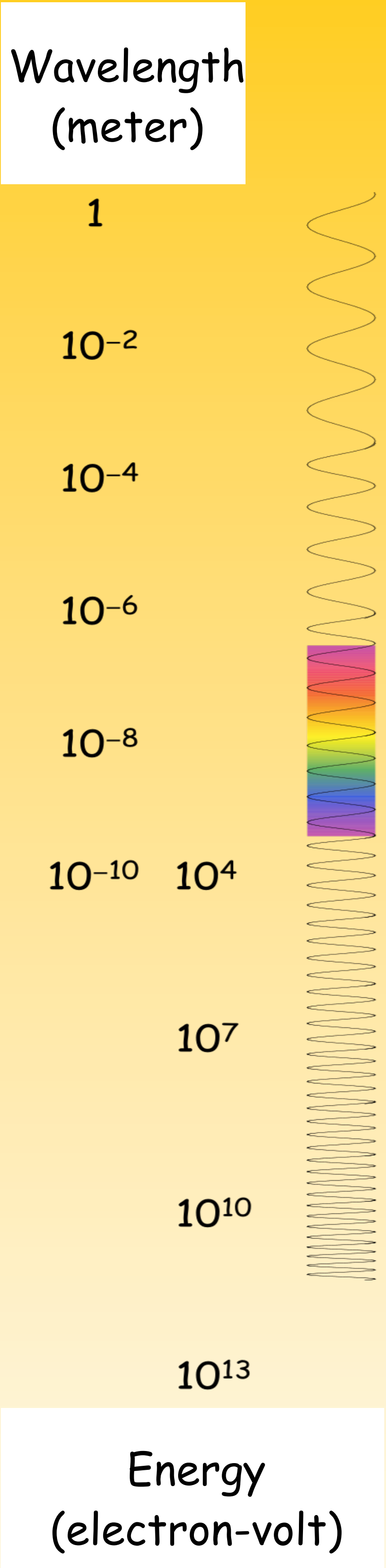
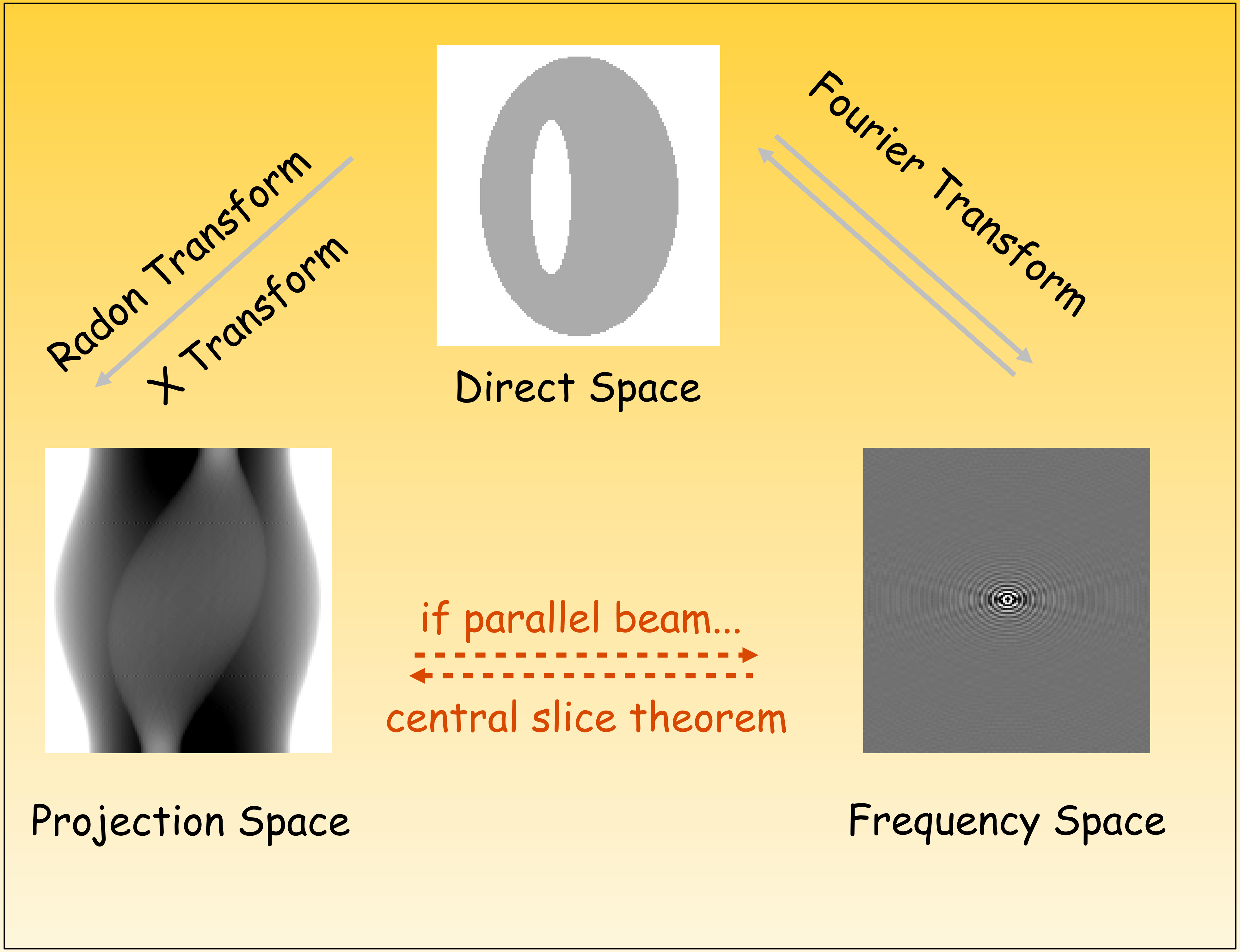


I - Recalls in tomography





I - Recalls in Tomography

Let's define $y \in \mathbb{R}^n$ the measures,

and $x \in \mathbb{R}^m$ the unknown to recover

and $S \in \mathcal{M}(\mathbb{R}^n, \mathbb{R}^m)$ a linear operator.

with $n << m$ in general ...

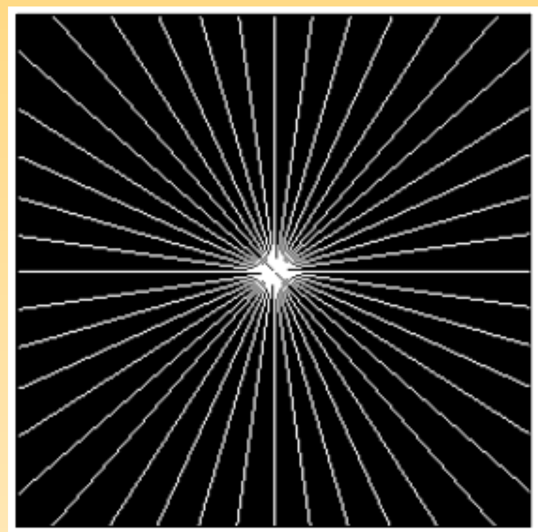
- Tomography is an ill-posed problem.
- System matrices are ill-conditioned operators...
- ... and their rank is almost always lower than n.

I - Then regularization !

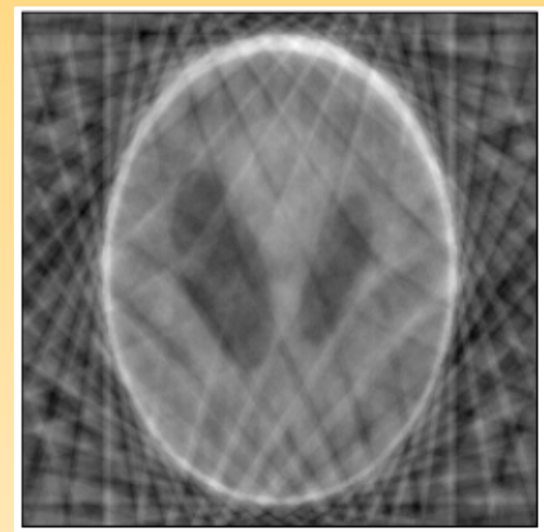
$\tilde{y} = SFx$ with: F Fourier Transform operator

S Selection operator

Shepp-Logan phantom



Fourier acquisition



Backprojection

BPDN - TV

Backprojection

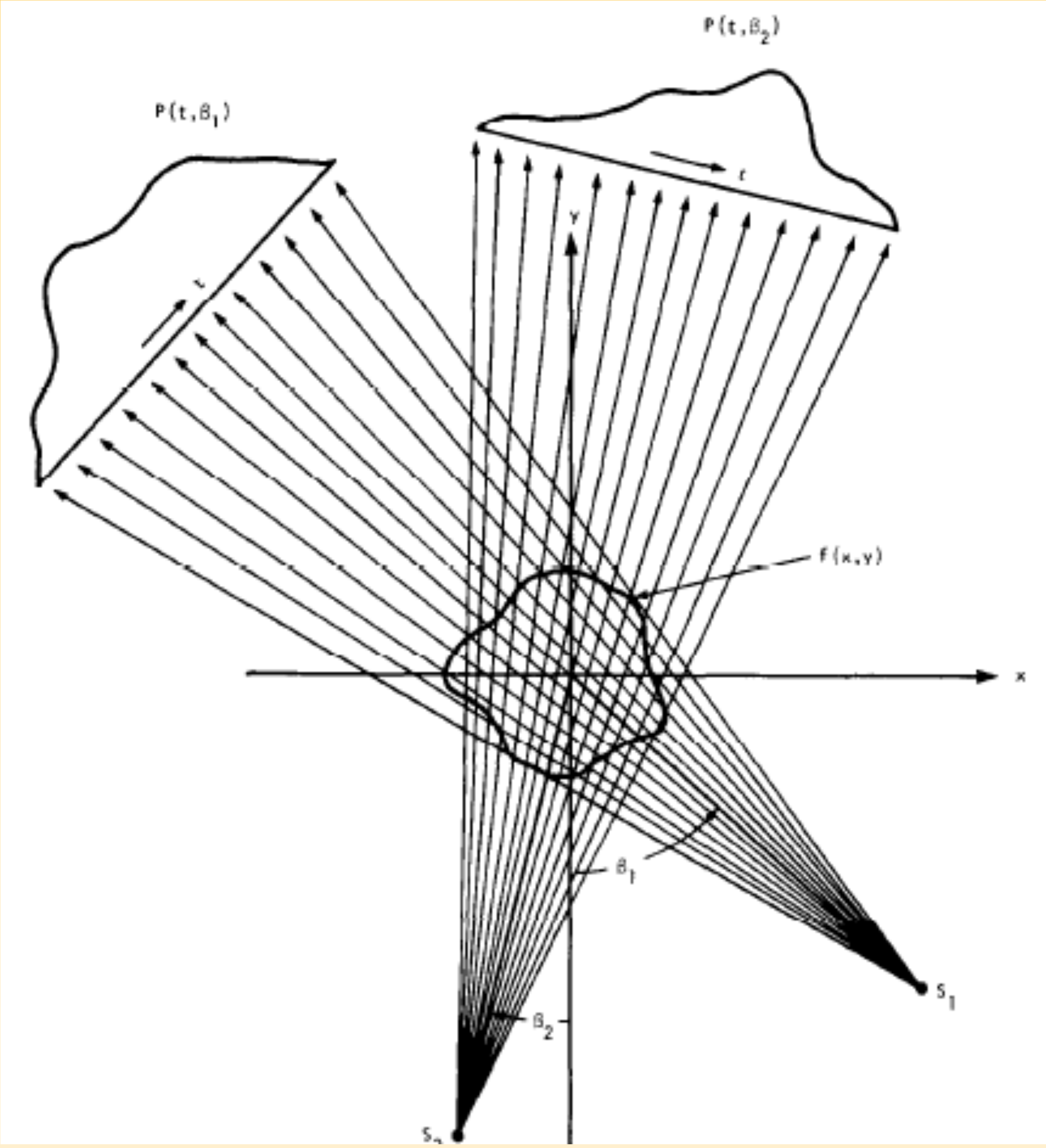
$$\hat{x} = \arg\min_{x} \|x\|_2 \quad \text{s.t.} \quad \tilde{y} = SFx$$

BPDN - TV

$$\hat{x} = \arg\min_{x} \|x\|_{TV} \quad \text{s.t.} \quad \tilde{y} = SFx$$

I - Real life is not parallel ...

- Fan-beam (2D), cone-beam (3D)
- No more Fourier equivalence
- System matrix $A \in \mathcal{M}(\mathbb{R}^n, \mathbb{R}^m)$



From Kak–Slaney :

Principles of Computerized Tomographic Imaging

Acquisition

$$y = Ax \qquad A \sim 10^6 \times \left[10^6 - 10^9\right]$$

Noiseless case

$$\hat{x} = \arg\min_x \|x\|_{TV} \quad \text{s.t.} \quad y = Ax$$

Gaussian Noise case

$$\hat{x} = \arg\min_x \|x\|_{TV} \quad \text{s.t.} \quad \|y - Ax\|_2 \leq \varepsilon$$

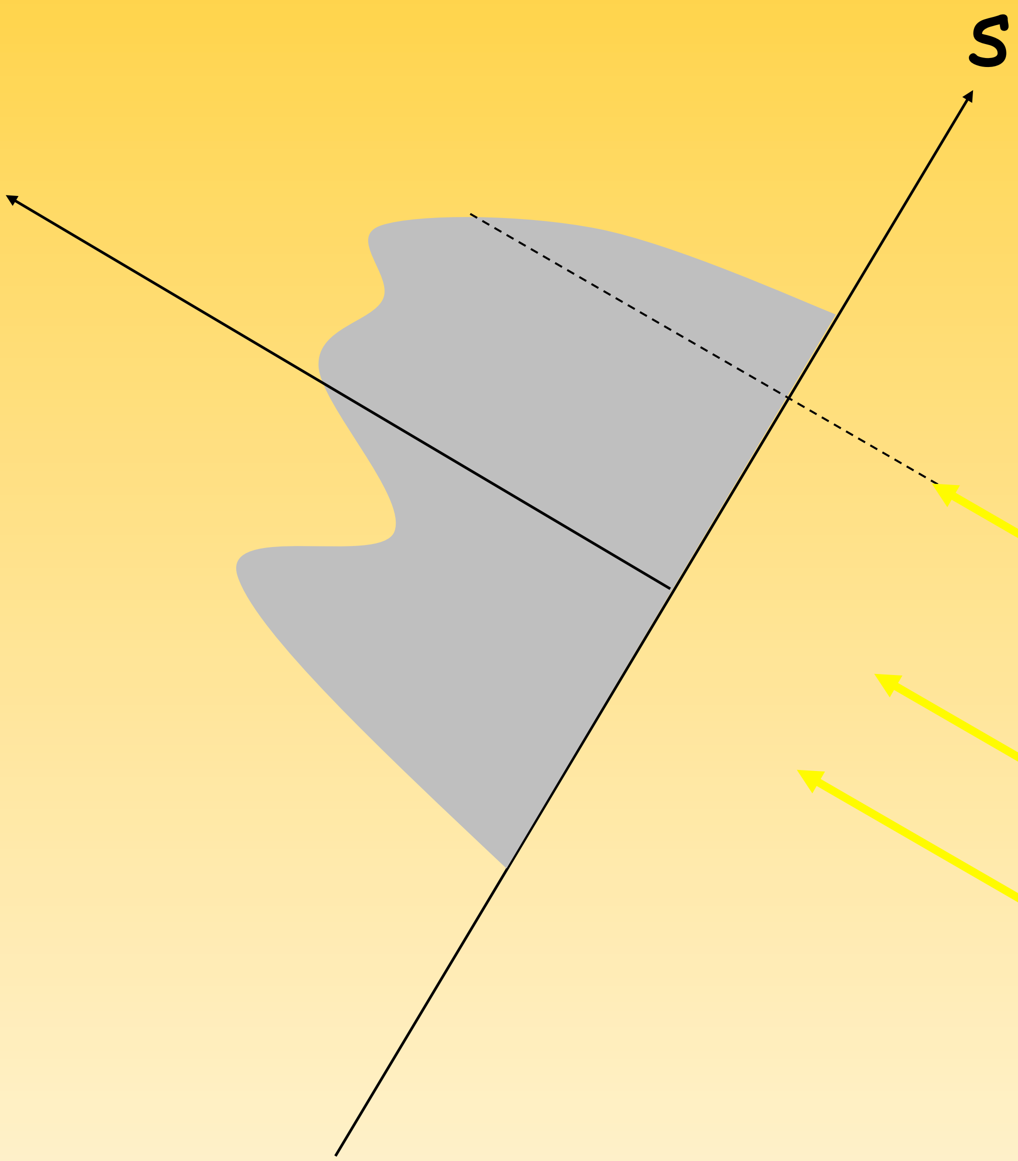
II - Biomedical applications

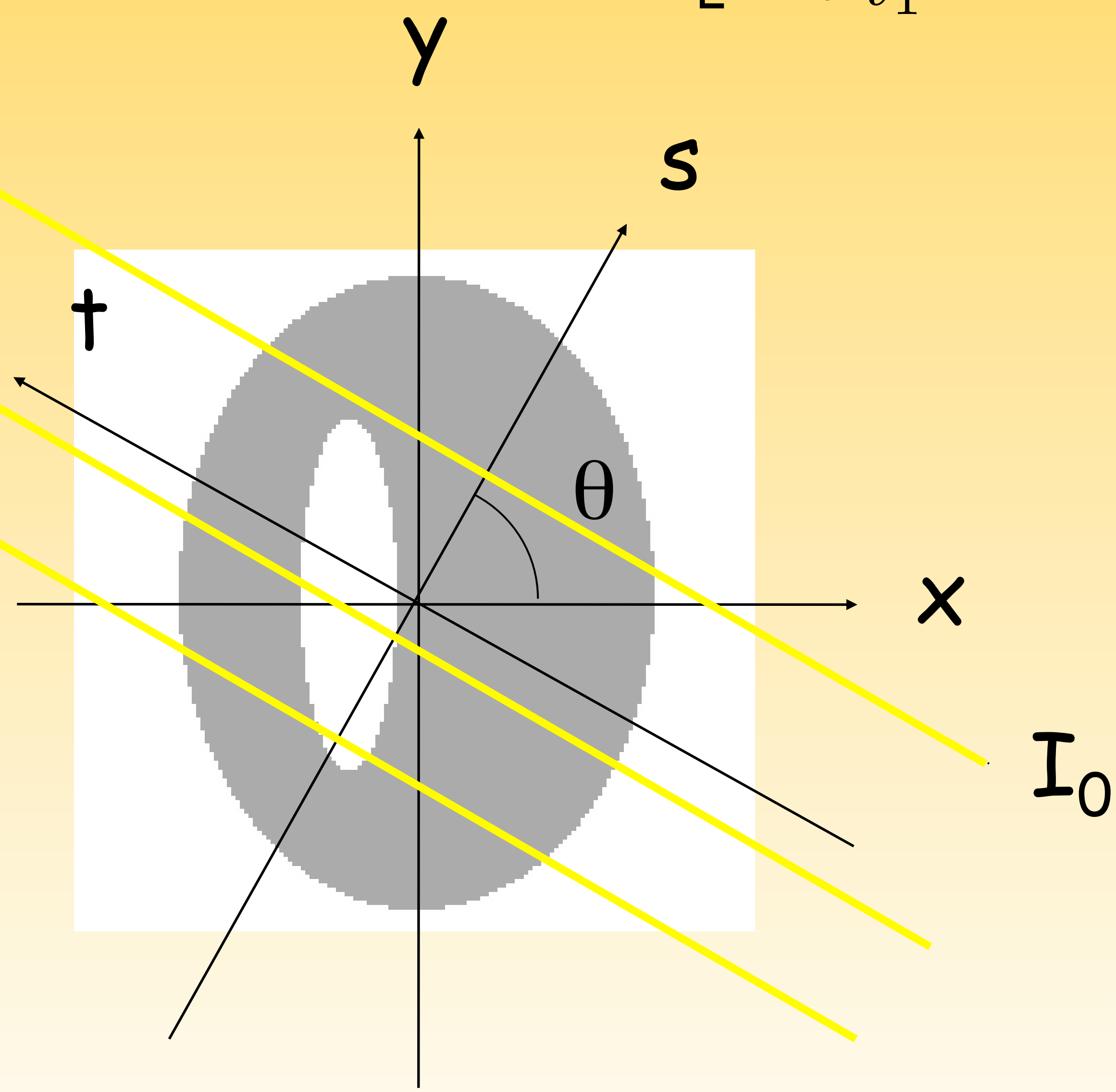
- a) Anatomical imaging : Cone-Beam Computerized Tomography
- b) Functional imaging : Positron Emission Tomography

I - Recalls on Computerized Tomography

The Beer-Lambert law describes the absorption of different materials along the line of sight.

$$I = I_0 \exp\left[-\int_{t_1}^{t_2} \mu_{E,m}(t)dt\right]$$





Absorption or attenuation coefficient

$$\mu = \mu_{E,m}$$

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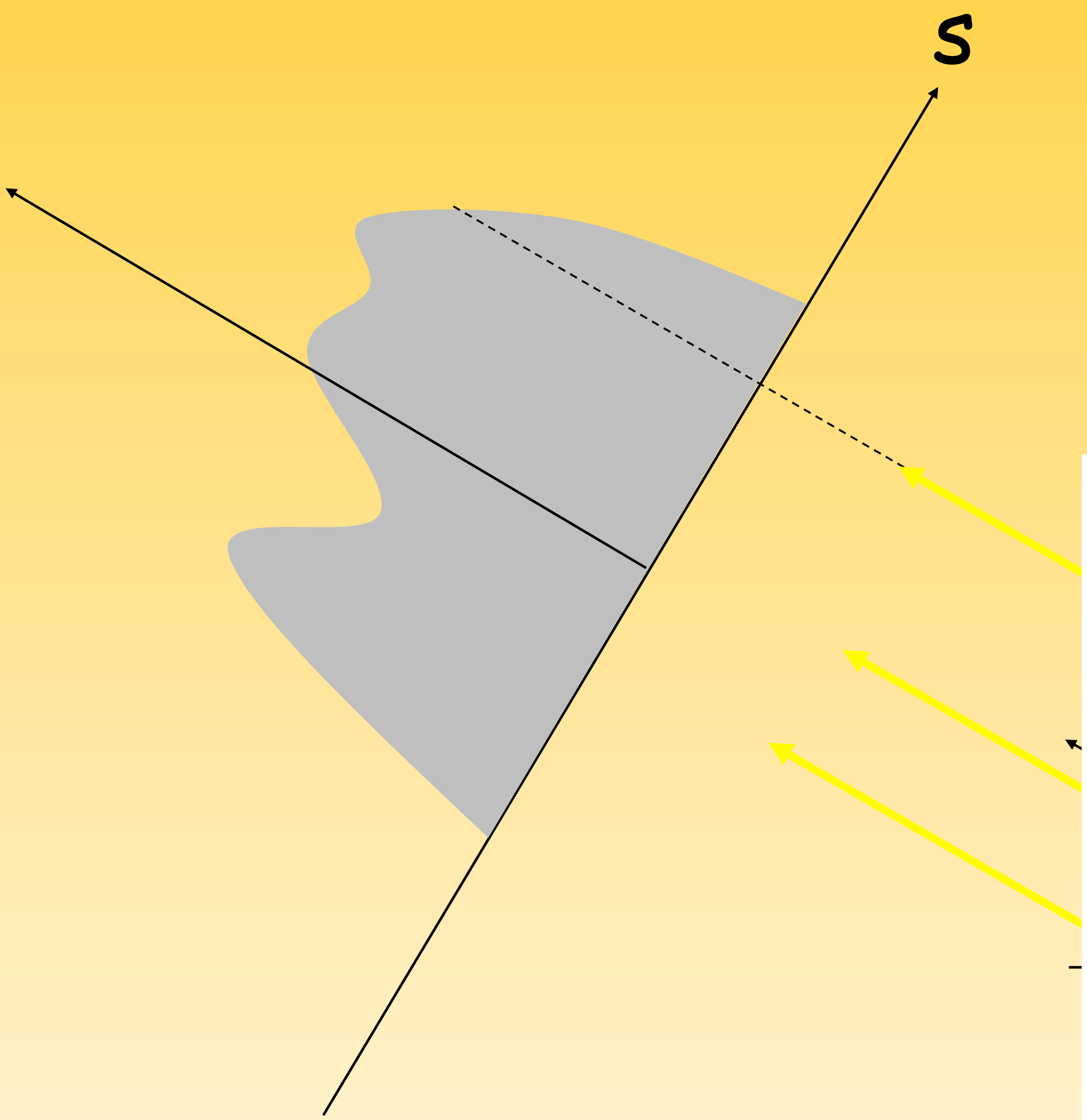

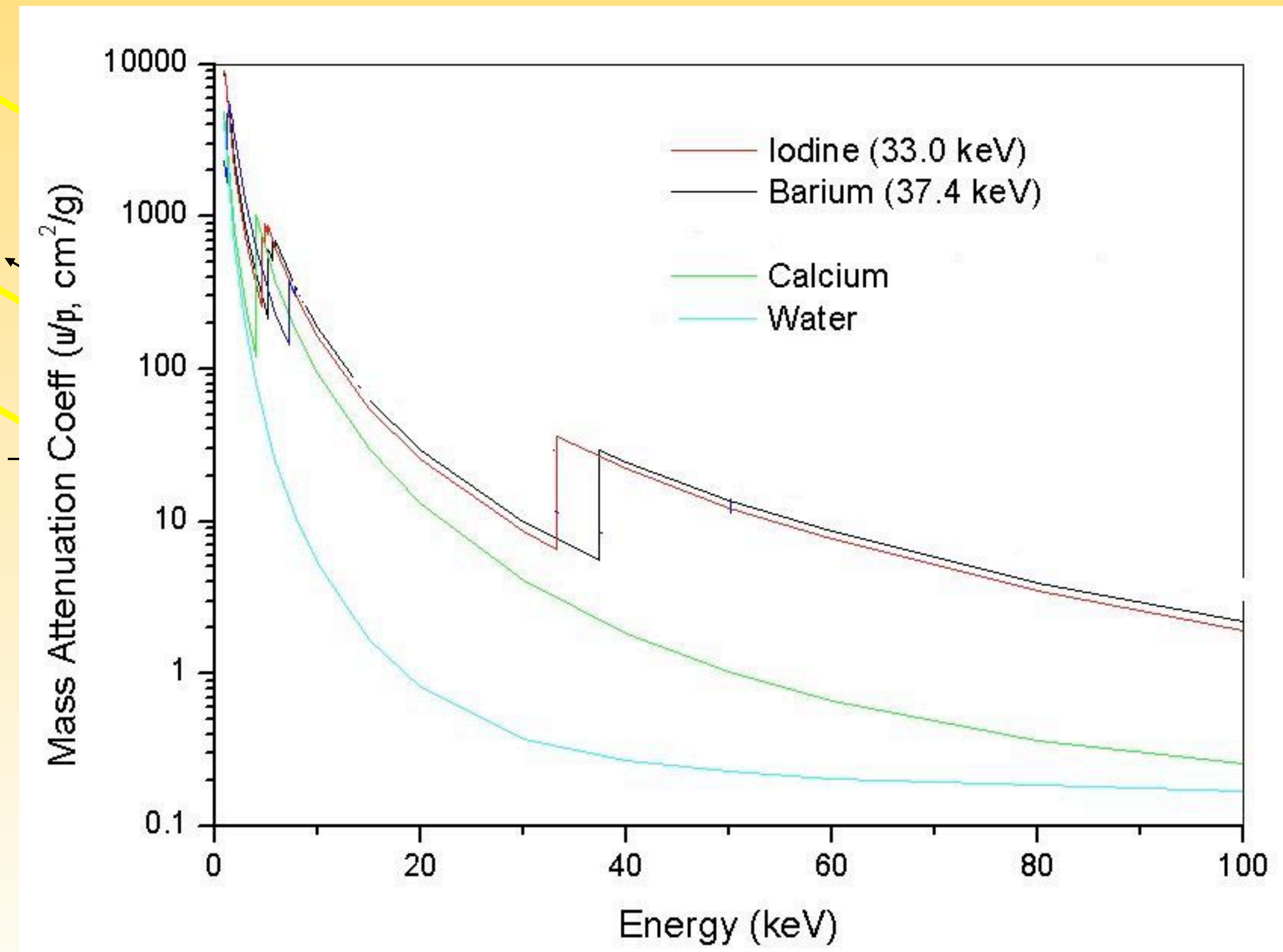

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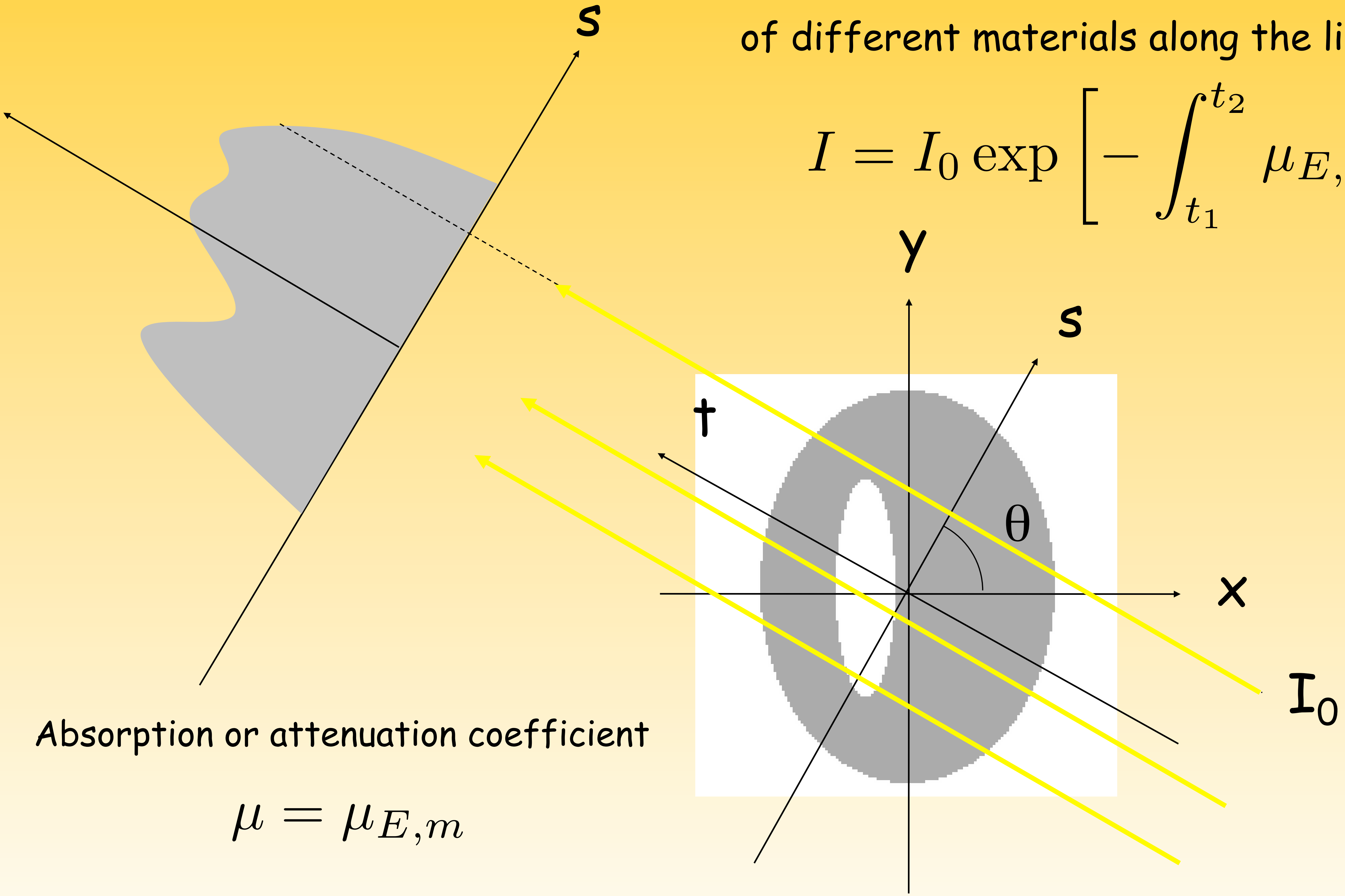
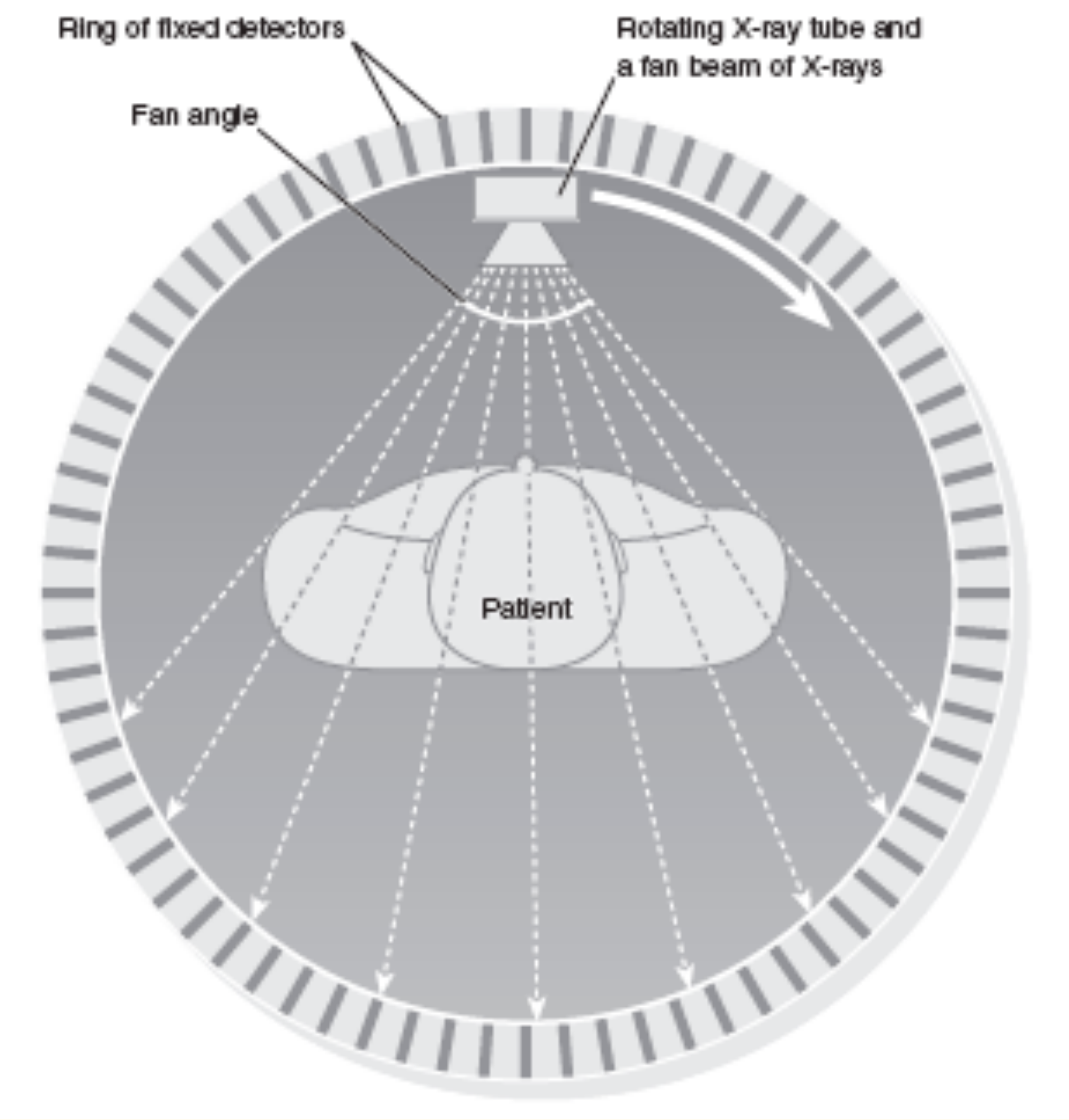


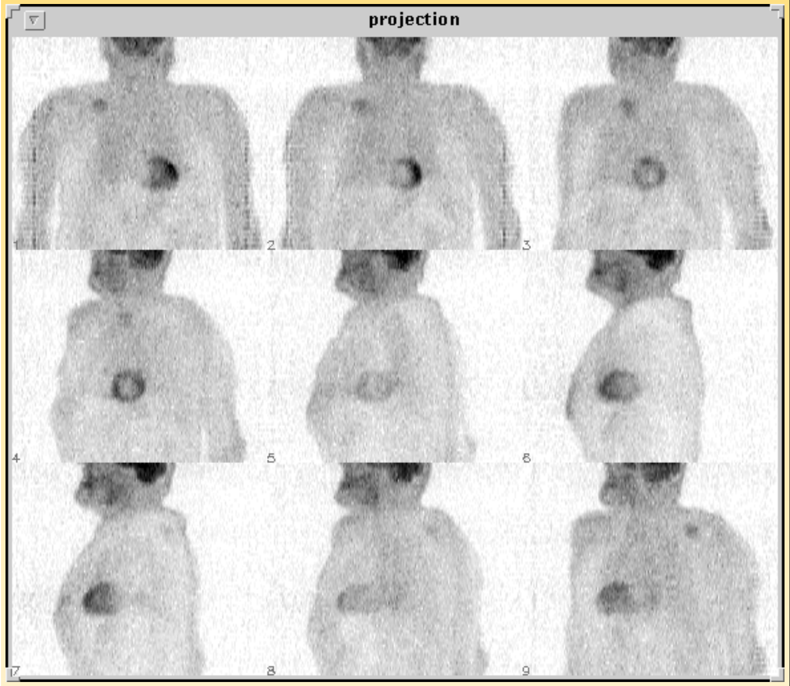
Absorption or attenuation coefficient

$$\mu = \mu_{E,m}$$

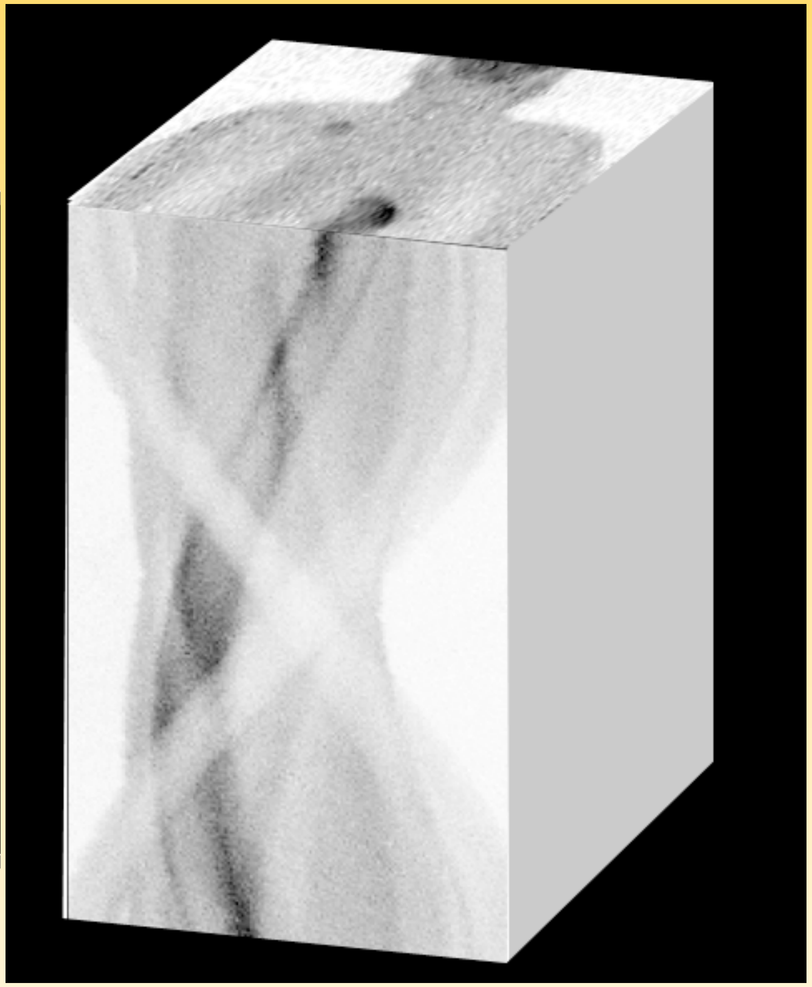
II - Recalls on Cone-Beam Computerized Tomography

- Basis of tomography : data in 2D + angle, object in 3D.





Sinogram



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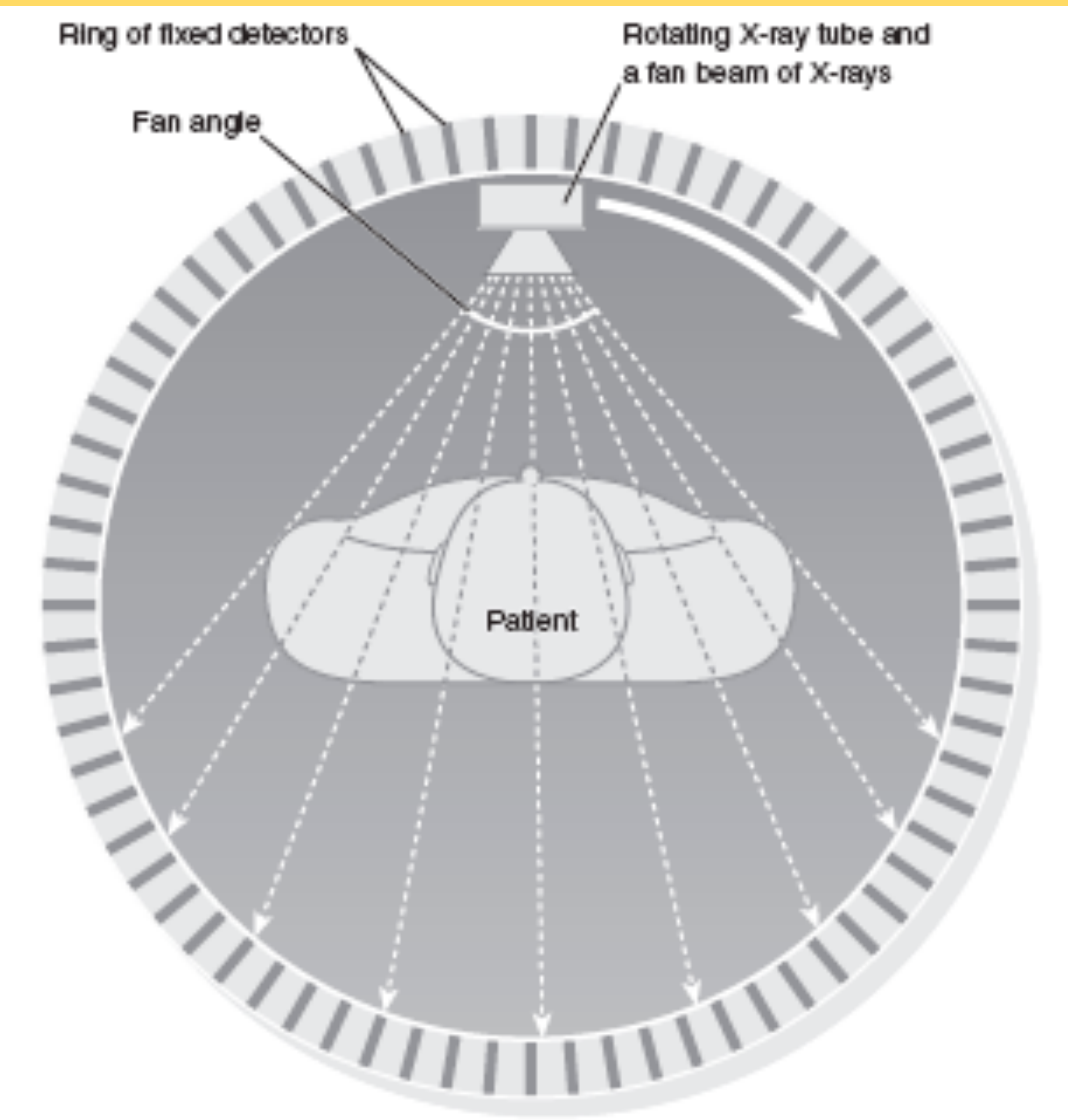

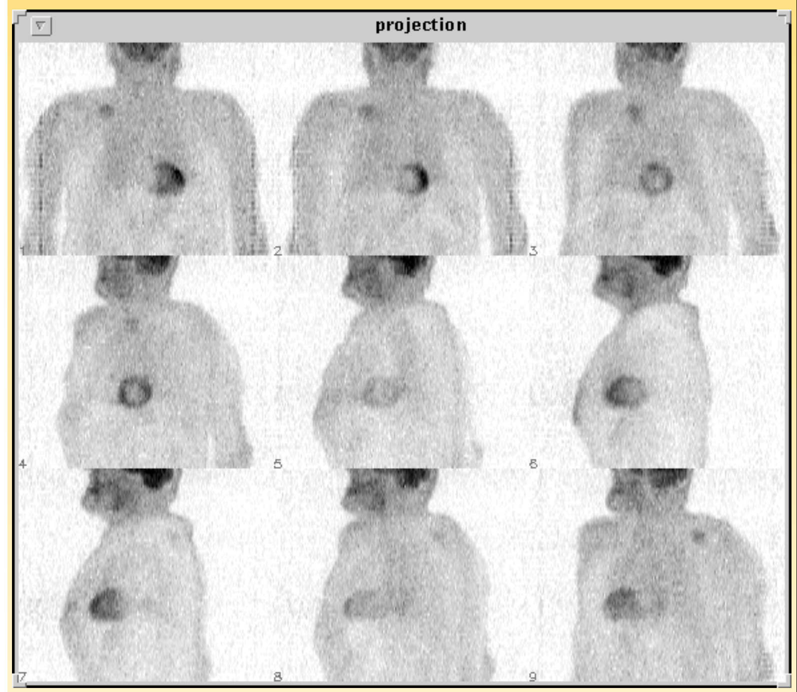

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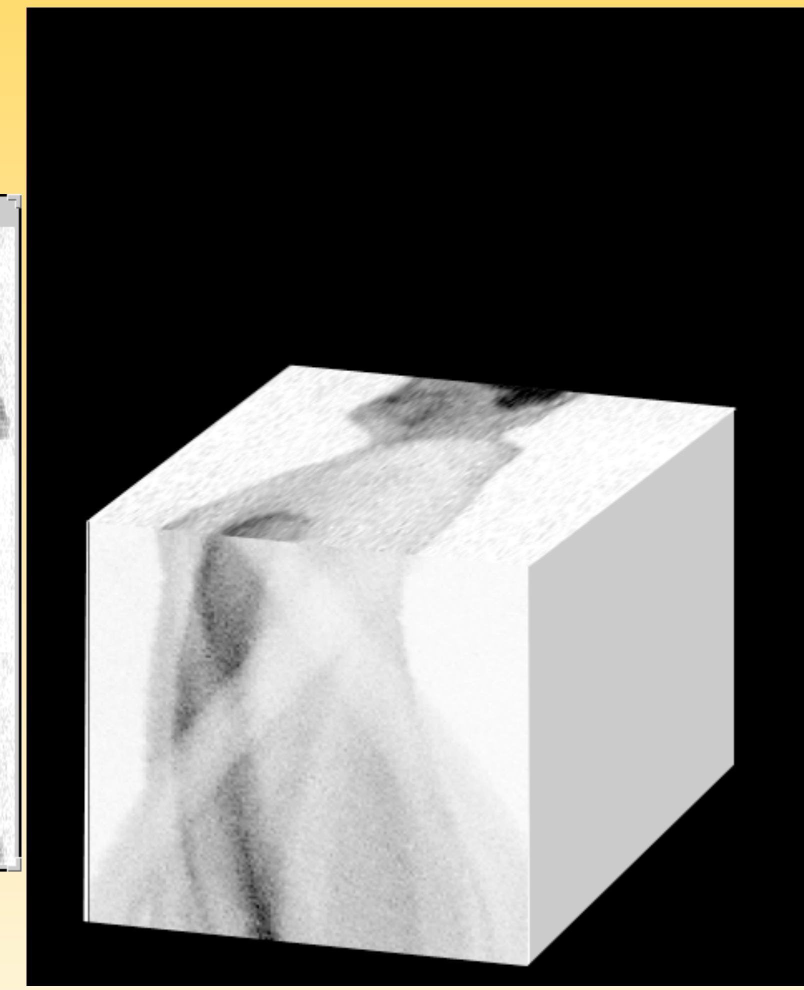
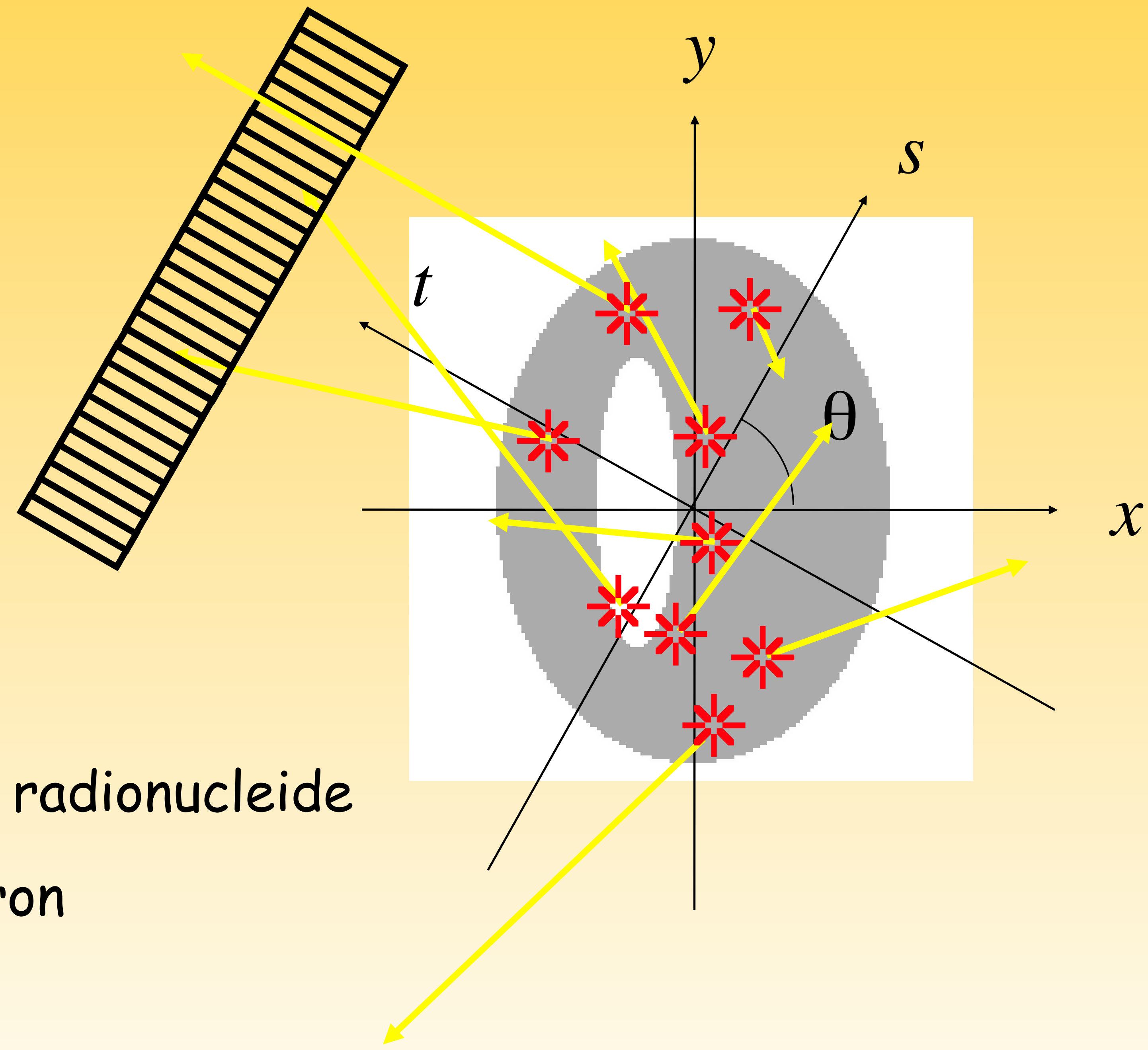


I - Recalls in Positron Emission Tomography

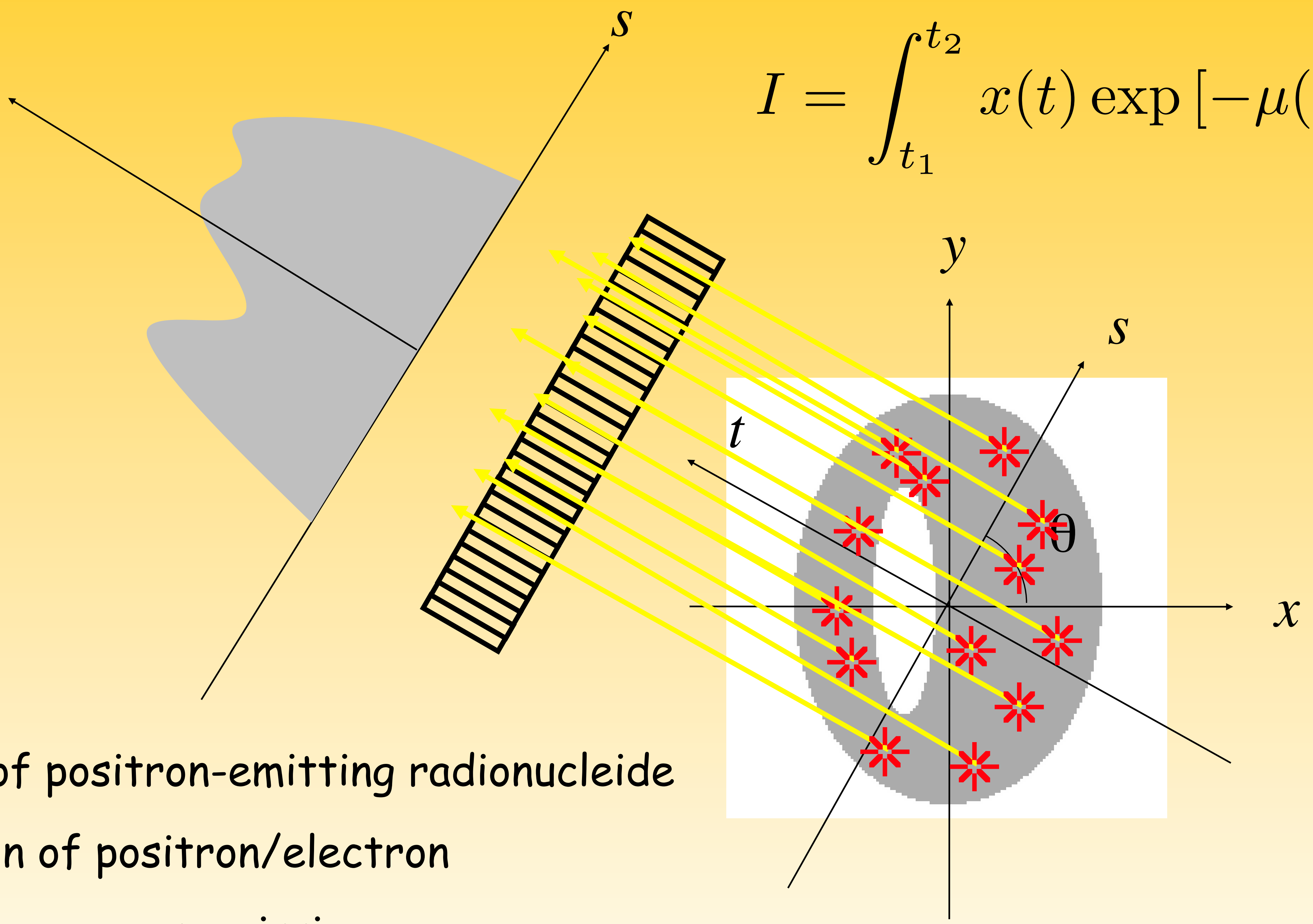


Injection of positron-emitting radionucleide

Annihilation of positron/electron

511 keV gamma rays emission

I - Recalls in Positron Emission Tomography



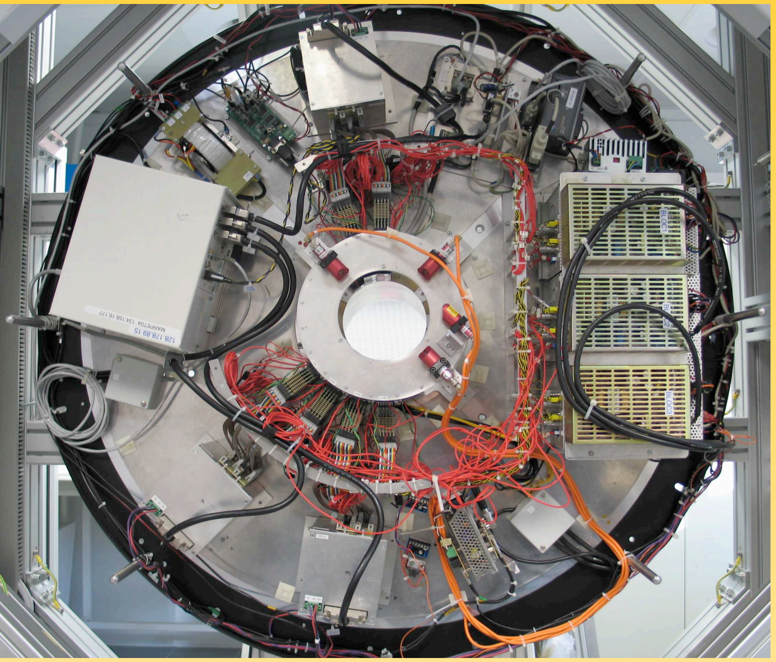
$$I = \int_{t_1}^{t_2} x(t) \exp\left[-\mu(t)\right] dt$$

Injection of positron-emitting radionucleide

Annihilation of positron/electron

511 keV gamma rays emission

ClearPET + XPAD = ClearPET/XPAD



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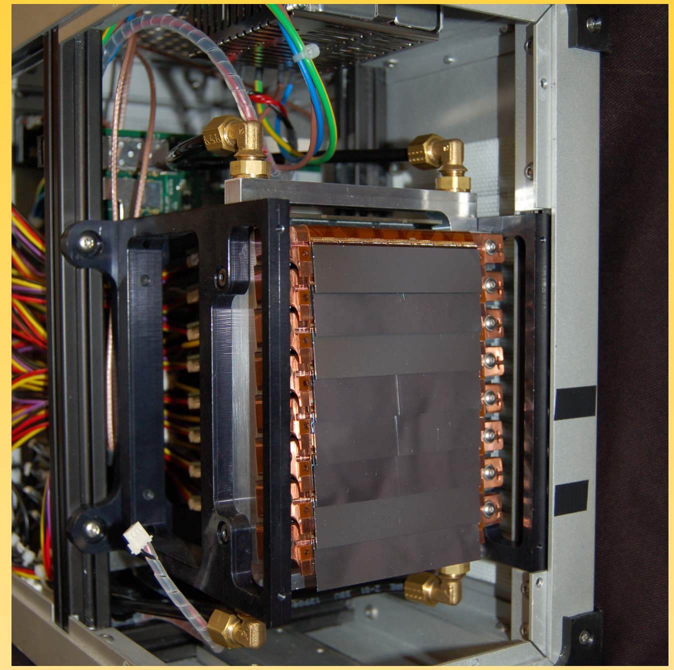

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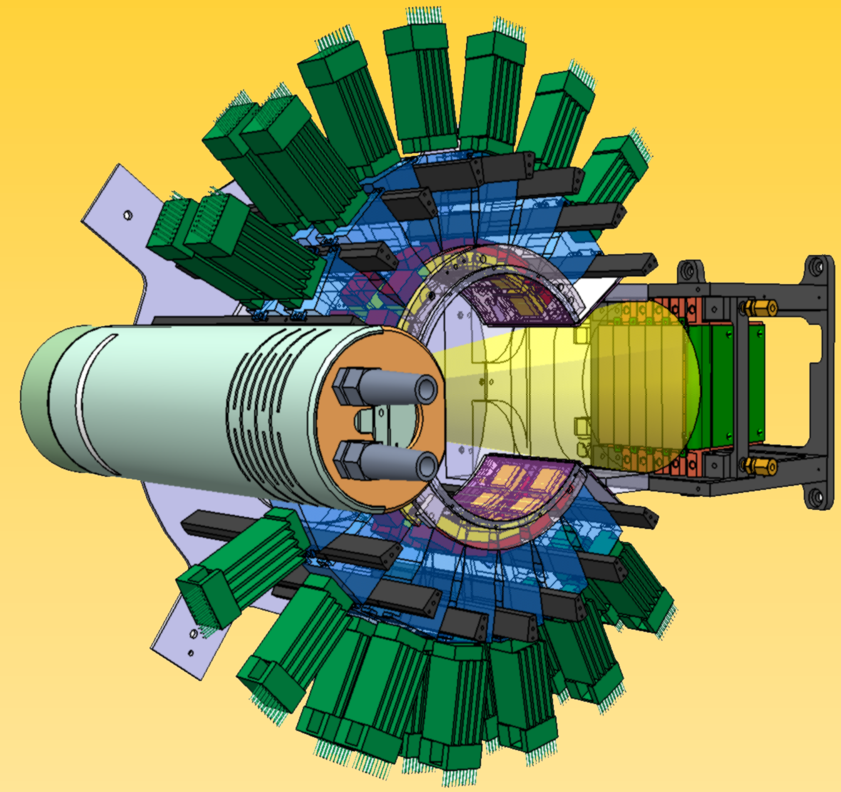


ClearPET (EPFL)

- Open geometry
- Phoswich LSO/LuYAP detectors
- 2 x 64 cristals of 2 x 2 x 8 mm^3
- PMT multi-anodes at 64 channels

XPAD (CPPM)

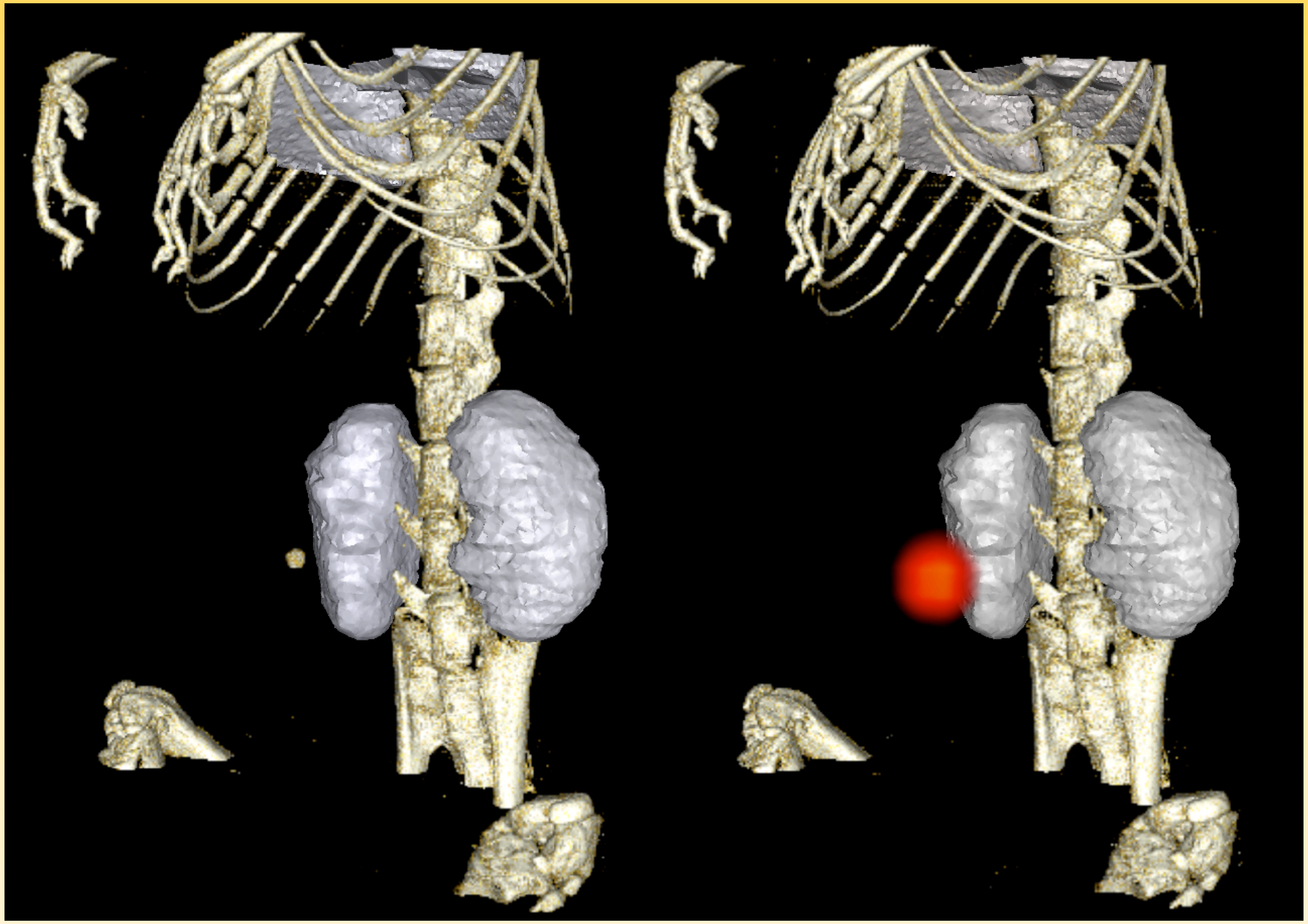
- XPAD3 camera
- 500 μm Si pixelized
- Pixels of 130 x 130 μm^2
- 0,5 Mpixels
- Energy selection 5-35 keV
- W X-ray source

CleartPET/XPAD

- Hybrid tomography
- Simultaneous TEP/TDM
- PET : 55 mm axial, 111 mm transverse
- CBCT : 59 mm axial, 38 mm transverse

PET/CT scan of a mouse : SIMULTANEOUS ACQUISITION

- Volume rendering
- Segmentation of lungs and kidneys
- 40 kV, 800 µA, filter Nb/Mo
- 360 projections
- 1 s/projection
- 10 000 photons/pixel



Challenges

- High quality of reconstruction while :
  - reducing the X-ray dose (CT)
  - reducing the radiotracer dose (PET)
  - reducing the exam duration (PET)

- Possible solutions :
  - Reduce the number of projections (CT)
  - Reduce the intensity of acquired signals (PET and CT)

Need to deal with pure Poisson Noise ...

II - Frameworks

For a monochromatic beam, a pixel of CBCT-scan measures

$$y_j \sim \mathcal{P}\left(z_j \exp\left(-\left[A\mu\right]_j\right)\right)$$

CBCT acquisition model

A crystal of the PET-scan measures

$$y_j \sim \mathcal{P}\left(\left[Bx\right]_j\right)$$

PET acquisition model

where the system matrices A and B incorporate geometry and corrections.

II - CBCT and PET Frameworks

CBCT acquisition model

$$y_j \sim \mathcal{P}\left(z_j \exp\left(-\left[A\mu\right]_j\right)\right)$$

CBCT General objective function

$$\hat{\mu} = \arg\min_{\mu} \sum_j \{y_j \left[A\mu\right]_j + z_j \exp\left(-\left[A\mu\right]_j\right)\} + \lambda J(\mu)$$

PET acquisition model

$$y_j \sim \mathcal{P}\left(\left[Bx\right]_j\right)$$

PET General objective function

$$\hat{x} = \arg\min_{x} \sum_j \{\left[Bx\right]_j - y_j \log\left(\left[Bx\right]_j + \epsilon\right)\} + \lambda J(x)$$

II - Sparse regularizations

Total-Variation

$$J_{TV}(u) = \sum_{1 \leq i,j \leq N} |(\nabla u)_{i,j}|$$

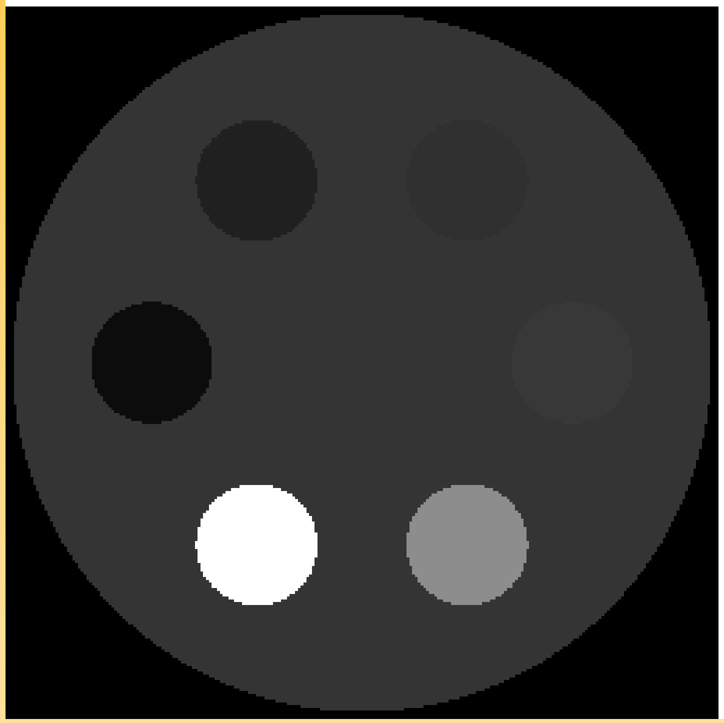
Regularized Total-Variation

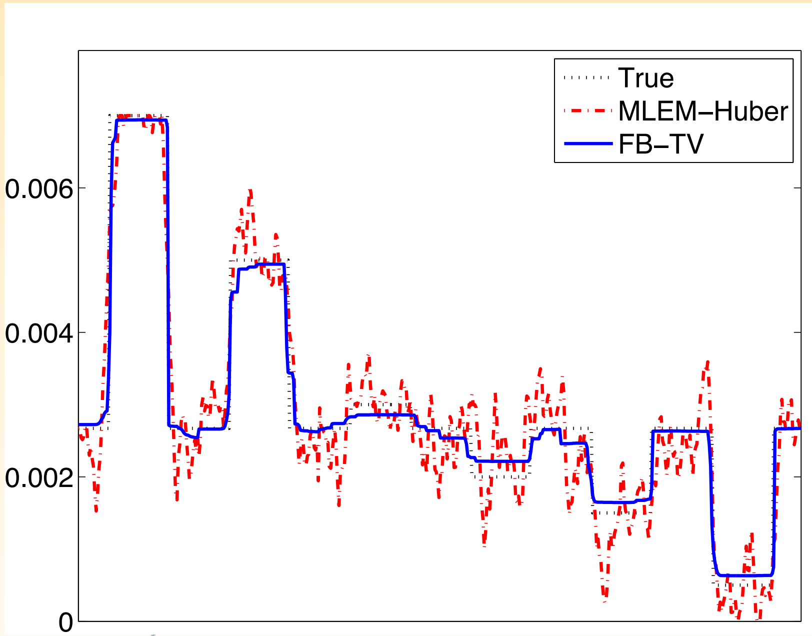
$$J_{TV}^{reg}(u) = \langle \sqrt{\alpha^2 + |\nabla u|^2}, 1 \rangle = \sum_{1 \leq i,j \leq N} \sqrt{\alpha^2 + |(\nabla u)_{i,j}|^2}$$

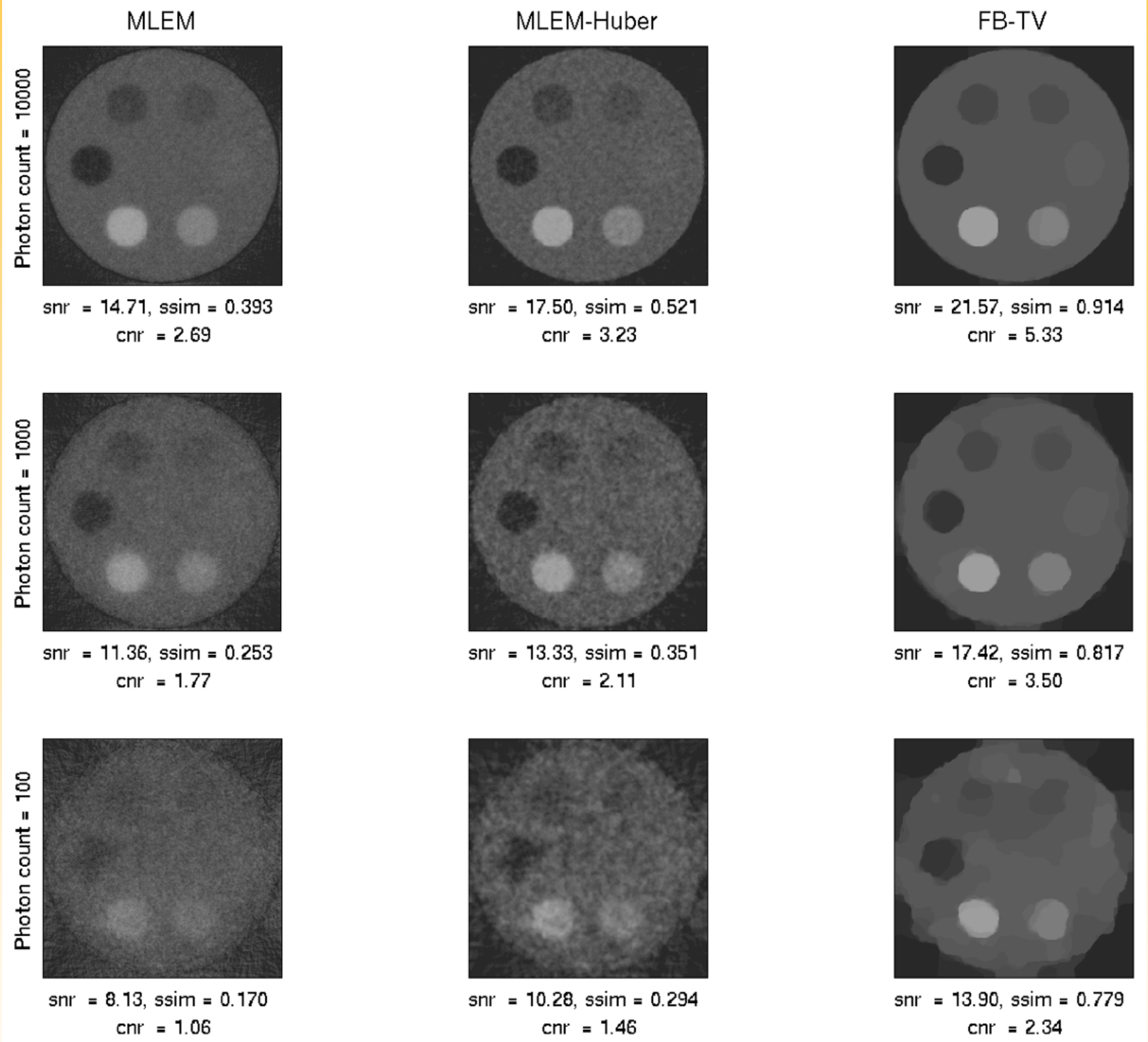
ℓ_1-norm inducing sparsity

$$J_{\ell_1,\varphi}(u) = \sum_{\lambda \in \Lambda} |\langle u, \varphi_\lambda \rangle| = \|R_{\varphi(u)}\|_{\ell_1}$$

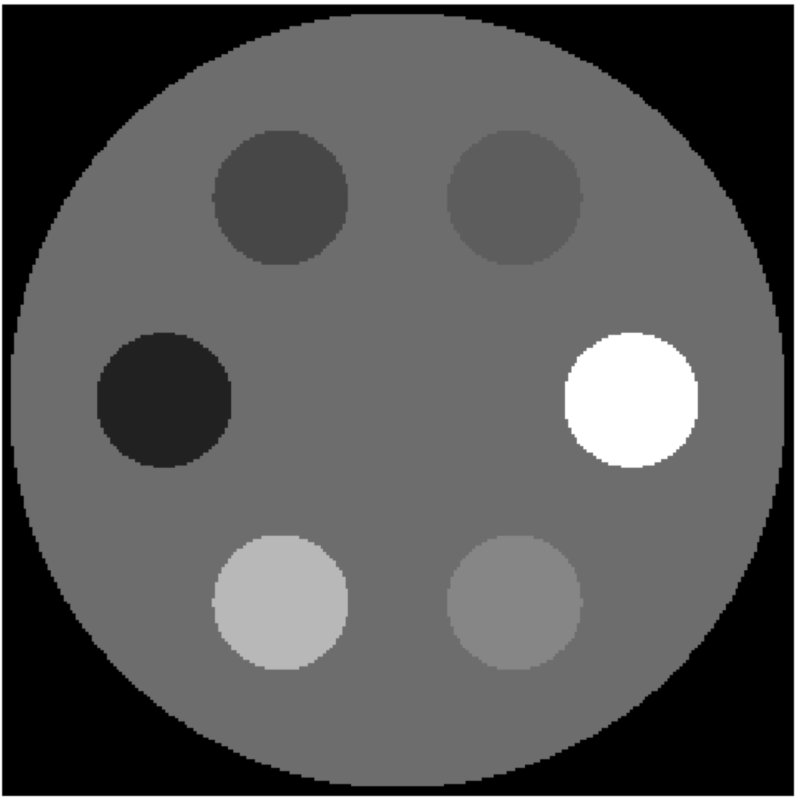
CBCT , Z = 1 000 photons, contrast phantom

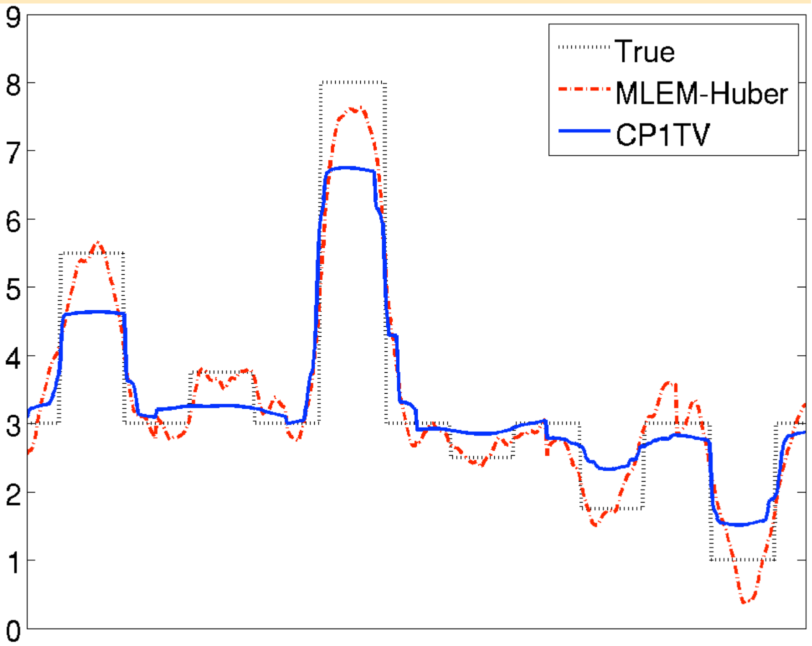


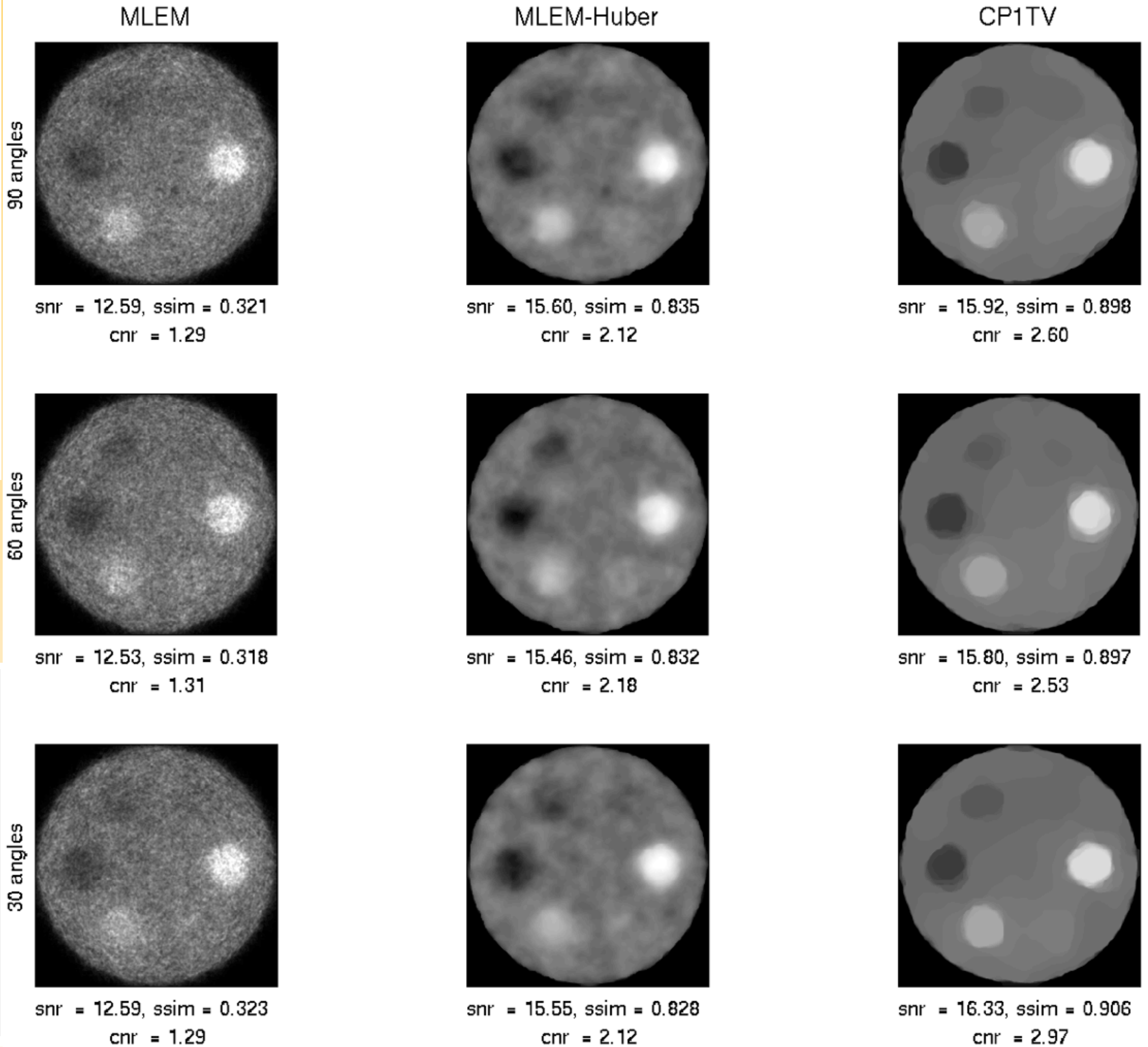




TEP , 200 000 counts (1500 /pixel), contrast phantom



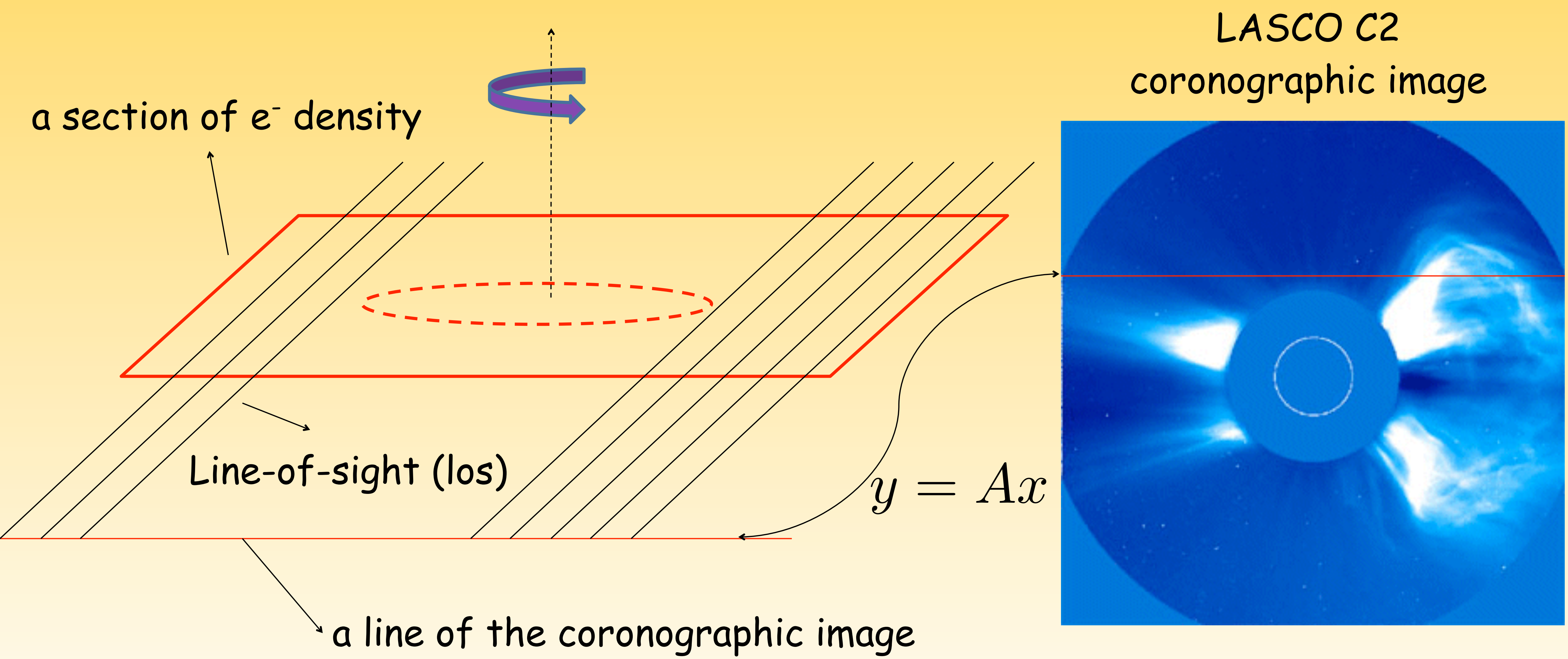




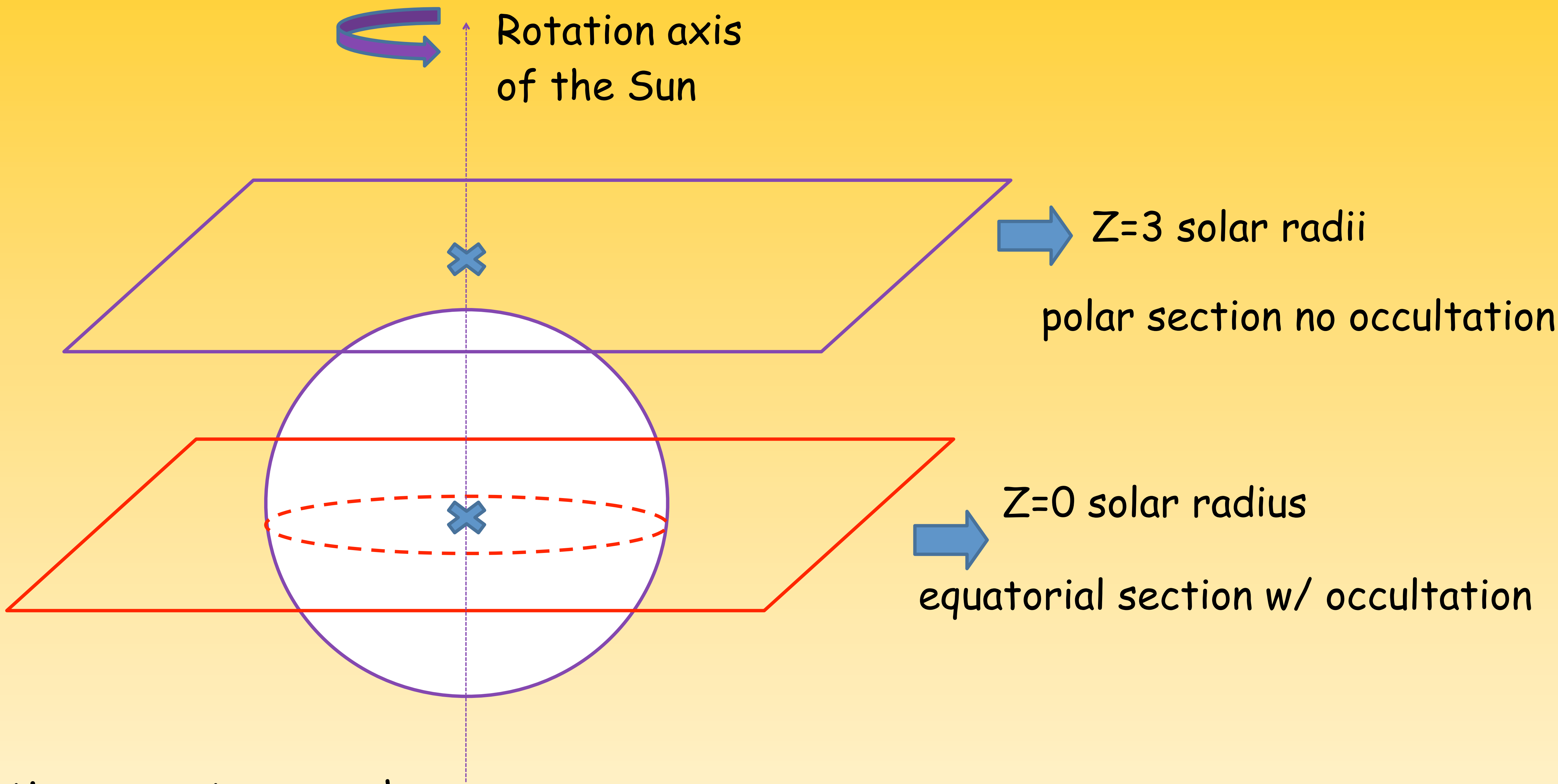
III - Astronomical application: Solar Rotational Tomography (SRT)

- Determining by tomographic reconstruction the electronic density of the coronal plasma. Huge implications for understanding physics of the corona.

- Coronographic images acquired during 1 rotation ... the Sun rotates for us !



III - Challenges



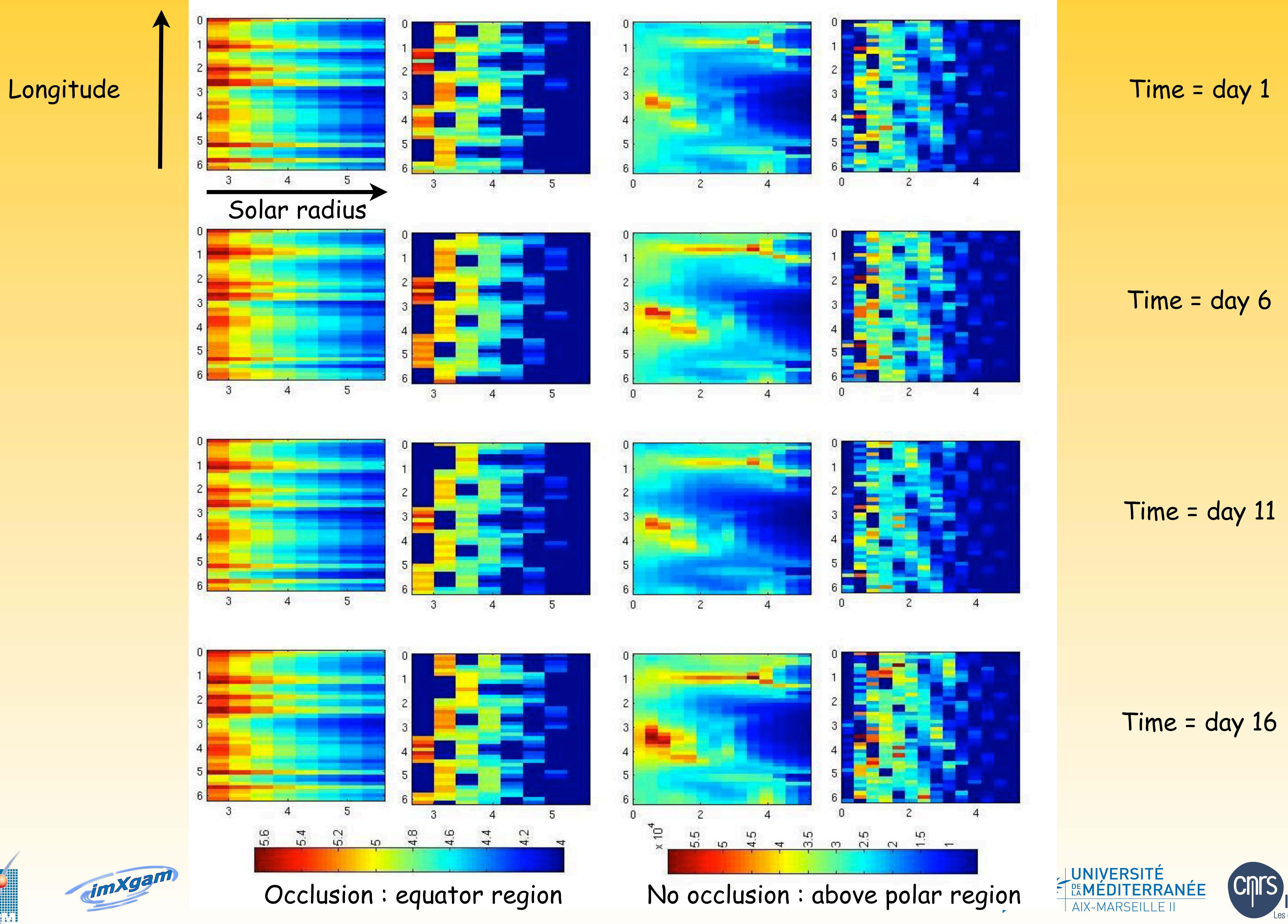
Static corona tomography

$$\hat{x} = \arg\min_{x} \|y - Ax\|_2^2 + \lambda \|L_s x\|_2^2$$

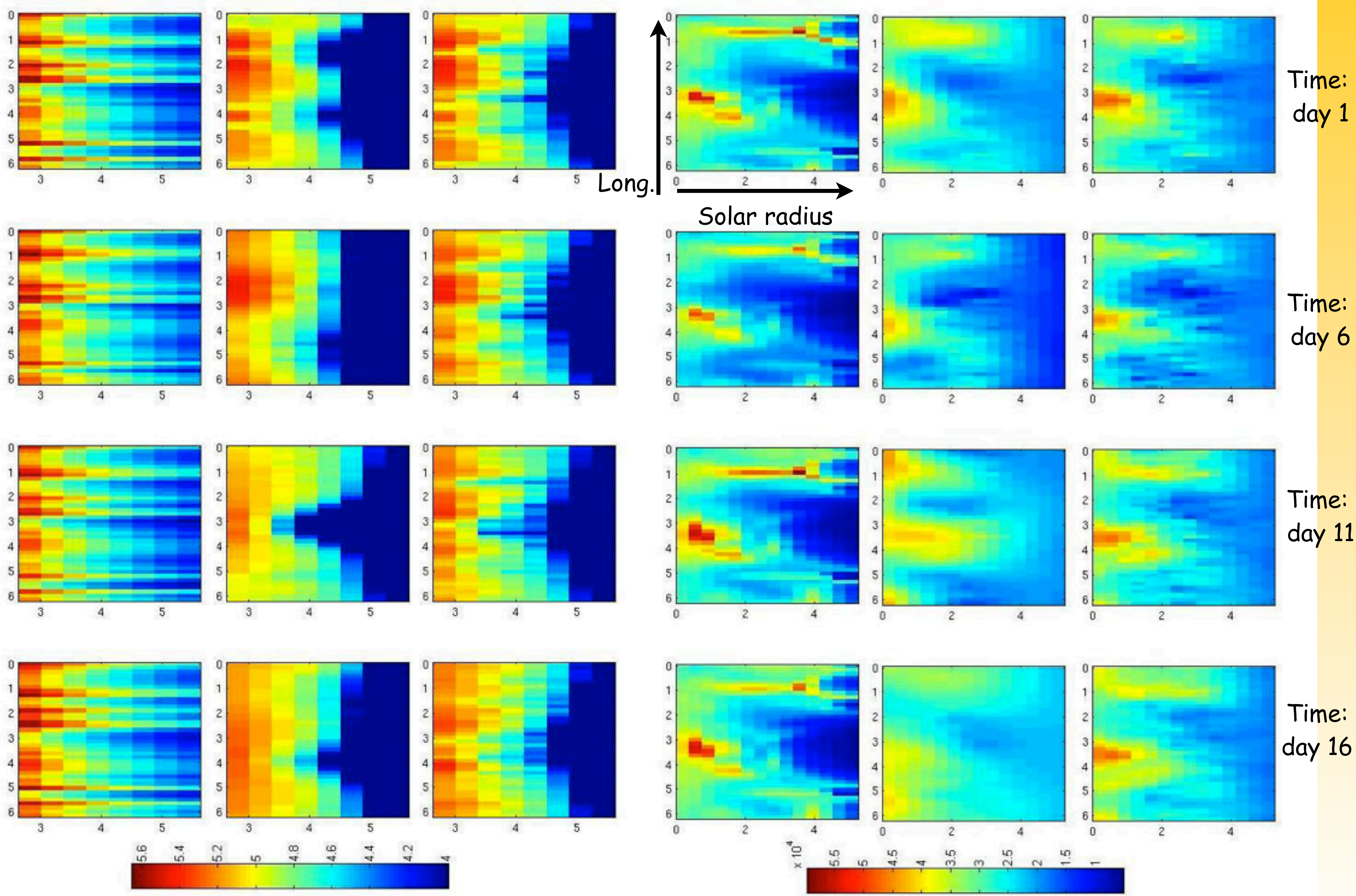
Time dependent tomography

$$\hat{x} = \arg\min_{x} \|y - Ax\|_2^2 + \lambda \|L_s x\|_2^2 + \nu \|L_t x\|_2^2$$

III - Preliminary results



III - Preliminary results



Occlusion : equator region

No occlusion : above polar region

Conclusion

Tomography is usually an ill-posed problem !

In case of parallel beams, consistant with Fourier sampling ...

Algorithms adapted to physics :

For nature of noise : Poisson or Gaussian noise taken into account, exact physical model.

For small number of projections : sparse regularizations enhance robustness. Choice of sparse decomposition is an current issue that is problem-dependent.

Reconstruction of real acquisitions in progress for PET, CBCT and SRT applications.

Implementation on GPUs strongly speeds-up the reconstruction (between 100 and 300 times faster compared to CPUs).

Announcement

4th - 9th September 2011 : BASP Frontiers 2011 meeting

in Villars, Switzerland

Biomedical and Astronomical Signal Processing Frontiers 2011

Fourier Sampling for Magnetic Resonance and Radio Interferometric imaging

More on http://imxgam.in2p3.fr/BASP2011/

Thanks for your attention !

I - What's new at CPPM ? ... Biomedical imaging !

CPPM : a lab from CNRS/IN2P3 for particle physics.

imXgam group : X and gamma imaging

Some imXgam projects :

XPIX : technological breakthrough !

Hybrid pixels for X-ray : XPAD cameras.

PIXSCAN :

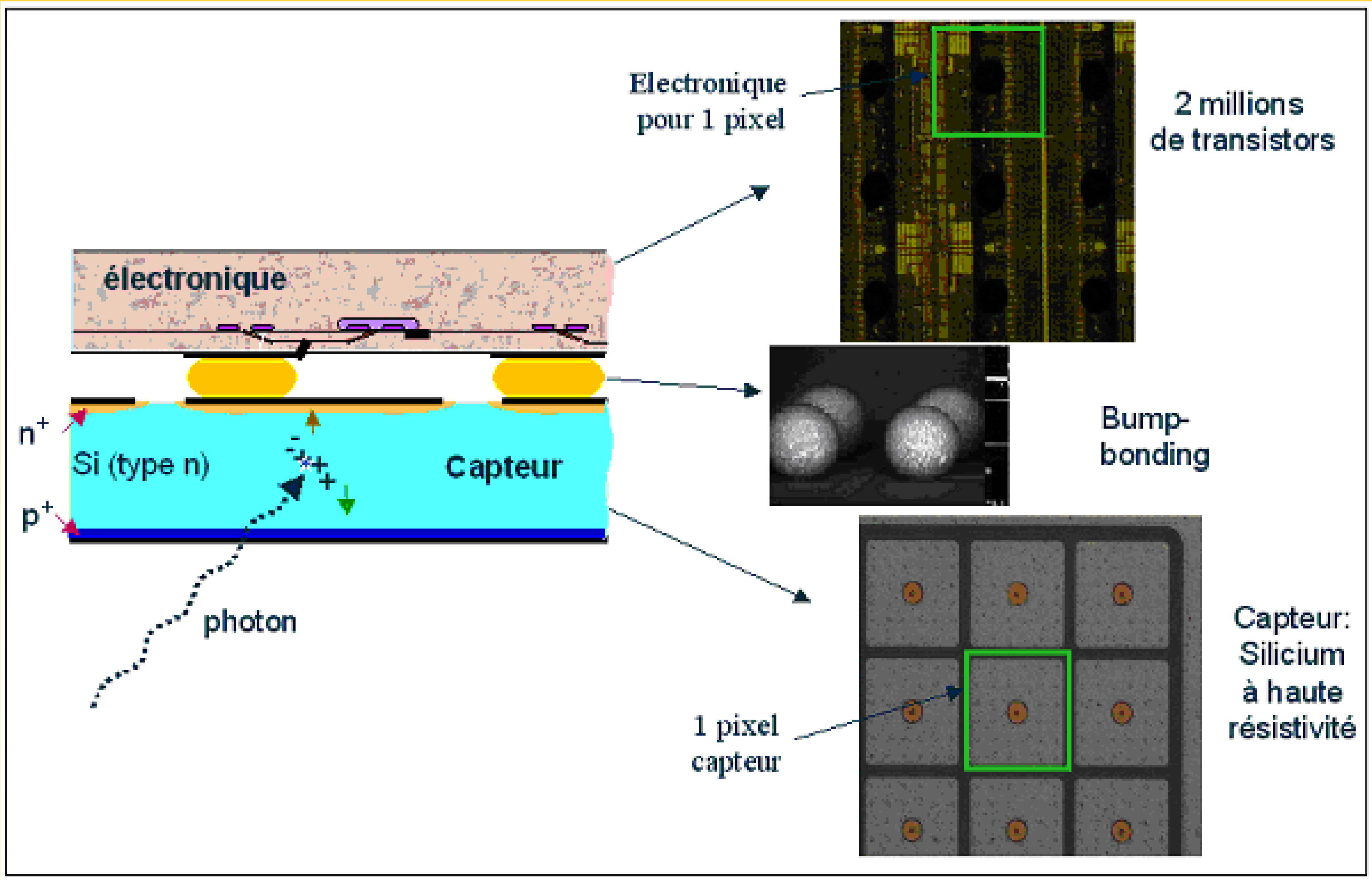
Micro CT-Scanner based on hybrid pixels.

ClearPET/XPAD :

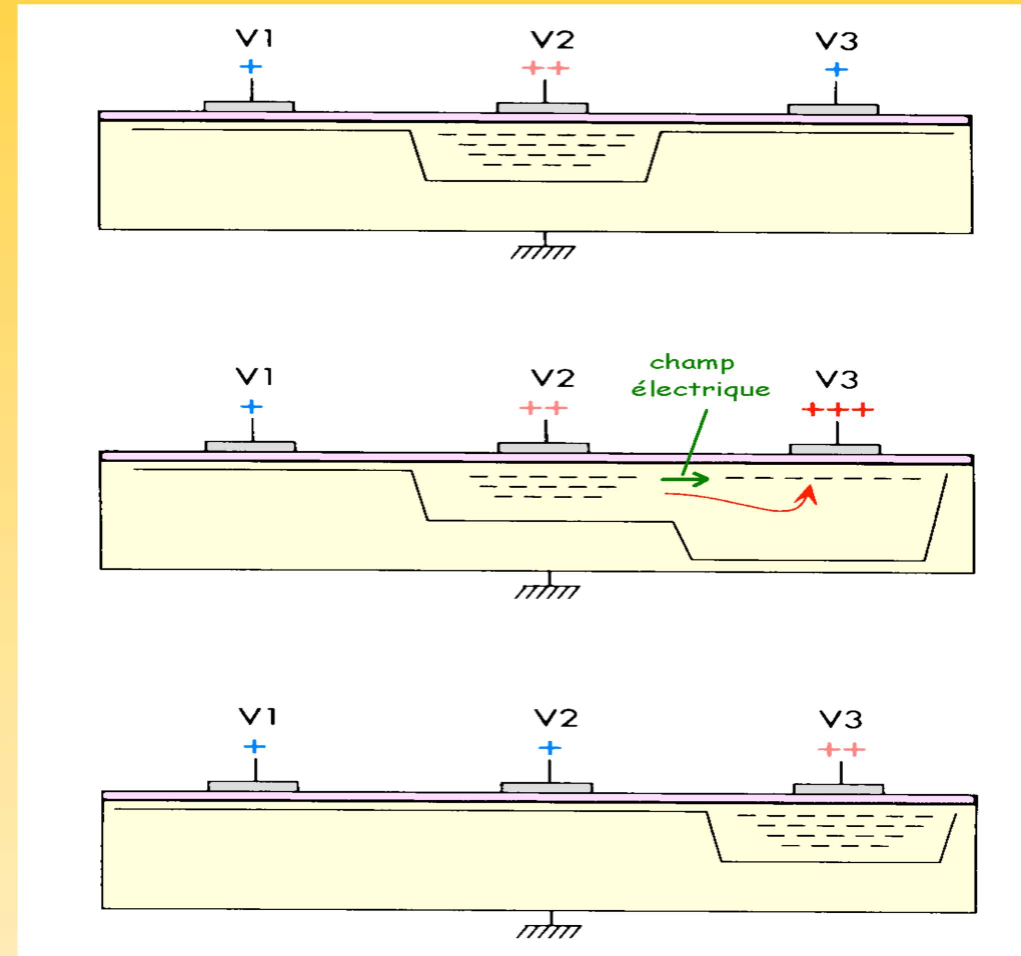
Simultaneous PET/CT imaging based on hybrid pixels.

Hybrid pixels

Hybrid pixel



CCD



- very fast data acquisition
- choice of substrat (Si, CdTE, AsGa)

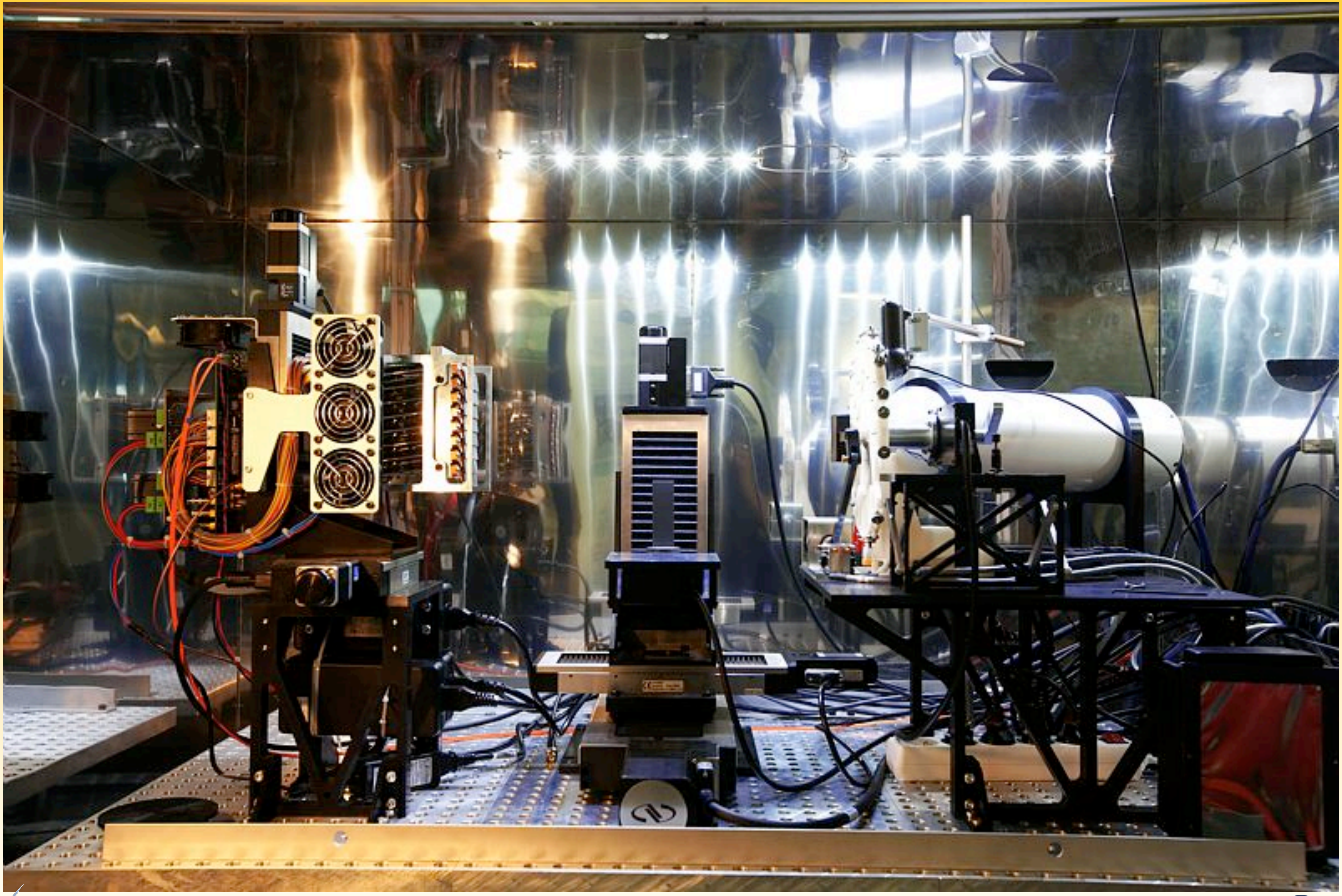
- No Dark noise
- Energy selection
- Very large dynamic range

Fundamental difference with other detectors (CCDs-like) :

Photon counting mode !

No charge integration !

micro-CT PIXSCAN II demonstrator

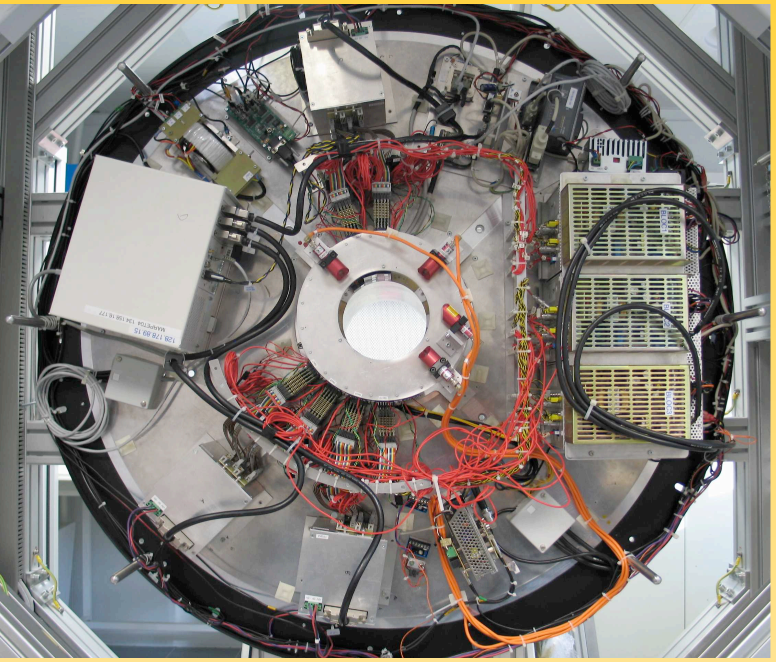


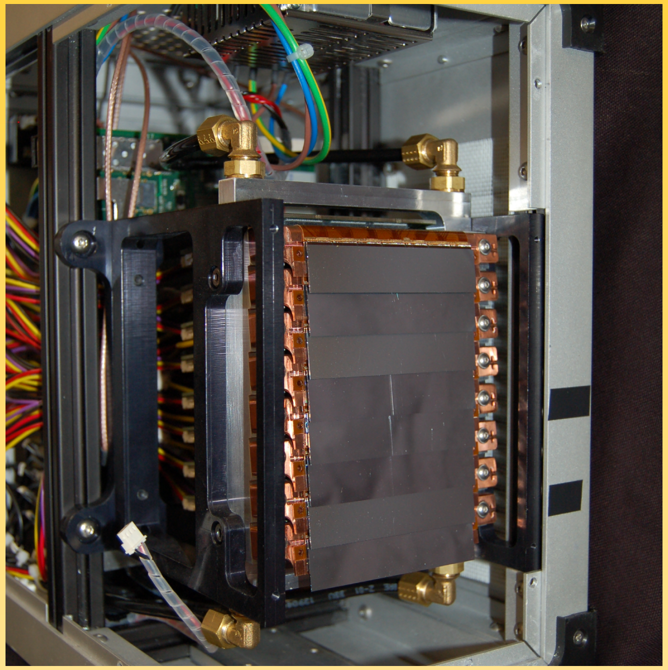
First light XPAD3/PIXSCAN II

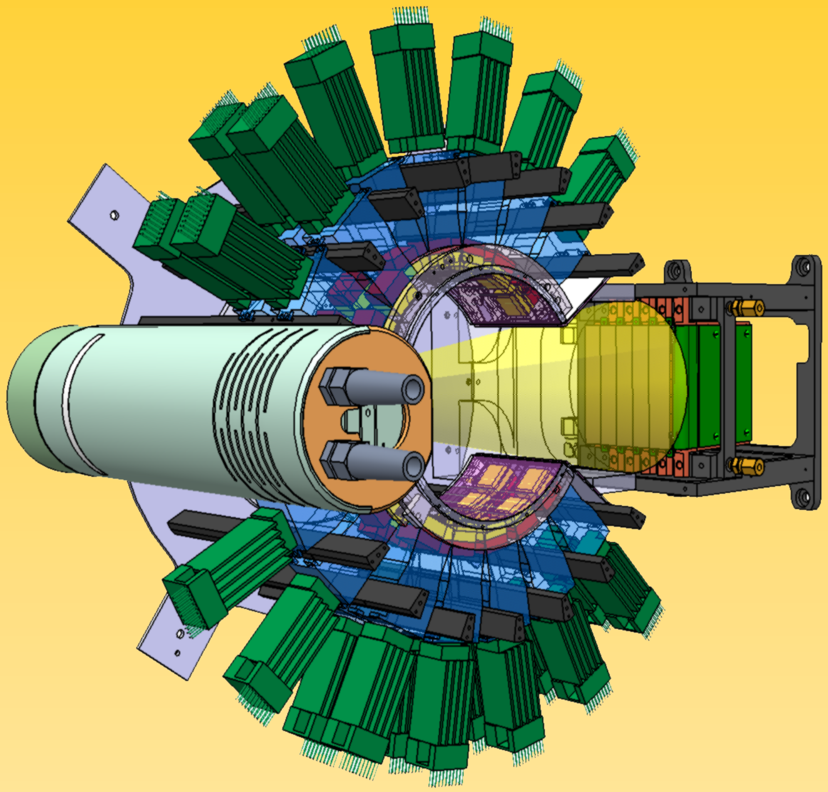
Reconstruction performed on a GPU AMD/ATI, Algorithm FDK.

But need of 720 projections and >1mGy/s at 160 mm

ClearPET + XPAD = ClearPET/XPAD







ClearPET (EPFL)

- Open geometry
- Phoswich LSO/LuYAP detectors
- 2 x 64 cristals of 2 x 2 x 8 mm^3
- PMT multi-anodes at 64 channels

XPAD (CPPM)

- XPAD3 camera
- 500 μm Si pixelized
- Pixels of 130 x 130 μm^2
- 0,5 Mpixels
- Energy selection 5-35 keV
- W X-ray source

CleartPET/XPAD

- Hybrid tomography
- Simultaneous TEP/TDM
- PET : 55 mm axial
111 mm transverse
- CBCT : 59 mm axial
38 mm transverse

II - Solvers

1 - Problem :

$$\arg\min_{x \in X} F(x) + G(x)$$

with F and G proper, convex, lower semi-continuous functions, and F diff.

Solution : Forward-Backward splitting iterations :

$$x_{k+1} = (I + h\partial G)^{-1}(x_k - h\nabla F(x_k))$$

2 - Problem :

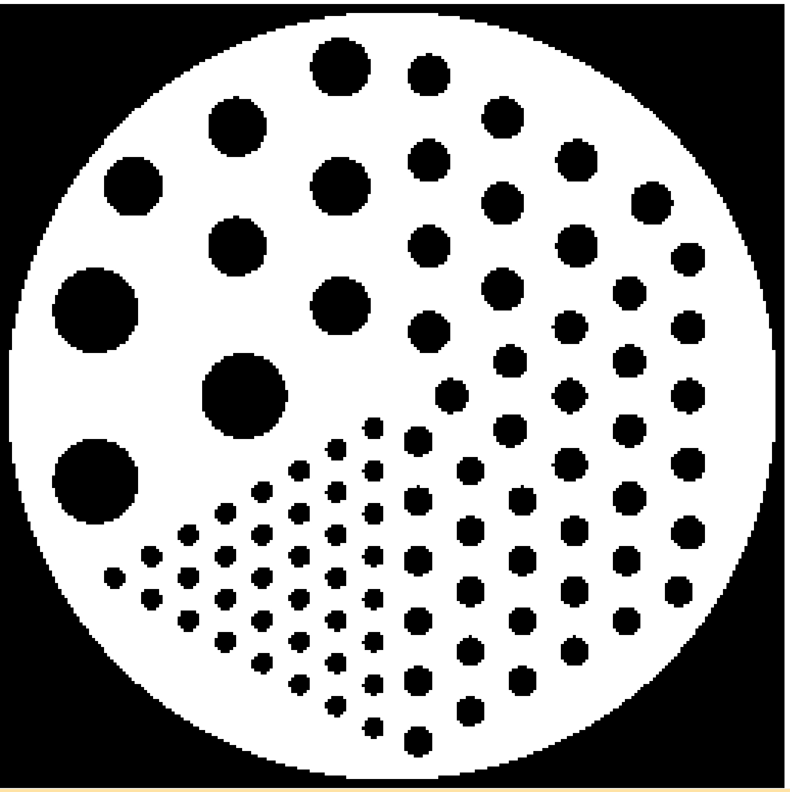
$$\arg\min_{x \in X} F(Kx) + G(x)$$

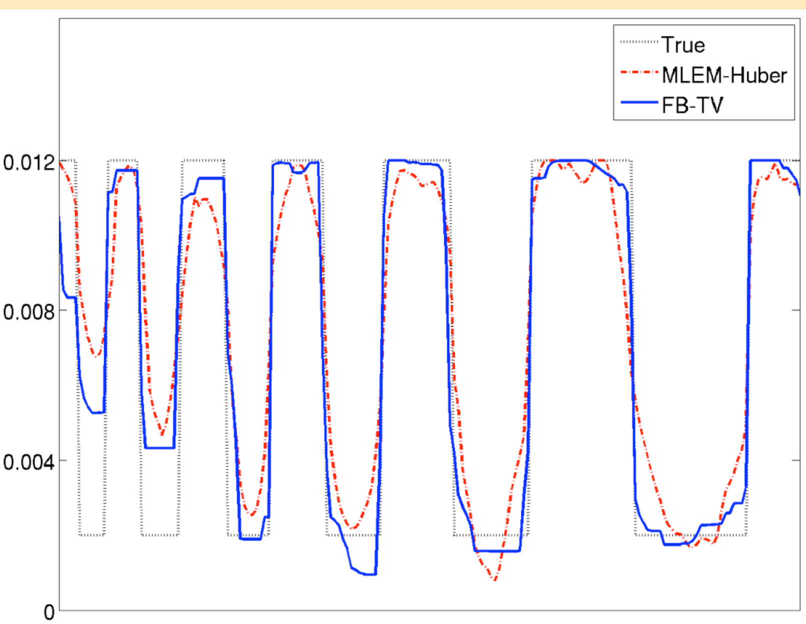
with K continuous linear operators and F no more diff.

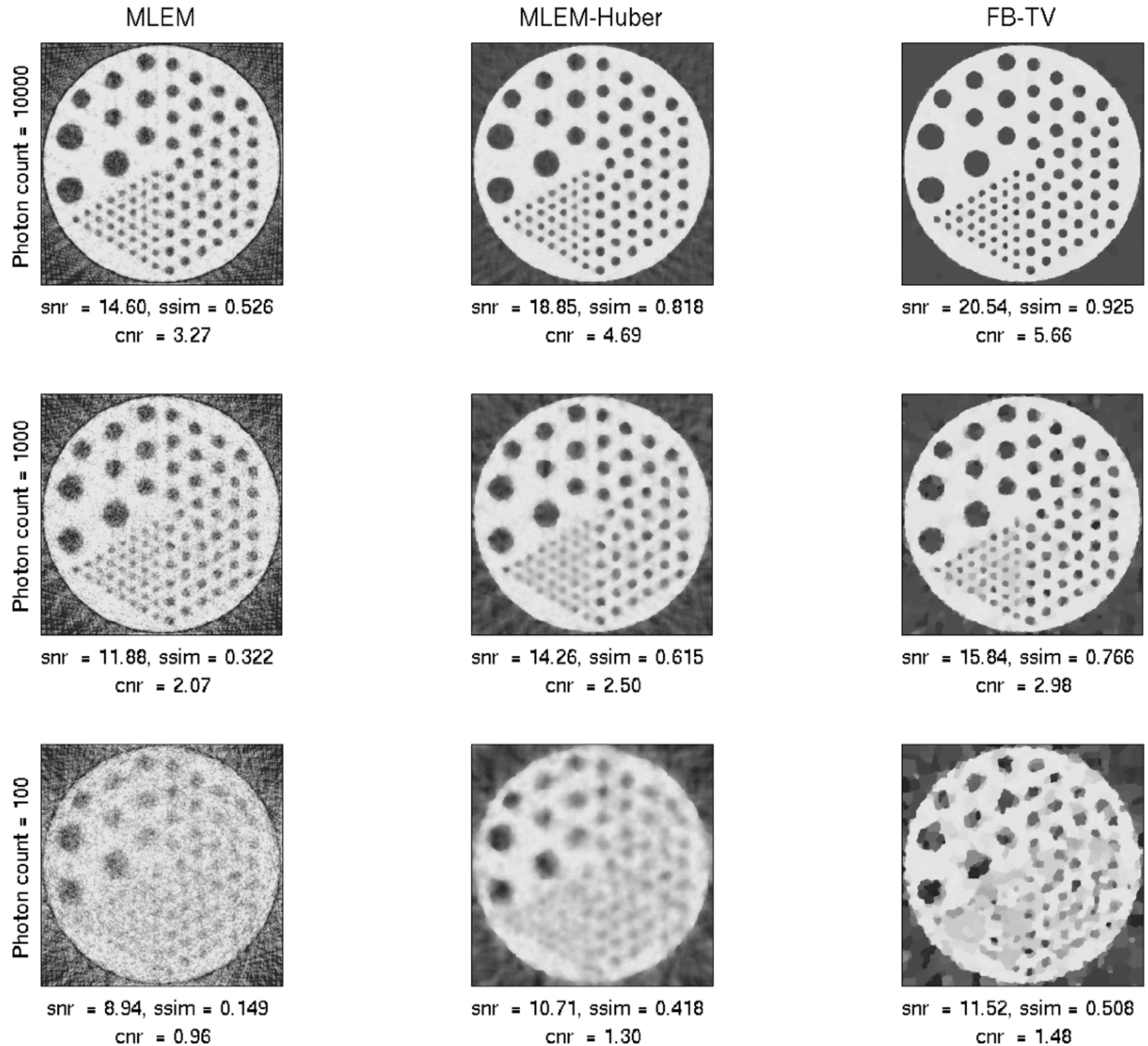
Solution : Chambolle-Pock iterations :

$$\begin{cases} y_{n+1} = (I + \sigma\partial F^*)^{-1}(y_n + \sigma K\bar{x}_n) \\ x_{n+1} = (I + \tau\partial G)^{-1}(x_n - \tau K^* y_{n+1}) \\ \bar{x}_{n+1} = 2x_{n+1} - x_n \end{cases}$$

CBCT , Z = 1 000 photons, resolution phantom







TEP , 200 000 counts (1500 /pixel), resolution phantom