AutoCalibrating Parallel MRI, with or without calibration lines, using Eigen-Vector analysis and structured low-rank matrix completion

igen

(ESPIRiT 2.0)

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The Need for Speed

- MRI data collection is inherently slow
- Faster imaging is essential in many applications
- Parallel Imaging
  - Faster imaging by reducing data
  - Exploit multiple receiver arrays

ultrasound

[1] cardiovascularultrasound.com
[2] siemensehealthcare.com
[3] Jim Pipe, BNI

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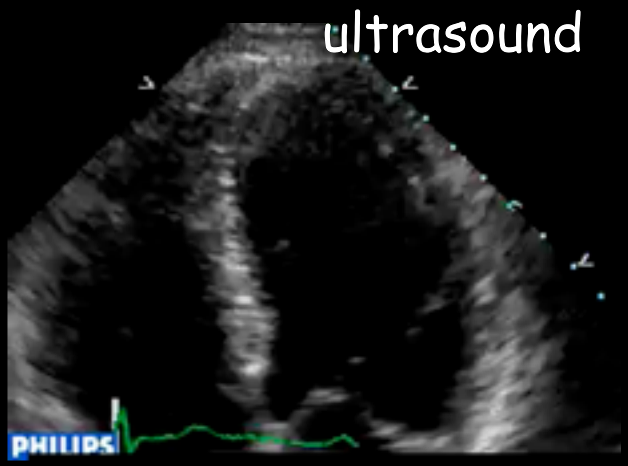

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The Book of
Parallel Imaging

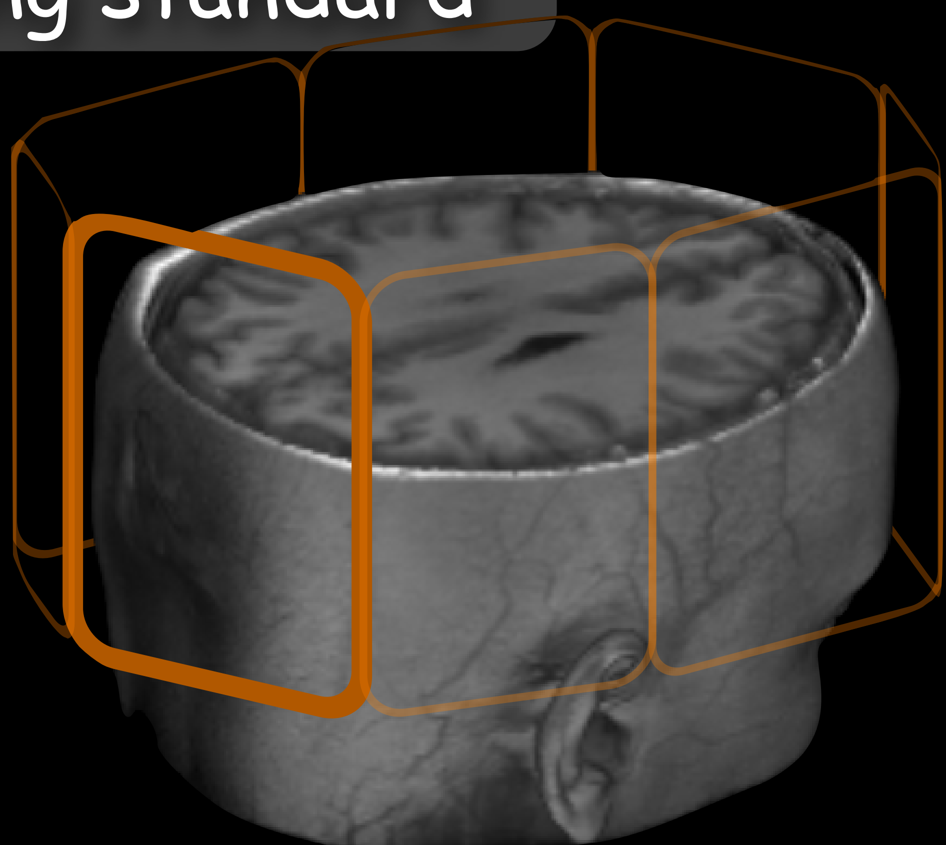
Preface

Coil Arrays

Used to:

- Increase SNR
- Acceleration

32 channels becoming standard



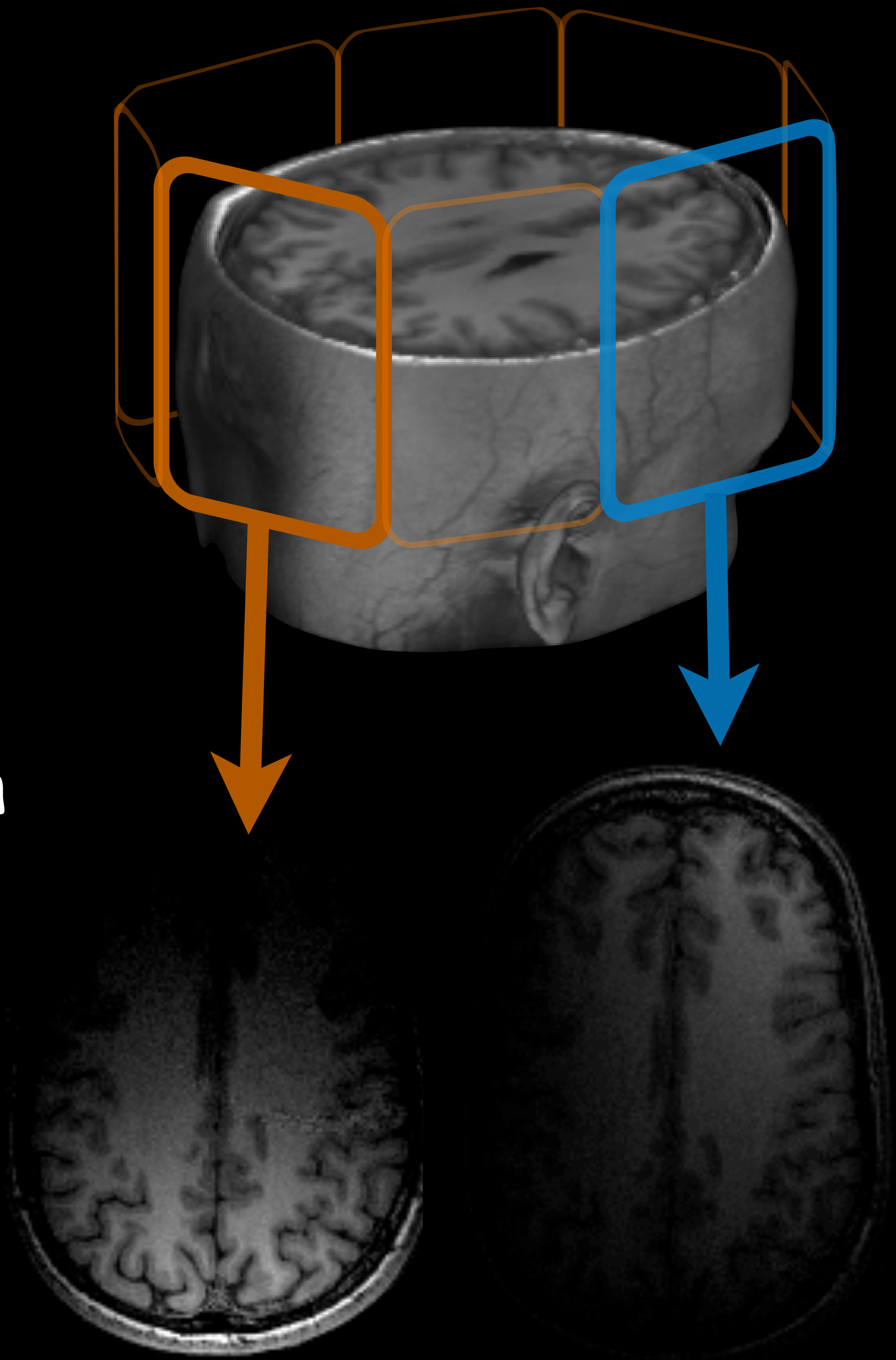
12ch body coil

8ch head coil

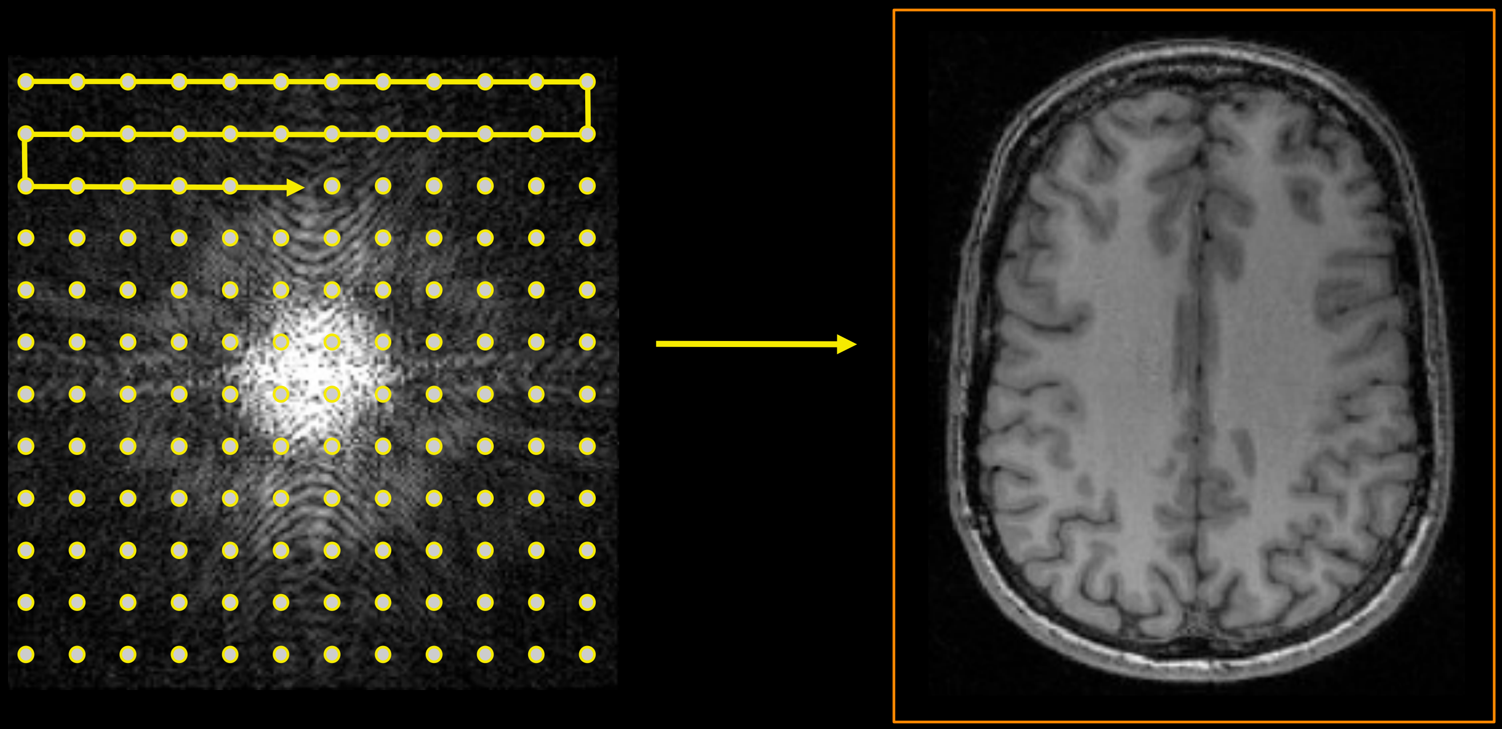
* Courtesy, Phil Beatty

Coil Sensitivities

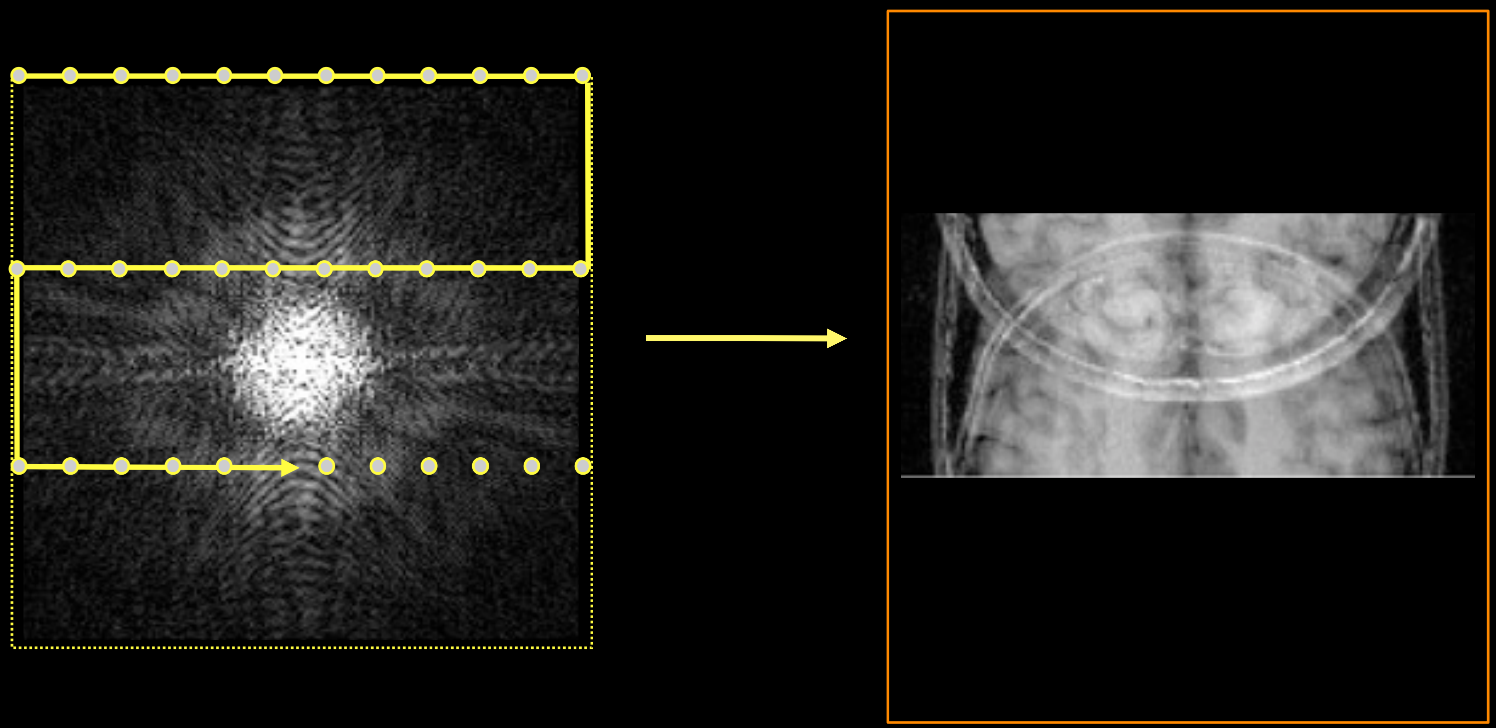
- Multiple local receiver coils
- Coil sensitivities provide additional information for reconstruction
- Allows undersampling/aliasing in k-space



Parallel receive coils reduce sampling requirements

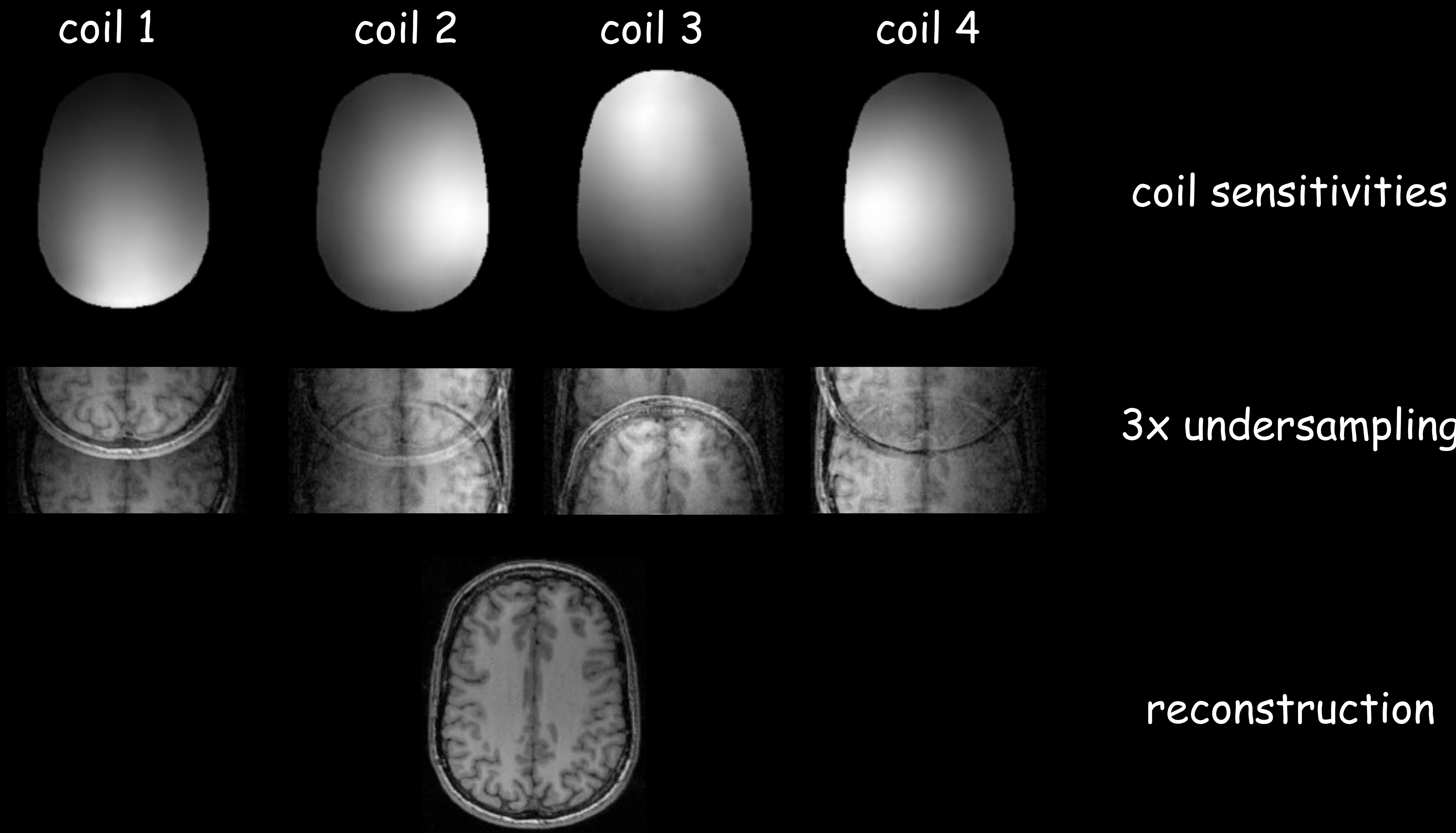


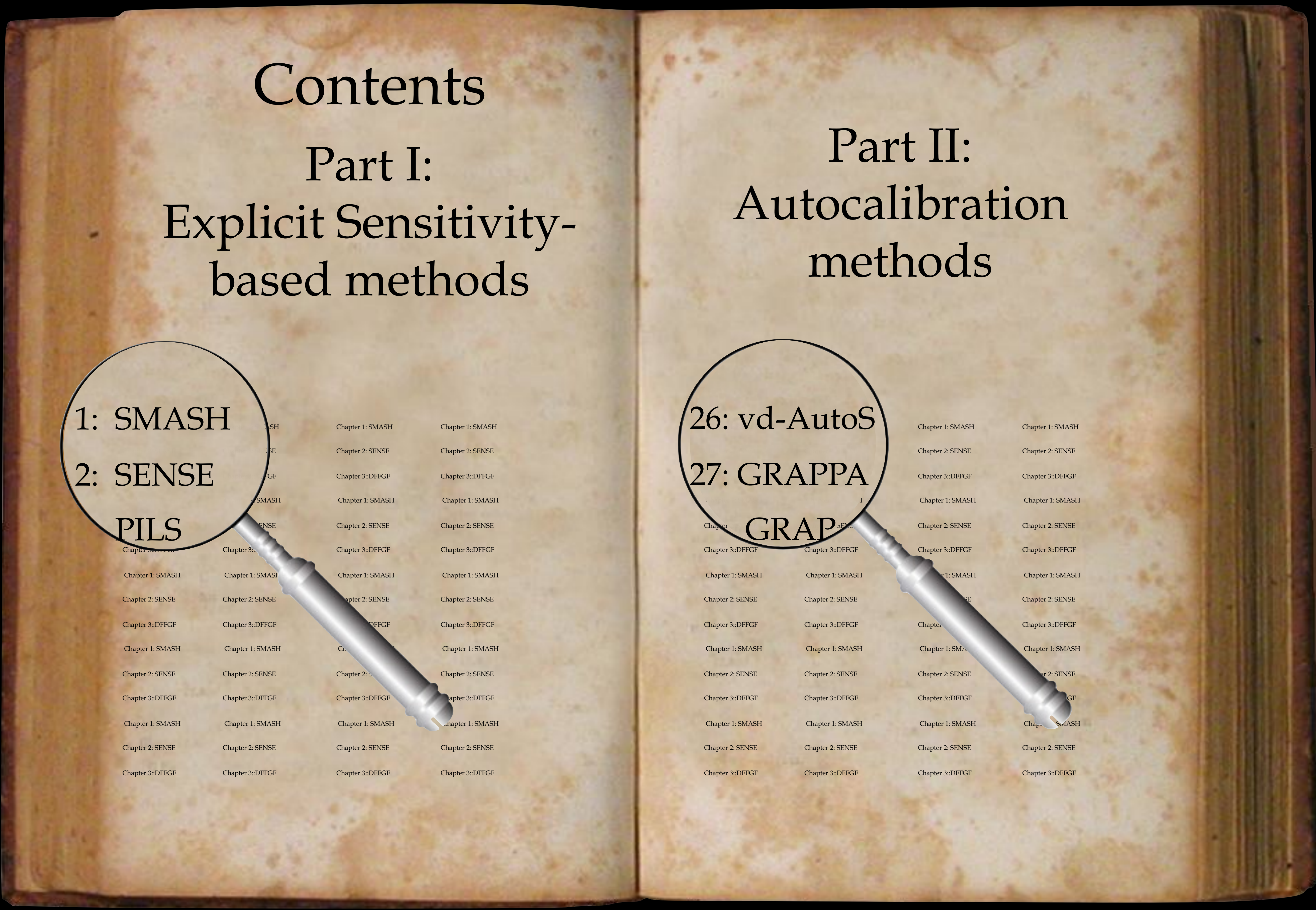
Standard k-space sampling



Reduced k-space sampling

Parallel Imaging




Contents
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Explicit Sensitivity-
based methods
1: SMASH
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PILS
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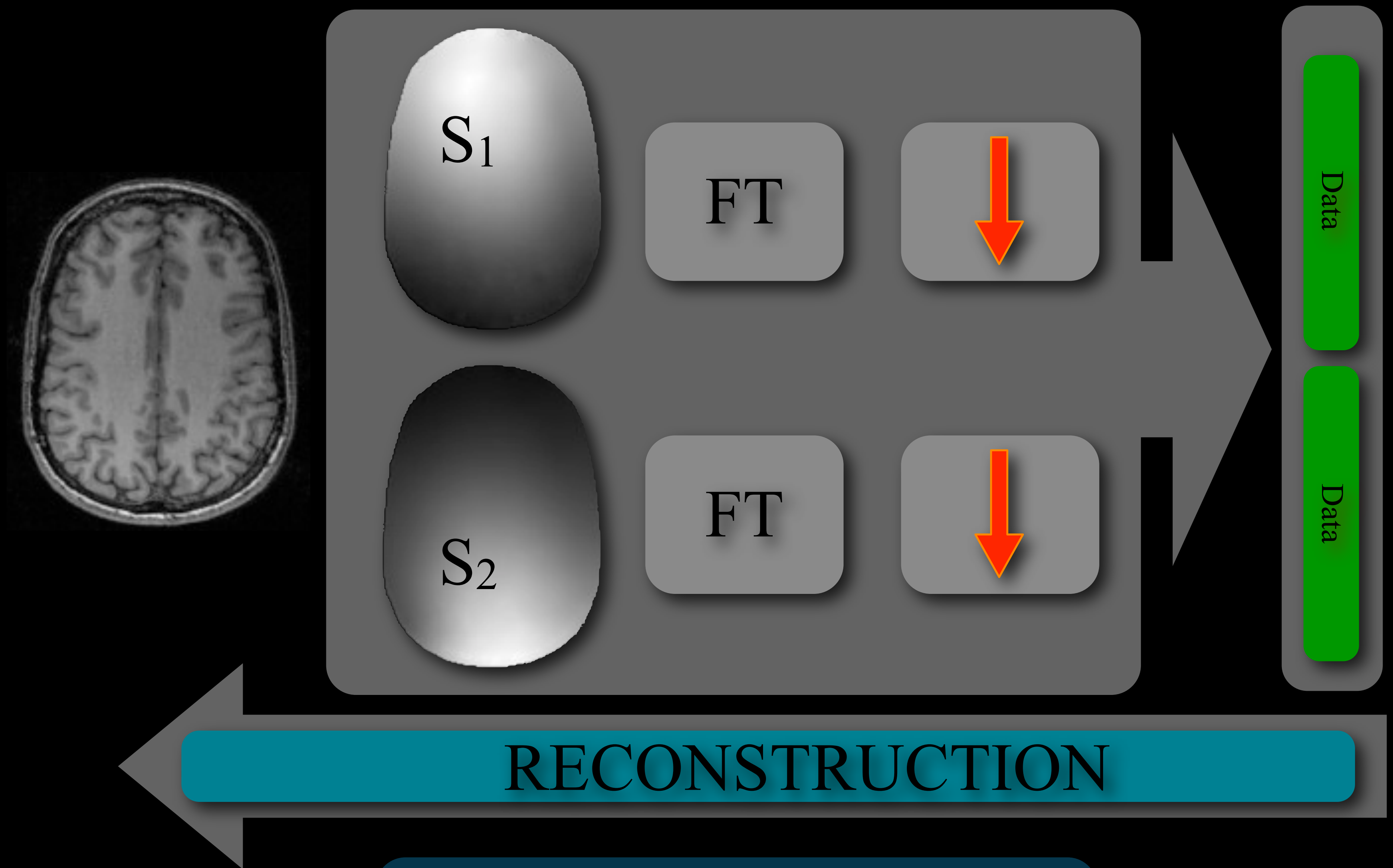
methods
26: vd-AutoS
27: GRAPPA
GRAP
Chapter 1: SMASH
Chapter 2: SENSE
Chapter 3::DFFGF

Sensitivity Encoding (SENSE)

SENSE model



$$Ex = y$$

Pruessmann
et. al., 1999

SENSE

*image, courtesy of Kevin King

SENSE

- Full inverse model

*image, courtesy of Kevin King

SENSE

- Full inverse model
- Noise optimal

*image, courtesy of Kevin King

SENSE

- Full inverse model
- Noise optimal
- One combined image

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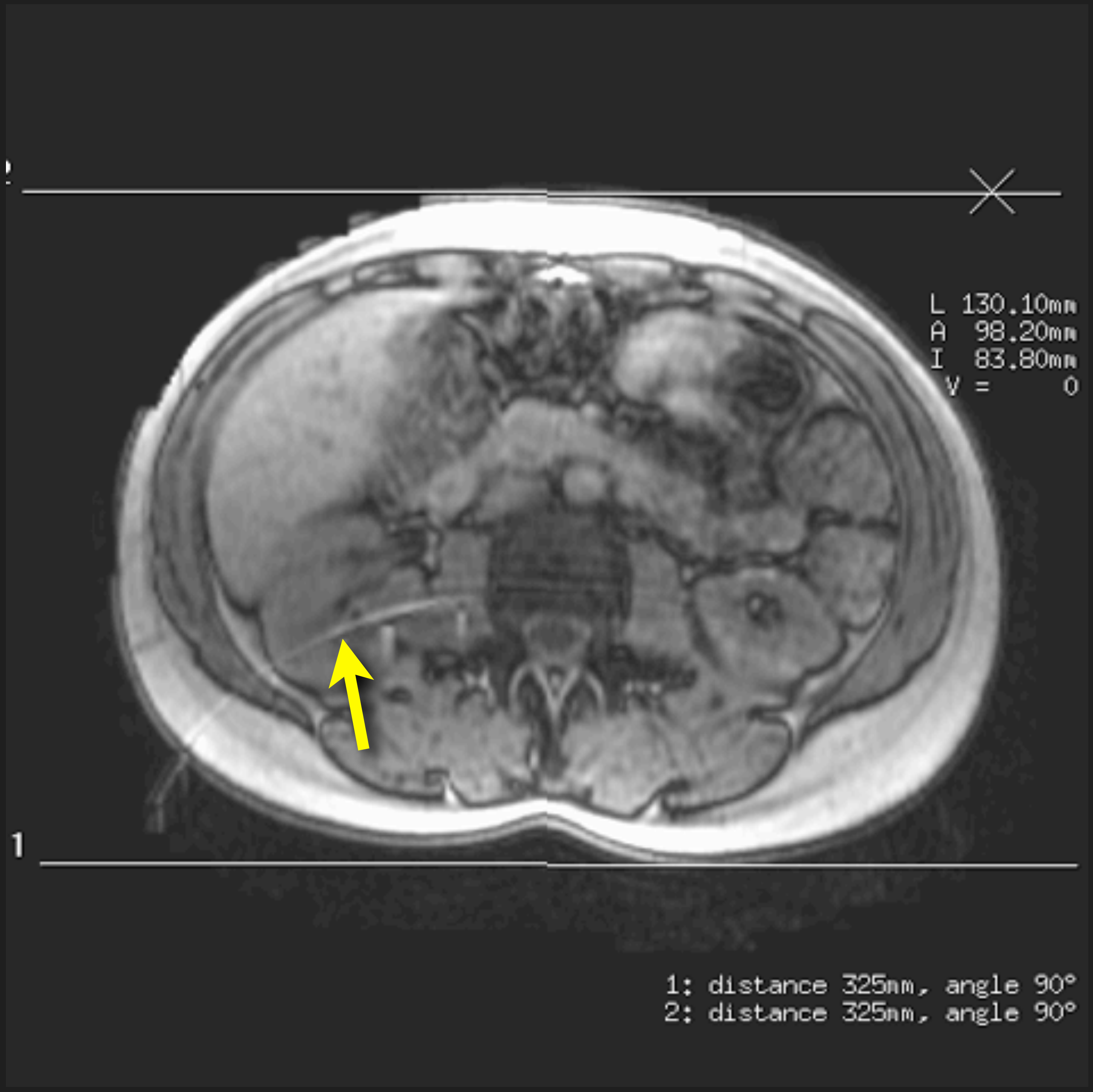
SENSE

- Full inverse model
- Noise optimal
- One combined image

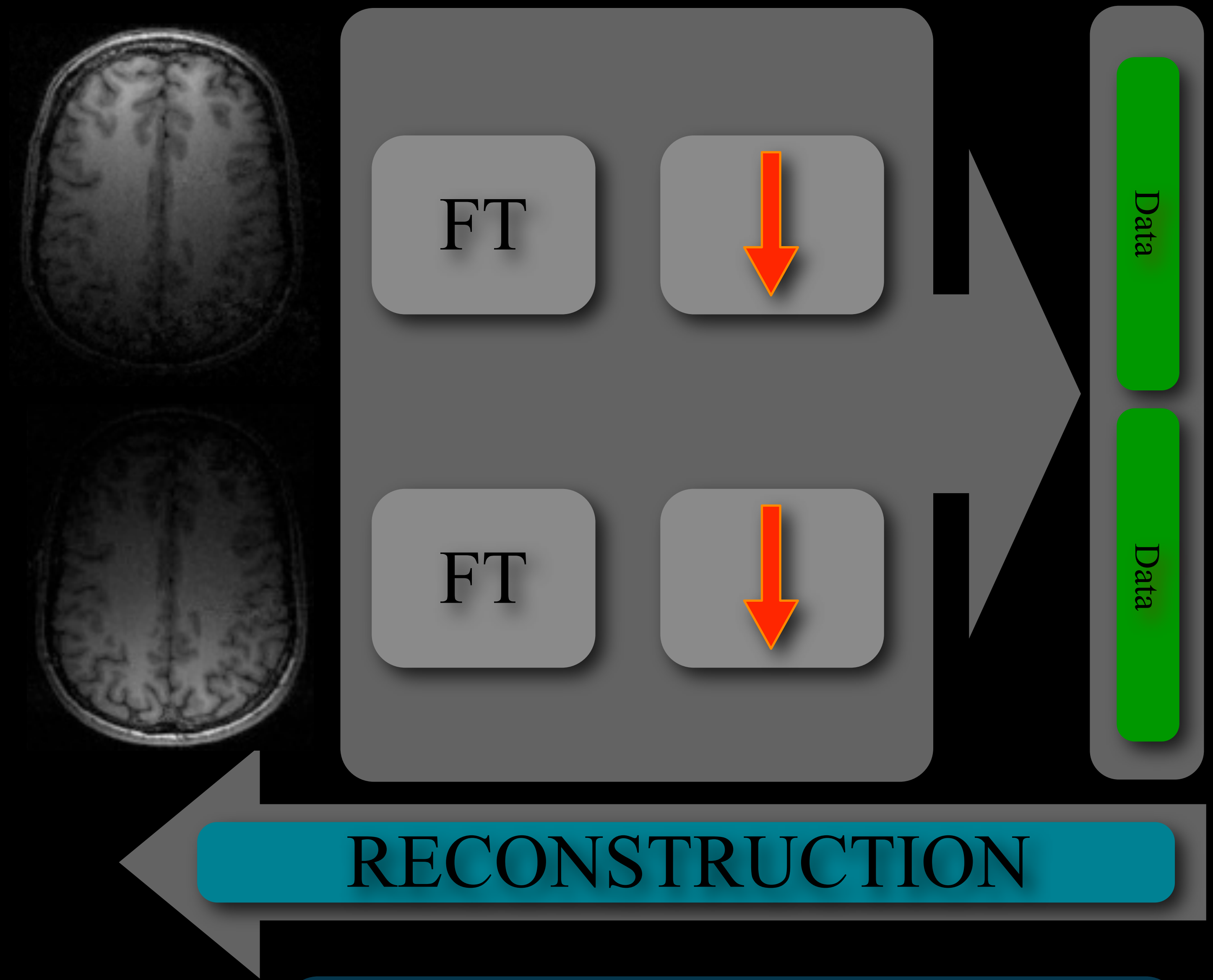
- Prone to errors in sensitivity map estimation.
  - Often less robust in practice



*image, courtesy of Kevin King

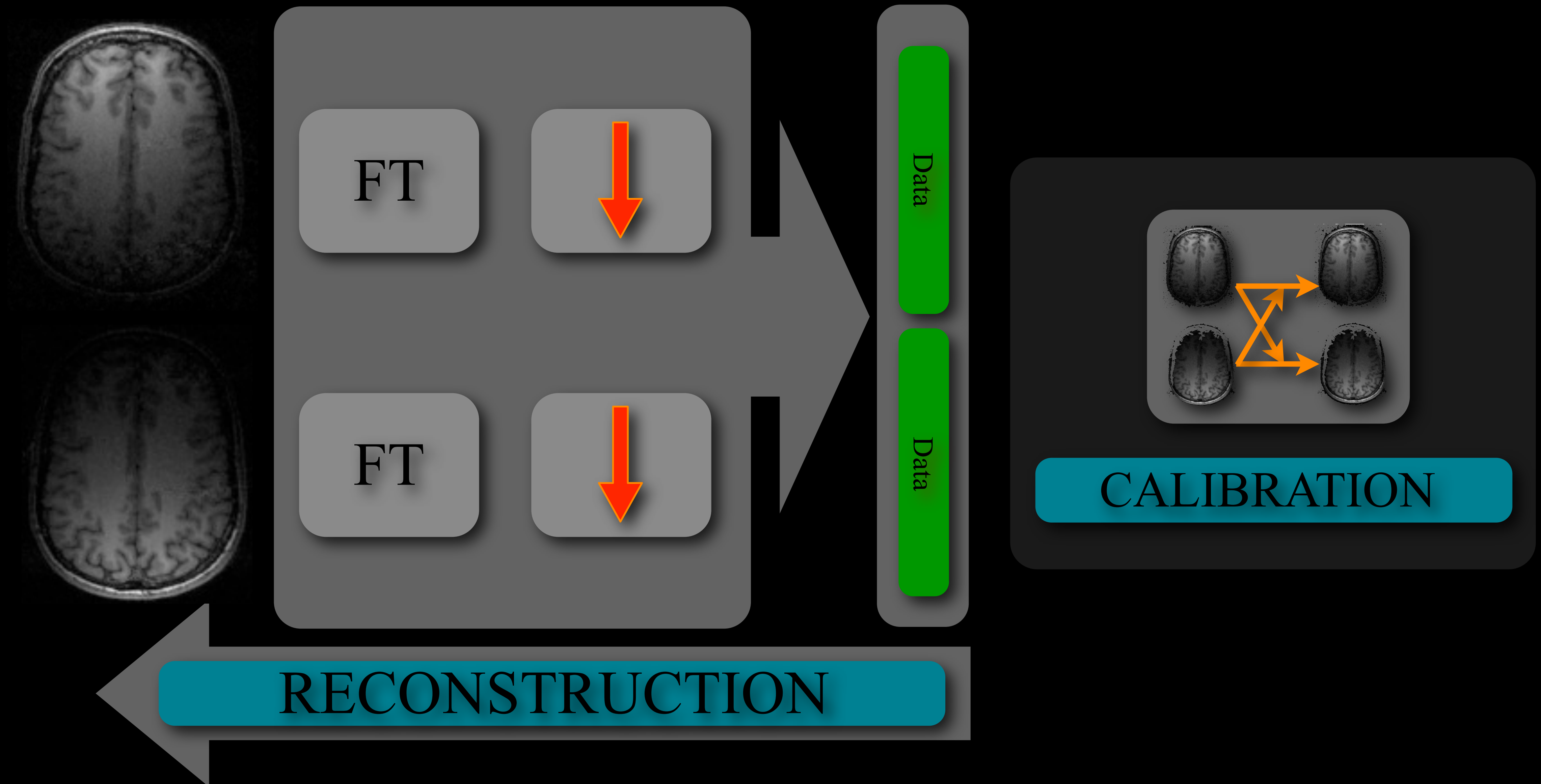
Generalized Autocalibrating Partially Parallel Acquisitions (GRAPPA)

Autocalibrating Model (GRAPPA)



$$F_u x = y$$

Autocalibrating Model (GRAPPA)

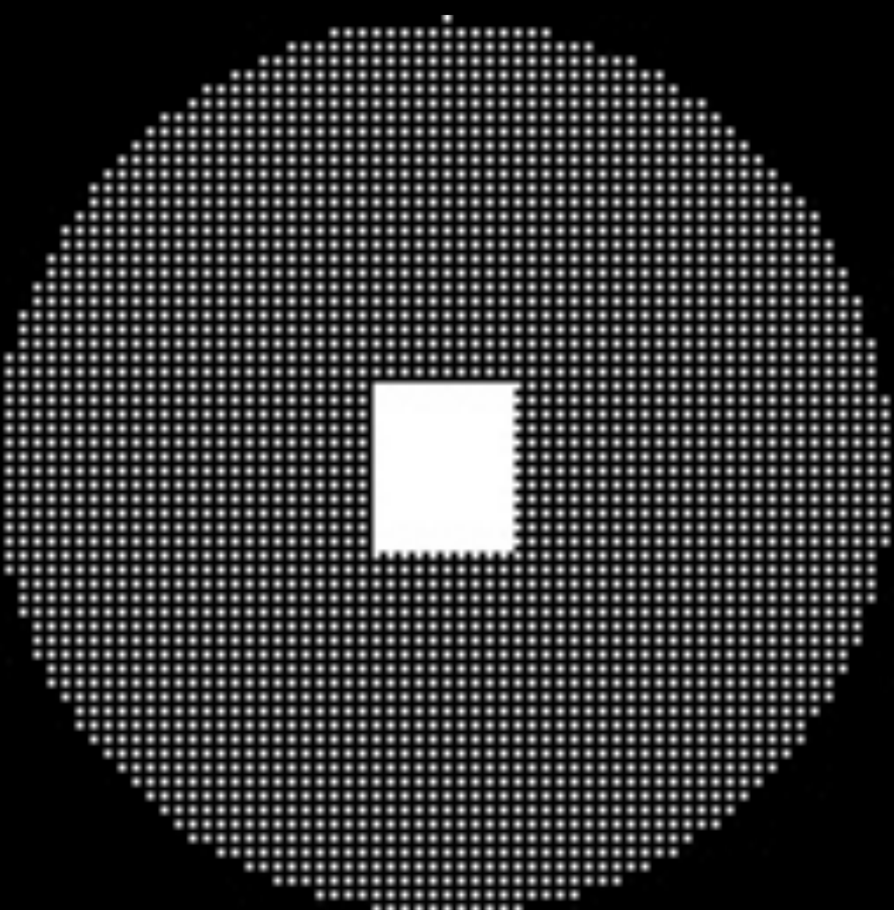
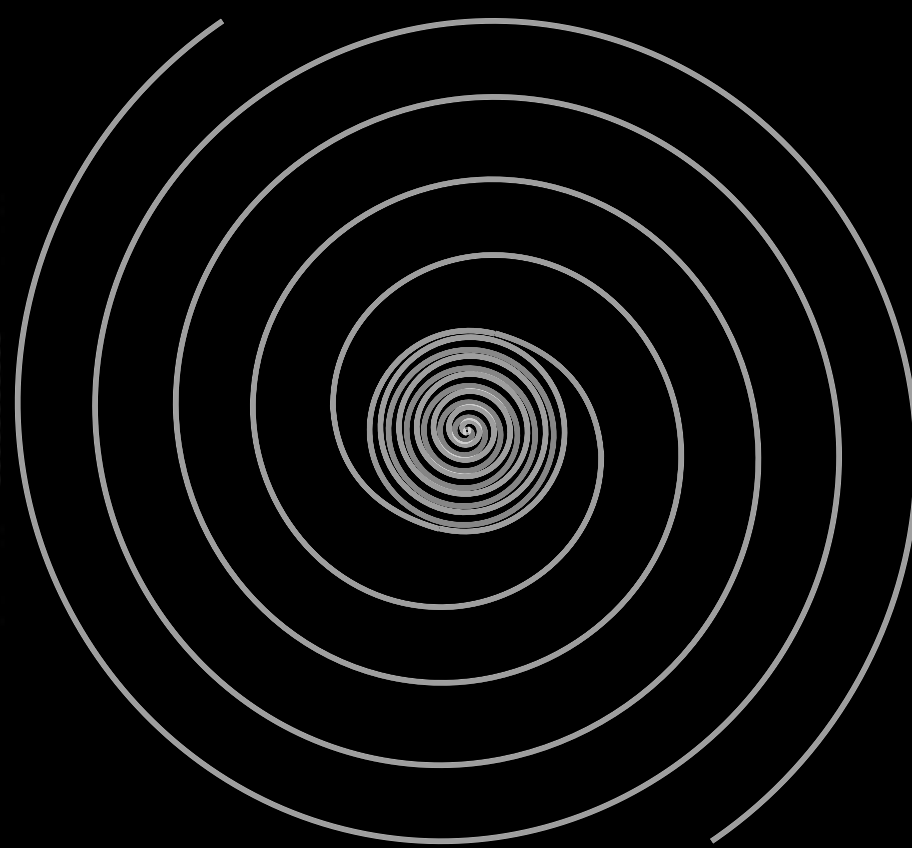
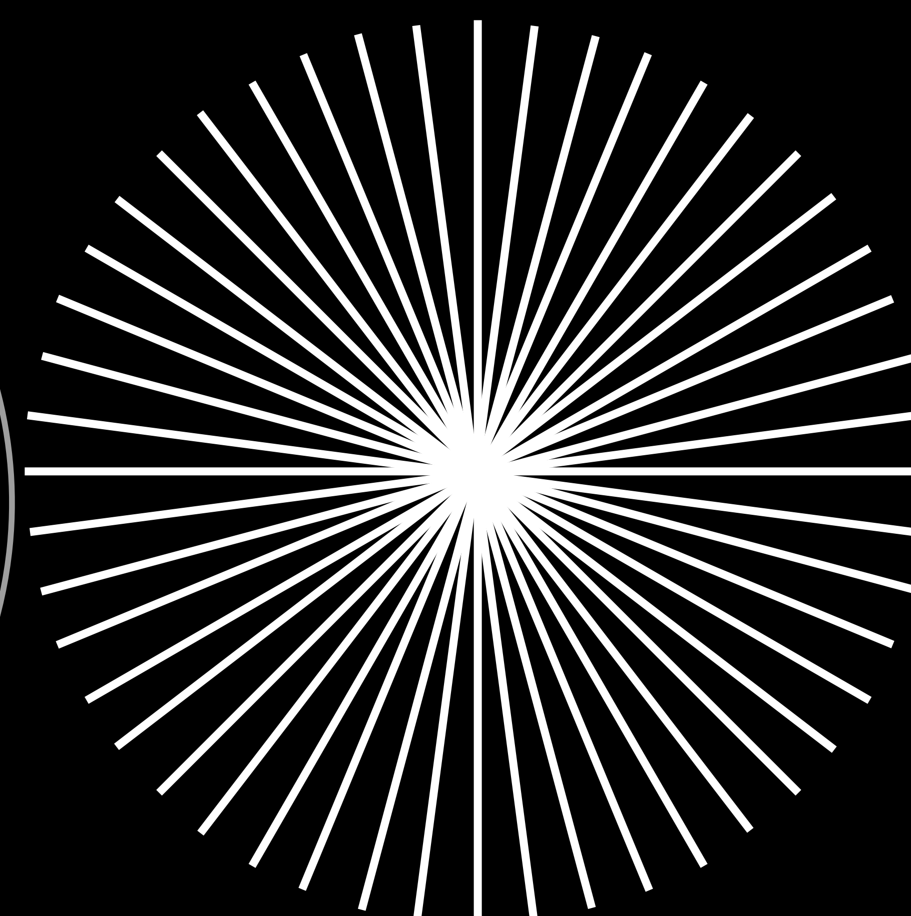


$$F_u x = y$$

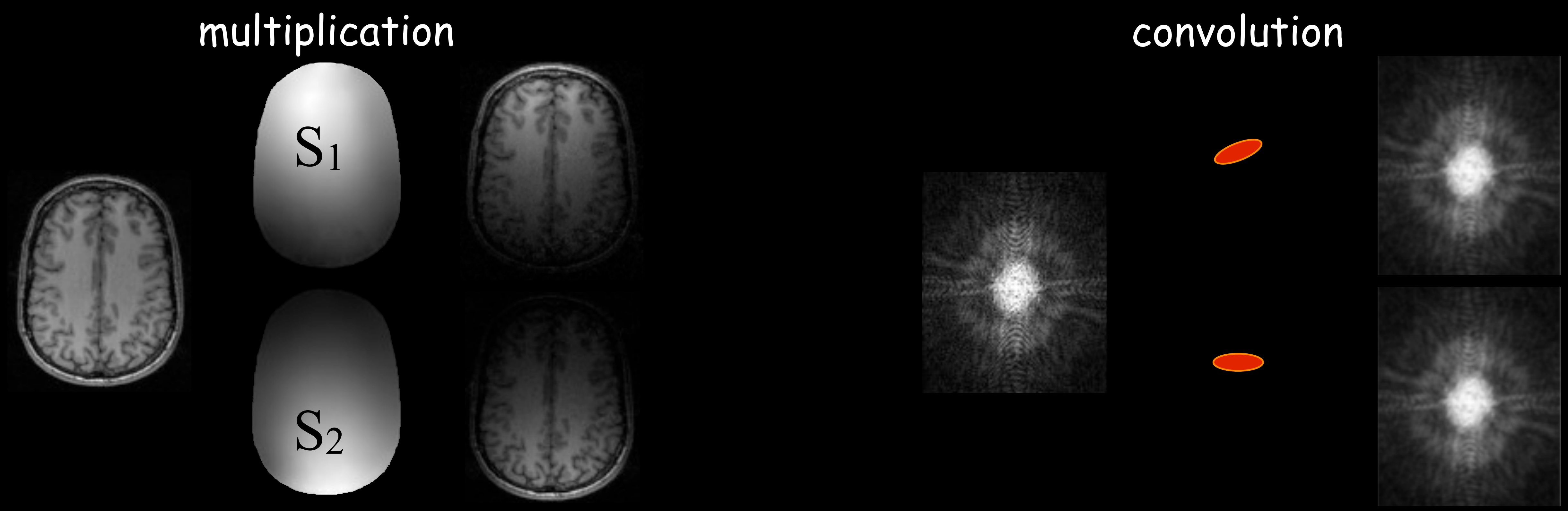
$$\vert x \in \mathbb{G}$$

AutoCalibration

- Autocalibration methods will have k-space center densely sampled
- Autocalibration tends to be more robust in practice

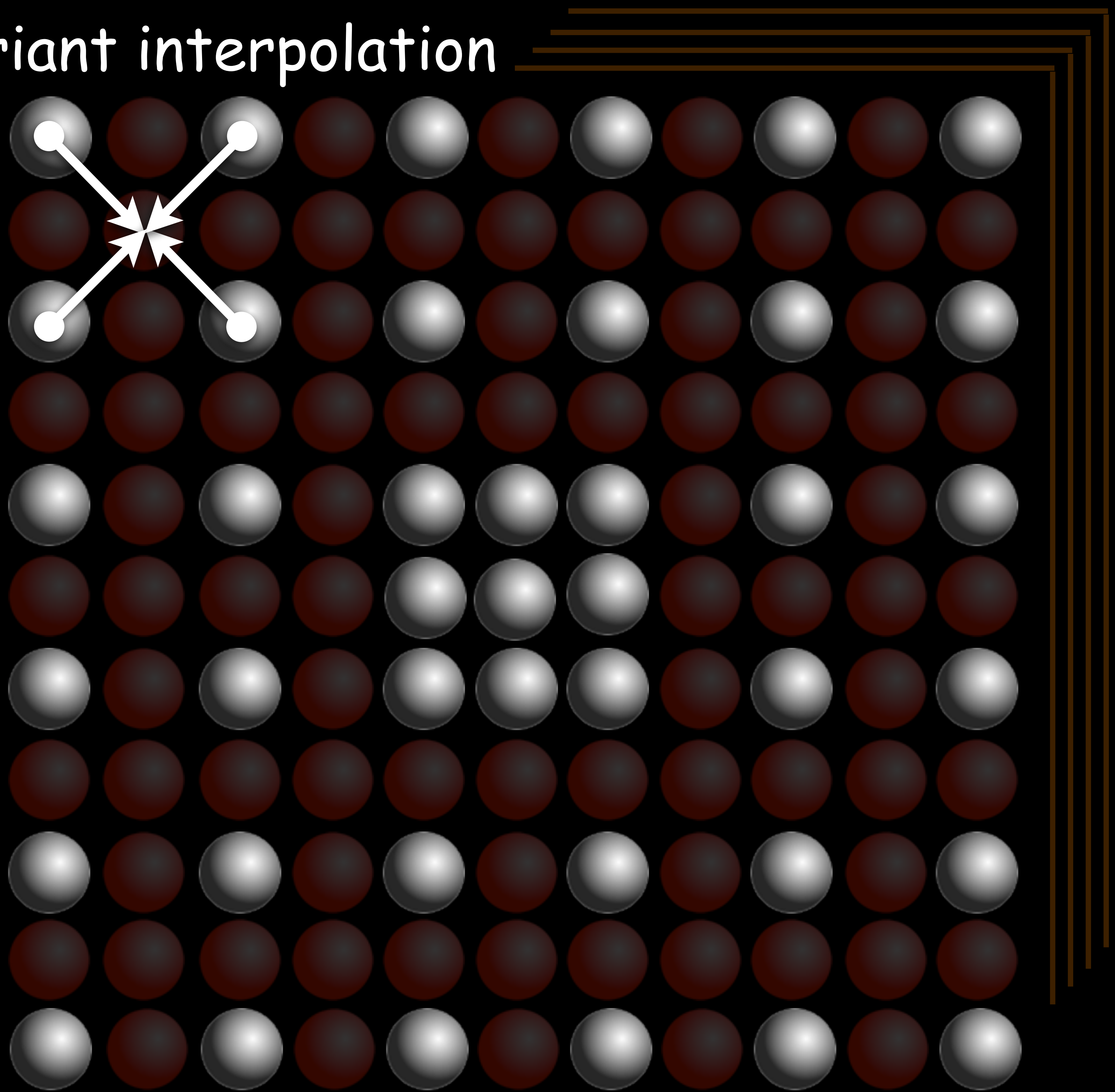
Correlation in k-space



- Image weighting is equivalent to k-space blurring
- Coil sensitivities are smooth, therefore the blurring kernel is compact.
- k-space becomes locally correlated.

GRAPPA

k-Space variant interpolation



Griswold
et. al., 2002

GRAPPA

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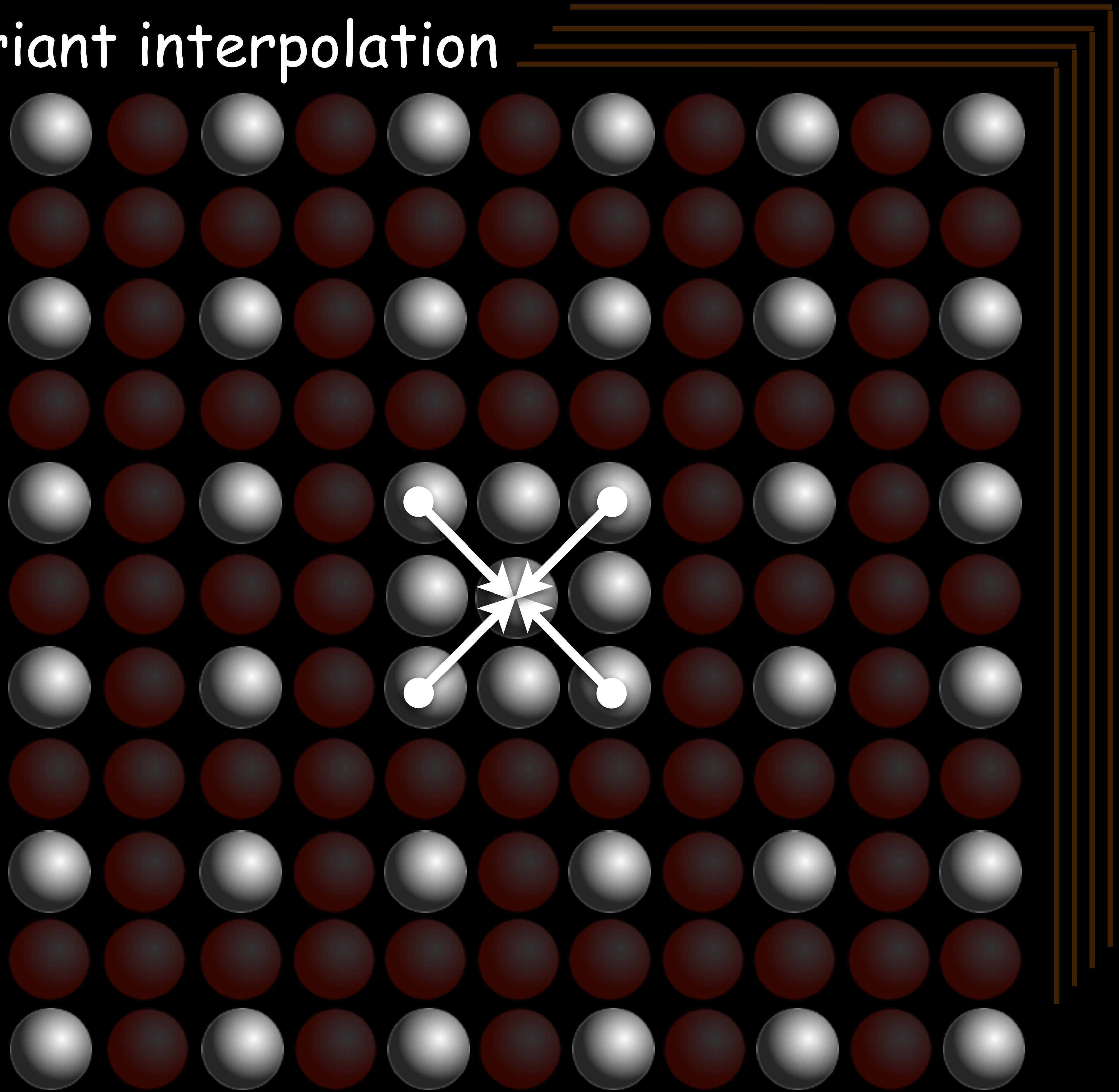

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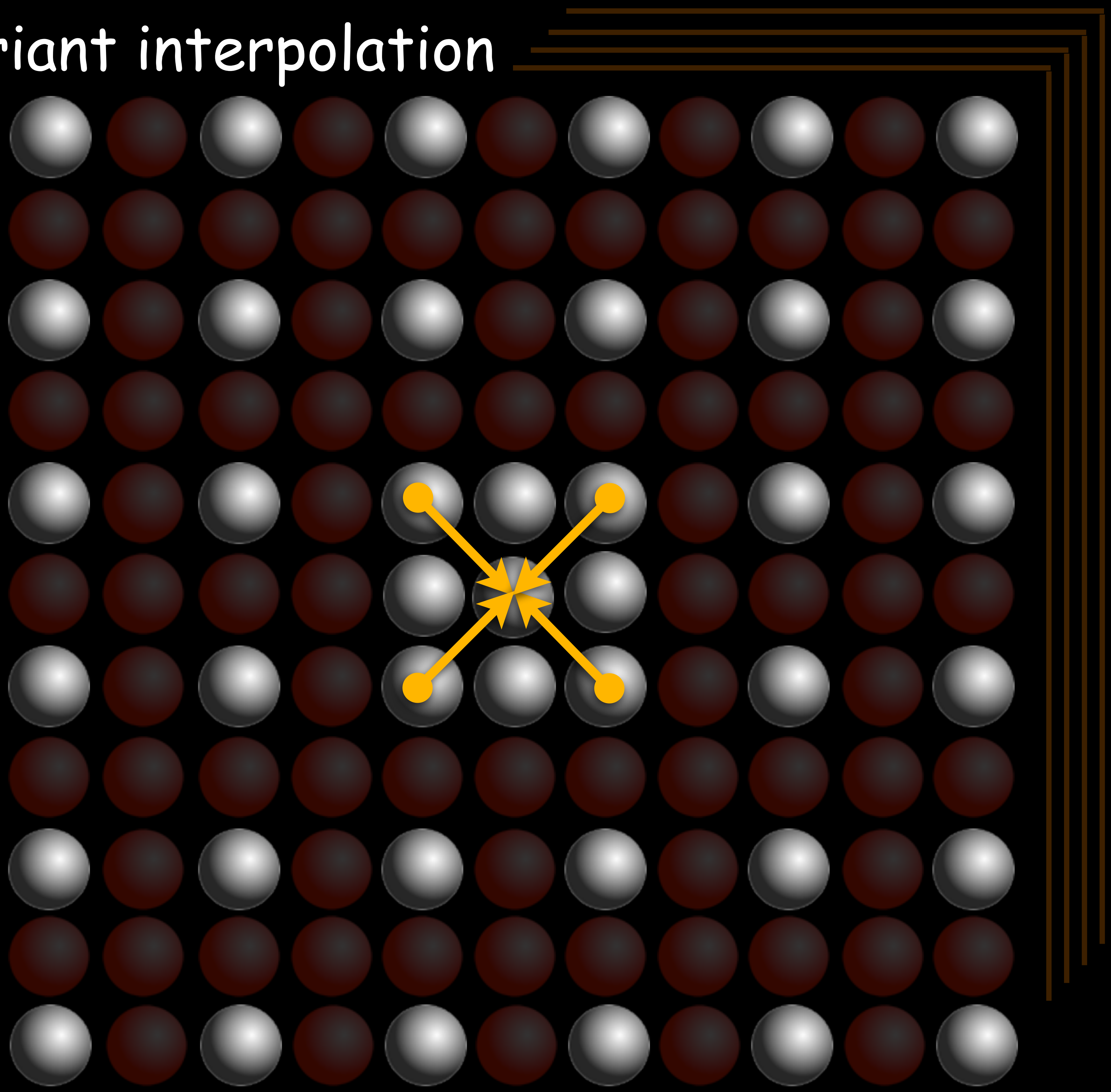

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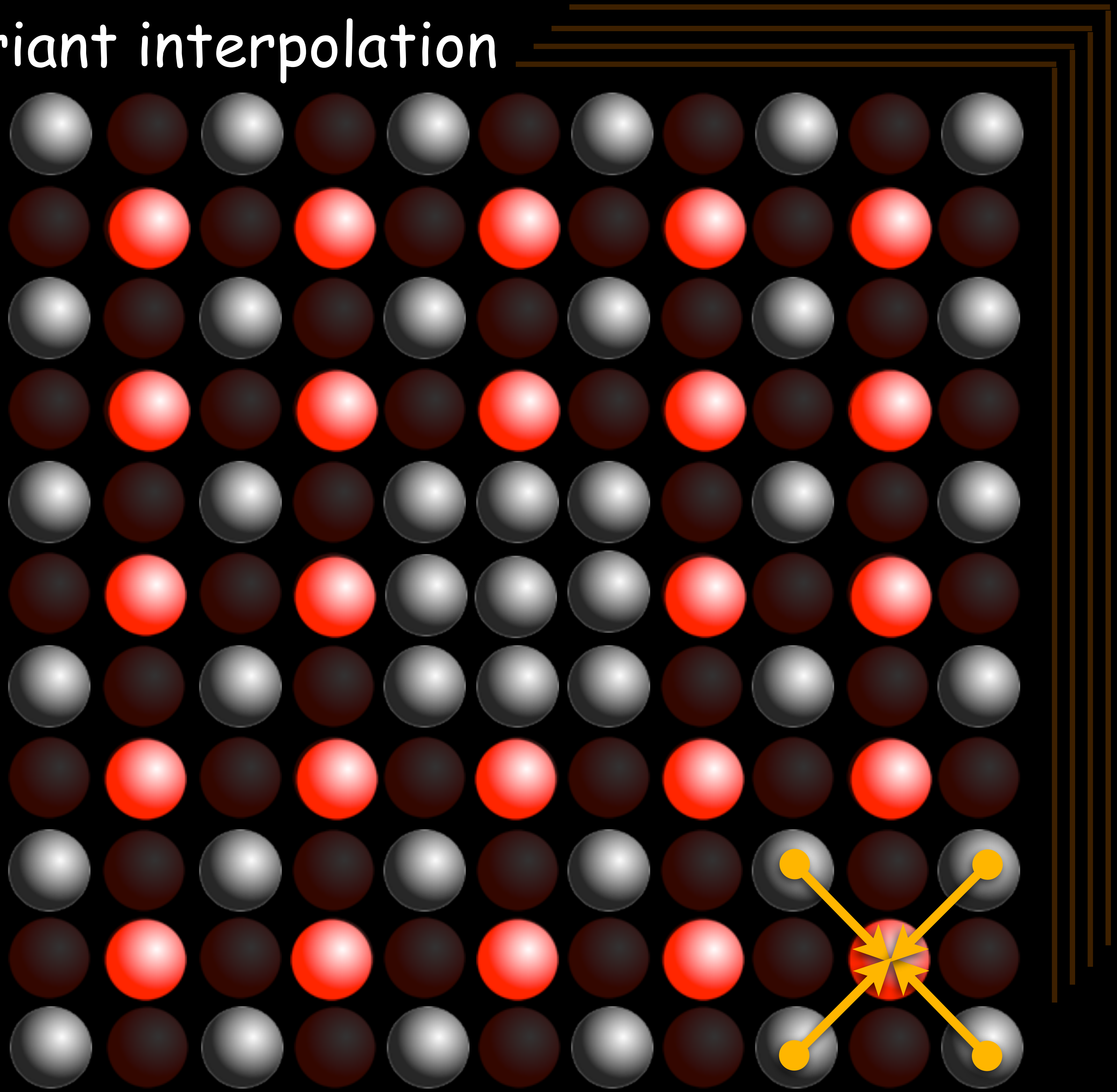

Griswold et. al., 2002

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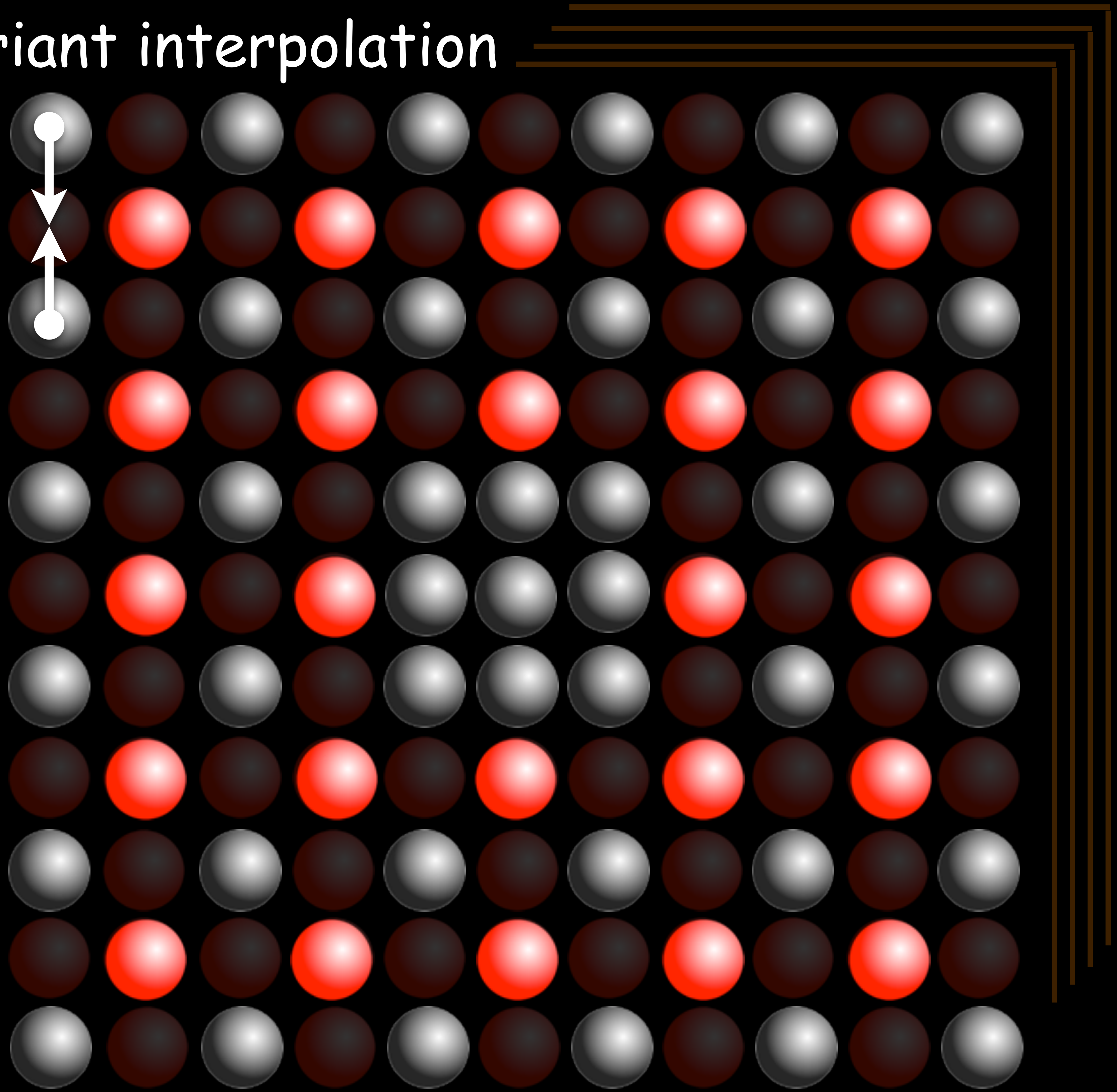

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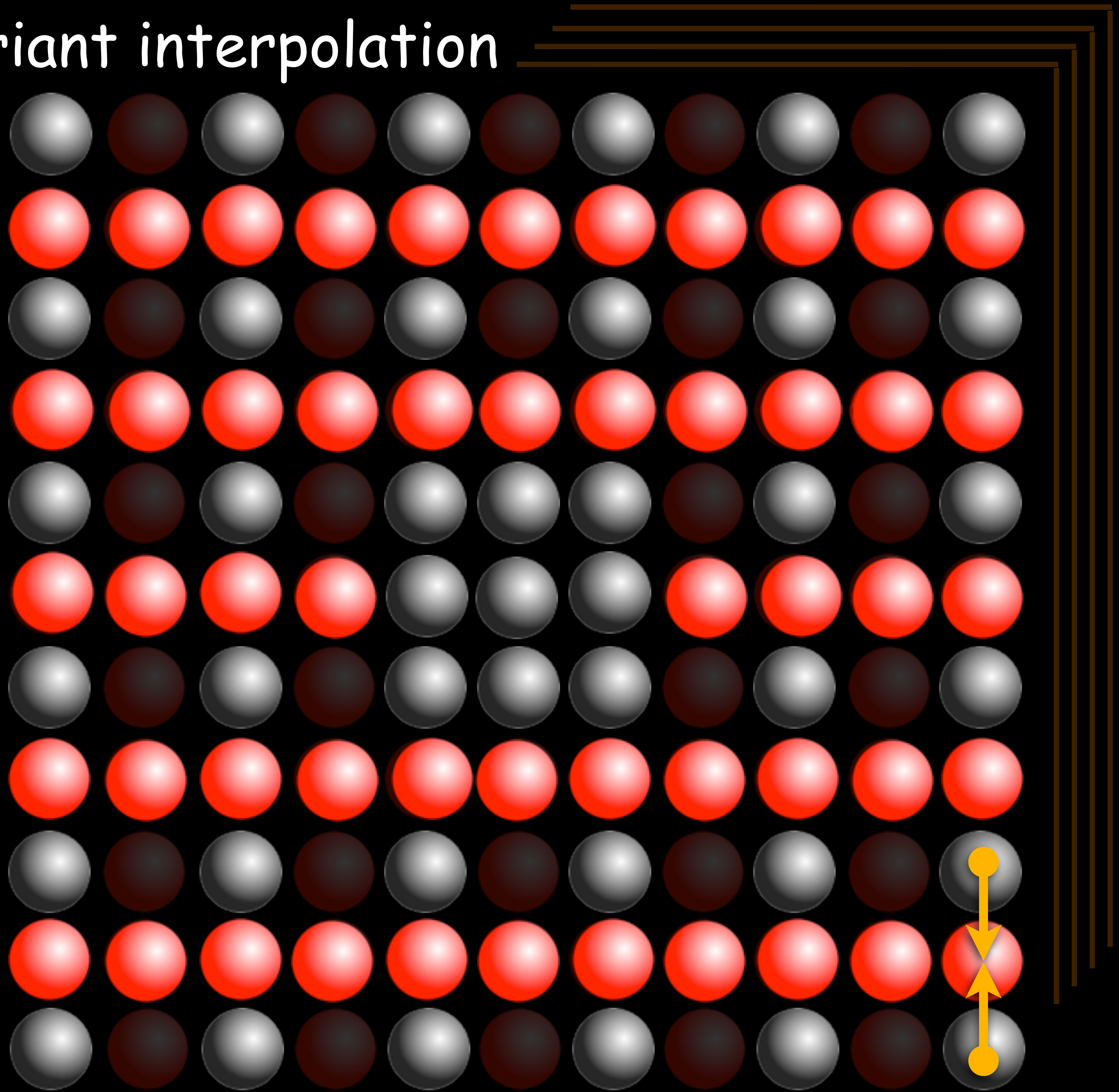

Griswold et. al., 2002

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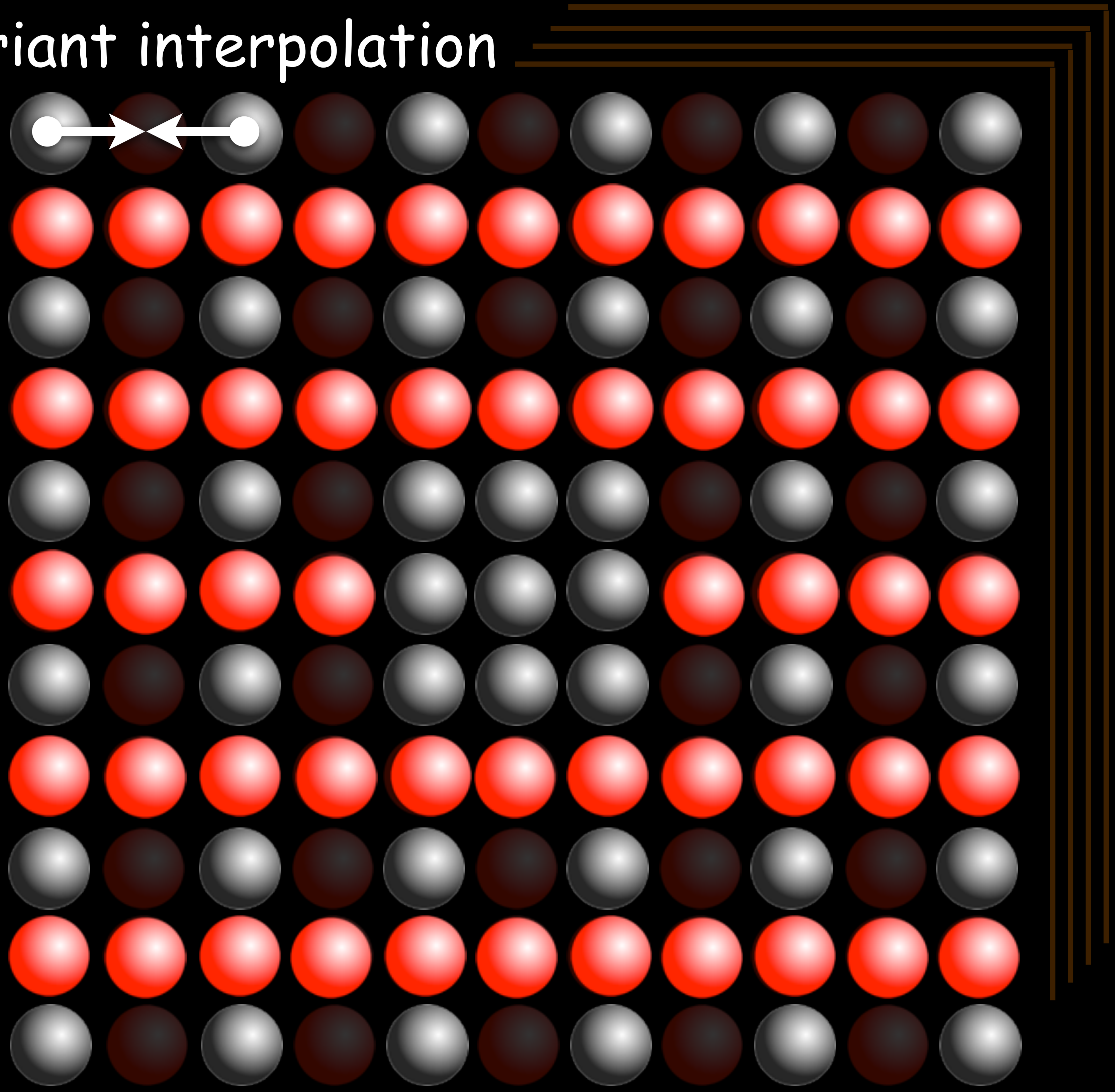

Griswold
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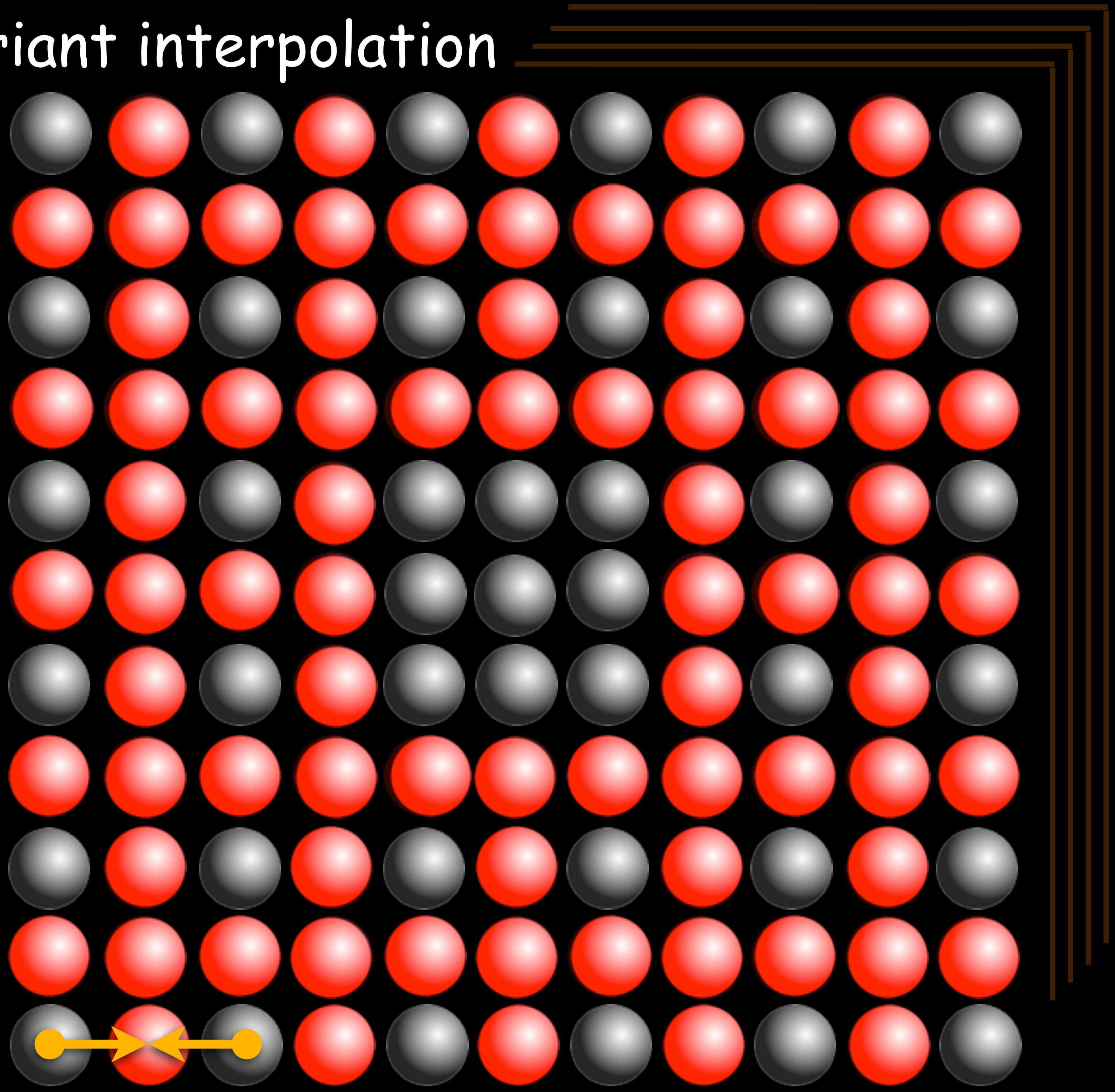
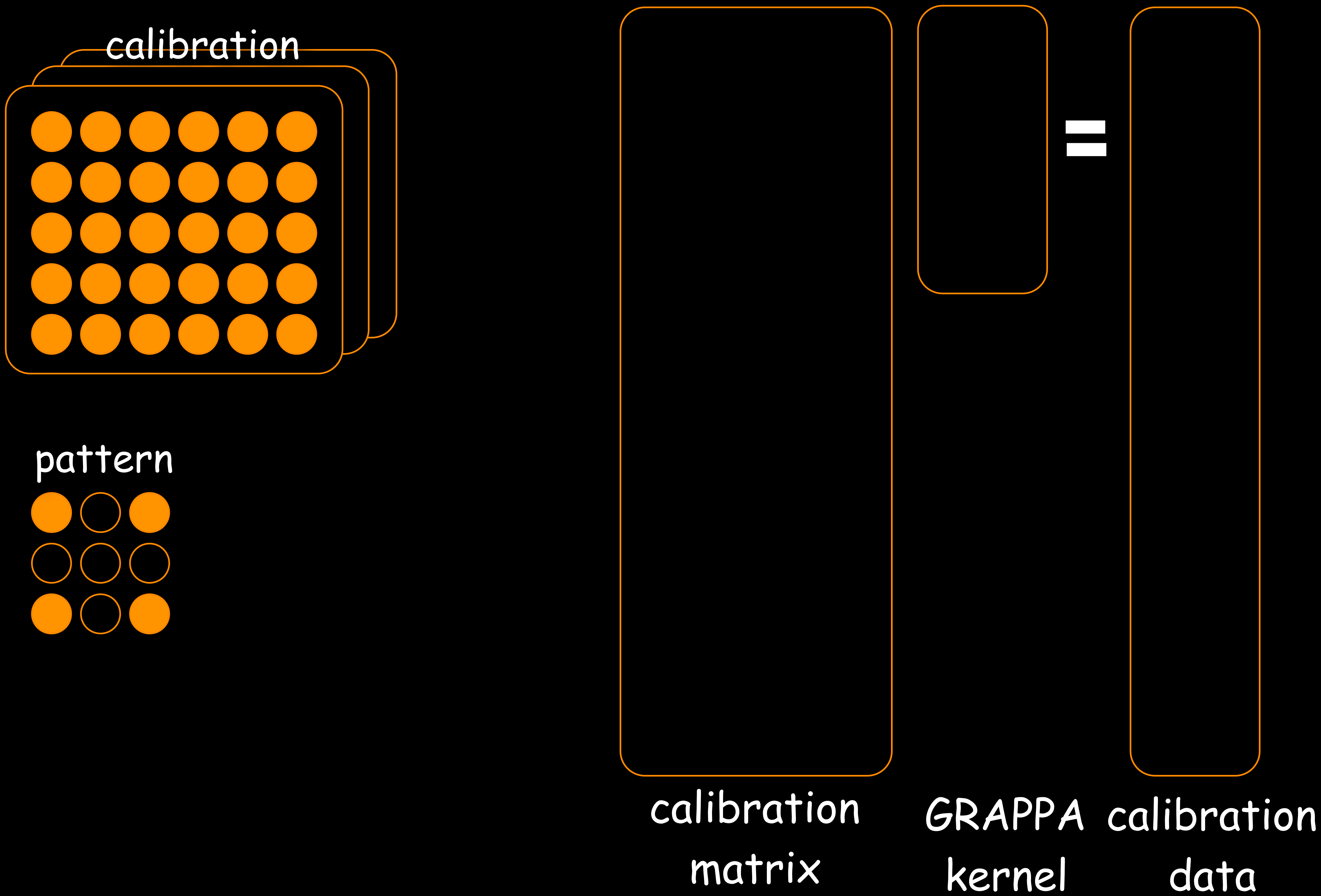
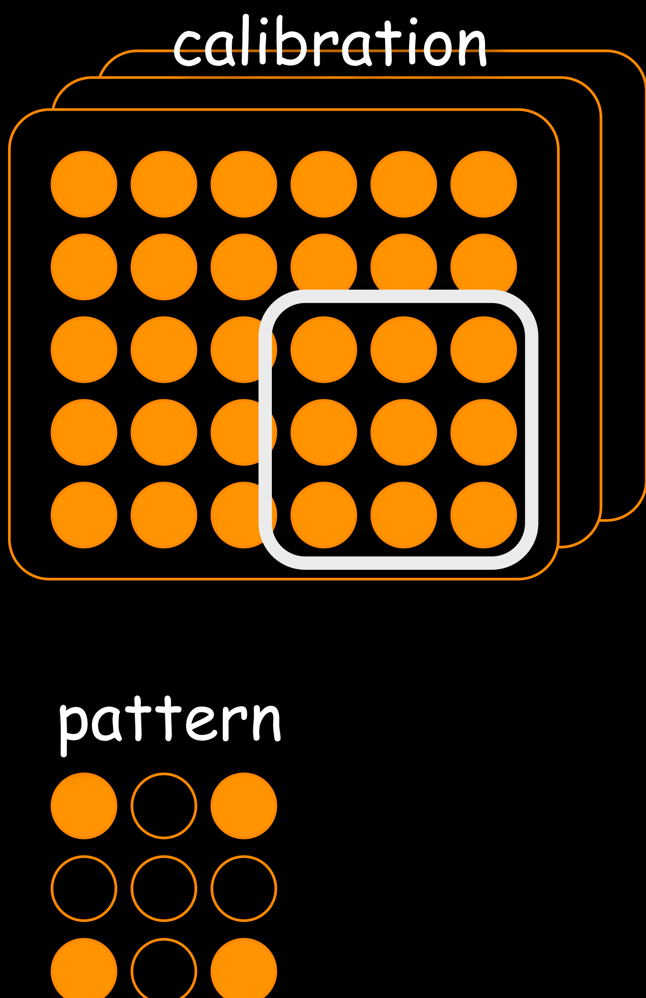


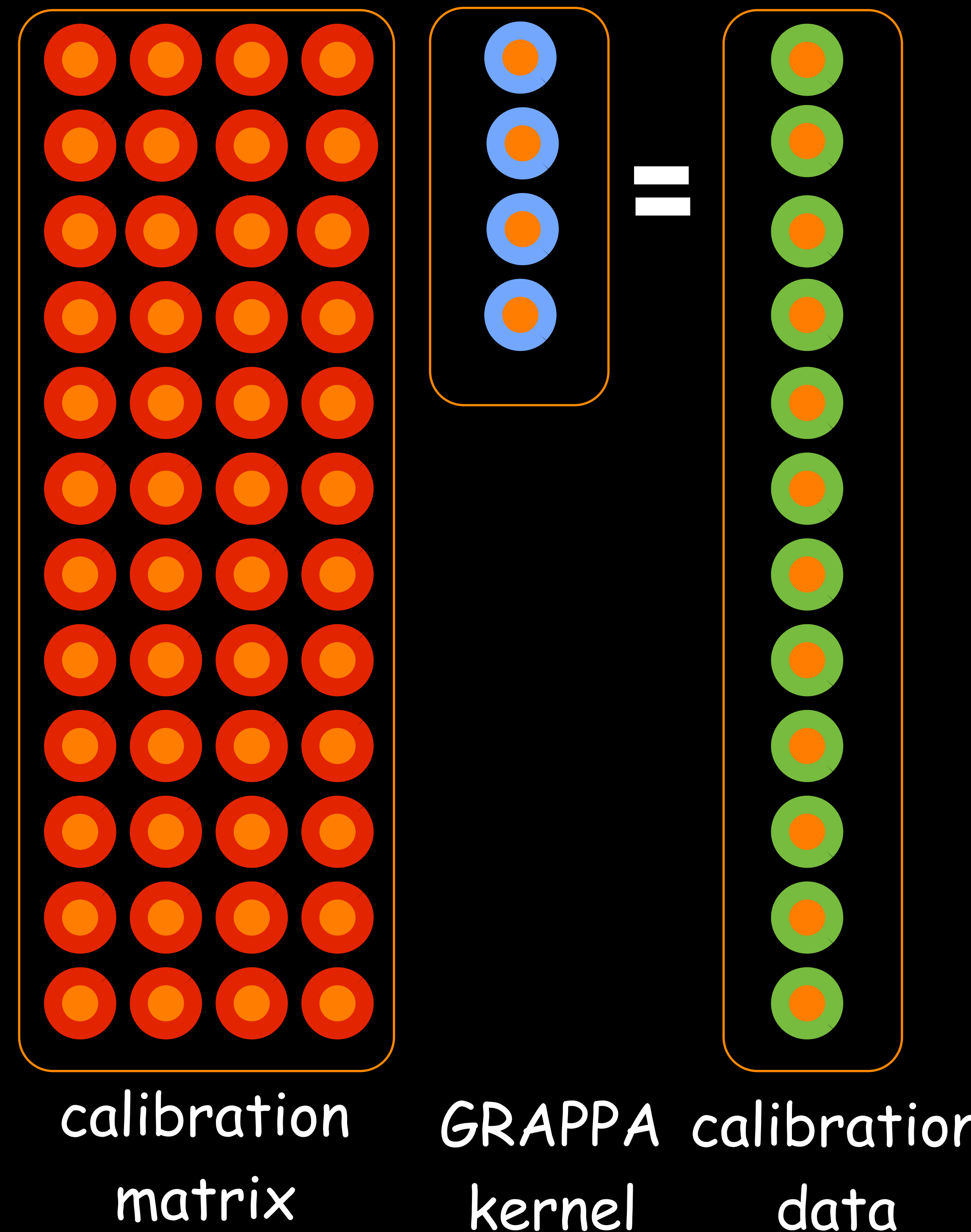
Griswold
et. al., 2002

GRAPPA Calibration



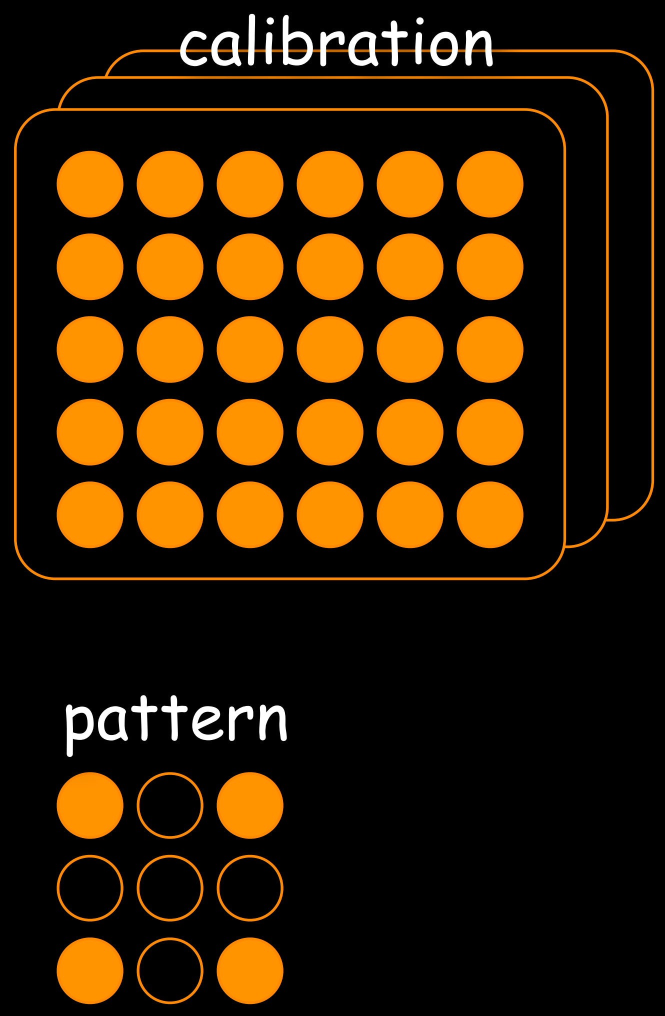
GRAPPA Calibration

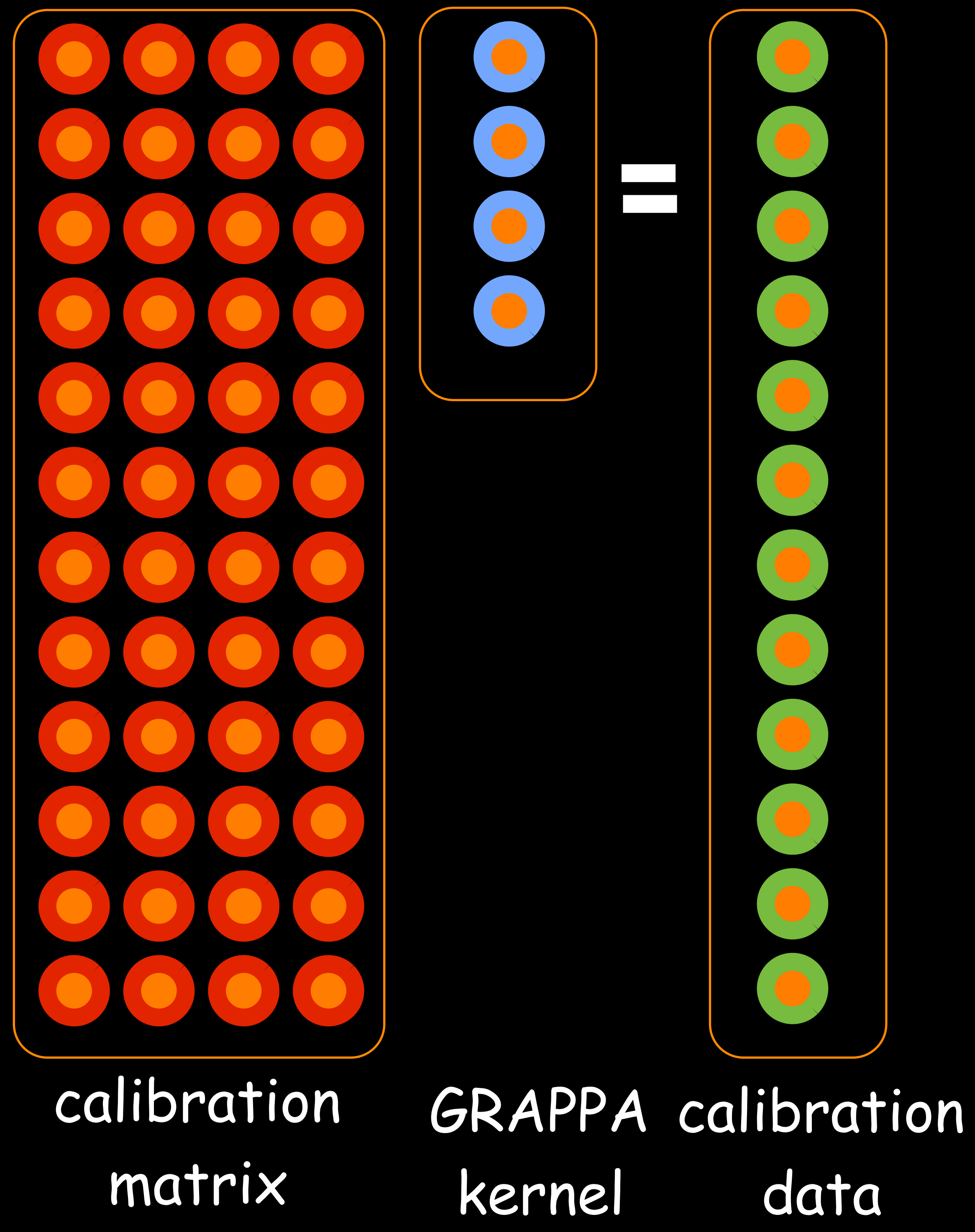




Autocalibration as a Subspace Method

A Different View





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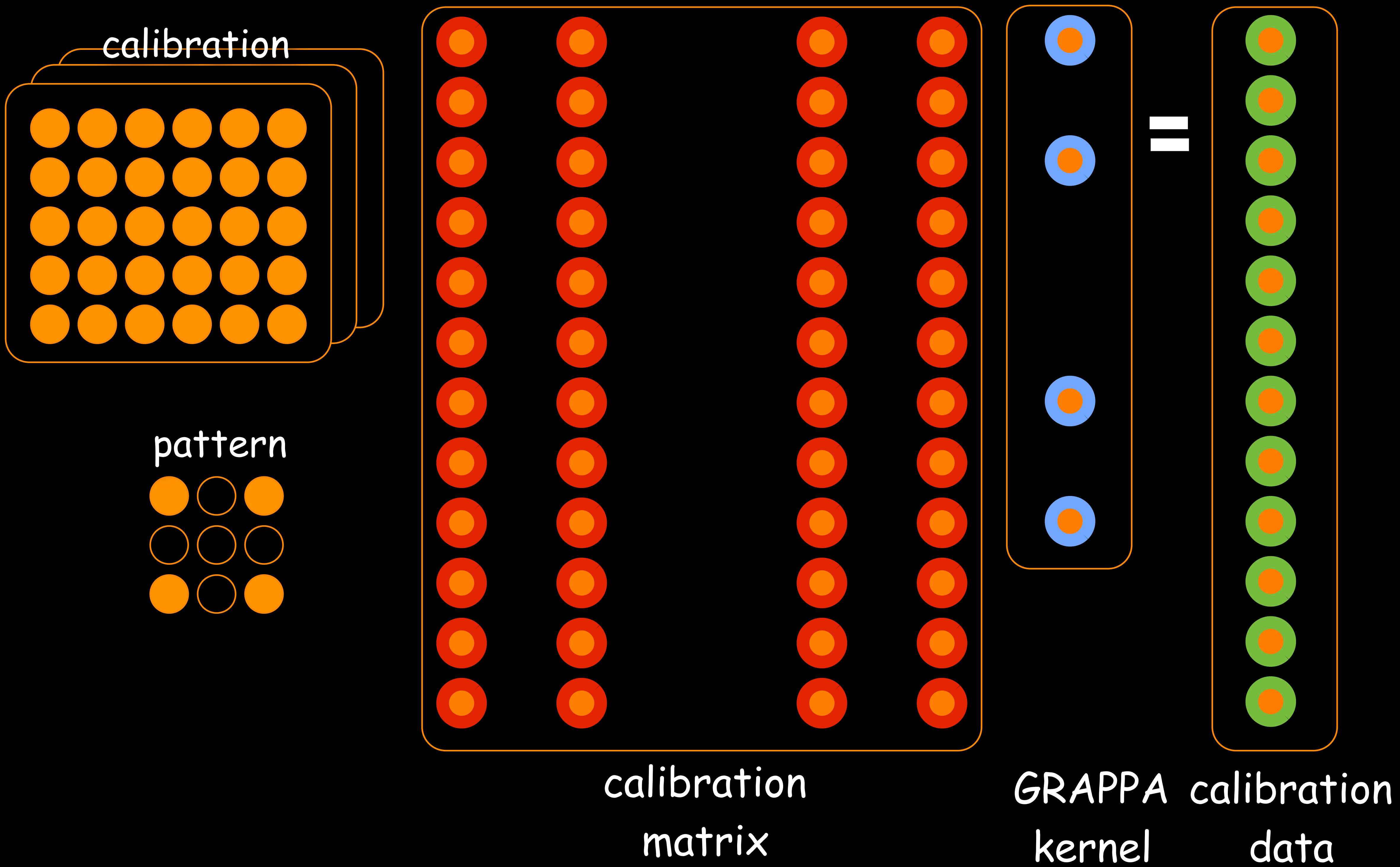

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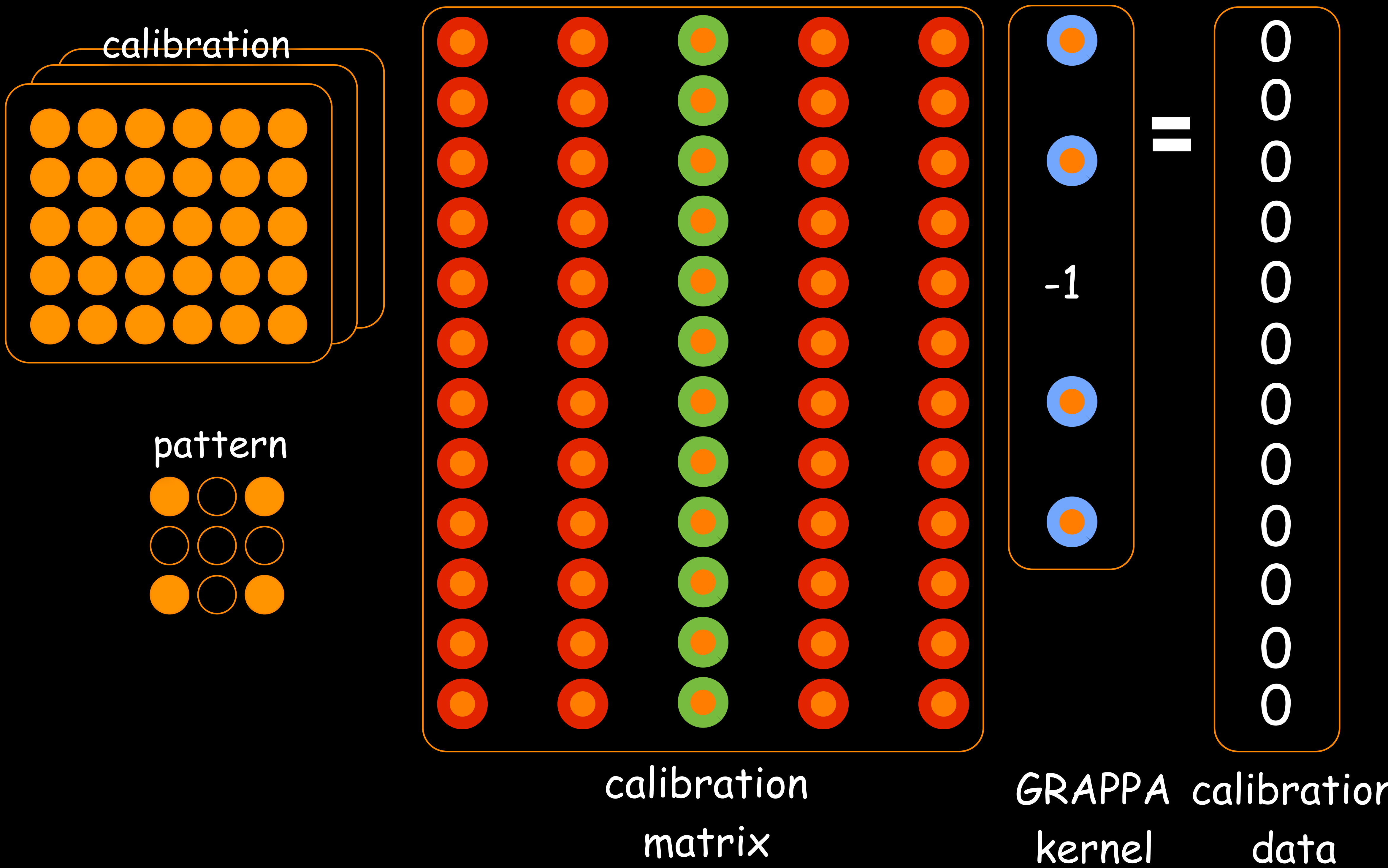

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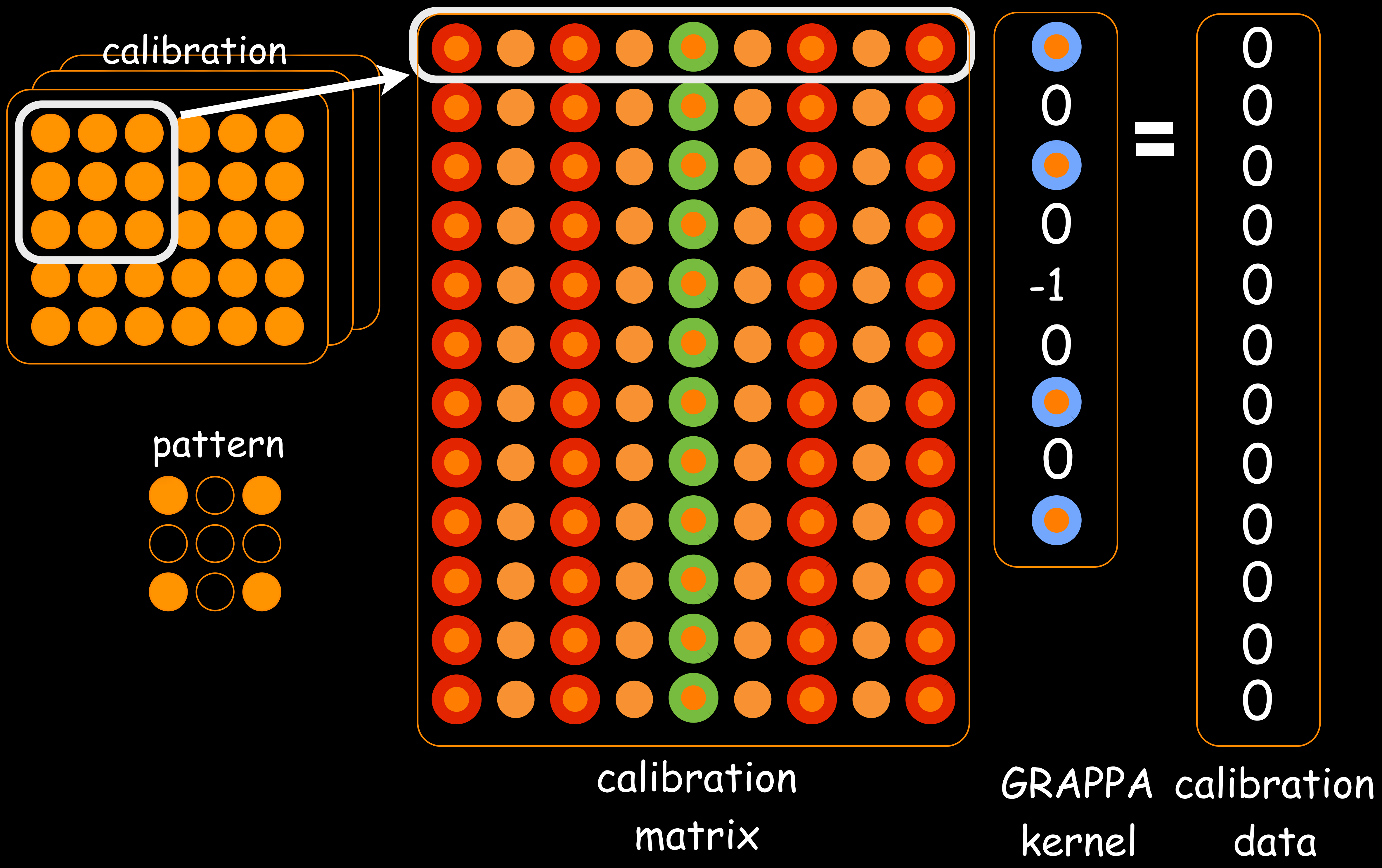

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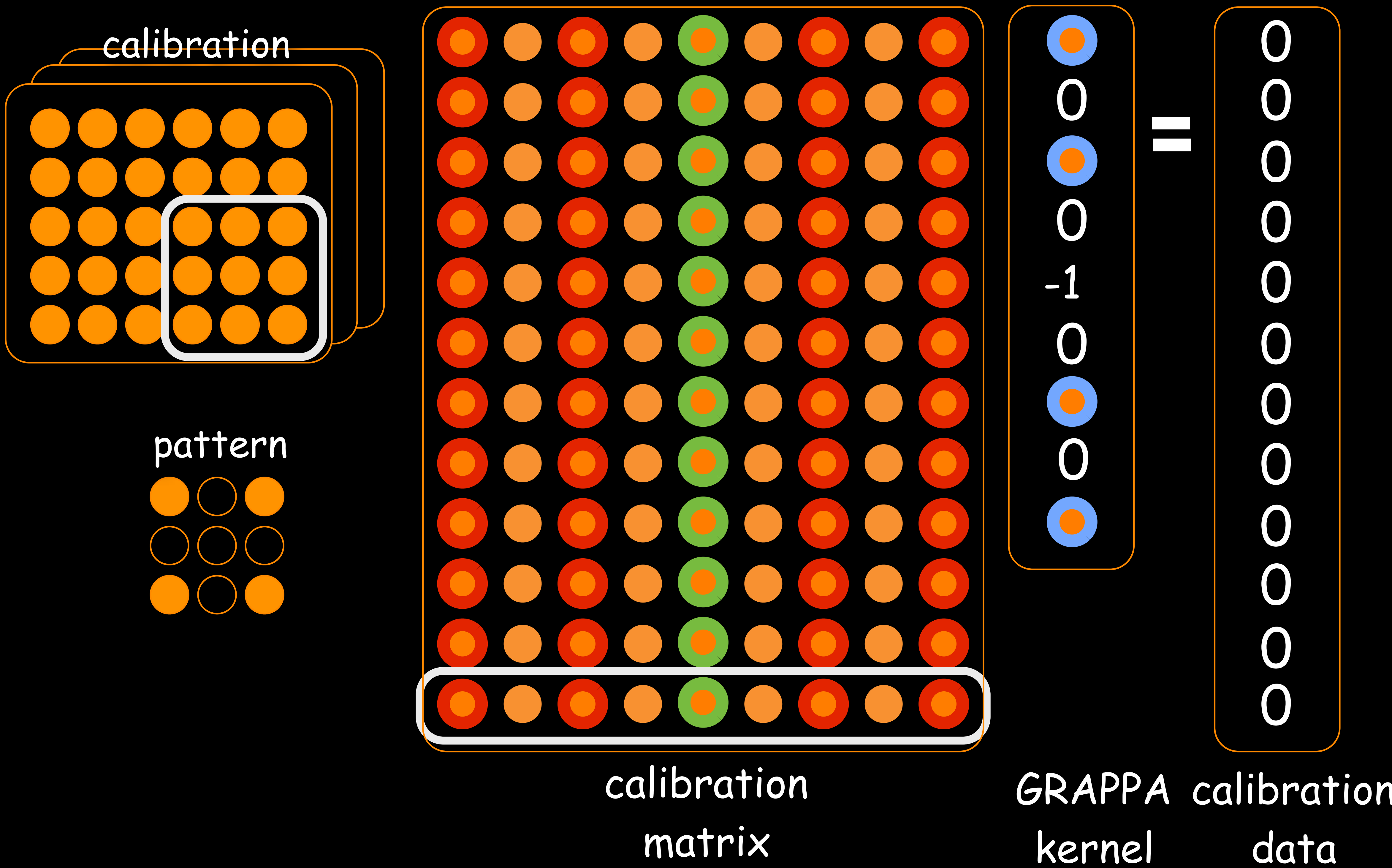

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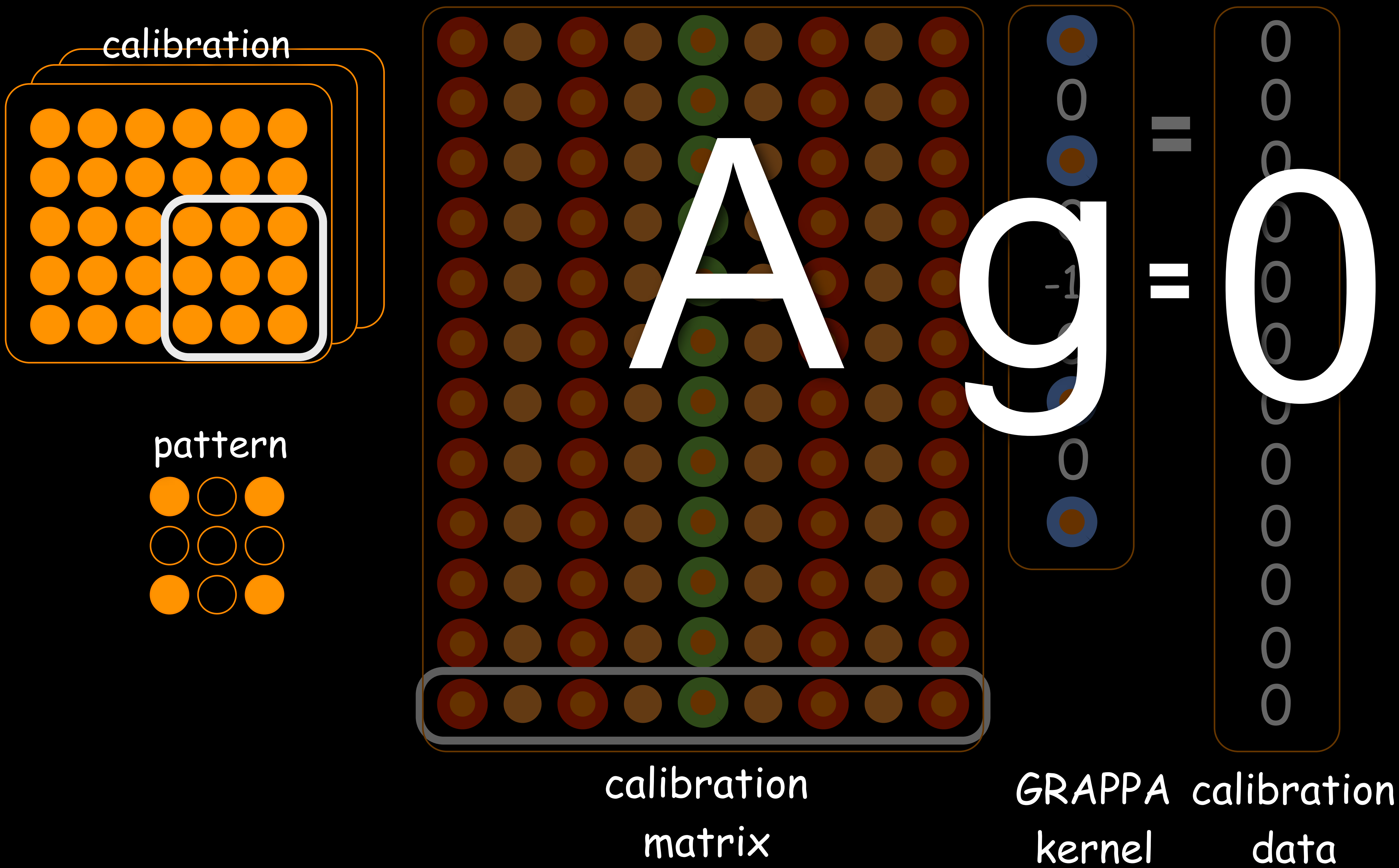
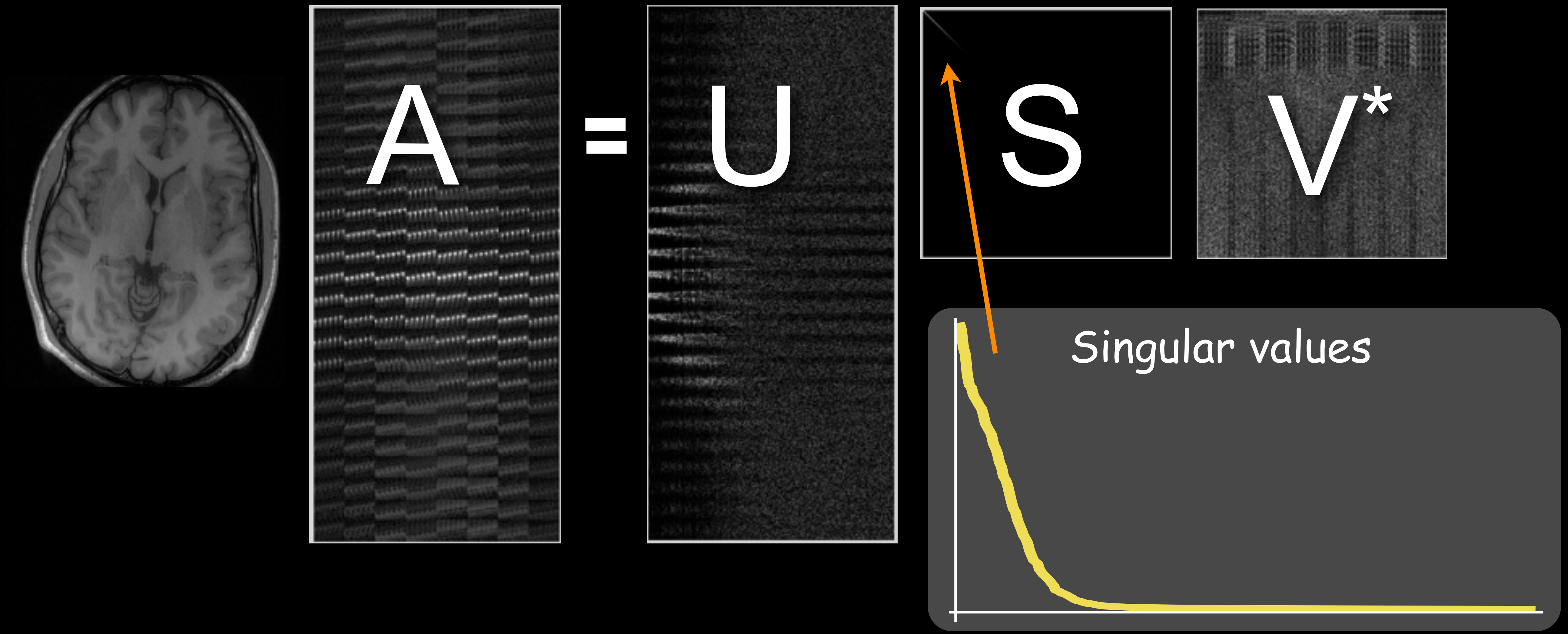


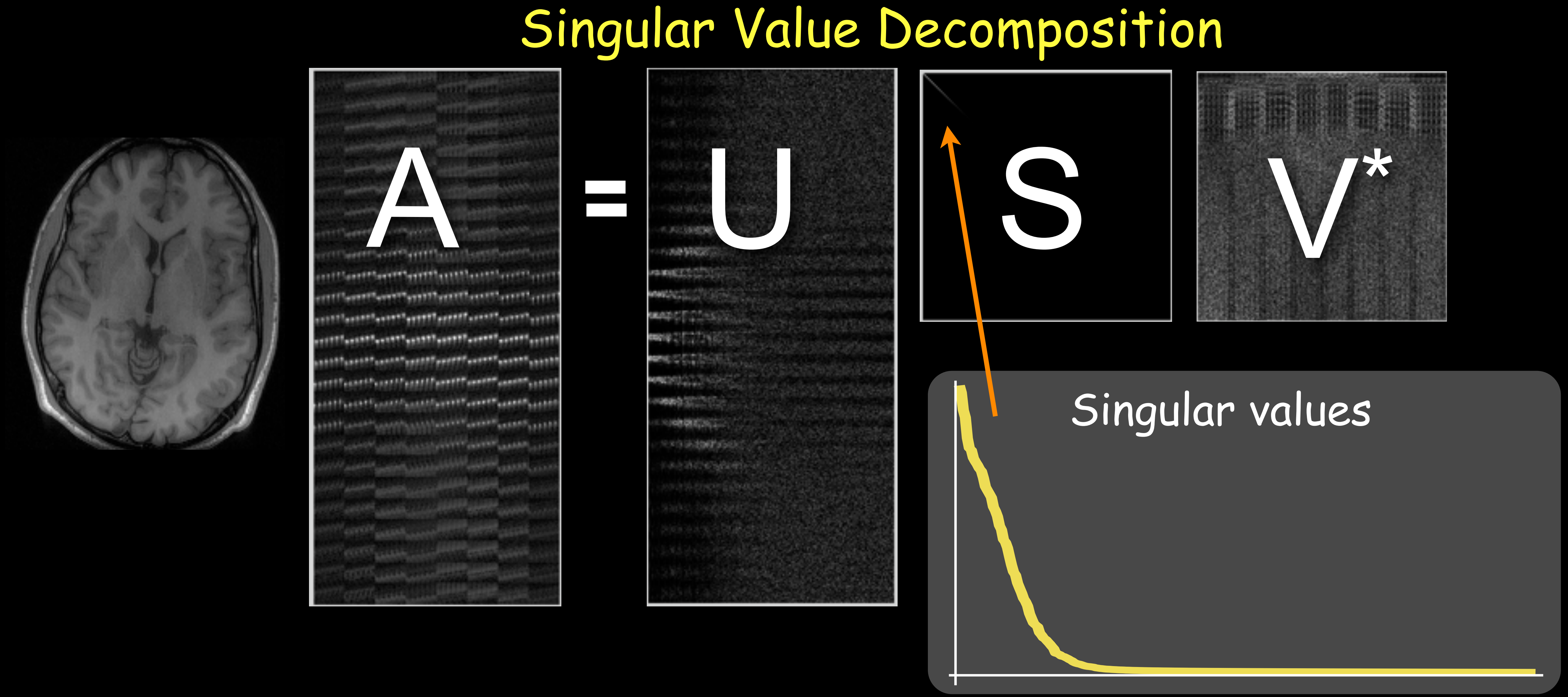
Null-Space of the Calibration Matrix

Singular Value Decomposition



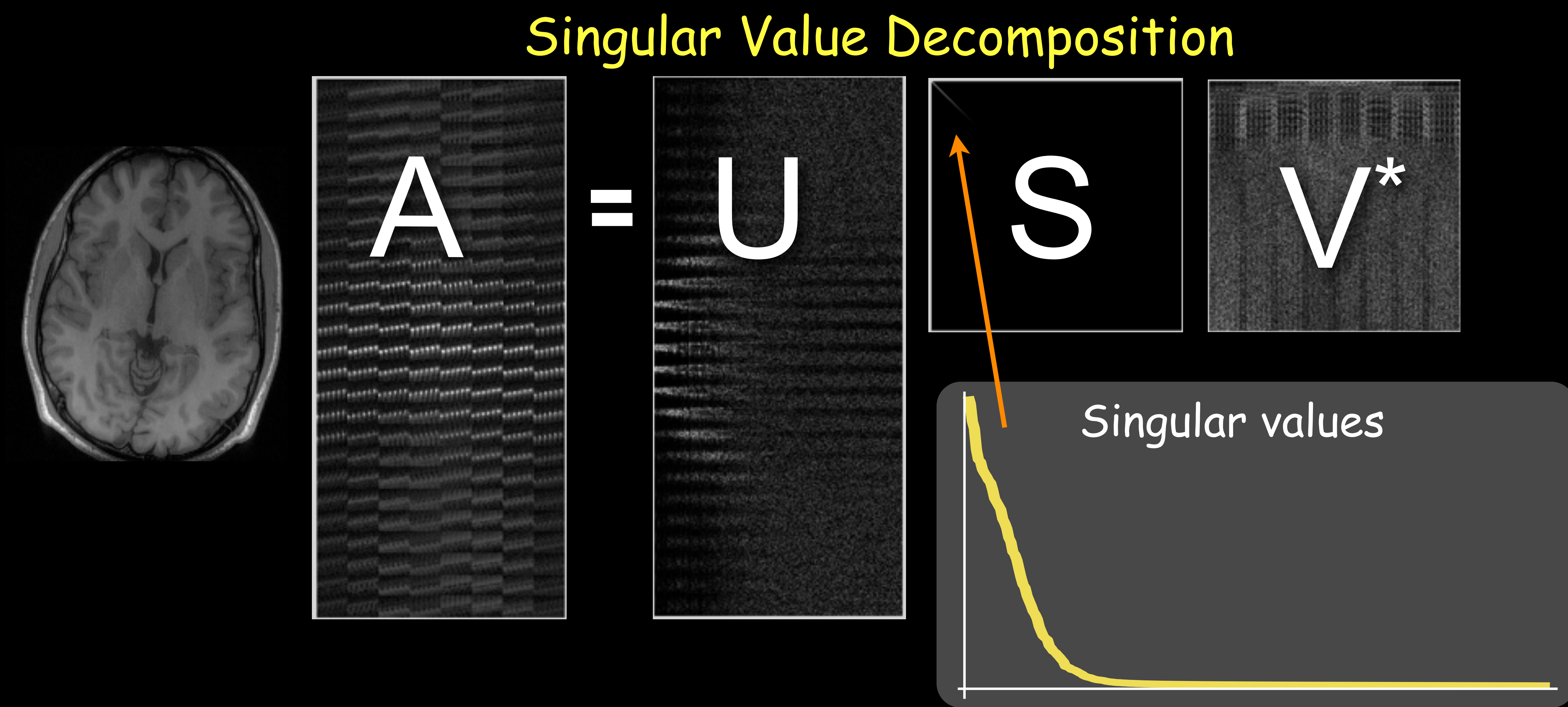
Null-Space of the Calibration Matrix

- Calibration Matrix has a Null-space



Null-Space of the Calibration Matrix

- Calibration Matrix has a Null-space
- The null-space <u>IS</u> our calibration information



Null-Space of the Calibration Matrix

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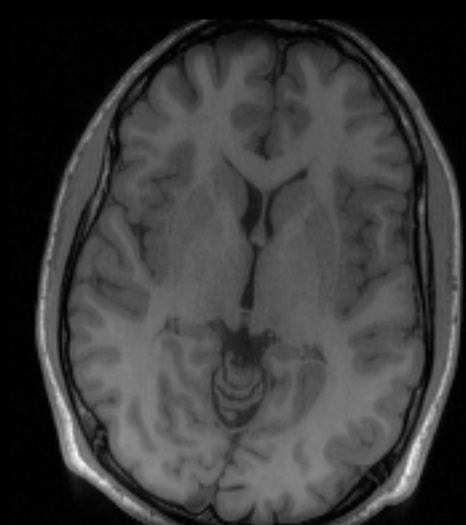
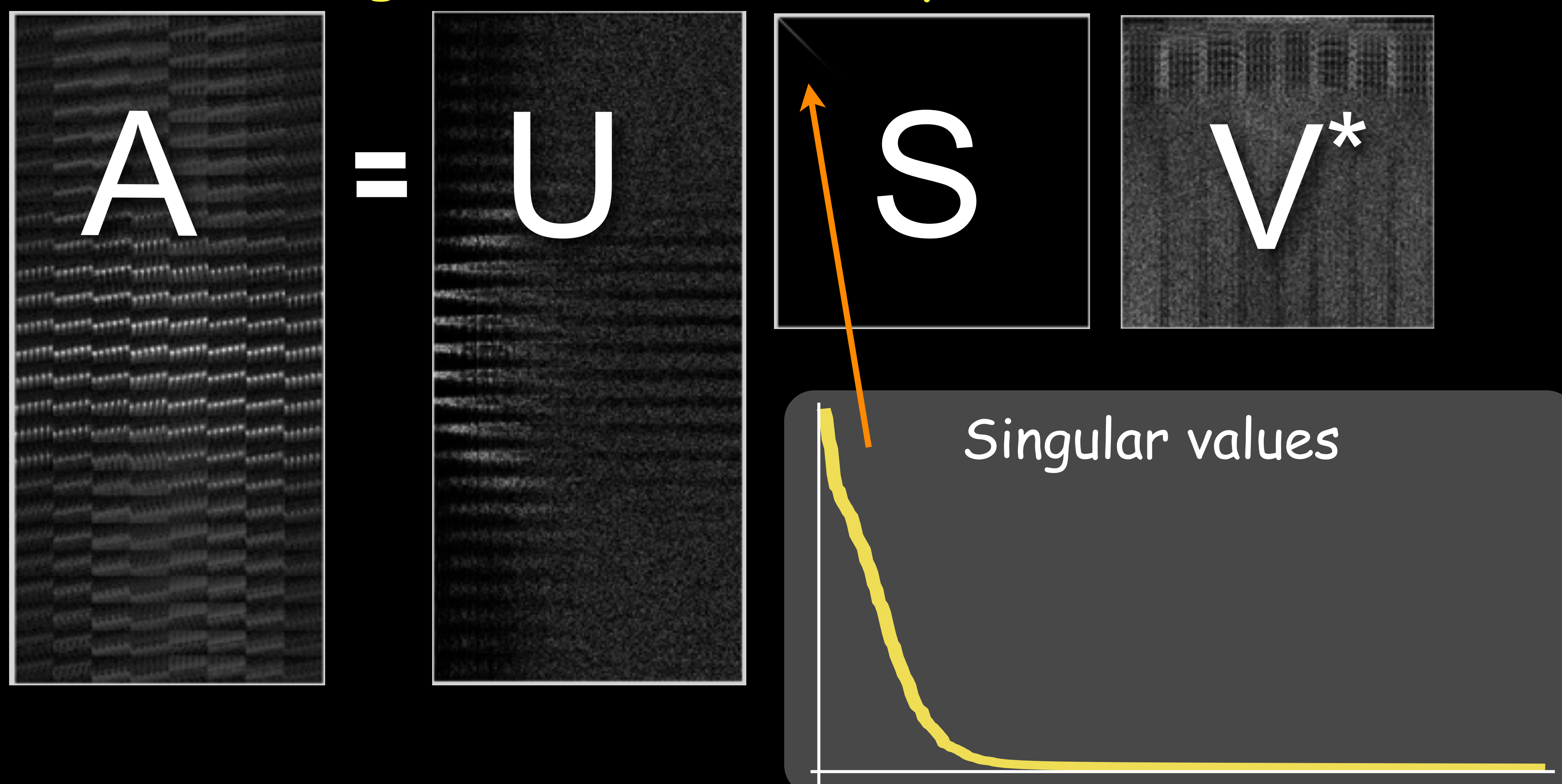
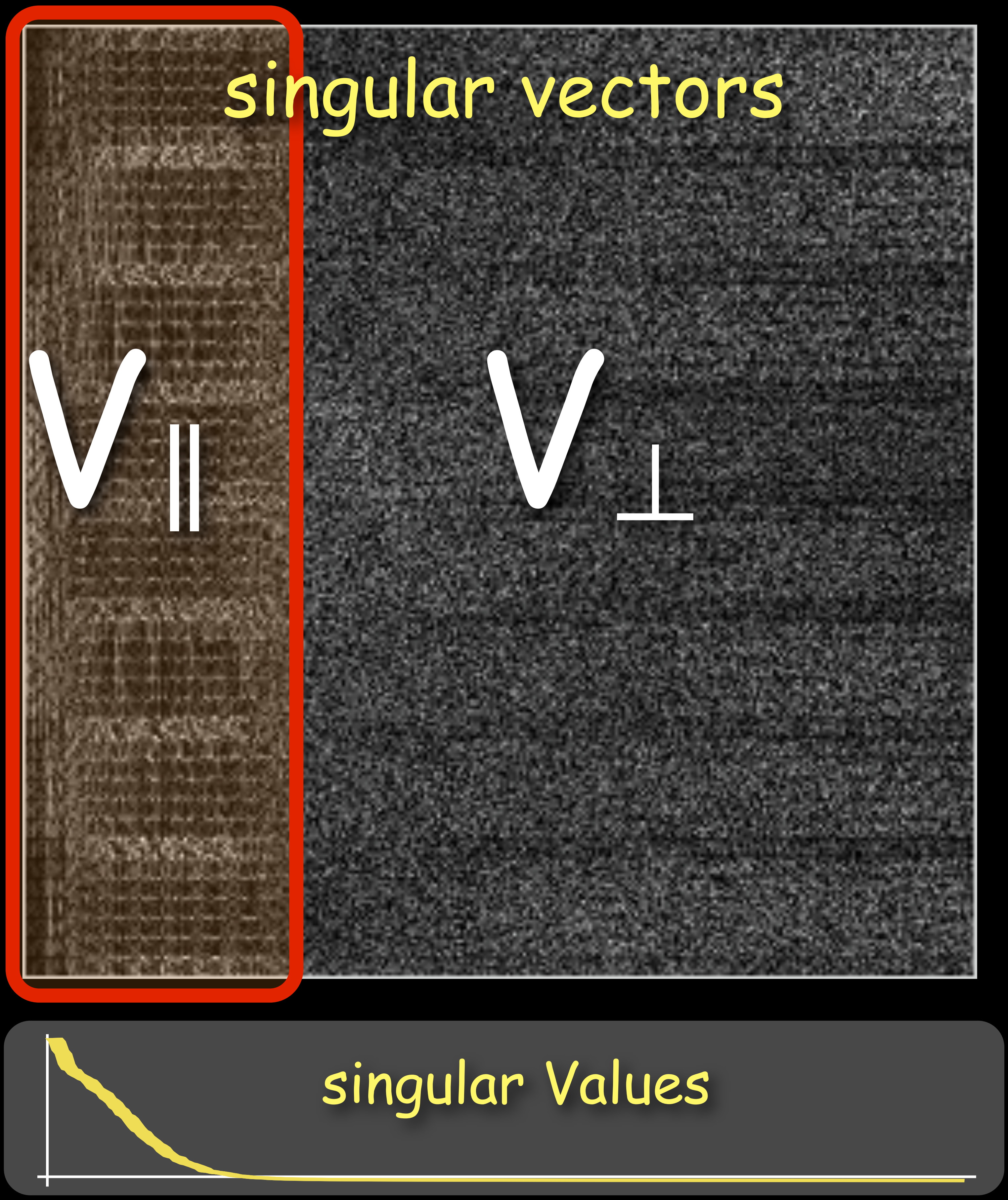
- The null-space <u>IS</u> our calibration information
- Same info used by GRAPPA/SPIRiT etc....



Singular Value Decomposition



Subspace of the DATA



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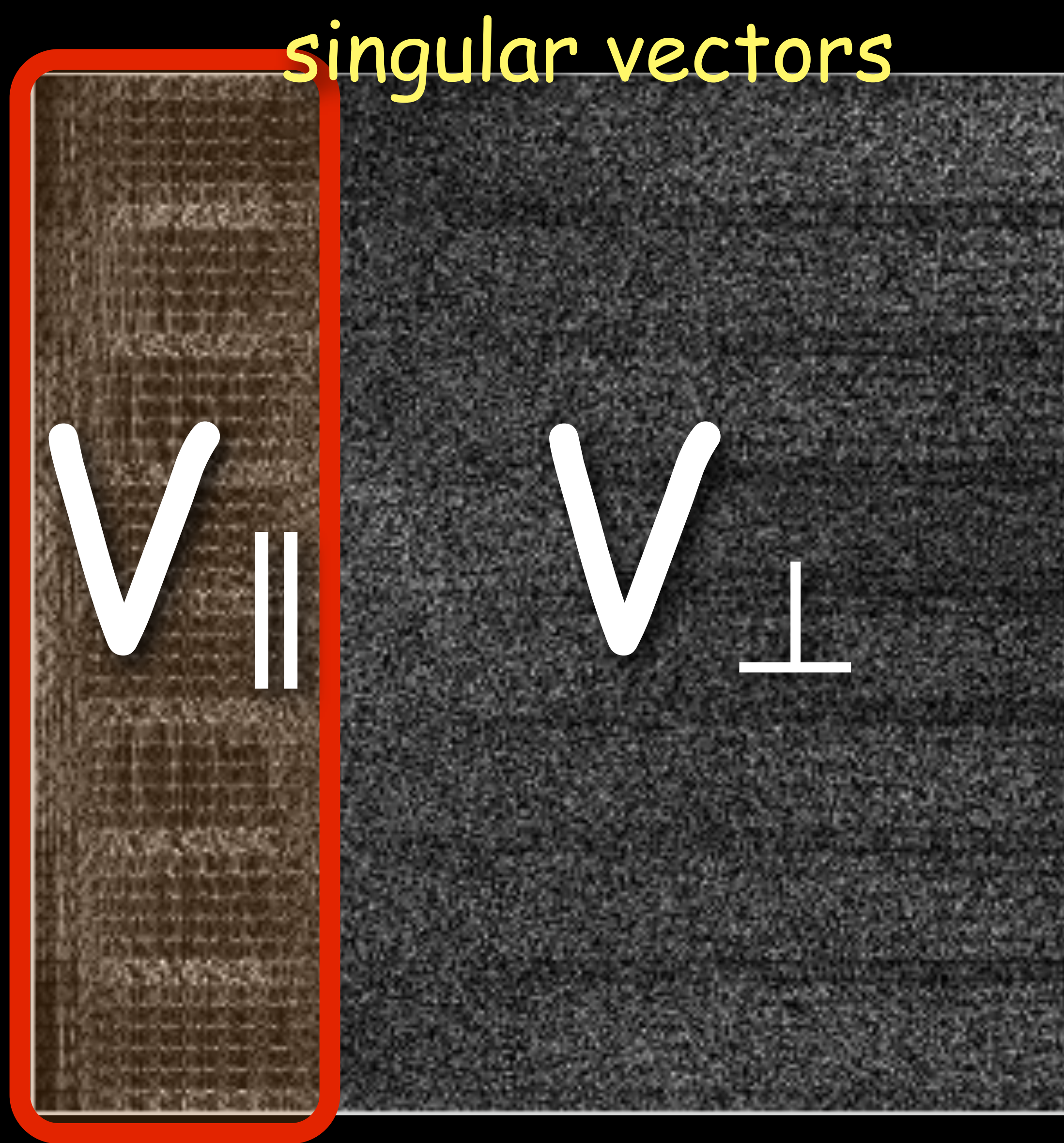
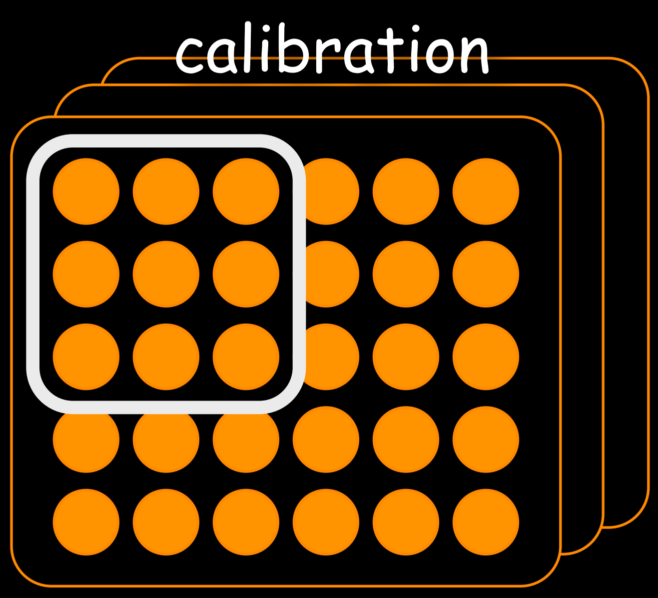
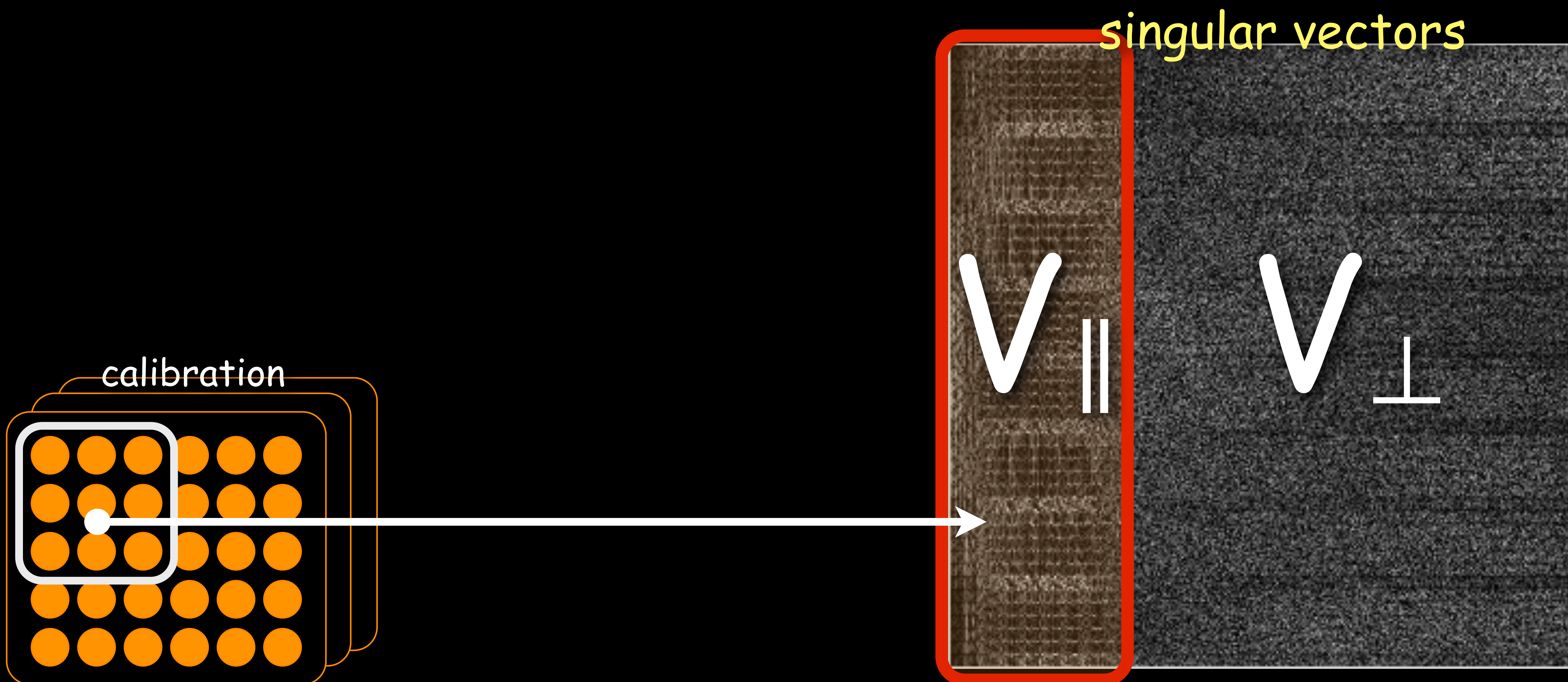




Subspace of the DATA

- Calibration blocks "live" in $V_\parallel$



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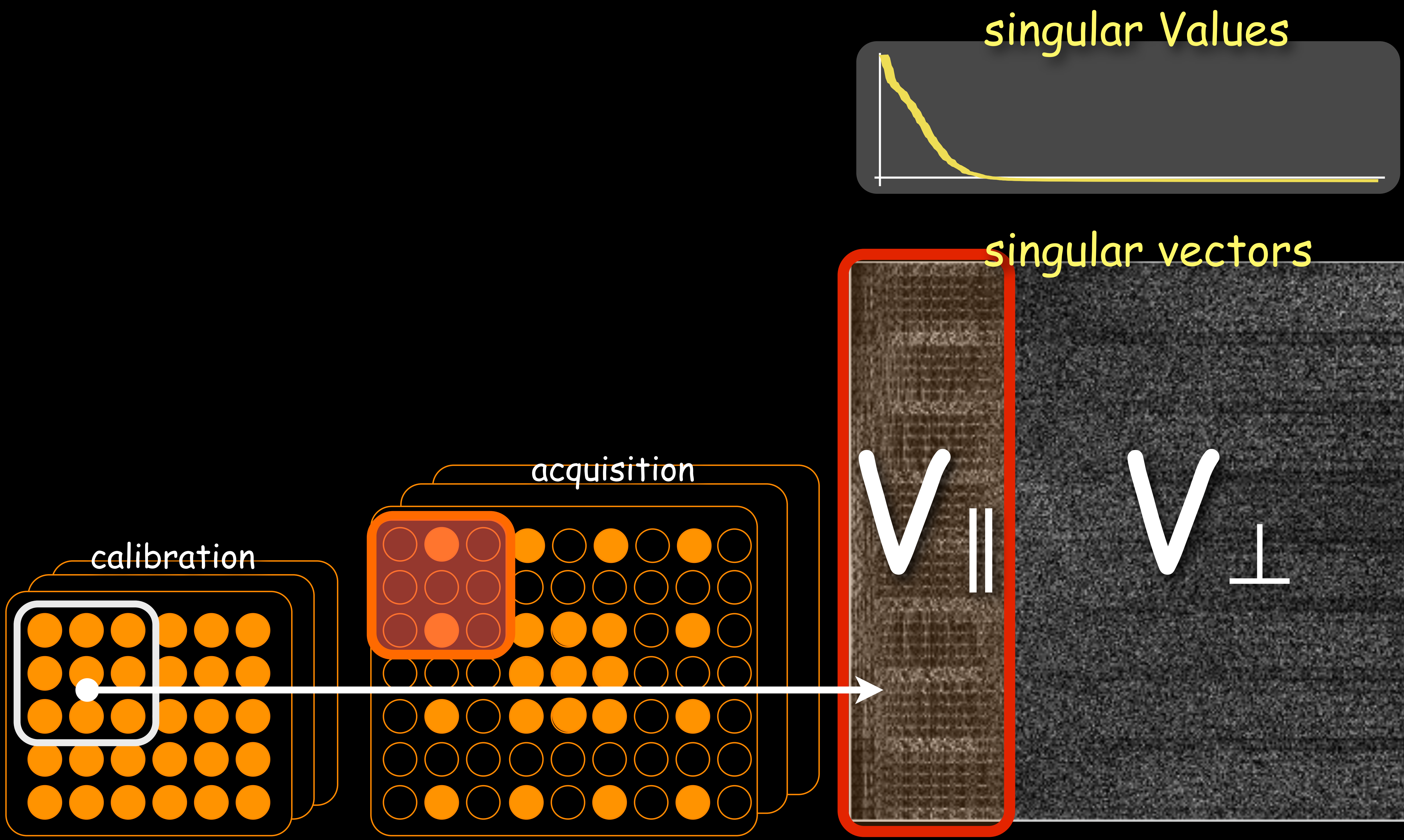
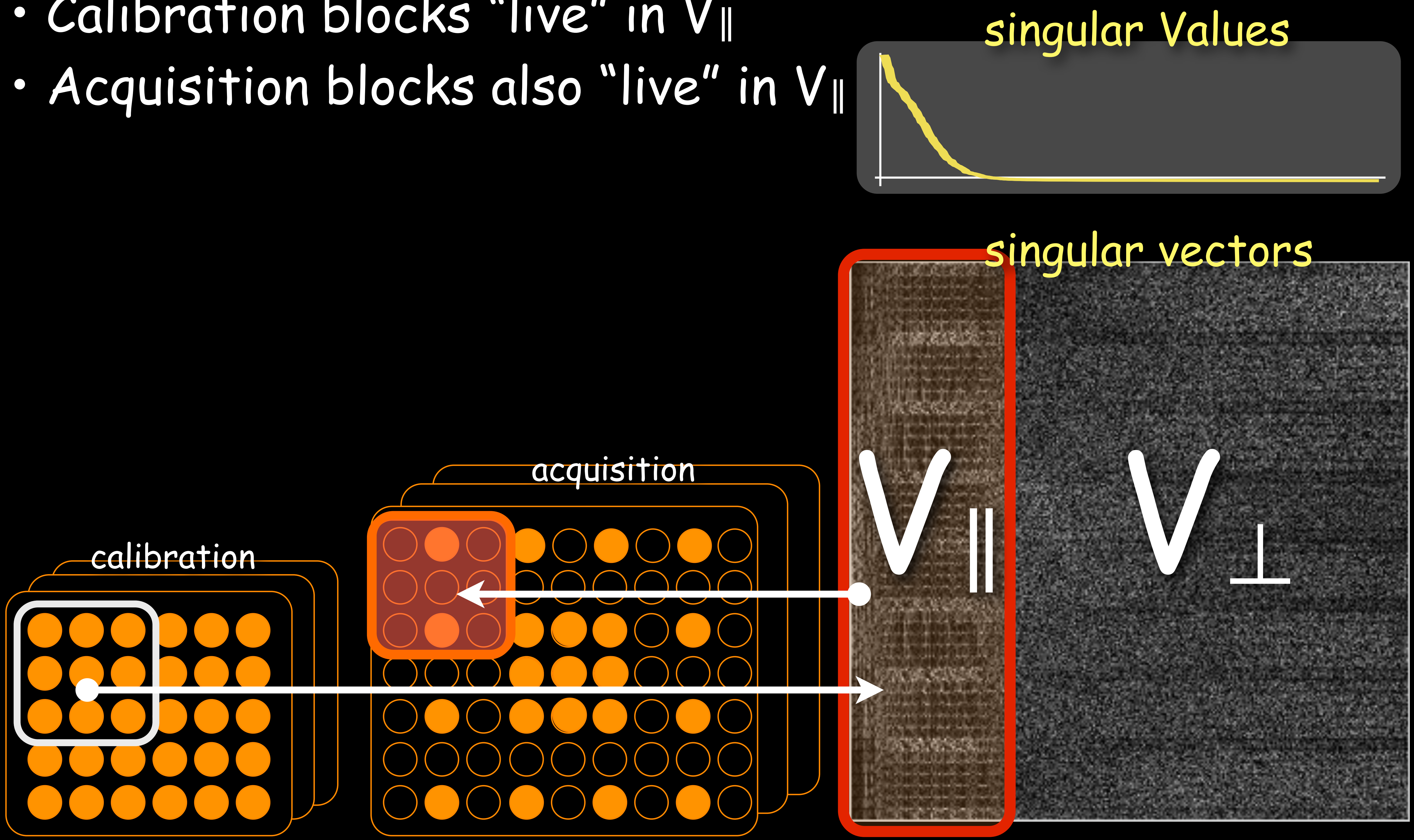


Subspace of the DATA

- Calibration blocks “live” in $V_{\parallel}$
- Acquisition blocks also “live” in $V_{\parallel}$



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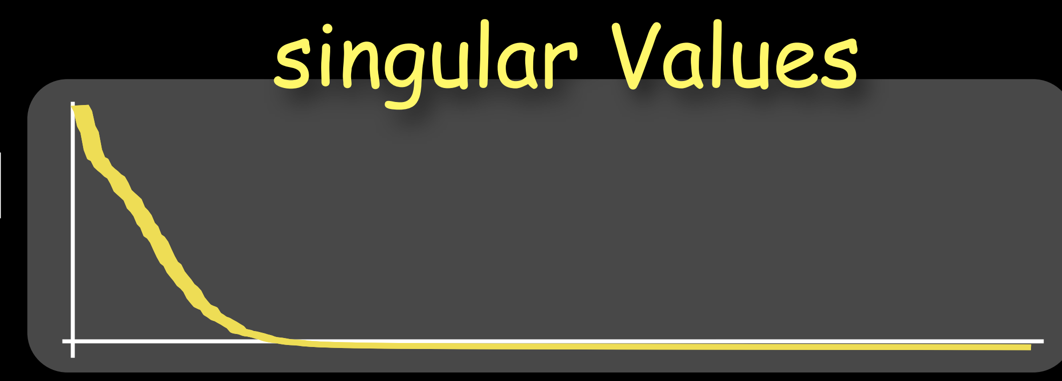
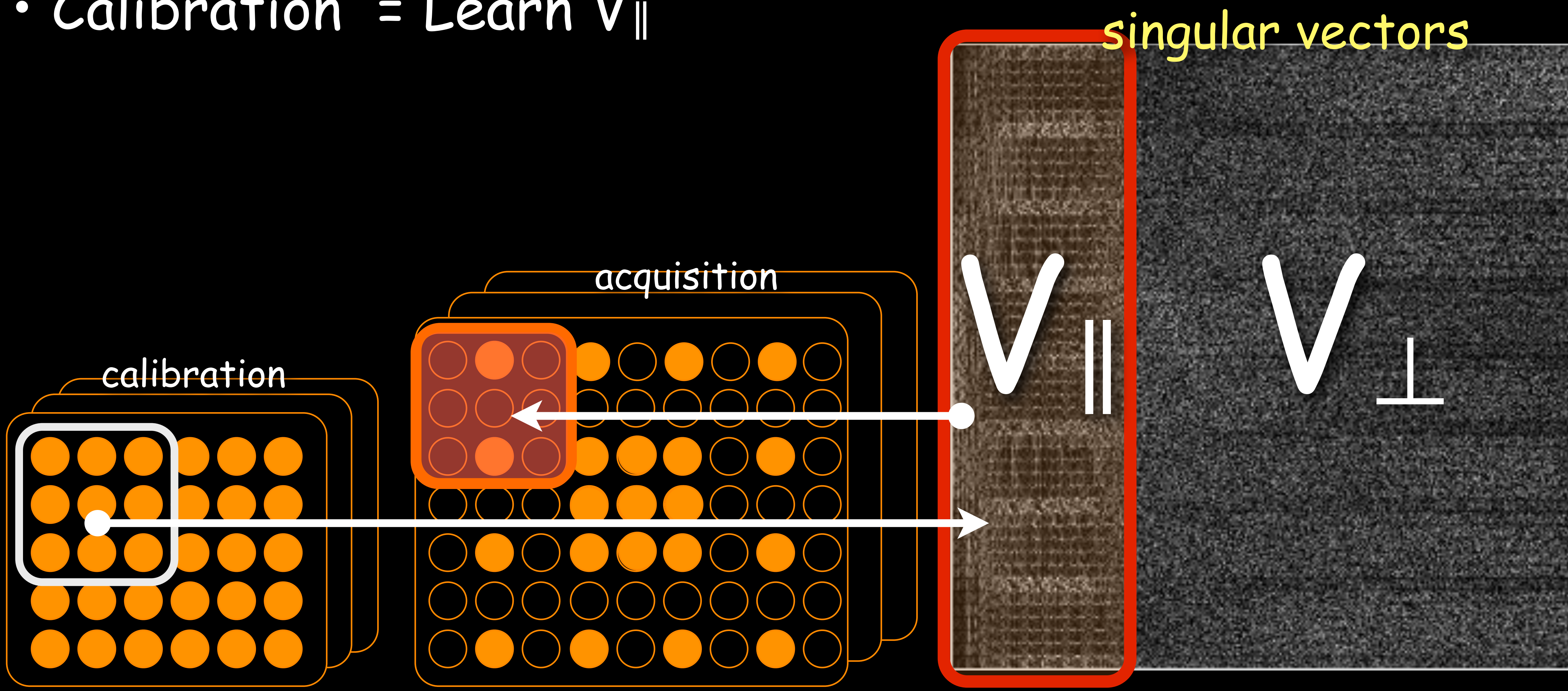


- Calibration = Learn $V_{\parallel}$



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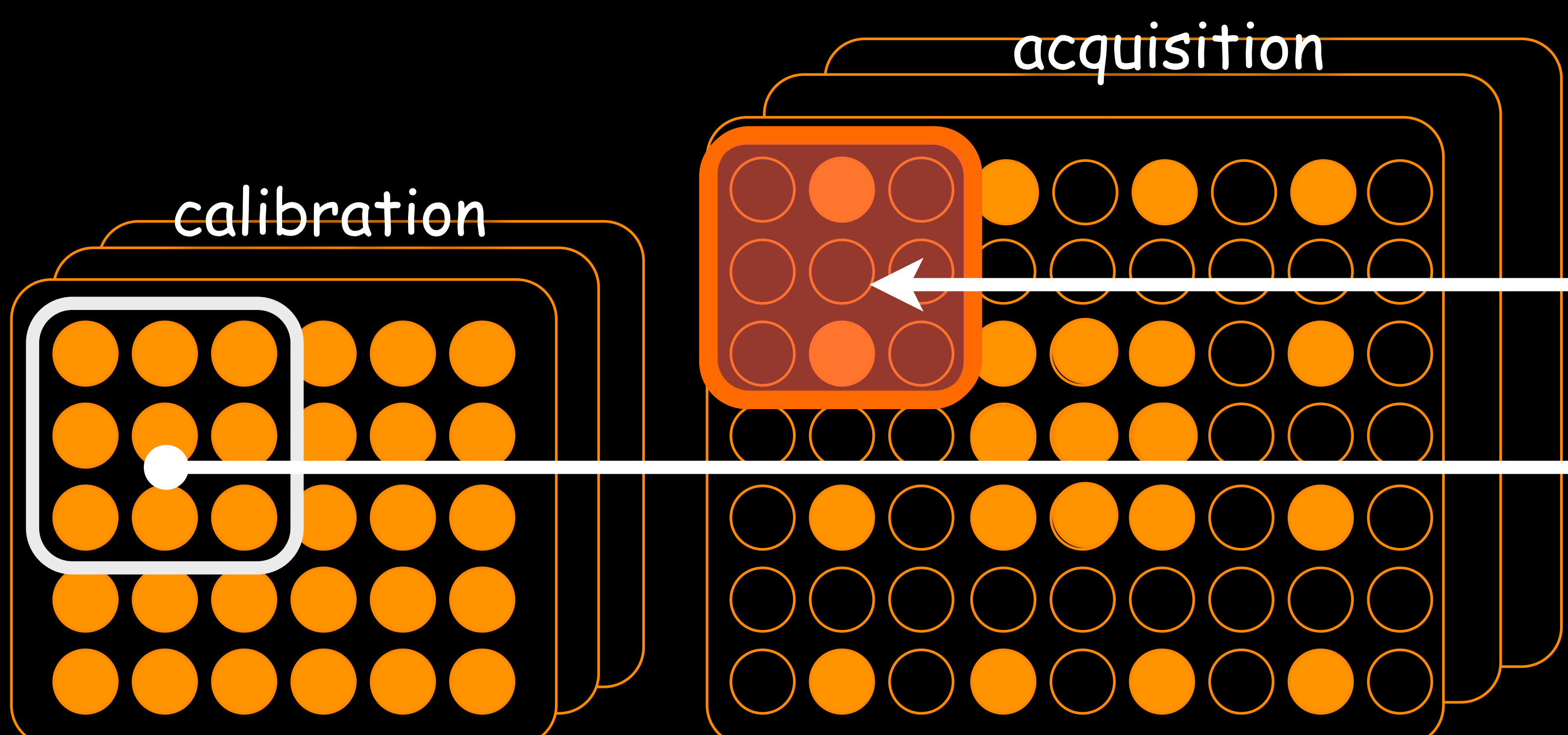
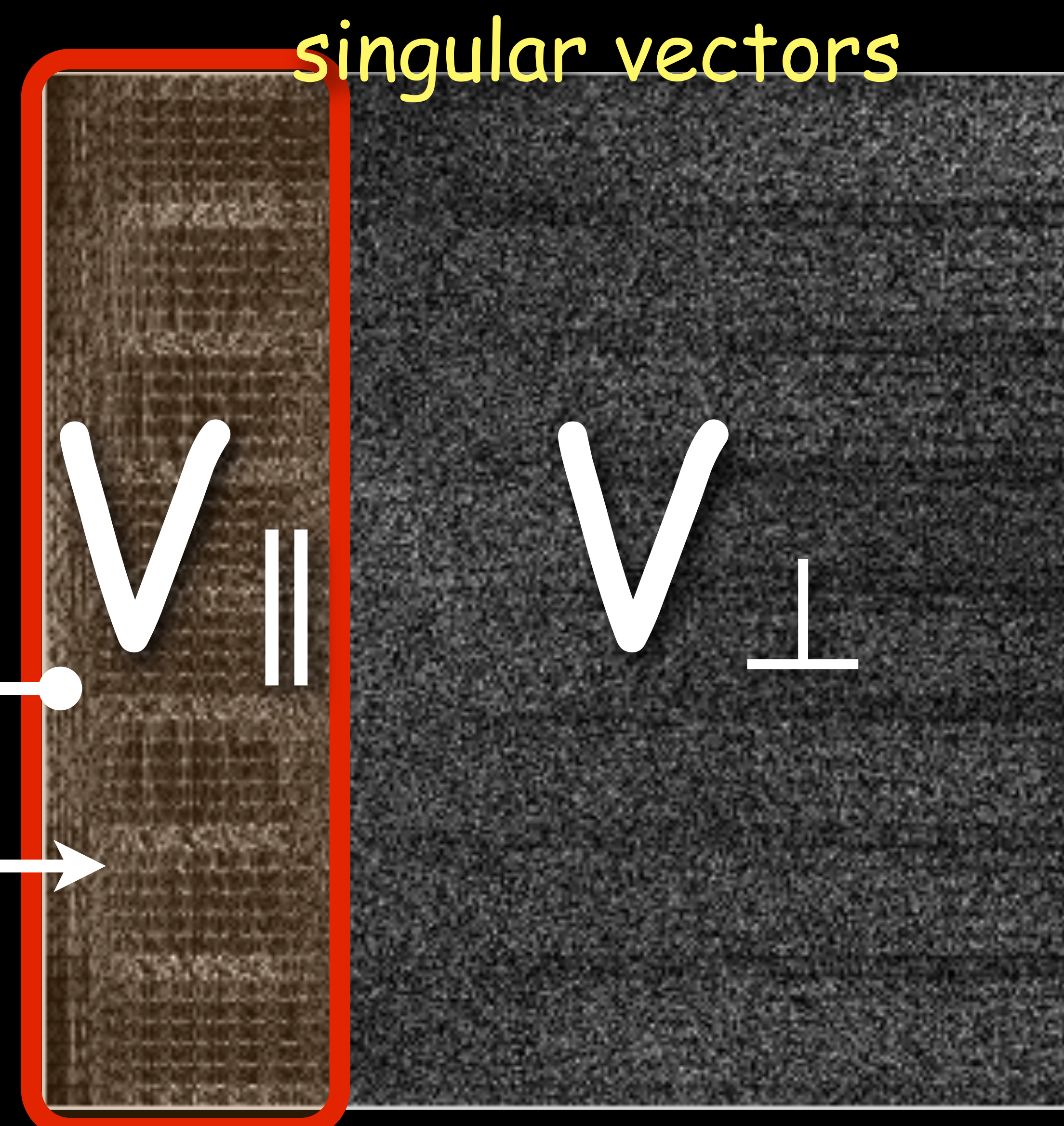
- Calibration = Learn $V_{\parallel}$
- Recovery = Enforce $V_{\parallel}$

(and data consistency)





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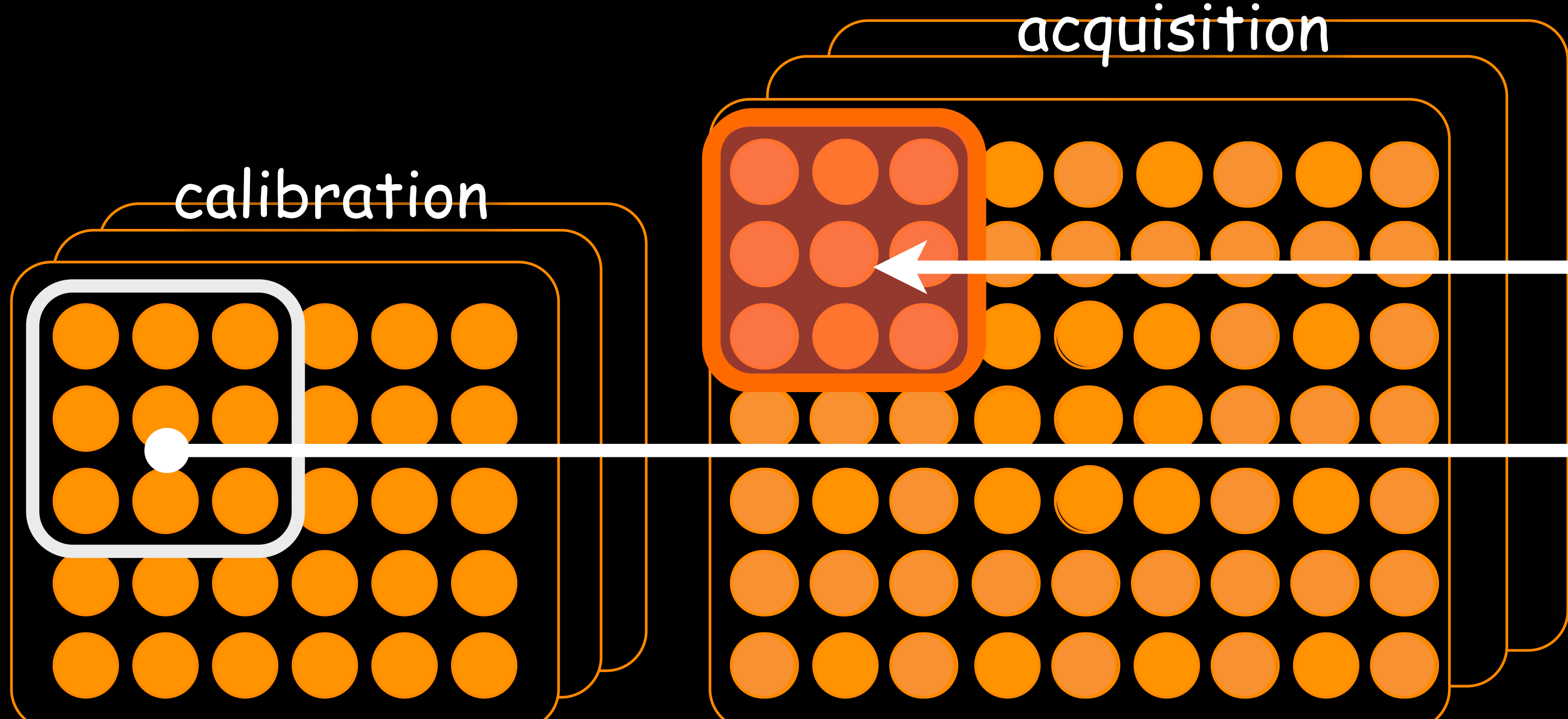
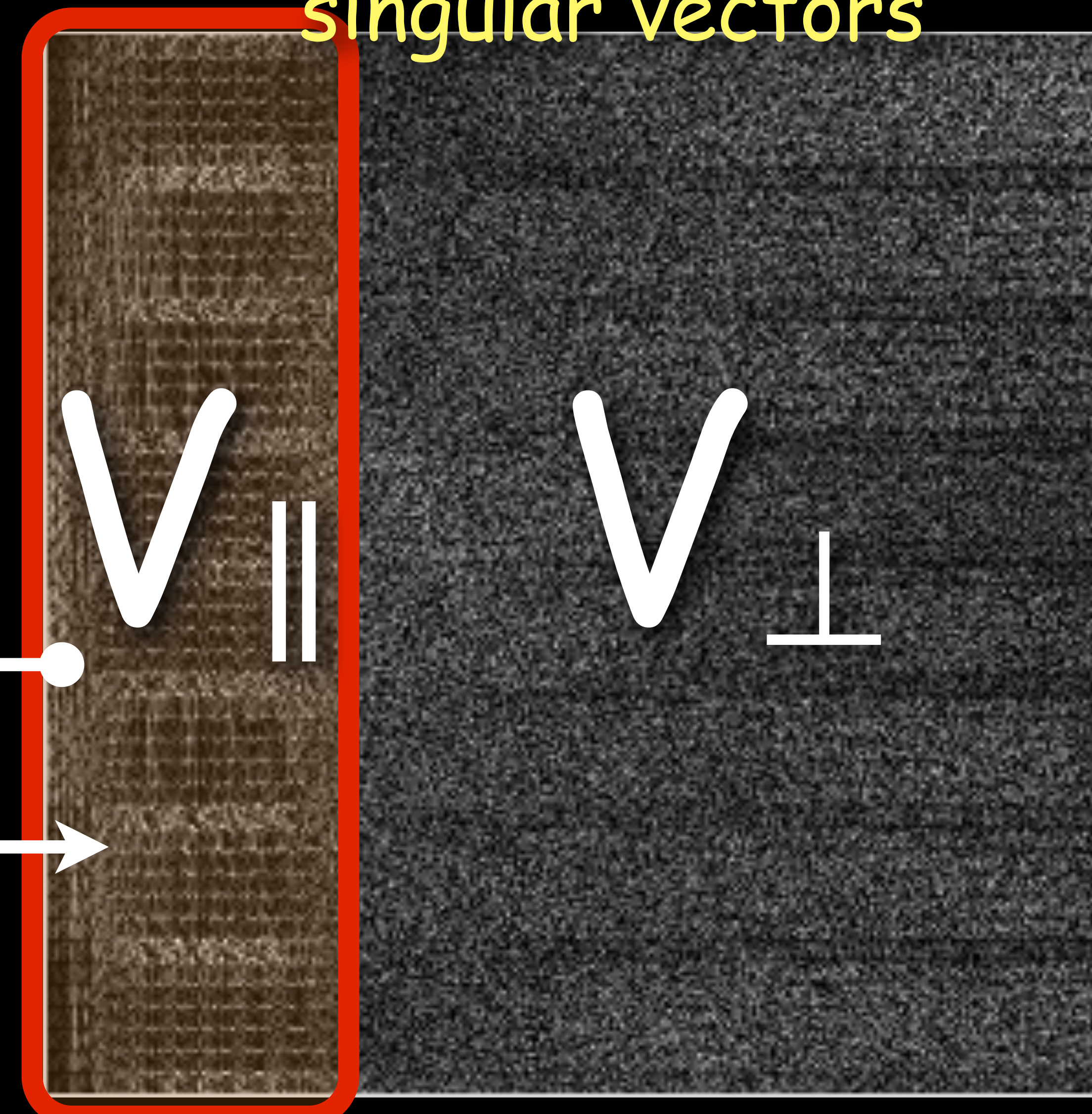
singular Values

- Calibration = Learn $V_{\parallel}$
- Recovery = Enforce $V_{\parallel}$

(and data consistency)

singular vectors





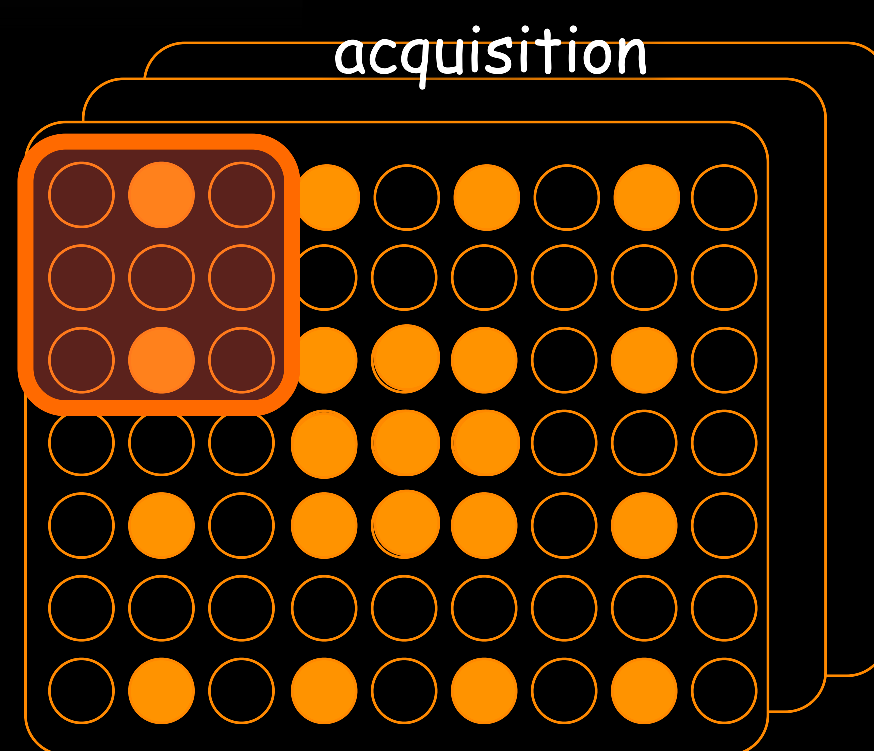
Reconstruction

Blockwise operation

$$\mathcal{V}_{||}x = x$$

$$x\big|_{\mathrm{acq}} = y$$

- Optimally use the calibration information
- Optimal reconstruction using calibration information
- Can be solved iteratively



Reconstruction

Blockwise operation → $V_{||}x$

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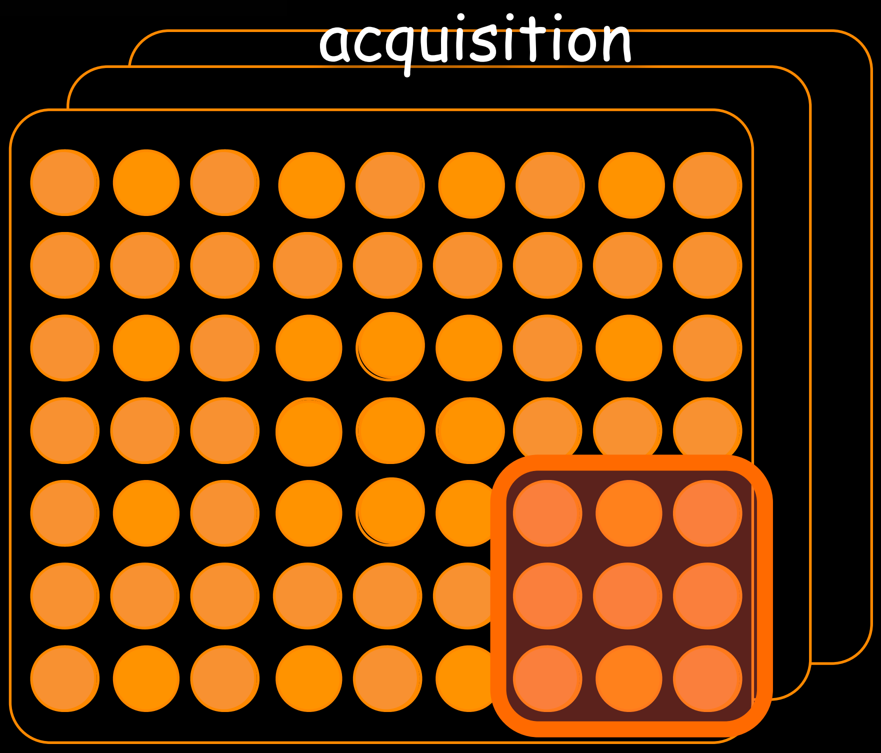

Back to SENSE with Eigen-Vector Analysis

Eigen-Vector solution

$$\mathcal{V}_{||}x = x$$

Eigen-Vector solution

$$V_{||}x = x$$

- Solution spanned by eigenVecs with eigenVals = 1

Eigen-Vector solution

$$V_{||} x = x$$

- Solution spanned by eigenVecs with eigenVals = 1
- Approach:
  - Compute eigenVecs explicitly
  - Project only on those with eigenVals = 1

Eigen-Vector solution

$$V_{||} x = x$$

- Solution spanned by eigenVecs with eigenVals = 1
- Approach:
  - Compute eigenVecs explicitly
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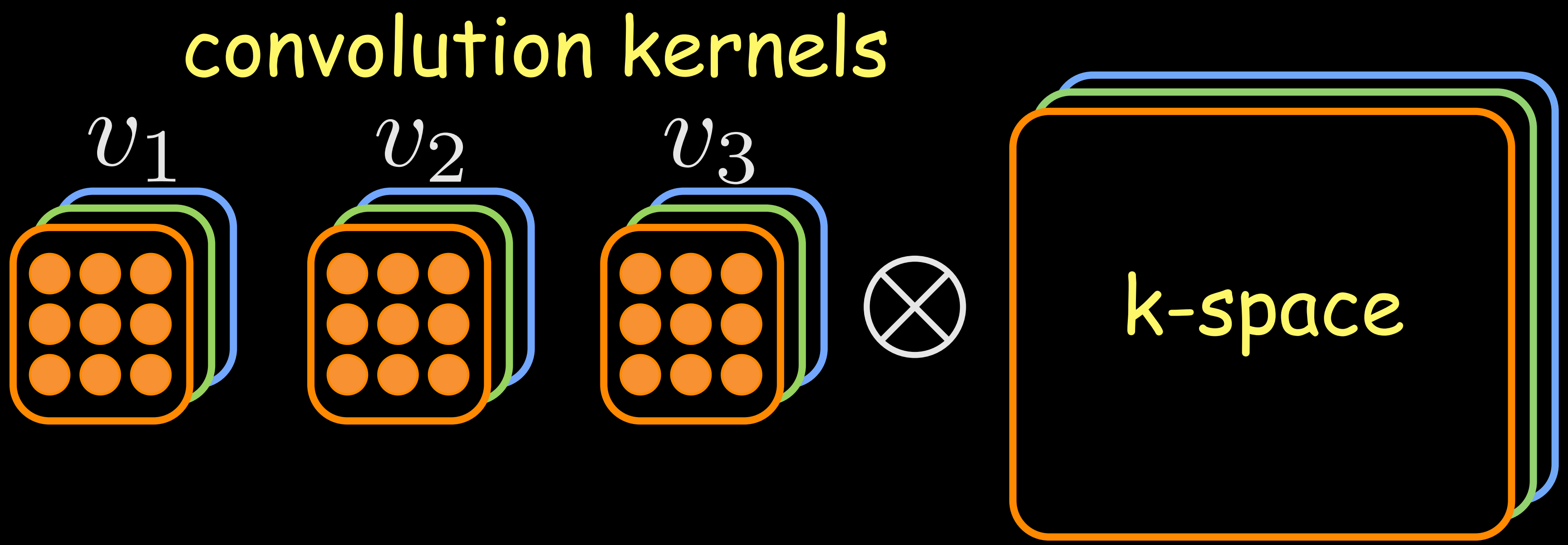
- Eigen-decomposition is fast in image domain.

Fast Eigen-Value Decomposition

$$V_{||} x$$



Fast Eigen-Value Decomposition

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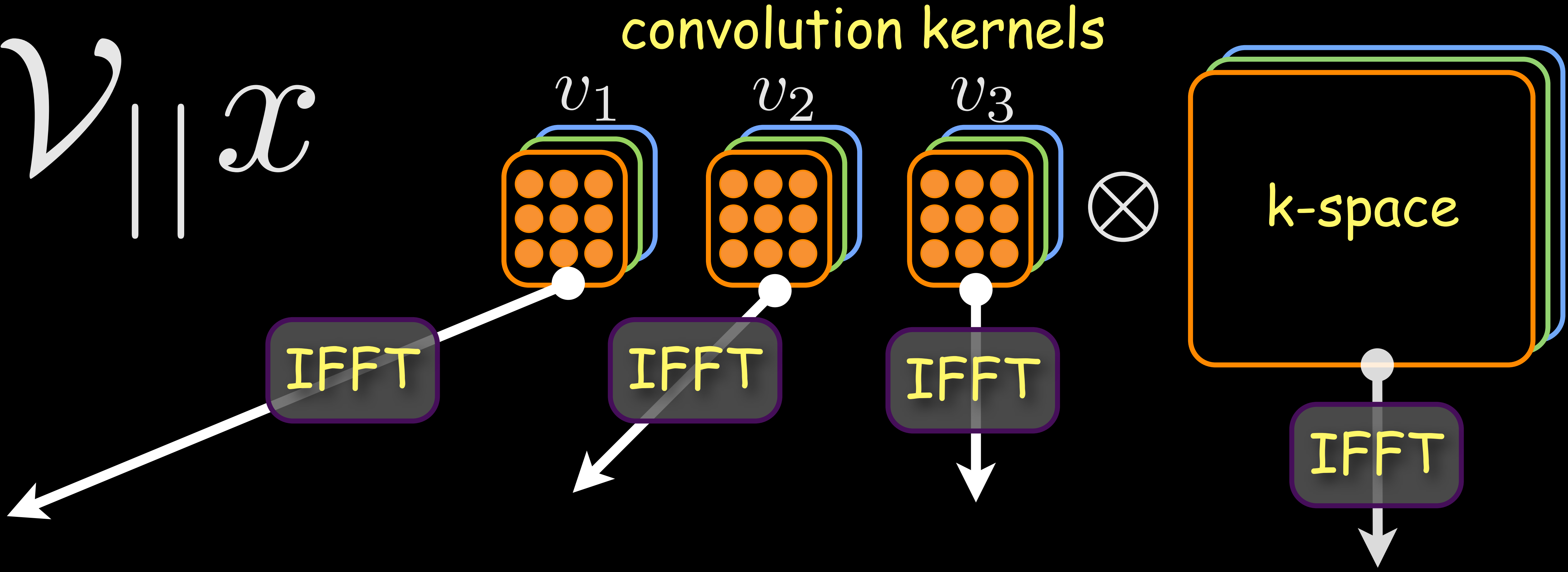
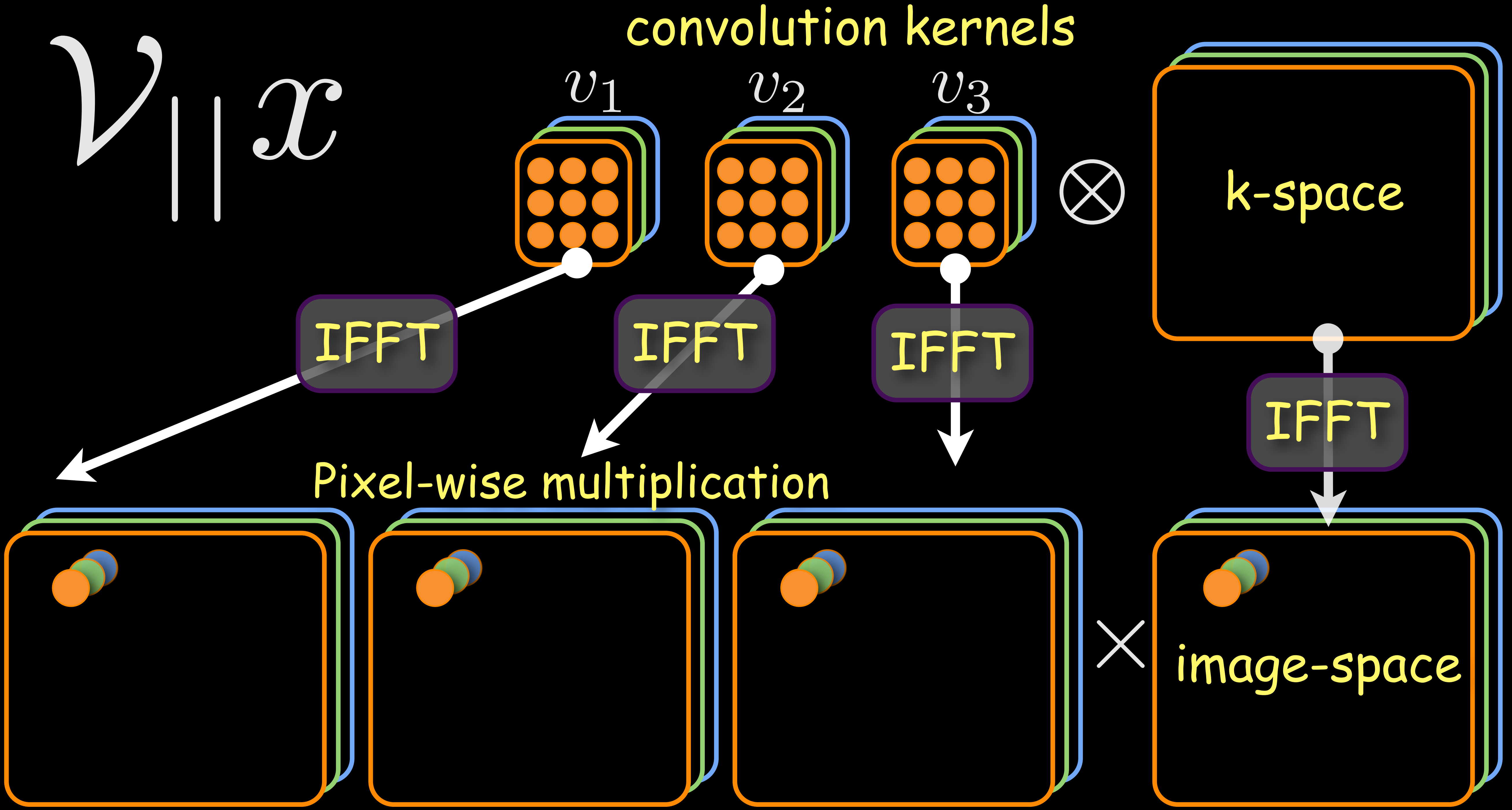
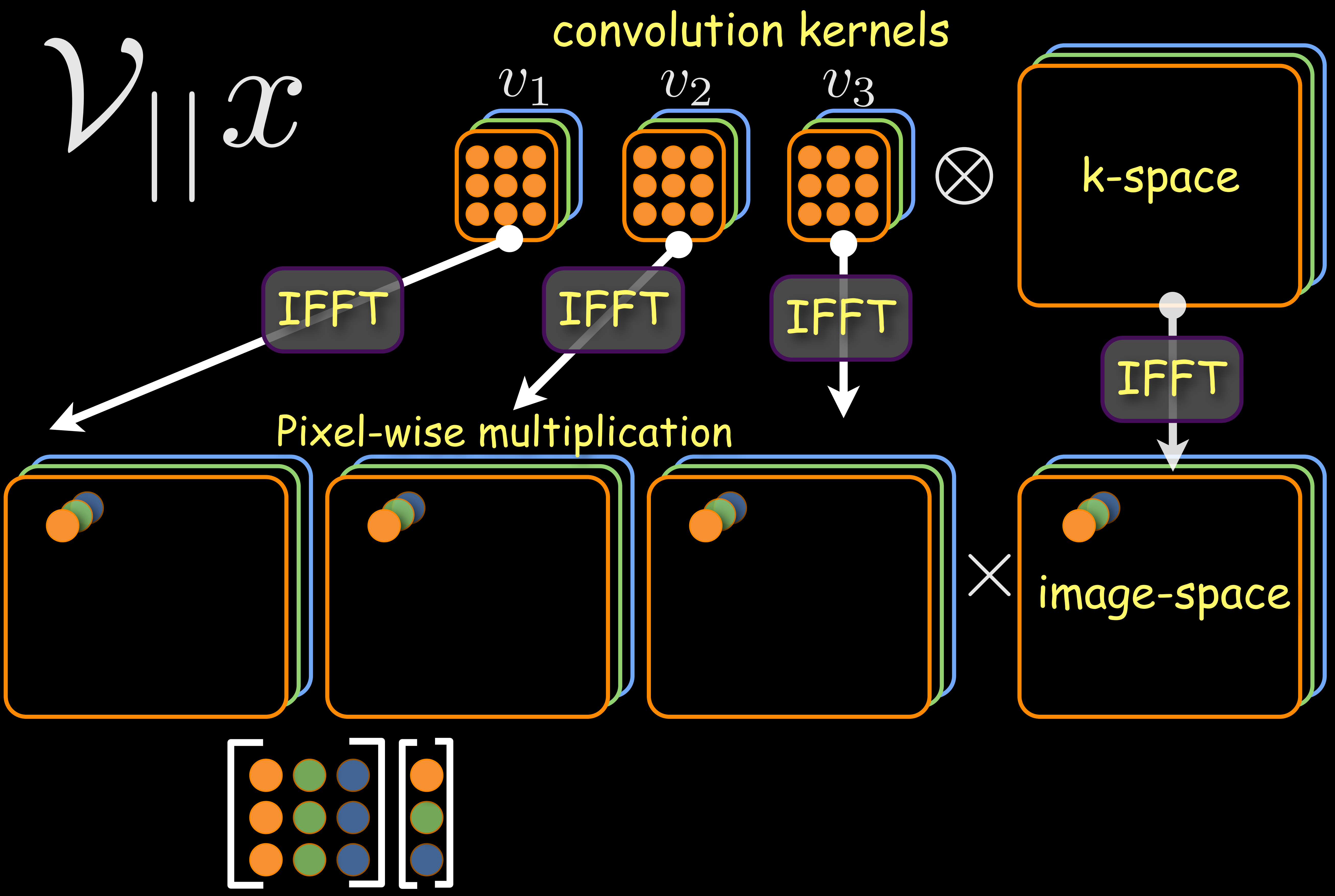


Fast Eigen-Value Decomposition

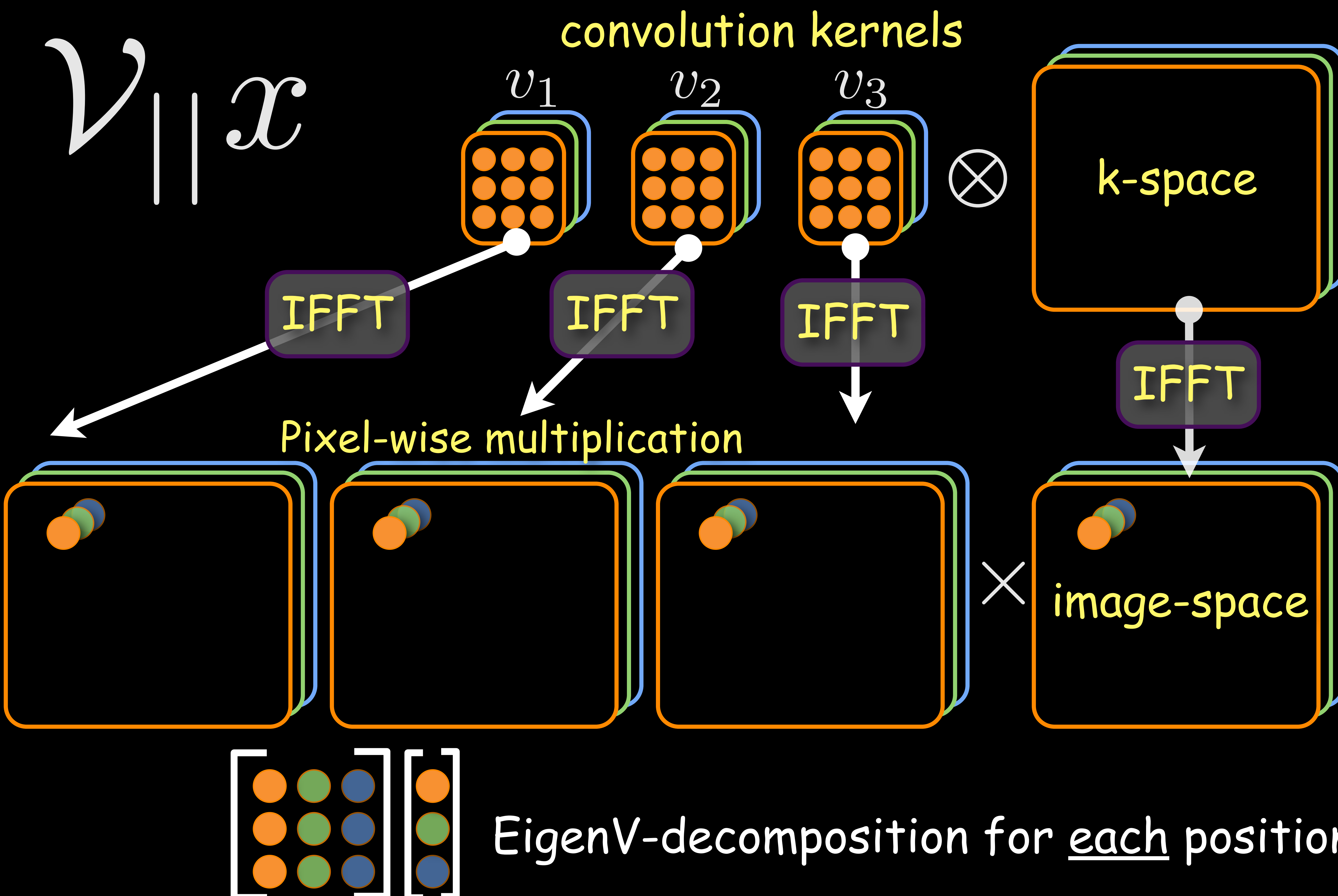


Fast Eigen-Value Decomposition



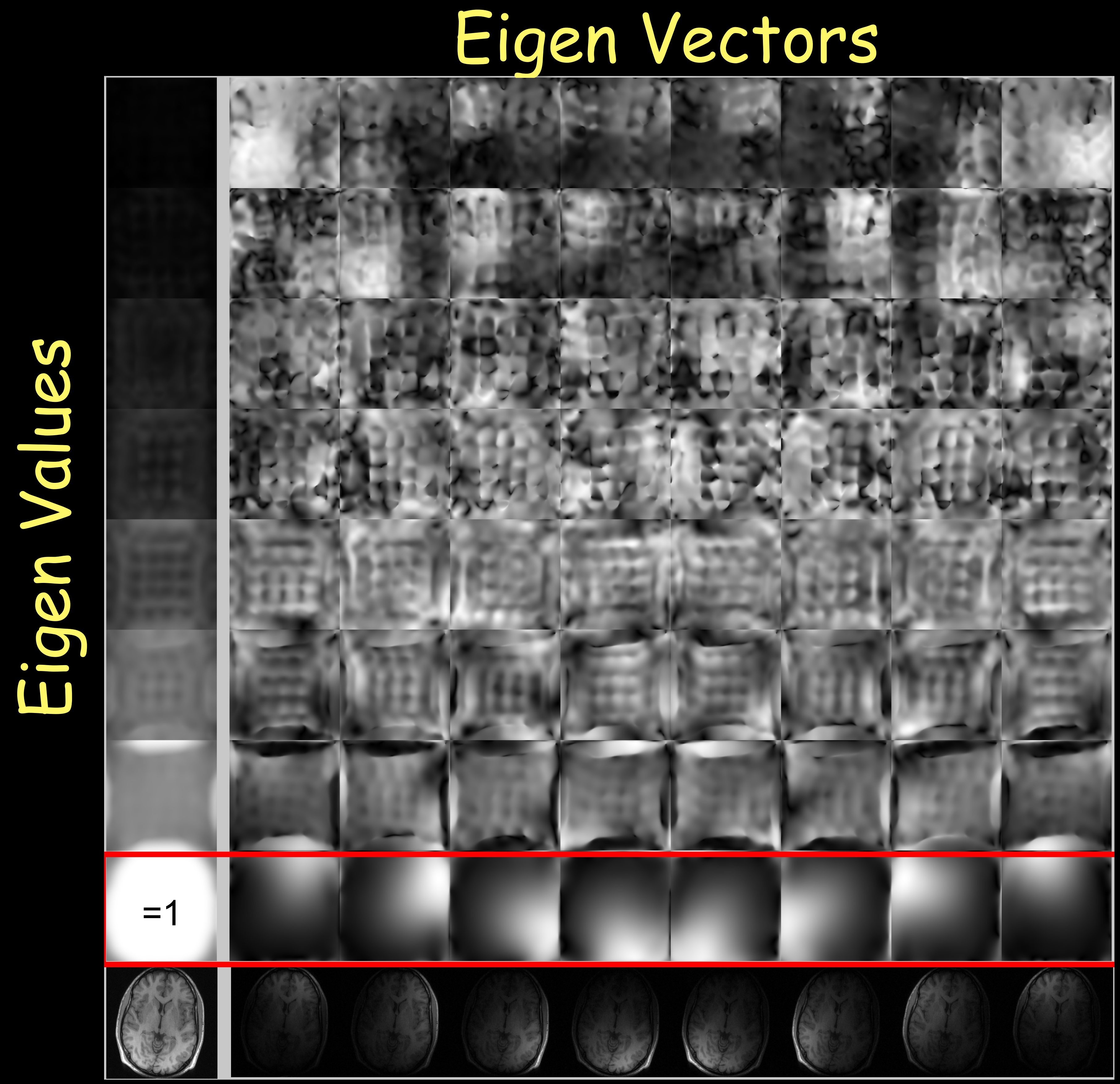
Fast Eigen-Value Decomposition

$V_{||}x$

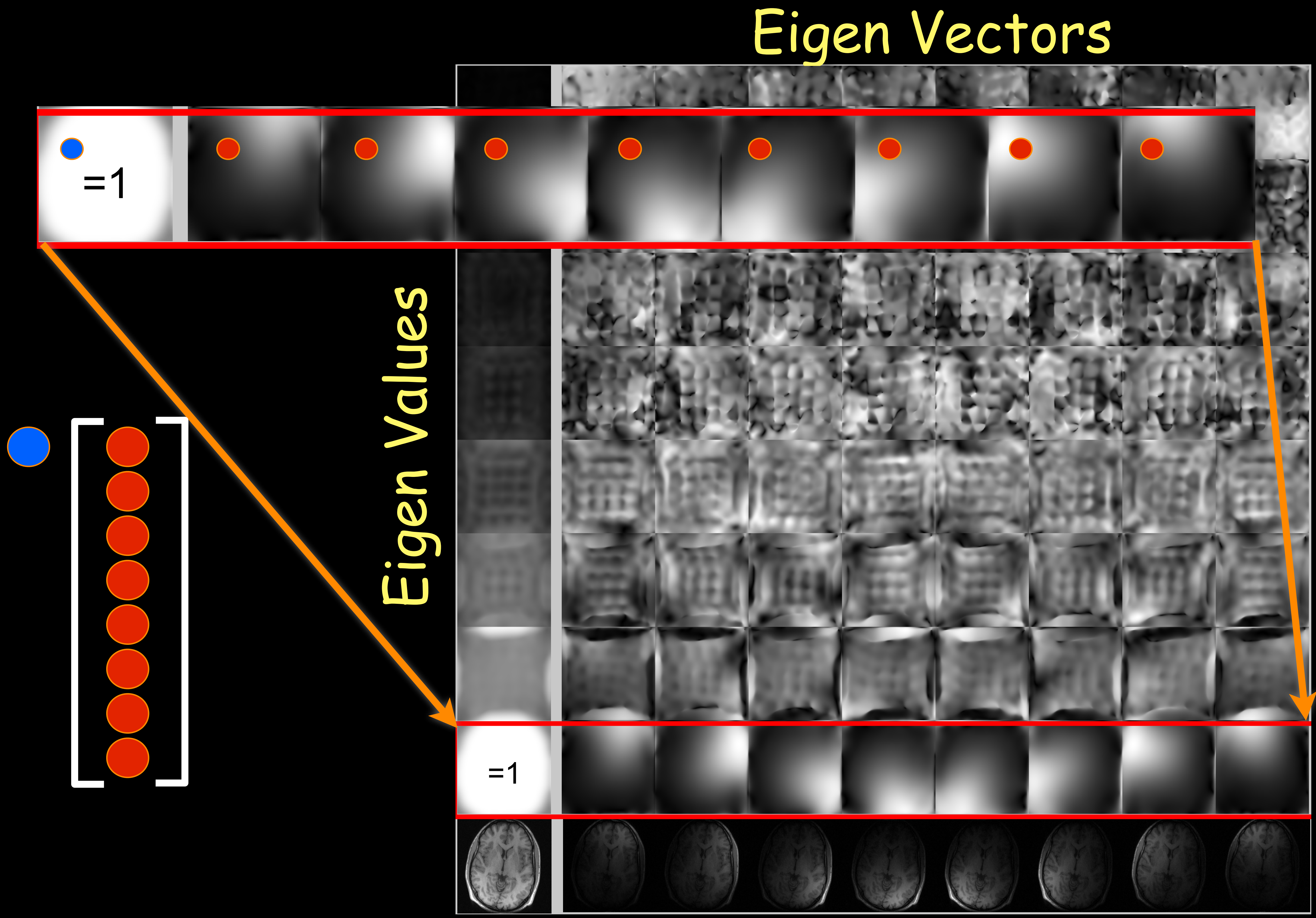


EigenV-decomposition for <u>each</u> position

Eigen-Vector Solution



Eigen-Vector Solution

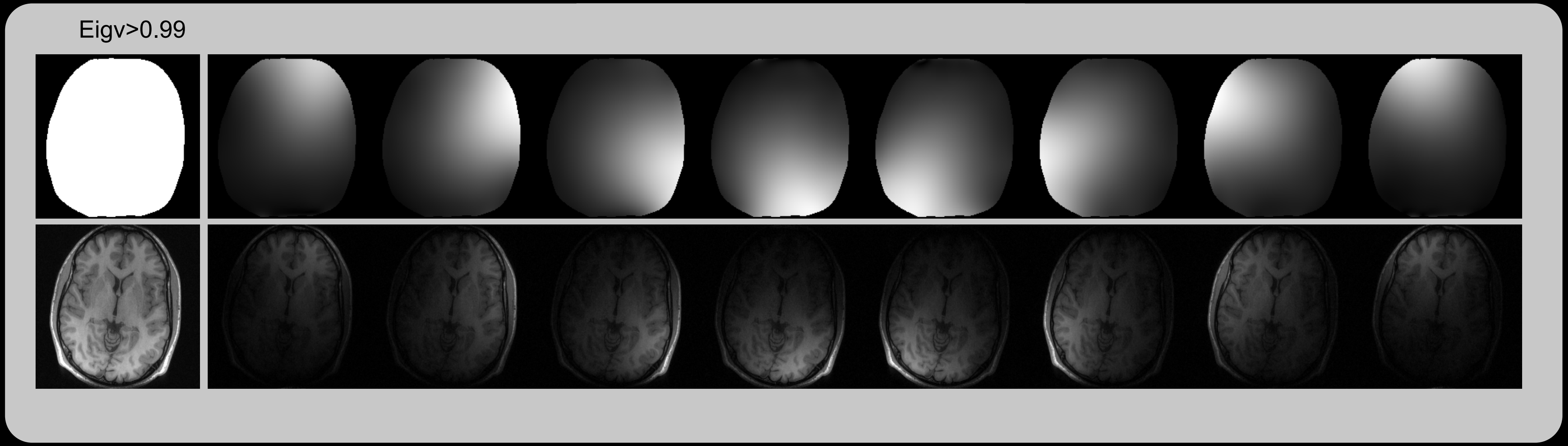


Eigen-Vector solution

$$V_{||}x = x$$

- EigenVecs with EigenVals = 1 are "sensitivity maps"

- Solution is spanned by sensitivity maps



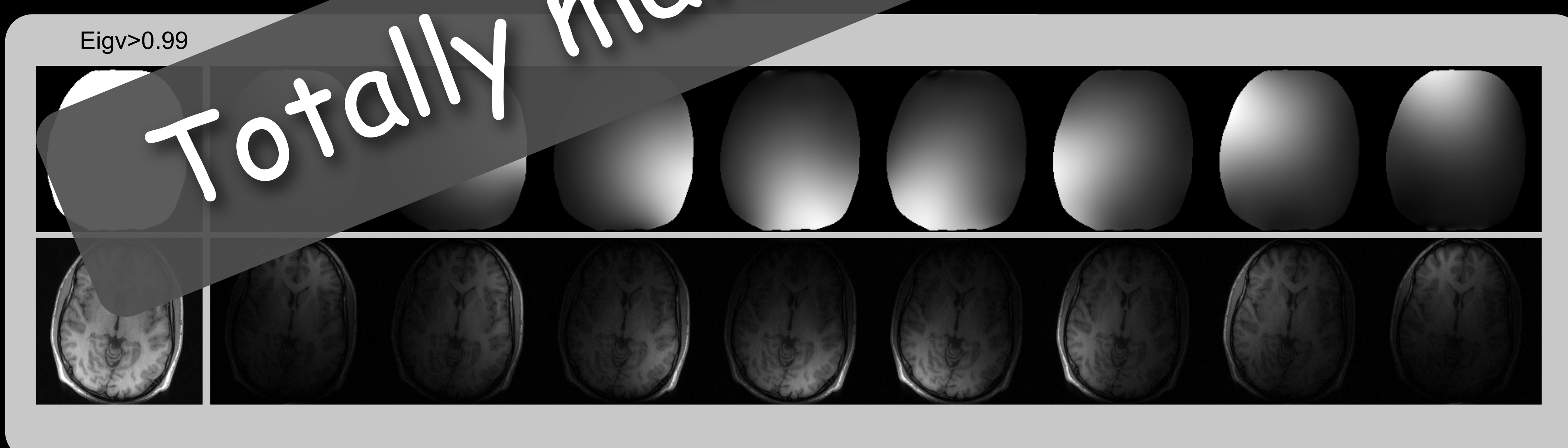
Eigen-Vector solution

$$V_{||}x = x$$

- EigenVecs with EigenVals = 1 are "sensitivity maps"

- Solution is spanned by sensitivity maps

Totally makes SENSE!



ESPIRiT 2.0

Calibration

Construct
A

$[U,S,V]=SVD(A)$

Construct
$V_{\parallel}$

Eigen-decomp
$\mathcal{V}_{\parallel}$

Maps with
EigenVal=1

Reconstruction

Use maps
with SENSE

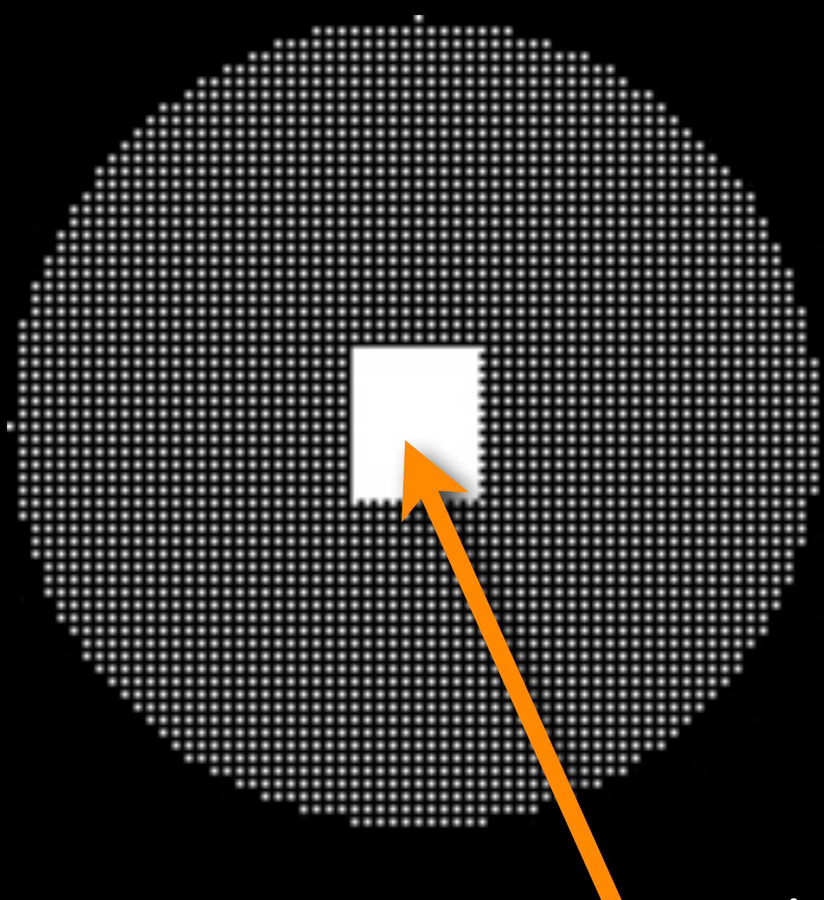
Overcoming the FOV limitation

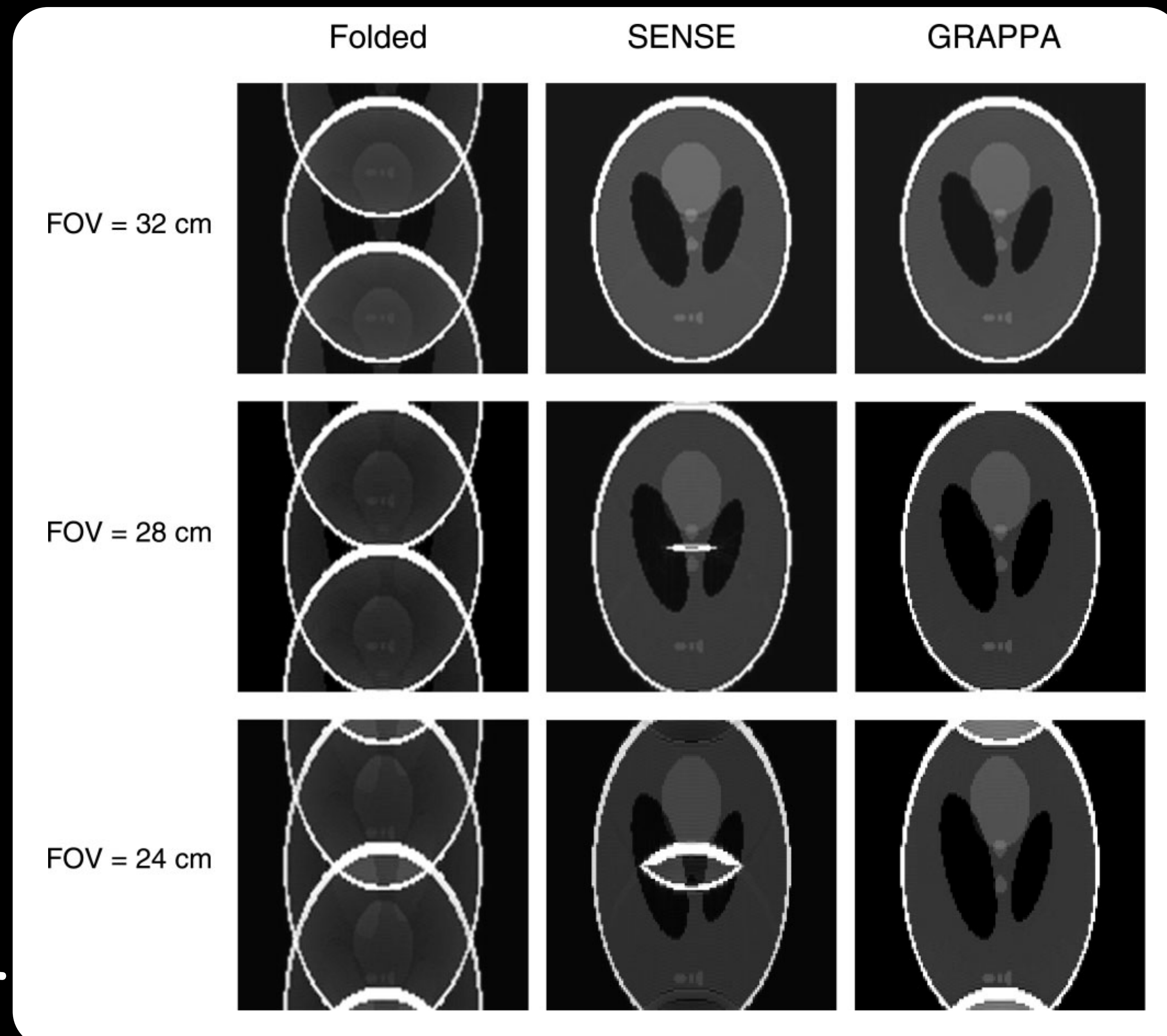
FOV limitations

Magnetic Resonance in Medicine 52:1118–1126 (2004)

Field-of-View Limitations in Parallel Imaging

Mark A. Griswold,[1*] Stephan Kannengiesser,[2] Robin M. Heidemann,[1] Jianmin Wang,[2] and Peter M. Jakob[1]

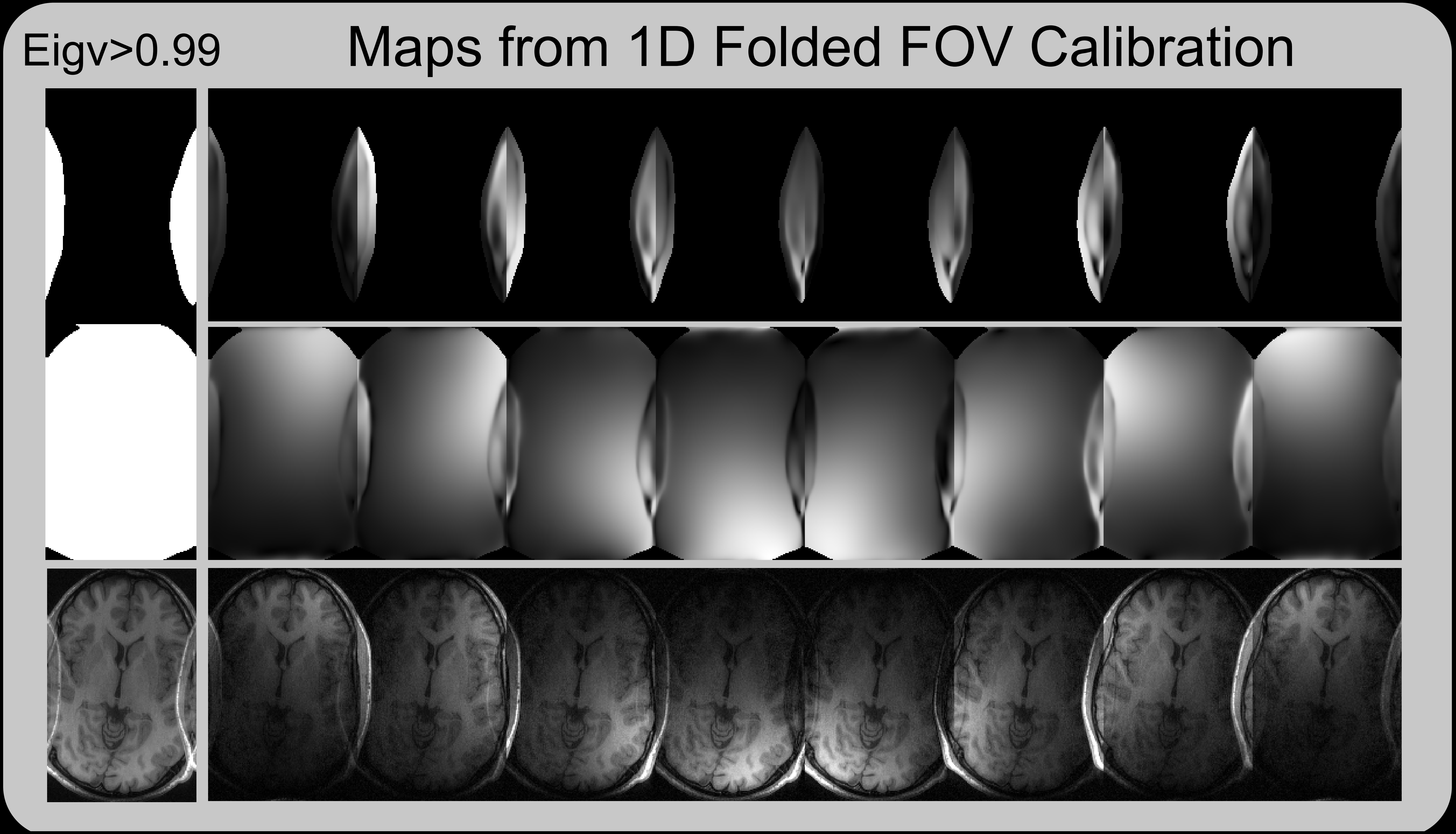




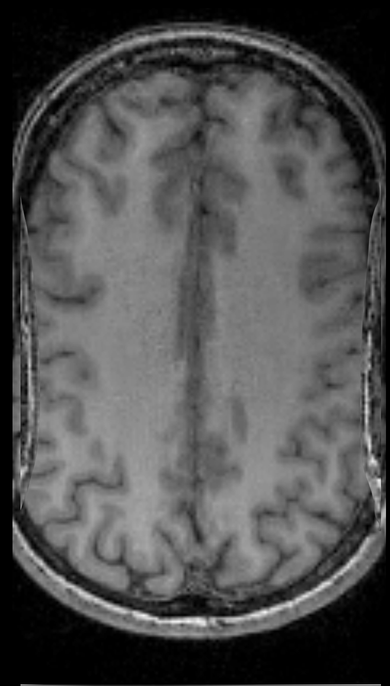
Supported FOV smaller than object

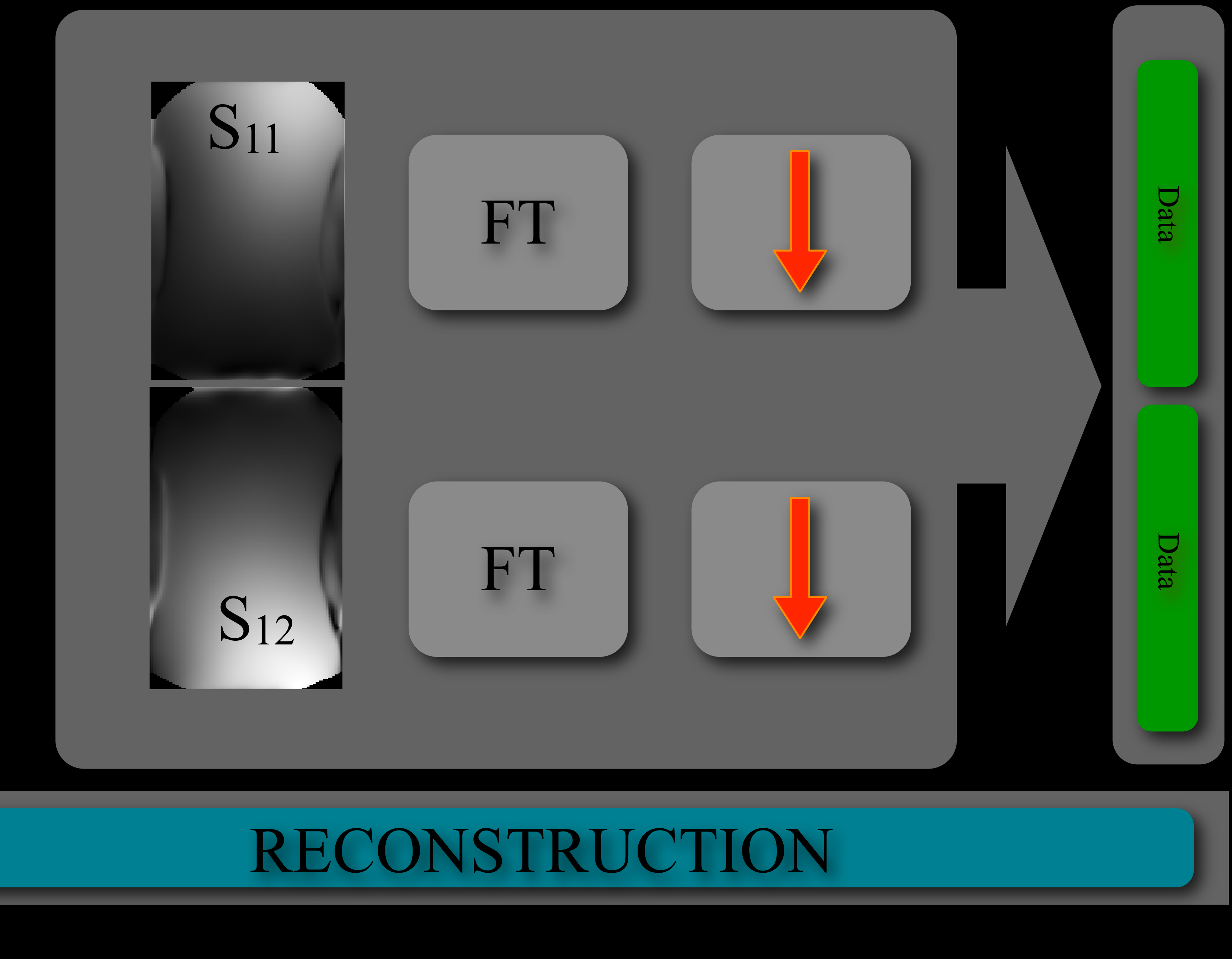
Folded FOV in Calibration

- Folded positions have <u>multiple</u> eigenVals=1



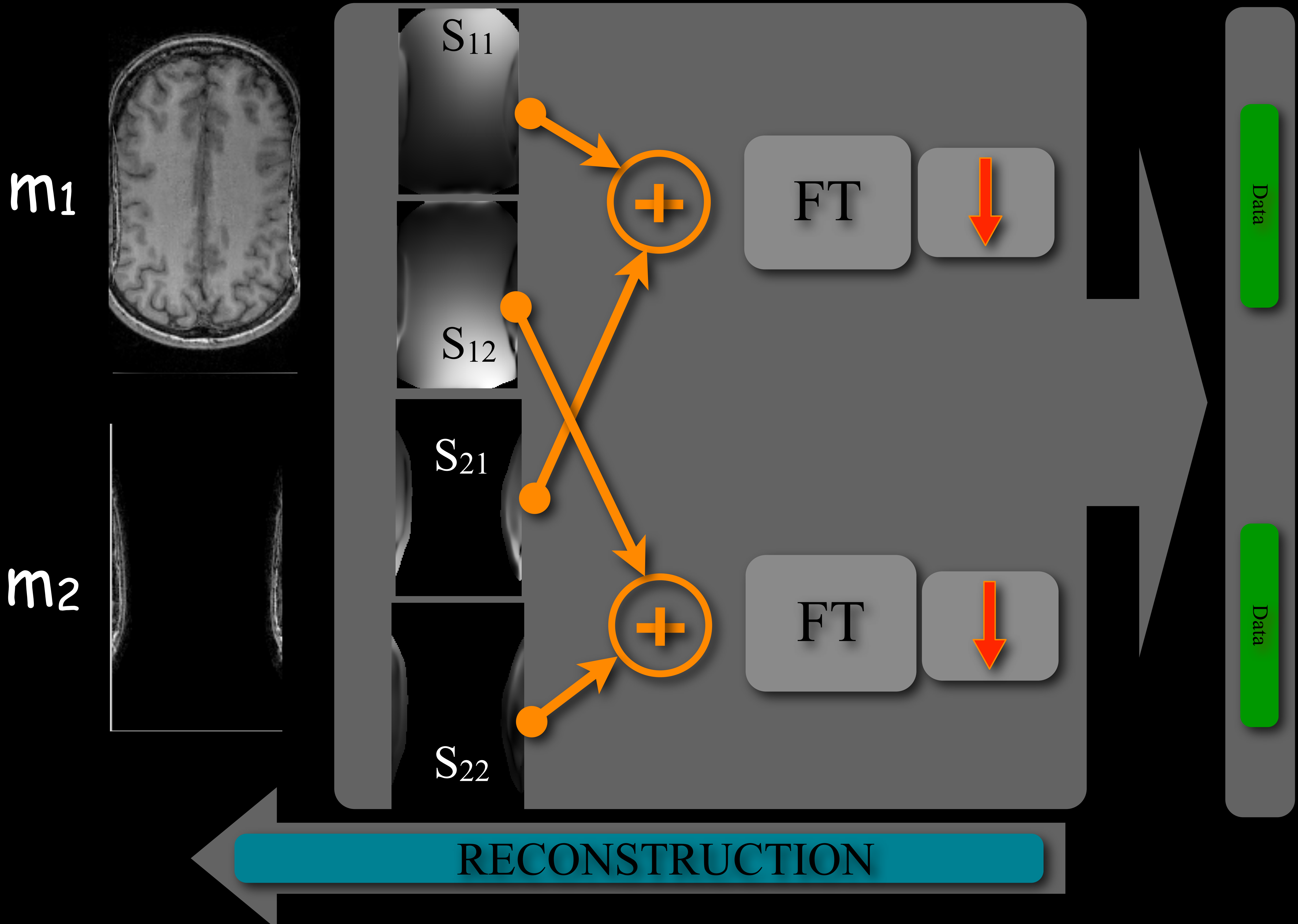
SENSE model





Pruessmann
et. al., 1999

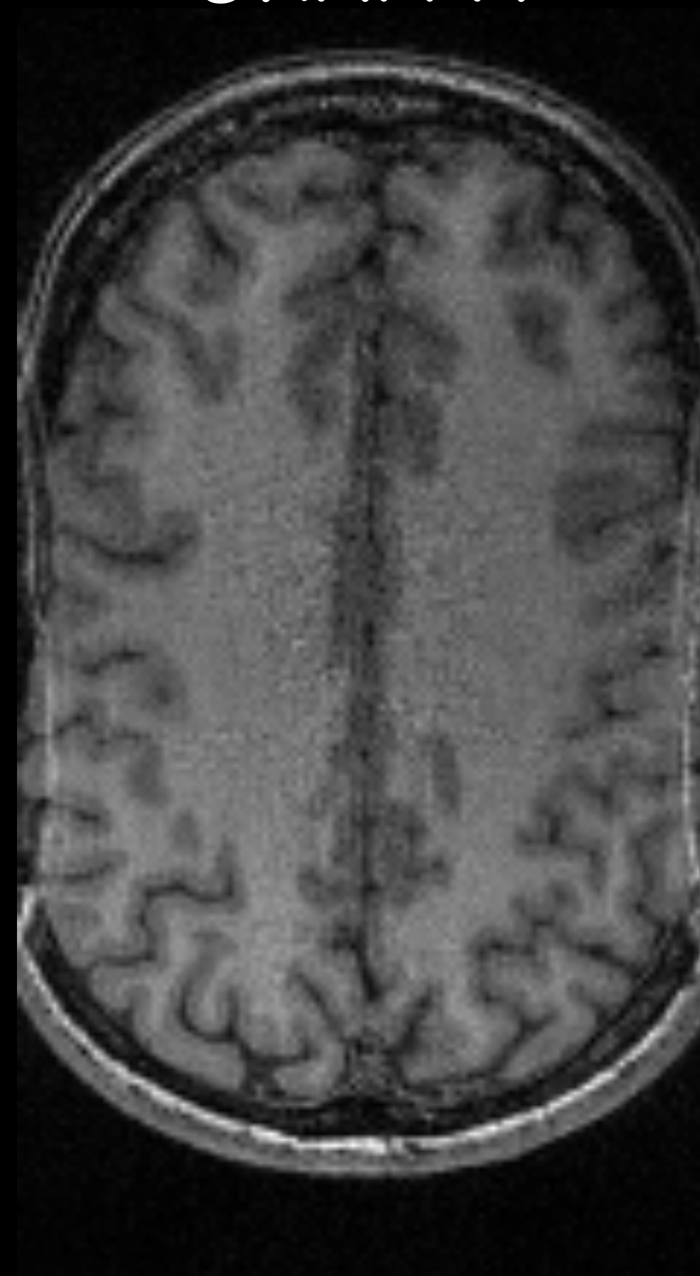
Modified SENSE model



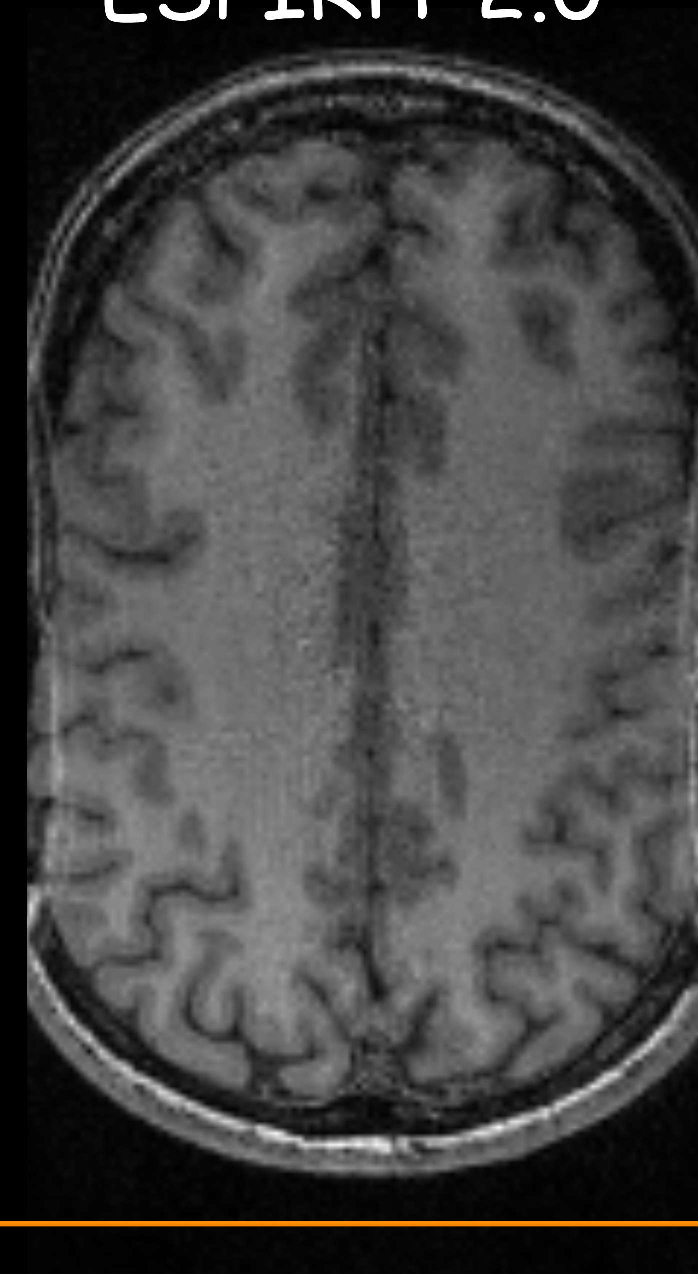
Folded Calibration ESPIRiT 2.0 vs mSENSE

mSENSE

GRAPPA



ESPIRiT 2.0



2-fold undersampling

8-channel head coil

Related Stuff

Magnetic Resonance in Medicine 43:682–690 (2000)

Adaptive Reconstruction of Phased Array MR Imagery

David O. Walsh,[1] Arthur F. Gmitro,[2*] and Michael W. Marcellin[3]

MULTICHANNEL ESTIMATION OF COIL SENSITIVITIES IN PARALLEL MRI

Robert L. Morrison, Jr.[†], *Mathews Jacob*[*], *and Minh N. Do*[†*]

Department of Electrical and Computer Engineering
[†]Coordinated Science Laboratory and [*]Beckman Institute
University of Illinois at Urbana-Champaign
rlmorris@uiuc.edu, mjacob@uiuc.edu, minhdo@uiuc.edu

202 IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 8, NO. 2, FEBRUARY 1999

Perfect Blind Restoration of Images Blurred by Multiple Filters: Theory and Efficient Algorithms

Gopal Harikumar, *Member, IEEE,* and Yoram Bresler, *Fellow, IEEE*

IMAGE RECONSTRUCTION FROM PHASED-ARRAY MRI DATA BASED ON MULTICHANNEL BLIND DECONVOLUTION

Huajun She[1], *Rong-Rong Chen*[1], *Dong Liang*[2], *Yuchou Chang*[2], *Leslie Ying*[2]

[1]Department of Electrical and Computer Engineering, University of Utah, Salt Lake City, UT, USA
[2]Department of Electrical Engineering and Computer Science, University of Wisconsin-Milwaukee, Milwaukee, WI, USA

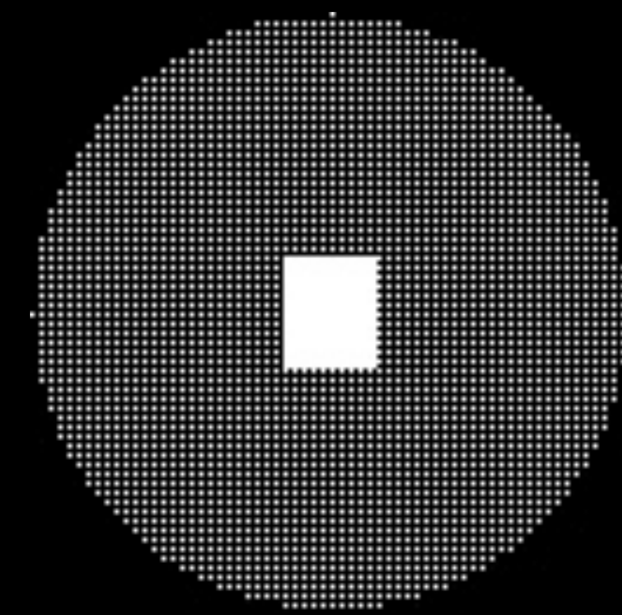
Some comments so far...

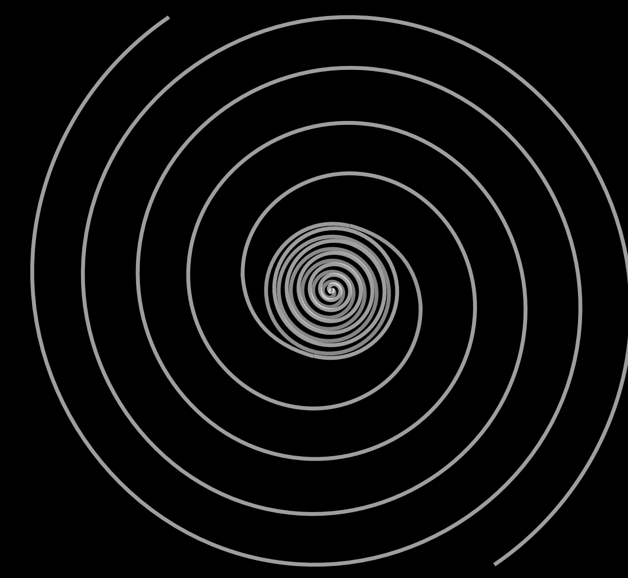
- Theory of optimal auto-calibration
  - Leads from GRAPPA-like acPI to SENSE
- Explained the FOV problem in terms of EigenVals/Vecs of operators
- Very robust and efficient coil combination
- Complexity of the reconstruction reduced from $O(n^2)$ to $O(n)$.
  n is # coils.

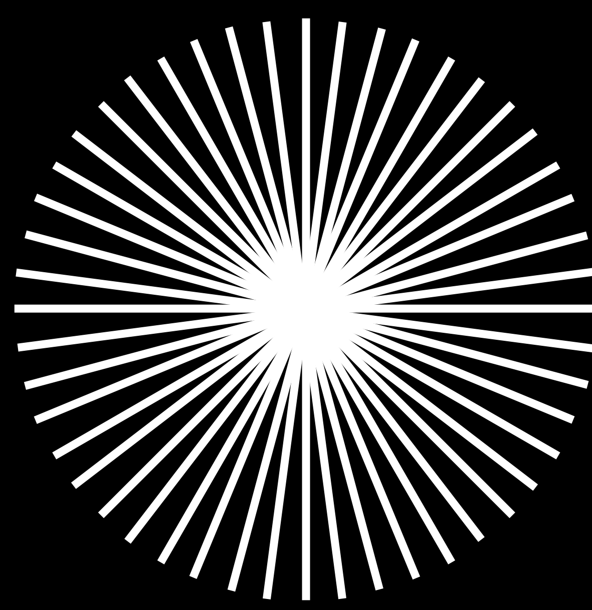
Going Calibrationless

AutoCalibration

- Autocalibration:
  - k-space center densely sampled
- But Sometimes..
  - Dense sampling can be $$$
  - Not enough of
  - Hard to acquire
- Can we autocalibrate from sparsely sampled k-space?

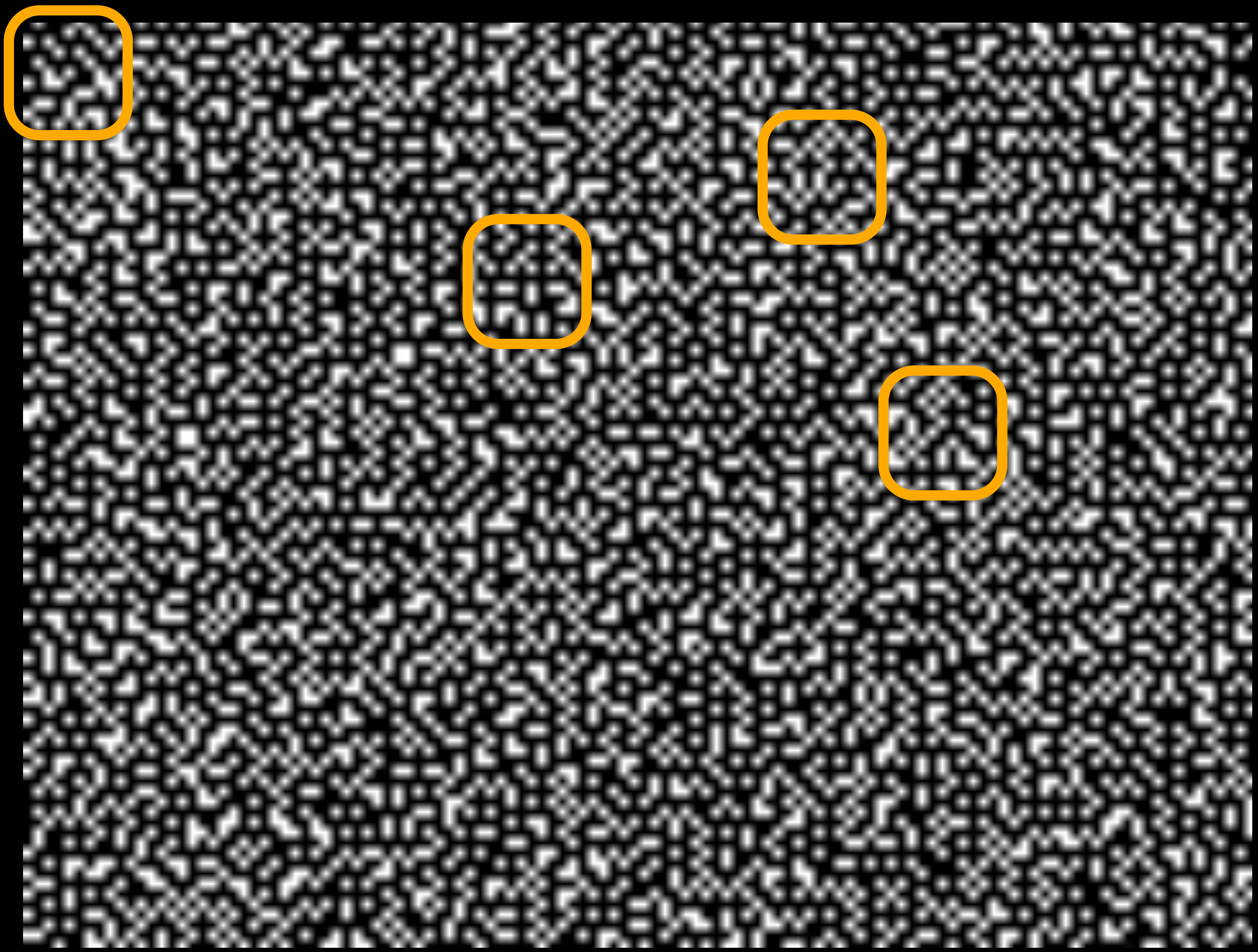




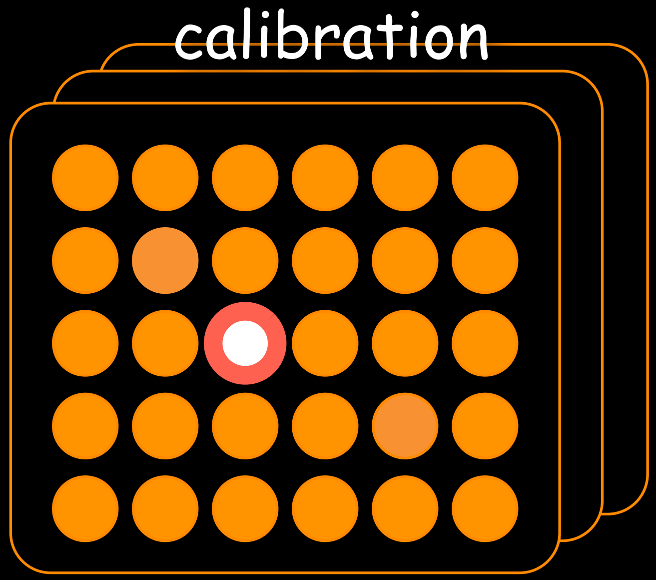


Cartesian Calibrationless

- What if there's no calibration?

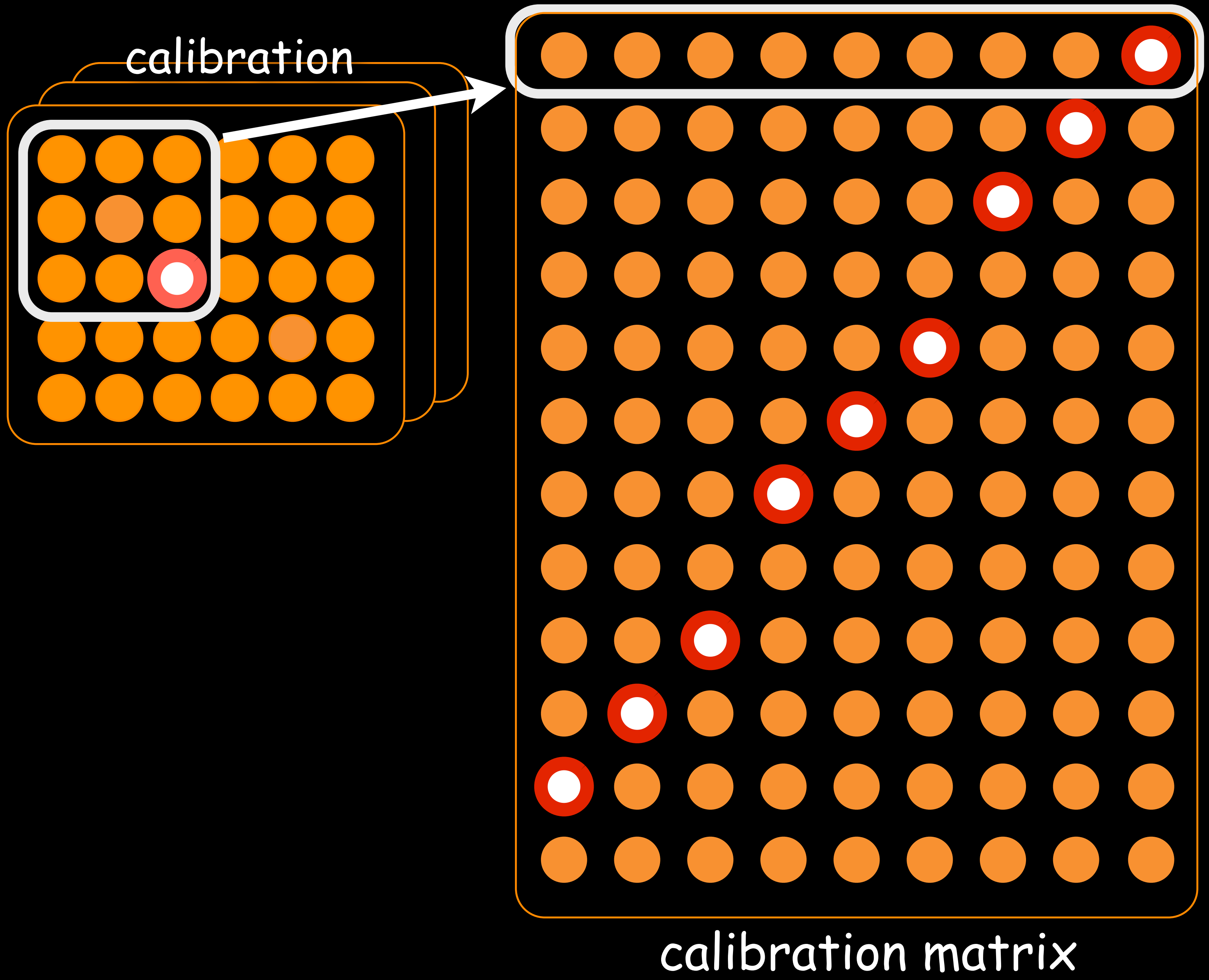


Calibration from dense sampling



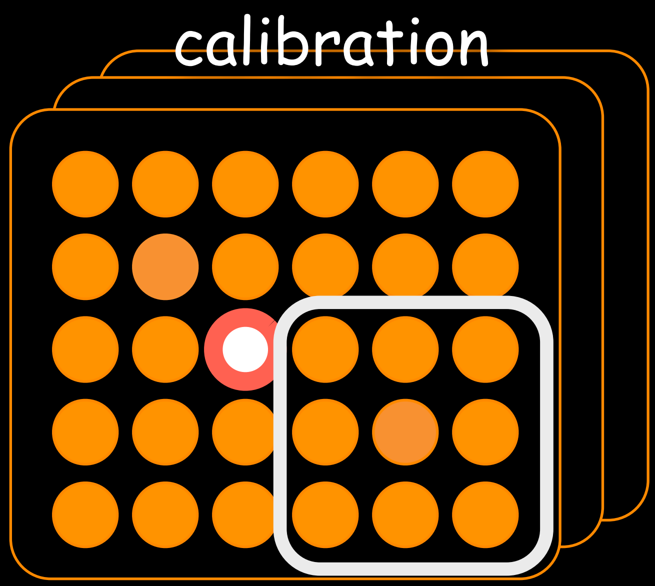
The Calibration Matrix:

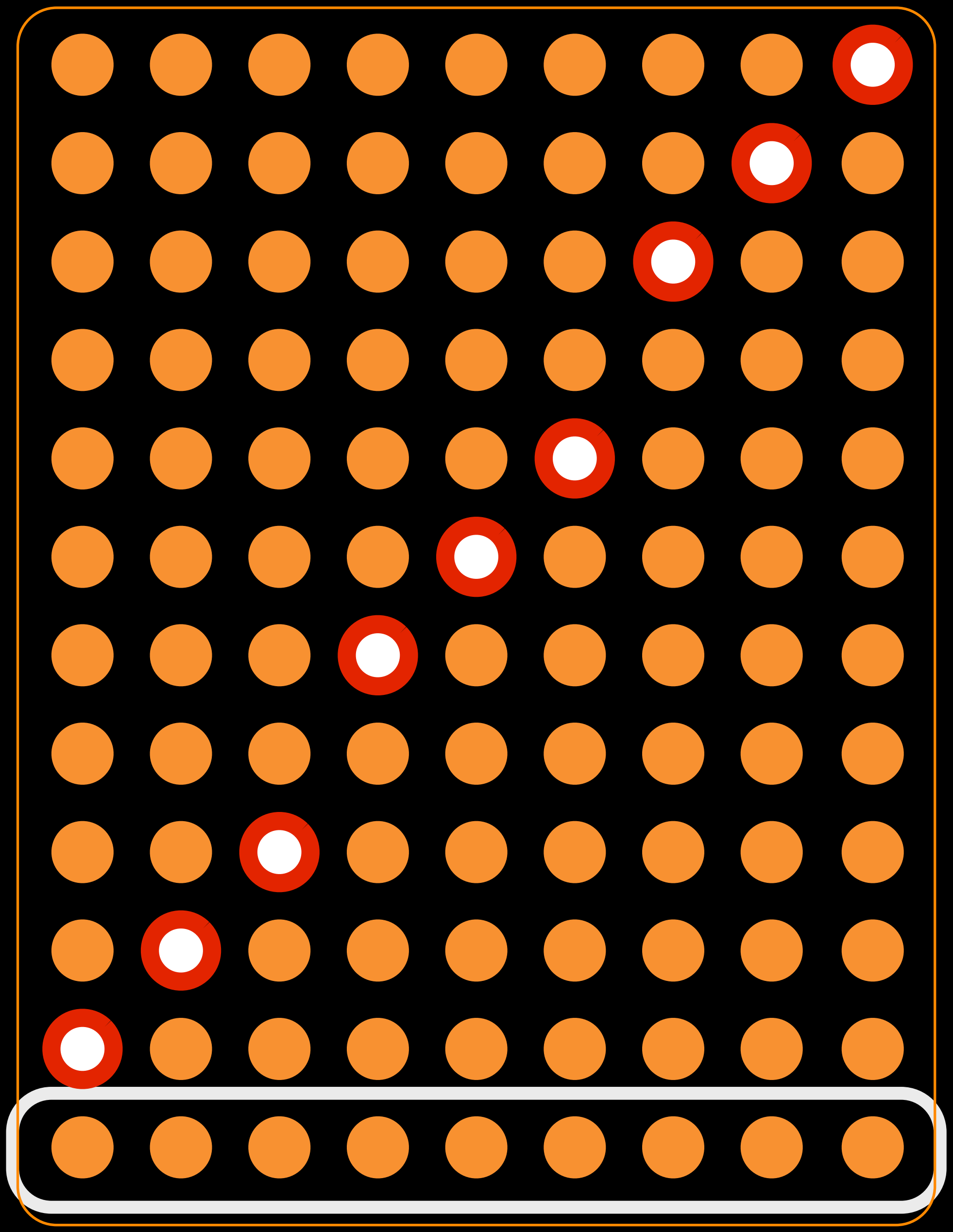
Calibration from dense sampling



The Calibration Matrix:

Calibration from dense sampling

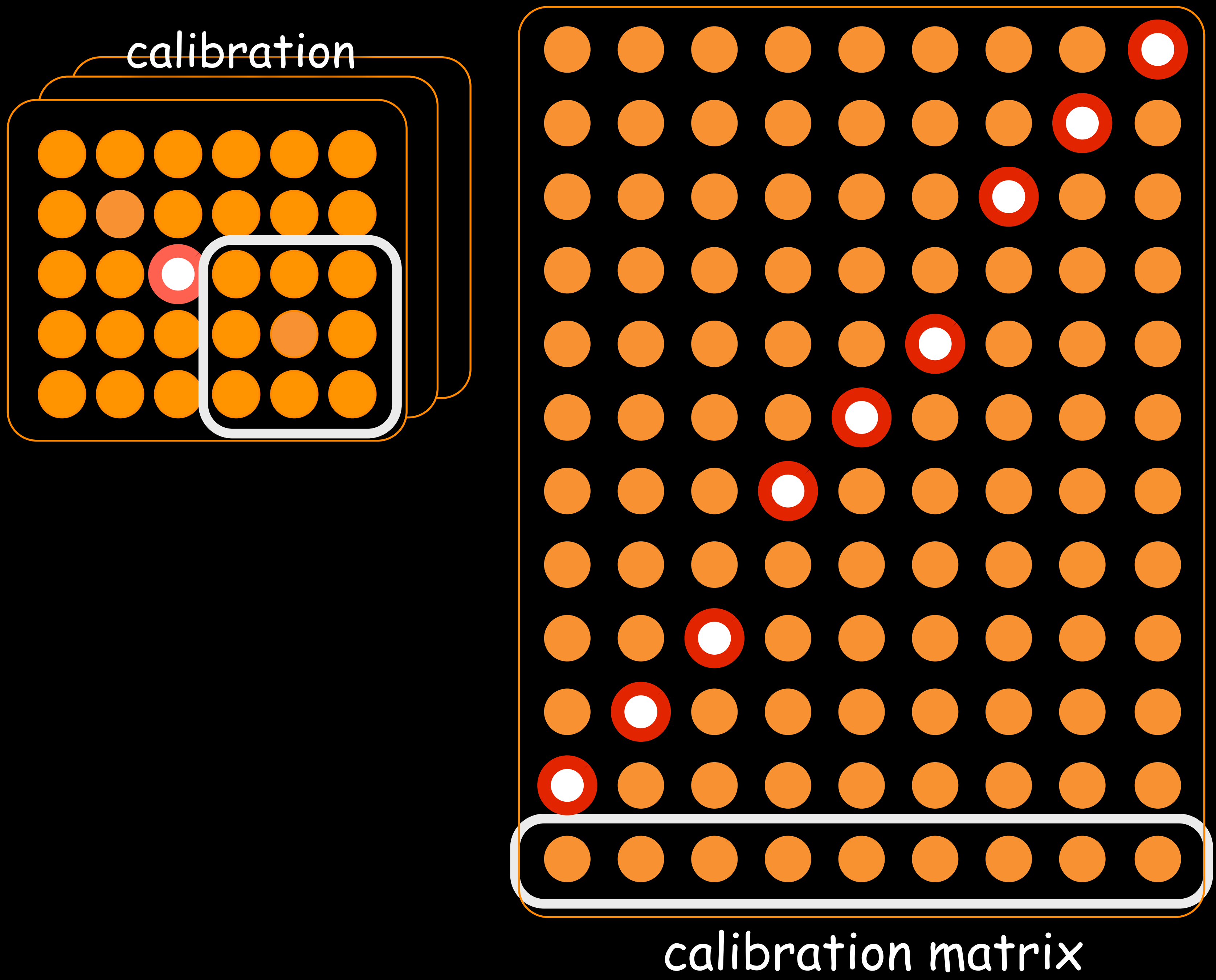




The Calibration Matrix:

- Has Hankel structure

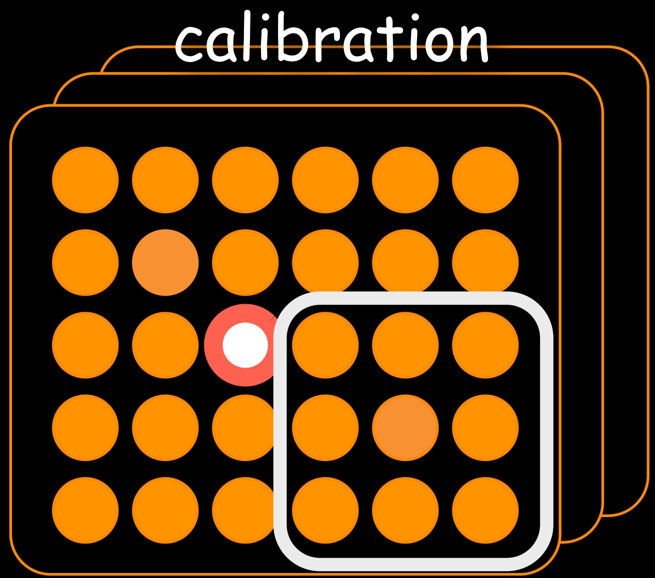
Calibration from dense sampling

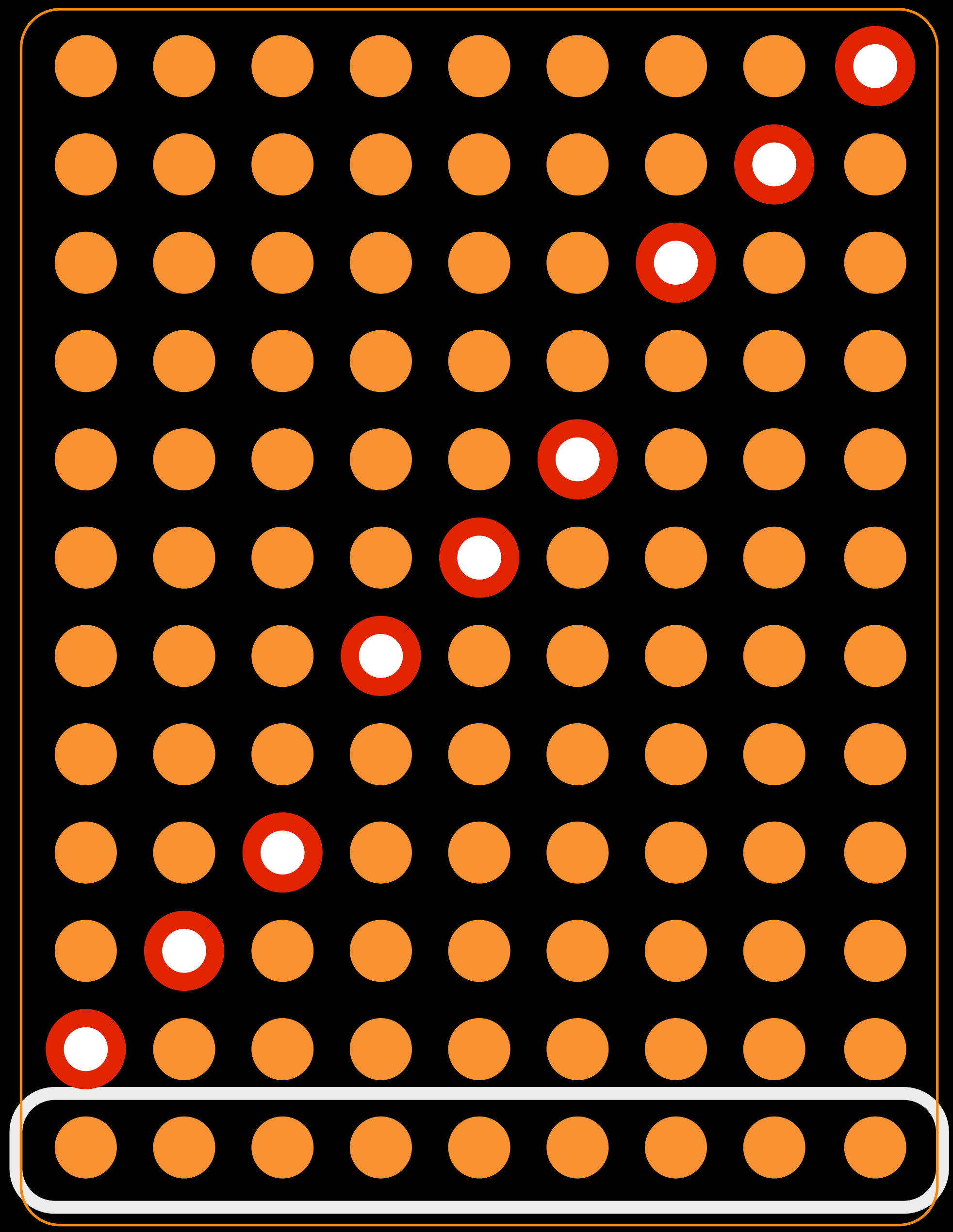


The Calibration Matrix:

- Has Hankel structure
- Rows are correlated

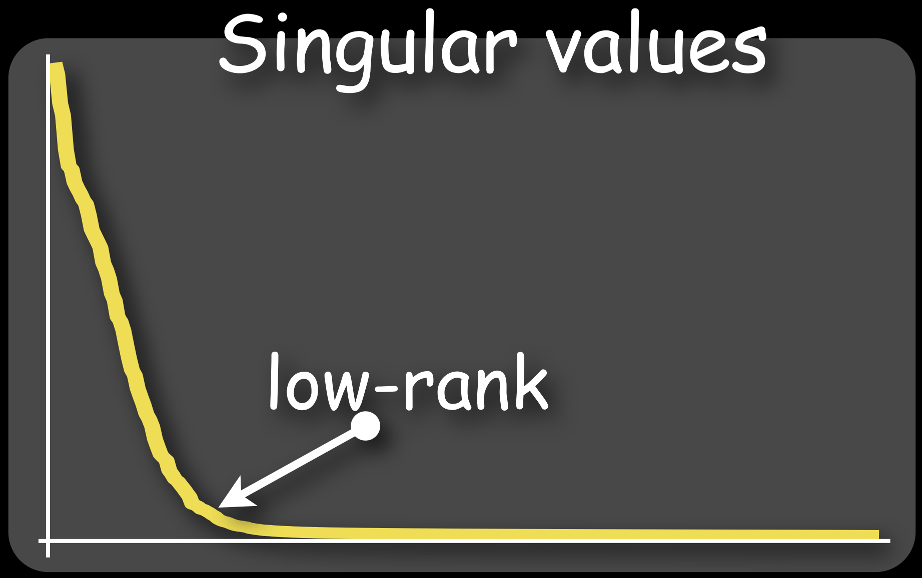
Calibration from dense sampling





calibration matrix

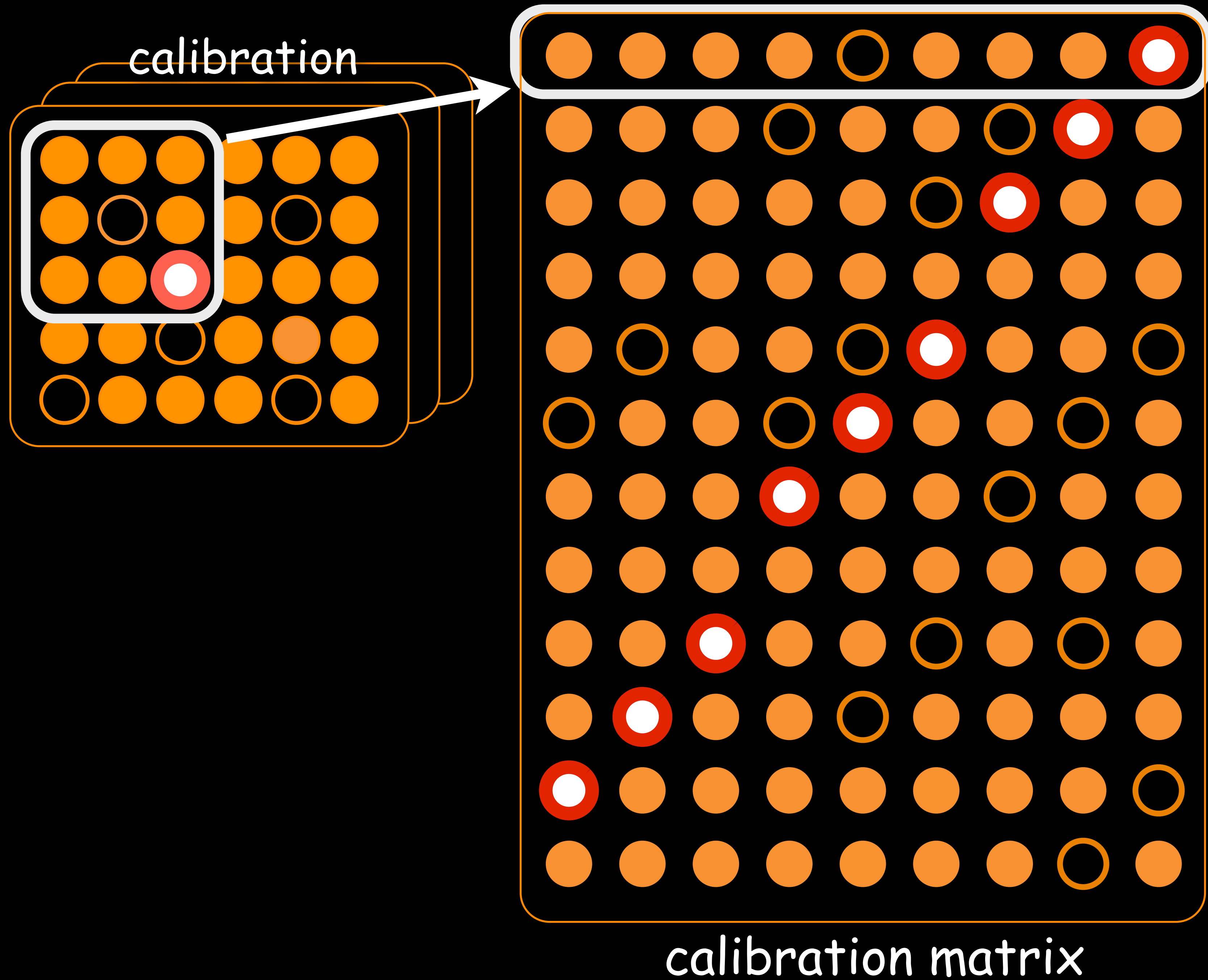
SVD(A)



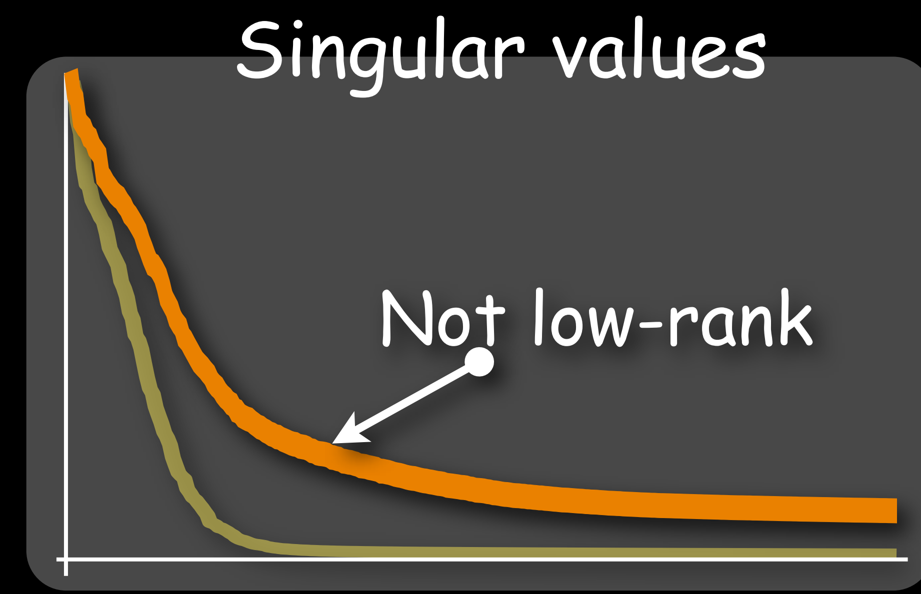
The Calibration Matrix:

- Has Hankel structure
- Rows are correlated

Calibrate from sparse sampling



SVD(A)



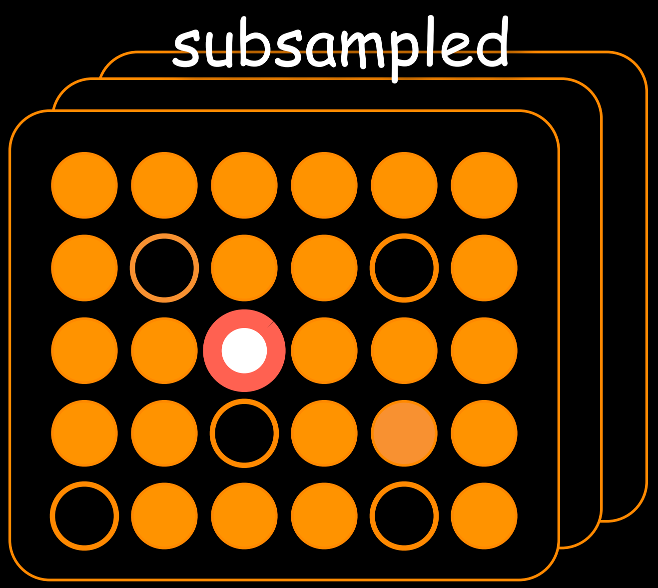
Subsampled Matrix:

- Has Hankel structure
- Less Correlation

The Cadzow Algorithm

Impose:

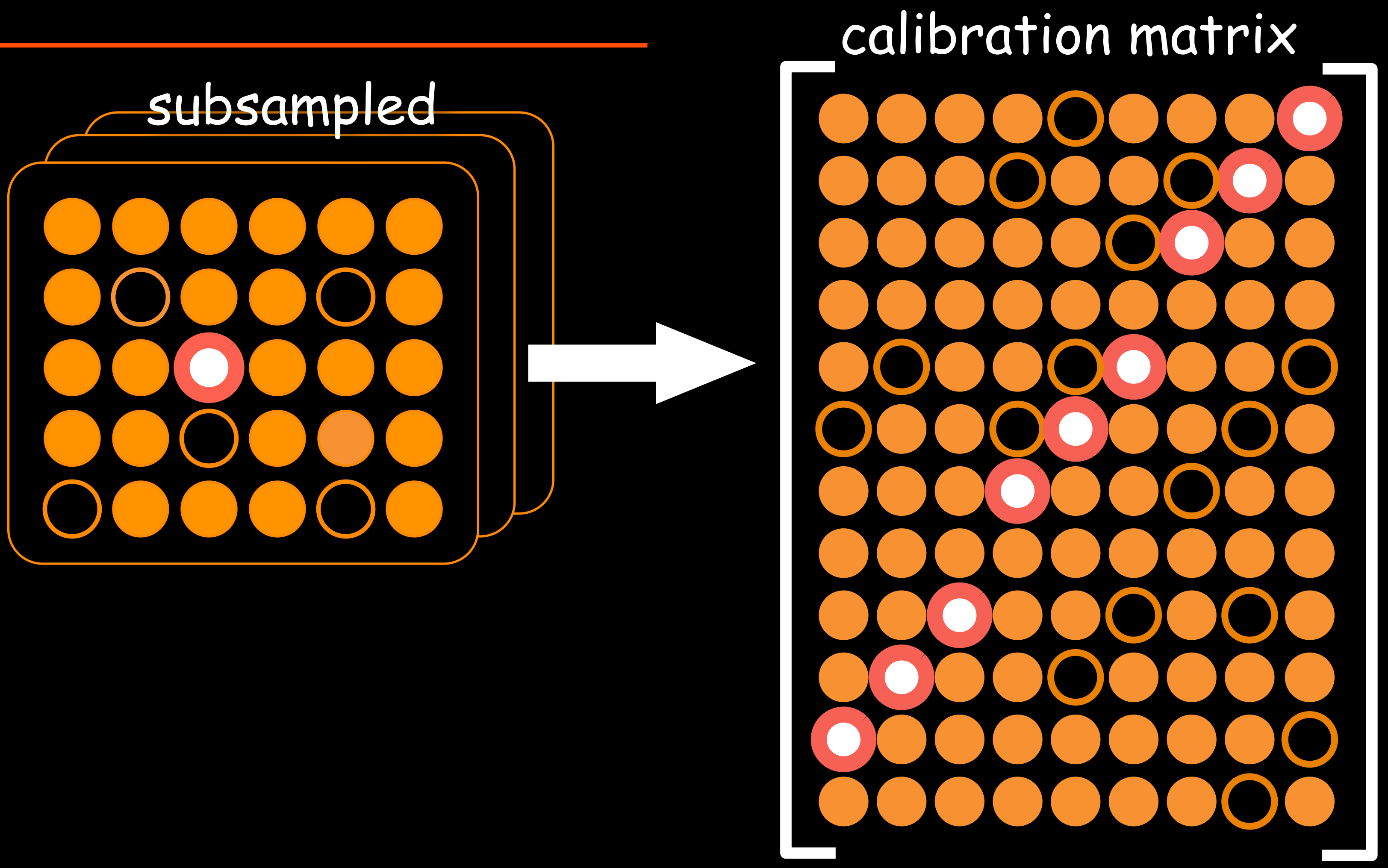
- Low-Rank
- Hankel Structure
- Data Consistency
- Iterate



The Cadzow Algorithm

Impose:

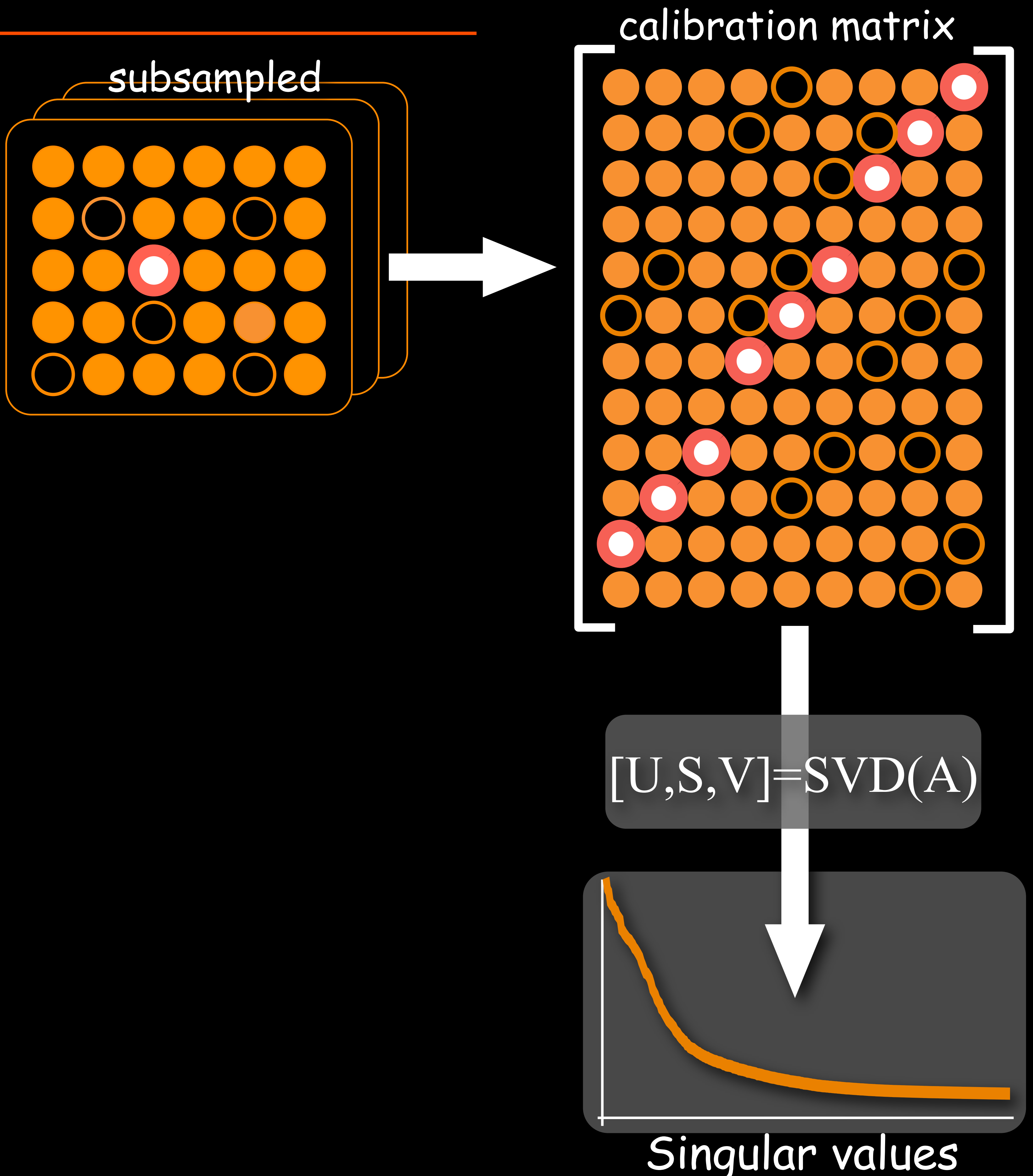
- Low-Rank
- Hankel Structure
- Data Consistency
- Iterate



The Cadzow Algorithm

Impose:

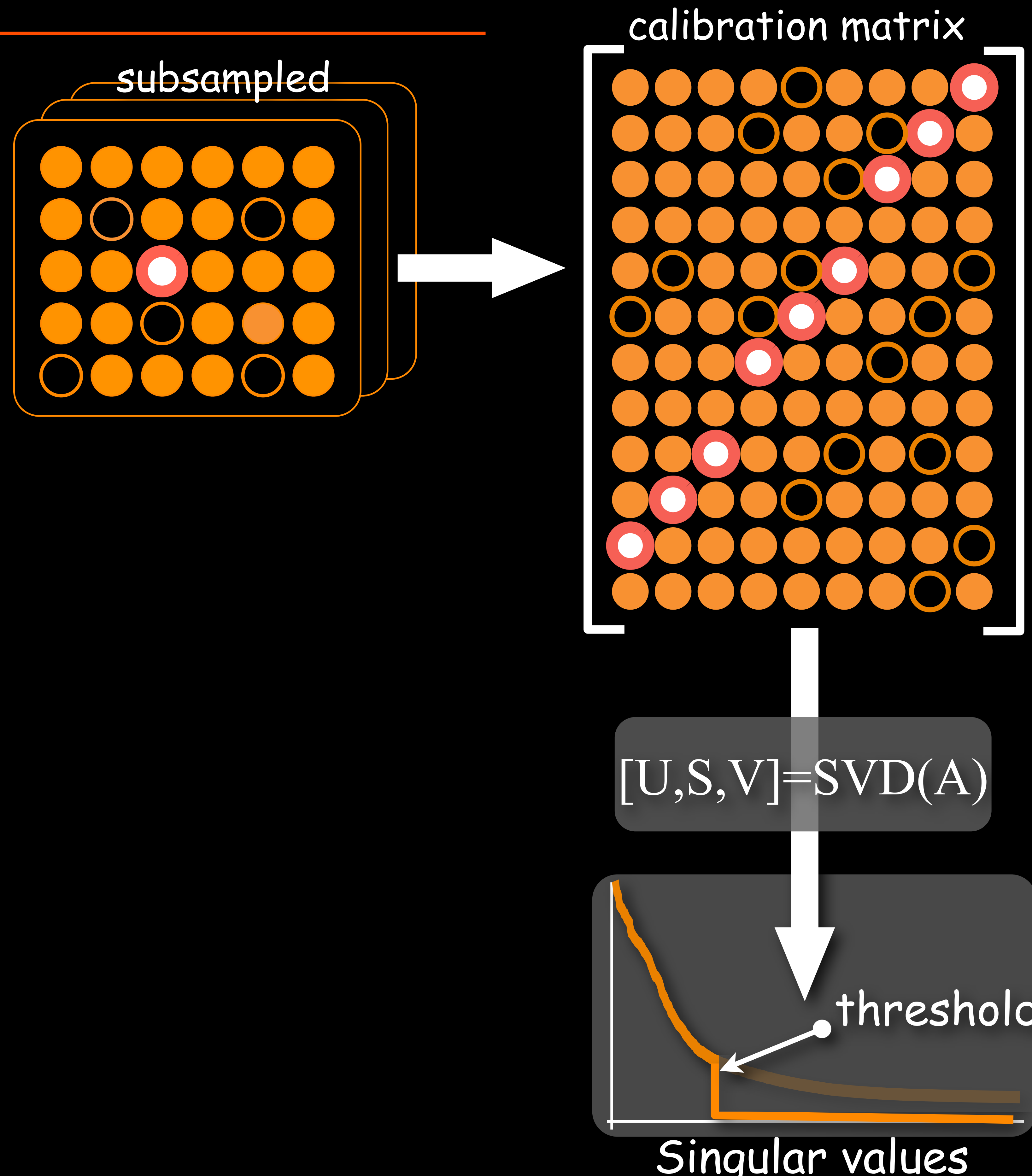
- Low-Rank
- Hankel Structure
- Data Consistency
- Iterate



The Cadzow Algorithm

Impose:

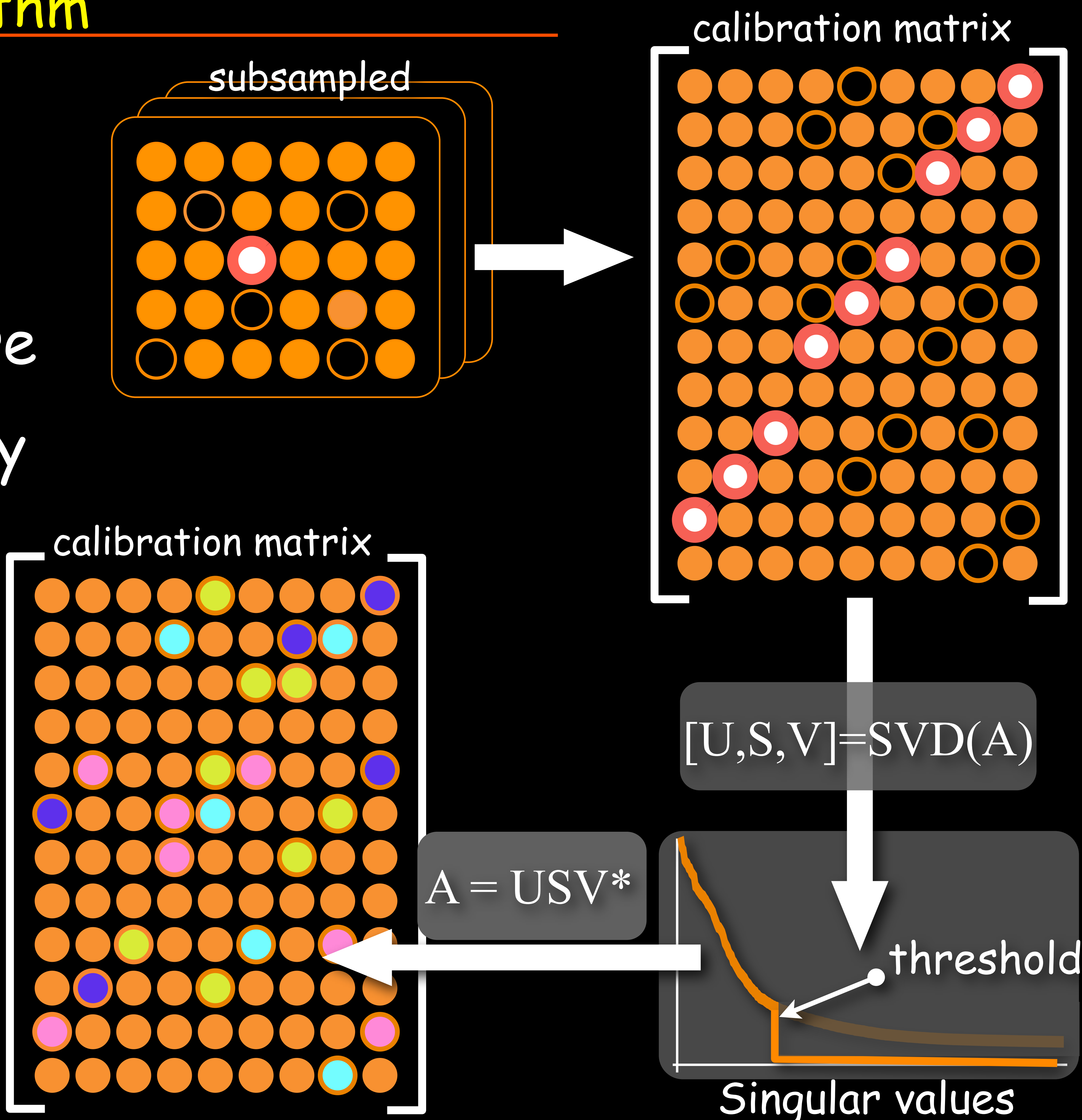
- Low-Rank
- Hankel Structure
- Data Consistency
- Iterate



The Cadzow Algorithm

Impose:

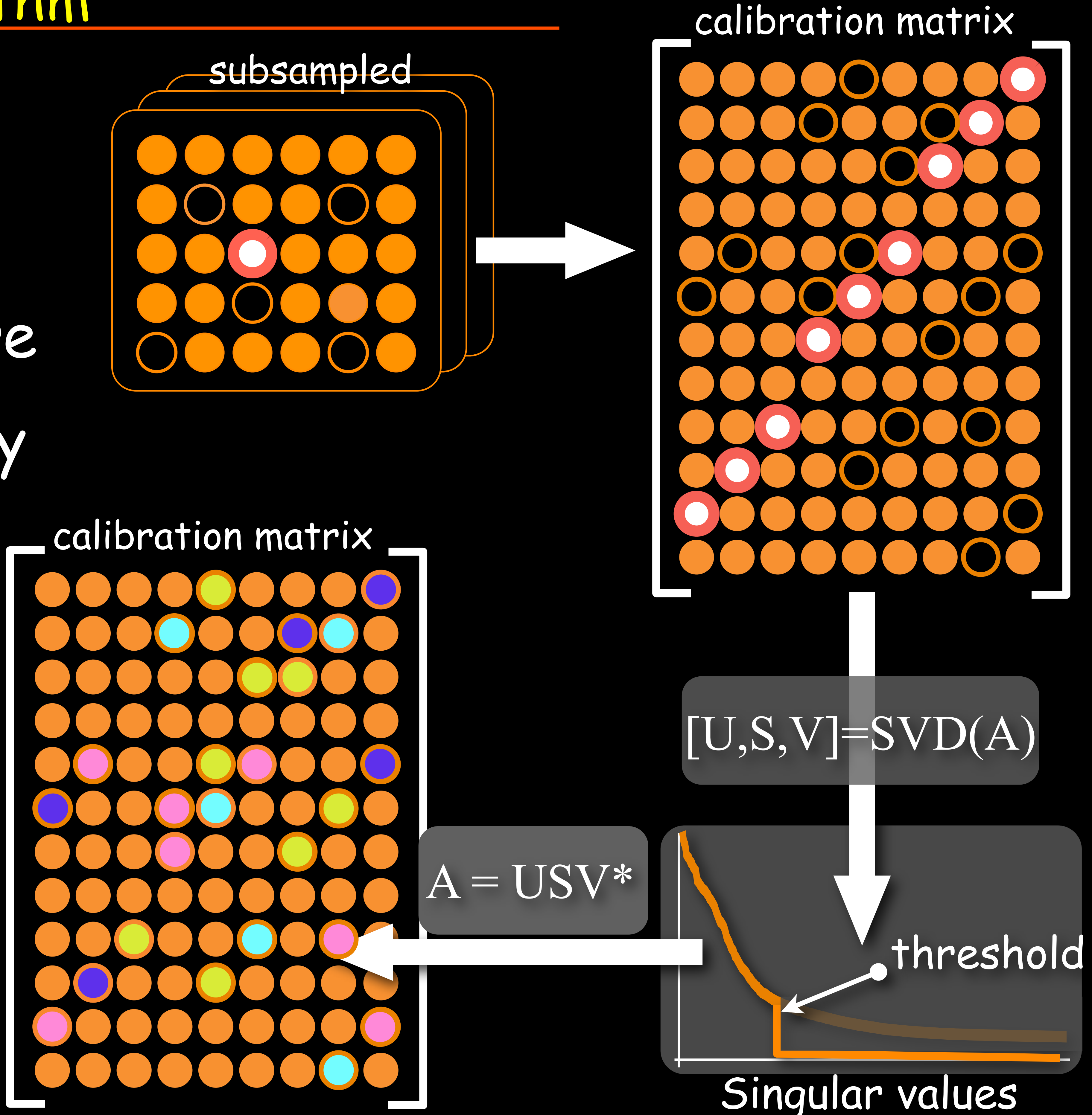
- Low-Rank
- Hankel Structure
- Data Consistency
- Iterate



The Cadzow Algorithm

Impose:

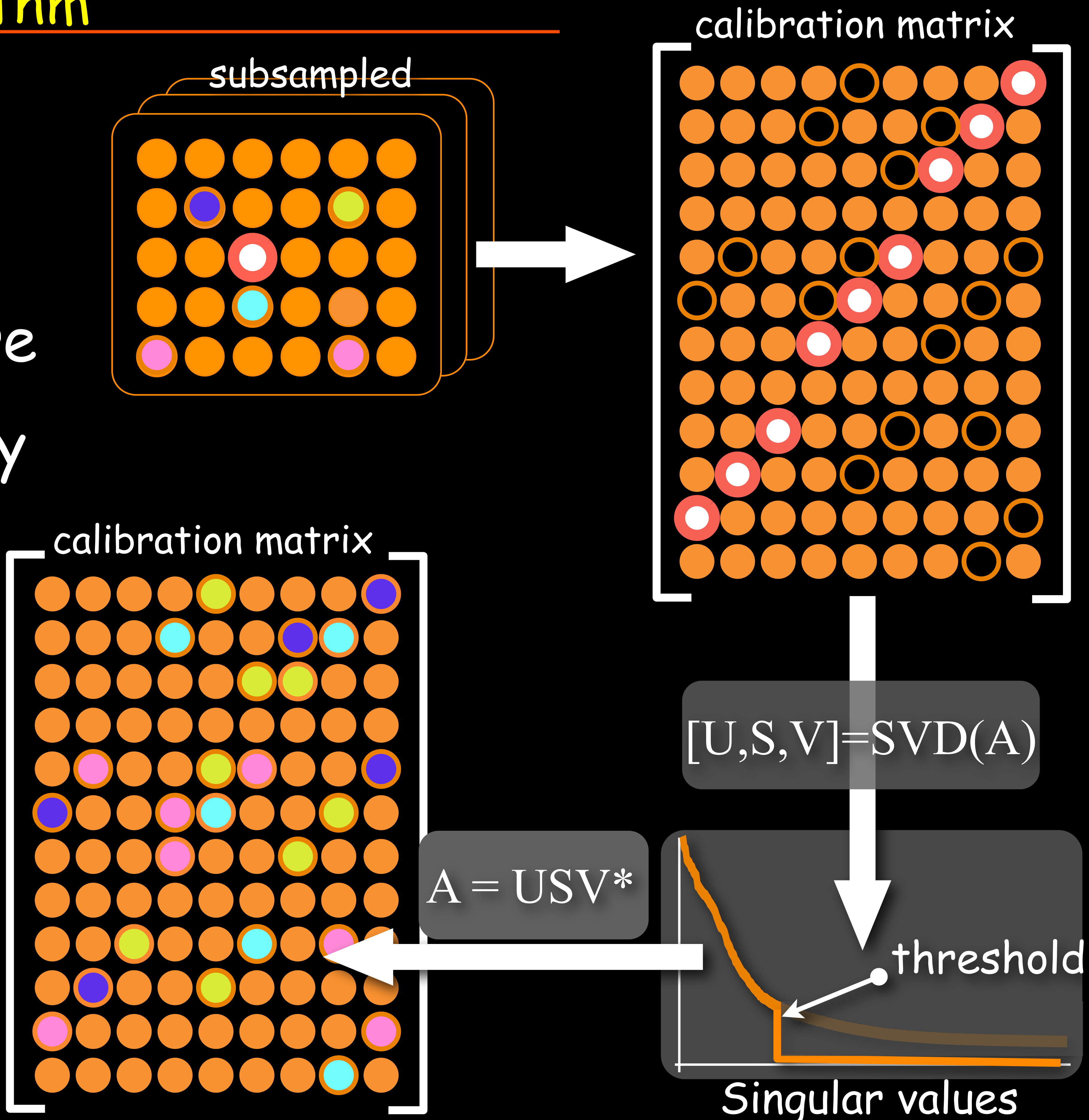
- Low-Rank
- Hankel Structure
- Data Consistency
- Iterate



The Cadzow Algorithm

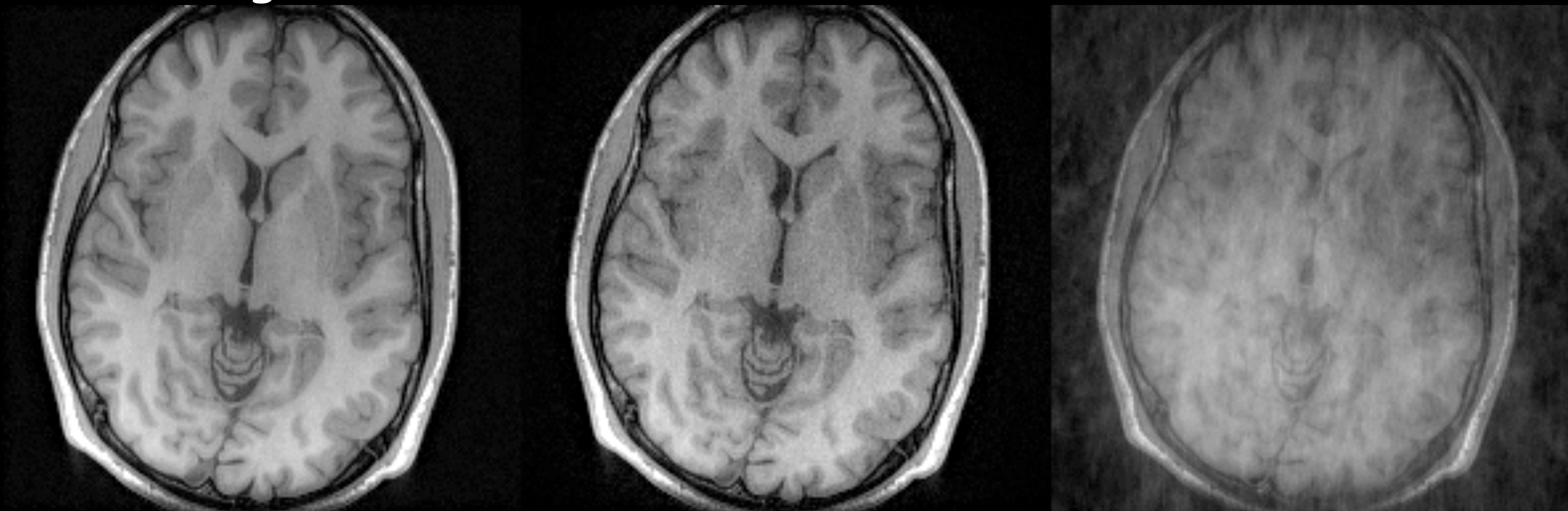
Impose:

- Low-Rank
- Hankel Structure
- Data Consistency
- Iterate



Results

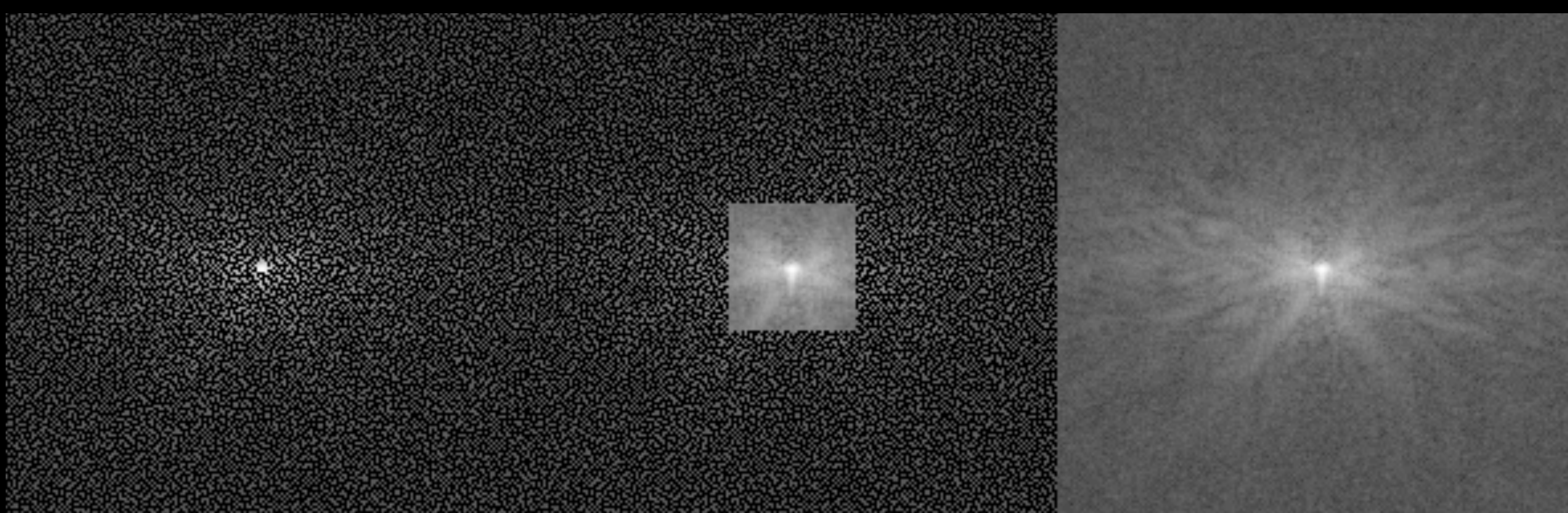
Original | x3 Calibrationless | x3 zero-fill



x3 undersampling

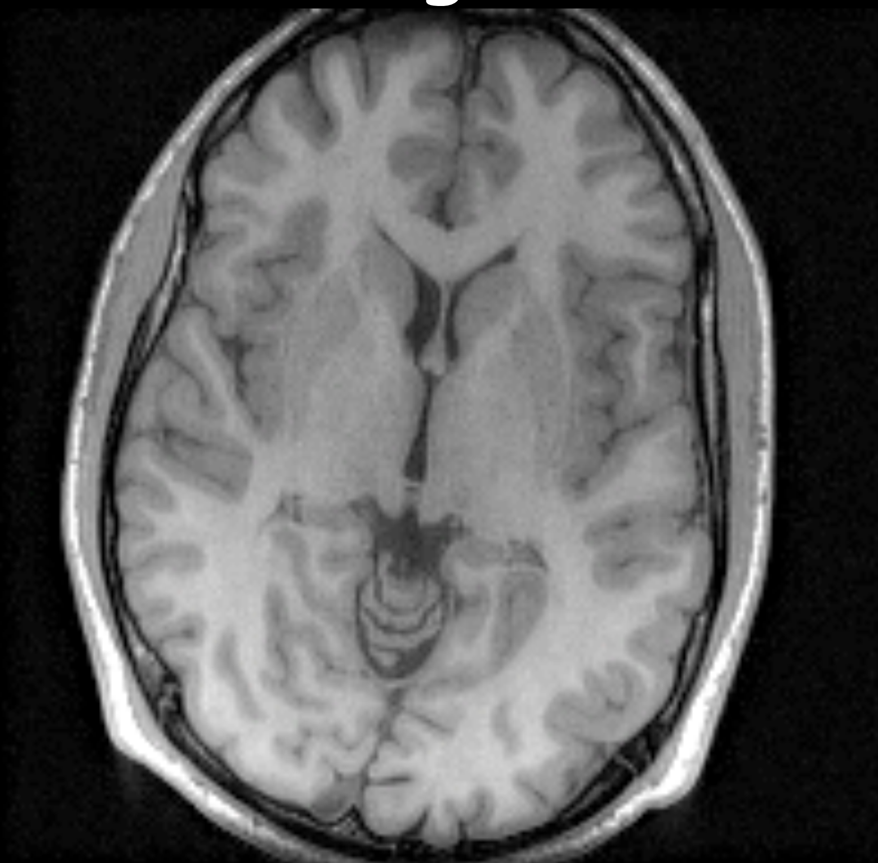
Poisson-Disc

8-chan head coil

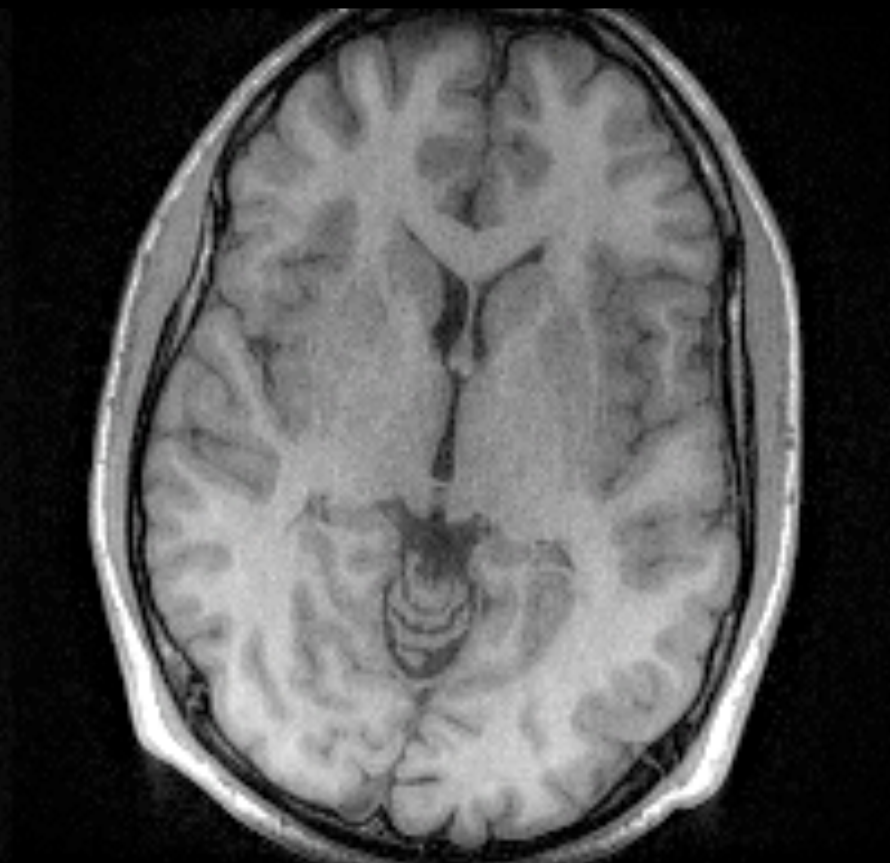


Results

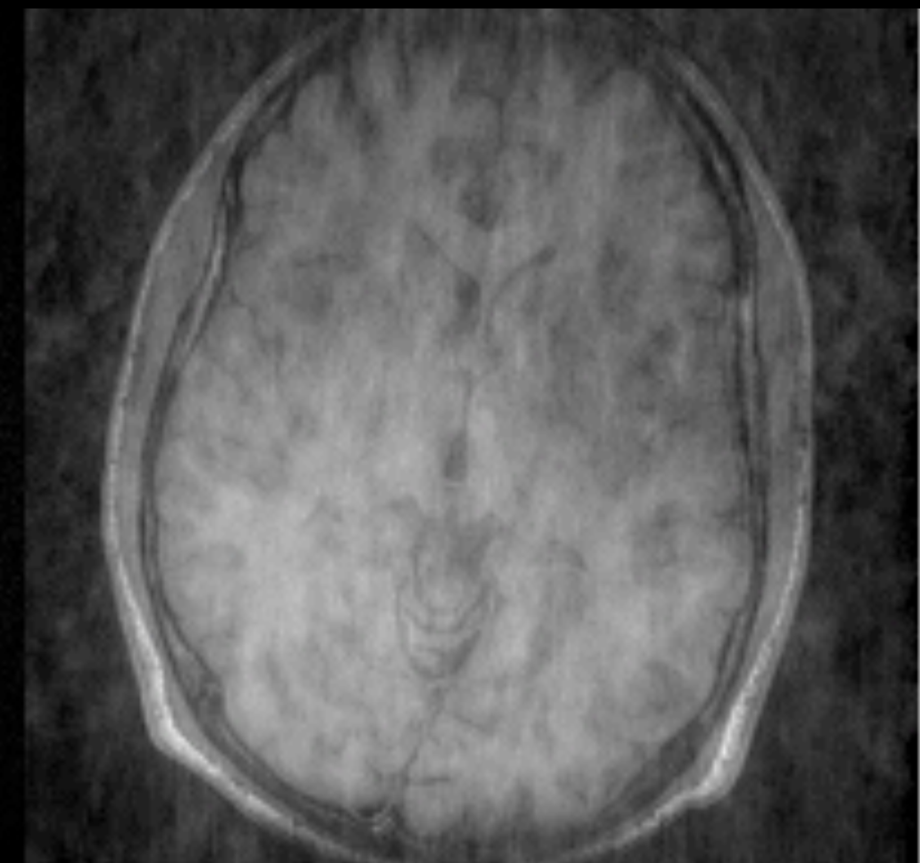
Original



x3 Calibrationless



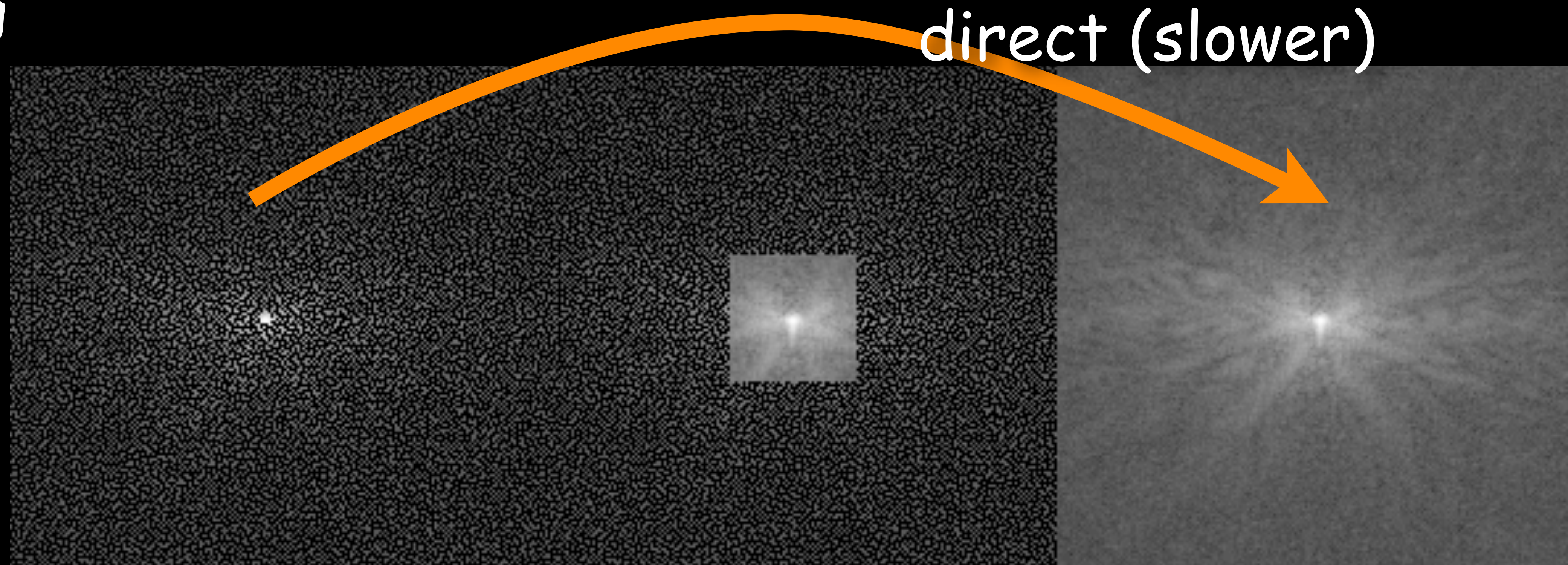
x3 zero-fill



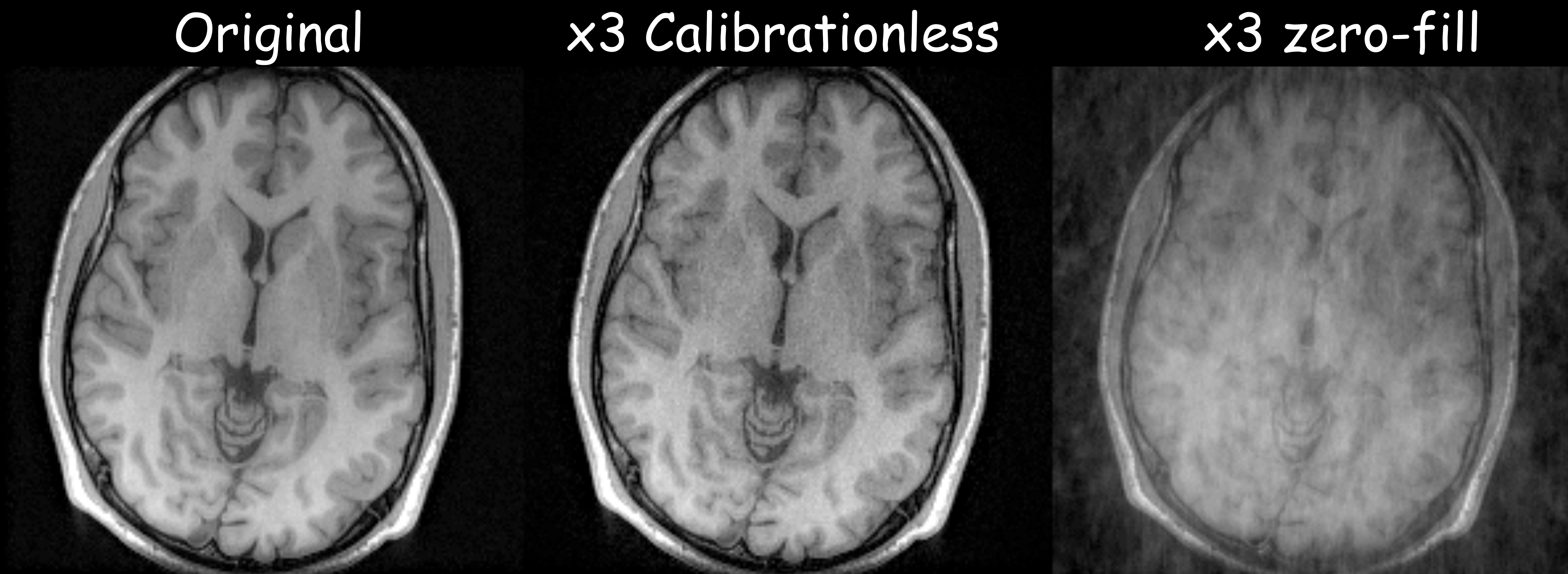
x3 undersampling

Poisson-Disc

8-chan head coil



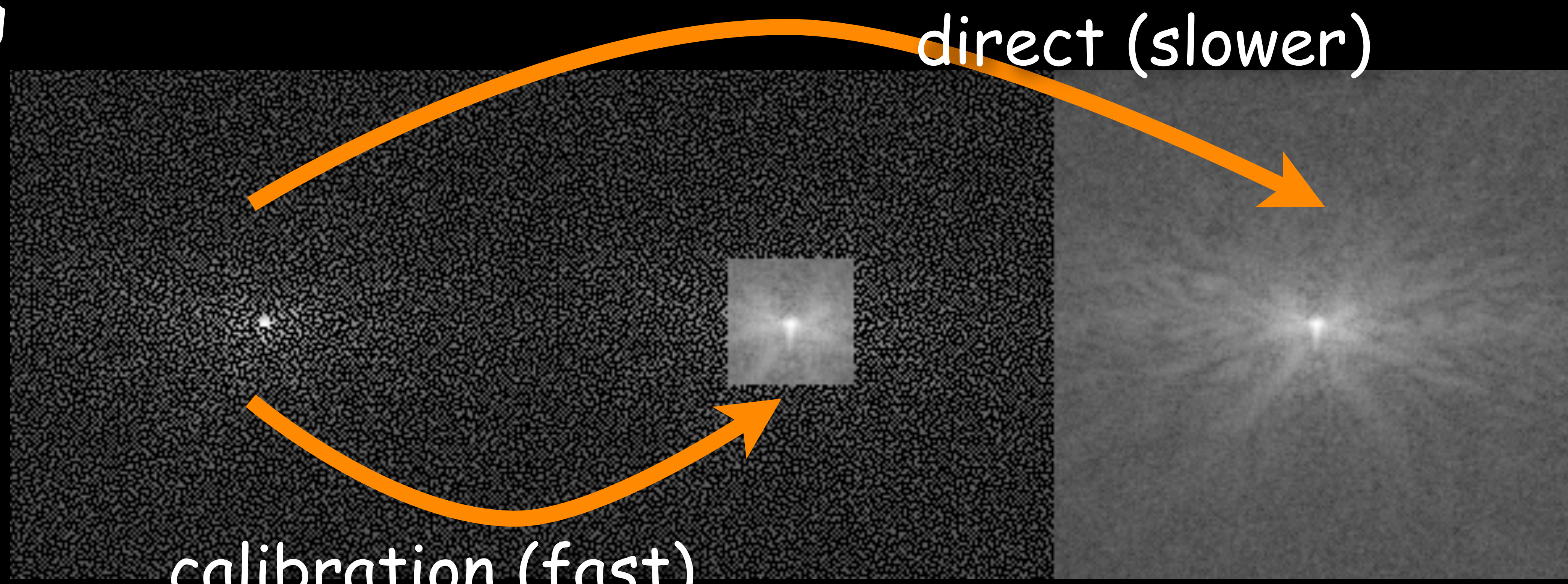
Results



x3 undersampling

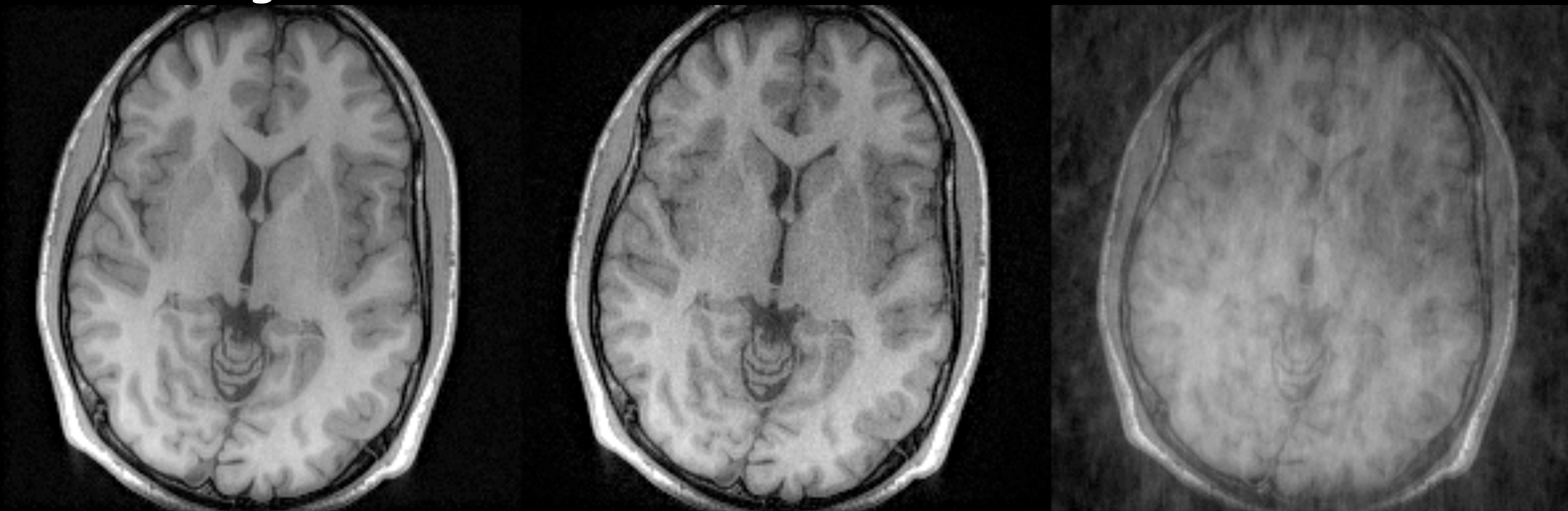
Poisson-Disc

8-chan head coil



Results

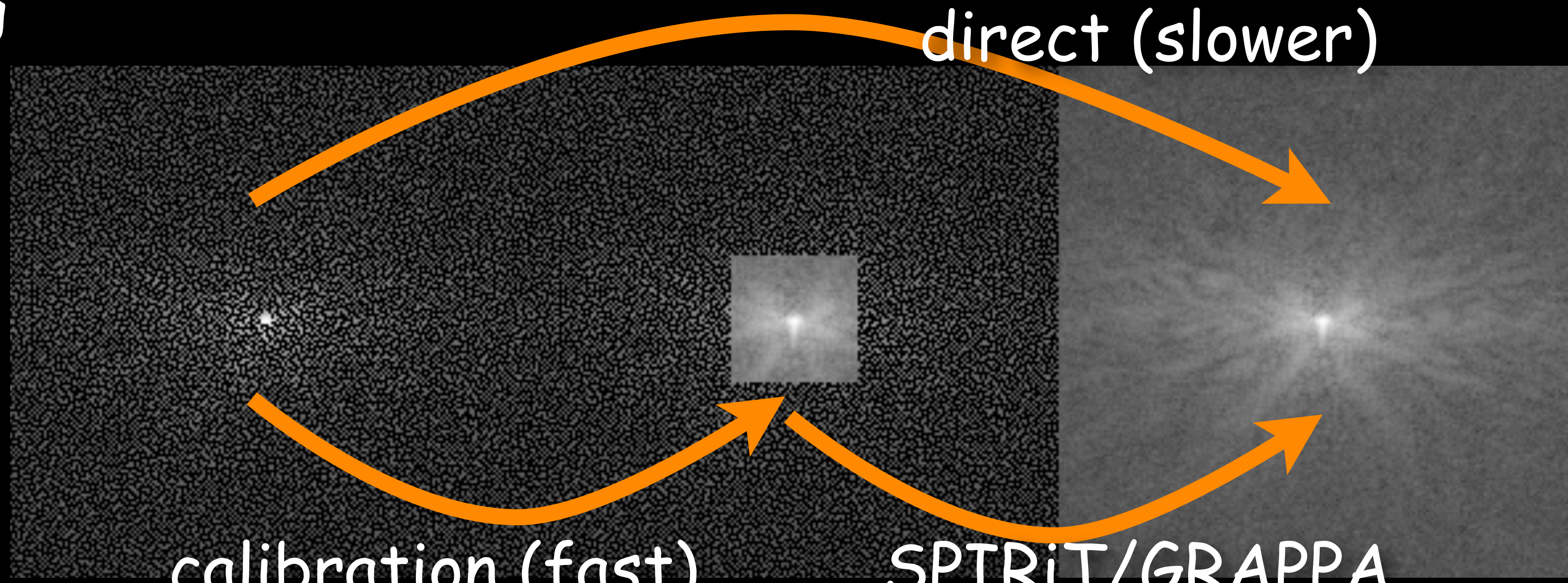
Original | x3 Calibrationless | x3 zero-fill



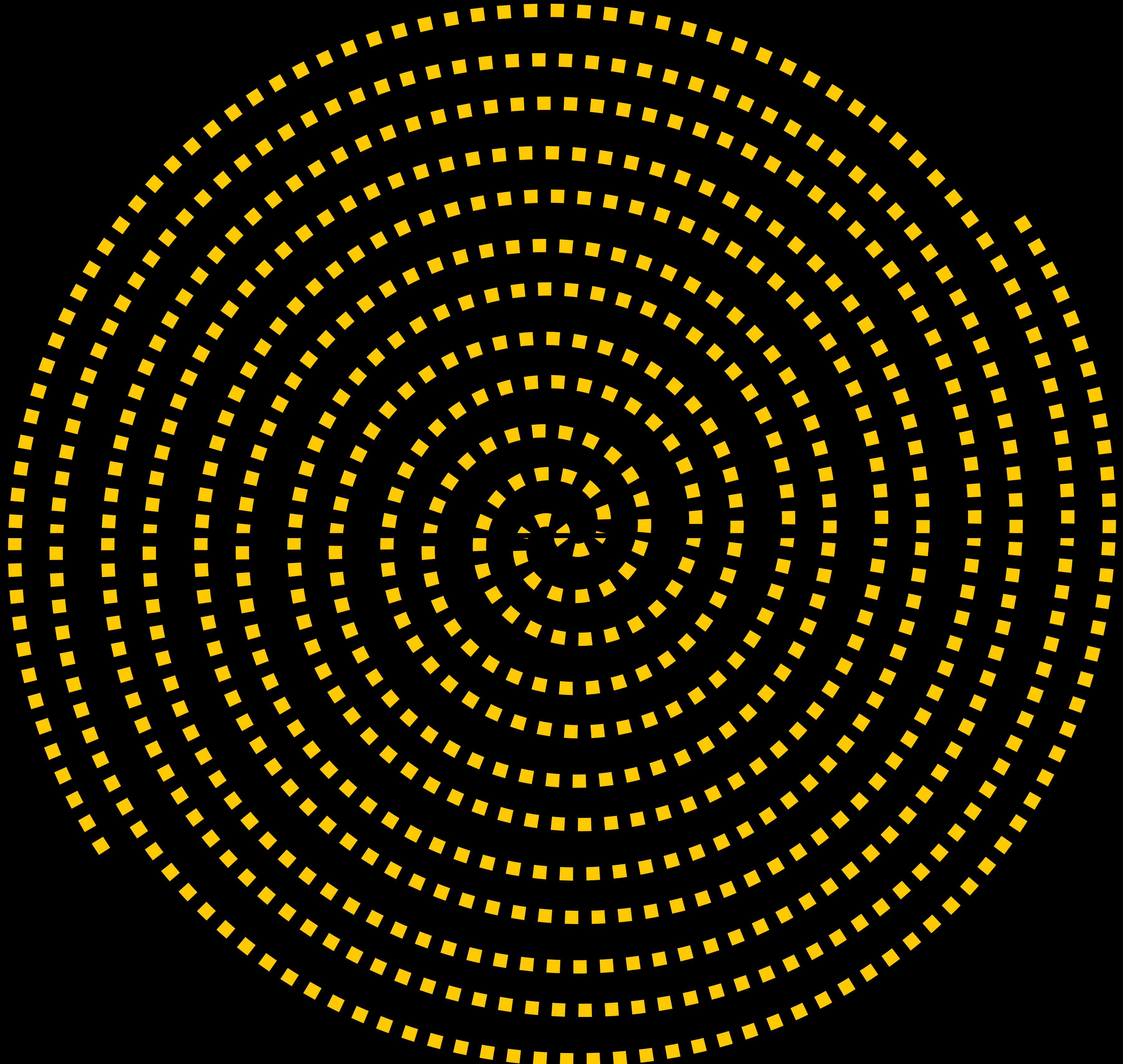
x3 undersampling

Poisson-Disc

8-chan head coil

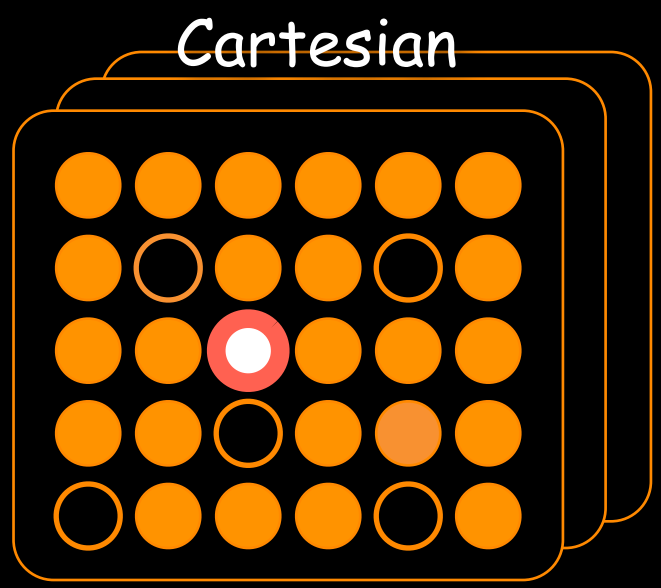


Post Cartesian

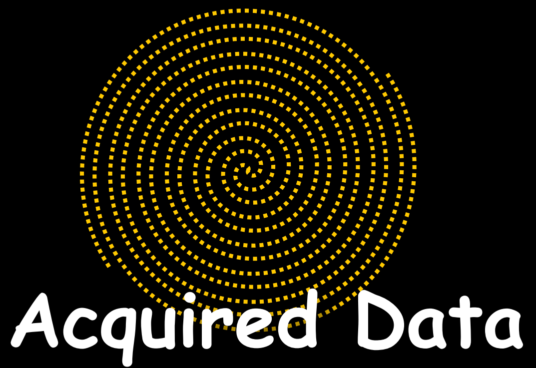


Post Cartesian

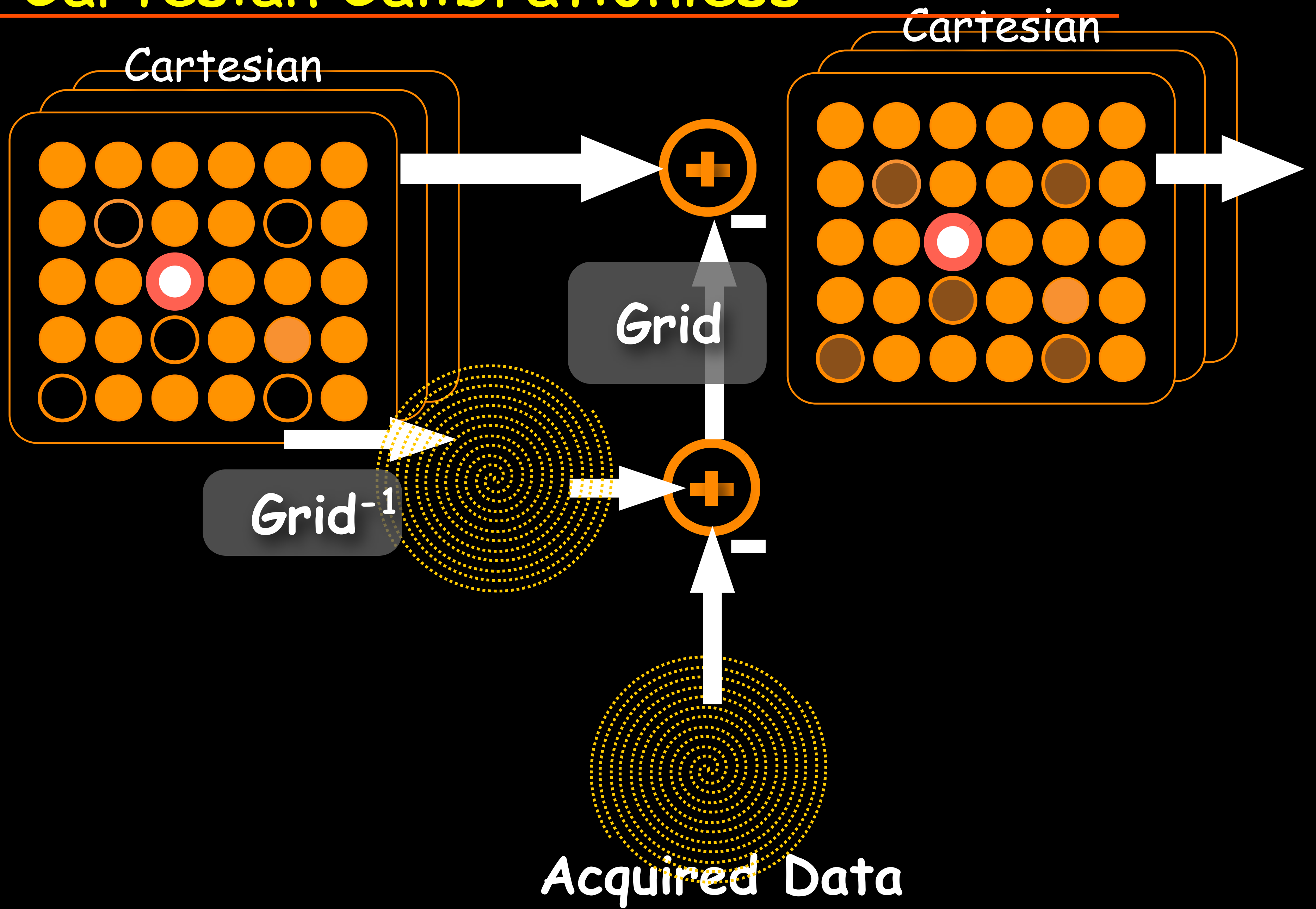
Post-Cartesian Calibrationless



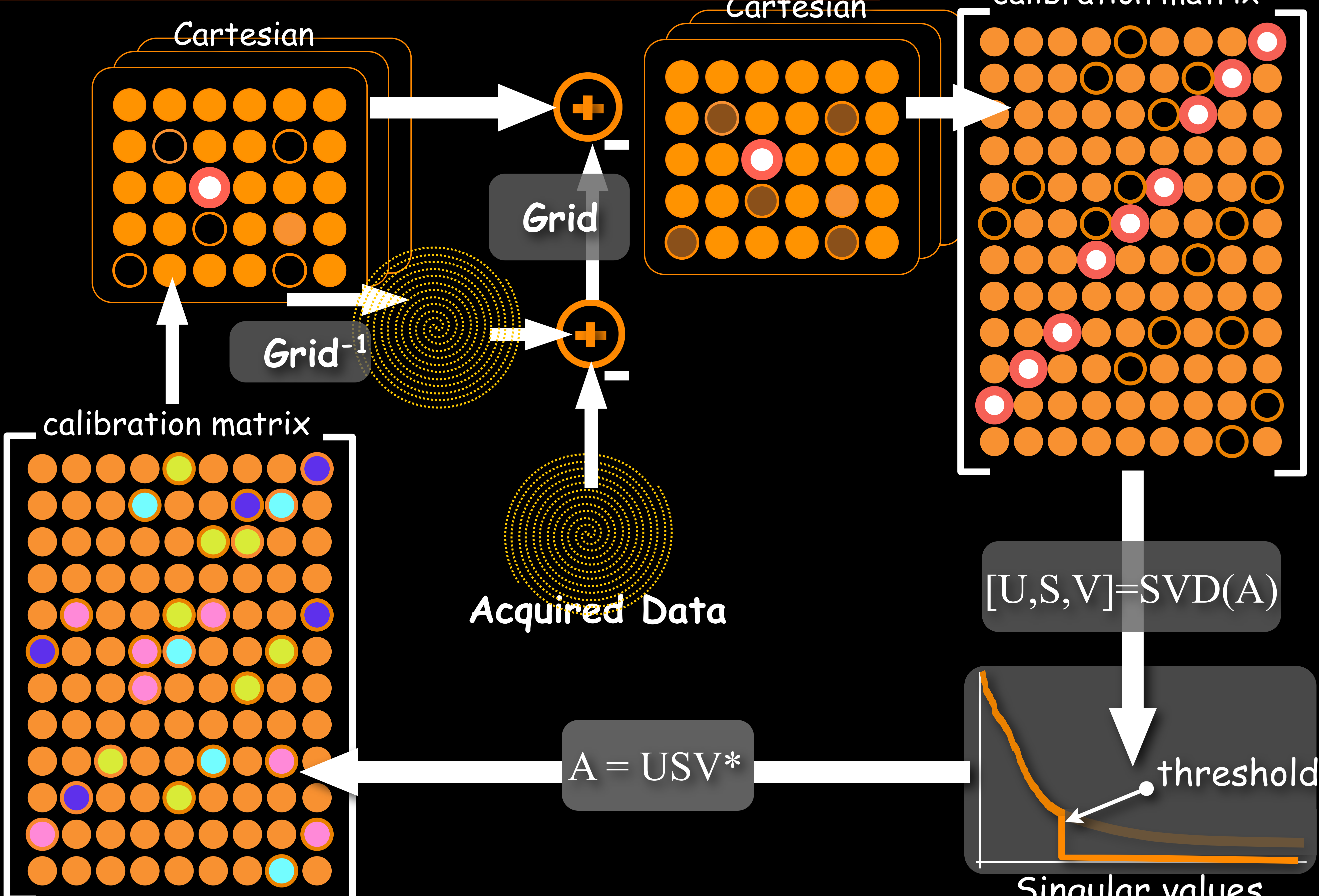
Data
Consistency?



Post-Cartesian Calibrationless



Post-Cartesian Calibrationless

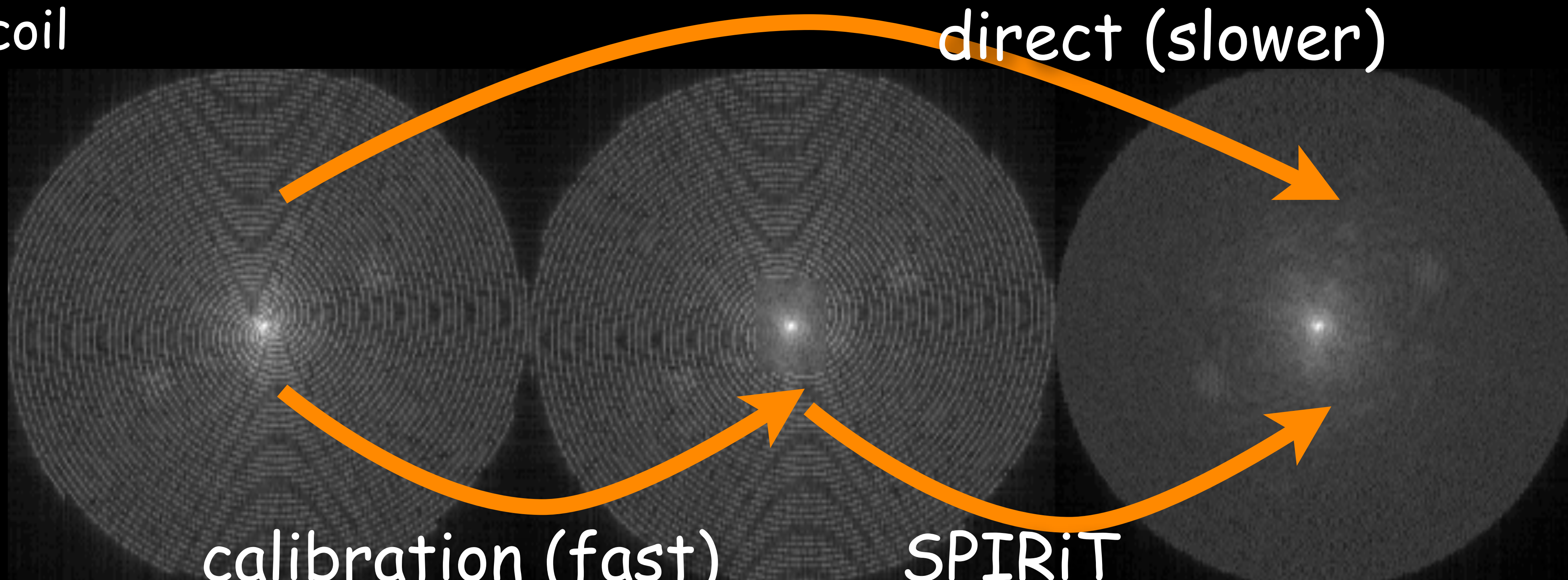


Calibrationless Post-Cartesian

3-fold undersampling

Uniform Spirals

8-chan Cardiac coil



Alternatives and Related

Magnetic Resonance in Medicine 57:1196–1202 (2007)

Joint Image Reconstruction and Sensitivity Estimation in SENSE (JSENSE)

Leslie Ying* and Jinhua Sheng

Magnetic Resonance in Medicine 63:1456–1462 (2010)

Nonlinear Inverse Reconstruction for Real-Time MRI of the Human Heart Using Undersampled Radial FLASH

Martin Uecker,* Shuo Zhang, and Jens Frahm

Magnetic Resonance in Medicine 59:903–907 (2008)

Iterative GRAPPA (iGRAPPA) for Improved Parallel Imaging Reconstruction

Tiejun Zhao and Xiaoping Hu*

Comments

- Low-rank completion is an extension of sparsity to matrices.
- Can also be thought of as blind system identification
- Here, rank is not extremely low
  - The Hankel structure is essential!
- Cadzow can fall to local minimum
  - Variable density helps **<u>A LOT</u>**

Acknowledgments

- John Pauly (Stanford)
- David Donoho (Stanford)
- Mark Murphy (UCB)
- Peter Shin (UCSF)
- Daniel Vigniron (UCSF)
- Peder Larson (UCSF)
- Anja Brau (GE ASL west)
- Peng Lei (GE ASL west)
- Michael Elad (Technion)

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NIH R01EB009690, R01EB007588, RR09794-15

UC Discovery Grant 193037

GE Healthcare

Thank you!

תודה רבה