

ILIAS-Double Beta Decay Meeting

Paris, November 19-20, 2007

Anatomy of the $0\nu\beta\beta$ -decay NME's

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and

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OUTLINE

- Recent results of the Bratislava-Caltech-Tuebingen, Strasbourg-Madrid, and Jyvaskyla-La Plata groups are discussed
- Anatomy of the $0\nu\beta\beta$ -decay NME is performed: two-nucleon s.r.c., pairing decomposition, higher order currents, r_{12} and q -dependences of $M^{0\nu}$...

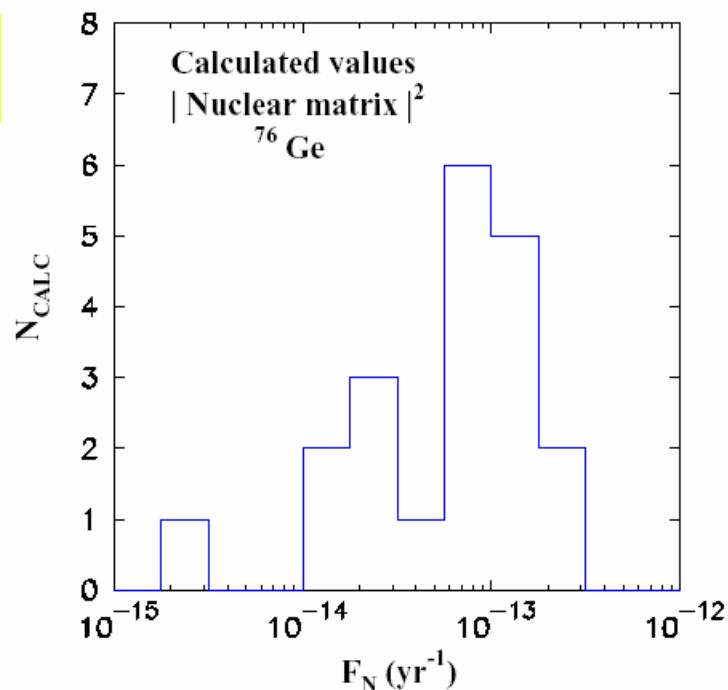
F. Š., A. Faessler V.Rodin,, P. Vogel, J. Engel, arXiv 0710.2055 (2007)

The situation is much better today!

Bahcall, Murayama, Pena-Garay,
Phys. Rev. D 70, 033012 (2004)

Authors put different NME calculations
at the same level

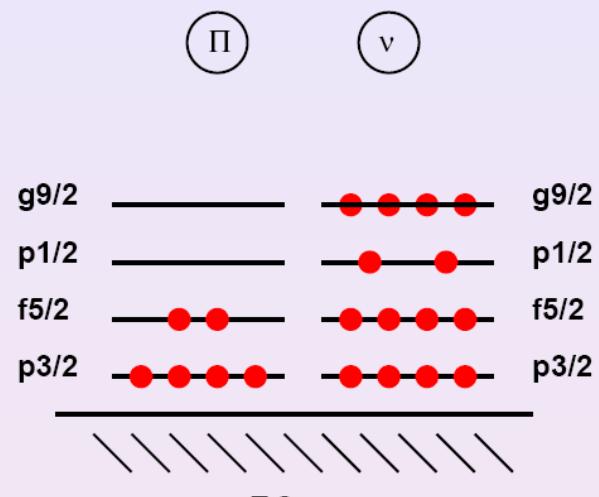
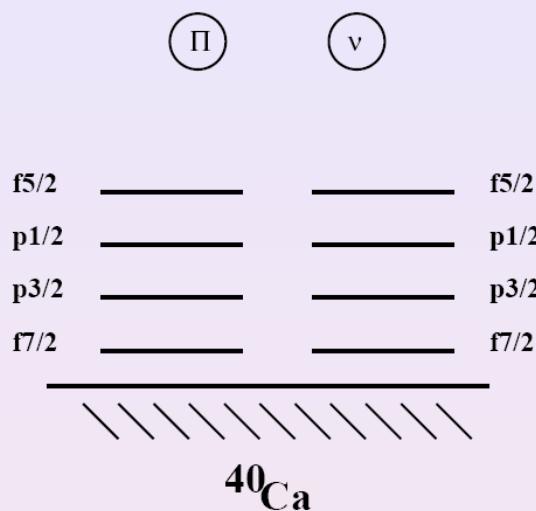
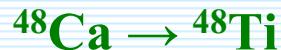
This suggest
an uncertainty
of NME
as much as
factor 5!



System	$G_1^{(0\nu)} \times 10^{14}$	N.M.E.	N.M.E. (this work)	$\langle m_\nu \rangle_{\text{max}}$
⁴⁸ Ca	6.43	1.08–2.38		8.70–19.0
⁷⁶ Ge	0.63	2.98–4.33	3.33	0.30–0.43
⁸² Se	2.73	2.53–3.98	3.44	4.73–7.44
⁹⁶ Zr	5.70	2.74	3.55	19.1–24.7
¹⁰⁰ Mo	4.57	0.77–4.67	2.97	2.18–13.2
¹¹⁶ Cd	4.68	1.09–3.46	3.75	2.37–8.18
¹²⁸ Te	0.16	2.51–4.58		9.51–17.4
¹³⁰ Te	4.14	2.10–3.59	3.49	1.87–3.20
¹³⁶ Xe	4.37	1.61–1.90	4.64	0.79–2.29

Shell Model

- Define a valence space
- Derive an effective interaction $H \Psi = E \Psi \rightarrow H_{\text{eff}} \Psi_{\text{eff}} = E \Psi_{\text{eff}}$
- Build and diagonalize Hamiltonian matrix (10^{10})
- Transition operator $\langle \Psi_{\text{eff}} | O_{\text{eff}} | \Psi_{\text{eff}} \rangle$
- Some phenomenological input needed
energy of states, systematics of $B(E2)$ and GT transitions (quenching f.)



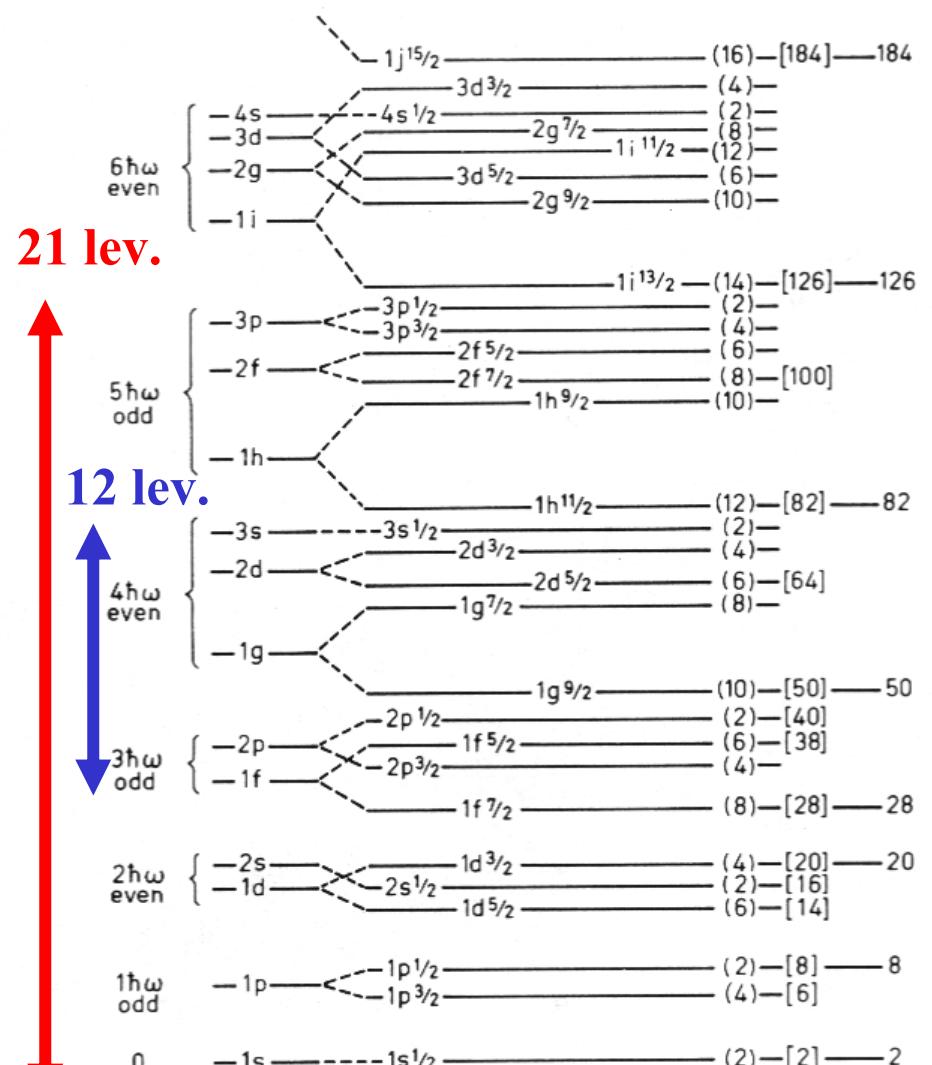
Small calculations

Fedor Simkovic

$^{76}\text{Se}_{42}$ in the valence
6 protons and 14 neutrons

QRPA

$2\nu\beta\beta$ -decay NME

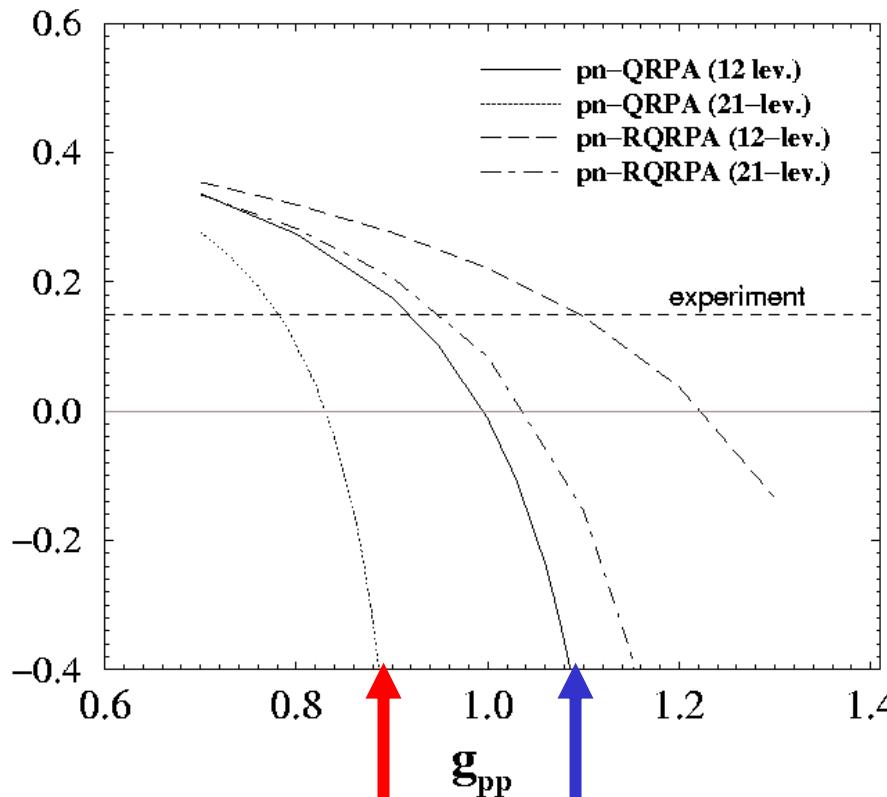


Only BCT group

Fedor Simkovic

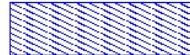
$$H = H_0 + g_{ph} H_{ph} + g_{pp} H_{pp}$$

${}^{76}\text{Ge} \rightarrow {}^{76}\text{Se}$



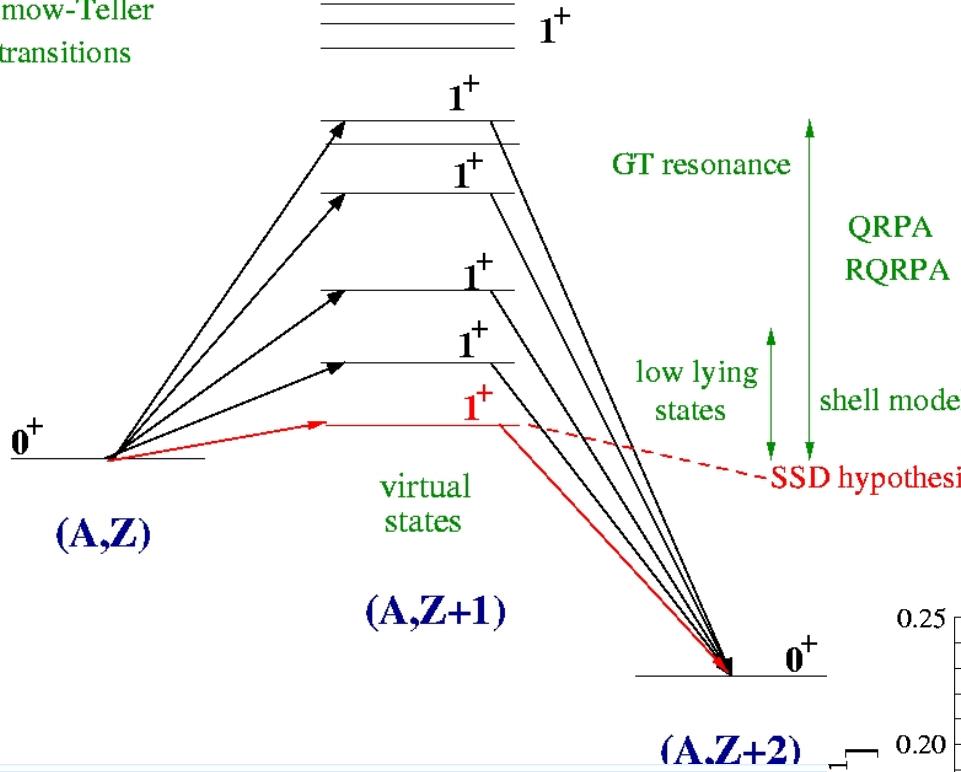
Collapse of the QRPA
21 l.m.s. 12 l.m.c

$2\nu\beta\beta$ -decay



Continuum states

Gamow-Teller transitions



$$M_{GT}^{2\nu} = \sum_m \frac{<0_f^+||\tau^+\sigma||1_m^+><1_m^+||\tau^+\sigma||0_i^+>}{E_m - E_i + \Delta}$$

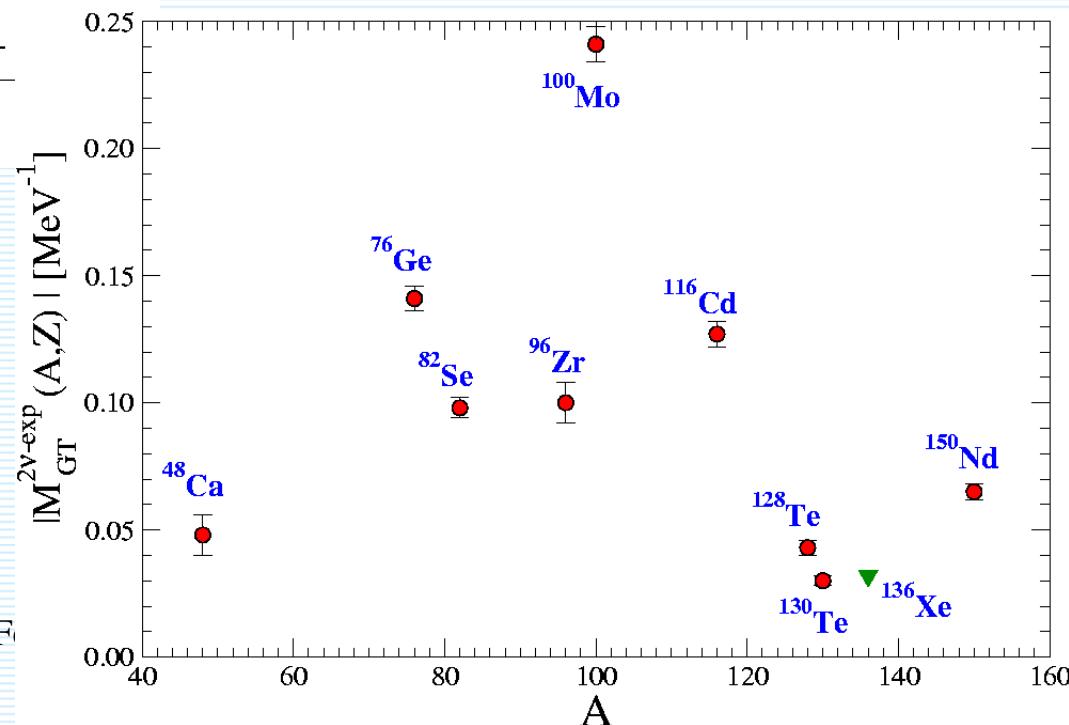
difference: by factor ~ 10

OEM

$2\nu\beta\beta$ -decay nuclear matrix elements

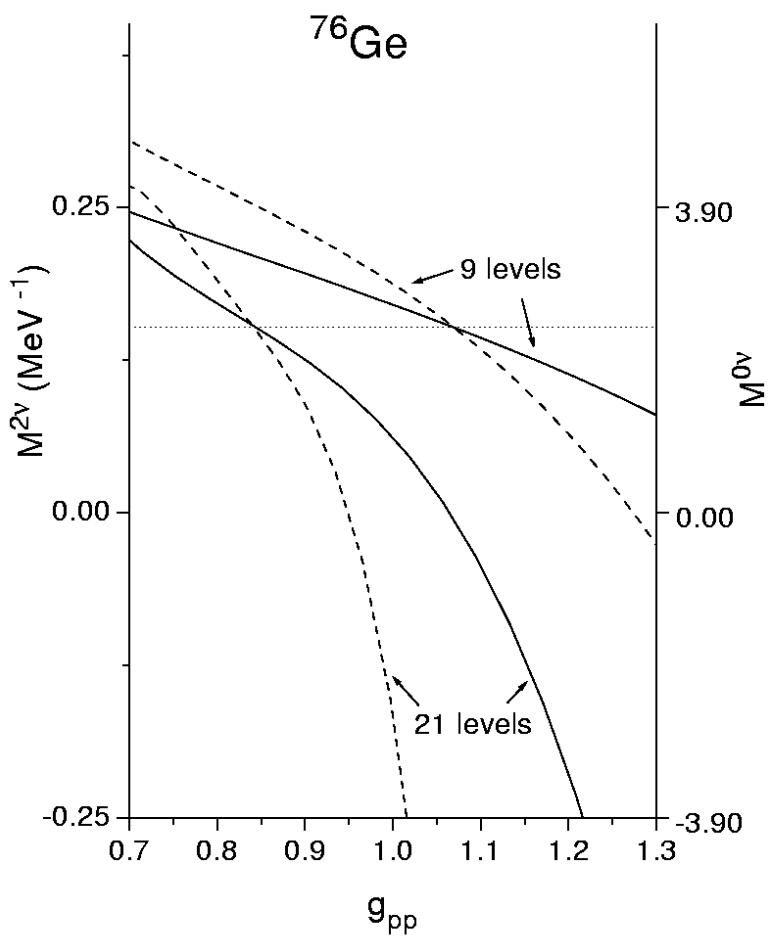
$$(T_{1/2}^{2\nu})^{-1} = G^{2\nu} |M_{GT}^{2\nu}|^2$$

Deduced from measured $T_{1/2}^{2\nu}$

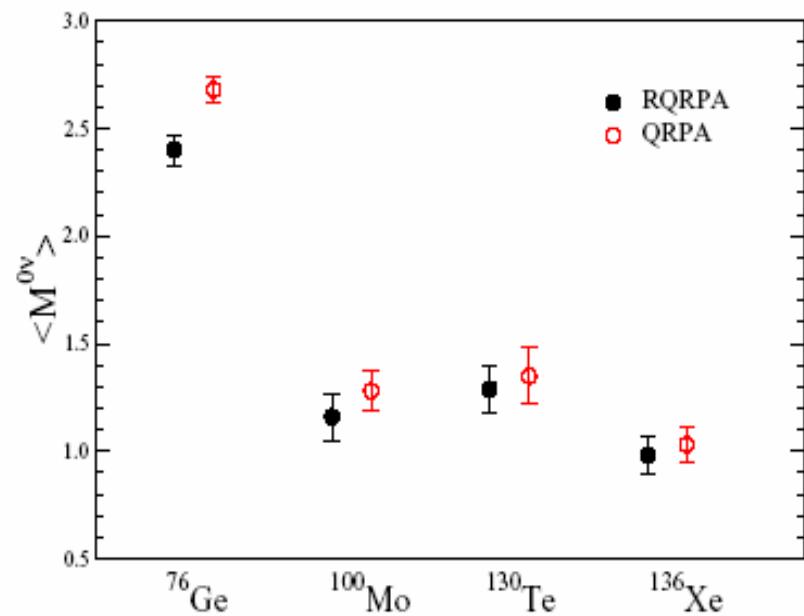


The $0\nu\beta\beta$ -decay NME: g_{pp} fixed to $2\nu\beta\beta$ -decay

Each point: (3 basis sets) \times (3 forces) = 9 values



By adjusting of g_{pp} to $2\nu\beta\beta$ -decay half-life
the dependence of the $0\nu\beta\beta$ -decay NME on
other things that are not a priori fixed
is essentially removed



Rodin, Faessler, Šimkovic, Vogel,
Phys. Rev. C 68, 044302 (2003)

Scheme of the calculation of the $0\nu\beta\beta$ -decay NME

F.Š., G. Pantis, J.D. Vergados, A.Faessler, Phys. Rev. C 60, 055502 (1999)

The $0\nu\beta\beta$ -decay half-life

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z) |M'^{0\nu}|^2 |\langle m_{\beta\beta} \rangle|^2 ,$$

NME= sum of Fermi, Gamow-Teller
and tensor contributions

$$M'^{0\nu} = \left(\frac{g_A}{1.25} \right)^2 \langle f | - \frac{M_F^{0\nu}}{g_A^2} + M_{GT}^{0\nu} + M_T^{0\nu} | i \rangle$$

Neutrino potential

$$H_K(r_{12}) = \frac{2}{\pi g_A^2} R \int_0^\infty f_K(qr_{12}) \frac{h_K(q^2) q dq}{q + E^m - (E_i + E_f)/2}$$

Finite nucleon size

$$f_{F,GT}(qr_{12}) = j_0(qr_{12}), \quad f_T(qr_{12}) = -j_2(qr_{12})$$

Higher order terms
of nucleon current
(hoc)

Form-factors:

(finite nucleon size)

$$h_F = g_V^2(q^2)$$

$$h_{GT} = g_A^2 \left[1 - \frac{2}{3} \frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} + \frac{1}{3} \left(\frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} \right)^2 \right]$$

$$h_T = g_A^2 \left[\frac{2}{3} \frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} - \frac{1}{3} \left(\frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} \right)^2 \right]$$

$$M_{K=F,GT,T} = \sum_{J^\pi, k_i, k_f, \mathcal{J}} \sum_{pn p' n'} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \sqrt{2\mathcal{J}+1} \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{Bmatrix} \langle p(1), p'(2); \mathcal{J} || f(r_{12}) O_K f(r_{12}) || n(1), n'(2); \mathcal{J} \rangle$$

Two-nucleon s.r.c.

$$\times \langle 0_f^+ || [\tilde{c}_{p'}^\dagger \tilde{c}_{n'}]_J || J^\pi k_f \rangle \langle J^\pi k_f | J^\pi k_i \rangle \langle J^\pi k_f | [c_p^+ \tilde{c}_n]_J || 0_i^+ \rangle$$

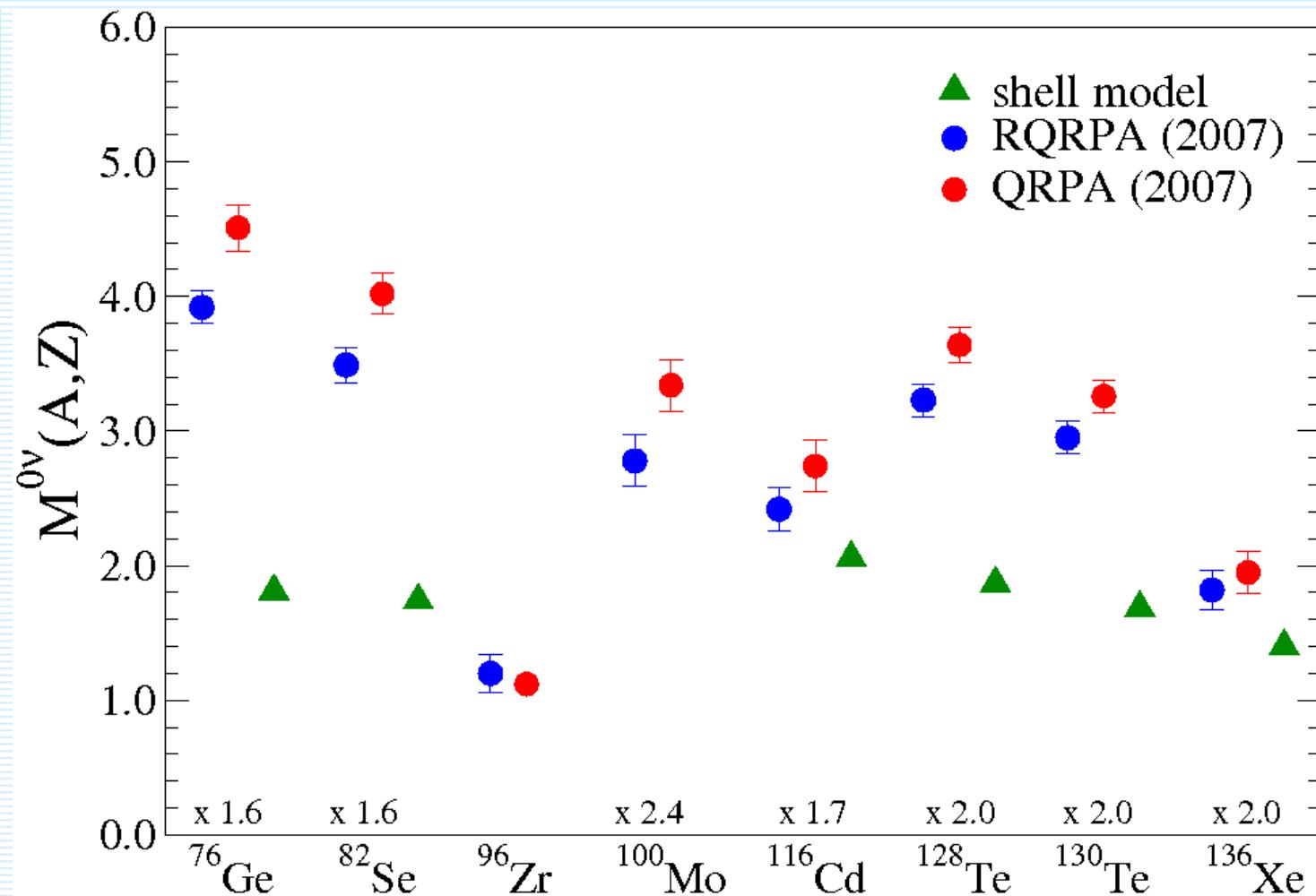
QRPA-like (2007) and NSM (2005) results

$g_A = 1.25$

Jastrow s.r.c.

NSM predicts
practically
the same value
for all NME's

NSM values
calculated
without
hoc

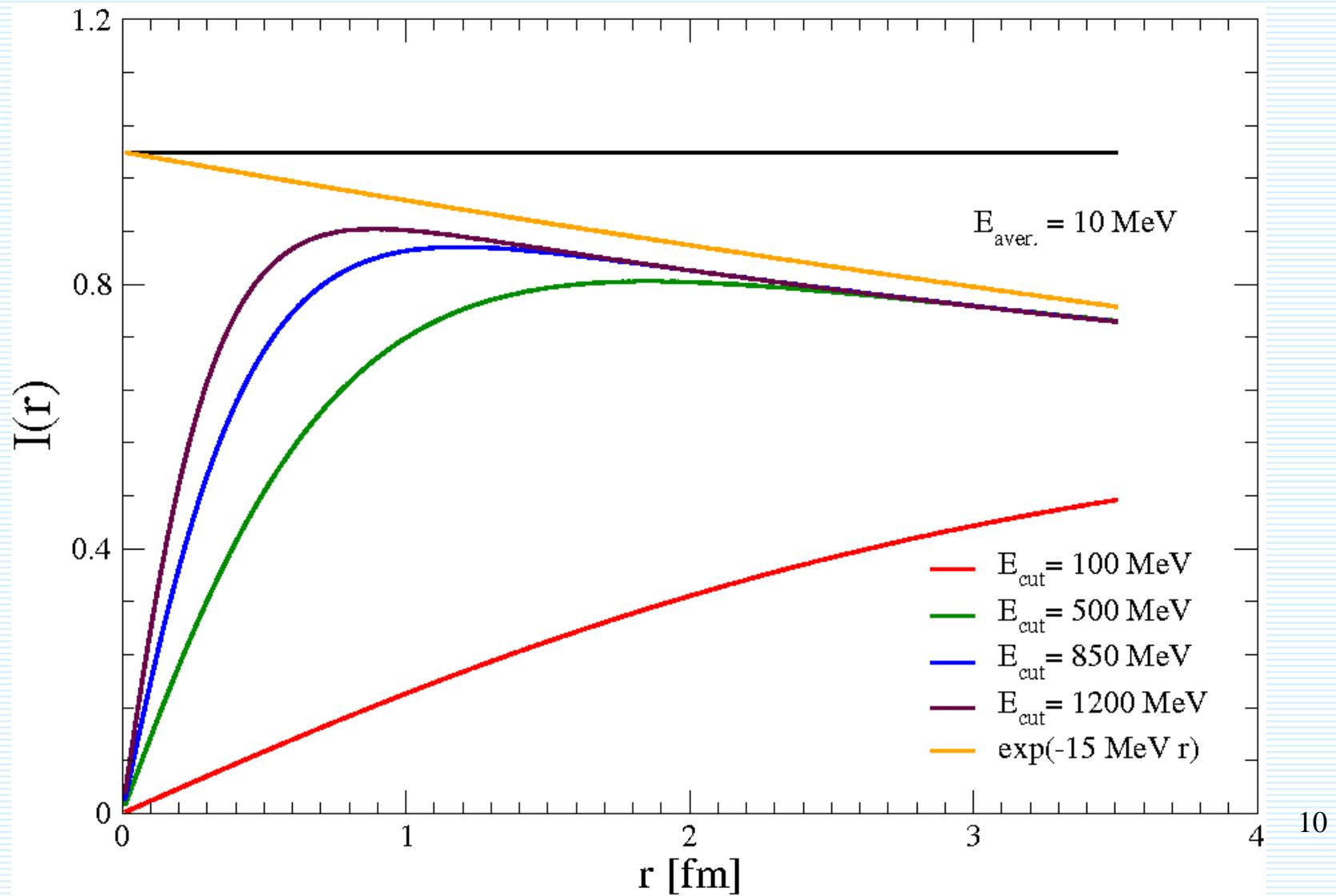


QRPA, RQRPA: V.Rodin, A. Faessler, F. Š., P. Vogel, Nucl. Phys. A 793, 213 (2007)
shell model: E. Caurier et al. Rev. Mod. Phys. 77, 427 (2005).

Finite nucleon size (formfactors) versus short range correlations.

Neutrino potential: $I(r)/r$

$$I(r) = \frac{2}{\pi} \int_0^\infty \frac{\sin(qr)}{(q + E_{\text{aver}})} \frac{dq}{(1 + q^2/E_{\text{cut}}^2)^4}$$

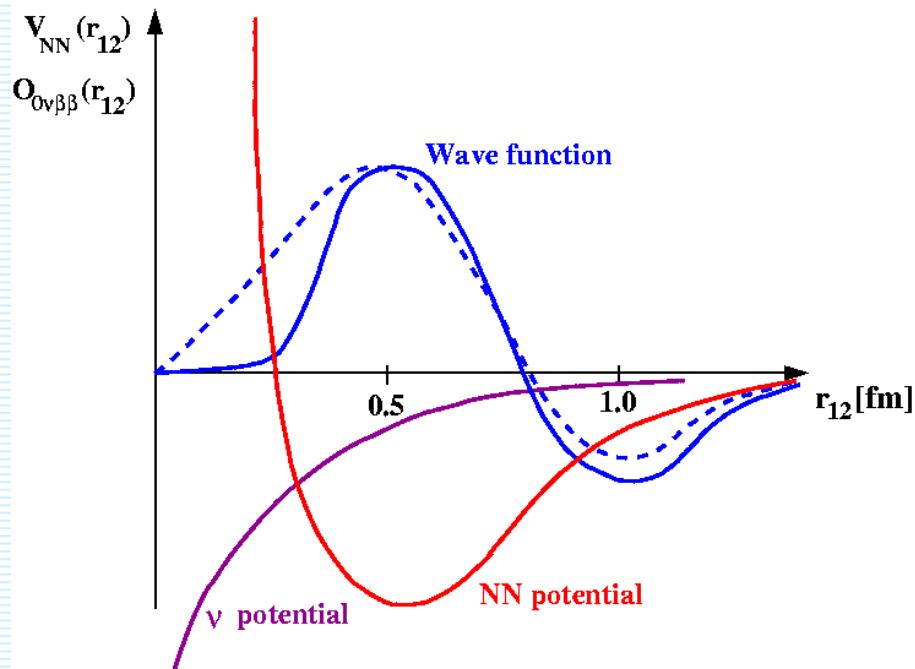


Two-nucleon short range correlations:

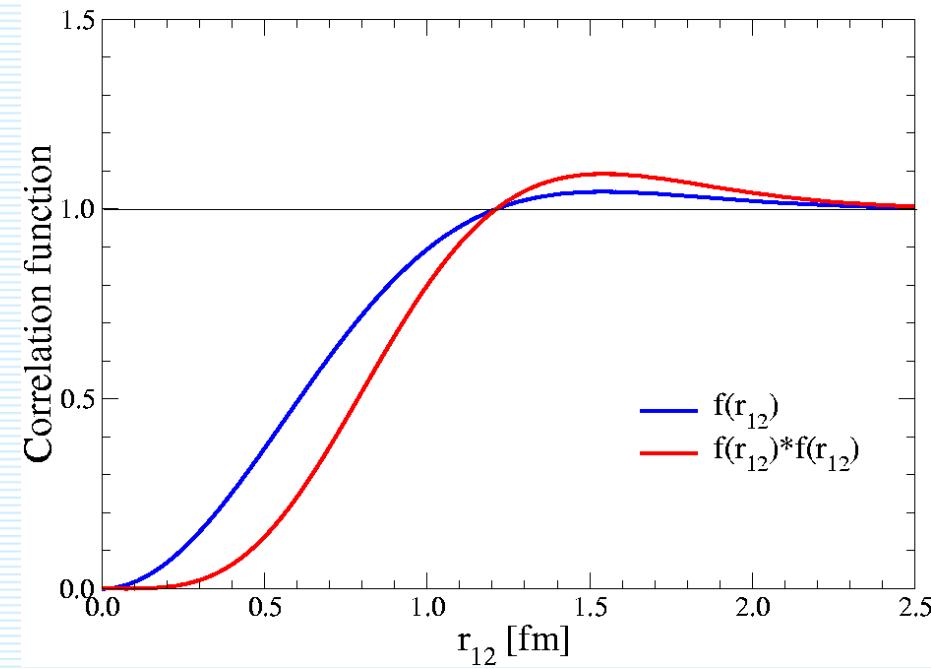
$$|\Psi\rangle_{corr.} = f_{Jastrow}(r) |\Psi\rangle_{uncorr.}$$

$$f_{Jastrow}(r) = e^{ar^2} (1 - br^2)$$

Nucleon–Nucleon Potential



Jastrow function $f(r_{12})$



0νββ-decay NME with UCOM s.r.c.

Korteleinen, Civitarese,
Suhonen, Toivanen,
Phys. Lett. B 647, 128 (2007)



	bare	Jastrow	UCOM Bonn-A
$M_{GT}^{(0\nu)}$	-6.755	-4.681	-6.265
$M_F^{(0\nu)}$	2.474	1.778	2.310
total	-8.328	-5.811	-7.734

v-potential: Closure approximation,
Finite nucleon size not considered

Feldmeier et al.: UCOM corelations

introduce correlations by means of a unitary transformation
with respect to the relative coordinate of all pairs

$$< nl(LS)JT | C_p C_\Omega H C_p C_\Omega | n'l'(L'S')JT >$$

$$C = \exp[-iG] = \exp[i \sum_{i < j} g_{ij}]$$

$$= < nl(LS)JT | T + U_{UCOM} | n'l'(L'S')JT >$$

$$|\Psi>_{corr.} = C |\Psi>_{uncorr.}$$

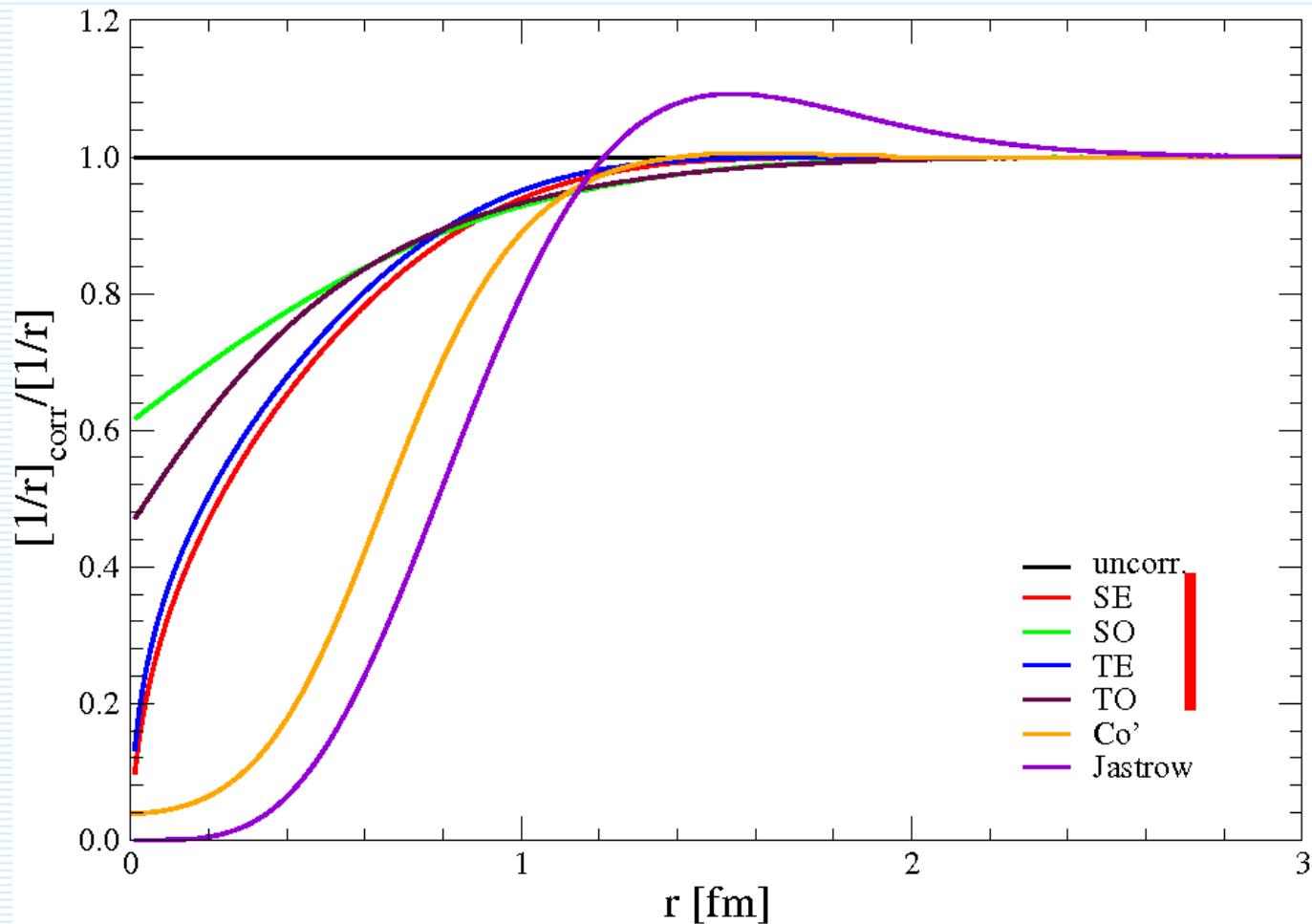
$$G^+ = G, \quad C^+ C = 1, \quad g = g(\vec{r}, \vec{q}, \vec{\sigma}_1, \vec{\sigma}_2, \vec{\tau}_1, \vec{\tau}_2)$$

$$\tilde{O}_{corr.} = C^+ \tilde{O}_{uncorr.} C$$

UCOM correlations:

- determined by two-body energy minimization
- V_{UCOM} and V_{bare} are phase shift equivalent

Comparison of UCOM and Spencer-Miller (Jastrow) s.r.c. (coulombic potential $\sim 1/r$)



UCOM

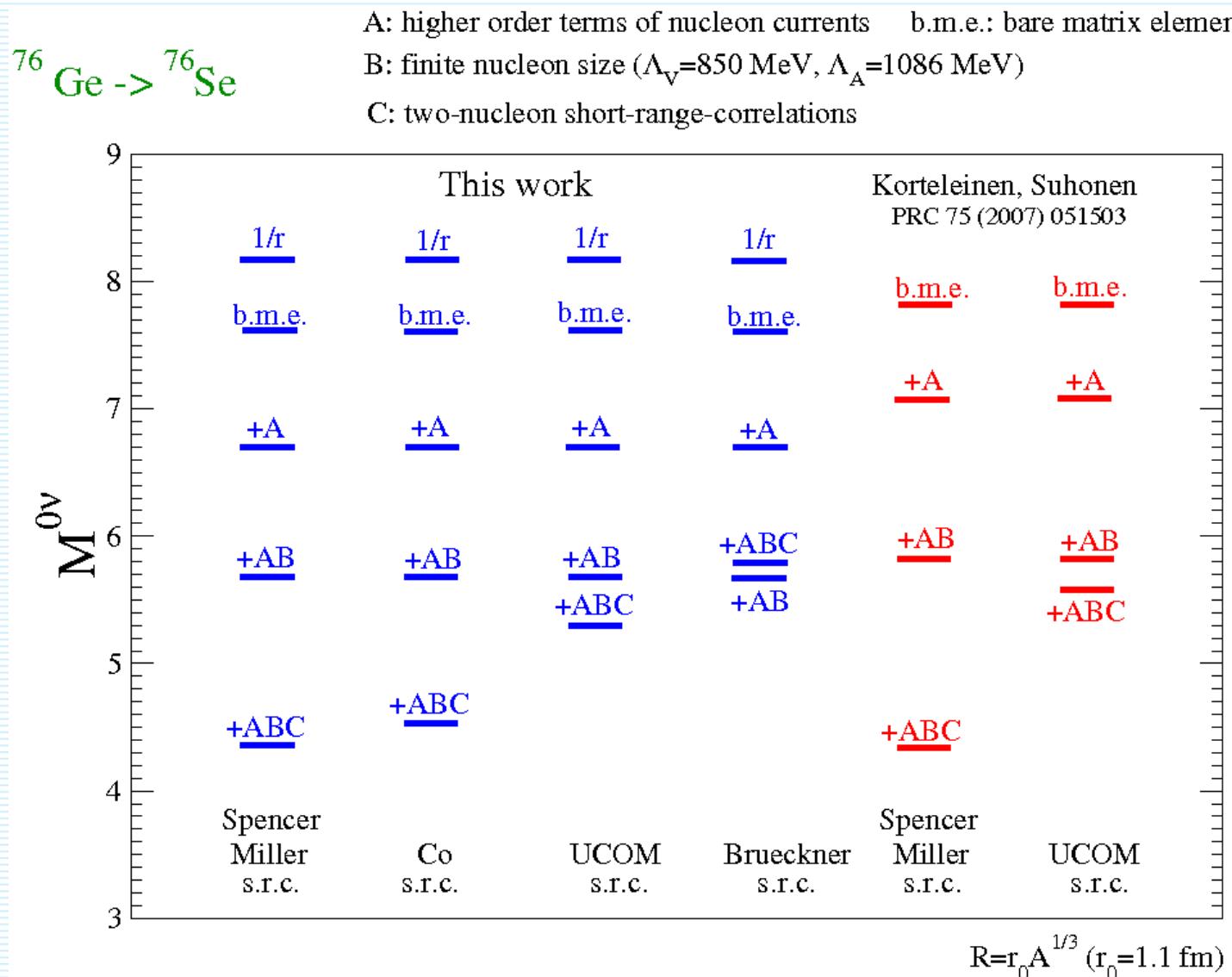
Spencer-Miller and Co s.r.c. (Jastrow like) the same for all channels $^{2S+1}L_J$
 UCOM s.r.c. different for singlet-even (SE), singlet-odd (SO), triplet-even (TE)
 and triplet-odd (TO) channels

Jyvaskyla group:

Changed of the formalism to our (new code), but no tensor term

Given for
 $r_0=1.1$ fm!

10 level model
space,
 $g_{pp}=1.00$

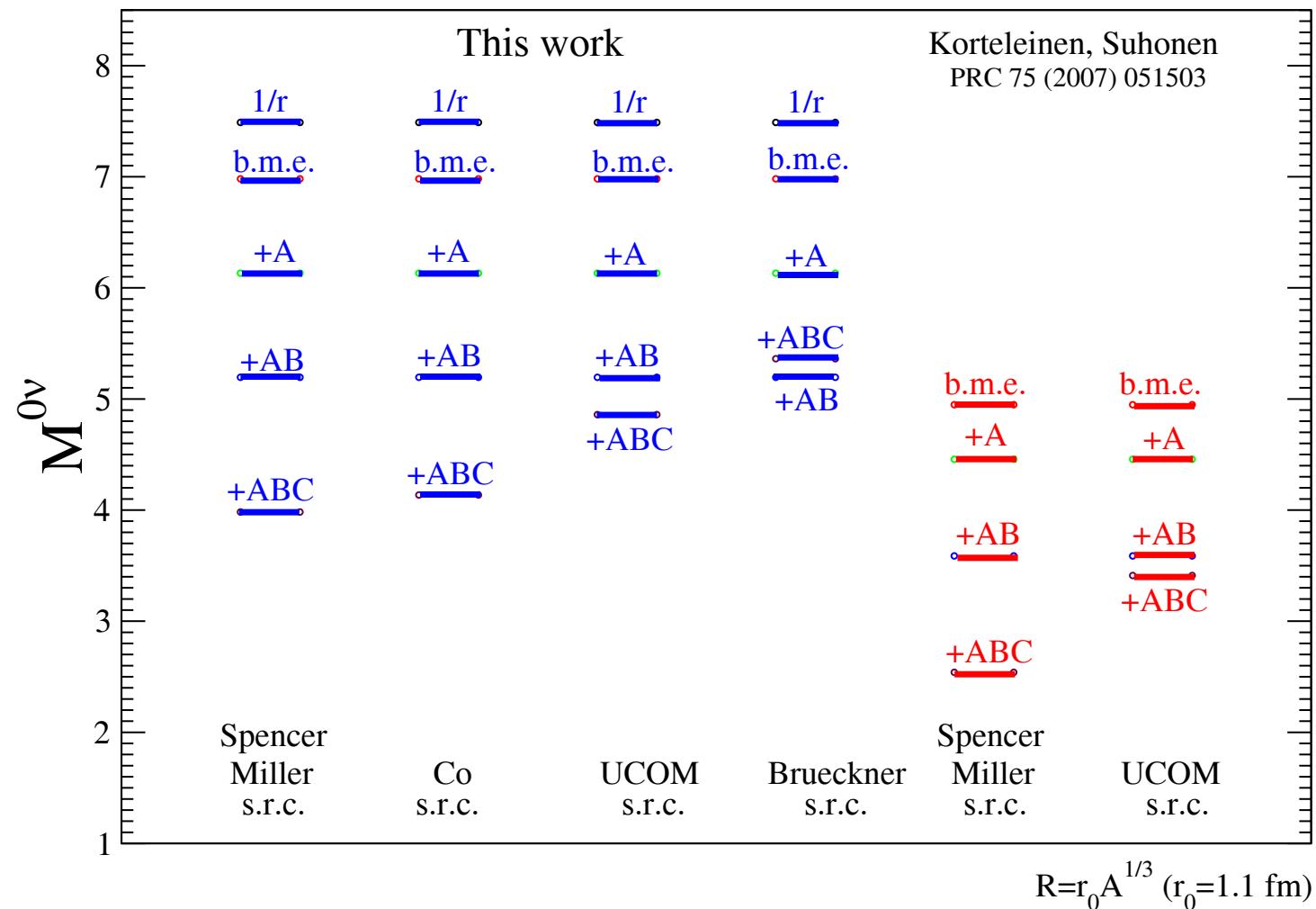


Jyvaskyla group:

Changed of the formalism to our (new code), but no tensor term

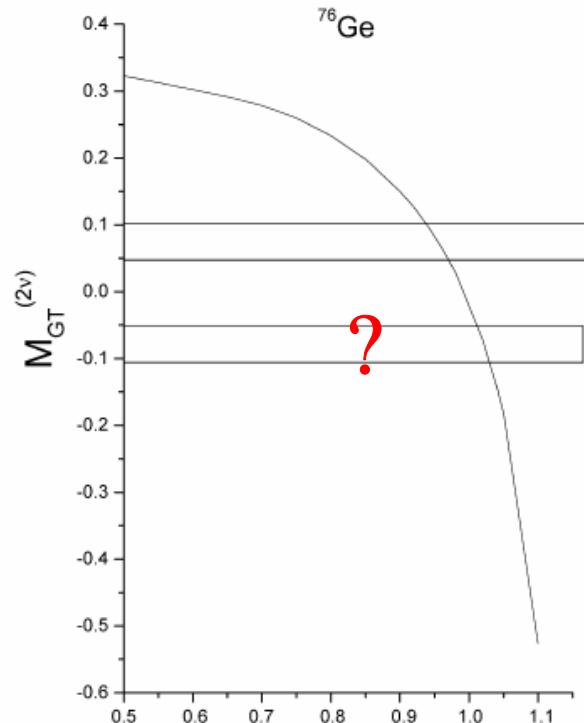


A: higher order terms of nucleon currents b.m.e.: bare matrix element
 B: finite nucleon size ($\Lambda_V=850$ MeV, $\Lambda_A=1086$ MeV)
 C: two-nucleon short-range-correlations



Given for
 $r_0=1.1$ fm!
 10 level modes
 space

Some comments on calculations of Jyvaskyla group



A clear message
is needed, what is correct?

11/23/2007

Before 2007:

- Fixing of g_{pp} to $2\nu\beta\beta$ -half lives is bad, fixing of g_{pp} to single β -decay observables is preferable
- Negative value of M_{GT} should be considered
- s.r.c. should not be taken into account (Durham meetings)
- formalism of Khadkikhar quark model (formfactors not presented)

Civitarese, Suhonen, NPA 761 (2005) 313; PLB 626 (2005) 80; Suhonen, NPA 752 (2005) 53

...

2007:

- Fixing of g_{pp} to $2\nu\beta\beta$ -half lives
 - Negative value of M_{GT} is not considered
 - s.r.c. are considered
 - Using our formalism with h.o.t of nucl. current
- Korteleinen, Civitarese, Suhonen, Toivainen,
PLB 647 (2007) 128; Korteleinen, Suhonen,
PRC 75 (2007) 051303

Origin of differences is unknown!

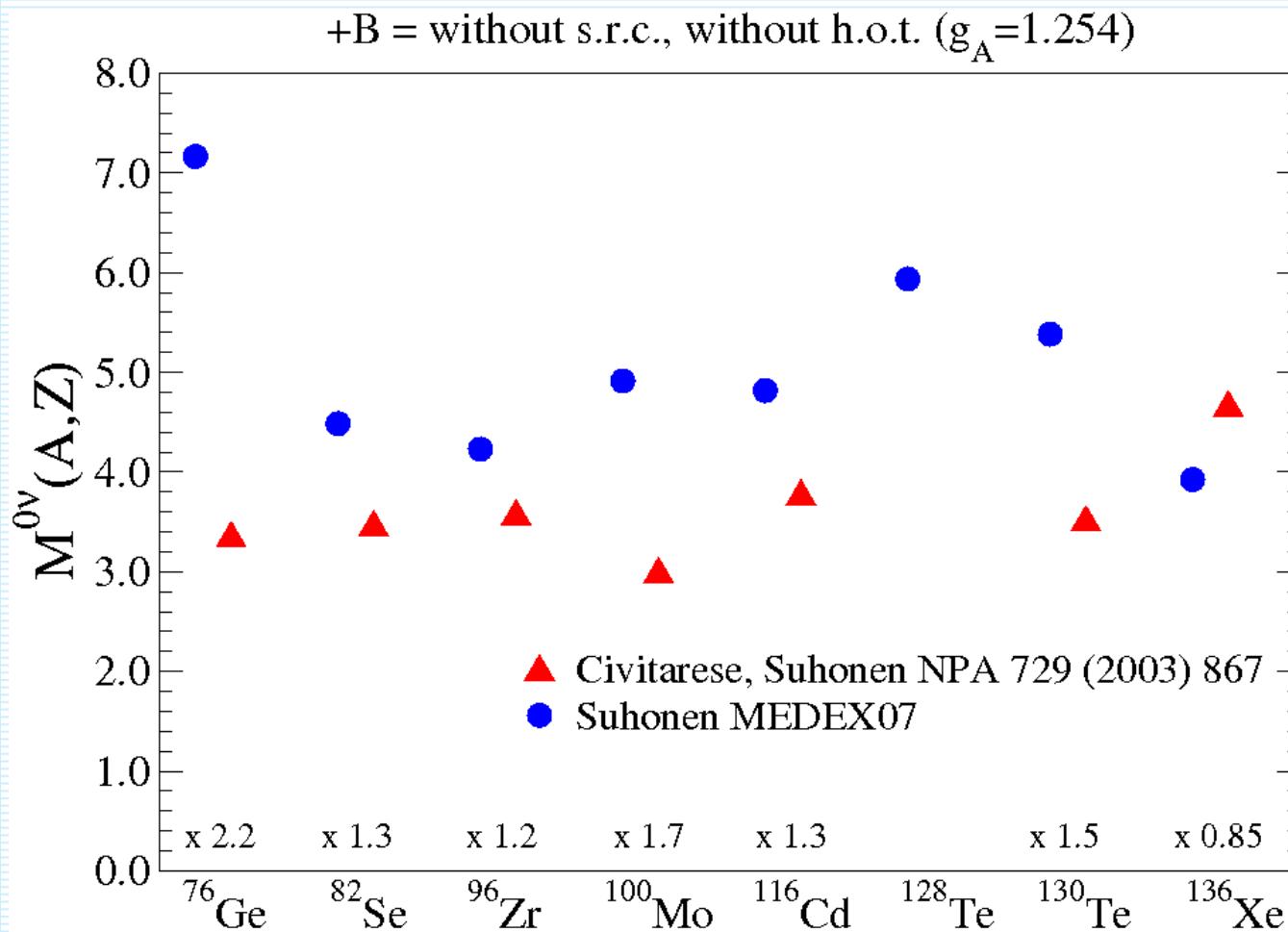
Quark model approach

of Khadkikhar ▲

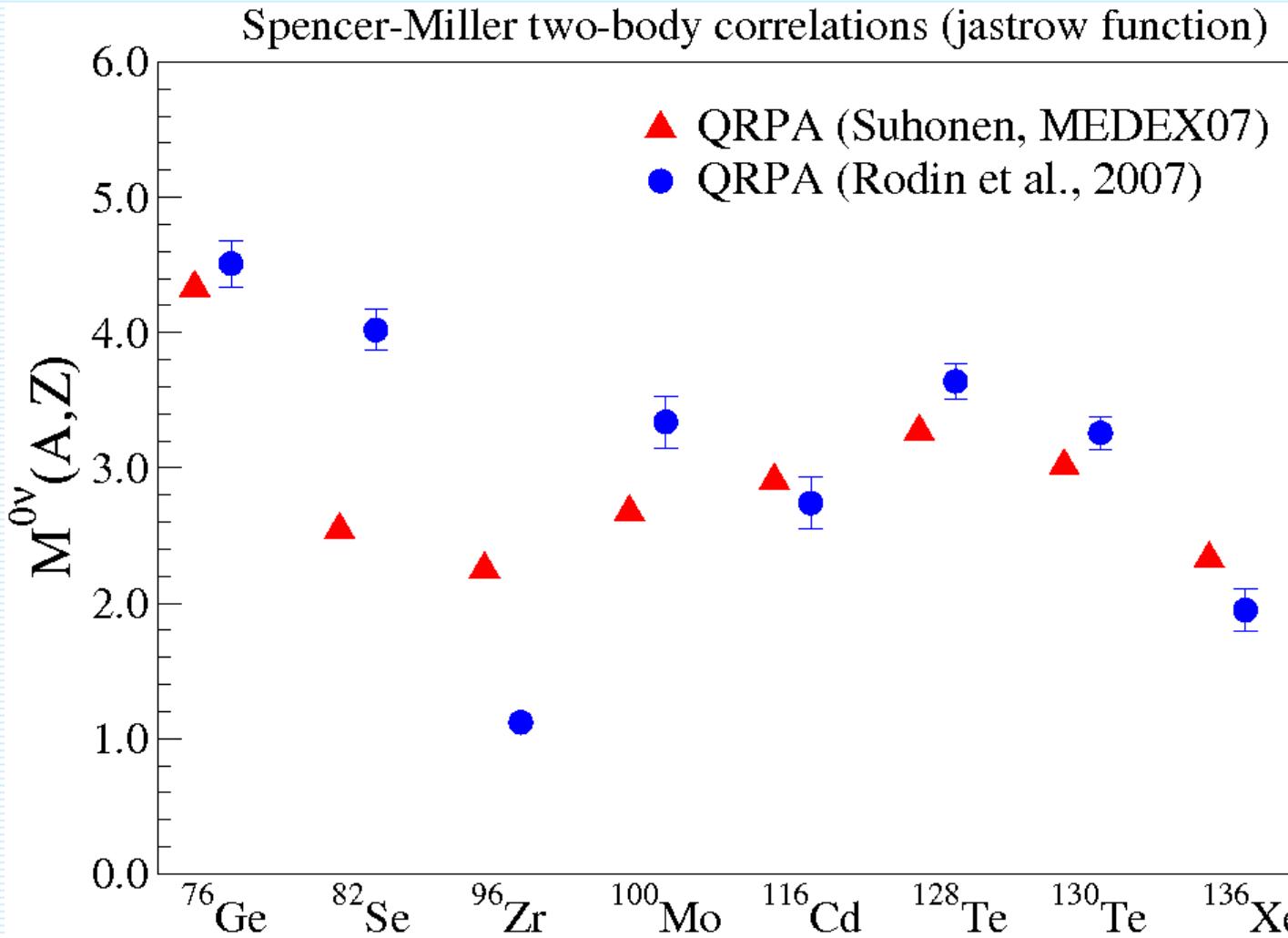
Jyvaskyla group [NPA
643, 207 (1998); NPA
729, 867 (2003) ...]
s.r.c. not considered

formfactor cutoff
unknown

Our approach ●
based
on formfactors and
inclusion of higher
order terms
of nucleon current



Is there a convergence of QRPA-like results?



Jyvaskyla group:
Results only for
one model space

$^{82}\text{Se}, ^{96}\text{Zr}$
Is it a
problem of
shifting of
single
particle
energies?

V.Rodin, A. Faessler, F. Š., P. Vogel, Nucl. Phys. A 793, 213 (2007)

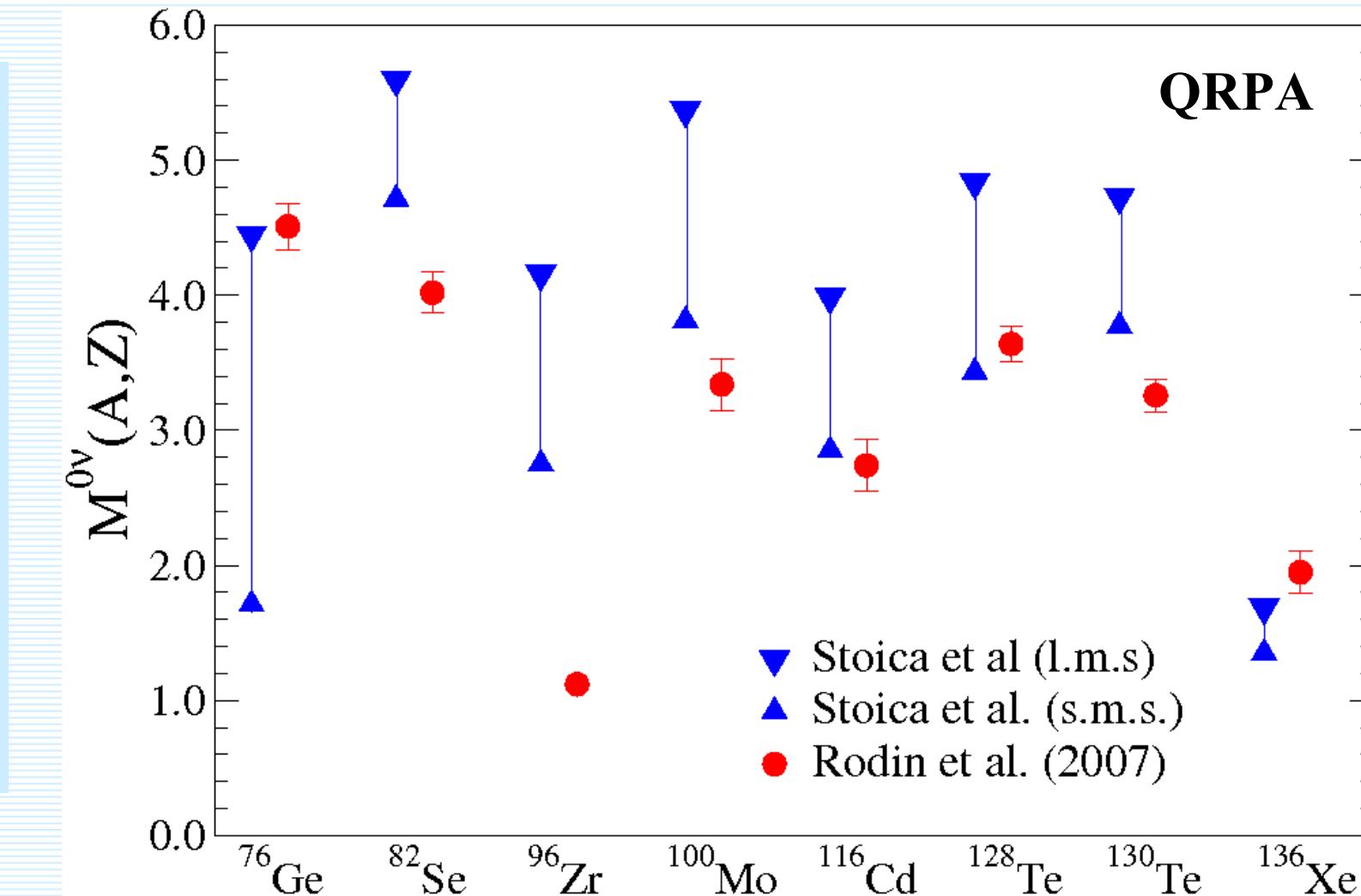
M. Korteleinen, J. Suhonen, Phys. Rev. C 76, 051303 (2007); 76, 6024315 (2007)

Is there a dependence on the size of the model space?

Stoica et al:
Yes
strong dep.

Rodin et al.
No
weak dep.

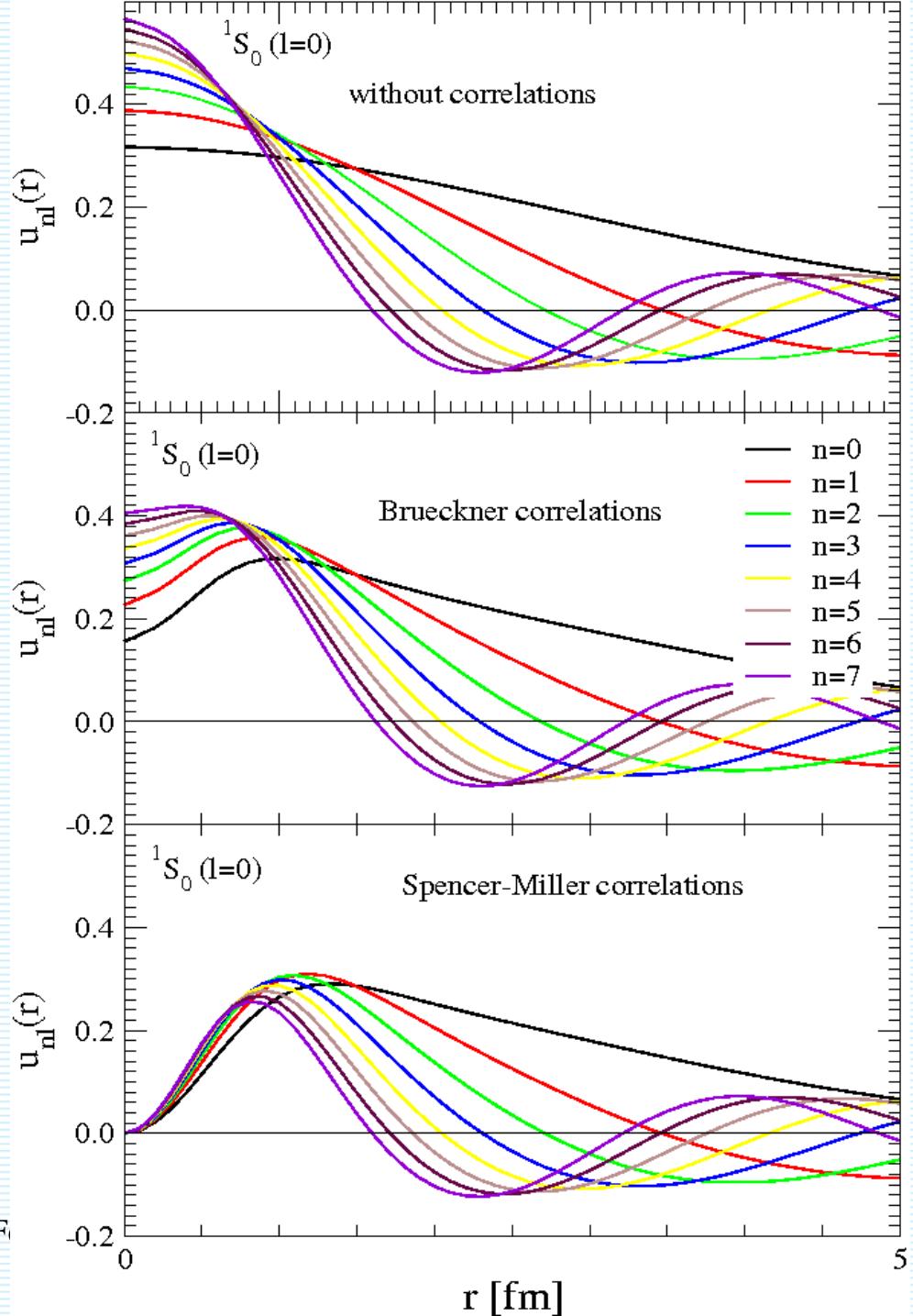
Jyvaskyla
group
can help
to solve
this problem



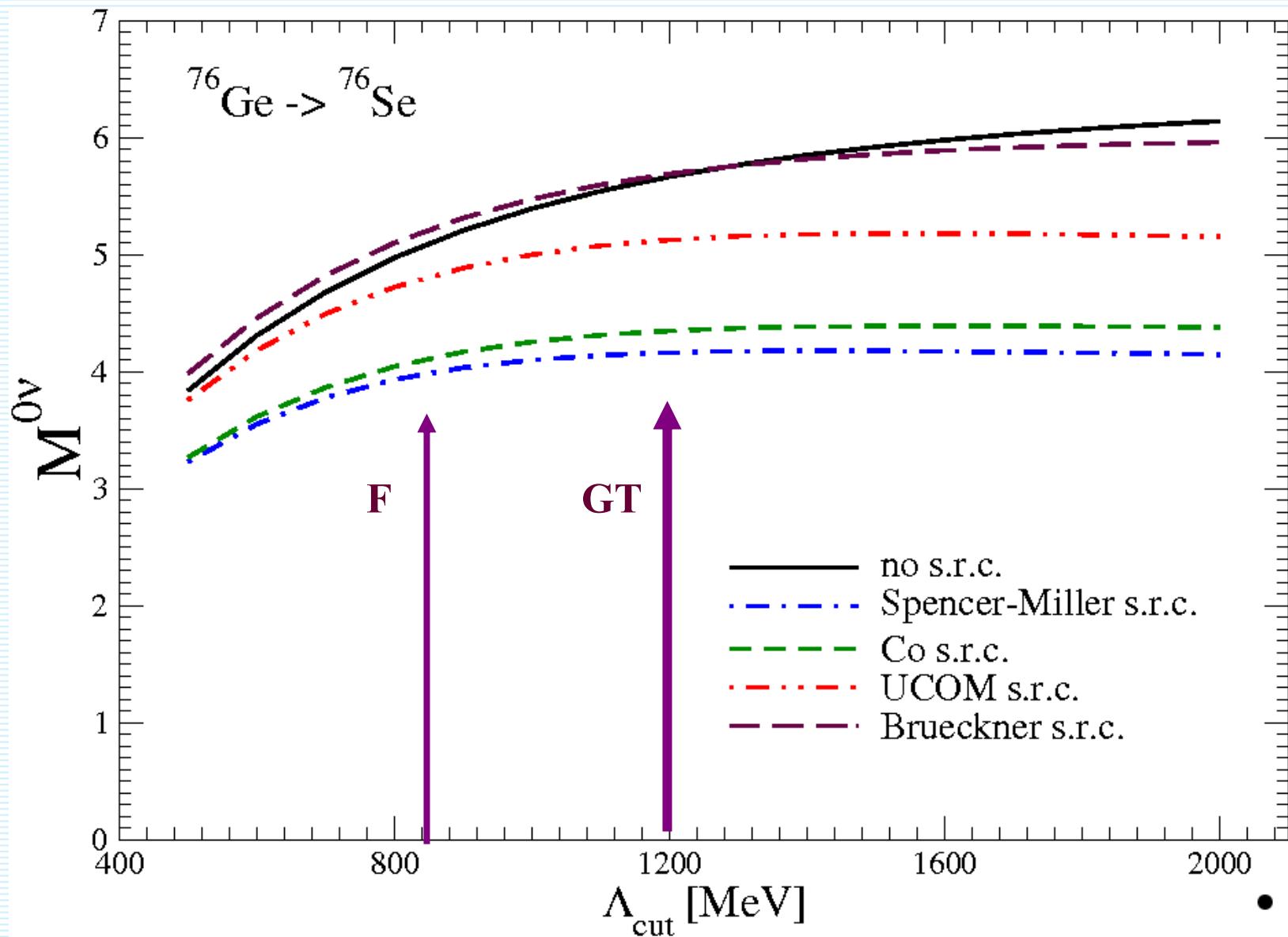
Two-body correlated wave functions

Numerical solution of
Bethe-Goldstone equation
H. Muether et al.

11/23/2007

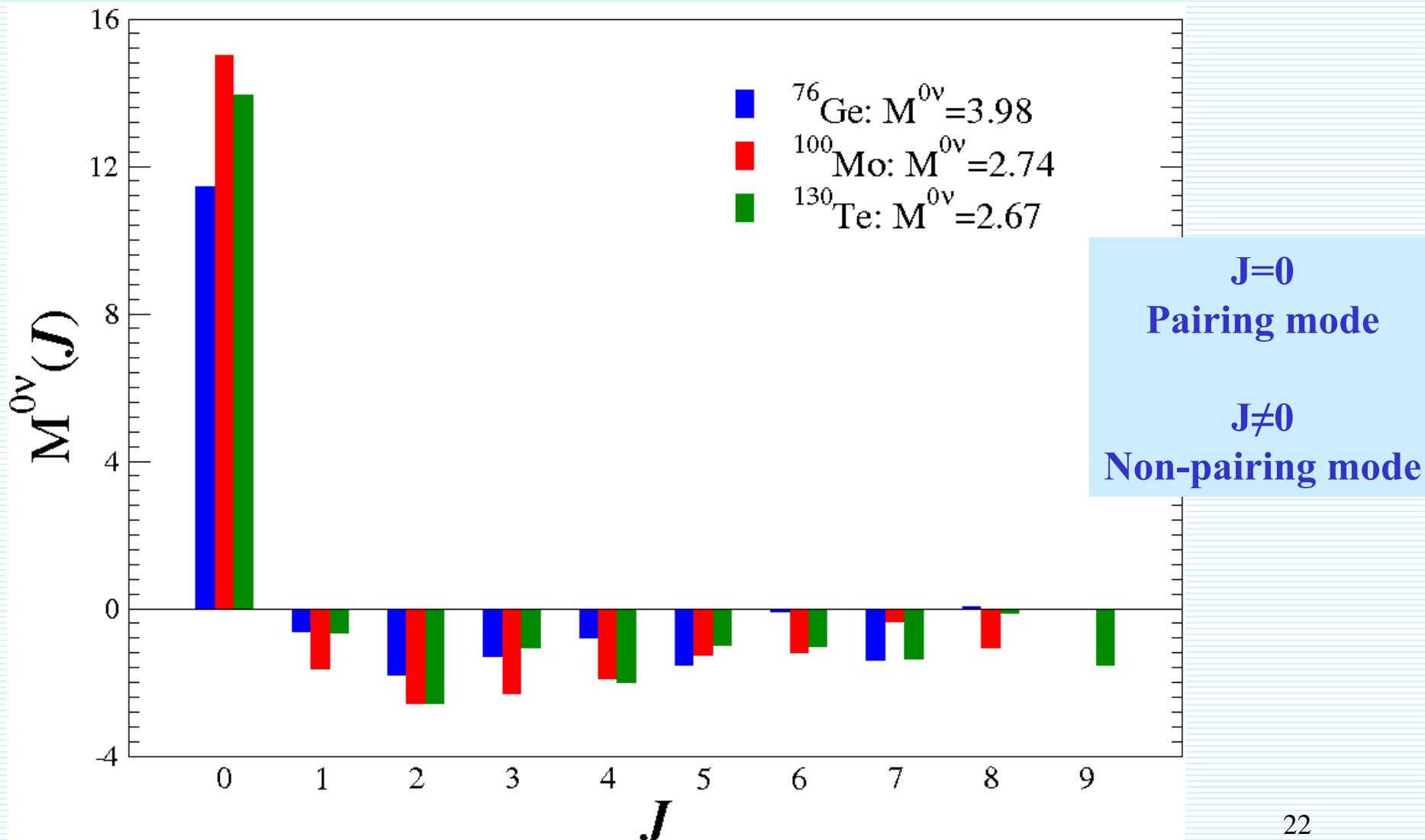


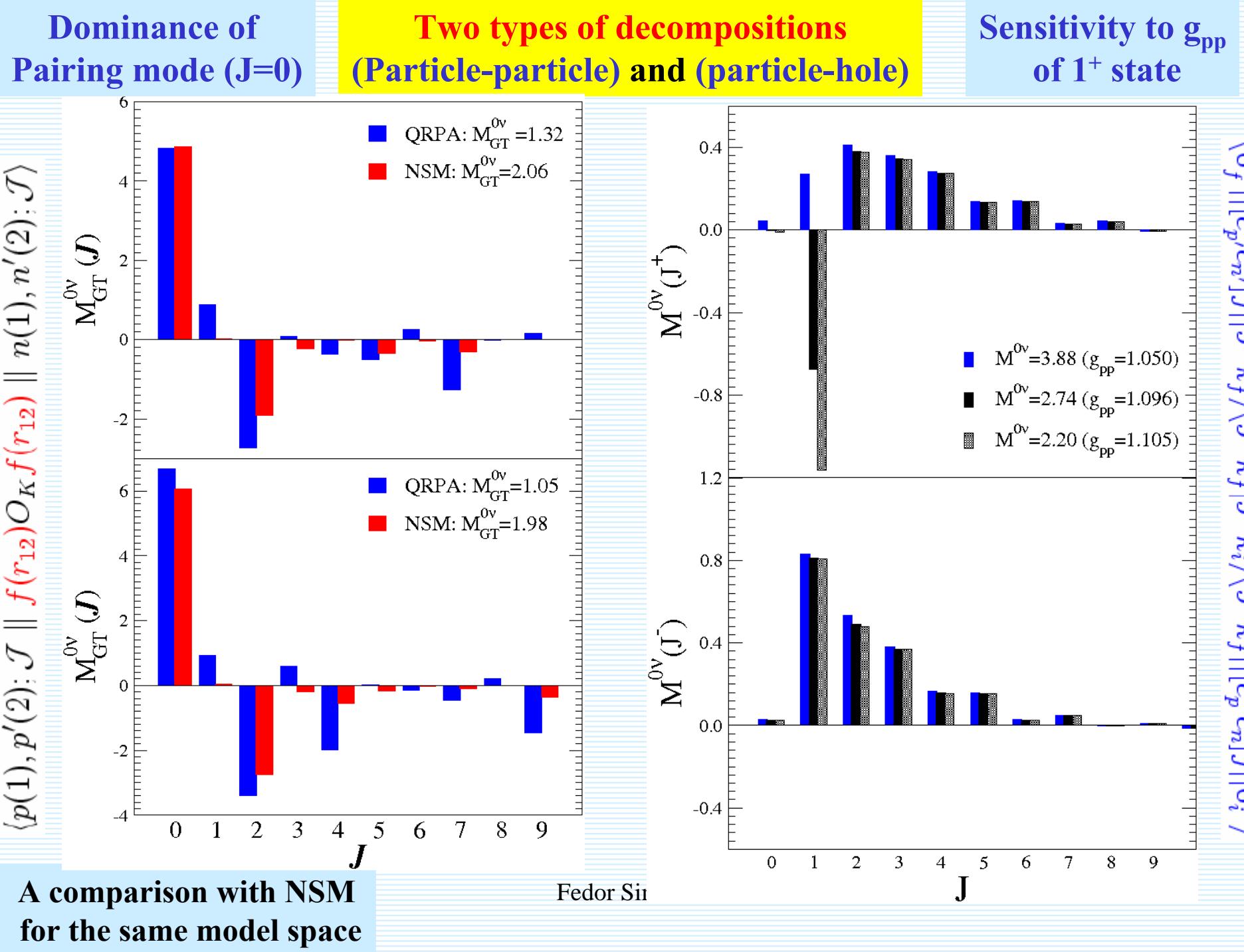
Finite nucleon size (formfactor) do the same job as the s.r.c.!!



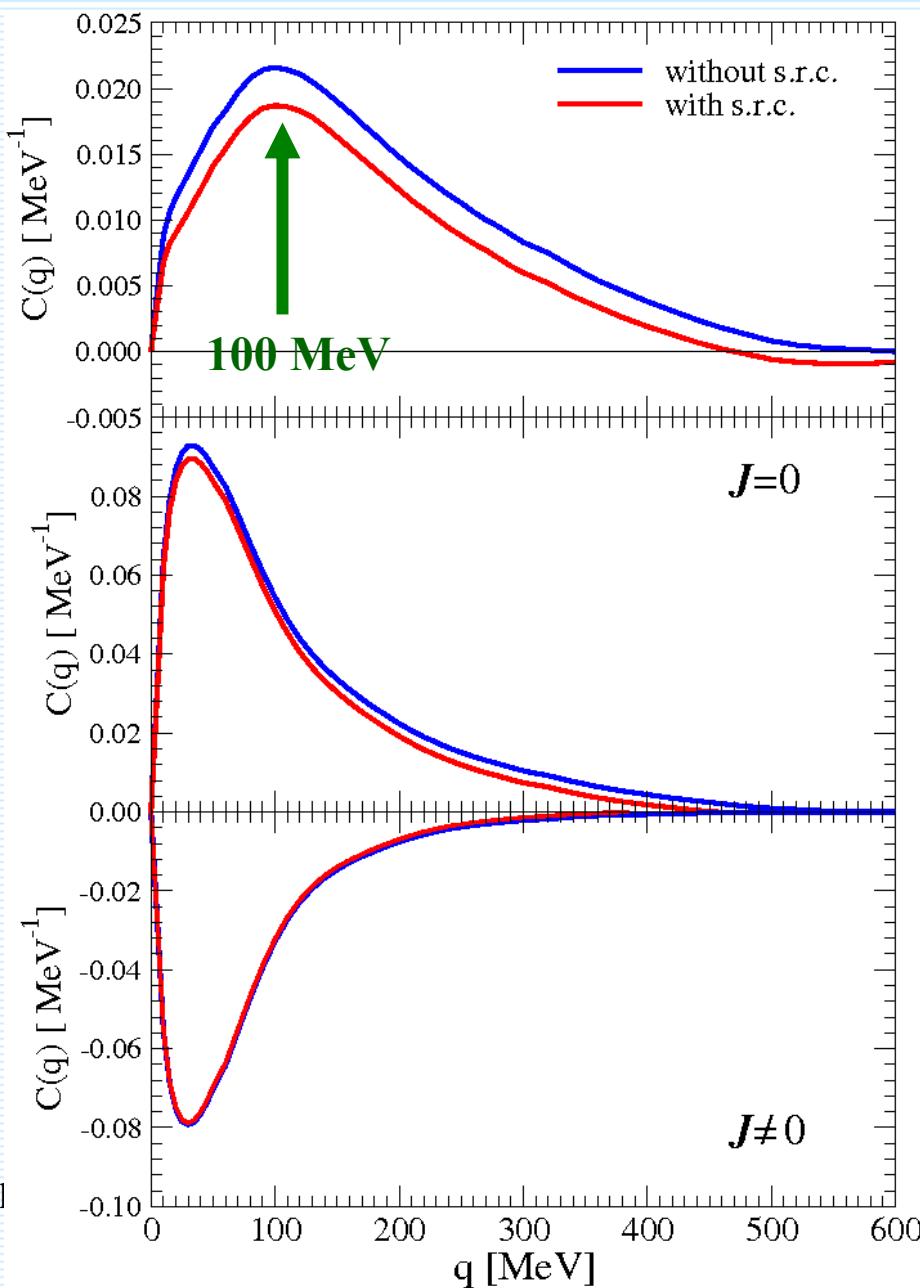
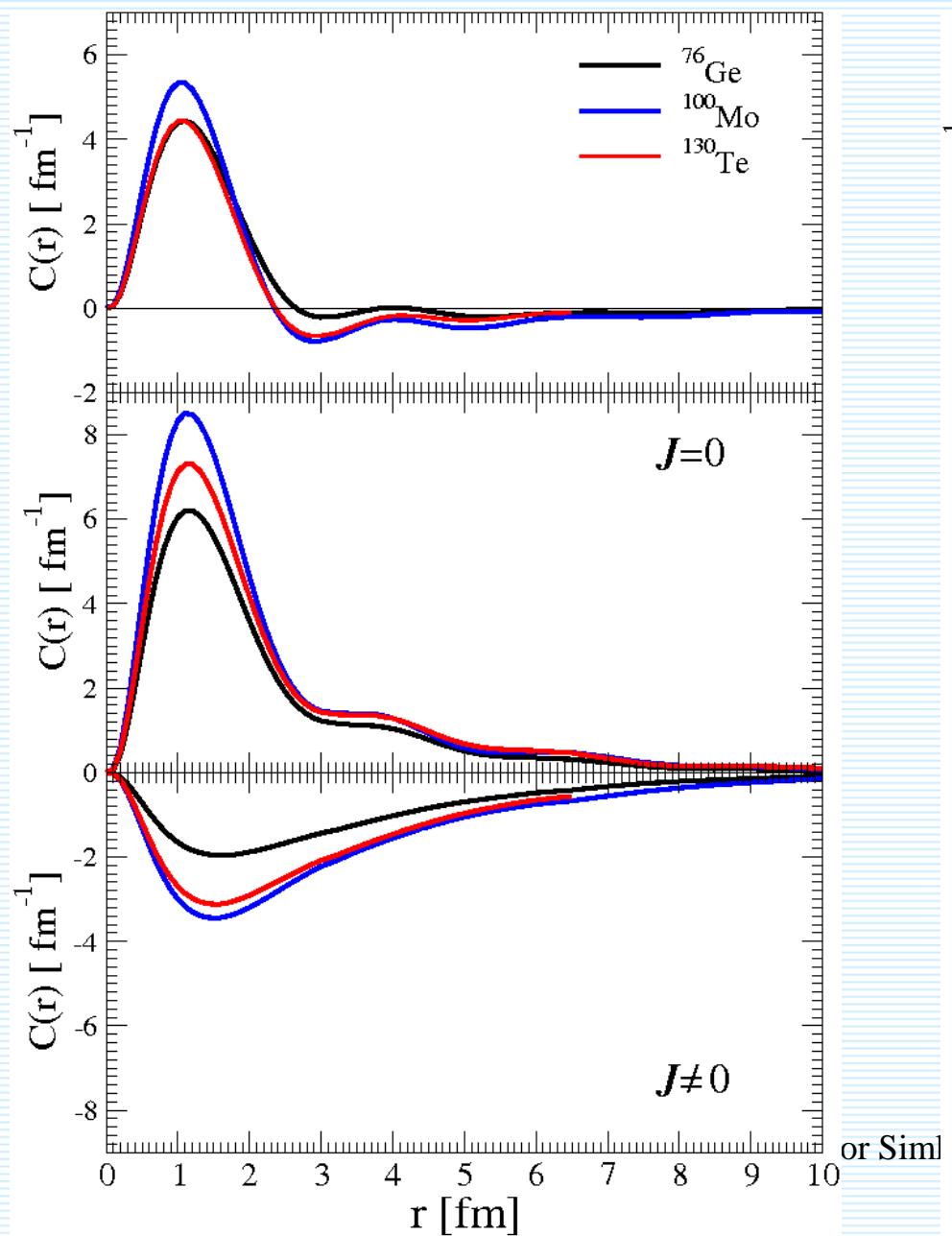
Decomposition in pp and nn channels

$$\langle p(1), p'(2); \mathcal{J} \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2); \mathcal{J} \rangle$$

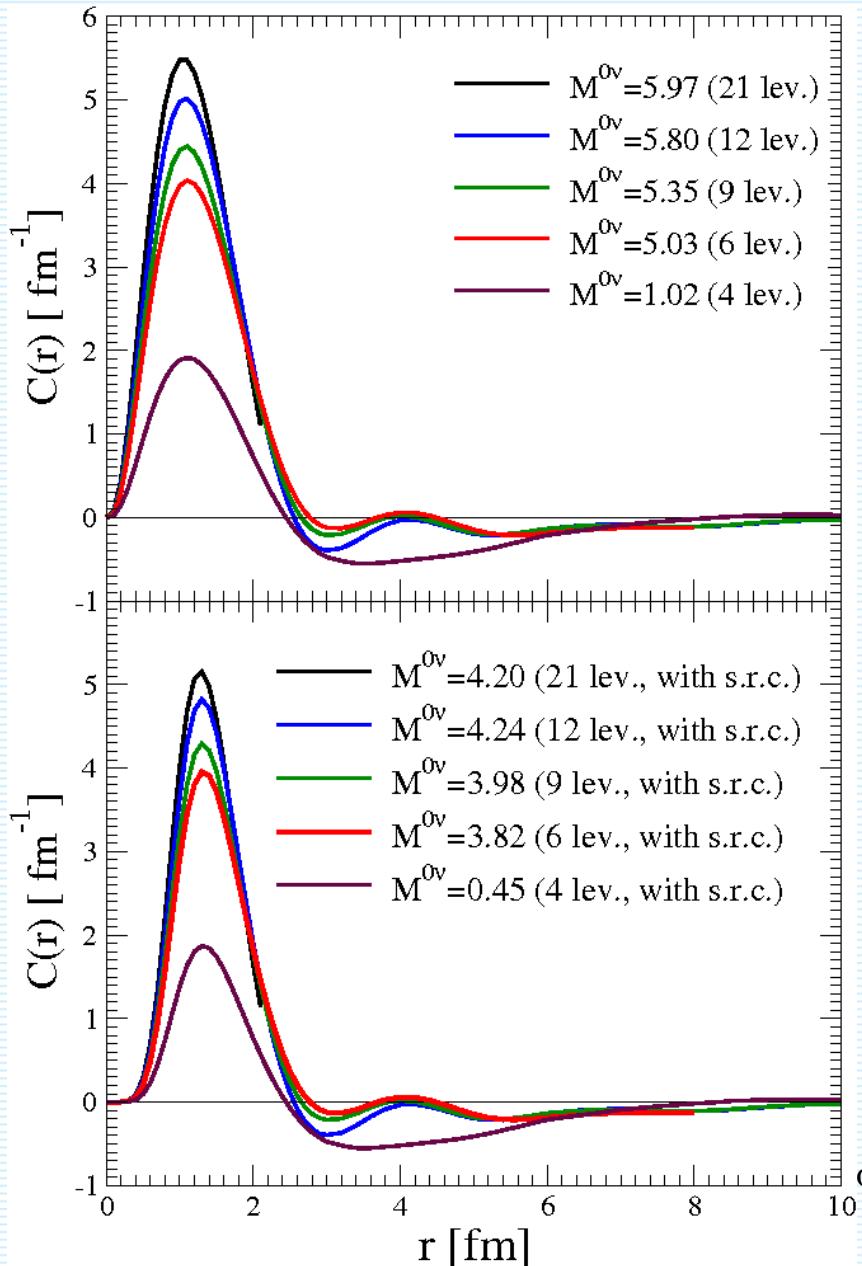




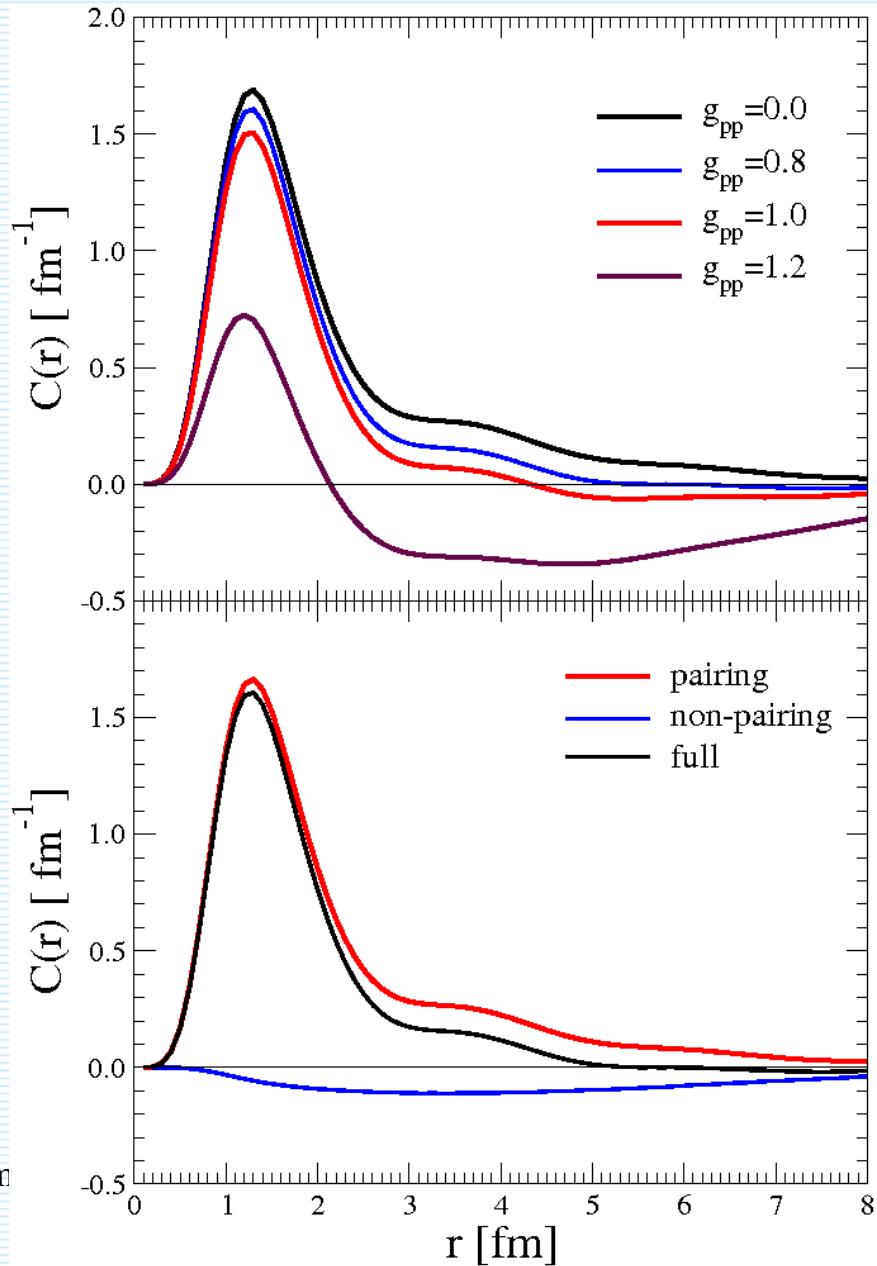
The r_{12} and momentum transfer dependences of $M^{0\nu}$



Dependence on the size of the model space

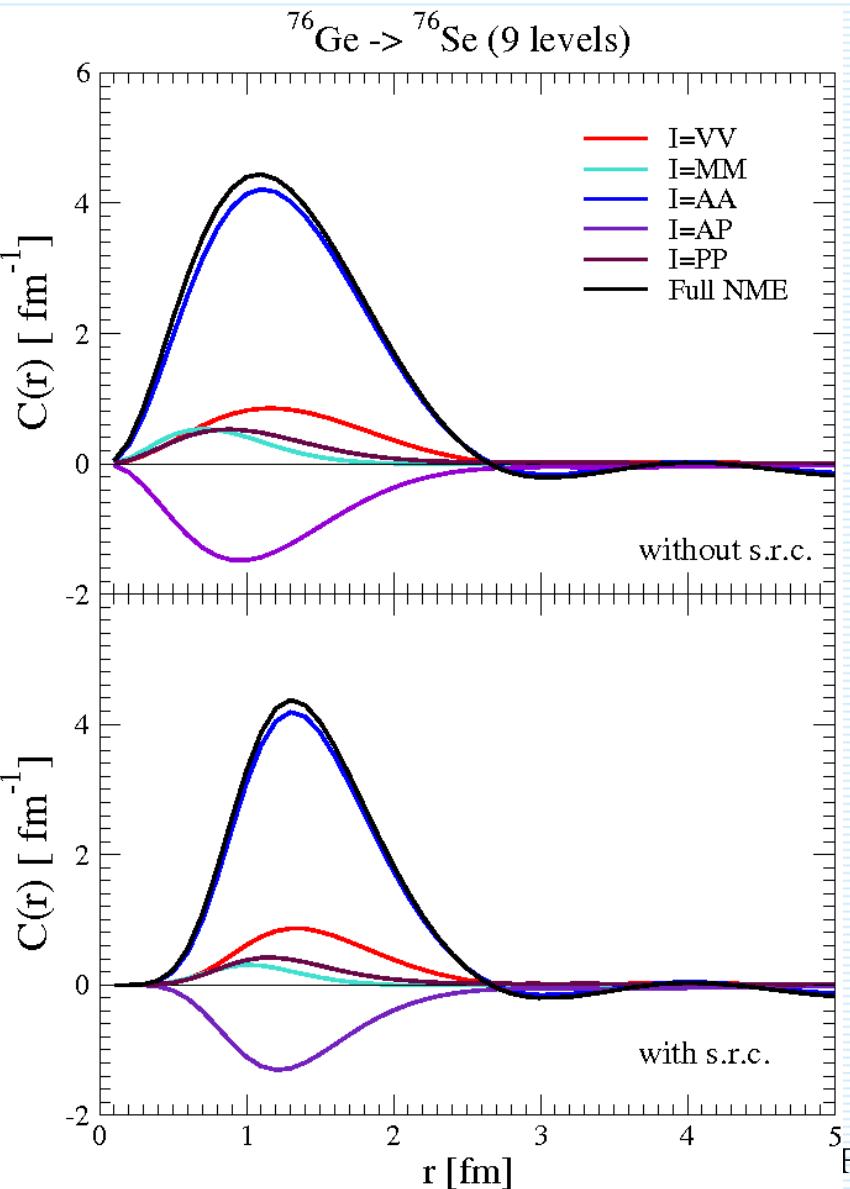


Dependence on g_{pp} in exactly solvable fermi-type model

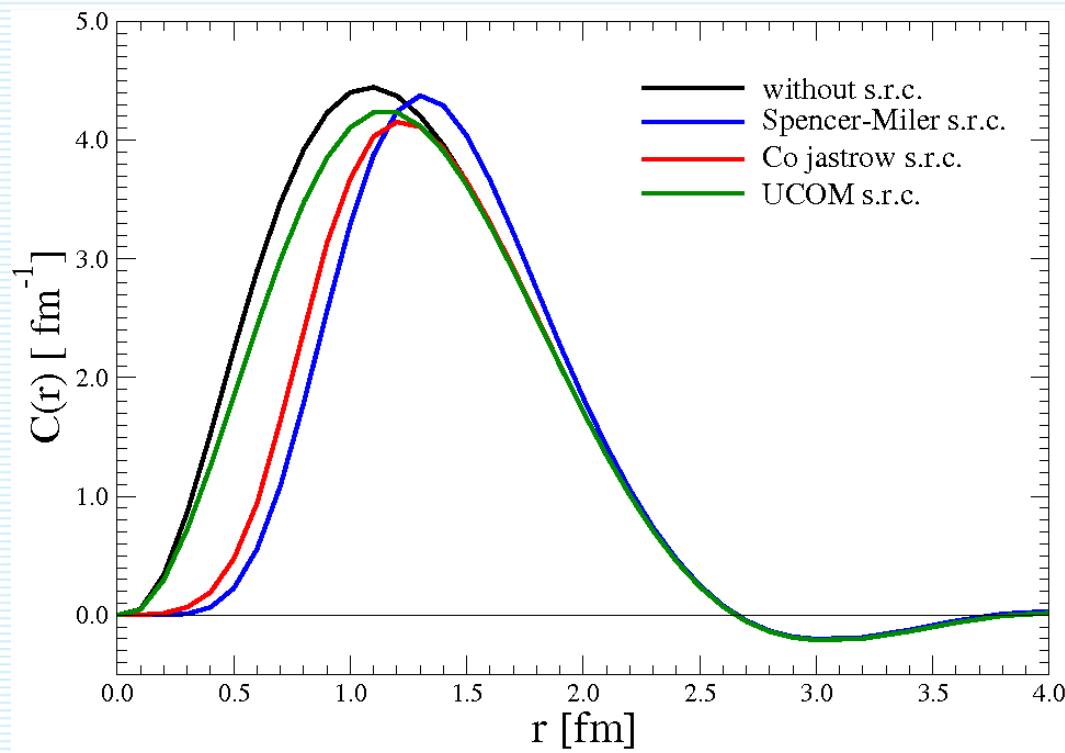


or Sim

The role of the constituents of the hadron current

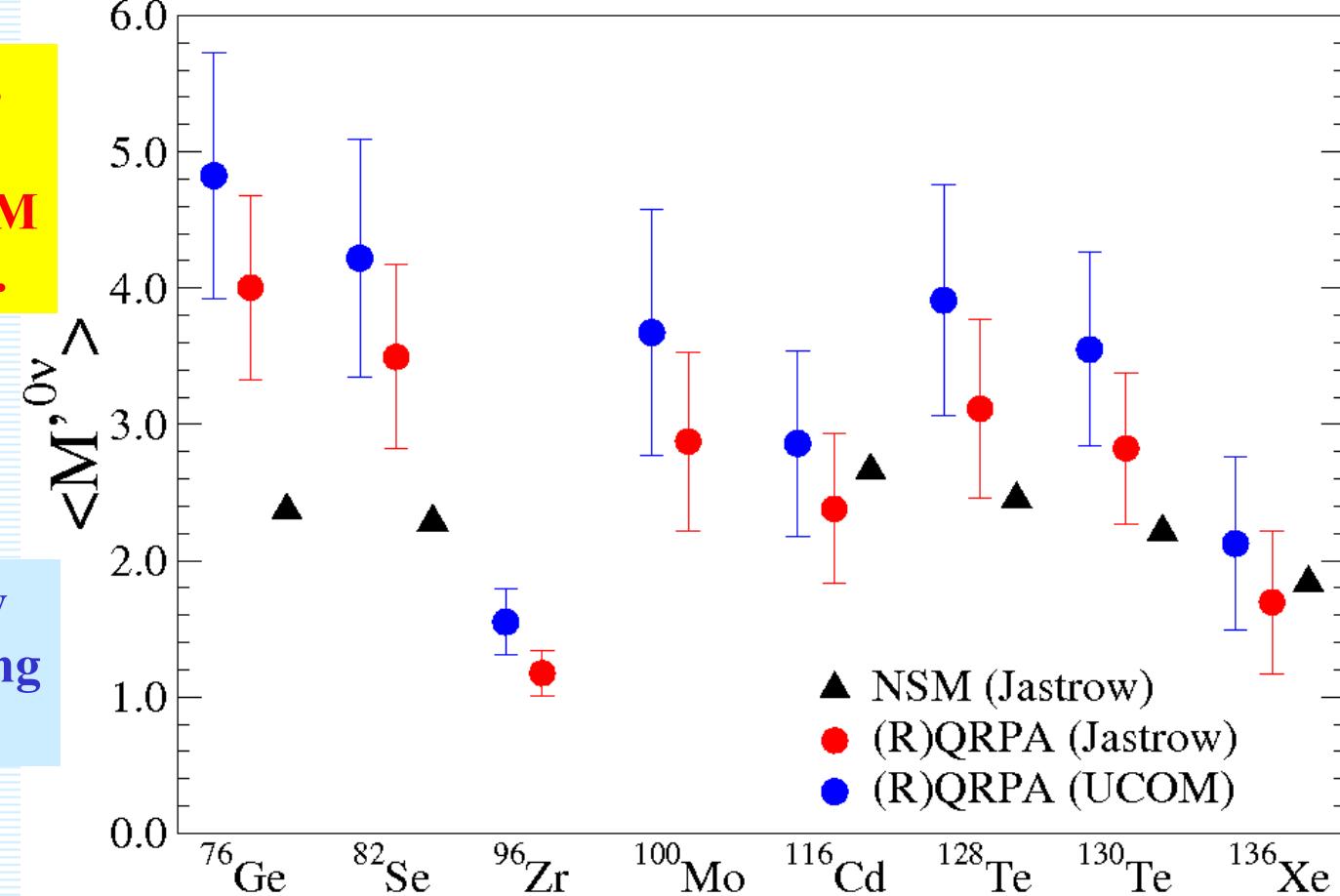


The effect of the two-nucleon short-range correlations



**(R)QRPA results
with
Jastrow and UCOM
two-nucleon s.r.c.**

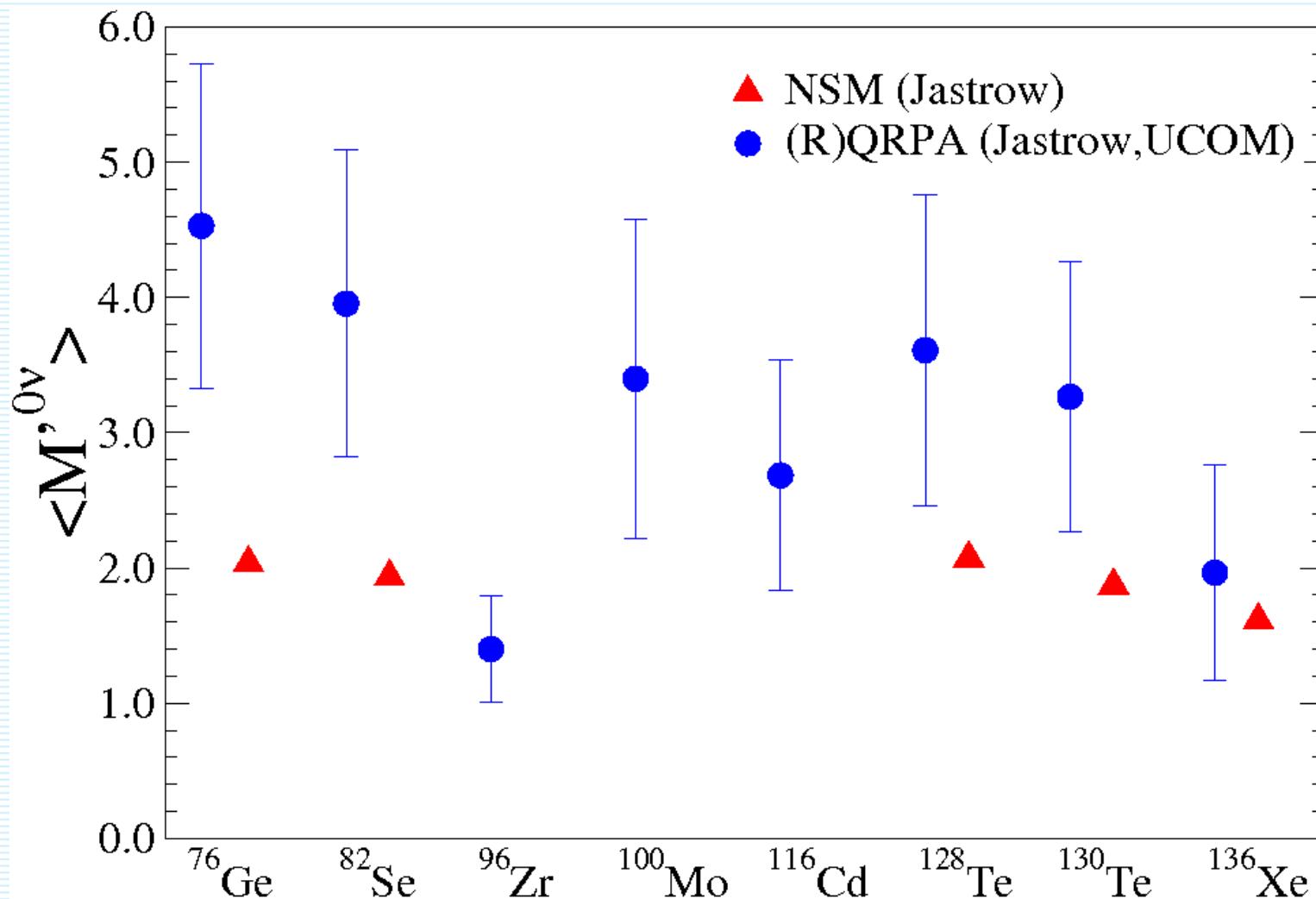
**Large uncertainty
due to weak coupling
constant g_A**



Nuclear transition	(R)QRPA (Jastrow s.r.c.)			(R)QRPA (UCOM s.r.c.)		
	$M'^{0\nu}$	$T_{1/2}^{0\nu}$ ($\langle m_{\beta\beta} \rangle = 50$ meV)		$M'^{0\nu}$	$T_{1/2}^{0\nu}$ ($\langle m_{\beta\beta} \rangle = 50$ meV)	
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	(3.33, 4.68)	$(6.01, 11.9) \times 10^{26}$		(3.92, 5.73)	$(4.01, 8.57) \times 10^{26}$	
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	(2.82, 4.17)	$(1.71, 3.73) \times 10^{26}$		(3.35, 5.09)	$(1.14, 2.64) \times 10^{26}$	
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	(1.01, 1.34)	$(7.90, 13.9) \times 10^{26}$		(1.31, 1.79)	$(4.43, 8.27) \times 10^{26}$	
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	(2.22, 3.53)	$(1.46, 3.70) \times 10^{26}$		(2.77, 4.58)	$(8.69, 23.8) \times 10^{25}$	
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	(1.83, 2.93)	$(1.95, 5.01) \times 10^{26}$		(2.18, 3.54)	$(1.34, 3.53) \times 10^{26}$	
$^{128}\text{Te} \rightarrow ^{128}\text{Xe}$	(2.46, 3.77)	$(3.33, 7.81) \times 10^{27}$		(3.06, 4.76)	$(2.09, 5.05) \times 10^{27}$	
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	(2.27, 3.38)	$(1.65, 3.66) \times 10^{26}$		(2.84, 4.26)	$(1.04, 2.34) \times 10^{26}$	
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	(1.17, 2.22)	$(3.59, 12.9) \times 10^{26}$		(1.49, 2.76)	$(2.32, 7.96) \times 10^{26}$	

Latest R(QRPA) and NSM results

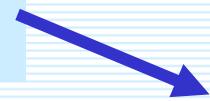
QRPA: Uncertainties due to s.r.c. (Jastrow,UCOM), weak coupling constant g_A , and approach (QRPA, RQRPA) are summed, NSM: hoc already included



11/ (R)QRPA: F. Š., A. Faessler V.Rodin,, P. Vogel, J. Engel, arXiv 0710.2055 (2007)
NSM: E. Caurier,J. Menendez, F. Nowacki, and A. Poves, arXiv 0709.2137 (2007) 8

0νββ-decay NME's might be smaller (deformations, overlap matrix)

**Estimation of the effect of the deformation
on the 0νββ-decay NME's**



Transition	$Q_{\beta\beta}$ [MeV]	$\beta_{initial}$		β_{final}		$i <BCS BCS>_f$
		ROM	B(E2)	ROM	B(E2)	
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	4.272	0.000	0.101	+0.17	0.269	0.51, 0.44
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	2.039	+0.095	0.262	+0.163	0.309	0.74, 0.67
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	2.995	+0.104	0.194		0.202	0.80
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	3.350		0.081	+0.068	0.172	0.46
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	3.034	+0.139	0.231	+0.136	0.217	0.85, 0.84
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	2.805	+0.113	0.191	+0.043	0.112	0.61, 0.52
$^{128}\text{Te} \rightarrow ^{128}\text{Xe}$	0.867	+0.011	0.136		0.184	0.66
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	2.529	+0.035	0.118		0.169	0.65
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	2.468		0.086		0.124	0.72
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	3.367	+0.367	0.285	+0.230	0.193	0.16, 0.37

Conclusions

0νββ–decay NME's (QRPA, from factors to tens of %)

Which way of calculation?

Two-nucleon aproach of Bratislava-Tuebingen group with finite nucleon size
and hoc. Both Jyvaskyla and Strassbourg-Madrid groups accepted this approach.

Which s.r.c.?

Spencer-Miller (Jastrow function), Co Jastrow-like, UCOM,
or something else? None of them are preferable. A source of uncertainty.

Which NME's to consider ? (the latest ones)

(R)QRPA:) V.Rodin, A. Faessler, F. Š., P. Vogel, Nucl. Phys. A 793, 213 (2007)

F. Š., A. Faessler V.Rodin,, P. Vogel, J. Engel, arXiv 0710.2055 (2007)

NSM: E. Caurier,J. Menendez, F. Nowacki, and A. Poves, arXiv 0709.2137 (2007)

[Jyvaskyla group should explain the differences between the old results (quark confinement model
aproach) and new results]

Usefull anatomy of the 0νββ-decay NME performed with many details.

The main contribution comes from the interaction of the closest nucleons.

It explains why 0νββ-decay NME' have approximately the same value.

Main source of uncertainty: s.r.c., g_A

There is no final answer yet (deformation, is there A-dependence of NME ...)?

What is the nature of neutrinos?



Theory

What is the origin of ν mass?



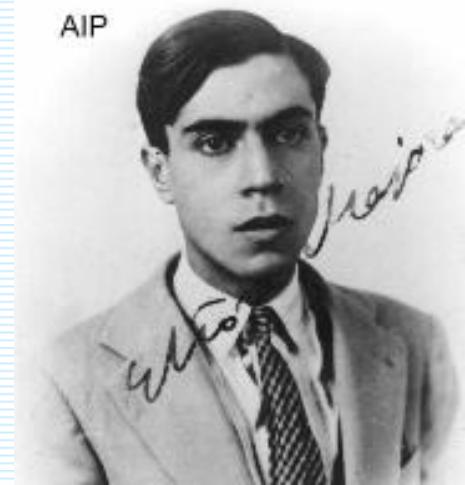
String theory
Extra dimen.

ν

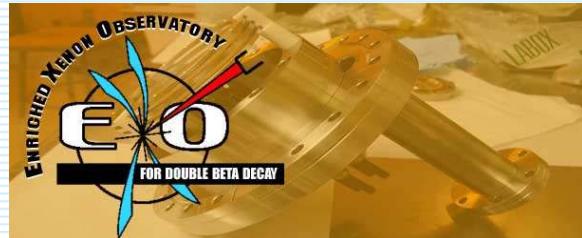
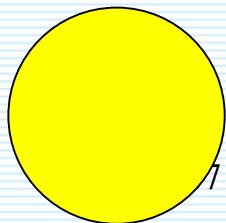
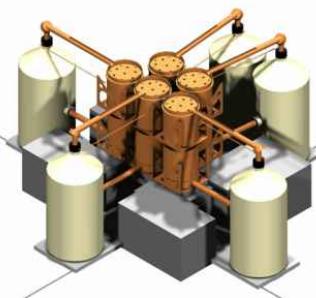
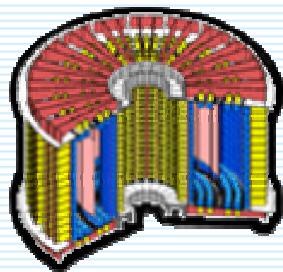


See-saw mech.
Radiative Higgs
R-parity viol. SUSY

AIP



Only the $0\nu\beta\beta$ -decay can answer this fundamental question



• • •

By product:

- Absolute ν mass scale
- CP Majorana phases