# Two-Loop QCD Corrections to Top-Antitop Production at Hadron Colliders

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# Plan of the Talk

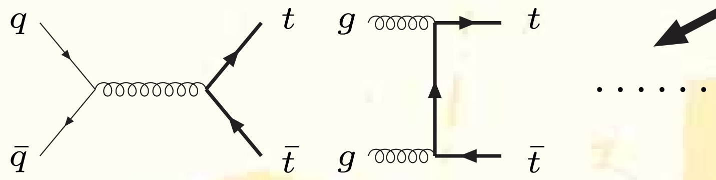
- General Introduction
  - Top Quark at the Tevatron and LHC Perspectives
- Status of the Theoretical calculations
  - Total Cross Section at NLO
  - Analytic Two-Loop QCD Corrections
- Conclusions

# Top Quark

- With a mass of $m_t = 173.1 \pm 1.3$ GeV, the TOP quark (the up-type quark of the third generation) is the heaviest elementary particle produced so far at colliders.
- Because of its mass, top quark is going to play a unique role in understanding the EW symmetry breaking $\Rightarrow$ Heavy-Quark physics crucial at the LHC.
- Two production mechanisms

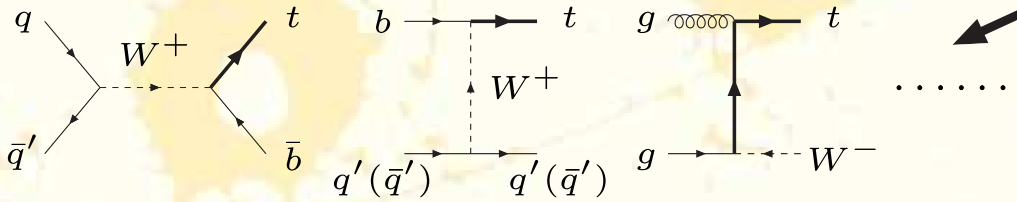
Pair Production

- $pp(\bar{p}) \rightarrow t\bar{t}$

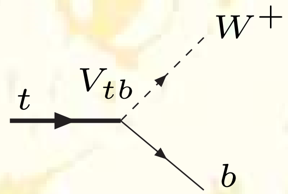



Single Top

- $pp(\bar{p}) \rightarrow t\bar{b}, tq'(\bar{q}'), tW^-$




- Top quark does not hadronize, since it decays in about $5 \cdot 10^{-25}$s (one order of magnitude smaller than the hadronization time) $\Longrightarrow$ opportunity to study the quark as single particle
  - Spin properties
  - Interaction vertices
  - Top quark mass
- Decay products: almost exclusively $t \rightarrow W^+ b$ $\quad (|V_{tb}| \gg |V_{td}|, |V_{ts}|)$



# Top Quark @ Tevatron

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Events measured at Tevatron

$\sigma_{t\bar{t}} \sim$ 7pb

- $p\bar{p} \to t\bar{t} \to W^+bW^-\bar{b} \to l\nu l\nu b\bar{b}$ — Dilepton $\sim 10\%$
- $p\bar{p} \to t\bar{t} \to W^+bW^-\bar{b} \to l\nu q\bar{q}' b\bar{b}$ — Lep+jets $\sim 44\%$
- $p\bar{p} \to t\bar{t} \to W^+bW^-\bar{b} \to q\bar{q}' q\bar{q}' b\bar{b}$ — All jets $\sim 46\%$

2 high-$p_T$ lept, $\geq$ 2 jets and ME

NO lept, $\geq$ 6 jets and low ME

1 isol high-$p_T$ lept, $\geq$ 4 jets and ME

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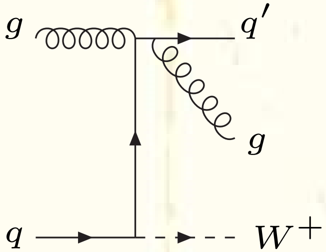
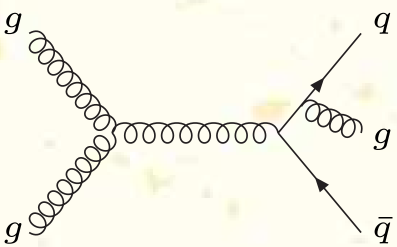
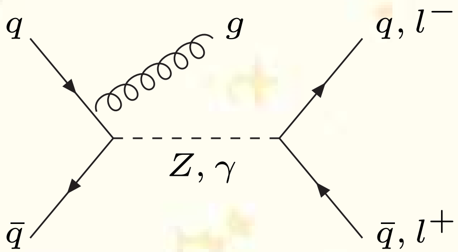
## Background Processes



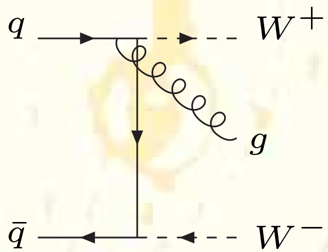

W+jets




QCD




Drell-Yan




Di-boson

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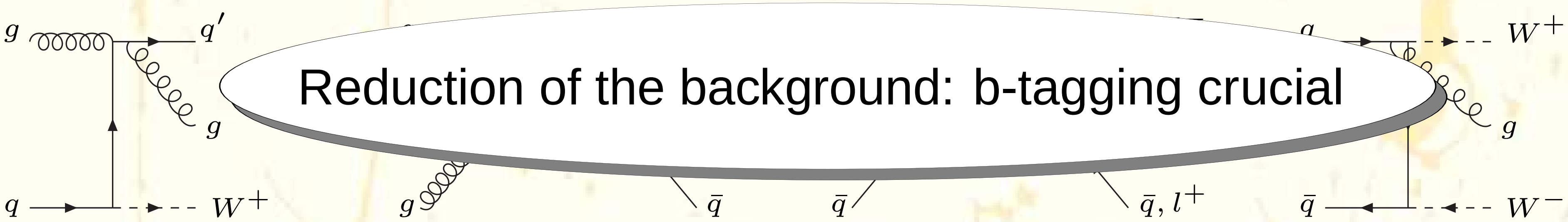


Reduction of the background: b-tagging crucial

W+jets | QCD | Drell-Yan | Di-boson

# Top Quark @ Tevatron

- Total Cross Section $$\sigma_{t\bar{t}} = \frac{N_{data} - N_{bkgr}}{\epsilon\, L} = 7.0 \pm 0.6\,\text{pb} \qquad (m_t = 175\,\text{GeV})$$

- Top-quark Mass $$m_t = 173.1 \pm 1.3\,\text{GeV}\ (0.75\%)$$

- $W$ helicity fractions $F_i = B(t \to bW^+(\lambda_W = i = -1, 0, 1)) \qquad (F_0 + F_+ + F_- = 1)$

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta^*} = \frac{3}{4}F_0 \sin^2\theta^* + \frac{3}{8}F_-(1-\cos\theta^*)^2 + \frac{3}{8}F_+(1+\cos\theta^*)^2$$

$$F_0 = 0.66 \pm 0.16 \pm 0.05 \quad F_+ = -0.03 \pm 0.06 \pm 0.03$$

- Spin correlations measured fitting the double distribution

$$\frac{1}{N}\frac{d^2N}{d\cos\theta_1\, d\cos\theta_2} = \frac{1}{4}(1 + \kappa\cos\theta_1\cos\theta_2)$$

$$-0.455 < \kappa < 0.865\,(68\%\,CL)$$

- Forward-Backward Asymmetry $$A_{FB} = (19.3 \pm 6.5(\text{sta}) \pm 2.4(\text{sys}))\%$$

# Top Quark @ LHC

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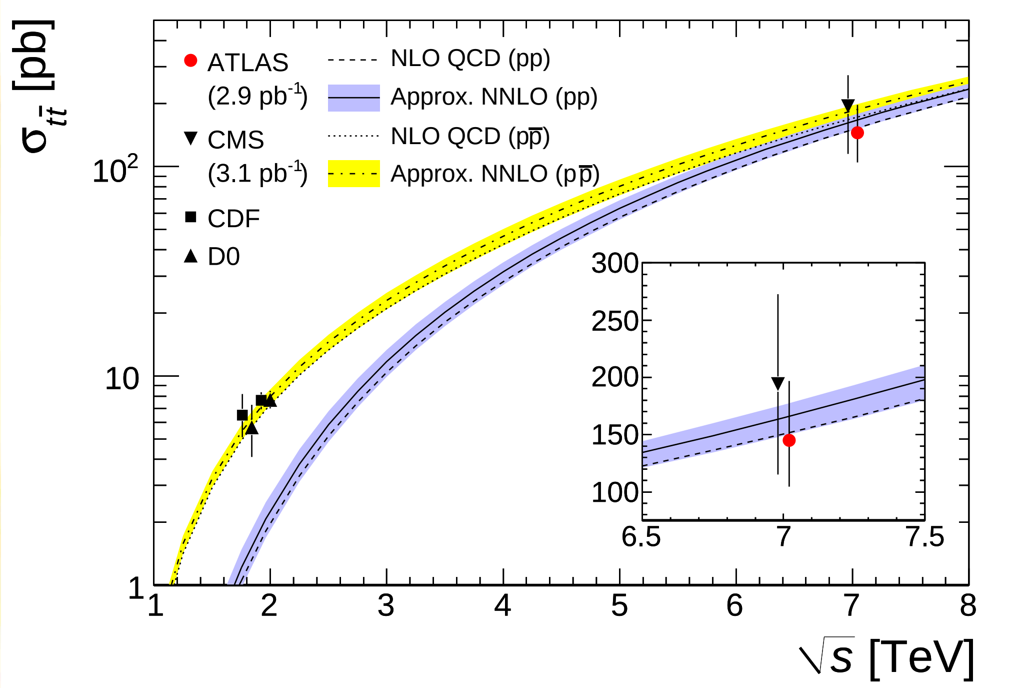
Very recently, new results became available from CMS and ATLAS collaborations, for $pp$ collisions at $\sqrt{s} = 7$ GeV, analysing almost 3 pb$^{-1}$:



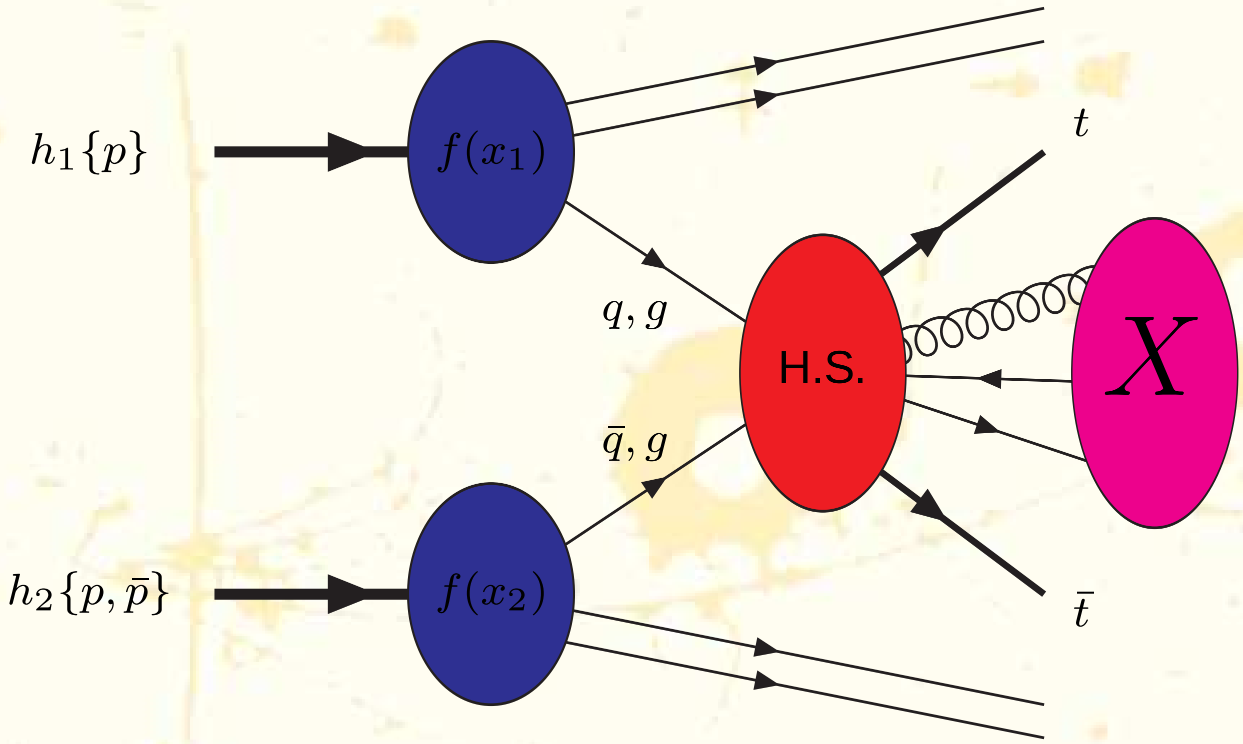

- CMS $\boxed{\sigma_{t\bar{t}} = 194 \pm 72(stat.) \pm 24(syst.) \pm 21(lumi.)\text{ pb}}$ arXiv:1010.5994
- ATLAS $\boxed{\sigma_{t\bar{t}} = 145 \pm 31^{+42}_{-27}\text{ pb}}$ arXiv:1012.1792

# Top Quark @ LHC: Perspectives

- Cross Section
  - With $100\,\text{pb}^{-1}$ of accumulated data an error of $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 15\%$ is expected (dominated by statistics!)
  - After 5 years of data taking an error of $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 5\%$ is expected
- Top Mass
  - With $1\,\text{fb}^{-1}$ Mass accuracy: $\Delta m_t \sim 1 - 3$ GeV
- Top Properties
  - W helicity fractions and spin correlations with $10\,\text{fb}^{-1}$ $\Longrightarrow$ 1-5%
  - Top-quark charge. With $1\,\text{fb}^{-1}$ we could be able to determine $Q_t = 2/3$ with an accuracy of $\sim 15\%$
- Sensitivity to new physics
  - all the above mentioned points
  - Narrow resonances: with $1\,\text{fb}^{-1}$ possible discovery of a $Z'$ of $M_{Z'} \sim 700$ GeV with $\sigma_{pp\to Z'\to t\bar{t}} \sim 11\,\text{pb}$

# Top-Anti Top Pair Production

According to the factorization theorem, the process $\boxed{h_1 + h_2 \to t\bar{t} + X}$ can be sketched as in the figure:

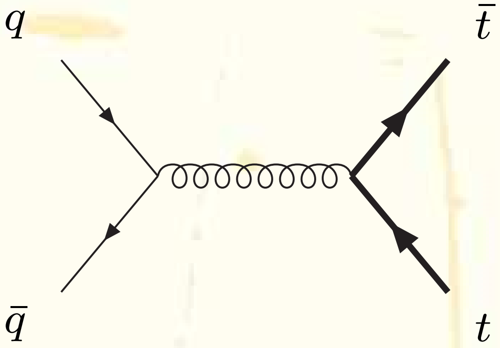



$$\sigma_{h_1,h_2}^{t\bar{t}} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_{h_1,i}(x_1, \mu_F) f_{h_2,j}(x_2, \mu_F)\ \hat{\sigma}_{ij}\left(\hat{s}, m_t, \alpha_s(\mu_R), \mu_F, \mu_R\right)$$

$$s = \left(p_{h_1} + p_{h_2}\right)^2 , \ \hat{s} = x_1 x_2 s$$

# Cross Section: Tree-Level

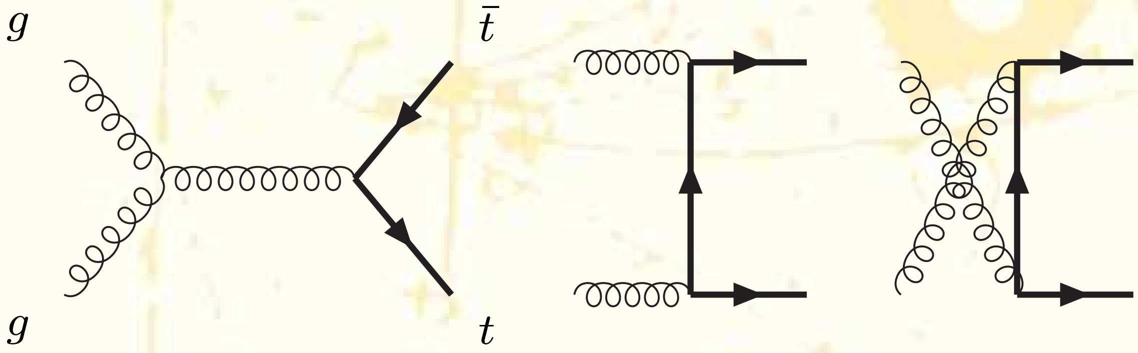
$$q(p_1) + \bar{q}(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$




Dominant at Tevatron

$\sim 85\%$

$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$




Dominant at LHC

$\sim 90\%$

$\sigma^{LO}_{t\bar{t}}(LHC, m_t = 171\,\text{GeV}) = 583\,\text{pb} \pm 30\%$

$\sigma^{LO}_{t\bar{t}}(Tev, m_t = 171\,\text{GeV}) = 5.92\,\text{pb} \pm 44\%$

# Cross Section: NLO

Fixed Order

- The NLO QCD corrections are quite sizable: + 25% at Tevatron and +50% at LHC. Scales variation $\pm 15\%$.

  Nason, Dawson, Ellis '88-'90; Beenakker, Kuijf, van Neerven, Smith '89-'91; Mangano, Nason, Ridolfi '92; Frixione et al. '95; Czakon and Mitov '08. Melnikov and Schulze '09; Bernreuther and Si '10

- Mixed NLO QCD-EW corrections are small: - 1% at Tevatron and -0.5% at LHC.

  Beenakker *et al.* '94 Bernreuther, Fuecker, and Si '05-'08
  Kühn, Scharf, and Uwer '05-'06; Moretti, Nolten, and Ross '06.

All-order Soft-Gluon Resummation

- Leading-Logs (LL)

  Laenen et al. '92-'95; Berger and Contopanagos '95-'96; Catani et al. '96.

- Next-to-Leading-Logs (NLL)

  Kidonakis and Sterman '97; R. B., Catani, Mangano, and Nason '98.

- Next-to-Next-to-Leading-Logs (NNLL) under study.

  Moch and Uwer '08; Beneke et al. '09; Czakon et al. '09; Kidonakis '09

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$$\sigma_{t\bar{t}}^{\mathrm{NLO+NLL}}(\mathrm{Tev}, m_t = 171\ \mathrm{GeV}, \mathrm{CTEQ6.5}) = 7.61\ {}^{+0.30(3.9\%)}_{-0.53(6.9\%)}\ (\mathrm{scales})\ {}^{+0.53(7\%)}_{-0.36(4.8\%)}\ (\mathrm{PDFs})\ \ \mathrm{pb}$$

$$\sigma_{t\bar{t}}^{\mathrm{NLO+NLL}}(\mathrm{LHC}, m_t = 171\ \mathrm{GeV}, \mathrm{CTEQ6.5}) = 908\ {}^{+82(9.0\%)}_{-85(9.3\%)}\ (\mathrm{scales})\ {}^{+30(3.3\%)}_{-29(3.2\%)}\ (\mathrm{PDFs})\ \ \mathrm{pb}$$

M. Cacciari, S. Frixione, M. Mangano, P. Nason, and G. Ridolfi, JHEP 0809:127,2008

# Measurement Requirements for $\sigma_{t\bar{t}}$

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Experimental requirements for $\sigma_{t\bar{t}}$:

- Tevatron $\Delta\sigma/\sigma \sim 9\% \Longrightarrow$ already $<$ current theoretical prediction
- LHC (14 TeV, high luminosity) $\Delta\sigma/\sigma \sim 5\% \ll$ current theoretical prediction!!

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Different groups presented approximated higher-order results for $\sigma_{t\bar{t}}$

- Including scale dep at NNLO, NNLL soft-gluon contributions, Coulomb corrections

$$\sigma_{t\bar{t}}^{\rm NNLOappr}({\rm Tev}, m_t = 173\ {\rm GeV}, {\rm MSTW2008}) = 7.04\ ^{+0.24}_{-0.36}\ ({\rm scales})\ ^{+0.14}_{-0.14}\ ({\rm PDFs})\ \ {\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NNLOappr}({\rm LHC}, m_t = 173\ {\rm GeV}, {\rm MSTW2008}) = 887\ ^{+9}_{-33}\ ({\rm scales})\ ^{+15}_{-15}\ ({\rm PDFs})\ \ {\rm pb}$$

Kidonakis and Vogt ’08; Moch and Uwer ’08; Langenfeld, Moch, and Uwer ’09

- Integration of the Invariant mass distribution at NLO+NNLL

$$\sigma_{t\bar{t}}^{\rm NLO+NNLL}({\rm Tev}, m_t = 173.1\ {\rm GeV}, {\rm MSTW2008}) = 6.48\ ^{+0.17}_{-0.21}\ ({\rm scales})\ ^{+0.32}_{-0.25}\ ({\rm PDFs})\ \ {\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO+NNLL}({\rm LHC}, m_t = 173.1\ {\rm GeV}, {\rm MSTW2008}) = 813\ ^{+50}_{-36}\ ({\rm scales})\ ^{+30}_{-35}\ ({\rm PDFs})\ \ {\rm pb}$$

V. Ahrens, A. Ferroglia, M. Neubert, B. D. Pecjak, L. L. Yang, arXiv:1006.4682

# Next-to-Next-to-Leading Order

The NNLO calculation of the top-quark pair hadro-production requires several ingredients:

- Virtual Corrections
  - two-loop matrix elements for $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$
  - interference of one-loop diagrams

Körner et al. '05-'08; Anastasiou and Aybat '08

- Real Corrections
  - one-loop matrix elements for the hadronic production of $t\bar{t}$ + $1$ parton
  - tree-level matrix elements for the hadronic production of $t\bar{t}$ + $2$ partons

Dittmaier, Uwer and Weinzierl '07-'08

- Subtraction Terms
  - Both matrix elements known for $t\bar{t} + j$ calculation, BUT subtraction up to 1 unresolved parton, while in a complete NNLO computation of $\sigma_{t\bar{t}}$ we need subtraction terms with up to 2 unresolved partons.

  Need of an extension of the subtraction methods at the NNLO.

Gehrmann-De Ridder, Ritzmann '09, Daleo et al. '09, Boughezal et al. '10, Glover, Pires '10

Very recently double real in $\sigma_{t\bar{t}}$.

Czakon '10, Anastasiou, Herzog, Lazopoulos '10

# Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$

$$\mathcal{A}_2^{(2\times 0)} = N_c C_F\left[N_c^2 A + B + \frac{C}{N_c^2} + N_l\left(N_c D_l + \frac{E_l}{N_c}\right) + N_h\left(N_c D_h + \frac{E_h}{N_c}\right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h\right]$$

218 two-loop diagrams contribute to the 10 different color coefficients

- The whole $\mathcal{A}_2^{(2\times 0)}$ is known numerically

  Czakon '08.

- The coefficients $D_i$, $E_i$, $F_i$, and $A$ are known analytically (agreement with num res)

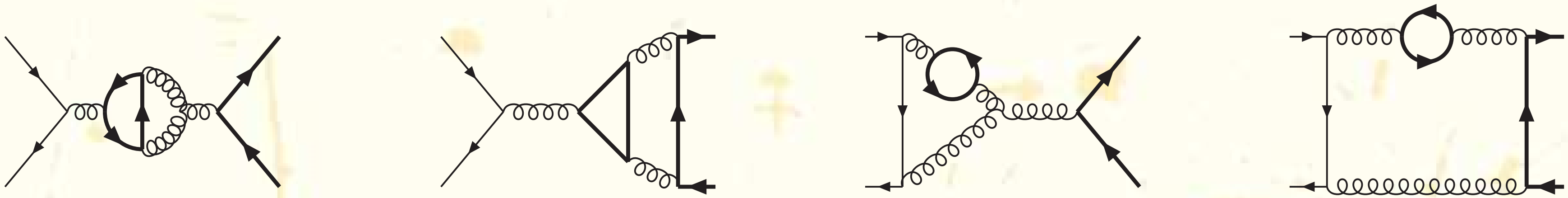
  R. B., Ferroglia, Gehrmann, Maitre, and Studerus '08-'09

- The poles of $\mathcal{A}_2^{(2\times 0)}$ (and therefore of $B$ and $C$) are known analytically

  Ferroglia, Neubert, Pecjak, and Li Yang '09

# Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

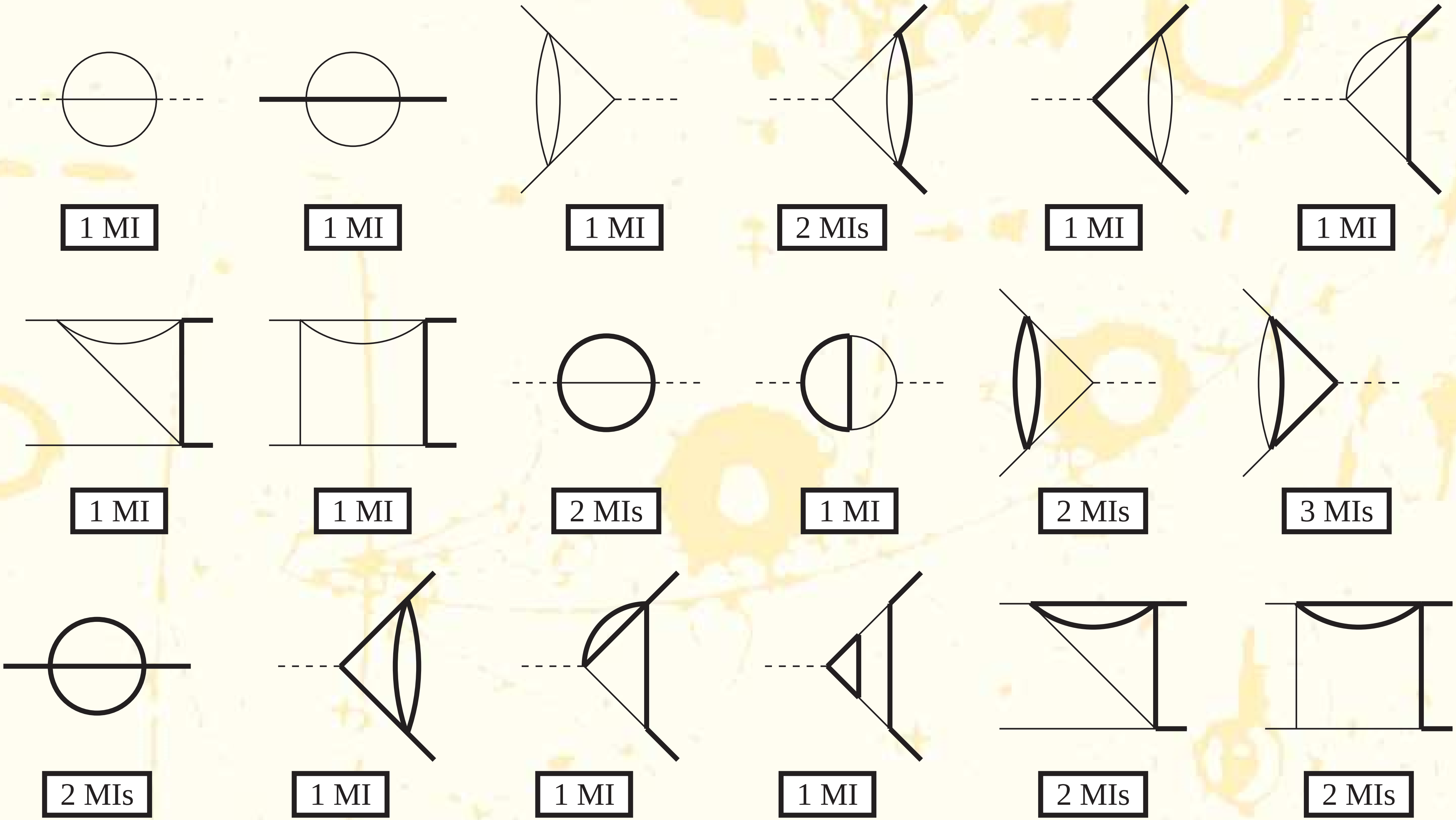
- $D_i$, $E_i$, $F_i$ come from the corrections involving a closed (light or heavy) fermionic loop:



- $A$ the leading-color coefficient, comes from the planar diagrams:

- The calculation is carried out analytically using:
  - Laporta Algorithm for the reduction of the dimensionally-regularized scalar integrals (in terms of which we express the $|\mathcal{M}|^2$) to the Master Integrals (MIs)
  - Differential Equations Method for the analytic solution of the MIs

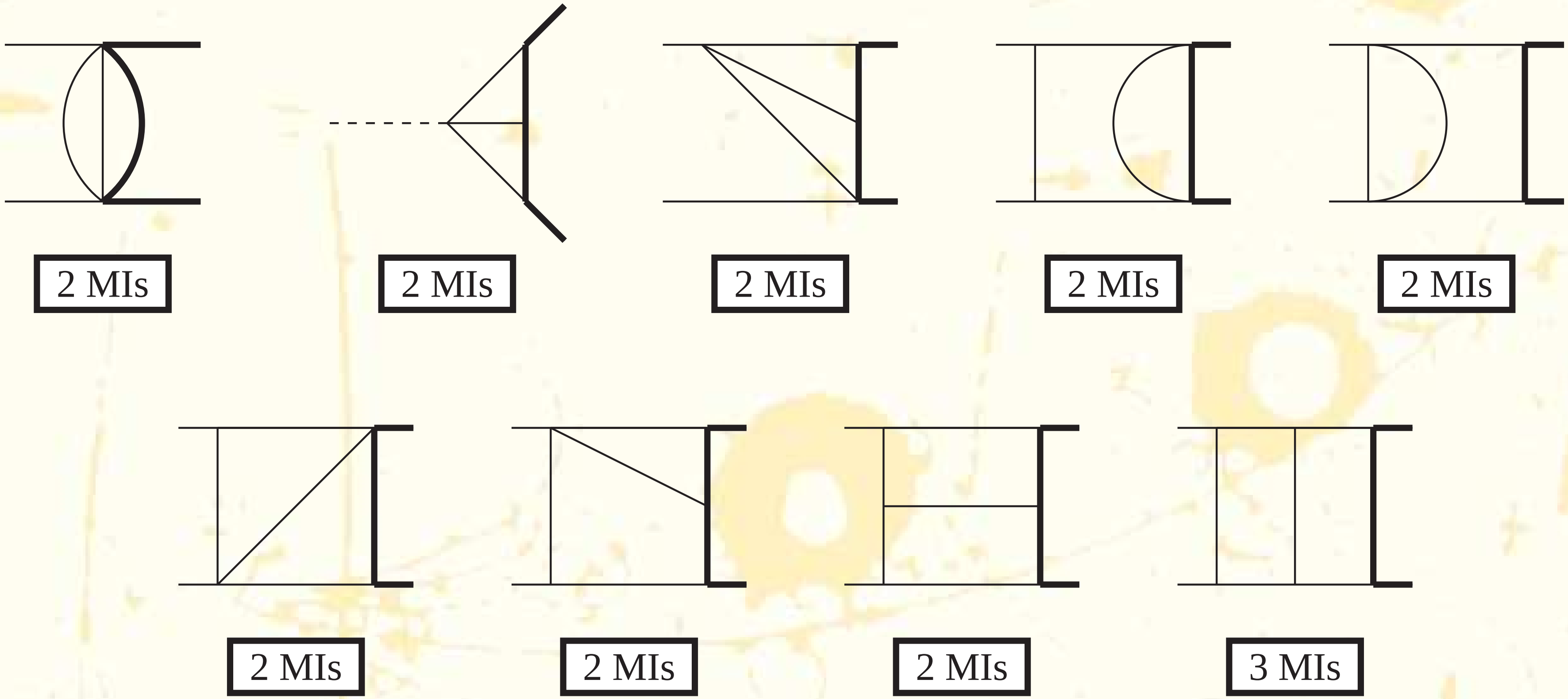
# Master Integrals for $N_l$ and $N_h$




18 irreducible two-loop topologies (26 MIs)

R. B., A. Ferroglia, T. Gehrmann, D. Maitre, and C. Studerus, JHEP **0807** (2008) 129.

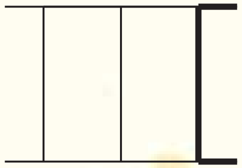
# Master Integrals for the Leading Color Coeff




For the leading color coefficient there are 9 additional irreducible topologies (19 MIs)

R. B., A. Ferroglia, T. Gehrmann, and C. Studerus, JHEP **0908** (2009) 067.

# Example: Box for the Leading Color Coeff



$$= \frac{1}{m^6} \sum_{i=-4}^{-1} A_i \epsilon^i + \mathcal{O}(\epsilon^0)$$

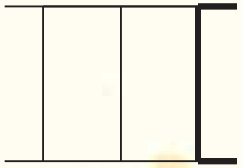
$$A_{-4} = \frac{x^2}{24(1-x)^4(1+y)},$$

$$A_{-3} = \frac{x^2}{96(1-x)^4(1+y)}\Big[-10G(-1;y) + 3G(0;x) - 6G(1;x)\Big],$$

$$A_{-2} = \frac{x^2}{48(1-x)^4(1+y)}\Big[-5\zeta(2) - 6G(-1;y)G(0;x) + 12G(-1;y)G(1;x) + 8G(-1,-1;y)\Big],$$

$$\begin{aligned} A_{-1} = \frac{x^2}{48(1-x)^4(1+y)}\Big[ & -13\zeta(3) + 38\zeta(2)G(-1;y) + 9\zeta(2)G(0;x) + 6\zeta(2)G(1;x) - 24\zeta(2)G(-1/y;x) \\ & +24G(0;x)G(-1,-1;y) - 24G(1;x)G(-1,-1;y) - 12G(-1/y;x)G(-1,-1;y) \\ & -12G(-y;x)G(-1,-1;y) - 6G(0;x)G(0,-1;y) + 6G(-1/y;x)G(0,-1;y) + 6G(-y;x)G(0,-1;y) \\ & +12G(-1;y)G(1,0;x) - 24G(-1;y)G(1,1;x) - 6G(-1;y)G(-1/y,0;x) + 12G(-1;y)G(-1/y,1;x) \\ & -6G(-1;y)G(-y,0;x) + 12G(-1;y)G(-y,1;x) + 16G(-1,-1,-1;y) - 12G(-1,0,-1;y) \\ & -12G(0,-1,-1;y) + 6G(0,0,-1;y) + 6G(1,0,0;x) - 12G(1,0,1;x) - 12G(1,1,0;x) + 24G(1,1,1;x) \\ & -6G(-1/y,0,0;x) + 12G(-1/y,0,1;x) + 6G(-1/y,1,0;x) - 12G(-1/y,1,1;x) + 6G(-y,1,0;x) \\ & -12G(-y,1,1;x)\Big] \end{aligned}$$

# Example: Box for the Leading Color Coeff



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$$A_{-2} = \frac{x^2}{48(1-x)^4(1+y)}\Big[-5\zeta(2) - 6G(-1;y)G(0;x) + 1 \ldots$$

$$A_{-1} = \frac{x^2}{48(1-x)^4(1+y)}\Big[-13\zeta(3) + 38\zeta(2)G(-1;y) + 9\zeta(2)G(0;x) + 6\zeta(2)G(1;x) - 24\zeta(2)G(-1/y;x)$$

$$+24G(0;x)G(-1,-1;y) - 24G(1;x)G(-1,-1;y) - 12G(-1/y;x)G(-1,-1;y)$$

$$-12G(-y;x)G(-1,-1;y) - 6G(0;x)G(0,-1;y) + 6G(-1/y;x)G(0,-1;y) + 6G(-y;x)G(0,-1;y)$$

$$+12G(-1;y)G(1,0;x) - 24G(-1;y)G(1,1;x) - 6G(-1;y)G(-1/y,0;x) + 12G(-1;y)G(-1/y,1;x)$$

$$-6G(-1;y)G(-y,0;x) - 12G(-1;y)G(-y,1;x) + 16G(-1,-1,-1;y) - 12G(-1,0,-1;y)$$

$$-12G(0,-1,-1;y) + 6G(0,0,-1;y) + 6G(1,0,0;x) - 12G(1,0,1;x) - 12G(1,1,0;x) + 24G(1,1,1;x)$$

$$-6G(-1/y,0,0;x) + 12G(-1/y,0,1;x) + 6G(-1/y,1,0;x) - 12G(-1/y,1,1;x) + 6G(-y,1,0;x)$$

$$-12G(-y,1,1;x)\Big]$$

$$\rho = \frac{4m_t^2}{\hat{s}} \to 1$$

1- and 2-dim GHPLs

# GHPLs

- One- and two-dimensional Generalized Harmonic Polylogarithms (GHPLs) are defined as repeated integrations over set of basic functions. In the case at hand

$$f_w(x) = \frac{1}{x-w}\,, \quad \text{with} \quad w \in \left\{0, 1, -1, -y, -\frac{1}{y}, \frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right\}$$

$$f_w(y) = \frac{1}{y-w}\,, \quad \text{with} \quad w \in \left\{0, 1, -1, -x, -\frac{1}{x}, 1 - \frac{1}{x} - x\right\}$$

- The weight-one GHPLs are defined as

$$G(0;x) = \ln x\,, \qquad G(w;x) = \int_0^x dt\, f_w(t)$$

- Higher weight GHPLs are defined by iterated integrations

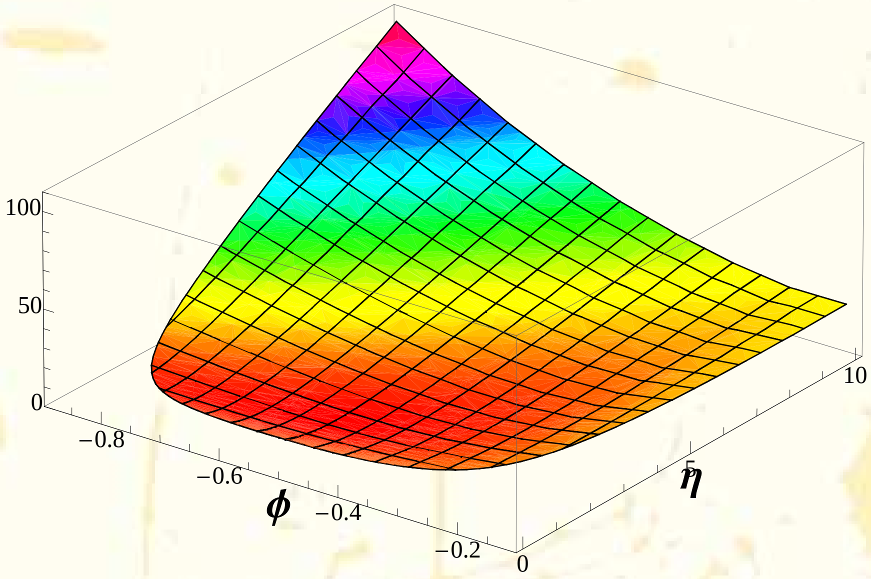
$$G(\underbrace{0,0,\cdots,0}_{n};x) = \frac{1}{n!}\ln^n x\,, \qquad G(w,\cdots;x) = \int_0^x dt\, f_w(t) G(\cdots;t)$$

- Shuffle algebra. Integration by parts identities

Remiddi and Vermaseren ’99, Gehrmann and Remiddi ’01-’02, Aglietti and R. B. ’03, Vollinga and Weinzierl ’04, R. B., A. Ferroglia, T. Gehrmann, and C. Studerus ’09

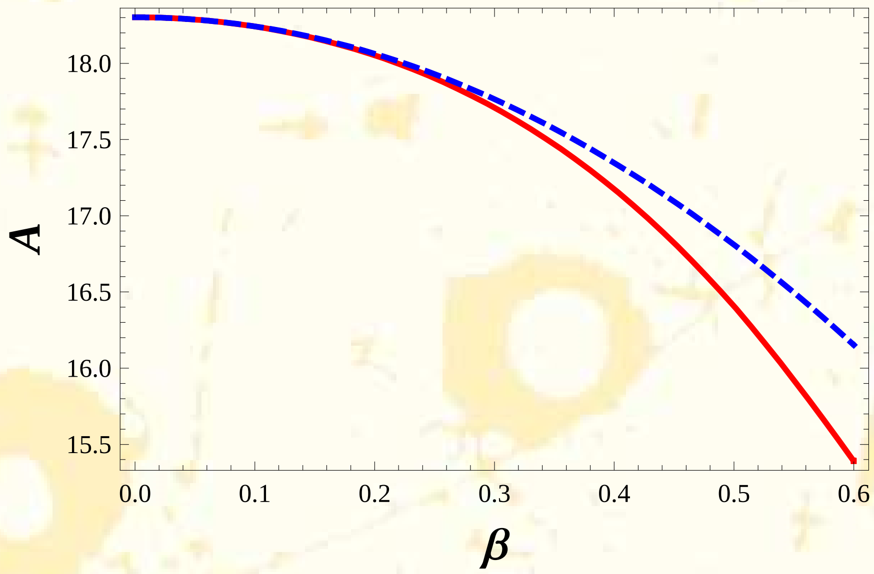
# Coefficient A in $q\bar{q}$

**Finite part of $A$**




$$\eta = \frac{s}{4m^2} - 1 \,, \quad \phi = -\frac{t-m^2}{s}$$

**Threshold expansion versus exact result**




$$\beta = \sqrt{1 - \frac{4m^2}{s}}$$

partonic c.m. scattering angle $= \frac{\pi}{2}$

Numerical evaluation of the GHPLs with GiNaC C++ routines (Vollinga and Weinzierl '04).

R. B., A. Ferroglia, T. Gehrmann, and C. Studerus, JHEP **0908** (2009) 067.

# Two-Loop Corrections to $gg \to t\bar{t}$

# Two-Loop Corrections to $gg \to t\bar{t}$

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times0)} + \mathcal{A}_2^{(1\times1)}$$

$$\begin{aligned}\mathcal{A}_2^{(2\times0)} \quad = \quad & (N_c^2-1)\Big(N_c^3 A + N_c B + \frac{1}{N_c} C + \frac{1}{N_c^3} D + N_c^2 N_l E_l + N_c^2 N_h E_h \\ & + N_l F_l + N_h F_h + \frac{N_l}{N_c^2} G_l + \frac{N_h}{N_c^2} G_h + N_c N_l^2 H_l + N_c N_h^2 H_h \\ & + N_c N_l N_h H_{lh} + \frac{N_l^2}{N_c} I_l + \frac{N_h^2}{N_c} I_h + \frac{N_l N_h}{N_c} I_{lh}\Big)\end{aligned}$$

789 two-loop diagrams contribute to 16 different color coefficients

- No numeric result for $\mathcal{A}_2^{(2\times0)}$ yet
- The poles of $\mathcal{A}_2^{(2\times0)}$ are known analytically

Ferroglia, Neubert, Pecjak, and Li Yang '09

- The coefficient $A$ is done. $E_l$–$I_l$ can be evaluated analytically as for the $q\bar{q}$ channel

R. B., Ferroglia, Gehrmann, von Manteuffel and Studerus '10, in preparation

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$$+ N_c N_l N_h H_{lh} + \frac{N_l^2}{N_c} I_l + \ldots$$

For the leading-color coefficient

**NO additional MI**

789 two-loop diagrams contribute to 16 different c...

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R. B., Ferroglia, Gehrmann, von Manteuffel and Studerus '10, in preparation

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- For the light-fermion contrib

14 additional MIs

different color coefficients

lytically

Ferroglia, Neubert, Pecjak, and Li Yang '09

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R. B., Ferroglia, Gehrmann, von Manteuffel and Studerus '10, in preparation

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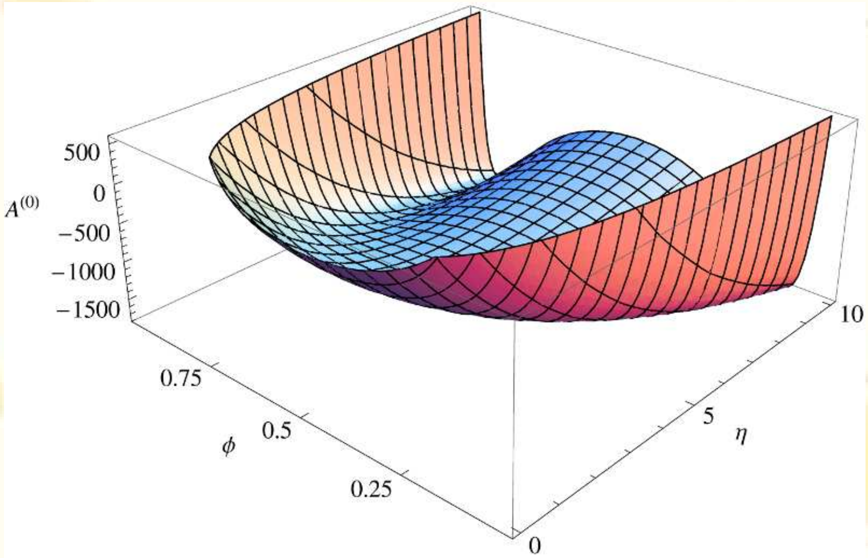


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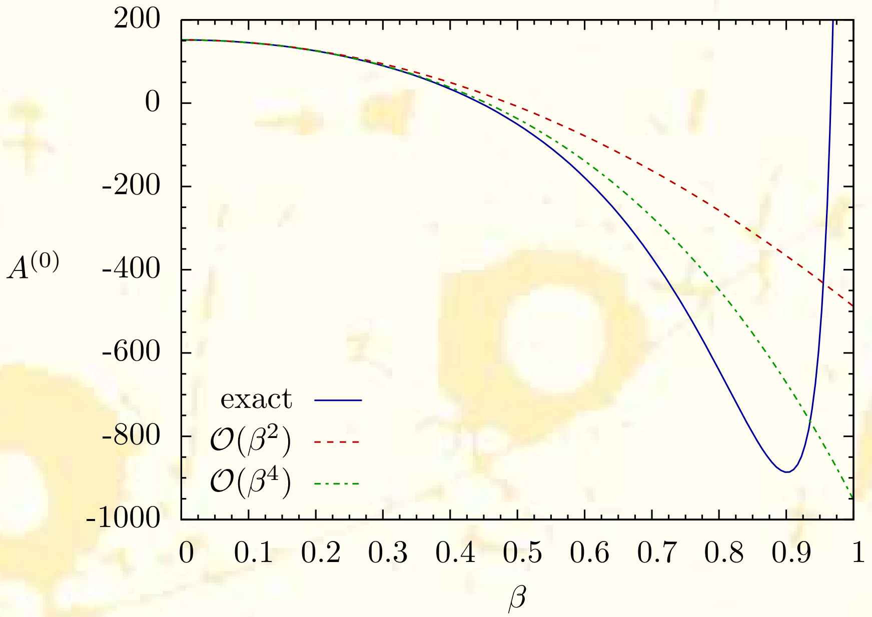


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R. B., A. Ferroglia, T. Gehrmann, and C. Studerus, arXiv:1011.6661

# Conclusions

- In the last 15 years, Tevatron explored top-quark properties reaching a remarkable experimental accuracy. The top mass could be measured with $\Delta m_t/m_t = 0.75\%$ and the production cross section with $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} = 9\%$. Other observables could be measured only with bigger errors.
- At the LHC the situation will further improve. The production cross section of $t\bar{t}$ pairs is expected to reach the accuracy of $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} = 5\%$!!
- This experimental precision demands for more accurate theoretical predictions. Quantum corrections have to be unavoidably taken into account.
- For the production cross section, $\sigma_{t\bar{t}}$, a complete NNLO analysis is mandatory in order to reach the experimental accuracy expected in 3-4 years from now.
- In spite of a big activity of different groups, many ingredients are still missing.
- In this talk I briefly reviewed the analytic evaluation of the two-loop matrix elements, afforded using the Laporta algorithm for the reduction to the MIs and the Differential Equations method for their analytic evaluation. To date, the corrections involving a fermionic loop (light or heavy) in the $q\bar{q}$ channel are completed, together with the leading color coefficients in both channels. Light-fermion corrections in the $gg$ channel can be calculated with the same technique and are at the moment under study.