Tribimaximal Mixing From Small Groups

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Based on arXiv:1012.2842 in collaboration w/Krishna Mohan Parattu

Neutrino Mixing Matrix

- Neutrinos have mass and the different flavors can mix

Super-Kamiokande Collaboration, Y. Fukuda *et al.*, "Evidence for oscillation of atmospheric neutrinos," *Phys. Rev. Lett.* **81** (1998) 1562–1567, hep-ex/9807003

SNO Collaboration, Q. R. Ahmad *et al.*, "Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory," *Phys. Rev. Lett.* **89** (2002) 011301, nucl-ex/0204008.

- Pontecorvo-Maki-Nakagawa-Sakata matrix

B. Pontecorvo, "Mesonium and antimesonium," *Sov. Phys. JETP* **6** (1957) 429.

Z. Maki, M. Nakagawa, and S. Sakata, "Remarks on the unified model of elementary particles," *Prog. Theor. Phys.* **28** (1962) 870–880.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\mathrm{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

- Completely analogous to the CKM matrix in the quark sector (except that for CKM, the down-type quarks are rotated)

Neutrino Mixing Matrix

What we know about the mixing angles ...

$$U_{\rm PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

M. C. Gonzalez-Garcia, M. Maltoni, and J. Salvado, "Updated global fit to three neutrino mixing: status of the hints of $\theta_{13} > 0$," *JHEP* **04** (2010) 056, 1001.4524.

Parameter	Mean Value	1σ range	3σ range
θ_{12}	34.4°	$33.4^\circ - 35.4^\circ$	$31.5^\circ - 37.6^\circ$
θ_{23}	42.8°	$39.9^\circ - 47.5^\circ$	$35.5^\circ - 53.5^\circ$
θ_{13}	5.6°	$2.9^\circ - 8.6^\circ$	$0^\circ - 12.5^\circ$

Harrison-Perkins-Scott Matrix

Presently our best guess ...

P. F. Harrison, D. H. Perkins, and W. G. Scott, "Tri-bimaximal mixing and the neutrino oscillation data," *Phys. Lett.* **B530** (2002) 167, hep-ph/0202074.

$$U_{\mathrm{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Suggestive of an underlying symmetry ...

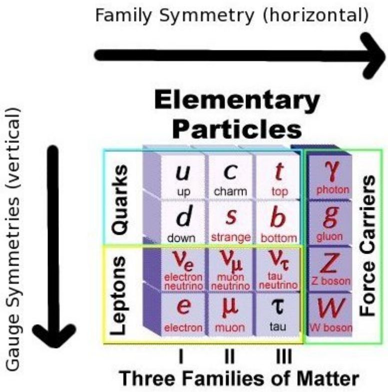
Some groups that have been considered in the literature:

Review: G. Altarelli and F. Feruglio, "Discrete Flavor Symmetries and Models of Neutrino Mixing," 1002.0211.

$$S_3,\quad D_4,\quad D_7,\quad A_4,\quad A_5,\quad \widetilde{T},\quad S_4,\quad (\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes_\varphi \mathbb{Z}_3,\quad C_7 \rtimes_\varphi \mathbb{Z}_3,\quad \mathrm{PSL}_2(7)$$

Horizontal Symmetries

➢ Introduce relations between families of quarks and leptons



Altarelli-Feruglio Model Revisited

G. Altarelli and F. Feruglio, "Tri-Bimaximal Neutrino Mixing, A4 and the Modular Symmetry," *Nucl. Phys.* **B741** (2006) 215–235, hep-ph/0512103.

❶ Symmetries of the model

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_R \times A_4 \times \mathbb{Z}_3$$

❷ Particle content and charges

Field	$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	$\mathrm{U}(1)_R$	A_4	$\mathbb{Z}_3$	$A_4 \times \mathbb{Z}_3$
L	$(\mathbf{2}, -1)$	1	$\mathbf{3}$	ω	$\mathbf{3'}$
e	$(\mathbf{1}, 2)$	1	$\mathbf{1}$	ω^2	$\mathbf{1'}$
μ	$(\mathbf{1}, 2)$	1	$\mathbf{1''}$	ω^2	$\mathbf{1^{(8)}}$
τ	$(\mathbf{1}, 2)$	1	$\mathbf{1'}$	ω^2	$\mathbf{1^{(5)}}$
h_u	$(\mathbf{2}, 1)$	0	$\mathbf{1}$	1	$\mathbf{1}$
h_d	$(\mathbf{2}, -1)$	0	$\mathbf{1}$	1	$\mathbf{1}$
φ_T	$(\mathbf{1}, 0)$	0	$\mathbf{3}$	1	$\mathbf{3}$
φ_S	$(\mathbf{1}, 0)$	0	$\mathbf{3}$	ω	$\mathbf{3'}$
ξ	$(\mathbf{1}, 0)$	0	$\mathbf{1}$	ω	$\mathbf{1''}$

❸ Breaking the family symmetry

$$\varphi_T = (v_T, v_T, v_T), \;\; \varphi_S = (v_S, 0, 0), \;\; \xi = v_\xi,$$

Group Information from GAP

The GAP Group, "GAP – Groups, Algorithms, and Programming, Version 4.4.12.", http://www.gap-system.org

"GAP is a system for computational discrete algebra, with particular emphasis on Computational Group Theory."

```
group := SmallGroup(36,11);;
Display(StructureDescription(group));
chartab := Irr(group);;
Display(chartab);
SizesConjugacyClasses(CharacterTable(group));
LoadPackage("repsn");;
for i in [1..Size(chartab)] do
  rep := IrreducibleAffordingRepresentation(chartab[i]);
  for el in Elements(group) do
    Display(el^rep);
  od;
od;
```

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➢ Specify the group that we will work with

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➢ The "human readable" name of the group

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➢ The character table

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➢ Dimensions of the conjugacy classes

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    Display(el^rep);
  od;
od;
```

➢ The matrices for the representations

The Character Table of $\mathbf{A_4} \times \mathbb{Z}_3$

	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	K_{10}	K_{11}	K_{12}
$\mathbf{1}$	1	1	1	1	1	1	1	1	1	1	1	1
$\mathbf{1'}$	1	1	ω^2	1	1	ω^2	ω	ω^2	ω^2	ω	ω	ω
$\mathbf{1''}$	1	1	ω	1	1	ω	ω^2	ω	ω	ω^2	ω^2	ω^2
$\mathbf{1'''}$	1	ω^2	1	1	ω	ω^2	1	1	ω	ω^2	1	ω
$\mathbf{1^{(4)}}$	1	ω	1	1	ω^2	ω	1	1	ω^2	ω	1	ω^2
$\mathbf{1^{(5)}}$	1	ω^2	ω^2	1	ω	ω	ω	ω^2	1	1	ω	ω^2
$\mathbf{1^{(6)}}$	1	ω	ω	1	ω^2	ω^2	ω^2	ω	1	1	ω^2	ω
$\mathbf{1^{(7)}}$	1	ω^2	ω	1	ω	1	ω^2	ω	ω^2	ω	ω^2	1
$\mathbf{1^{(8)}}$	1	ω	ω^2	1	ω^2	1	ω	ω^2	ω	ω^2	ω	1
$\mathbf{3}$	3	0	3	-1	0	0	3	-1	0	0	-1	0
$\mathbf{3'}$	3	0	3ω	-1	0	0	$3\omega^2$	ω	0	0	$1+\omega$	0
$\mathbf{3''}$	3	0	$3\omega^2$	-1	0	0	3ω	$1+\omega$	0	0	ω	0

$\omega = e^{2\pi i/3}$ is the primitive third root of unity

Decomposition of Tensor Products

From the character table and the dimensions of the conjugacy classes:

$$\begin{gathered}
\mathbf{1} \otimes \mathbf{1} = \mathbf{1} \\
\mathbf{1} \otimes \mathbf{1}^{(5)} = \mathbf{1}^{(5)} \\
\mathbf{1} \otimes \mathbf{3}' = \mathbf{3}' \\
\mathbf{1}' \otimes \mathbf{1}^{(4)} = \mathbf{1}^{(8)} \\
\mathbf{1}' \otimes \mathbf{3} = \mathbf{3}'' \\
\mathbf{1}'' \otimes \mathbf{1}^{(4)} = \mathbf{1}^{(6)} \\
\mathbf{1}'' \otimes \mathbf{3} = \mathbf{3}' \\
\mathbf{1}''' \otimes \mathbf{1}^{(5)} = \mathbf{1}^{(8)} \\
\mathbf{1}''' \otimes \mathbf{3}' = \mathbf{3}' \\
\mathbf{1}^{(4)} \otimes \mathbf{1}^{(7)} = \mathbf{1}'' \\
\mathbf{1}^{(5)} \otimes \mathbf{1}^{(5)} = \mathbf{1}^{(6)} \\
\mathbf{1}^{(5)} \otimes \mathbf{3}' = \mathbf{3} \\
\mathbf{1}^{(6)} \otimes \mathbf{3} = \mathbf{3}' \\
\mathbf{1}^{(7)} \otimes \mathbf{3} = \mathbf{3}' \\
\mathbf{1}^{(8)} \otimes \mathbf{3}' = \mathbf{3}
\end{gathered}$$

$$\begin{gathered}
\mathbf{1} \otimes \mathbf{1}' = \mathbf{1}' \\
\mathbf{1} \otimes \mathbf{1}^{(6)} = \mathbf{1}^{(6)} \\
\mathbf{1} \otimes \mathbf{3}'' = \mathbf{3}'' \\
\mathbf{1}' \otimes \mathbf{1}^{(5)} = \mathbf{1}^{(7)} \\
\mathbf{1}' \otimes \mathbf{3}' = \mathbf{3} \\
\mathbf{1}'' \otimes \mathbf{1}^{(5)} = \mathbf{1}''' \\
\mathbf{1}'' \otimes \mathbf{3}' = \mathbf{3}'' \\
\mathbf{1}''' \otimes \mathbf{1}^{(6)} = \mathbf{1}'' \\
\mathbf{1}''' \otimes \mathbf{3}'' = \mathbf{3}'' \\
\mathbf{1}^{(4)} \otimes \mathbf{1}^{(8)} = \mathbf{1}^{(5)} \\
\mathbf{1}^{(5)} \otimes \mathbf{1}^{(6)} = \mathbf{1} \\
\mathbf{1}^{(5)} \otimes \mathbf{3}'' = \mathbf{3}' \\
\mathbf{1}^{(6)} \otimes \mathbf{3}' = \mathbf{3}'' \\
\mathbf{1}^{(7)} \otimes \mathbf{3}' = \mathbf{3}'' \\
\mathbf{1}^{(8)} \otimes \mathbf{3}'' = \mathbf{3}'
\end{gathered}$$

$$\begin{gathered}
\mathbf{1} \otimes \mathbf{1}'' = \mathbf{1}'' \\
\mathbf{1} \otimes \mathbf{1}^{(7)} = \mathbf{1}^{(7)} \\
\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}'' \\
\mathbf{1}' \otimes \mathbf{1}^{(6)} = \mathbf{1}^{(4)} \\
\mathbf{1}' \otimes \mathbf{3}'' = \mathbf{3}' \\
\mathbf{1}'' \otimes \mathbf{1}^{(6)} = \mathbf{1}^{(8)} \\
\mathbf{1}'' \otimes \mathbf{3}'' = \mathbf{3} \\
\mathbf{1}''' \otimes \mathbf{1}^{(7)} = \mathbf{1}^{(6)} \\
\mathbf{1}^{(4)} \otimes \mathbf{1}^{(4)} = \mathbf{1}''' \\
\mathbf{1}^{(4)} \otimes \mathbf{3} = \mathbf{3} \\
\mathbf{1}^{(5)} \otimes \mathbf{1}^{(7)} = \mathbf{1}^{(4)} \\
\mathbf{1}^{(6)} \otimes \mathbf{1}^{(6)} = \mathbf{1}^{(5)} \\
\mathbf{1}^{(6)} \otimes \mathbf{3}'' = \mathbf{3} \\
\mathbf{1}^{(7)} \otimes \mathbf{3}'' = \mathbf{3}
\end{gathered}$$

$$\begin{gathered}
\mathbf{1} \otimes \mathbf{1}''' = \mathbf{1}''' \\
\mathbf{1} \otimes \mathbf{1}^{(8)} = \mathbf{1}^{(8)} \\
\mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1} \\
\mathbf{1}' \otimes \mathbf{1}^{(7)} = \mathbf{1}''' \\
\mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}' \\
\mathbf{1}'' \otimes \mathbf{1}^{(7)} = \mathbf{1}^{(5)} \\
\mathbf{1}''' \otimes \mathbf{1}''' = \mathbf{1}^{(4)} \\
\mathbf{1}''' \otimes \mathbf{1}^{(8)} = \mathbf{1}' \\
\mathbf{1}^{(4)} \otimes \mathbf{1}^{(5)} = \mathbf{1}' \\
\mathbf{1}^{(4)} \otimes \mathbf{3}' = \mathbf{3}' \\
\mathbf{1}^{(5)} \otimes \mathbf{1}^{(8)} = \mathbf{1}'' \\
\mathbf{1}^{(6)} \otimes \mathbf{1}^{(7)} = \mathbf{1}' \\
\mathbf{1}^{(7)} \otimes \mathbf{1}^{(7)} = \mathbf{1}^{(8)} \\
\mathbf{1}^{(8)} \otimes \mathbf{1}^{(8)} = \mathbf{1}^{(7)}
\end{gathered}$$

$$\begin{gathered}
\mathbf{1} \otimes \mathbf{1}^{(4)} = \mathbf{1}^{(4)} \\
\mathbf{1} \otimes \mathbf{3} = \mathbf{3} \\
\mathbf{1}' \otimes \mathbf{1}''' = \mathbf{1}^{(5)} \\
\mathbf{1}' \otimes \mathbf{1}^{(8)} = \mathbf{1}^{(6)} \\
\mathbf{1}'' \otimes \mathbf{1}''' = \mathbf{1}^{(7)} \\
\mathbf{1}'' \otimes \mathbf{1}^{(8)} = \mathbf{1}^{(4)} \\
\mathbf{1}''' \otimes \mathbf{1}^{(4)} = \mathbf{1} \\
\mathbf{1}''' \otimes \mathbf{3} = \mathbf{3} \\
\mathbf{1}^{(4)} \otimes \mathbf{1}^{(6)} = \mathbf{1}^{(7)} \\
\mathbf{1}^{(4)} \otimes \mathbf{3}'' = \mathbf{3}'' \\
\mathbf{1}^{(5)} \otimes \mathbf{3} = \mathbf{3}'' \\
\mathbf{1}^{(6)} \otimes \mathbf{1}^{(8)} = \mathbf{1}''' \\
\mathbf{1}^{(7)} \otimes \mathbf{1}^{(8)} = \mathbf{1} \\
\mathbf{1}^{(8)} \otimes \mathbf{3} = \mathbf{3}''
\end{gathered}$$

$$\begin{gathered}
\mathbf{3} \otimes \mathbf{3} = \mathbf{1} + \mathbf{1}''' + \mathbf{1}^{(4)} + 2 \otimes \mathbf{3} \\
\mathbf{3} \otimes \mathbf{3}'' = \mathbf{1}' + \mathbf{1}^{(5)} + \mathbf{1}^{(8)} + 2 \otimes \mathbf{3}'' \\
\mathbf{3}' \otimes \mathbf{3}'' = \mathbf{1} + \mathbf{1}''' + \mathbf{1}^{(4)} + 2 \otimes \mathbf{3}
\end{gathered}$$

$$\begin{gathered}
\mathbf{3} \otimes \mathbf{3}' = \mathbf{1}'' + \mathbf{1}^{(6)} + \mathbf{1}^{(7)} + 2 \otimes \mathbf{3}' \\
\mathbf{3}' \otimes \mathbf{3}' = \mathbf{1}' + \mathbf{1}^{(5)} + \mathbf{1}^{(8)} + 2 \otimes \mathbf{3}'' \\
\mathbf{3}'' \otimes \mathbf{3}'' = \mathbf{1}'' + \mathbf{1}^{(6)} + \mathbf{1}^{(7)} + 2 \otimes \mathbf{3}'
\end{gathered}$$

Invariant Lagrangian

- Terms that are invariant, have 2 leptons and mass dimension ≤ 5:

$$L\,L\,h_u\,h_u\,\varphi_S + L\,L\,h_u\,h_u\,\xi + L\,e\,h_d\,\varphi_T + L\,\mu\,h_d\,\varphi_T + L\,\tau\,h_d\,\varphi_T$$

Invariant Lagrangian

➢ Terms that are invariant, have 2 leptons and mass dimension ≤ 5:

$$L\,L\,h_u\,h_u\,\varphi_S + L\,L\,h_u\,h_u\,\xi + L\,e\,h_d\,\varphi_T + L\,\mu\,h_d\,\varphi_T + L\,\tau\,h_d\,\varphi_T$$

$$\mathbf{3'}\otimes\mathbf{3'}\otimes\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{3'}=\left(\mathbf{1'}+\mathbf{1^{(5)}}+\mathbf{1^{(8)}}+2\times\mathbf{3''}\right)\otimes\mathbf{3'}=2\times\mathbf{1}+2\times\mathbf{1'''}+2\times\mathbf{1^{(4)}}+7\times\mathbf{3}$$

Invariant Lagrangian

➢ Terms that are invariant, have 2 leptons and mass dimension ≤ 5:

$$L\,L\,h_u\,h_u\,\varphi_S + L\,L\,h_u\,h_u\,\xi + L\,e\,h_d\,\varphi_T + L\,\mu\,h_d\,\varphi_T + L\,\tau\,h_d\,\varphi_T$$

$$\mathbf{3'}\otimes\mathbf{3'}\otimes\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{3'}=\left(\mathbf{1'}+\mathbf{1^{(5)}}+\mathbf{1^{(8)}}+2\times\mathbf{3''}\right)\otimes\mathbf{3'}=2\times\mathbf{1}+2\times\mathbf{1'''}+2\times\mathbf{1^{(4)}}+7\times\mathbf{3}$$

➢ Contract family indices (need to know Clebsch-Gordan coefficients):

$$\frac{1}{\sqrt{3}}L_2\,L_3\,h_u\,h_u\,\varphi_{S,1}+\frac{1}{\sqrt{3}}L_3\,L_1\,h_u\,h_u\,\varphi_{S,2}+\frac{1}{\sqrt{3}}L_1\,L_2\,h_u\,h_u\,\varphi_{S,3}+\frac{1}{\sqrt{3}}L_1\,L_1\,h_u\,h_u\,\xi$$
$$+\frac{1}{\sqrt{3}}L_2\,L_2\,h_u\,h_u\,\xi+\frac{1}{\sqrt{3}}L_3\,L_3\,h_u\,h_u\,\xi$$

Invariant Lagrangian

- Terms that are invariant, have 2 leptons and mass dimension ≤ 5:

$$L\,L\,h_u\,h_u\,\varphi_S + L\,L\,h_u\,h_u\,\xi + L\,e\,h_d\,\varphi_T + L\,\mu\,h_d\,\varphi_T + L\,\tau\,h_d\,\varphi_T$$

$$\mathbf{3'}\otimes\mathbf{3'}\otimes\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{3'}=\left(\mathbf{1'}+\mathbf{1^{(5)}}+\mathbf{1^{(8)}}+2\times\mathbf{3''}\right)\otimes\mathbf{3'}=2\times\mathbf{1}+2\times\mathbf{1'''}+2\times\mathbf{1^{(4)}}+7\times\mathbf{3}$$

- Contract family indices (need to know Clebsch-Gordan coefficients):

$$\tfrac{1}{\sqrt{3}}L_2\,L_3\,h_u\,h_u\,\varphi_{S,1}+\tfrac{1}{\sqrt{3}}L_3\,L_1\,h_u\,h_u\,\varphi_{S,2}+\tfrac{1}{\sqrt{3}}L_1\,L_2\,h_u\,h_u\,\varphi_{S,3}+\tfrac{1}{\sqrt{3}}L_1\,L_1\,h_u\,h_u\,\xi$$
$$+\tfrac{1}{\sqrt{3}}L_2\,L_2\,h_u\,h_u\,\xi+\tfrac{1}{\sqrt{3}}L_3\,L_3\,h_u\,h_u\,\xi$$

- Contract $\mathrm{SU}(2)$ indices and substitute *vevs* $\langle\varphi_S\rangle = (v_S, 0, 0)$, etc:

$$\tfrac{1}{\sqrt{3}}L_2^{(1)}\,L_3^{(1)}\,v_u\,v_u\,v_S+\tfrac{1}{\sqrt{3}}L_1^{(1)}\,L_1^{(1)}\,v_u\,v_u\,\xi+\tfrac{1}{\sqrt{3}}L_2^{(1)}\,L_2^{(1)}\,v_u\,v_u\,\xi+\tfrac{1}{\sqrt{3}}L_3^{(1)}\,L_3^{(1)}\,v_u\,v_u\,\xi$$

Finally, the Mixing Angles

- Mass matrices

$$M_{\ell^+} = \begin{array}{c} \\ L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} \begin{array}{c} \begin{array}{ccc} e & \mu & \tau \end{array} \\ \begin{pmatrix} -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \end{pmatrix} \end{array}, \qquad M_\nu = \begin{array}{c} \\ L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} \begin{array}{c} \begin{array}{ccc} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{array} \\ \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ 0 & \frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \end{array}$$

Finally, the Mixing Angles

➢ Mass matrices

$$M_{\ell^+} = \begin{matrix} & e & \mu & \tau \\ L_1^{(2)} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ L_2^{(2)} & -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ L_3^{(2)} & -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \end{matrix}, \qquad M_\nu = \begin{matrix} & L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \\ L_1^{(1)} & \frac{1}{\sqrt{3}} & 0 & 0 \\ L_2^{(1)} & 0 & \frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ L_3^{(1)} & 0 & \frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \end{matrix}$$

➢ Singular value decomposition: $\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$

$$D_\mathrm{L} = \begin{pmatrix} -0.5774+i\,0.0000 & -0.5774+i\,0.0000 & -0.5774+i\,0.0000 \\ 0.5738-i\,0.0636 & -0.2319+i\,0.5287 & -0.3420-i\,0.4652 \\ 0.5731-i\,0.0702 & -0.3474-i\,0.4612 & -0.2257+i\,0.5314 \end{pmatrix}, \quad U_\mathrm{L} = \begin{pmatrix} 0.0000 & -0.7071 & -0.7071 \\ -1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7071 & -0.7071 \end{pmatrix}$$

Finally, the Mixing Angles

➢ Mass matrices

$$M_{\ell^+} = \begin{array}{c} \\ L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} \begin{array}{c} \begin{array}{ccc} e & \mu & \tau \end{array} \\ \begin{pmatrix} -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \end{pmatrix} \end{array}, \qquad M_\nu = \begin{array}{c} \\ L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} \begin{array}{c} \begin{array}{ccc} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{array} \\ \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ 0 & \frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \end{array}$$

➢ Singular value decomposition: $\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$

$$D_\mathrm{L} = \begin{pmatrix} -0.5774+i\,0.0000 & -0.5774+i\,0.0000 & -0.5774+i\,0.0000 \\ 0.5738-i\,0.0636 & -0.2319+i\,0.5287 & -0.3420-i\,0.4652 \\ 0.5731-i\,0.0702 & -0.3474-i\,0.4612 & -0.2257+i\,0.5314 \end{pmatrix}, \quad U_\mathrm{L} = \begin{pmatrix} 0.0000 & -0.7071 & -0.7071 \\ -1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7071 & -0.7071 \end{pmatrix}$$

➢ Neutrino mixing matrix: $U_\mathrm{PMNS} = D_\mathrm{L} U_\mathrm{L}^\dagger$ (needs rephasing)

$$U_\mathrm{PMNS} = \begin{pmatrix} 0.8165 + i\,0.0000 & 0.5774 + i\,0.0000 & 0.0000 + i\,0.0000 \\ 0.4058 - i\,0.0449 & -0.5738 + i\,0.0636 & 0.0778 + i\,0.7028 \\ 0.4052 - i\,0.0497 & -0.5731 + i\,0.0702 & -0.0860 - i\,0.7019 \end{pmatrix}$$

➢ Mixing angles: $\theta_{12} = 35.26, \quad \theta_{23} = 45.00, \quad \theta_{13} = 0.00$ Tribimaximal ✔

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$$|U_{\rm PMNS}| = \begin{pmatrix} 0.8165 & 0.5774 & 0.0000 \\ 0.4082 & 0.5774 & 0.7071 \\ 0.4082 & 0.5774 & 0.7071 \end{pmatrix}$$

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Finally, the Mixing Angles

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➢ Neutrino mixing matrix: $U_\mathrm{PMNS} = D_\mathrm{L} U_\mathrm{L}^\dagger$ (needs rephasing)

$$|U_\mathrm{PMNS}| = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

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First Generalization

❶ Generalize family symmetry:

$$A_4 \times \mathbb{Z}_3 \;\rightarrow\; \text{1048 groups of order} \leq 100 \;\rightarrow\; \text{90 w/3-dim irreps}$$

❷ Keep same particle content:

$$L, \quad e, \quad \mu, \quad \tau, \quad h_u, \quad h_d, \quad \varphi_T, \quad \varphi_S, \quad \xi$$

❸ Generalize family charge assignments:

$$(L, e, \mu, \tau, h_u, h_d, \varphi_T, \varphi_S, \xi) \rightarrow (\mathbf{3}', \mathbf{1}', \mathbf{1}^{(8)}, \mathbf{1}^{(5)}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{3}', \mathbf{1}'')$$
$$\rightarrow (*, \; *, \; *, \; *, \; *, \; *, \; *, \; *, \; *)$$

❹ Generalize symmetry breaking patterns:

$$\langle \varphi_T \rangle = (*, *, *), \quad \langle \varphi_S \rangle = (*, *, *), \quad \langle \xi \rangle = (*, *, *)$$

Results for $\mathbf{A_4} \times \mathbb{Z}_3$

➢ We consider 2 models equivalent, if their Lagrangians are the same after contracting the family indices, but before the vevs are substituted

➢ In this sense, we have 39,900 inequivalent models/Lagrangians

➢ 22,932 models have non-singular charged lepton and neutrino mass matrices:

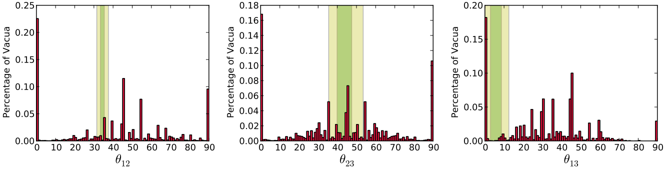
$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \qquad \hat{M}_\nu = U_L M_\nu U_R^\dagger, \qquad U_{\mathrm{PMNS}} \equiv D_{\mathrm{L}} U_{\mathrm{L}}^\dagger$$

➢ 4,481 consistent w/experiment at 3σ level (19.5%)

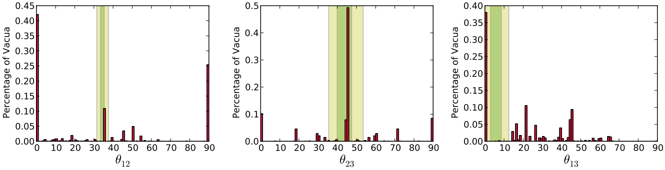
➢ 4,233 are tribimaximal (18.5%)

➢ Probably largest set of viable neutrino models ever constructed!

Most Likely Mixing Angles

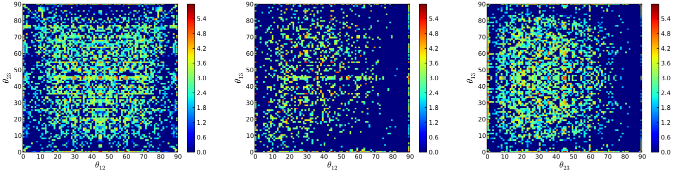


(a) Number of models that give θ_{ij} with no constraints on the other 2 angles. Each histogram has 15992118 entries.

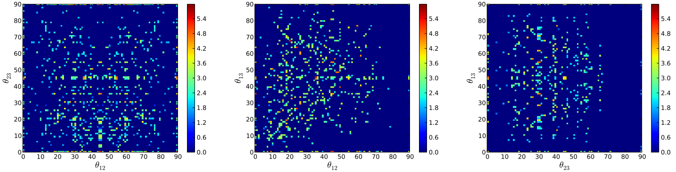


(b) Number of models that give θ_{ij} with the other 2 angles restricted to their 3σ interval. The histograms have 838289, 148886 and 225844 entries, respectively.

Correlation Between Pairs of Mixing Angles

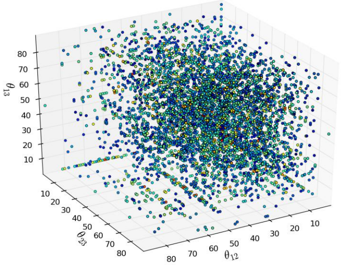


(c) Number of models that give θ_{ij} and θ_{mn} with no constraint on the remaining angle. Each histogram has 15992118 entries.

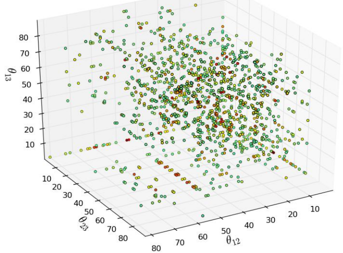


(d) Number of models that give θ_{ij} and θ_{mn} with the remaining angle restricted to its 3σ interval. The histograms have 2941000, 3675600 and 1057170 entries, respectively.

Correlation Between All Mixing Angles



(e) The 5528 bins that are ≥ 1.



(f) The 1287 bins that are ≥ 1000.

Are We Looking Under the Lamppost?

Second Generalization

❶ Generalize family symmetry:

$$A_4 \times \mathbb{Z}_3 \;\rightarrow\; 1048 \text{ groups of order} \leq 100 \;\rightarrow\; 90 \text{ w/3-dim irreps}$$

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$$L, \quad e, \quad \mu, \quad \tau, \quad h_u, \quad h_d, \quad \varphi_T, \quad \varphi_S, \quad \xi$$

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$$(L, e, \mu, \tau, h_u, h_d, \varphi_T, \varphi_S, \xi) \rightarrow (\mathbf{3}', \mathbf{1}', \mathbf{1}^{(8)}, \mathbf{1}^{(5)}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{3}', \mathbf{1}'')$$
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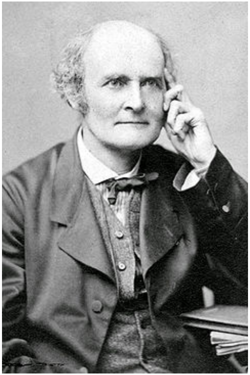
Small Groups

Arthur Cayley (1821-1895) is the first to systematically construct groups; in 1854, he determined all groups of order 4 and 6 . . .



Small Groups

The first few of the 1048 groups of order ≤ 100
✔ $=$ U(n) and ✔ $=$ SU(n) for $n = 2, 3$

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[1, 1]	1	✗	---	---	---	✗
[2, 1]	C_2	✗	---	---	---	✗
[3, 1]	C_3	✗	---	---	---	✗
[4, 1]	C_4	✗	---	---	---	✗
[4, 2]	$C_2 \times C_2$	✗	---	---	---	✗
[5, 1]	C_5	✗	---	---	---	✗
[6, 1]	S_3	✗	✔ (SU)	✔ (U)	✔ (U)	✗
[6, 2]	C_6	✗	---	---	---	✗
[7, 1]	C_7	✗	---	---	---	✗
[8, 1]	C_8	✗	---	---	---	✗

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[8, 2]	$C_4 \times C_2$	✗	---	---	---	✗
[8, 3]	D_4	✗	✔	✔	✔	✗
[8, 4]	Q_8	✗	✔	✔	✔	✗
[8, 5]	$C_2 \times C_2 \times C_2$	✗	---	---	---	✗
[9, 1]	C_9	✗	---	---	---	✗
[9, 2]	$C_3 \times C_3$	✗	---	---	---	✗
[10, 1]	D_5	✗	✔	✔	✔	✗
[10, 2]	C_{10}	✗	---	---	---	✗
[11, 1]	C_{11}	✗	---	---	---	✗
[12, 1]	$C_3 \rtimes_\varphi C_4$	✗	✔	✔	✔	✗

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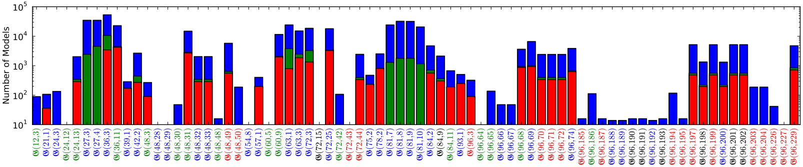
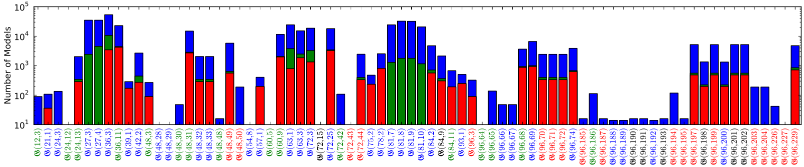
[12, 2]	C_{12}	✗	---	---	---	✗
[12, 3]	A_4	✔	✔	✗	✗	✔
[12, 4]	D_6	✗	✔	✔	✔	✗
[12, 5]	$C_6 \times C_2$	✗	---	---	---	✗
[13, 1]	C_{13}	✗	---	---	---	✗
[14, 1]	D_7	✗	✔	✔	✔	✗
[14, 2]	C_{14}	✗	---	---	---	✗
[15, 1]	C_{15}	✗	---	---	---	✗
[16, 1]	C_{16}	✗	---	---	---	✗
[16, 2]	$C_4 \times C_4$	✗	---	---	---	✗

All Models



- ➢ 76 of 90 groups can be scanned in less than 60 days

All Models



- ➢ 76 of 90 groups can be scanned in less than 60 days
- ➢ 9 groups (12%) only have singular mass matrices

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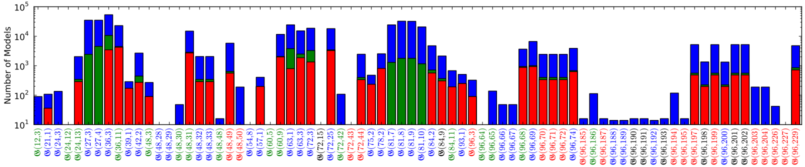

- ➢ 76 of 90 groups can be scanned in less than 60 days
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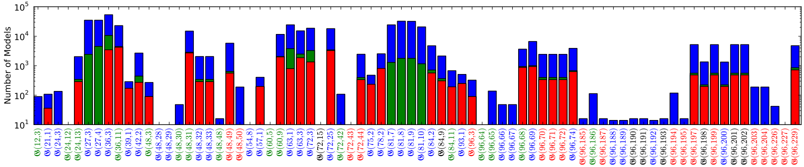

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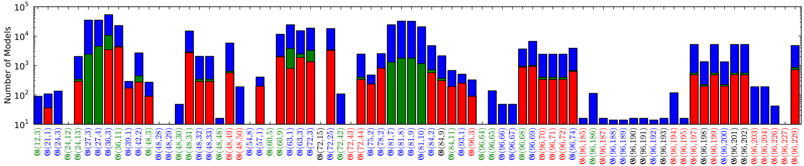

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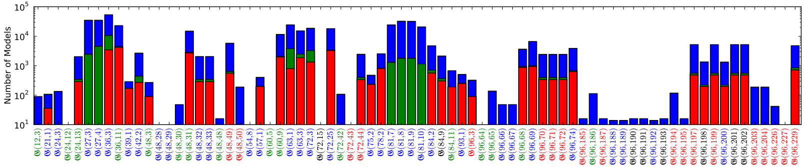

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- ➢ Smallest group that can produce TBM: $\mathfrak{G}(21,1) = T_7$
- ➢ Largest fraction of TBM models: $\mathfrak{G}(39,1) = T_{13}$. Special?

Conclusions

- Constructed thousands of new models of tribimaximal mixing
  - 18.5% of all $A_4 \times \mathbb{Z}_3$ models are TBM. Encouraging!
  - Prediction for θ_{13}: If A_4 and $\theta_{13} \lesssim 12^\circ \rightsquigarrow \theta_{13} = 0^\circ$
  - Prediction for δ: 0°
  - Correlations between mixing angles: Fix two, predict the third
- Constructed specific models
  - $\theta_{13} \neq 0$ possible: $\theta_{12} \simeq 34^\circ$, $\theta_{23} \simeq 41^\circ$ and $\theta_{13} \simeq 5^\circ$
  - Altarelli-Feruglio model works with $\mathbb{Z}_2$: $A_4 \times \mathbb{Z}_2 \simeq \Sigma(24)$
  - TBM possible for T_7, $\Sigma(24)$, T_{13}, T_{14}, $\Delta(48)$, T_{19}, ...
- Is A_4 special? Are TBM and A_4 connected?
  - 50% of the 76 groups we scanned can accommodate TBM
  - Metacyclic group T_{13} has larger fraction of TBM models
  - Smallest group w/TBM is T_7
- `GAP` as a new tool for model builders