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Fragmentation models with quark spin. Application to quark polarimetry

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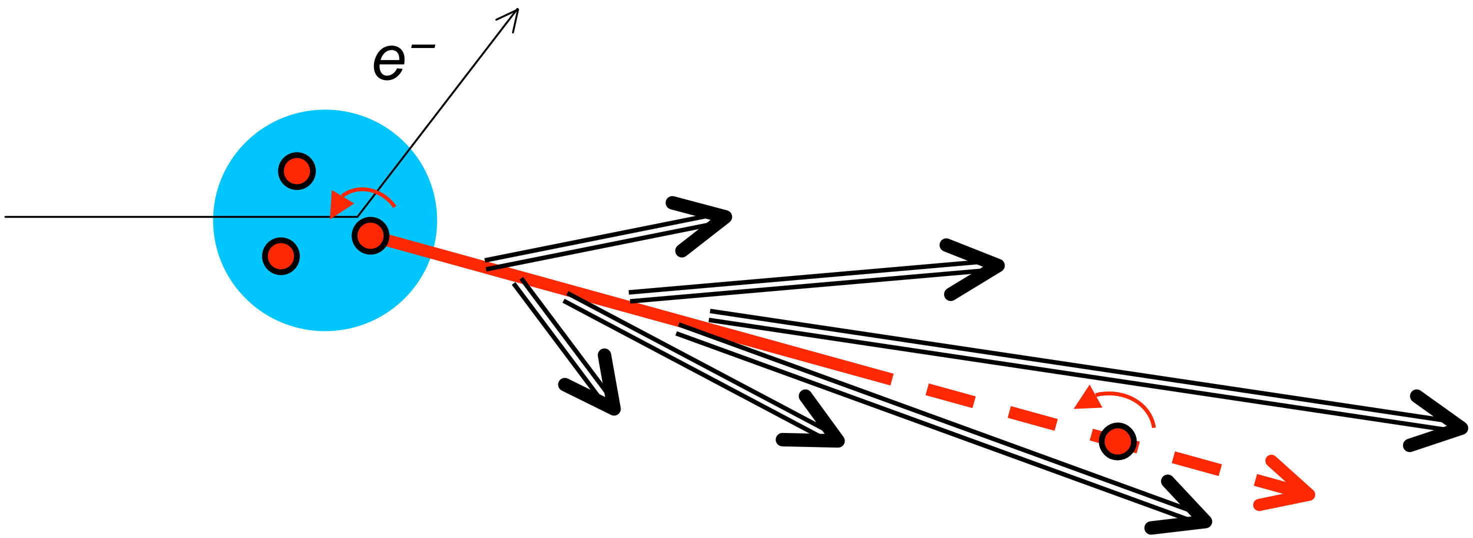
Summary

- quark polarimetry
- **Collins** and "**jet handedness**" spin asymmetries
- current fragmentation models (**without spin**)
- **string + $^3P_0$** mechanism
- **multiperipheral** model with quark exchanges
- introduction of **vector mesons**
- interferences between **permuted** string diagrams

Usefulness of quark polarimetry

An example : **semi**-inclusive deep inelastic scattering (SIDIS)

e^- *(unpolarised)* + N↑ → q↑ → jet of hadrons



If we have a ***quark polarimetre*** one can measure :

$\Delta q(x)$ = quark helicity distribution in the nucleon

$\delta q(x)$ = quark transversity (transverse spin) distribution

Possible polarimeters

- $q\uparrow \rightarrow \Lambda\uparrow + X$

  $\quad\quad \hookrightarrow p + \pi^-$ (self-analysing decay)

  = simple, but small efficiency.

- azimutal asymmetry (*Collins* effect) in

  $q\uparrow$ (**transversely** polarised) $\rightarrow$ 1 or 2 mesons + X

- mirror asymmetry (*jet handedness*) in

  $q\uparrow$ (**longitudinaly** polarised) $\rightarrow$ 2 or 3 mesons + X

  [Nachtman1977; Efremov et al 1992]

The 1-particle Collins effect

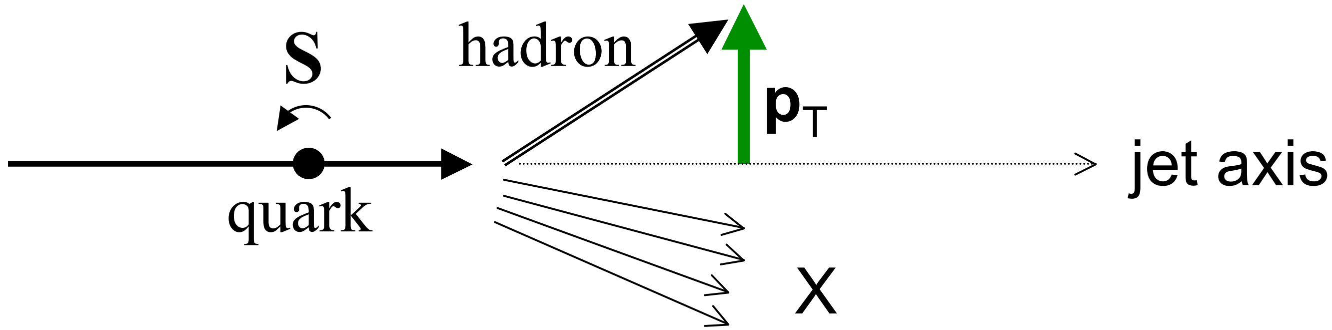
Fragmentation function of a transversely polarised quark

$$F(z, \mathbf{p}_T; \mathbf{S}_T) = F_0(z, p_T) \; [\, 1 + A_T(z, p_T)\; \underbrace{\mathbf{q}.(\mathbf{p}_T \times \mathbf{S}_T)/(q p_T)}_{\sin(\phi_{Spin} - \phi_{hadron})} \,]$$

A_T: *analysing power*

$z = |\mathbf{p}| / |\mathbf{q}|$

$A_T \in [-1,+1]$

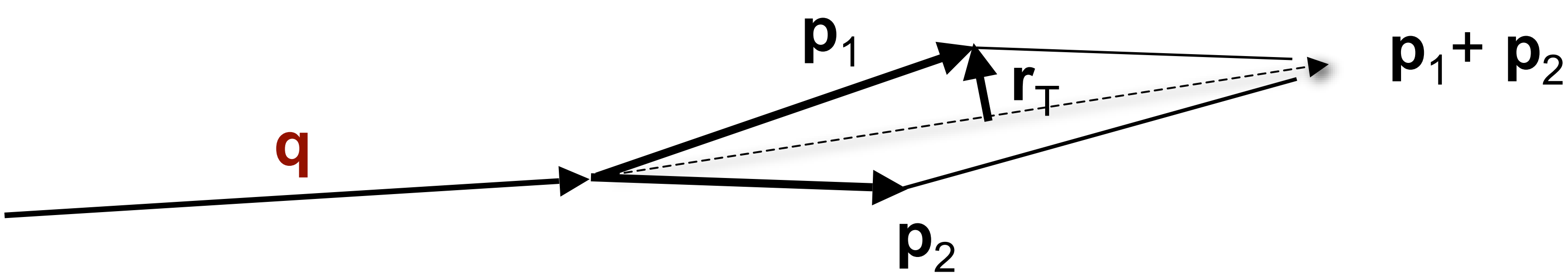


Analogy : *Magnus effect* on a ping-pong ball

The *relative* 2-particle Collins effect (also called *interference fragmentation* [Radicci et al])

Replace $\mathbf{p}_T$ by the *relative* transverse momentum of two fast particles,

$$\mathbf{r}_T \approx (z_2\, \mathbf{p}_1 - z_1\, \mathbf{p}_2) /(z_1+z_2)$$



Analysing power : $A_T(z_1 , z_2 , |\mathbf{r}_T|)$

Advantage :

the asymmetry is not degraded by gluon emission

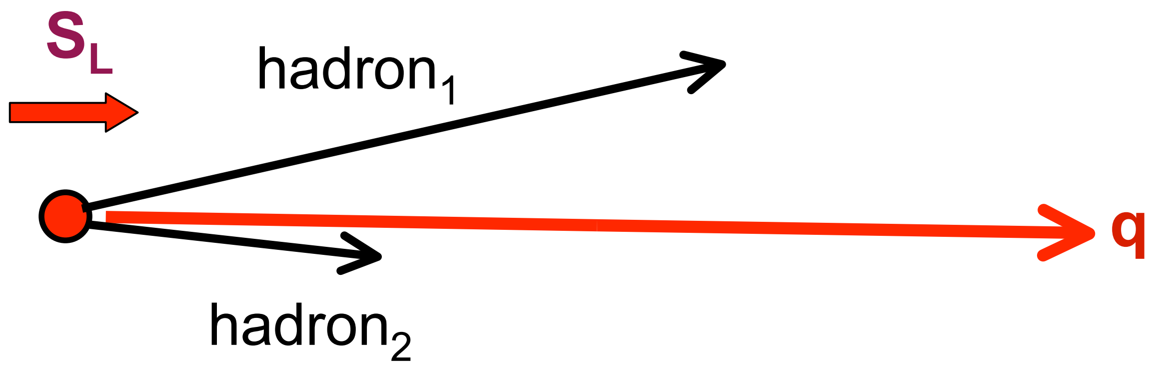
Jet handedness (minimal version : with 2 particles)

$q\uparrow(\text{helicity } \mathbf{S_L}) \rightarrow 2 \text{ hadrons} + X$

$$F(z_1\,,\,|p_{T1}|\,,\,z_2\,,\,|p_{T2}|\,,\,\phi_{12}\,;\,\mathbf{S_L}\,) \;=$$

$$F_0(\ldots|\phi_{12}|) \;\times\; [\;1+A_L(\ldots\;|\phi_{12}|)\,\mathbf{S_L}\;\text{sign}\,\{\mathbf{q}.(\mathbf{p}_1\times\mathbf{p}_2)\}\;]$$

analysing power



Jet handedness with 3 particles

Take a **third** particle in the jet and replace

$$\text{sign}\{\,\mathbf{q}.(\mathbf{p}_1\times\mathbf{p}_2)\,\}$$

by

$$\text{sign}\{\,\mathbf{p}_3\,.\,(\mathbf{p}_1\times\mathbf{p}_2)\,\}$$

not degraded by gluon emission

Need of calibration

The analysing powers are *non-perturbative* functions of z and $\mathbf{p}_T$:

$A_T(z, |p_T|)$ (simple Collins)

$A_T(z_1, z_2, |\mathbf{r}_T|)$ (relative Collins)

$A_L(z_1, |\mathbf{p}_{T1}|, z_2, |\mathbf{p}_{T2}|, |\phi_{12}|)$ (2-particle handedness)

$A_L(z_1, z_2, z_3, |\mathbf{r}_{T12}|, |\mathbf{r}_{T23}|, |\mathbf{r}_{T31}|)$ (3-particle handedness)

They have to be measured by a *calibration* experiment.

Many variables !… A **model** is needed to guide the calibration experiment and to suggest a parametrization.

Recall on current models **without spin**.
1) the recursive model

[Krzywicki, Field & Feynman, Peterson, …]

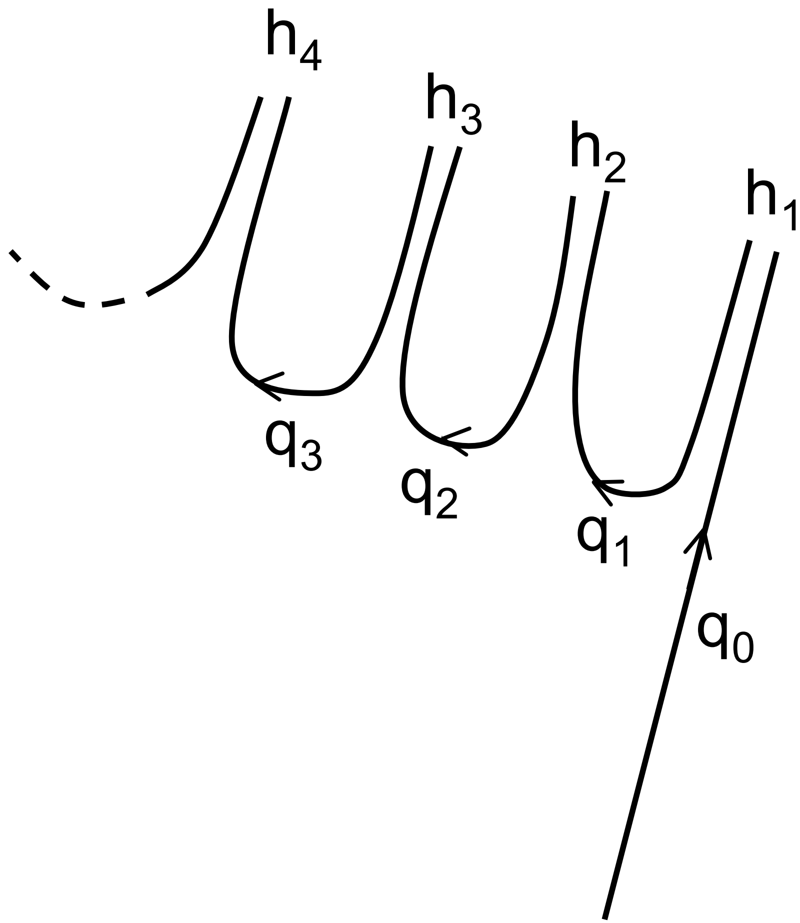
$q_0 \rightarrow h_1 + q_1$,
$q_1 \rightarrow h_2 + q_2$, etc.

Conservation of 4-momentum:

$$\boldsymbol{k}_n = \boldsymbol{p}_{n+1} + \boldsymbol{k}_{n+1}$$

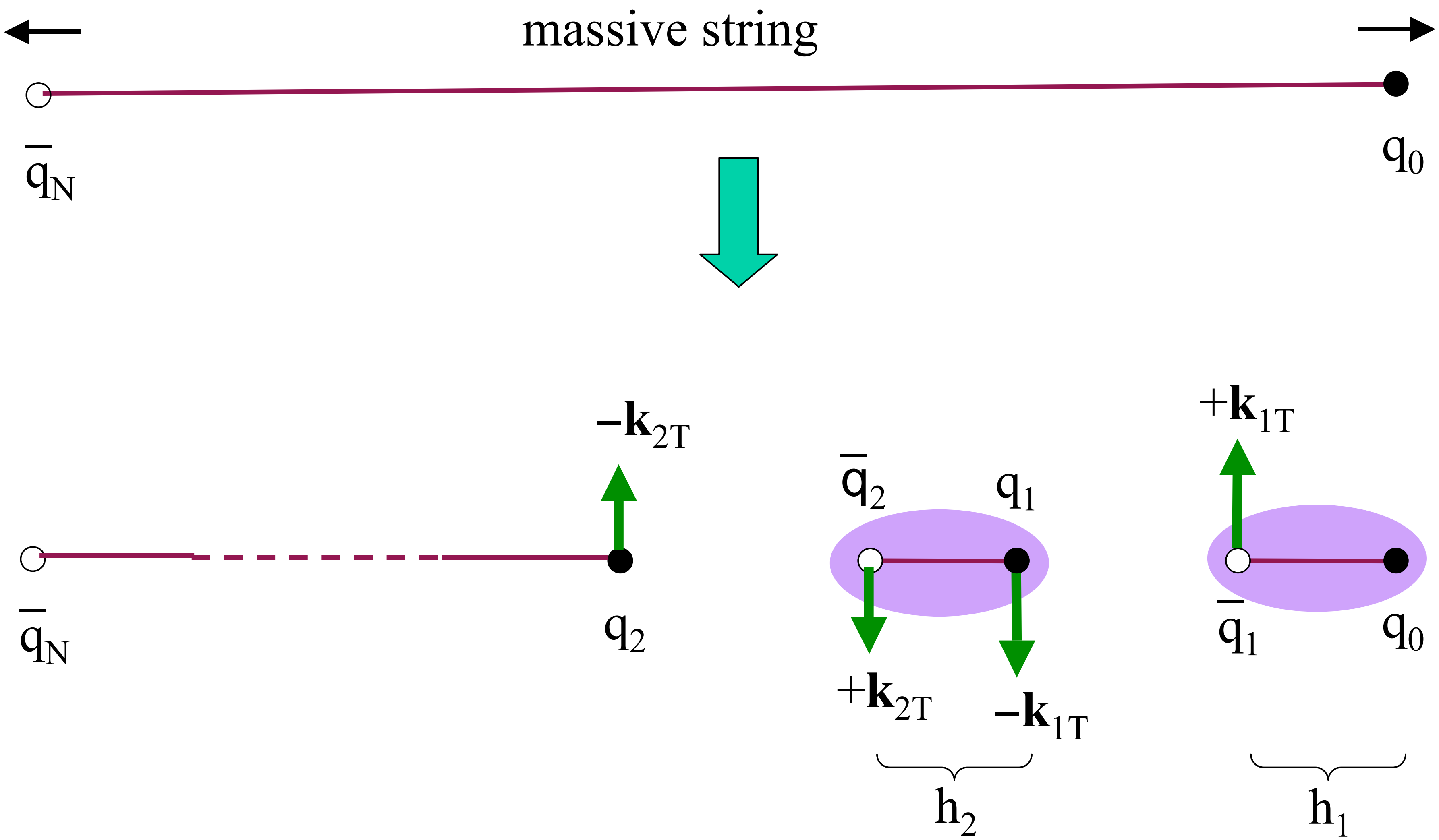
Splitting function for $q_{n-1} \rightarrow h_n + q_n$:

d(probability) = $f_n(\zeta_n, \boldsymbol{k}_{nT})\, d\zeta_n\, d^2\boldsymbol{k}_{nT}$

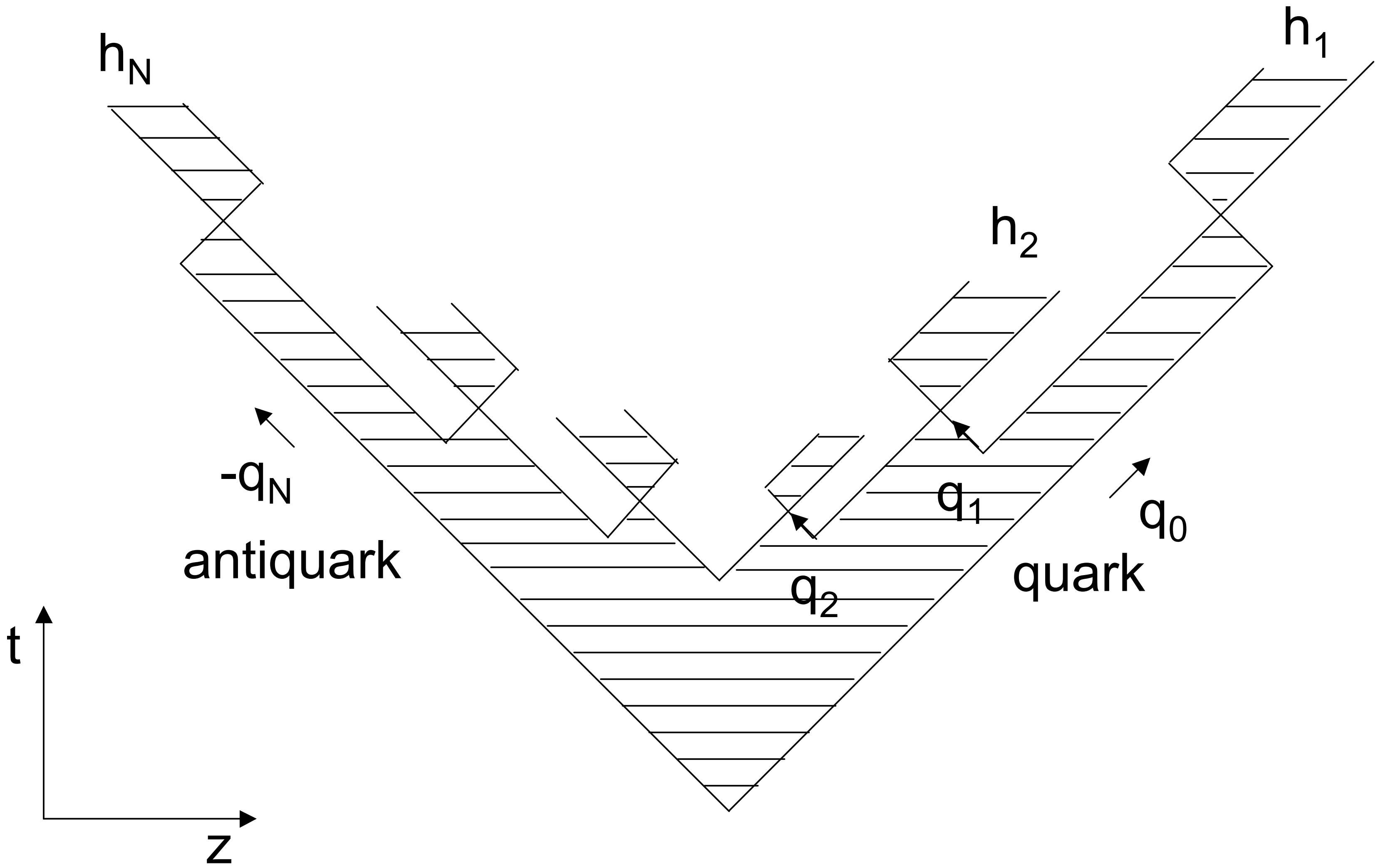


$\zeta_n = |k_n| / |k_{n-1}|$

2) the string model [e.g., Phys. Reports 97 (1983)]



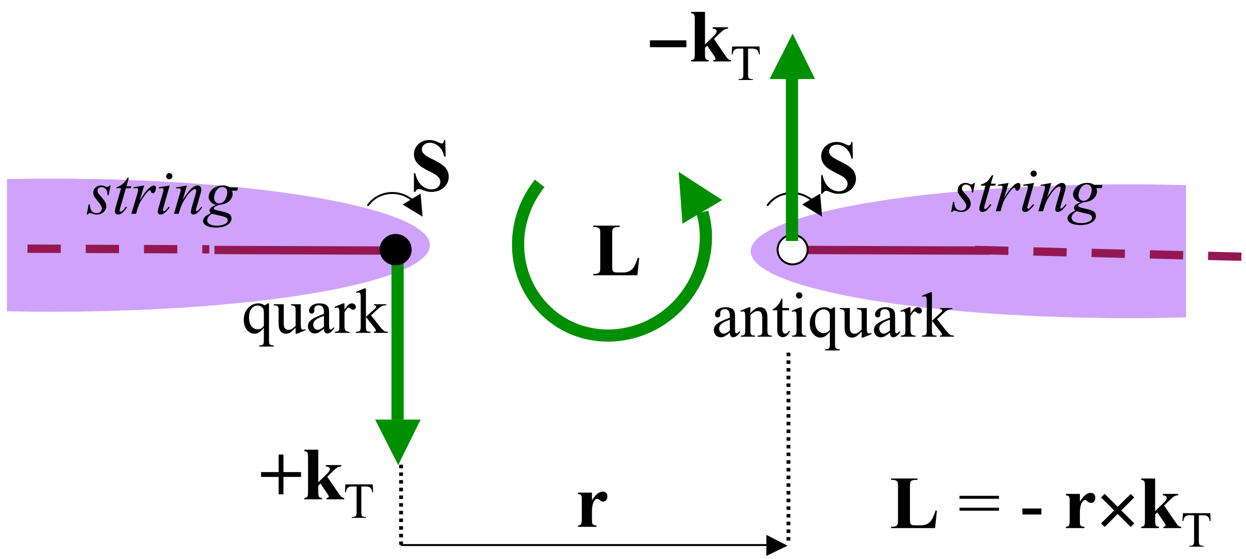
space-time view



A semi-classical spin mechanism : *string + $^3P_0$*

At a string break, the quark-antiquark pair is created in the state $^3P_0 = 0^{++}$ = vacuum quantum numbers

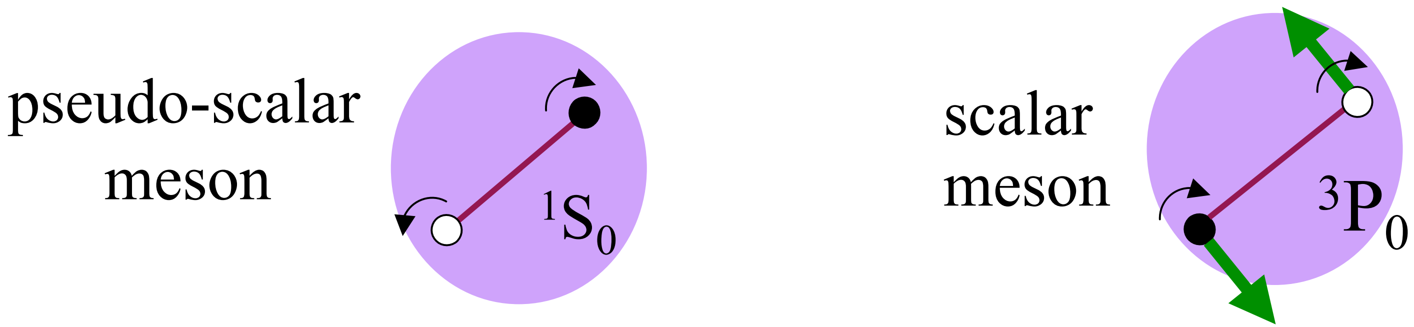
[*Le Yaouang et al*]



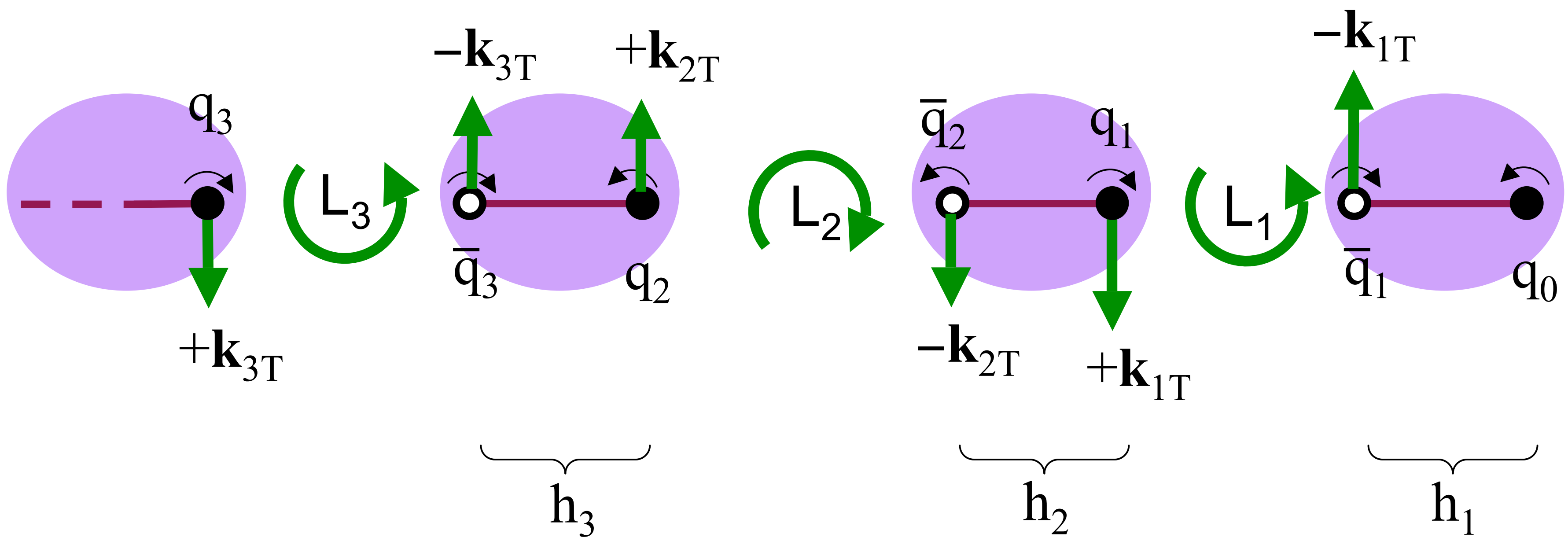
→ correlation between $\mathbf{k}_T$ and $\mathbf{S}_{quark}$

It explains the polarisation of inclusive hyperons [Lund]

Application to Collins effect [X.A., Czyzewski, Yabuki]



String decay into **pseudoscalar** mesons :



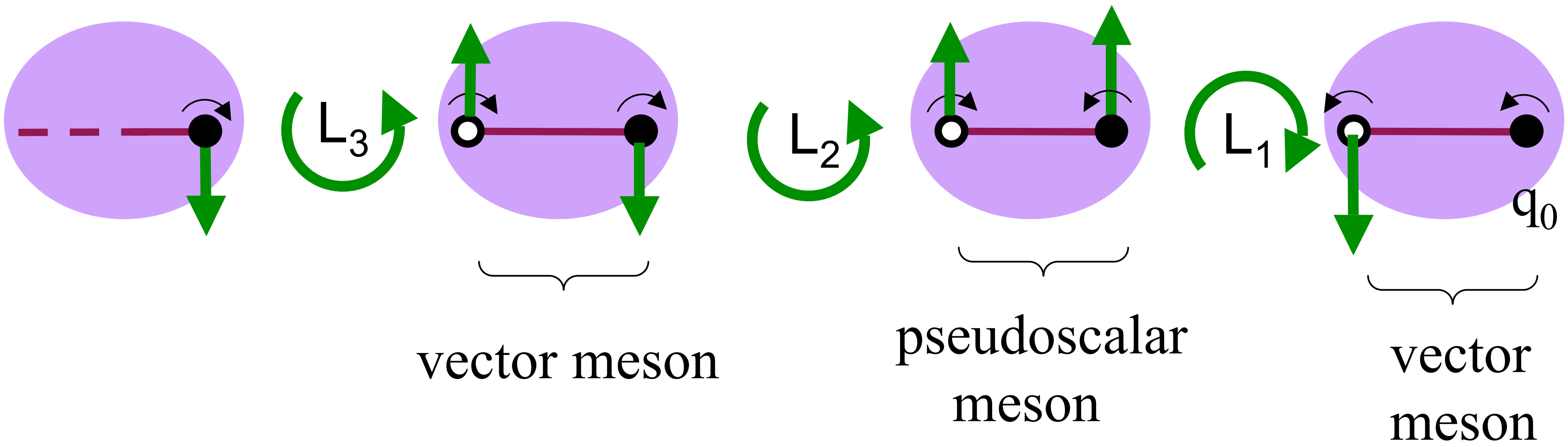
Predictions of *string* + $^3P_0$ (1)

- **Alternate** Collins effects for *pseudo-scalar* mesons

- Strong Collins effect **for the subleading particle**
  (‘‘unfavored fragmentation’’)
  This is observed in experiment !

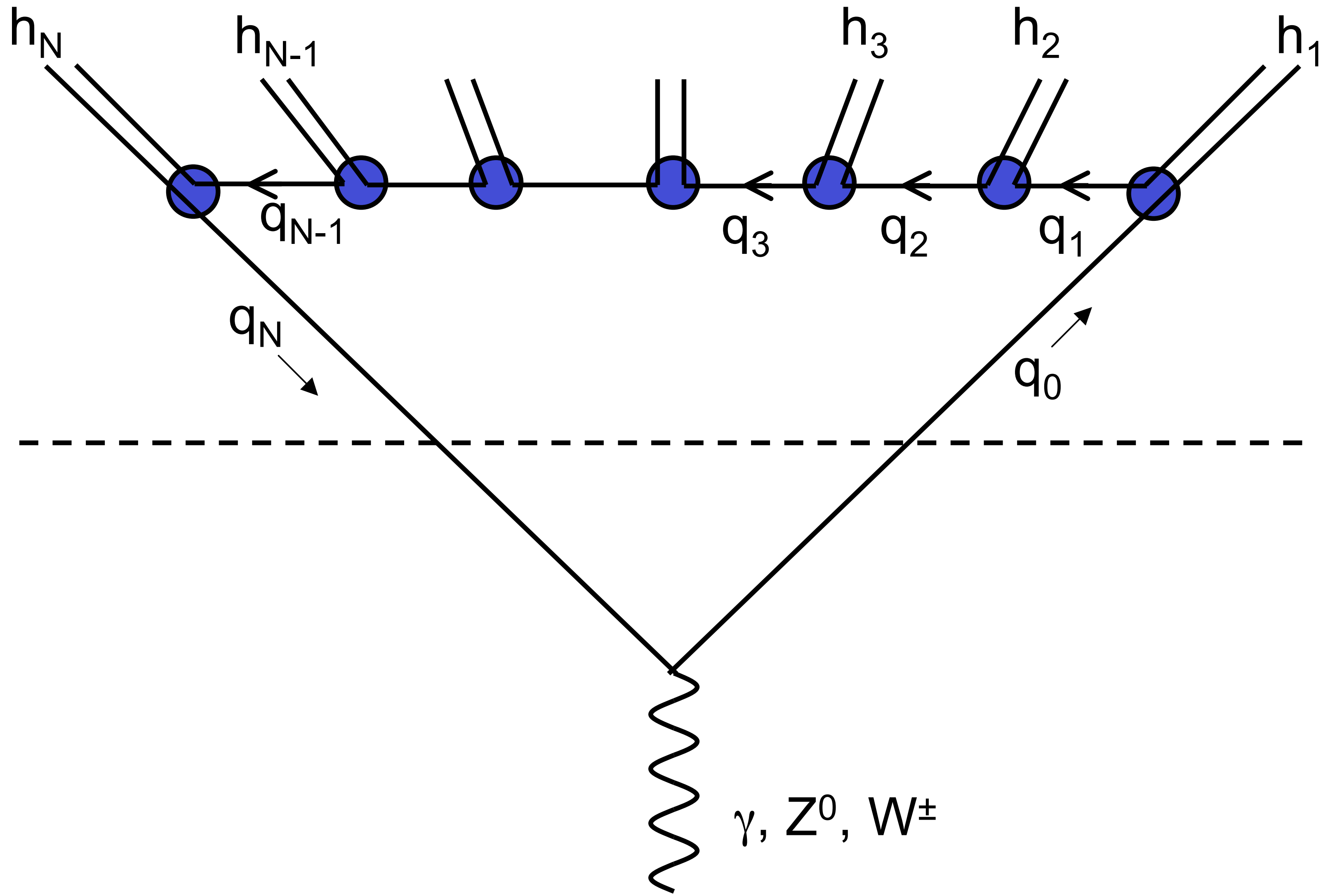
Predictions of the *string* + $^3P_0$ (2)

- Collins effects for a leading *pion* and for a leading *ρ-meson* are **opposite** [Czyzewski]

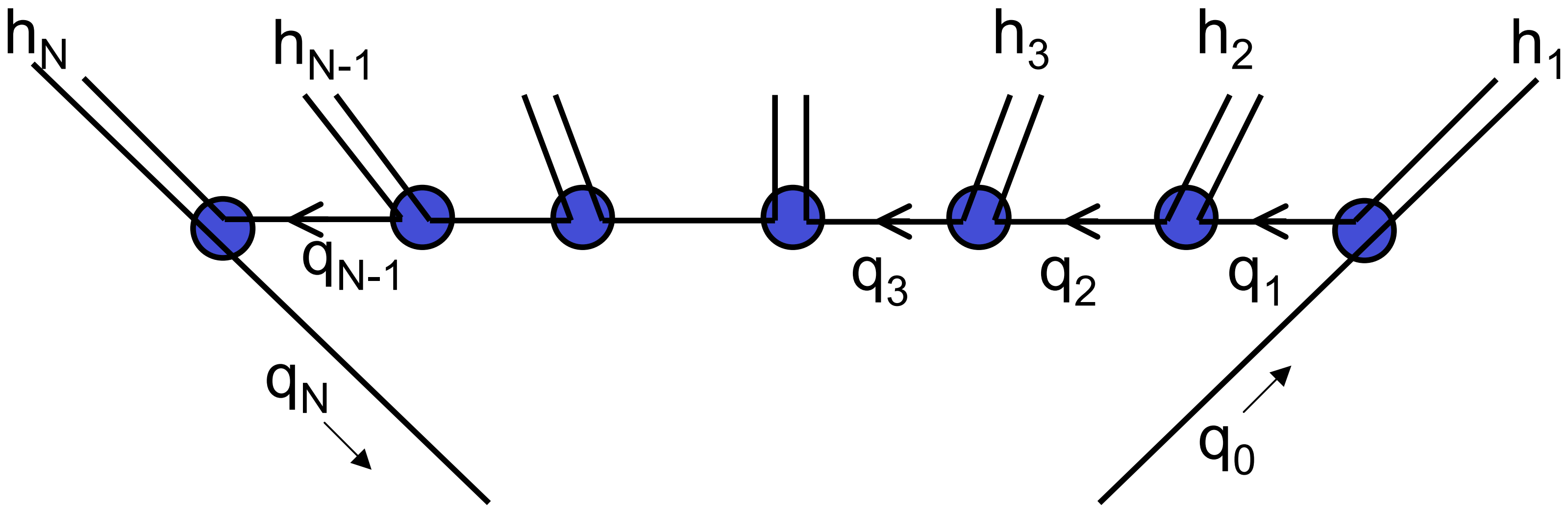


- $\langle \mathbf{p}_T^2 \rangle$ of a ρ-meson is smaller than $\langle \mathbf{p}_T^2 \rangle$ of a pion, ***whether or not q_0 is polarised !*** (spin is necessary anyway)

A full quantum model : multiperipheral model with quark exchanges



Invariance laws



- Lorentz boost along the jet axis
- rotation about the jet axis
- reflection about the (xz) or (yz) plane (~ parity)
- reversal of the quark chain (charge conjugation)

Invariance under transverse boost or rotation is not required.

1rst drastic simplification

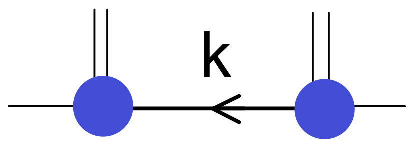
- use **Pauli** spinors, instead of Dirac spinors

> *Vertex* : replace γ_5 by σ_z

> *Propagator* : replace $m + \gamma_\mu k^\mu$ by $\mu + \sigma_z\, \sigma.\mathbf{k}_T$

μ → complex number (kind of mass)

2^{nd} drastic simplification



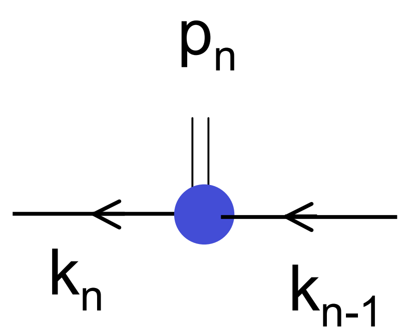
- factorise the propagators in $\Delta(k) = \Delta_L(k_L)\,\Delta_T(\mathbf{k}_T)$

$$k_L = (k^0, k^z) \quad , \quad \mathbf{k}_T = (k^x, k^y)$$

k_L dependence : $\Delta_L(k_L) = \exp\{-\, b\, |k_L|^2/2\}$

$\mathbf{k}_T$ dependence : $\Delta_T(\mathbf{k}_T) = \exp\{-(\mathbf{k}_T)^2/2\} \quad (\mu + \sigma_z\, \sigma.\mathbf{k}_T)$

3^{rd} drastic simplification



- the hadron mass-shell contraint

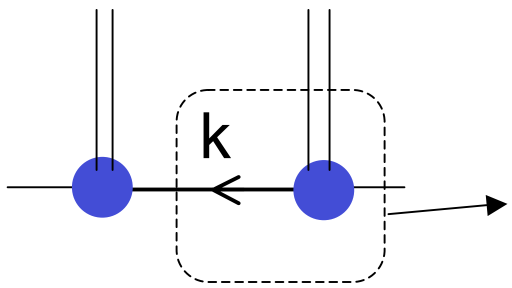
$$p_n^2 = (k_{n-1,L} - k_{n,L})^2 - (\mathbf{k}_{n-1,T} - \mathbf{k}_{n,T})^2 = m^2$$

is **ignored**.

It allows to disentangle k_L and $\mathbf{k}_T$ completely.

From now on we will concentrate on $\mathbf{k}_T$ dependences.

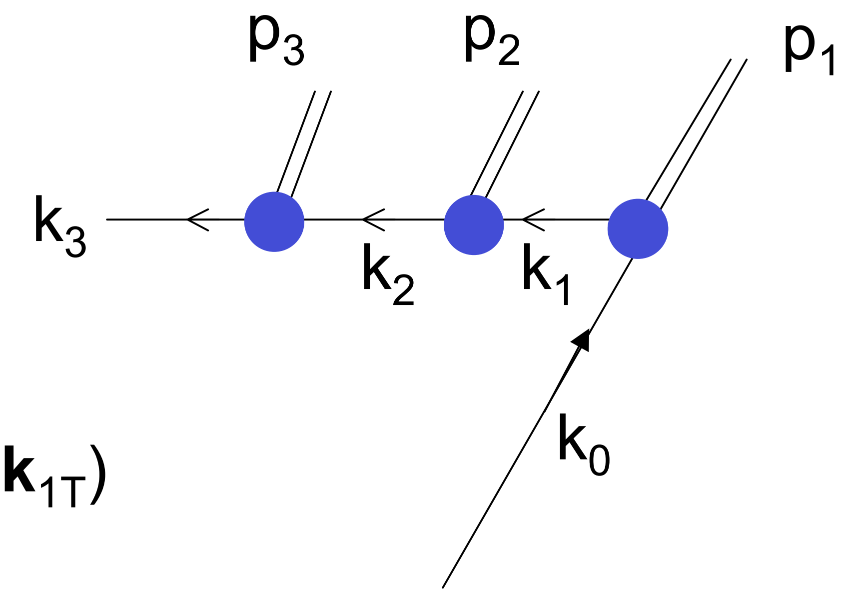
The "interrupted" multiperipheral process



propagator × vertex : $M(\mathbf{k}_T) =$

$$\exp\{-(\mathbf{k}_T)^2/2\}\ \ (\mu + \sigma_z\, \sigma.\mathbf{k}_T)\ \ \sigma_z$$

Interrupted diagram :



amplitude = $M(\mathbf{k}_{3T})\ \ M(\mathbf{k}_{2T})\ \ M(\mathbf{k}_{1T})$

It factorises like the amplitude of a cascade **decay**.

n-meson p_T - distributions

Joint p_T - distribution of the *n* leading particles :

$$J_n\,(\mathbf{k}_{1T},\,\mathbf{k}_{2T}\,,\ldots\,\mathbf{k}_{nT,}\,) =$$

$$\text{trace}\,\{\,\underbrace{M(\mathbf{k}_{nT})\,\ldots\,M(\mathbf{k}_{1T})}\quad \rho_0\quad M^\dagger(\mathbf{k}_{1T})\,\ldots\,M^\dagger(\mathbf{k}_{nT})\,\}$$

$\rho_0 \rightarrow$ $(1+\,\sigma.\mathbf{S}_0)/2$ = density matrix of the initial quark

$M(\mathbf{k}_{nT})\ldots M(\mathbf{k}_{1T}) \rightarrow$ interrupted amplitude

p_T - spectrum of the 1rst rank meson

$$J_1(\mathbf{k}_{1T}) = \exp\{-\mathbf{k}_{1T}{}^2\} \times$$

$$\mathrm{trace}\{ \ (\mu + \sigma_z\, \sigma.\mathbf{k}_{1T})\, \sigma_z \quad \rho_0 \quad \sigma_z\, (\mu^* + \sigma_z\, \sigma.\mathbf{k}_{1T}) \ \}$$

$$= \exp(-\mathbf{p}_{1T}{}^2) \times \{ |\mu|^2 + \mathbf{p}_{1T}{}^2 + 2\, \underbrace{\mathrm{Im}(\mu)\, \mathbf{S}.(\mathbf{z}\times\mathbf{p}_{1T})}_{\text{Collins effect}} \}$$

$\mathbf{z} = \mathbf{q} / |\mathbf{q}|$

Analysing power : $A_T = 2\, \mathrm{Im}(\mu)\, |\mathbf{z} \times \mathbf{p}_{1T}| \ / \ (|\mu|^2 + \mathbf{p}_{1T}{}^2)$

For Im(μ) >0, A_T has the sign predicted by string + $^3P_0$

Joint $\mathbf{p}_T$ distribution for ranks 1 et 2

$$J_2(\mathbf{k}_{1T},\mathbf{k}_{2T}) = \exp\{-\mathbf{k}_{1T}^2 - \mathbf{k}_{2T}^2\} \times \{$$

$$(|\mu|^2 + \mathbf{k}_{1T}^2)(|\mu|^2 + \mathbf{k}_{2T}^2) - 4\,\mathbf{k}_{1T}\,.\,\mathbf{k}_{2T}\,\mathrm{Im}^2(\mu)$$ ← **unpolarised**

$$+\,2\,\mathrm{Im}(\mu)\,\mathbf{S_T}\,.\,(\mathbf{z}\times\mathbf{k}_{1T})\;\;(2\,\mathbf{k}_{1T}\,.\,\mathbf{k}_{2T} - |\mu|^2 - \mathbf{k}_{2T}^2)$$

$$+\,2\,\mathrm{Im}(\mu)\,\mathbf{S_T}\,.\,(\mathbf{z}\times\mathbf{k}_{2T})\;\;(|\mu|^2 - \mathbf{k}_{1T}^2)$$

} **global and relative Collins**

$$-\,2\,\mathrm{Im}(\mu^2)\,\mathbf{S_L}\,.\,(\mathbf{k}_{1T}\times\mathbf{k}_{2T})\;\}$$ ← **handedness**

Results of the simplified multiperipheral model

- it reproduces the string + $^3P_0$ results :

  * alternate Collins effects,

  * strong Collins effect for the ‘‘unfavored’’ fragmentation

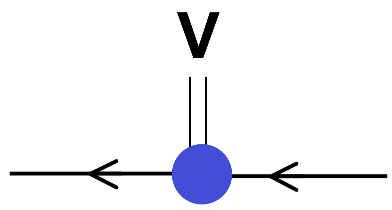
  * $\langle \mathbf{p}_T^2 \rangle$ (vector meson) $<$ $\langle \mathbf{p}_T^2 \rangle$ (pseudoscalar meson)

- it generates *jet handedness*

Inclusion of spin-1 mesons

Vertex for vector mesons emission

$$\Gamma = G_L V_z + G_T\, \sigma.\mathbf{V}_T\, \sigma_z$$



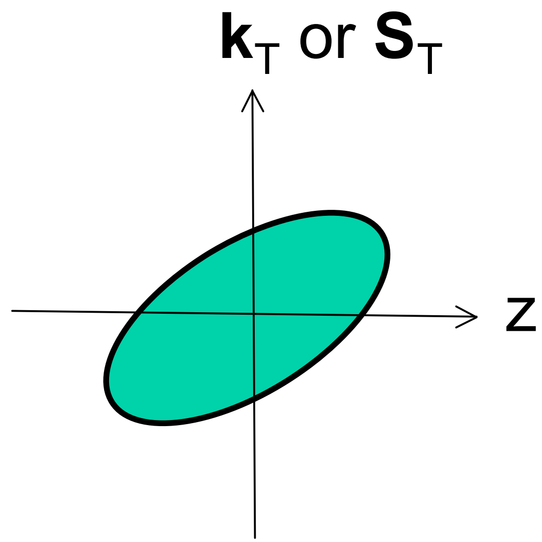
V = spin wave function (vector representation)

$G_L \neq G_T$: *longitudinal* and *transverse* coupling constants

- $\alpha = G_L/G_T$ = complex number

Polarised quark $\rightarrow \rho$ - meson

$\mathrm{Im}(\mu) \neq 0$ and $\mathrm{Im}(\alpha) \neq 0$ produce *oblique alignments* of the ρ-meson,

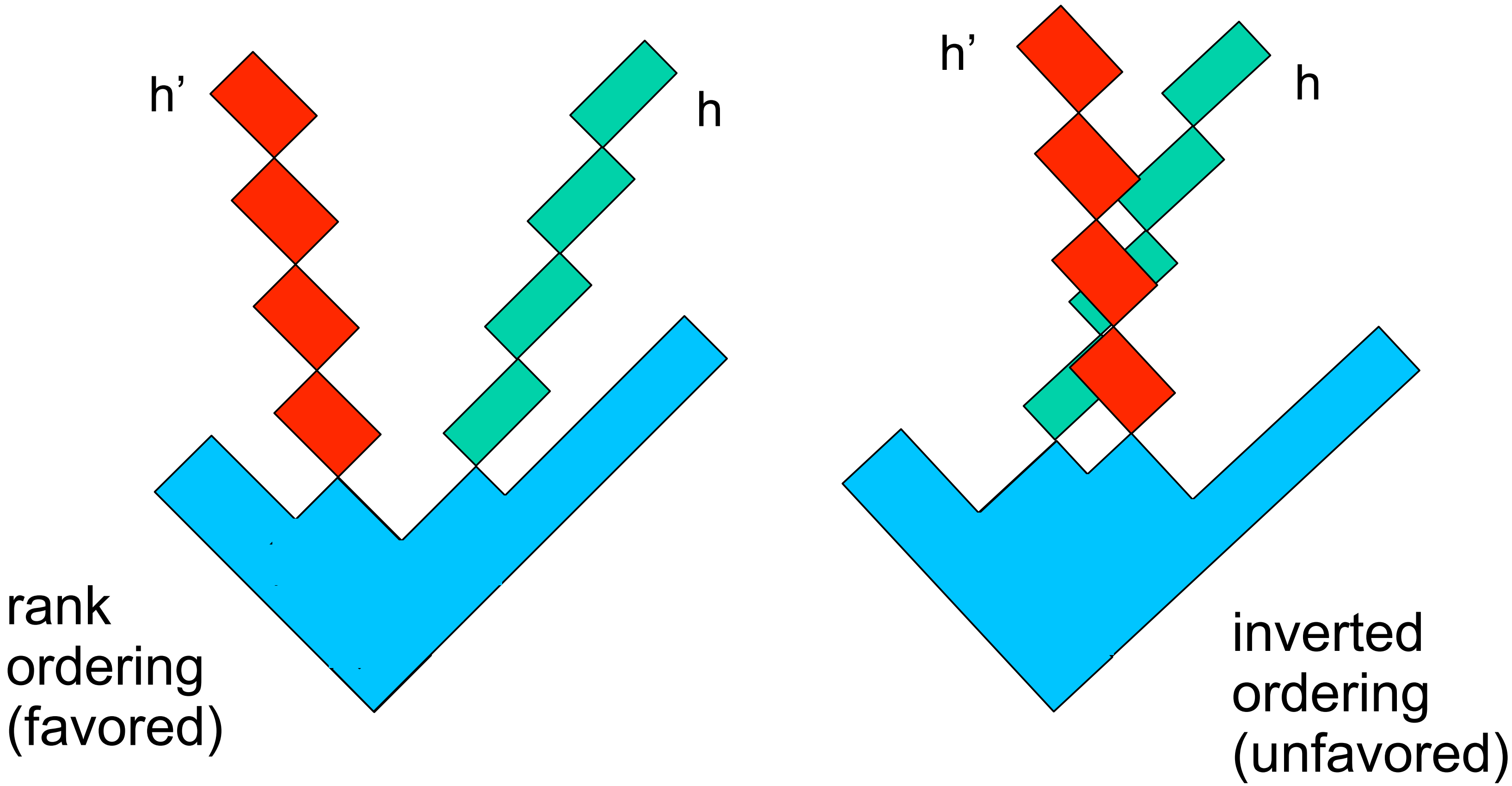


The decay pions possess
- global and relative Collins effects
- jet-handedness

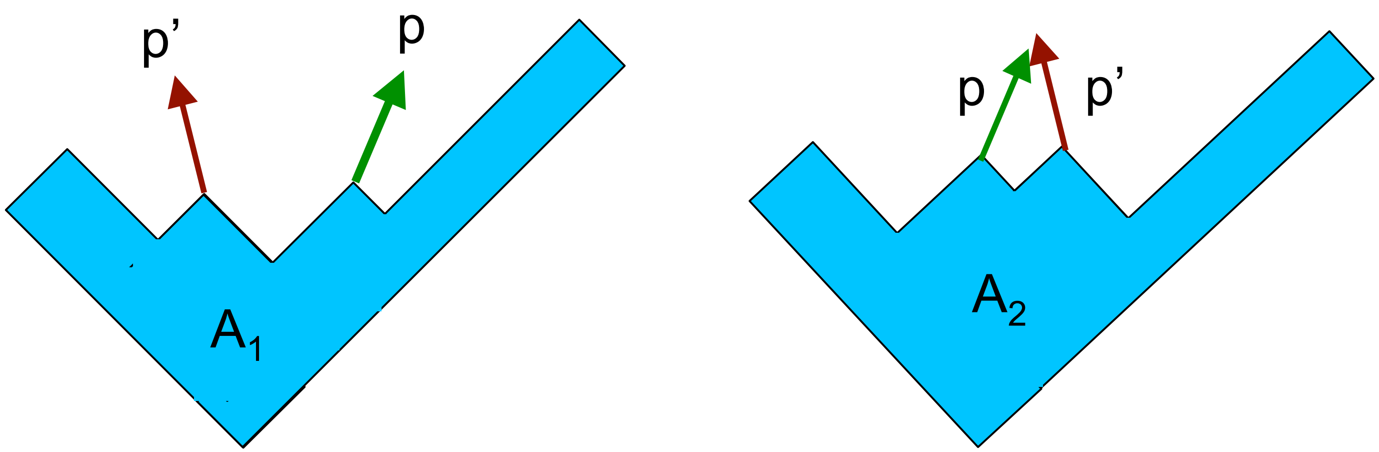
~ like direct pions (duality ?)

3rd mechanism : interference between permuted string diagrams

Rapidity ordering : h > h'



… permuted string diagrams (2)



Amplitude of a string diagram : $\exp(i\kappa A)$ (κ= string tension)

$\rightarrow$ interference phase : $\kappa(A_2\text{-}A_1)$

- If h and h' are **identical** hadrons the interference gives Bose-Einstein correlations [Andersson & Hofmann ; Bowler & X.A.]

- introducing quark spin, one also gets a **Collins effect**.

Main results (1)

- a simple model, inspired from the multiperipheral model of *Amati, Fubini and Stanghelini*, can include the spin degree of freedom in quark jet simulation.

- it is a quantum realisation of the string + $^3P_0$ model.

- Both *direct* pion emissions and 1^- *resonance decay* lead to Collins and jet-handedness effects

Main results (2)

- The model was drastically over-simplified (we neglected the mass-shell constraint)

- The model with only pseudoscalar mesons is easiest to implement in a Monte-Carlo

- Even if you are not interested in polarised experiments, quark spin plays a role in jet fragmentation

(cf. the prediction $\langle \mathbf{p}_T^2 \rangle$ of ρ-meson $<$ $\langle \mathbf{p}_T^2 \rangle$ of pion)

Main results (3)

- An other mechanism of spin effects is the interference between permuted string diagrams.

  It has more theoretical ground and no new parameter,

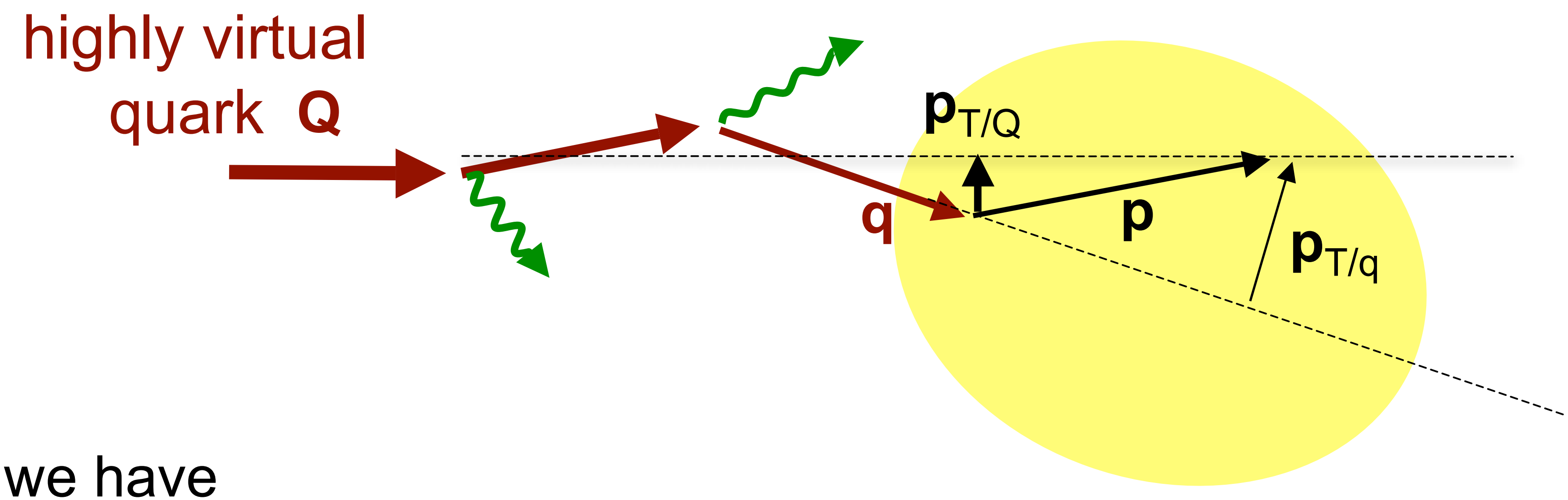
  but is more difficult to implement in a Monte - Carlo.

Thank you for your attention !

Defects of the simple Collins effect

- error on the jet axis $\rightarrow$ error on $\mathbf{p}_T$

- at high Q^2, **gluon emission** washes out the asymmetry (like an uncertainty on the jet axis)



we have

- a **plain** Collins effect in $\mathbf{p}_{T/q} \perp \mathbf{q}$ (yellow area)
- a **degraded** Collins effect in $\mathbf{p}_{T/Q} \perp \mathbf{Q}$

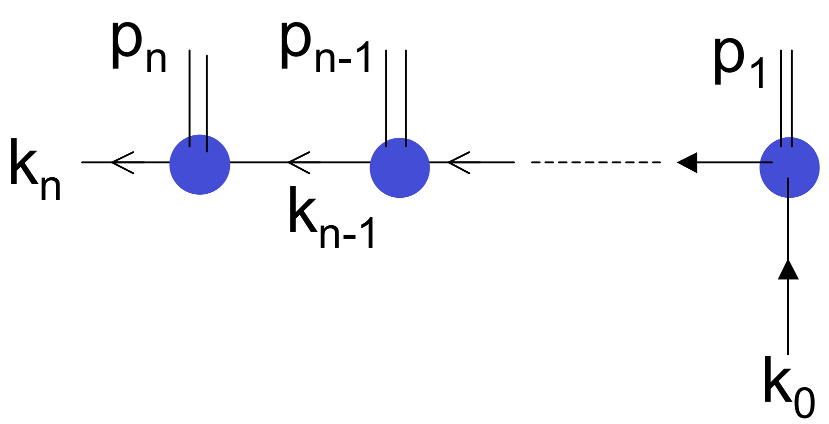
Relations between ζ and z, $\boldsymbol{k}_T$ and $\boldsymbol{p}_T$

For the nth meson, $z_n = (1-\zeta_n) \times \zeta_{n-1}\,\zeta_{n-2}\,\ldots\,\zeta_1$

$$\boldsymbol{k}_{nT} = -\boldsymbol{p}_{nT} - \boldsymbol{p}_{n-1,T} \ldots - \boldsymbol{p}_{1T}$$

Implementation in a Monte-Carlo

Suppose that we got
$\mathbf{k}_{1T}, \mathbf{k}_{2T}, \ldots \mathbf{k}_{n-1T}$
and the density matrix
$\rho_{n-1} = (1+ \sigma.\mathbf{S}_{n-1})/2$
of the $(n-1)^{th}$ quark.



1) we choose $\mathbf{k}_{nT}$ at random with the distribution

$$J(\mathbf{k}_{nT}) = \text{trace}\ \{ M(\mathbf{k}_{nT})\ \rho_{n-1}\ M^{\dagger}(\mathbf{k}_{nT}) \}$$

2) we calculate the density matrix of the n^{th} quark by

$$\rho_n = M(\mathbf{k}_{nT})\ \rho_{n-1}\ M^{\dagger}(\mathbf{k}_{nT}) \ /\ \text{trace}\{ \text{idem} \}$$

and so on ...