# UNIVERSAL EXTRA DIMENSIONS ON THE REAL PROJECTIVE PLANE

## Model and phenomenology

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# Outline

- Universal extra-dimensions and the 5D case
- 6D on the Real Projective Plane
- Unitarity: the cutoff of the model
- Phenomenology of the model

# Universal Extra Dimensions

All SM fields can propagate in the full D-dimensional background

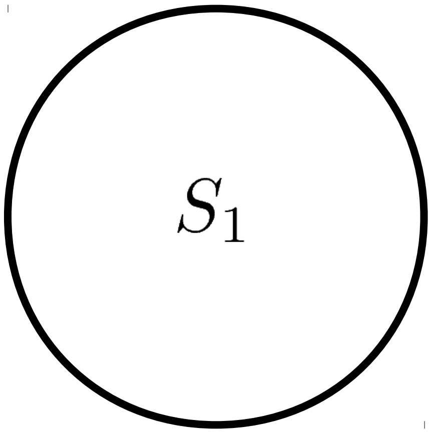
**Assumption: flat metric**

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu - \delta_{ab}dy^a dy^b$$

How many extra-dimensions?

How to compactify such extra-dimensions?

# 5-Dimensional Model




**Compactification on a circle**

$$x_5 \in \{0, 2\pi R_5\}$$

Non-chiral 4D zero modes for fermions
(the chirality projector $P_{L,R}$ is missing)

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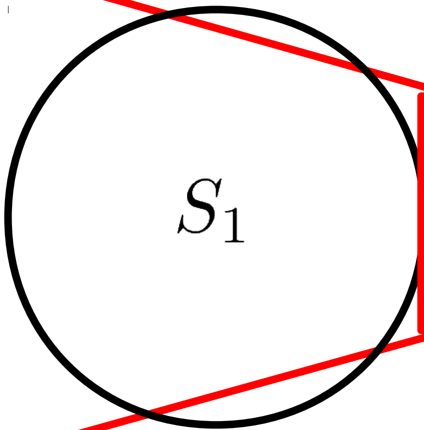


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$\ldots_5\}$

Not a good description of low-energy physics

s for fermions

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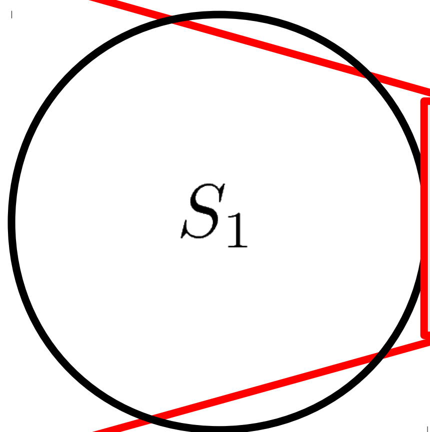


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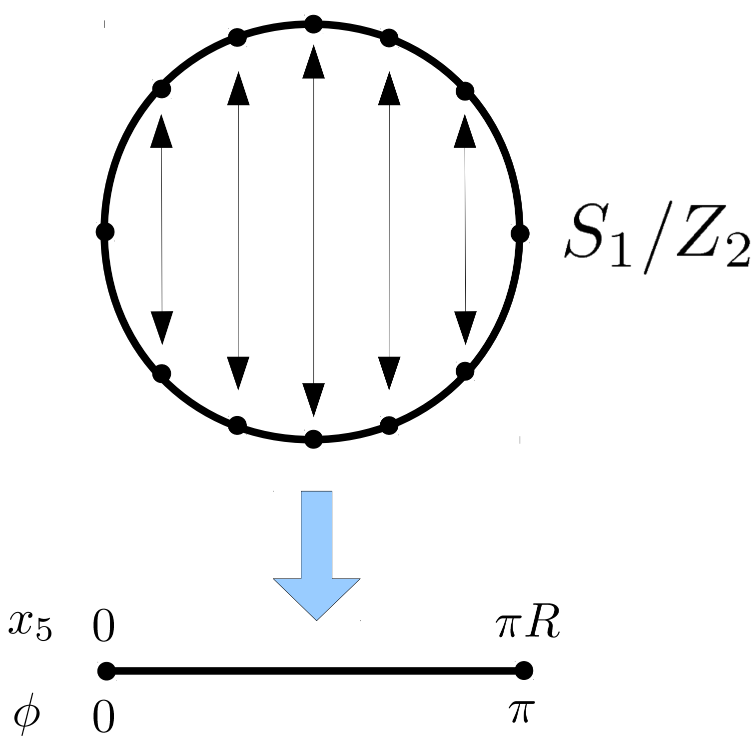


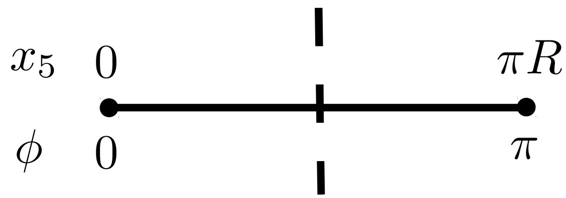

**Compactification on an interval (orbifold)**

Identification of opposite points

**Zero mode chiral Fermions**

# 5-Dimensional Model

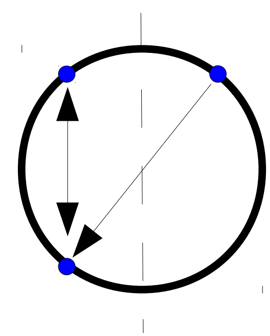
## KK parity




Discrete symmetry about the midpoint

It is a remnant of 5D continuous translational invariance:

$$x_5 \to x_5 + \pi R$$



$$x_5 \to \pi R - x_5$$

# 5-Dimensional Model

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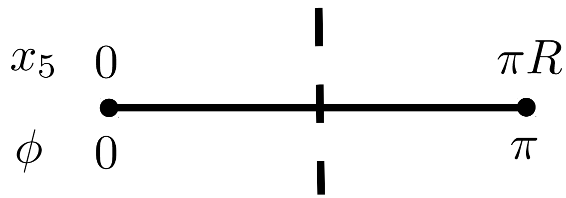


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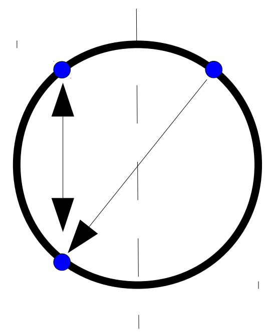

➡ $$x_5 \to \pi R - x_5$$

Invariance of the action under KK symmetry ➡ Interactions must contain an even number of modes with odd n

The lightest KK-odd level is stable ➡ Dark Matter candidate

Automatic in the bulk, but not for the <u>physically different</u> fixed points 0 and $\pi$R

KK-parity imposed by hand on the fixed points

If

orbifold without fixed points

so that

KK parity unbroken globally

then

Natural Dark Matter candidate

Is it possible?

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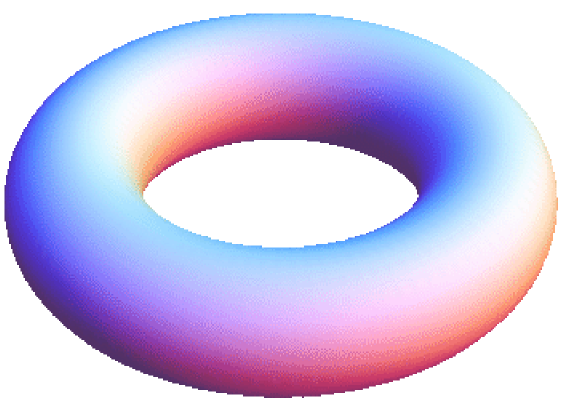
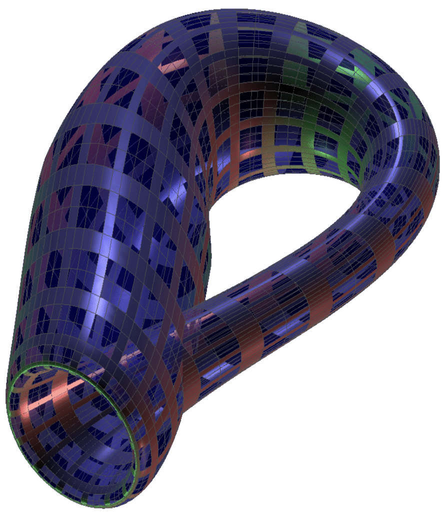
# 6-Dimensional Orbifolds

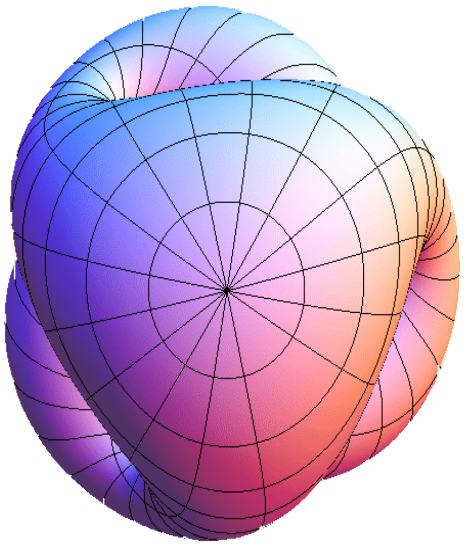
17 possible ways to orbifold the extra $\mathbf{R}^2$ ➡ Only 3 of them are **WITHOUT** fixed points or lines



**Torus**



**Klein Bottle**



**Real Projective Plane**

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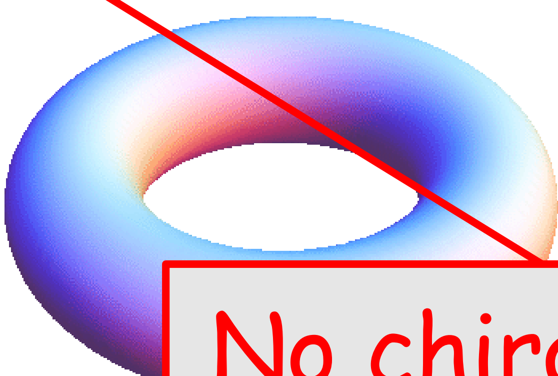
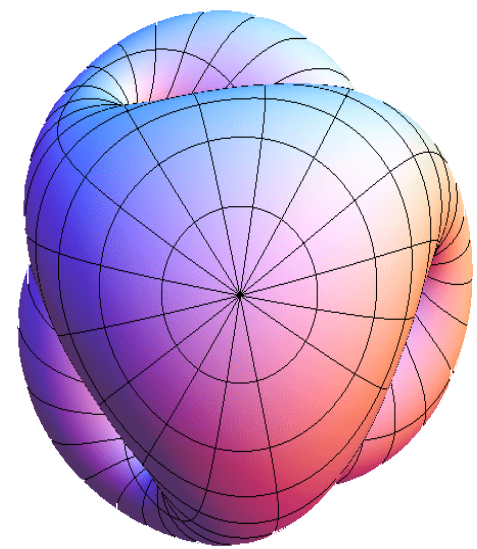


No chiral fermions

Torus

Klein Bottle



Real Projective Plane

# 6D on the Real Projective Plane

Mathematical definition $\mathbf{R}/\mathbf{pgg}$ ⇒ $\mathbf{pgg} = \langle r, g | r^2 = (g^2 r)^2 = 1 \rangle$

glide $g : \begin{cases} x_5 \sim x_5 + \pi R_5 \\ x_6 \sim -x_6 + \pi R_6 \end{cases}$

rotation $r : \begin{cases} x_5 \sim -x_5 \\ x_6 \sim -x_6 \end{cases}$

The **rotation** has 4 fixed-points

$(0,0) \quad (\pi,\pi) \quad (0,\pi) \quad (\pi,0)$

The **glide** identifies them in pairs

$(0,0) \to (\pi,\pi) \quad (0,\pi) \to (\pi,0)$

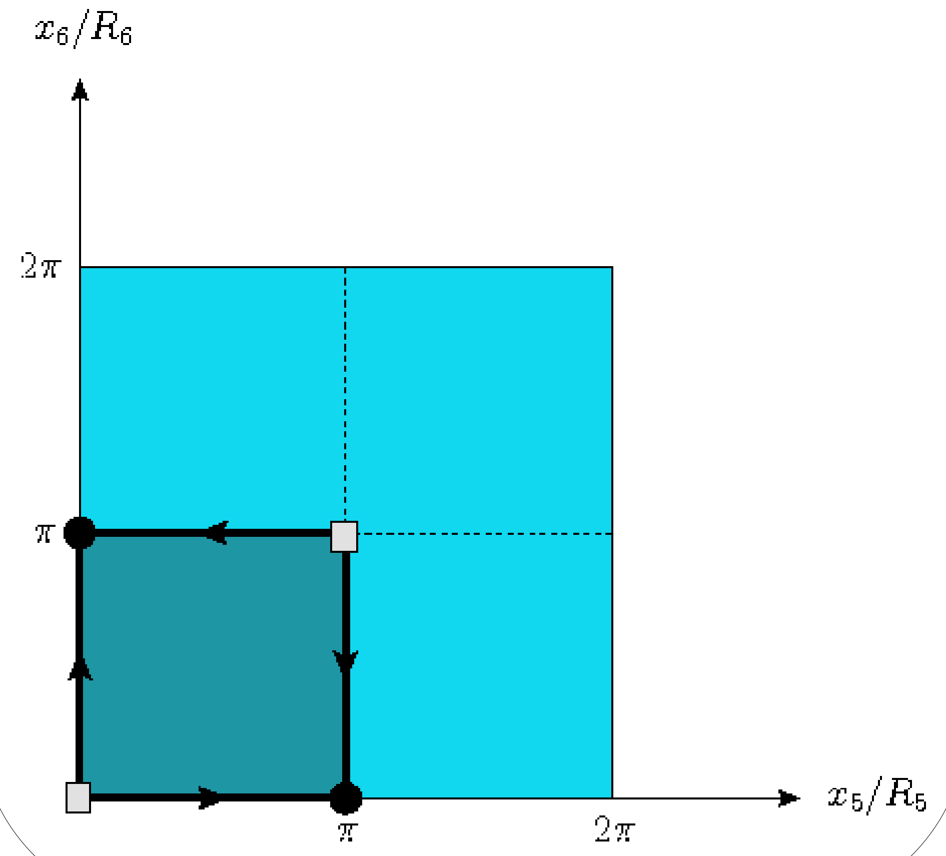
**No fixed points!**

Only 2 conical singularities

where local interactions can be added

Construction of the fundamental domain

Torus + Rotation + Glide

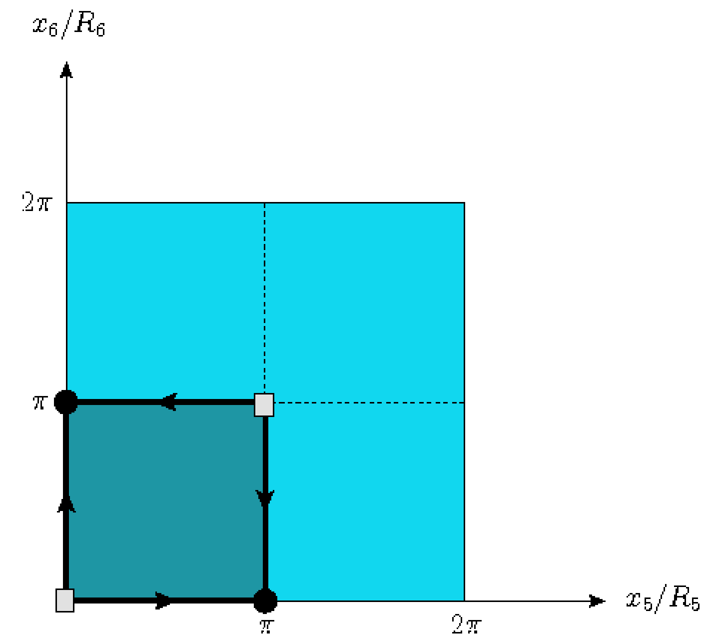


# KK-parity and Dark Matter

A discrete symmetry is left unbroken

even on the singular points!!

$$p_{KK} : \begin{cases} x_5 \sim x_5 + \pi R_5 \\ x_6 \sim x_6 + \pi R_6 \end{cases}$$




Fields on the RPP can be KK expanded into combinations of sines and cosines:

e.g. $$\cos(k(x_5 + \pi R_5))\sin(l(x_6 + \pi R_6)) = (-1)^{(k+l)} \cos(kx_5)\sin(lx_6)$$

Modes can be classified by their parity under KK symmetry ➡ Odd KK modes can only be produced in pairs

**The lightest KK tiers (k,l)=(0,1) and (1,0) contain a NATURAL Dark Matter candidate**

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# Unitarity in extra-dimensions

Pure Yang-Mills theory

Infinite tower of KK modes ➡ Finite terms in the amplitude, which grow with energy

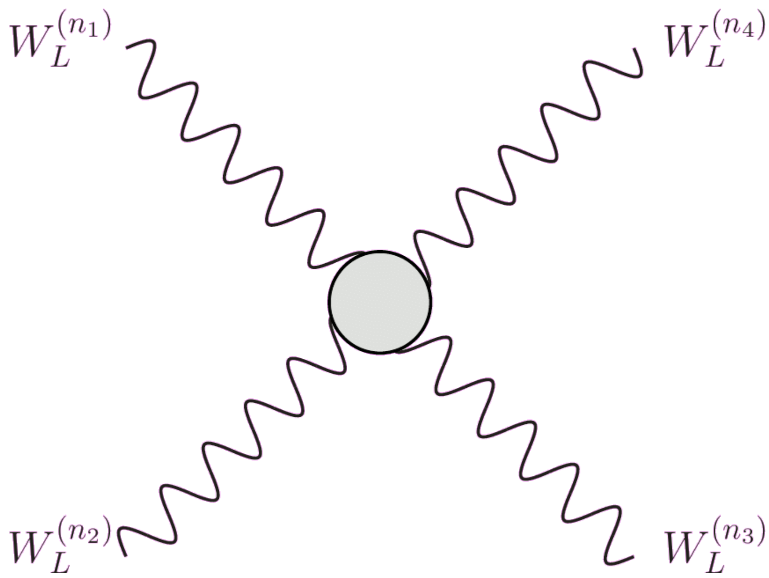
Extra-dimensional theories are not renormalizable

⬇

Unitarity will be broken at energies comparable to the cutoff of the theory

**Unitarity constraints are a powerful tool to determine the cutoff**

# The Unitarity Matrix



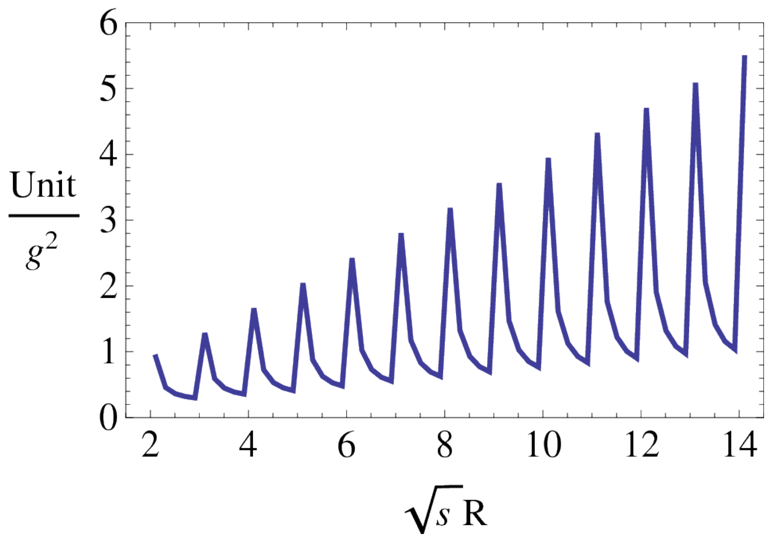

Generic Process

$$W_L(n_1)W_L(n_2) \to W_L(n_3)W_L(n_4)$$

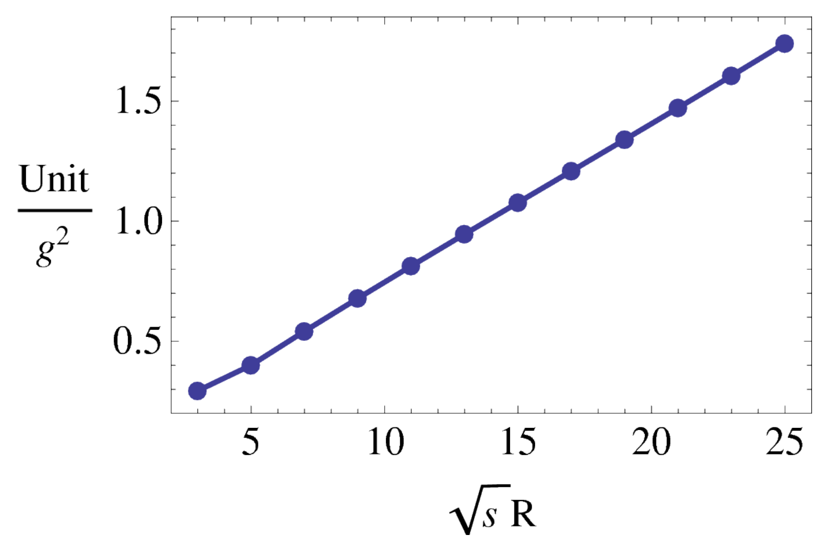
$E \gtrsim m_1 + m_2 = \frac{n1+n2}{R}$ ➡ $T_{n_1 n_2 \to n_1 n_2}$ Only elastic channel

$E \gtrsim m_3 + m_4 \gg m_1 + m_2$ ➡ $\begin{cases} T_{n_1 n_2 \to n_1 n_2} \\ T_{n_1 n_2 \to n_3 n_4} \\ T_{n_3 n_4 \to n_1 n_2} \\ T_{n_3 n_4 \to n_3 n_4} \end{cases}$ Many different processes are allowed

# Unitarity in 5D




Threshold effects are relevant, but they can be safely taken into account with a proper analysis




Taking the points just below thresholds the unitarity parameter grows **linearly** with the energy

Fitting the linear behaviour
( $g=g_{ew}(M_Z)$ )

$$\Lambda_{\text{cutoff 5D}} \sim \frac{34}{R}$$

# Unitarity in 6D on the RPP

2 KK indices (k,l) → The number of open channels increases very rapidly with energy!

| $\sqrt{s}$ | $5D$ | $6D$ |
|---|---|---|
| $2M_{KK}$ | $1\times 1$ | $4\times 4$ |
| $3M_{KK}$ | $3\times 3$ | $17\times 17$ |
| $4M_{KK}$ | $6\times 6$ | $49\times 49$ |
| $5M_{KK}$ | $10\times 10$ | $107\times 107$ |
| $6M_{KK}$ | $15\times 15$ | $226\times 226$ |

Challenge for the computation of the eigenvalues

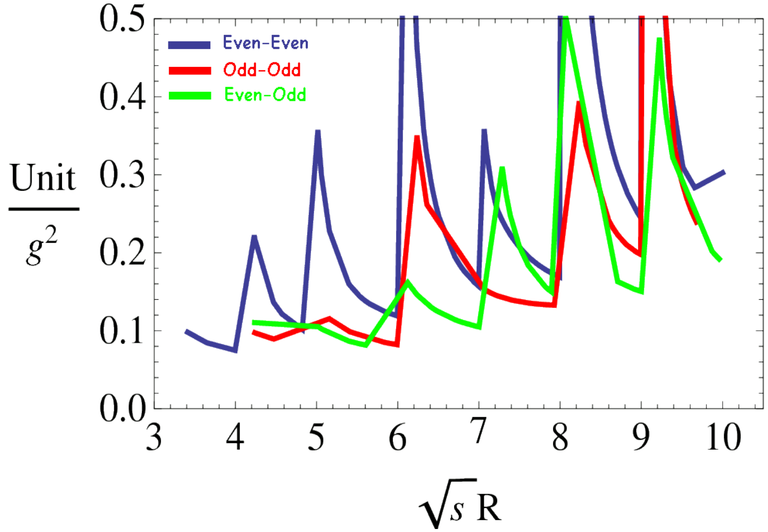
Conservation of extra-dimensional momentum in both coordinates

$$|l_1 \pm l_2| = |l_3 \pm l_4| \longrightarrow \begin{cases} \text{even} \to \text{even} \\ \text{odd} \to \text{odd} \end{cases}$$

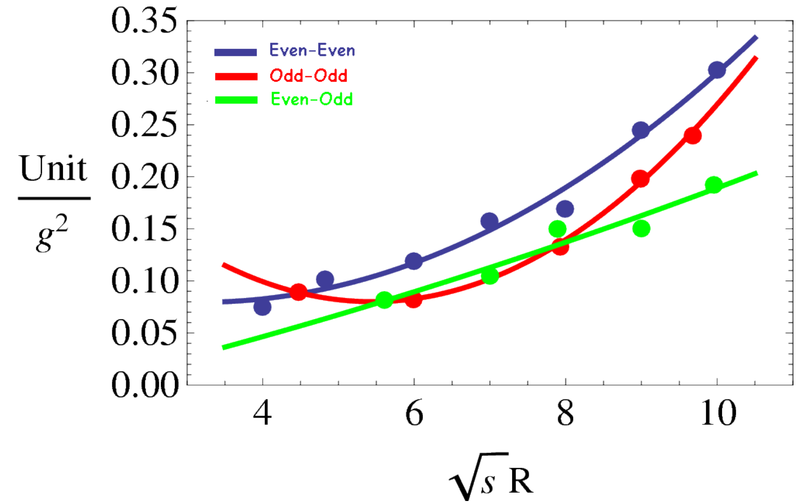
4 independent submatrices →

$$\begin{cases} |l_1 \pm l_2|_{\text{even}}, |k_1 \pm k_2|_{\text{even}} \\ |l_1 \pm l_2|_{\text{odd}}, |k_1 \pm k_2|_{\text{odd}} \\ |l_1 \pm l_2|_{\text{even}}, |k_1 \pm k_2|_{\text{odd}} \\ |l_1 \pm l_2|_{\text{odd}}, |k_1 \pm k_2|_{\text{even}} \end{cases}$$

# Unitarity in 6D on the RPP




Threshold effect structures are much more complicated than in the 5D case!




Taking the points below threshold in the four independent configurations the behaviour is **quadratic**

Fitting the quadratic behaviour
( $g = g_{ew}(M_Z)$ )

$\Rightarrow$ $\Lambda_{\text{cutoff 6D}} \sim \frac{21}{R}$

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# The Spectrum of the Model

| Levels | Mass | $P_{KK}=(-1)^{l+k}$ | Gauge Vectors $(A,W,Z,g)^{\mu}$ | Gauge Scalars $(A,W,Z,g)^{5,6}$ | Fermions | Higgs |
|---|---|---|---|---|---|---|
| (0,0) | 0 | + | ✔ | ✘ | ✔ (chiral) | ✔ |
| (1,0) (0,1) | 1/R | - | ✘ | ✔ | ✔ (Dirac) | ✘ |
| (1,1) | $\sqrt{2}/R$ | + | ✔ | ✔ | ✔ (Dirac) | ✔ |
| (2,0) (0,2) | 2/R | + | ✔ | ✘ | ✔ (Dirac) | ✔ |
| (2,1) (1,2) | $\sqrt{5}/R$ | - | ✔ | ✔ | ✔ (Dirac) | ✔ |

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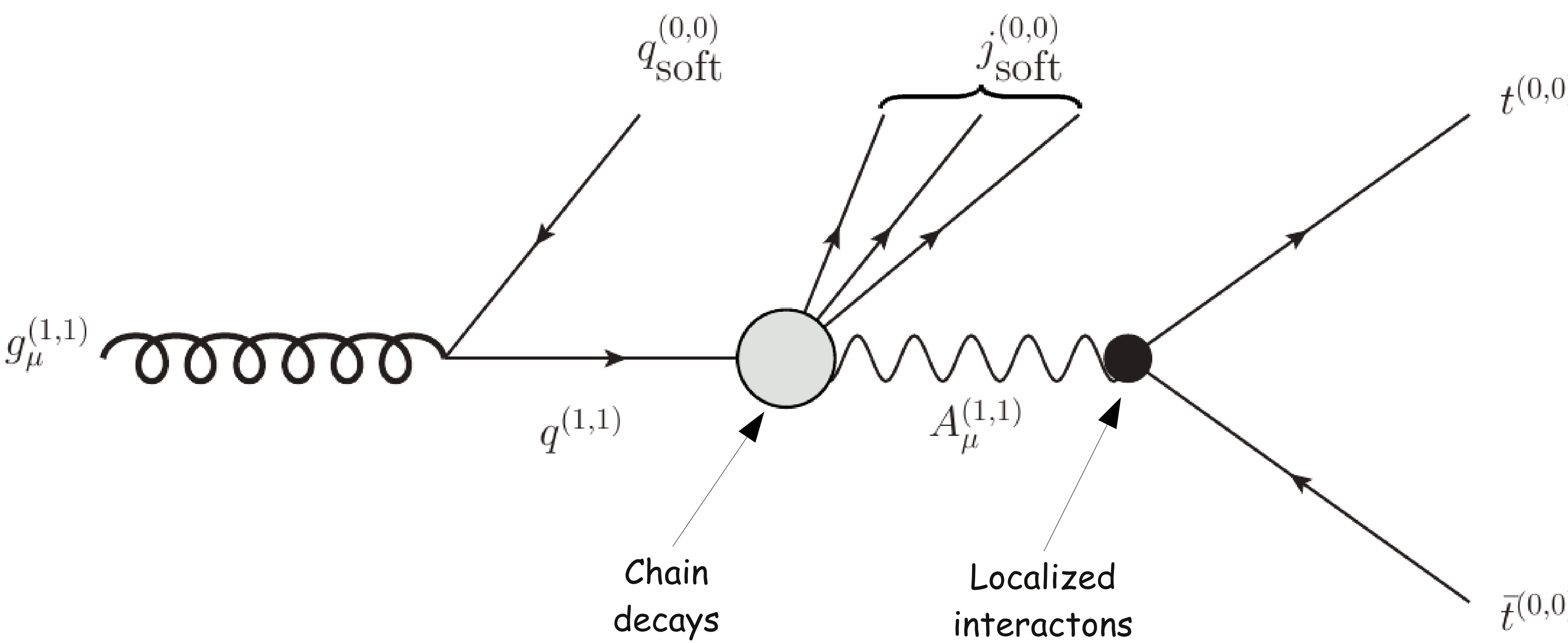
| (0,0) | 0 | + | ✔ | ✘ | ✔ | ✔ |
| (1,0)<br>(0,1) | 1/R | - | ✘ | ✔ | ✔ | ✘ |
| (1,1) | $\sqrt{2}/R$ | + | ✔ | ✔ | ✔ | ✔ |

Degenerate masses ➡ { Radiative Corrections; EWSB: Higgs VEV; Higher order localized operators } ➡ Mass Splittings

# The tier (1,1)

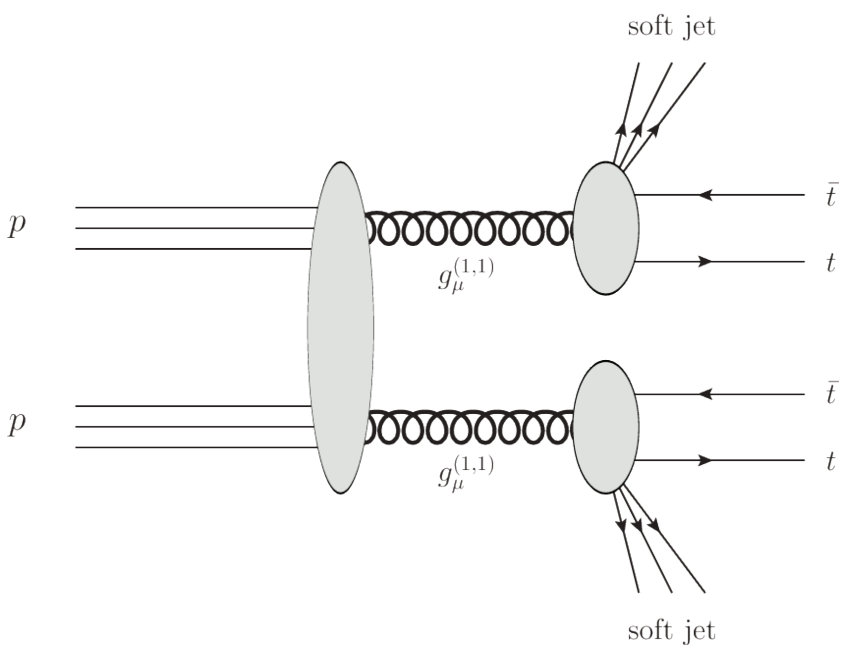
(1,1) particles can decay directly into SM states
through localized interactions which mix different levels

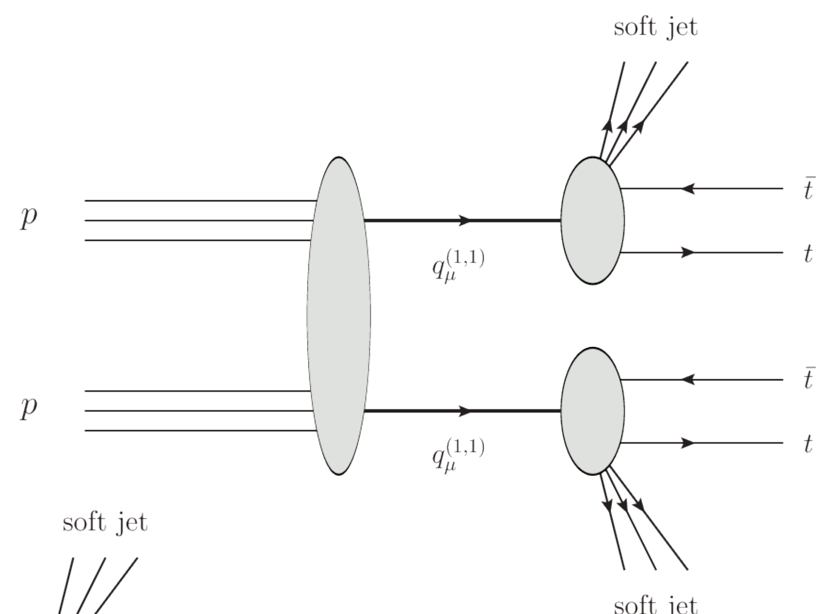
**Interesting signature**

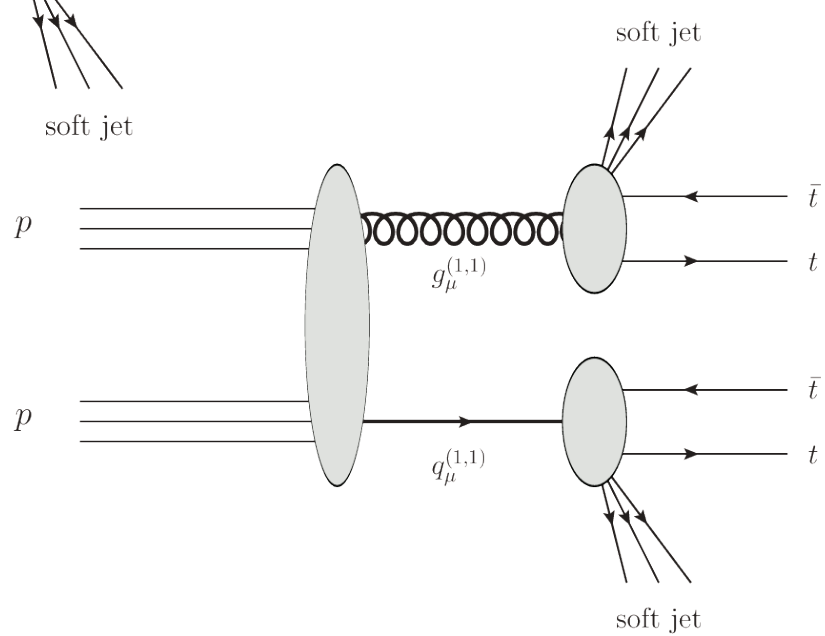


# The tier (1,1)

At the LHC it will be possible to pair-produce
(1,1) gluons or quarks and obtain a final state containing 4 tops + soft jets



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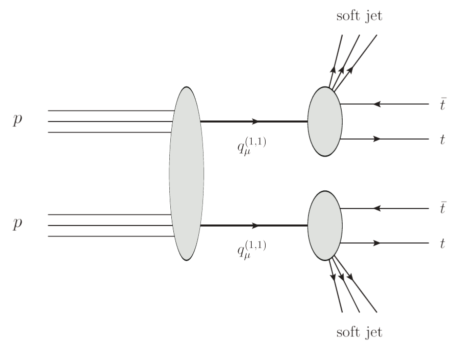
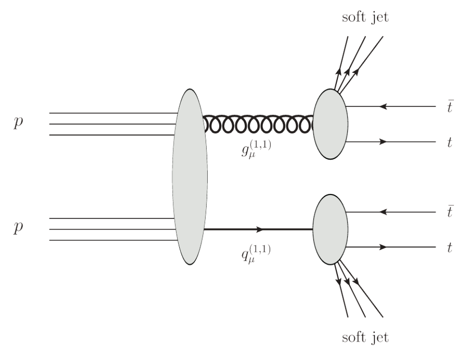
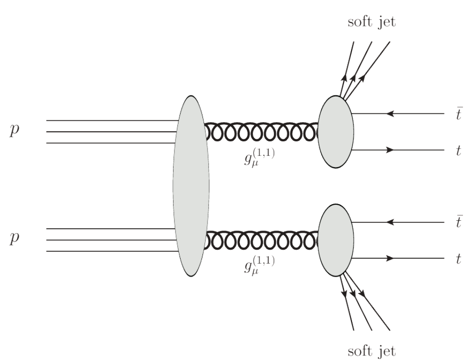



$$\sigma_{qq} = 7.247 \pm 0.012(pb) \ @ \ 7TeV$$




$$\sigma_{gq} = 5.614 \pm 0.007(pb) \ @ \ 7TeV$$




$$\sigma_{gg} = 0.688 \pm 0.001(pb) \ @ \ 7TeV$$

# The tier (1,1)

How can we see this signal at the LHC?

$$pp \to 2t + 2\bar{t} \to 4b\text{-jets} + 4W \to \begin{cases} 0l + 4b\text{-jets} + 8 \text{ jets} \\ 1l + 4b\text{-jets} + 6 \text{ jets} \\ 2l + 4b\text{-jets} + 4 \text{ jets} \\ 3l + 4b\text{-jets} + 2 \text{ jets} \\ 4l + 4b\text{-jets} \end{cases} + \not{E}_T + \text{jet}_{\text{soft}}$$

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Main backgrounds

$$\sigma^{SM}_{pp\to 4t} \lesssim 1\, fb$$

Standard Model cross section is negligible

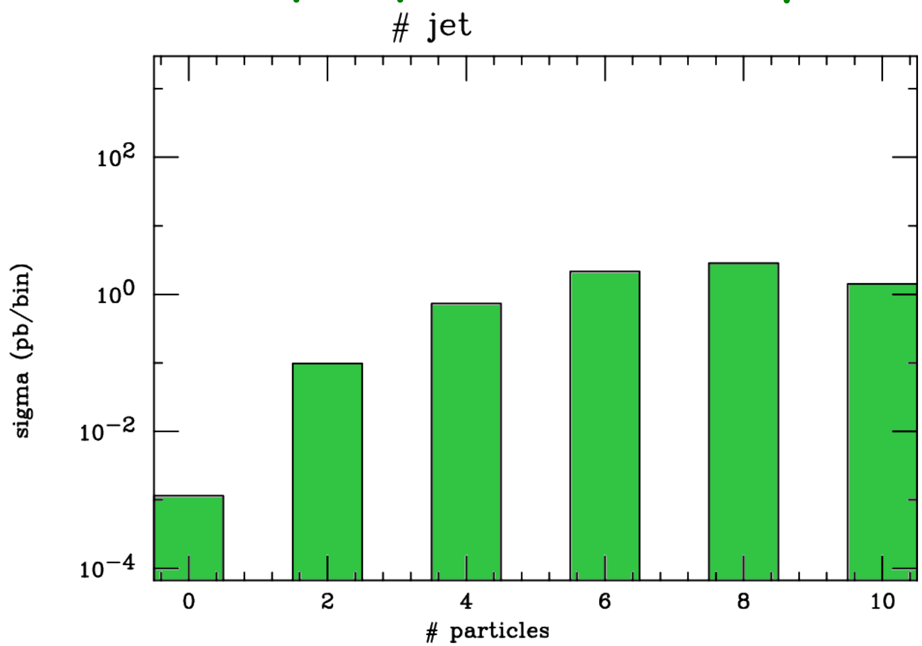
Misidentification is the main problem. Need to compute processes such as

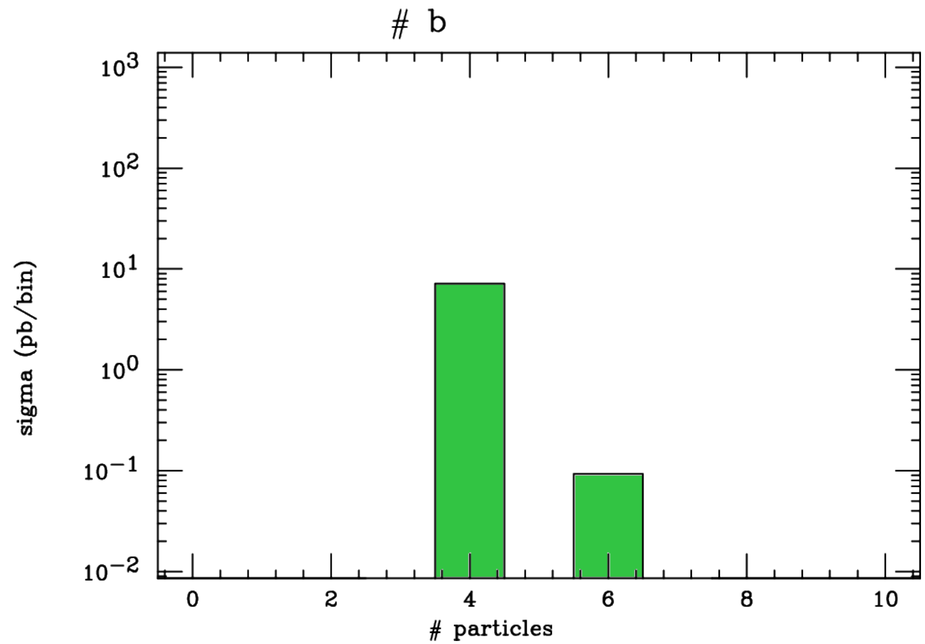
$$pp \to \begin{cases} ZZ + \text{jets} \\ WW + \text{jets} \\ t\bar{t} + \text{jets} \end{cases}$$

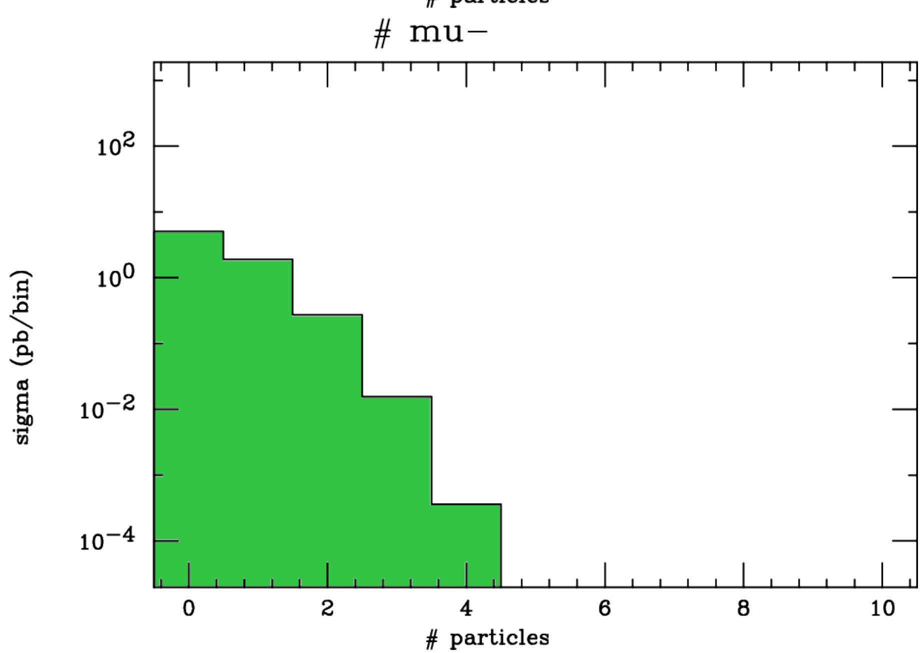
Work in progress with CMS group in Lyon...

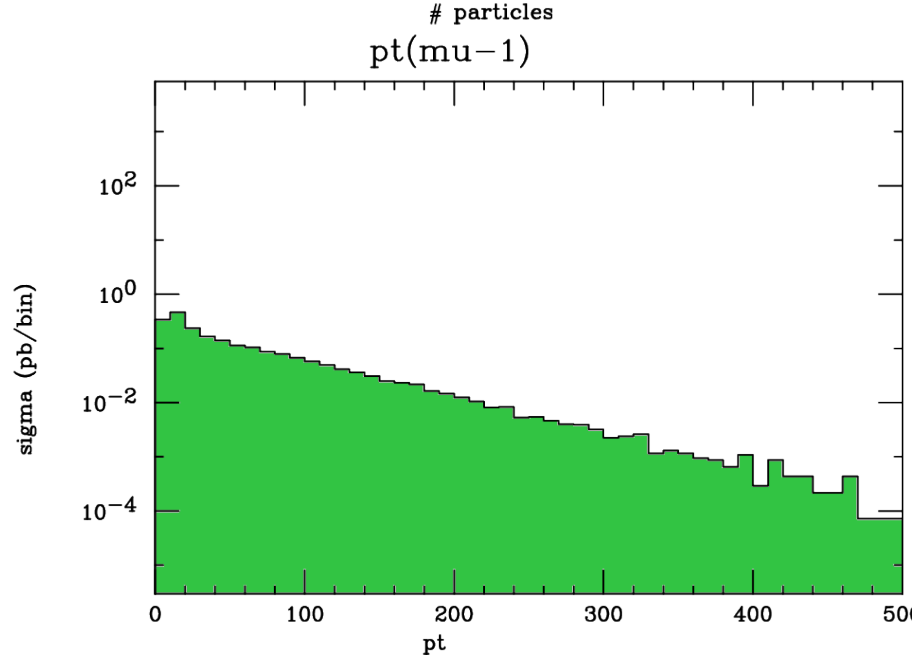
# The tier (1,1)

## $q^{(1,1)}q^{(1,1)}$-initiated process (preliminary results)



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In **5D** KK parity must be imposed **by hand**
on the **fixed points** of the orbifold

In **6D** only three orbifolds **without fixed points/lines**
but **chiral zero mode fermions** only on the **Real Projective Plane**

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In **6D** only three orbifolds **without fixed points/lines**
but **chiral zero mode fermions** only on the **Real Projective Plane**

**Interesting phenomenology at LHC** with peculiar signals
from the next-to-lightest, KK-even **tier (1,1)**

# Work in progress...

**Gauge-Higgs Unification**

various possibilities already analyzed

but complete computation of unitarity bounds in progress

**Phenomenology**

of heavier tiers (2,0) and (2,1)

Computation of additional bounds on localized operators from

**Electroweak Precision Tests**

# Backup...

# More than four Dimensions

Infinite or Compact?

**Infinite**

Just like ordinary dimensions,
but one must explain why SM fields
are confined to 4D at low energy

**Compact**

"$x_{5,6,\ldots}$" are limited to a finite interval $\{0, 2\pi R_{5,6,\ldots}\}$:
effective 4D theory up to distance scales
of the order of the compactification radii $R_{5,6,\ldots}$

Assumption: extra dimensions are compact!

# More than four Dimensions

A field that propagates in D dimensions can be Fourier-expanded

$$\Phi(x_\mu, x_5, x_6, \dots) = \sum_{k_5, k_6, \dots} \phi^{(k_5, k_6, \dots)}(x_\mu) e^{i\left(\frac{k_5}{R_5} x_5 + \frac{k_6}{R_6} x_6 + \dots\right)}$$

Compact space ➡ Discrete sum

From quadri-momentum to D-momentum: $\tilde{p} = (p_\mu, p_5, p_6, \dots)$

Equation of motion

$$0 = \tilde{p}^2 = p^2 - \sum_i p_i^2 = p^2 - \frac{k_5^2}{R_5} - \frac{k_6^2}{R_6} - \dots$$

➡

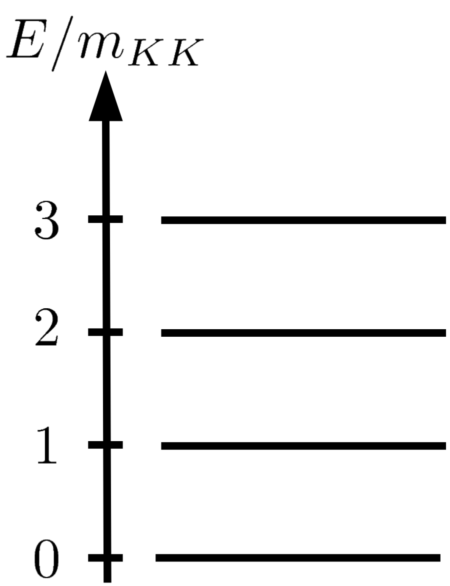
KK mass

$$m^2_{k_5, k_6, \dots} = \frac{k_5^2}{R_5^2} + \frac{k_6^2}{R_6^2} + \dots$$

# More than four Dimensions

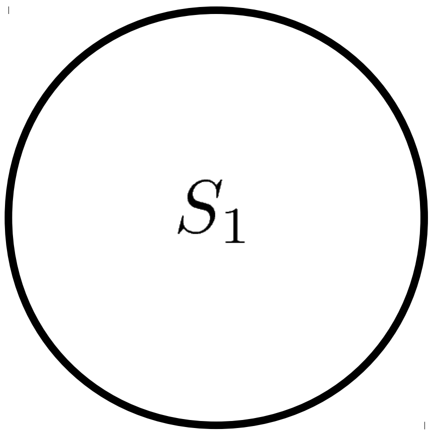
## The 5D case

Extra-dimensional field in 5D $\Phi(x_\mu, x_5)$




Tower of 4D particles with masses related to $M_{kk}$=1/R

# 5-Dimensional Model




**Compactification on a circle**

$$x_5 \in \{0, 2\pi R_5\}$$

## Fermions

Clifford Algebra in 5D contains the 4 Dirac matrices and the $\gamma_5$:

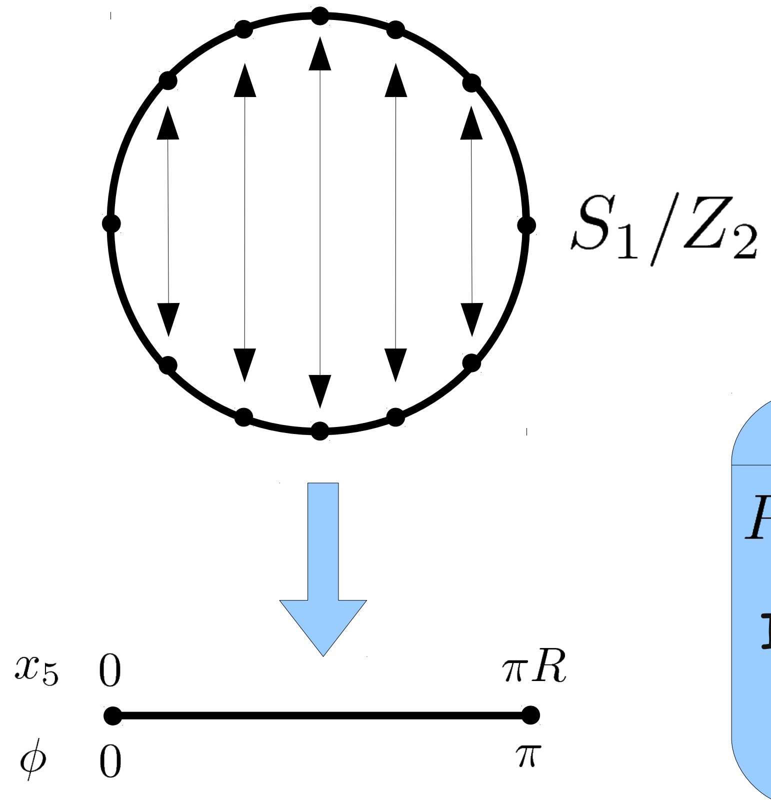
$$\{\Gamma_M, \Gamma_N\} = 2\eta_{MN} \quad \text{with} \quad \Gamma_\mu \equiv \gamma_\mu, \Gamma_5 \equiv -i\gamma_5$$

A chirality projector $P_{L,R}$ is missing

Fermions are 4-component Dirac spinors $\xrightarrow{\text{KK expansion}}$ Non-chiral 4D zero modes

Not a good description of low-energy physics!

# 5-Dimensional Model




**Compactification on an interval (orbifold)**

Identification of opposite points

**Orbifold and parity of fermions**

$$P(x_5) = -x_5 \qquad \Phi(x^\mu, -x_5) = P(\Phi)(x^\mu, x_5)$$

Invariance of the action requires:

$$P(\Psi_L) = +\Psi_L \qquad P(\Psi_R) = -\Psi_R$$

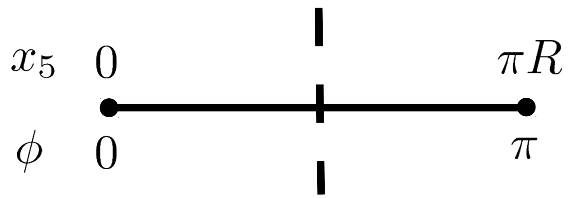
$$\Psi_L(x, x_5) \sim \sum_{n=0}^{\infty} \psi_L^{(n)}(x) \cos\left(\frac{n}{R} x_5\right) \qquad \Psi_R(x, x_5) \sim \sum_{n=1}^{\infty} \psi_R^{(n)}(x) \sin\left(\frac{n}{R} x_5\right)$$

No n=0 R-fermions

**Zero mode chiral Fermions**

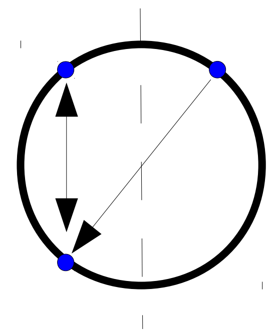
# 5-Dimensional Model

## KK parity




Discrete symmetry about the midpoint

It is a remnant of 5D continuous translational invariance:

$x_5 \to x_5 + \pi R$ ➡  ➡ $x_5 \to \pi R - x_5$

Under this symmetry: $\begin{cases} \cos(\frac{n}{R}(x_5 + \pi R)) \\ \sin(\frac{n}{R}(x_5 + \pi R)) \end{cases} = (-1)^n \begin{cases} \cos(\frac{n}{R} x_5) \\ \sin(\frac{n}{R} x_5) \end{cases}$ Modes with odd n flip sign

Invariance of the action under KK symmetry ➡ Interactions must contain an even number of modes with odd n

The lightest KK-odd level is stable ➡ Dark Matter candidate

# 6D on the Real Projective Plane

Mathematical definition $\mathbf{R}/\mathbf{pgg}$ ⇨ $\mathbf{pgg} = \langle r, g | r^2 = (g^2 r)^2 = 1 \rangle$

glide $g: \begin{cases} x_5 \sim x_5 + \pi R_5 \\ x_6 \sim -x_6 + \pi R_6 \end{cases}$

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⇩

$$t_5 = g^2 : \begin{cases} x_5 \sim x_5 + 2\pi R_5 \\ x_6 \sim x_6 \end{cases}$$

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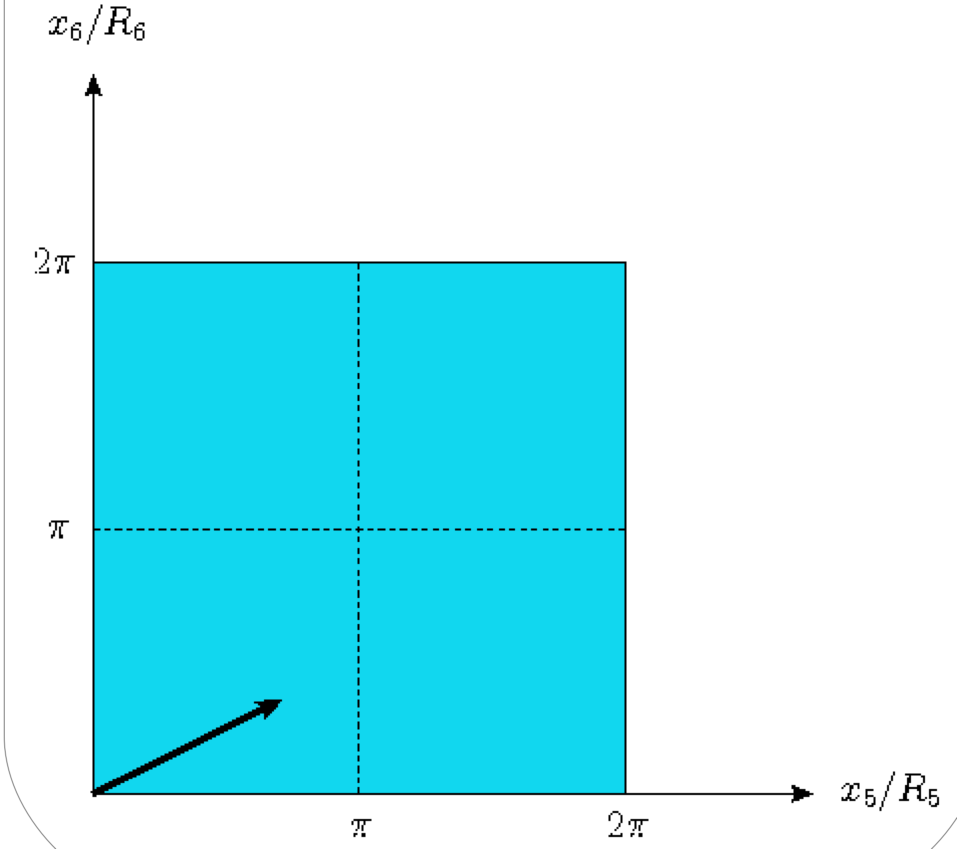
Construction of the fundamental domain

Torus



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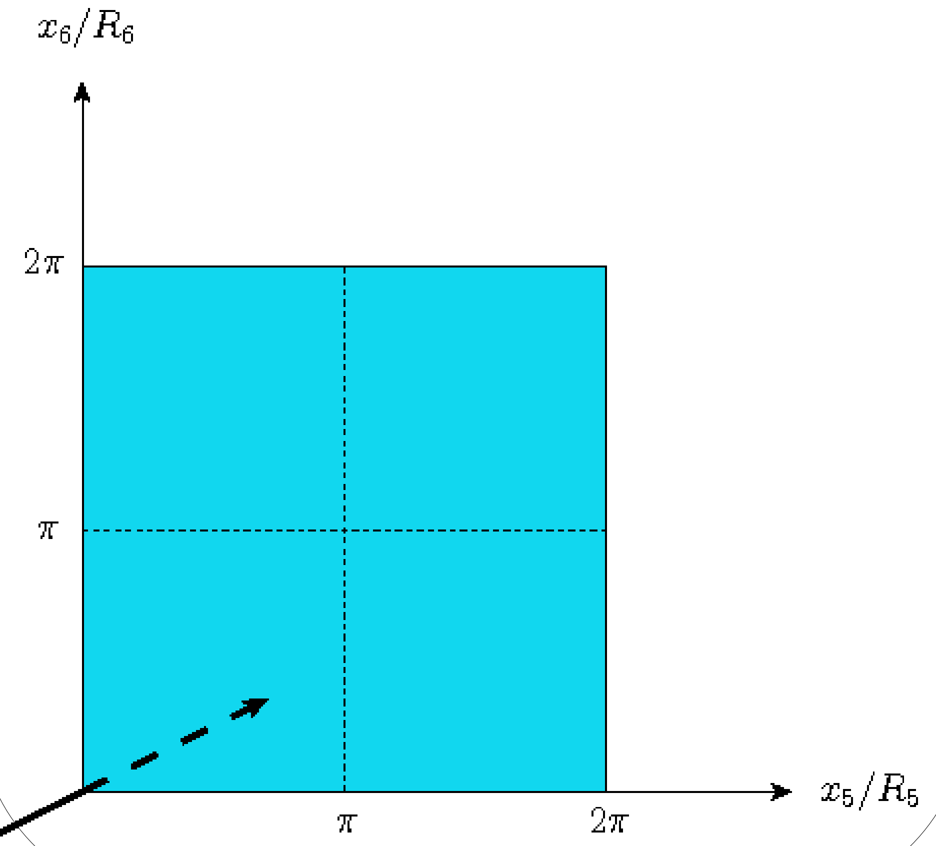
Construction of the fundamental domain

Torus + Rotation



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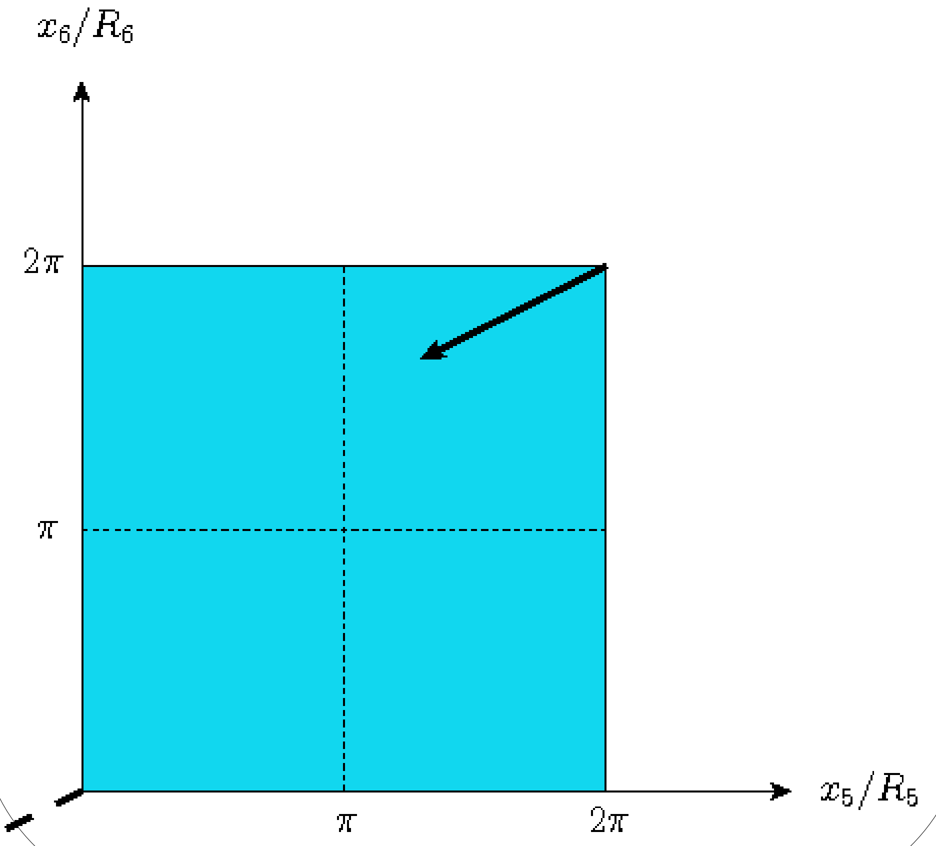

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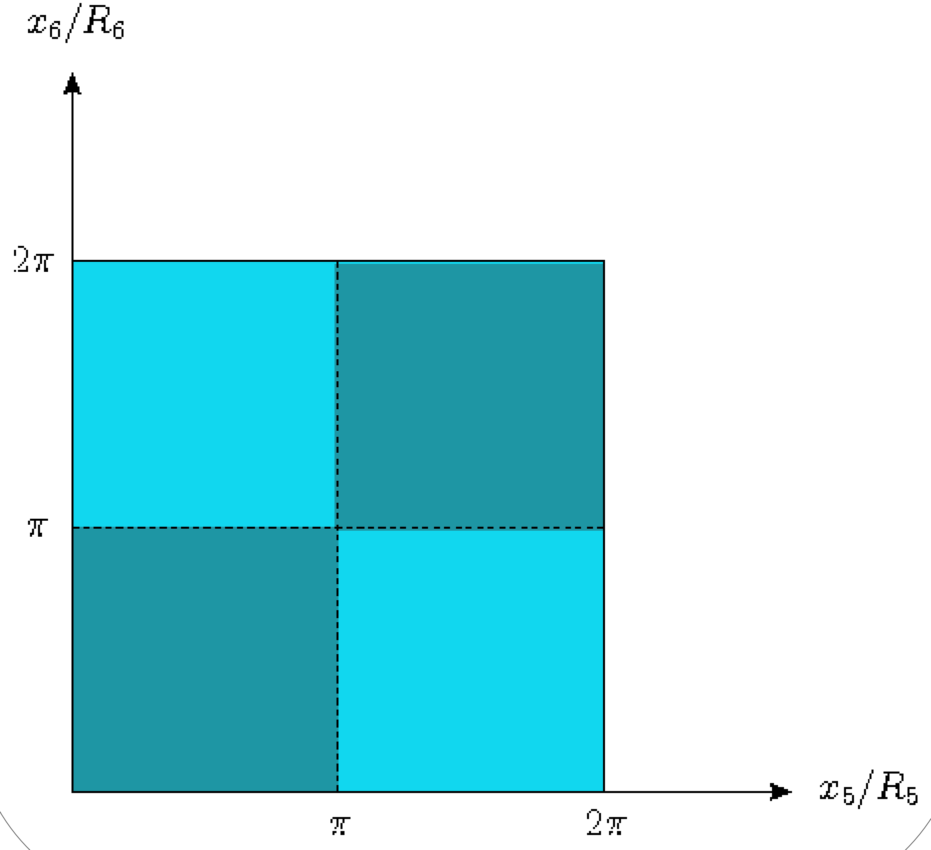

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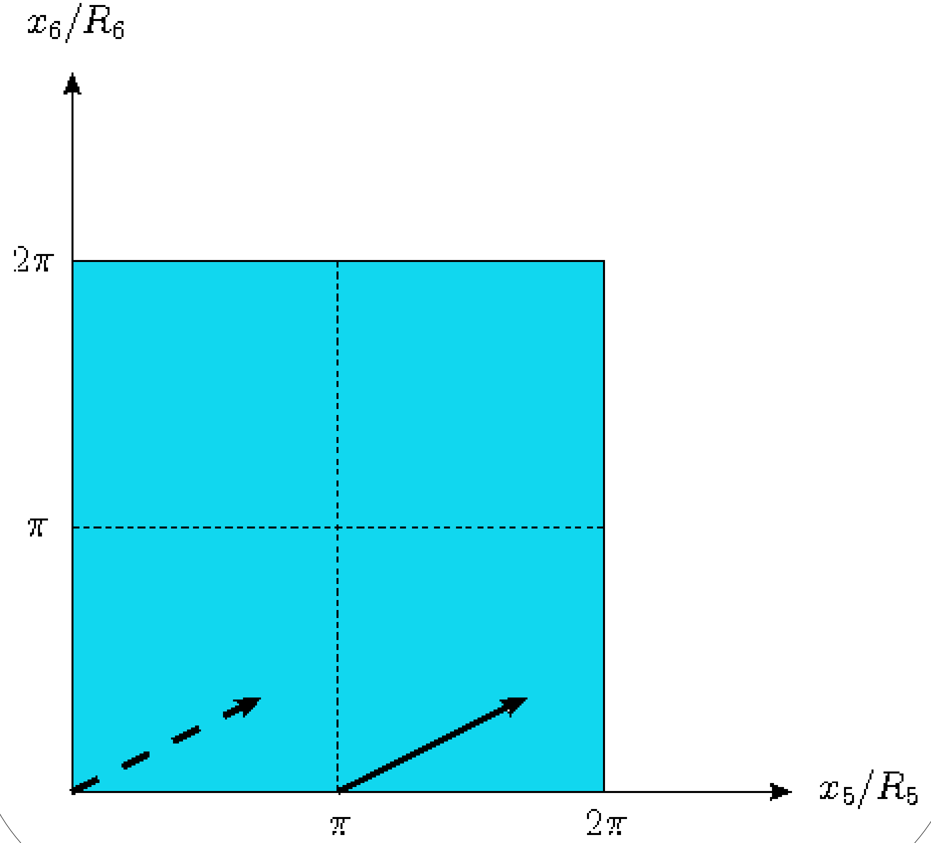
Torus + Glide



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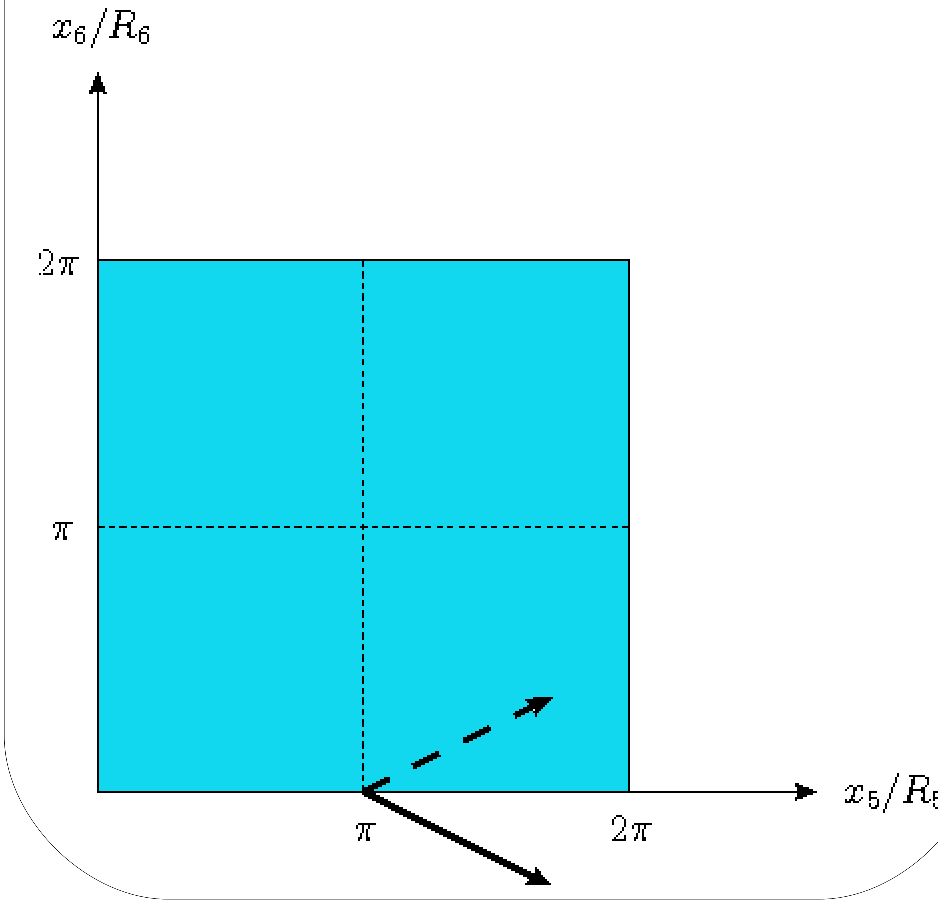

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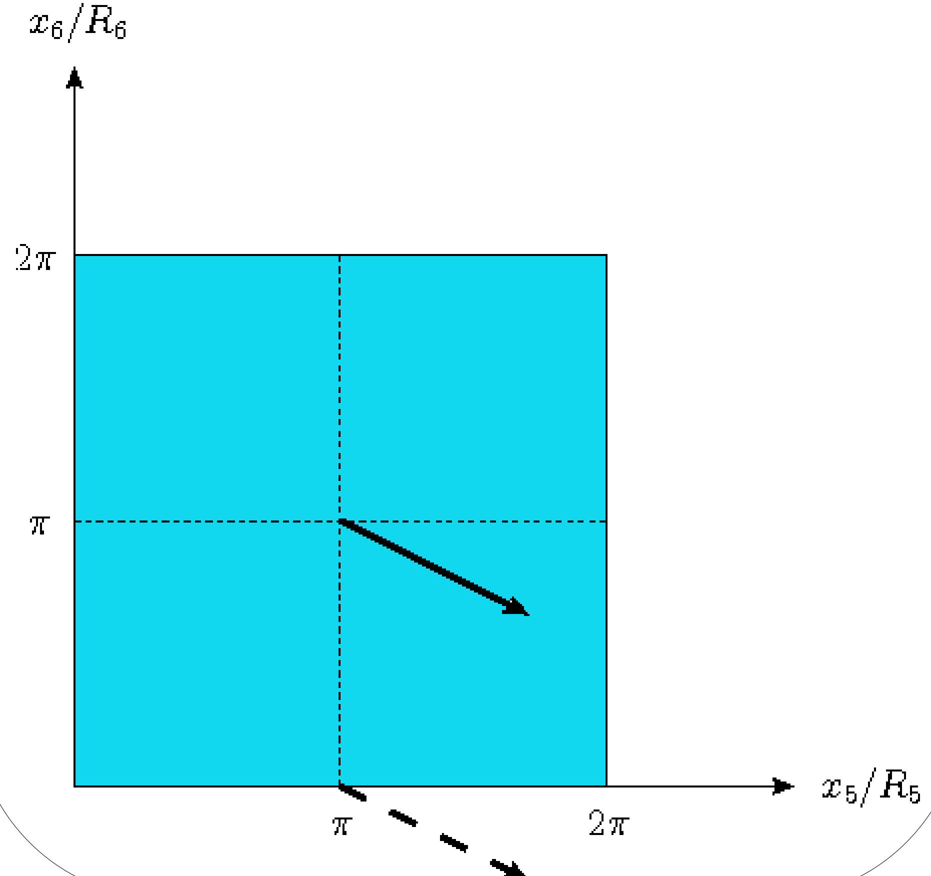

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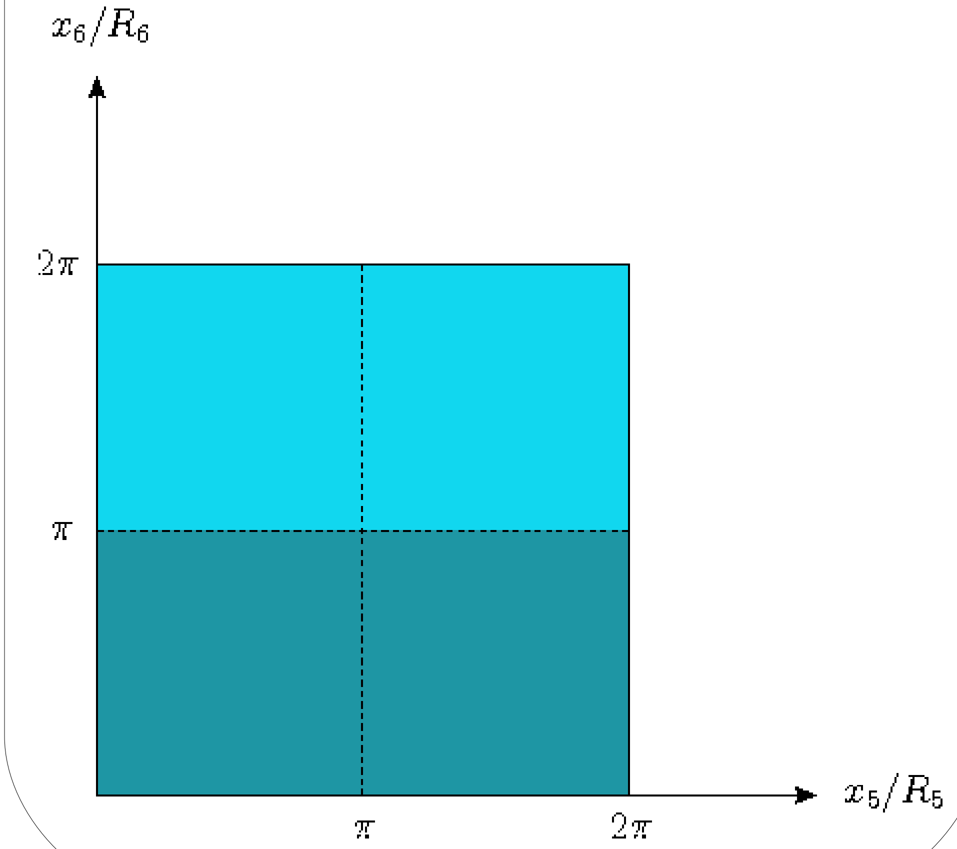

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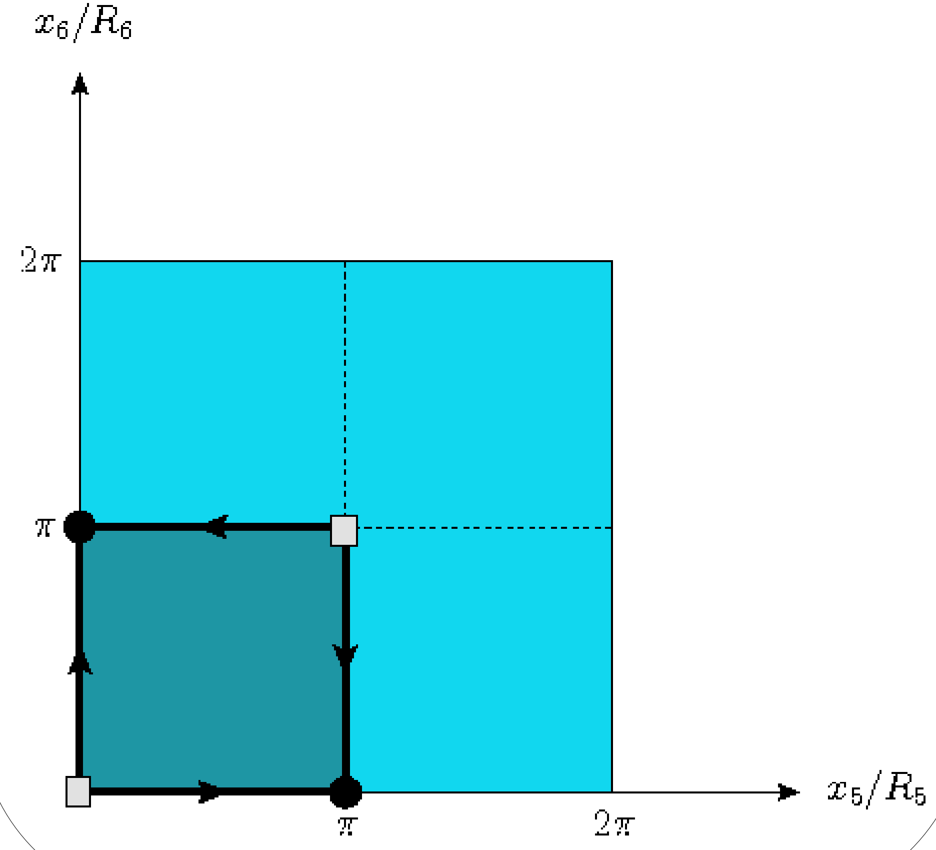
Construction of the fundamental domain

Torus + Rotation + Glide



# Scalars

$$S_{\text{scalar}} = \int_0^{2\pi} dx_5 dx_6 \left\{ \partial_\alpha \Phi^\dagger \partial^\alpha \Phi - M^2 \Phi^\dagger \Phi \right\} \qquad R_5 = R_6 = 1$$

The general solutions of the equation of motion are given by:

$$\Phi = \sum_{k,l} \phi^{(k,l)} (A \cos kx_5 \cos lx_6 + B \sin kx_5 \sin lx_6 + C \sin kx_5 \cos lx_6 + D \cos kx_5 \sin lx_6)$$

The modes can be characterized by their parities under rotation and glide:

| | $p_r$ | $p_g$ |
|---|---|---|
| $\cos kx_5 \ \cos lx_6$ | $+$ | $(-1)^{k+l}$ |
| $\sin kx_5 \ \sin lx_6$ | $+$ | $(-1)^{k+l+1}$ |
| $\sin kx_5 \ \cos lx_6$ | $-$ | $(-1)^{k+l}$ |
| $\cos kx_5 \ \sin lx_6$ | $-$ | $(-1)^{k+l+1}$ |

$$\begin{aligned} \cos(kx_5)\cos(lx_6) &\xrightarrow{r} \cos(-kx_5)\cos(-lx_6) \\ &= \cos(kx_5)\cos(lx_6) \\ \cos(kx_5)\cos(lx_6) &\xrightarrow{g} \cos\left(k(x_5+\pi)\right)\cos\left(l(-x_6+\pi)\right) \\ &= \cos\left((k+l)\pi\right)\cos(kx_5)\cos(lx_6) \end{aligned}$$

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| $\sin kx_5 \;\; \cos lx_6$ | $-$ | $(-1)^{k+l}$ |
| $\cos kx_5 \;\; \sin lx_6$ | $-$ | $(-1)^{k+l+1}$ |

The classification of the modes is:

| $(k,l)$ | $p_{KK}$ | $(p_r,p_g)=(++)$ | $(+-)$ | $(-+)$ | $(--)$ |
|---|---|---|---|---|---|
| $(0,0)$ | $+$ | $\frac{1}{2\pi}$ | | | |
| $(0,2l)$ | $+$ | $\frac{1}{\sqrt{2}\pi}\cos 2lx_6$ | | | $\frac{1}{\sqrt{2}\pi}\sin 2lx_6$ |
| $(0,2l-1)$ | $-$ | | $\frac{1}{\sqrt{2}\pi}\cos(2l-1)x_6$ | $\frac{1}{\sqrt{2}\pi}\sin(2l-1)x_6$ | |
| $(2k,0)$ | $+$ | $\frac{1}{\sqrt{2}\pi}\cos 2kx_5$ | | $\frac{1}{\sqrt{2}\pi}\sin 2kx_5$ | |
| $(2k-1,0)$ | $-$ | | $\frac{1}{\sqrt{2}\pi}\cos(2k-1)x_5$ | | $\frac{1}{\sqrt{2}\pi}\sin(2k-1)x_5$ |
| $(k,l)_{\text{k+l even}}$ | $+$ | $\frac{1}{\pi}\cos kx_5\cos lx_6$ | $\frac{1}{\pi}\sin kx_5\sin lx_6$ | $\frac{1}{\pi}\sin kx_5\cos lx_6$ | $\frac{1}{\pi}\cos kx_5\sin lx_6$ |
| $(k,l)_{\text{k+l odd}}$ | $-$ | $\frac{1}{\pi}\sin kx_5\sin lx_6$ | $\frac{1}{\pi}\cos kx_5\cos lx_6$ | $\frac{1}{\pi}\cos kx_5\sin lx_6$ | $\frac{1}{\pi}\sin kx_5\cos lx_6$ |

# Gauge Bosons

$$S_{\text{gauge}} = \int_0^{2\pi} dx_5 dx_6 \left\{ -\frac{1}{4} F_{MN} F^{MN} - \frac{1}{2\xi} (\partial_\mu A^\mu - \xi(\partial_5 A_5 + \partial_6 A_6))^2 \right\}$$

$$A_M = \{A_\mu, A_5, A_6\}$$

## Vector boson

$$\underbrace{-\partial^\mu F_{\mu\nu} - \frac{1}{\xi}\partial_\nu \partial^\mu A_\mu}_{=p^2 A_\mu} + (\partial_5^2 + \partial_6^2) A_\nu = 0$$

Same spectrum and wave functions as scalar fields with M=0

## Scalar Components

In Unitary gauge: $\xi \to \infty$

$$\partial_5 A_5 + \partial_6 A_6 = 0$$

The two fields are not independent

$$A_{5,6} = \sum \phi_{5,6}(x_5, x_6) A^{(k,l)}$$

# Gauge Bosons

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Parities are related

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| $(k,l)_{\text{k+l even}}$ | $+$ | $\frac{1}{\pi} \cos kx_5 \cos lx_6$ | $\frac{l}{\pi\sqrt{k^2+l^2}} \sin kx_5 \cos lx_6$ | $-\frac{k}{\pi\sqrt{k^2+l^2}} \cos kx_5 \sin lx_6$ |
| $(k,l)_{\text{k+l odd}}$ | $-$ | $\frac{1}{\pi} \sin kx_5 \sin lx_6$ | $\frac{l}{\pi\sqrt{k^2+l^2}} \cos kx_5 \sin lx_6$ | $-\frac{k}{\pi\sqrt{k^2+l^2}} \sin kx_5 \cos lx_6$ |

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Zero mode
only for vectors!

# Fermions

Why the RPP contains zero mode chiral fermions?

## Clifford algebra in 6D

$$\Gamma^1 \ldots \Gamma^6 \quad + \quad \Gamma^7 = \Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5\Gamma^6$$

6 8x8 Gamma matrices + 6D Chirality Matrix

$$\Gamma^\mu = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 0 & i\gamma^5 \\ i\gamma^5 & 0 \end{pmatrix}, \quad \Gamma^6 = \begin{pmatrix} 0 & \gamma^5 \\ -\gamma^5 & 0 \end{pmatrix}$$

$$\Gamma^7 = \begin{pmatrix} -\gamma^5 & 0 \\ 0 & \gamma^5 \end{pmatrix}$$

# Fermions

Why the RPP contains zero mode chiral fermions?

$$P_\pm = \frac{1}{2}(1 \pm \Gamma^7) = \begin{pmatrix} \frac{1}{2}(1 \mp \gamma^5) & 0 \\ 0 & \frac{1}{2}(1 \pm \gamma^5) \end{pmatrix} = \begin{pmatrix} P_{L/R} & 0 \\ 0 & P_{R/L} \end{pmatrix}$$

6D Chirality → 4D Chirality

$$\Psi = \begin{pmatrix} \Psi_- \\ \Psi_+ \end{pmatrix} \qquad \Psi_- = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}_- \qquad (\psi_L)_\pm = \begin{pmatrix} \chi \\ 0 \end{pmatrix}_\pm$$

$$\Psi_\pm = P_\pm \Psi \qquad \Psi_+ = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}_+ \qquad (\psi_R)_\pm = \begin{pmatrix} 0 \\ \bar{\eta} \end{pmatrix}_\pm$$

# Fermions

Why the RPP contains zero mode chiral fermions?

Kinetic term ⇒ $$\int dx_5 dx_6 i\bar{\Psi}\Gamma^M\partial_M\Psi = \int dx_5 dx_6 i(\bar{\Psi}_+\Gamma^M\partial_M\Psi_+ + \bar{\Psi}_-\Gamma^M\partial_M\Psi_- + \dots)$$

$$i\bar{\Psi}_\pm\Gamma^M\partial_M\Psi_\pm = i\bar{\psi}_\pm\gamma^\mu\partial_\mu\psi_\pm + \underbrace{\frac{1}{2}[\bar{\psi}_{L\pm}\gamma^5(\partial_5 \mp i\partial_6)\psi_{R\pm} + \bar{\psi}_{R\pm}\gamma^5(\partial_5 \pm i\partial_6)\psi_{L\pm}]}$$

Glide

$$\frac{1}{2}[\bar{\psi}_{L\pm}\gamma^5(\partial_5 \pm i\partial_6)\psi_{R\pm} + \bar{\psi}_{R\pm}\gamma^5(\partial_5 \mp i\partial_6)\psi_{L\pm}]$$

⇓

$$\psi_{\{L,R\}\pm}(g(x)) = \psi_{\{L,R\}\mp}$$

Non Chiral 4D zero mode

Rotation

$$\frac{1}{2}[\bar{\psi}_{L\pm}\gamma^5(-\partial_5 \pm i\partial_6)\psi_{R\pm} + \bar{\psi}_{R\pm}\gamma^5(-\partial_5 \mp i\partial_6)\psi_{L\pm}]$$

⇓

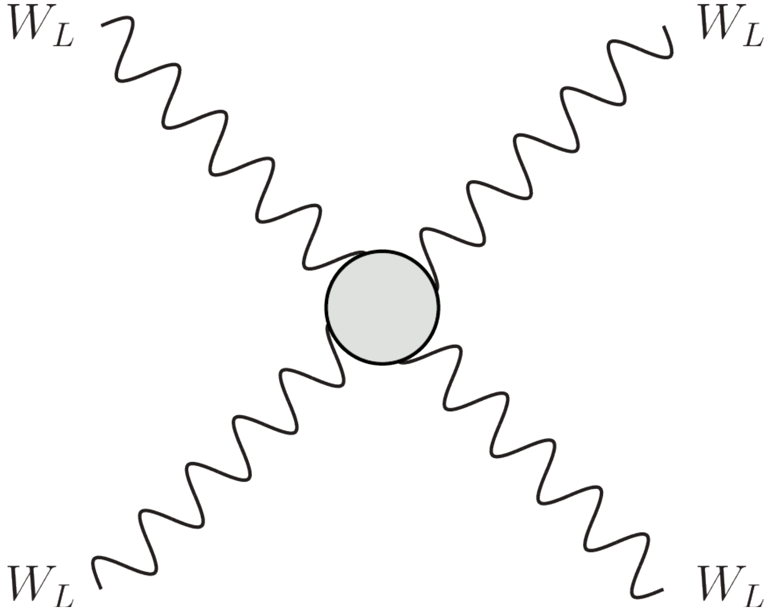
Global sign compensated if

$$p_r(\psi_L) = -p_r(\psi_R)$$

**Chiral 4D theory!**

# The SM case

Scattering of longitudinally polarized gauge bosons




4-leg + s-channel + t-channel + u-channel

$$\mathcal{T} = \underbrace{A\left(\frac{s}{M_W^2}\right)^2 + B\,\frac{s}{M_W^2}}+C$$

Explicitly divergent high-energy behaviour

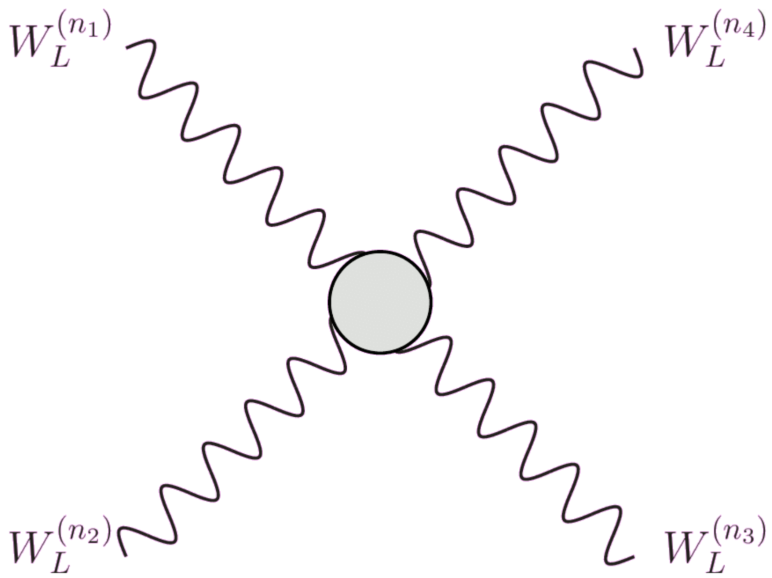
$A = 0$ Cancellations among pure gauge diagrams

$B = 0$ Cancellations involving Higgs boson

$C \underset{s \gg M_h}{\longrightarrow}$ constant

Unitarity condition: $M_H^2 \leq \frac{4\pi\sqrt{2}}{G_F}$

# The Unitarity Matrix




Generic Process

$$W_L(n_1)W_L(n_2) \to W_L(n_3)W_L(n_4)$$

**Elastic channel**

Optical Theorem

↓

Unitarity Condition

$$\text{Im } T_{n_1 n_2 \to n_1 n_2} = \sigma_{n_1 n_2} |T_{n_1 n_2 \to n_1 n_2}|^2$$

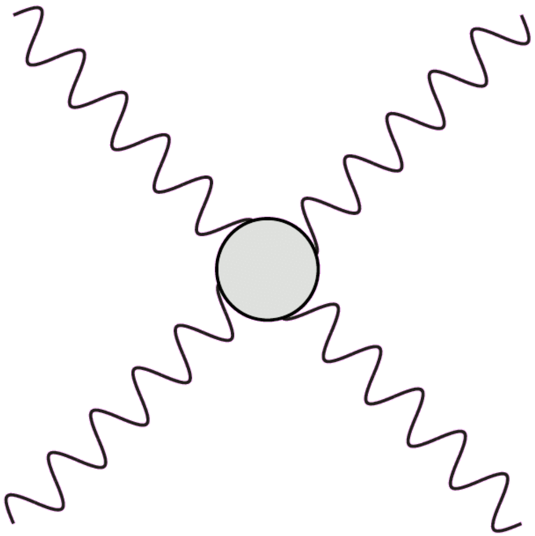
↓

$$\text{Im } T_{n_1 n_2 \to n_1 n_2} \leq |T_{n_1 n_2 \to n_1 n_2}|$$

↓

$$\sigma_{n_1 n_2} T_{n_1 n_2 \to n_1 n_2} \leq 1$$

# The Unitarity Matrix




Generic Process

$$W_L(n_1)W_L(n_2) \to W_L(n_3)W_L(n_4)$$

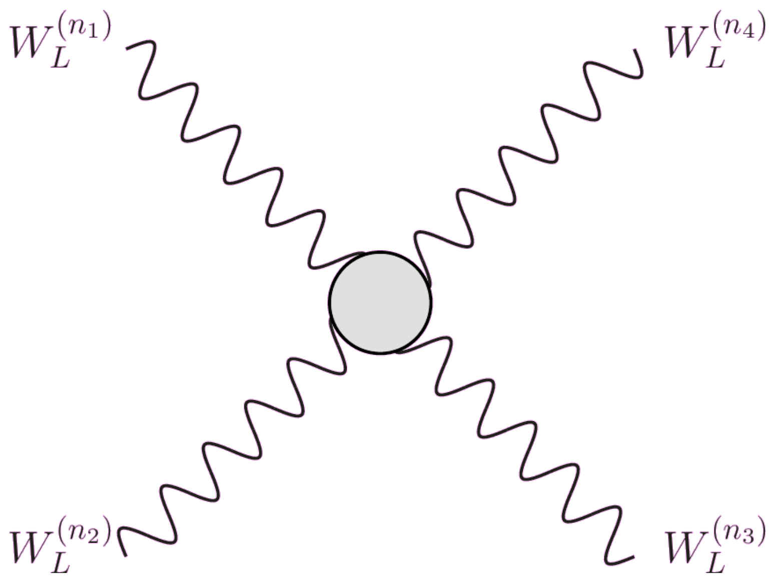
## Higher energies

$\alpha, \beta =$ 2-particle states

$$\left.\begin{aligned} \mathrm{Im}\ T_{\alpha\alpha} &= \sigma_\alpha |T_{\alpha\alpha}|^2 + \sigma_\beta |T_{\alpha\beta}|^2 \\ \mathrm{Im}\ T_{\alpha\beta} &= \sigma_\alpha T_{\alpha\alpha} T^\dagger_{\alpha\beta} + \sigma_\beta T_{\alpha\beta} T^\dagger_{\beta\beta} \\ \mathrm{Im}\ T_{\beta\beta} &= \sigma_\alpha |T_{\alpha\beta}|^2 + \sigma_\beta |T_{\beta\beta}|^2 \\ &\vdots \end{aligned}\right\} \Rightarrow \mathrm{Im}\ T = T\Sigma T^\dagger$$

De Curtis et al. hep-ph/0301059

# The Unitarity Matrix




Generic Process

$$W_L(n_1)W_L(n_2) \to W_L(n_3)W_L(n_4)$$

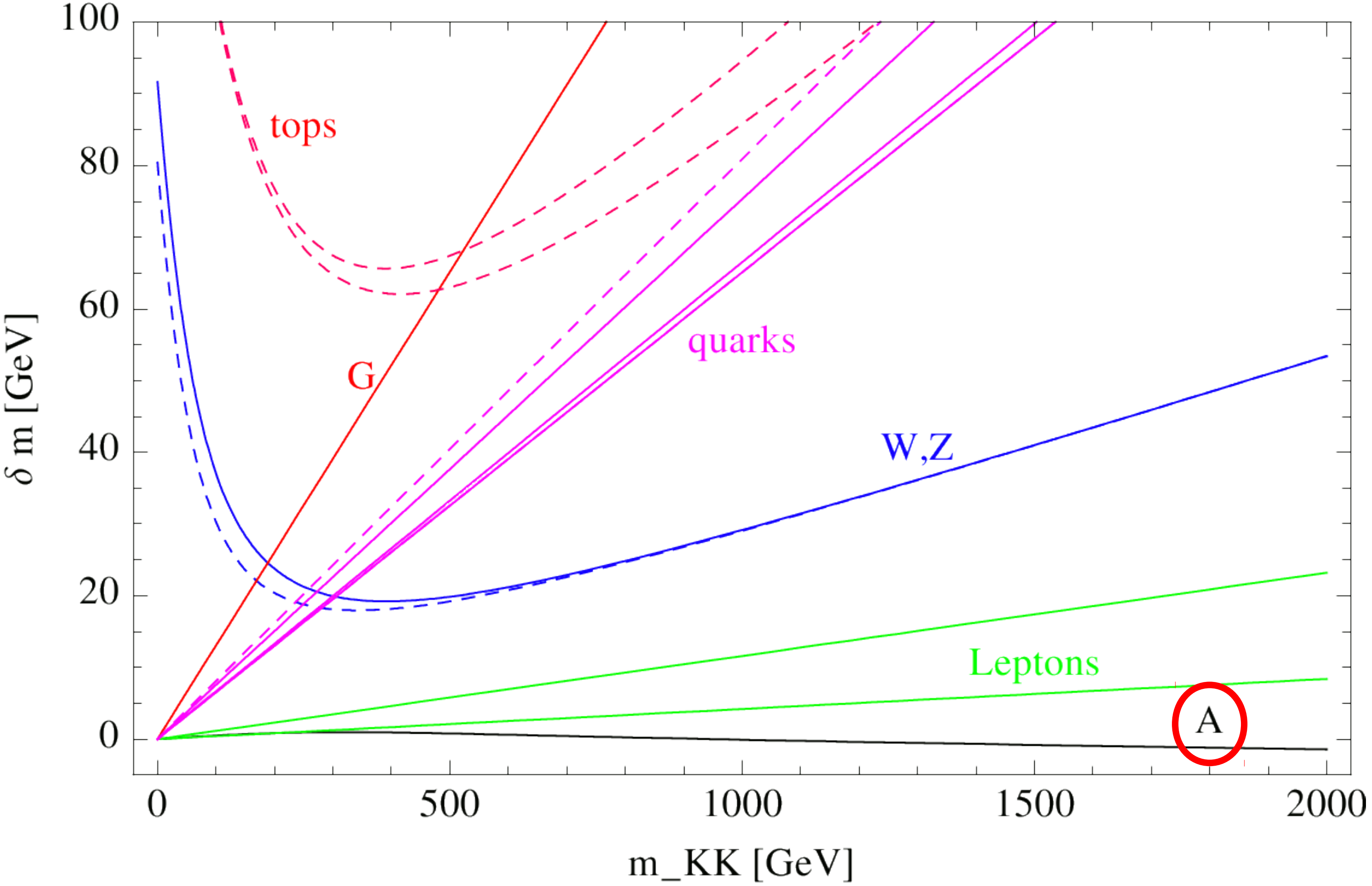
**Higher energies**

$$\text{Im}\, T = T\Sigma T^\dagger \begin{cases} \Sigma_{ii} \underset{s \gg \text{threshold}}{\longrightarrow} 1 \;\Rightarrow\; \Sigma \underset{s\to\infty}{\sim} \mathbf{1} \\ T = \text{diag}(\lambda_1, \lambda 2, \dots) \end{cases} \Rightarrow \max\{\lambda_i\} \leq 1$$

**Cutoff** ➡ energy for which the maximum eigenvalue of the scattering matrix violates the unitarity bound

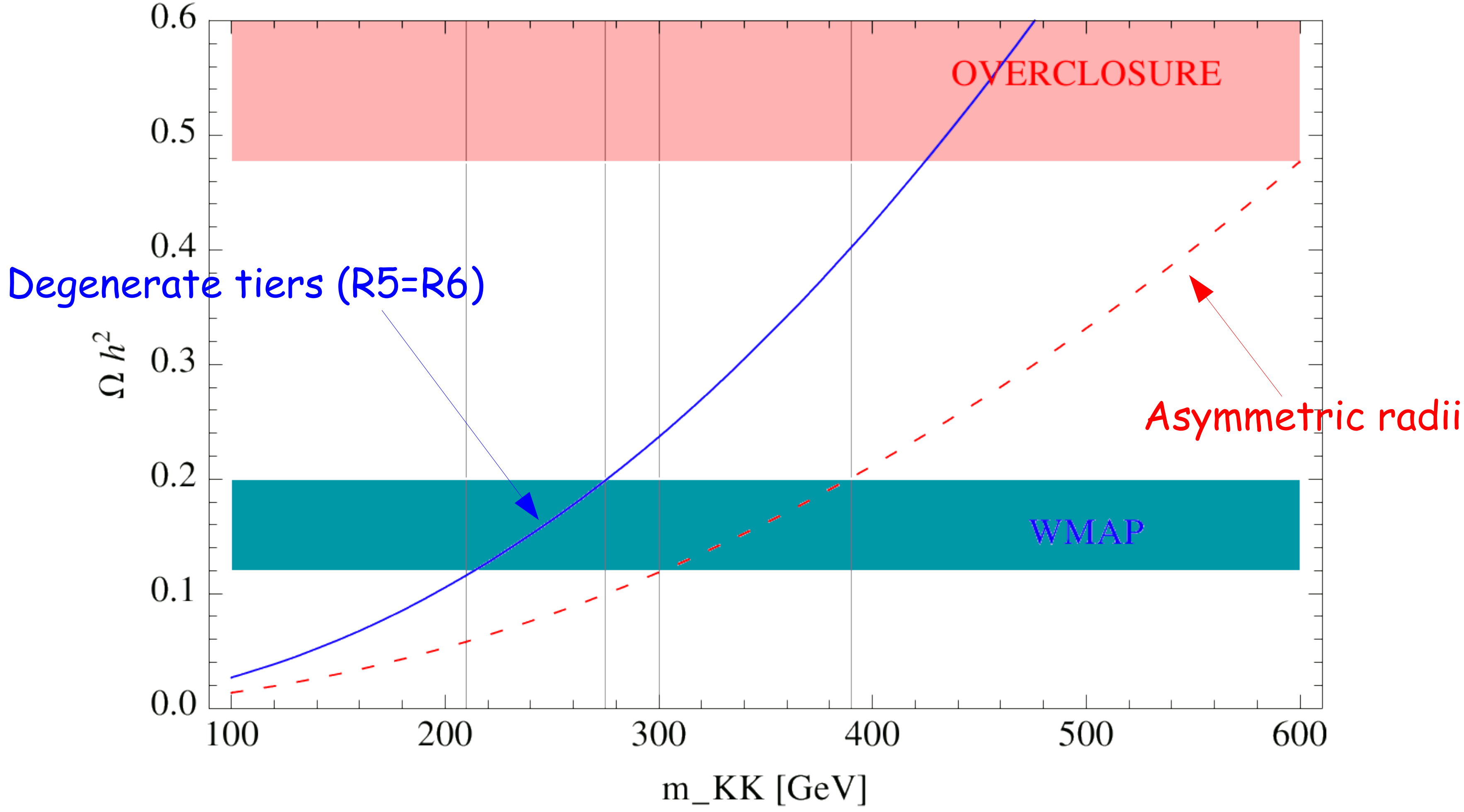
# The tier (1,0)-(0,1)

The lightest (1,0)-(0,1) state is the natural DM candidate



# The tier (1,0)-(0,1)

## (Rough) Estimation of $M_{KK}$ from Dark Matter abundance

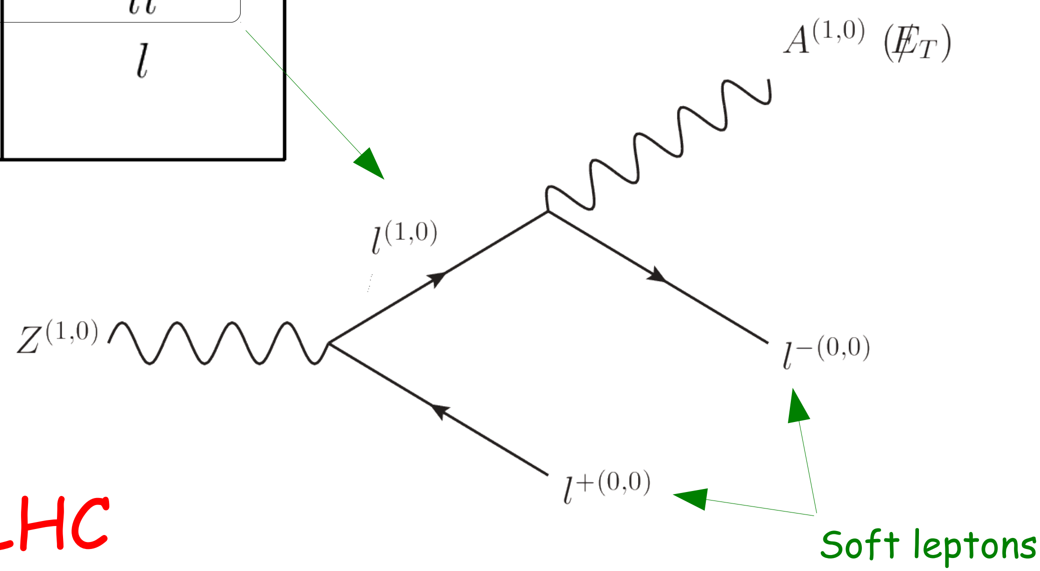


# The tier (1,0)-(0,1)

Production at LHC

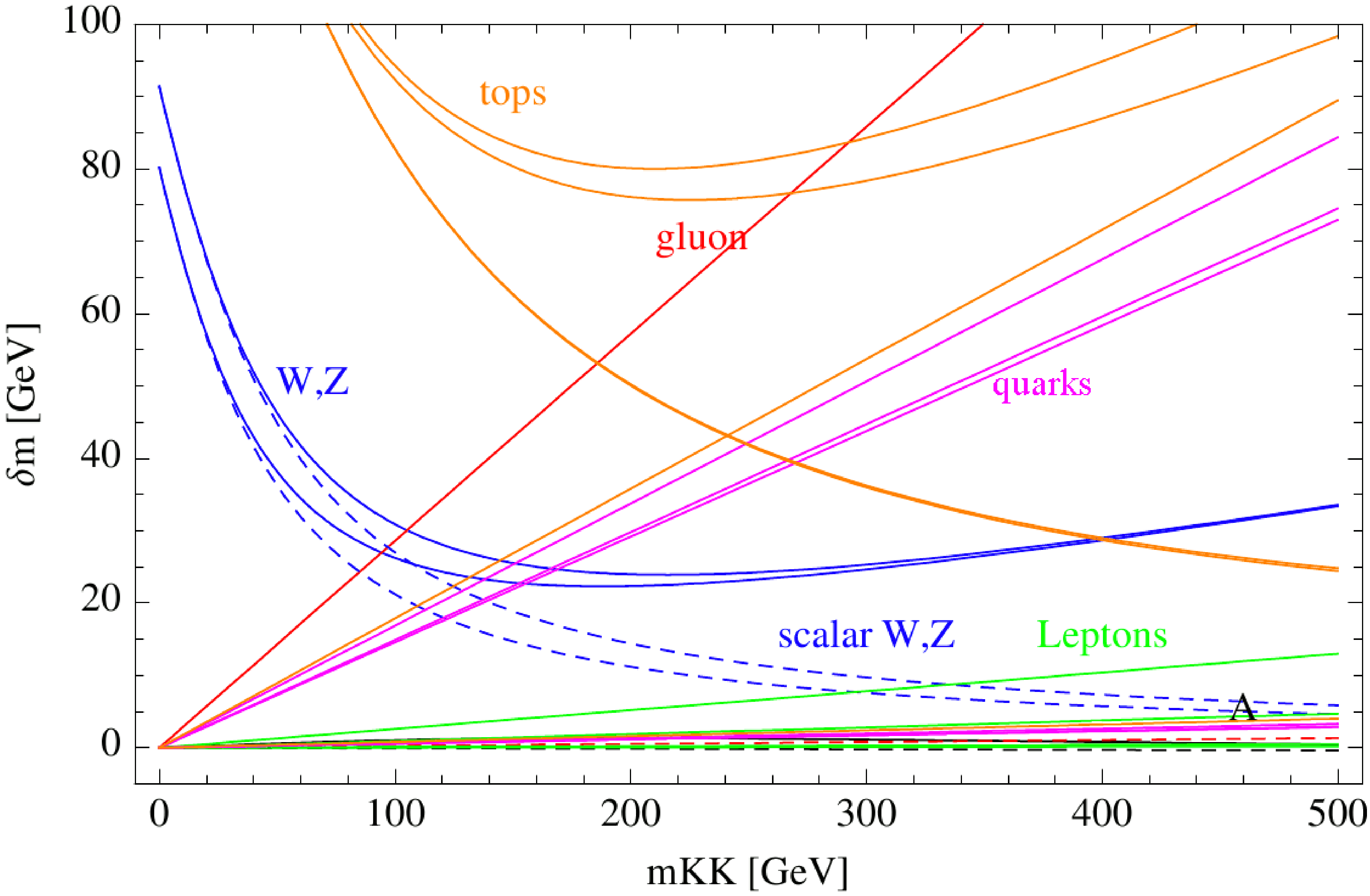
| | $m_X - m_{LLP}$ in GeV | decay mode | final state + MET |
|---|---|---|---|
| $t^{(1,0)}$ | 70 | $bW^{(1,0)}$ | $bjj$ <br> $bl\nu$ |
| $G^{(1,0)}$ | 40-70 | $qq^{(1,0)}$ | $jj$ |
| $q^{(1,0)}$ | 20-40 | $qA^{(1,0)}$ | $j$ |
| $W^{(1,0)}$ | 20 | $l\nu^{(1,0)}$, $\nu l^{(1,0)}$ | $l\nu$ |
| $Z^{(1,0)}$ | 20 | $ll^{(1,0)}$ | $ll$ |
| $l^{(1,0)}$ | $< 5$ | $lA^{(1,0)}$ | $l$ |
| $A^{(1,0)}$ | 0 | - | |

Splittings are small




Difficult to observe at LHC

# The tier (1,1)



100
80
60
40
20
0
δm [GeV]
0
100
200
300
400
500
mKK [GeV]
tops
gluon
W,Z
quarks
scalar W,Z
Leptons
A

# Conclusions

**Universal Extra-dimensions** can accommodate
a **Dark Matter candidate** through conservation of **KK parity**

In **5D** KK parity must be imposed **by hand**
on the **fixed points** of the orbifold

In **6D** only three orbifolds **without fixed points/lines**
but **chiral zero mode fermions** only on the **Real Projective Plane**

Few parameters: $M_{KK}$ and $\Lambda$ that can be estimated
from computation of **unitarity bounds** and **Dark Matter abundance**